

PACE IIT & Medical

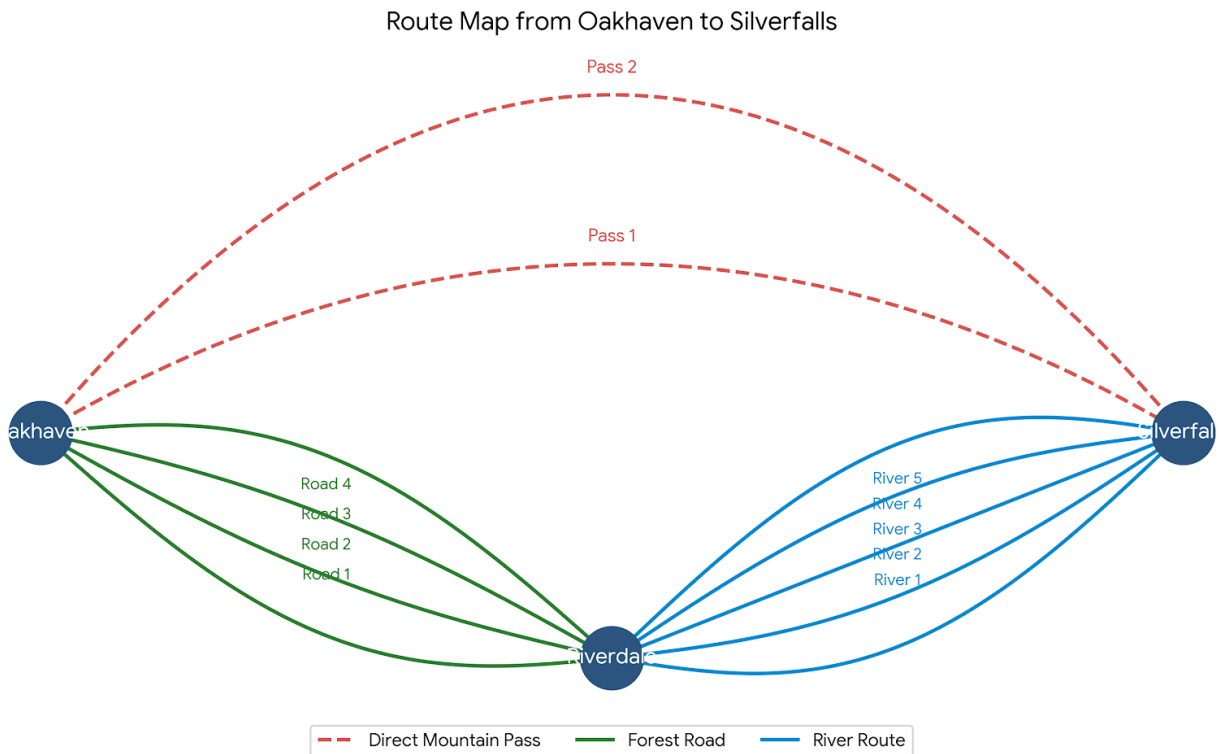
ACE of PACE

Answer Key

1. A
2. A
3. C
4. D
5. B
6. C
7. D
8. B
9. D
10. D
11. A
12. B
13. D
14. A
15. C
16. C
17. B
18. A
19. B
20. C

Solutions

Q.1.



Q.2.

CLOCK HAND ALIGNMENT: Grade 10 Investigation

(0:00:00 < t < 12:00:00)

THE CHALLENGE: Can all three hands overlap perfectly between noon and midnight?

STEP 1: ALIGN HOUR & MINUTE HANDS:
They overlap exactly 10 times in 12 hours (excluding 12 o'clock).
We use fraction algebra:

$$t = \frac{12}{11}k.$$
Show a table for k=1 to 10 with rounded times.
 (e.g., k=1: 1:05:27, k=2: 2:10:54).

STEP 2: CHECK THE SECOND HAND:
For all three to overlap, the Second Hand *must* also be at the exact same fraction.
We check if the Second Hand position at these times is an integer (exact mark).

$$59 \times \frac{12k}{11} = \frac{708k}{11}$$
 (no remainder).

THE CONCLUSION
RESULT: NO.
 The fraction $\frac{708k}{11}$ *never* gives a whole number for k between 1 and 10. There are no triple-overlaps!

SNAP SHOT
MINUTE & HOUR OVERLAP

EXAMPLE 1: MINUTE & HOUR OVERLAP
Red hand is shown 3 seconds away with a clear gap.

Close, but Second Hand is off.

EXAMPLE 2: ALL THREE DISTINCT

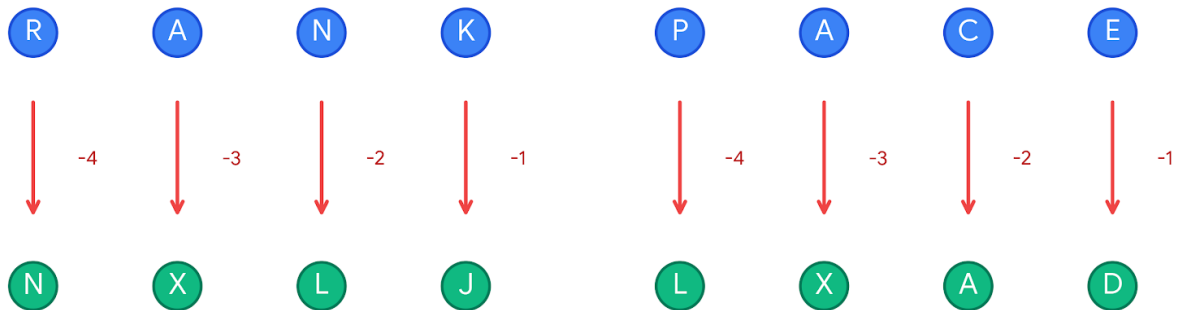
Typical distinct positions.

X ——— ← → X
 HIGH NOON OBSERVATION MIDNIGHT'S

Q.3.

Example: RANK → NXLJ

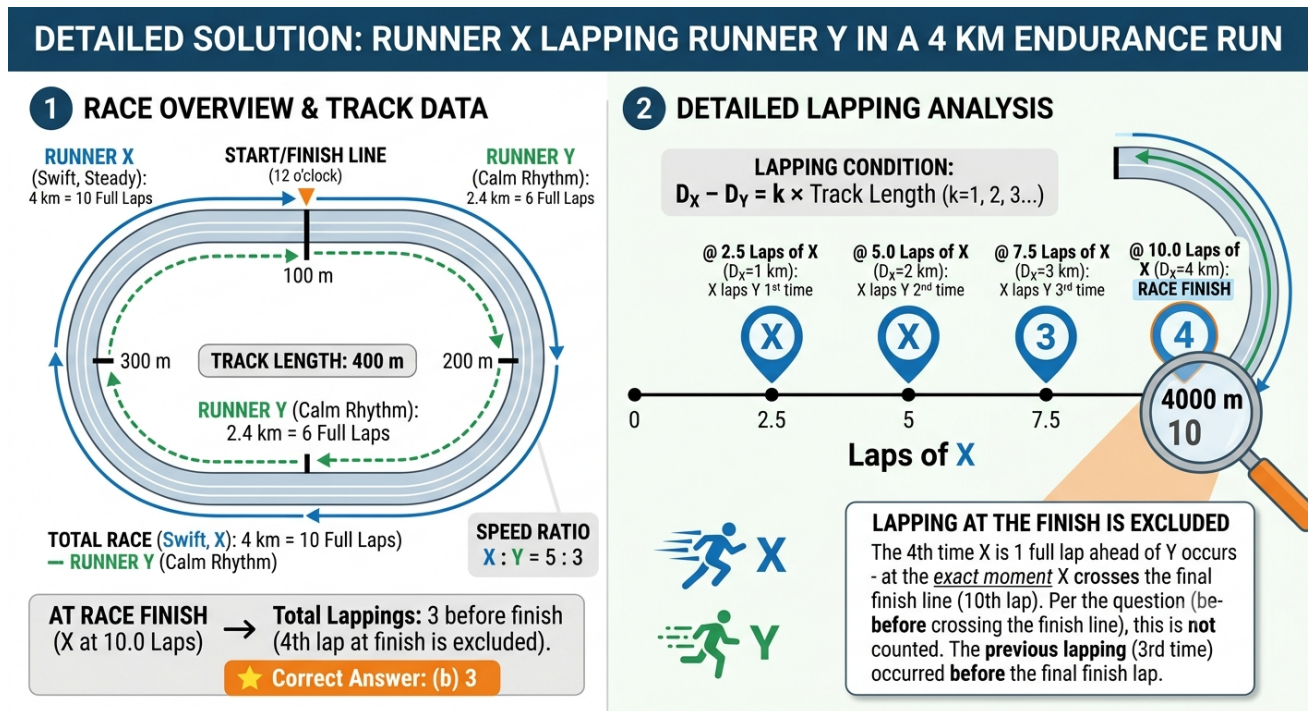
Target: PACE → LXAD



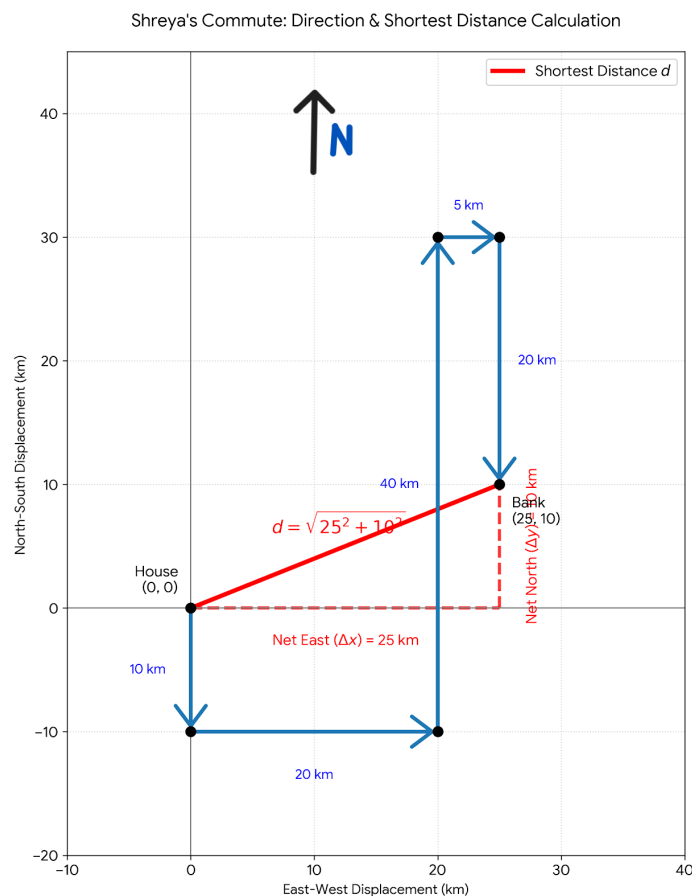
Q.4.

Initial Position Initial State Top=3, Front=1, Right=5	turn until the front shows a 2 Rotated Position Rotated State (Horizontal turn) Top=3, Front=2, Right=6
Analysis of Rotation Analysis: Rotated 90° about Top/Bottom axis.	Opposite Face Pairs Resulting Opposite Pairs: 1↔2, 3↔4, 5↔6
Conclusion: The side opposite 5 is 6.	

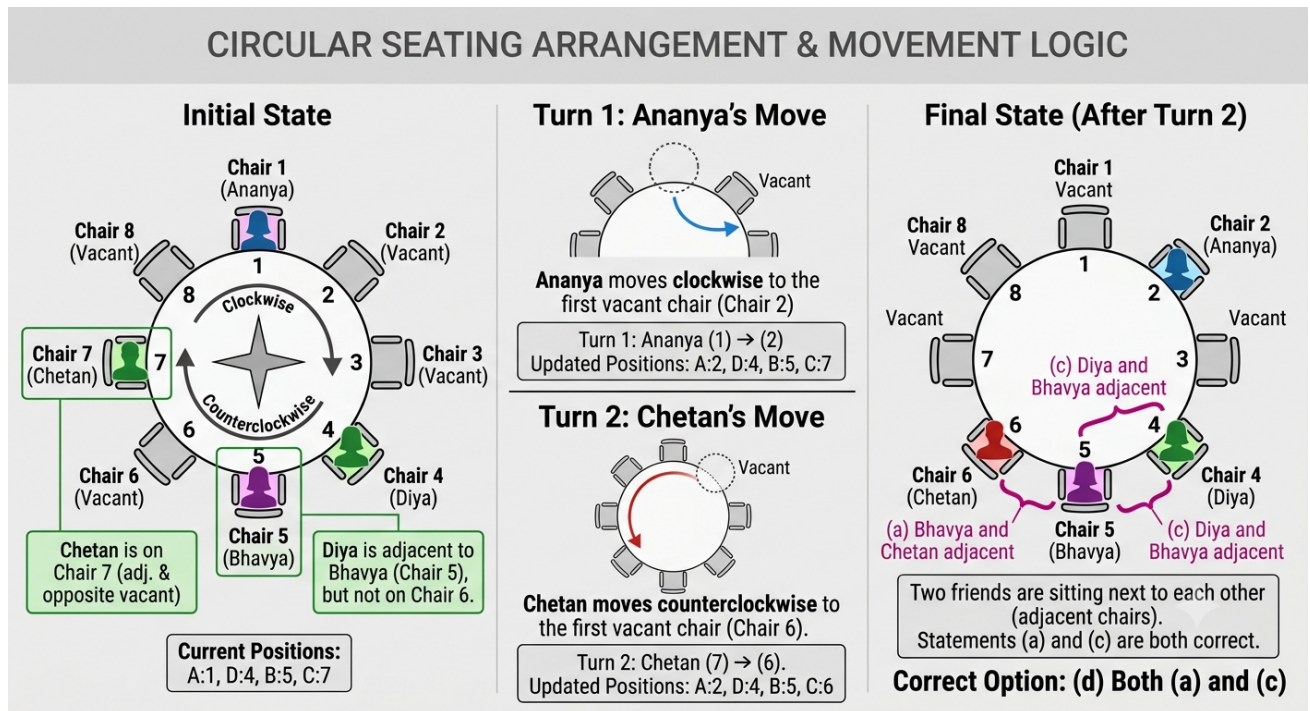
Q.5.



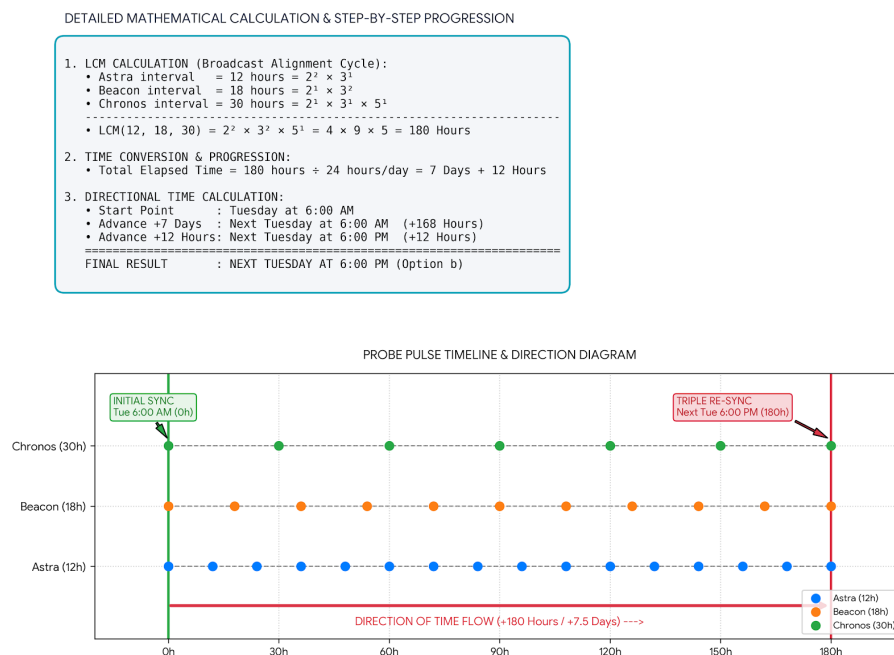
Q.6.



Q.7.



Q.8.



Q.9

SOLUTION DIAGRAM: IDENTIFYING THE FALLACY IN THE "2 = 3" PROOF

STEP-BY-STEP BREAKDOWN

- 1

$-6 = -6$

VALID
(Start with a true statement)
- 2

$4 - 10 = 9 - 15$

VALID
(Both sides equal -6)
- 3

$4 - 10 + \frac{25}{4} = 9 - 15 + \frac{25}{4}$

VALID
(Added 25/4 to both sides)
- 4

$\left(2 - \frac{5}{2}\right)^2 = \left(3 - \frac{5}{2}\right)^2$

VALID
(Both sides are $(-0.5)^2$ and $(0.5)^2 = 0.25$)
- 5

$2 - \frac{5}{2} = 3 - \frac{5}{2}$

FALLACY!
ERROR LOCATED HERE!
- 6

$2 = 3$

INVALID CONCLUSION

EXPLAINING THE MATHEMATICAL ERROR IN STEP 5

THE PROBLEM:
IGNORING ABSOLUTE VALUES IN SQUARE ROOTS

GENERAL RULE: $\sqrt{x^2} = |x|$

Applying to Step 4:

$$\sqrt{\left(2 - \frac{5}{2}\right)^2} = \sqrt{\left(3 - \frac{5}{2}\right)^2}$$

$$\left|2 - \frac{5}{2}\right| = \left|3 - \frac{5}{2}\right|$$

$$|-0.5| = |0.5|$$

$$0.5 = 0.5 \text{ (True)}$$

Ah, the key was the absolute value!

Symmetric points $(-x \text{ and } x)$ map to the same y -value, but have different x -values. Because $(-0.5)^2 = (0.5)^2$, it **DOES NOT MEAN** $-0.5 = 0.5$. A fallacy arises by discarding the negative root.

The fallacy is in Step 5 where the square roots were simplified incorrectly, leading to a false equality of different values $((-0.5) \text{ and } (0.5))$. The correct answer is (D).

Mr. Sharma

Q.10.

STORAGE CONTAINER DESIGN

DIAGONAL & AREA RELATIONS

A)

$$D = \sqrt{a^2 + b^2 + c^2} \Rightarrow D^2 = a^2 + b^2 + c^2$$

B)

$$S = 2(ab + bc + ca)$$

(Total Surface Area)

METAL EDGING STRIP TOTAL LENGTH

$$L = 4a + 4b + 4c = 4(a + b + c)$$

(Total Edge Length)

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a + b + c)^2 = D^2 + S \Rightarrow a + b + c = \sqrt{D^2 + S} \Rightarrow \boxed{L = 4\sqrt{D^2 + S}}$$

Total Edging Required

Q.11.

ADVENTURE PARK: CALCULATING JUNCTION DISTANCE

Minimum wire length between track intersections.

1 THE RED & BLUE LINES

2 FINDING INTERSECTION POINTS (JUNCTIONS)

SET EQUATIONS EQUAL:

$$2x^2 - 4x + 3 = x^2 - 3x + 9$$

SIMPLIFY & FACTOR:

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

3 CALCULATING THE DIRECT DISTANCE

DISTANCE FORMULA:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3 - (-2))^2 + (9 - 19)^2}$$

4 FINAL SOLUTION

MINIMUM WIRE LENGTH:

$5\sqrt{5}$ **UNITS**
 ≈ 11.18 **UNITS**
 THIS IS **OPTION (a)**

Q.12.

UNCOVERING THE ALCHEMIST'S SPELL: $(P+R) \div Q$ SOLUTION

1. UNDERSTAND THE PROBLEM

Original: PQ
Transformed: RQQ

Find P, Q, R, then calculate $(P+R) \div Q$

Each letter is a **UNIQUE** digit (0-9). R cannot be 0.

2. FORMULATE THE EQUATION

$3 * (PQ)$ vs RQQ
 $3 * (10P + Q)$ vs $100R + 10Q + Q$
 $100R + 10Q + Q = 100R + 11Q$

The simplified equation:
 $30P - 8Q = 100R$,
 leading to $15P - 4Q = 50R$

3. DEDUCE THE VALUES

IF Q = 0:

 $15P = 50R$
 $3P = 10R \rightarrow$ Impossible for unique digits

IF Q = 5:

 $15P - 20 = 50R$
 $3P - 4 = 10R$
 $3P = 10R + 4$

If R=1,	3P=14	(X)
If R=2,	3P=24 $\rightarrow P=8$	(✓)
If R=3,	3P=34	(X)

P=8 Q=5 R=2

4. FINAL CALCULATION & VERIFICATION

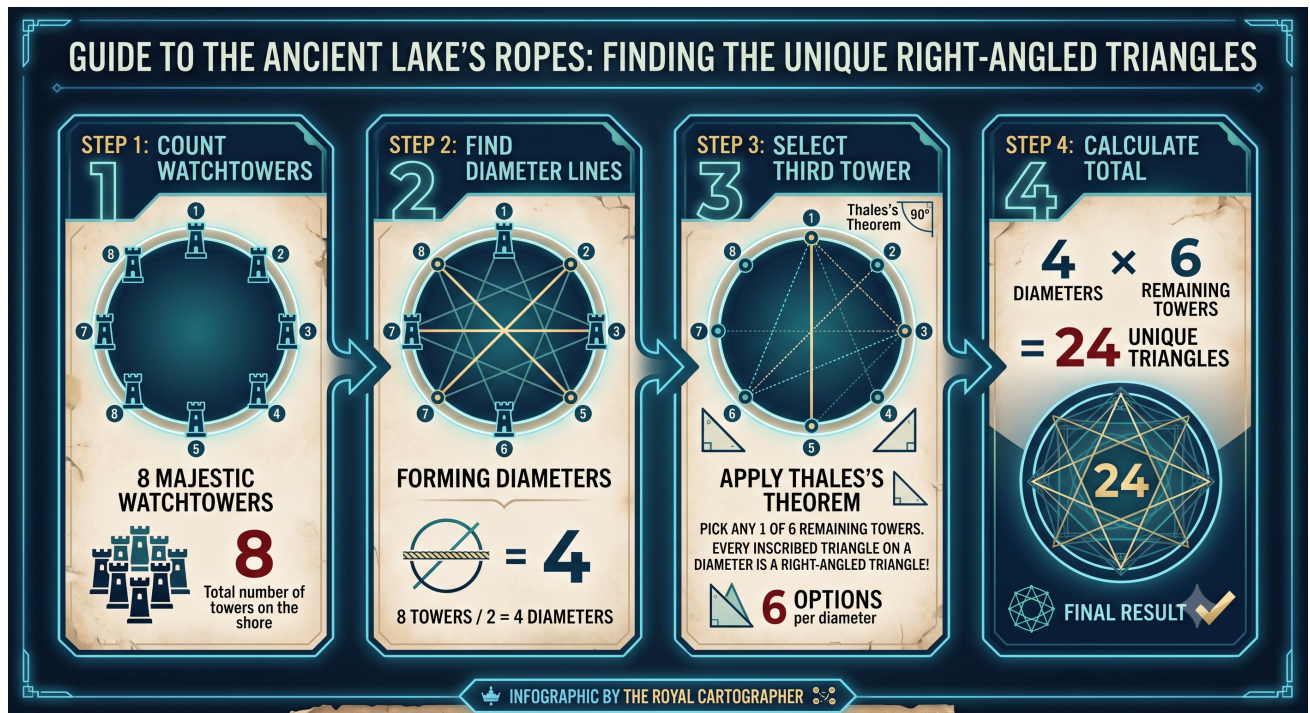
VALUES FOUND:
 P = 8, Q = 5, R = 2

Verification:
 $3 * (85) = 255$
 (Correct RQQ)

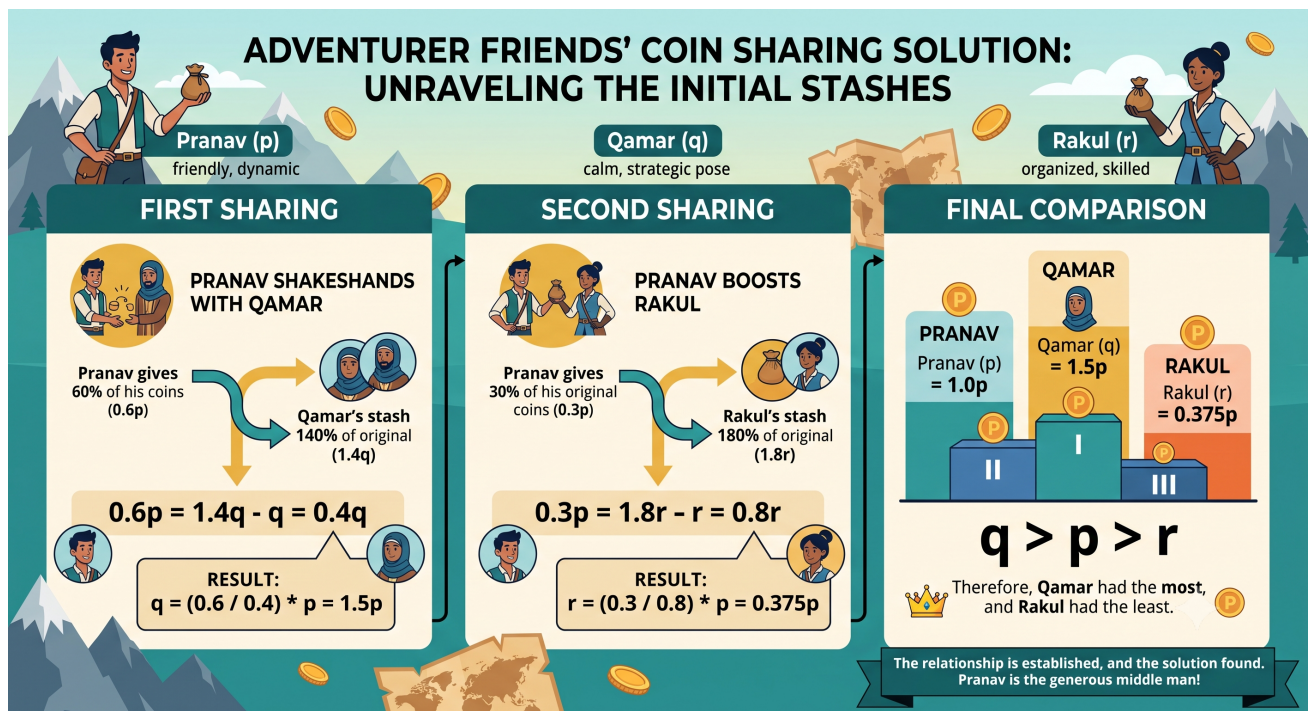
SOLVE THE SPELL:
 $(P + R) \div Q$
 $(8 + 2) \div 5 = 10 \div 5 = 2$

2

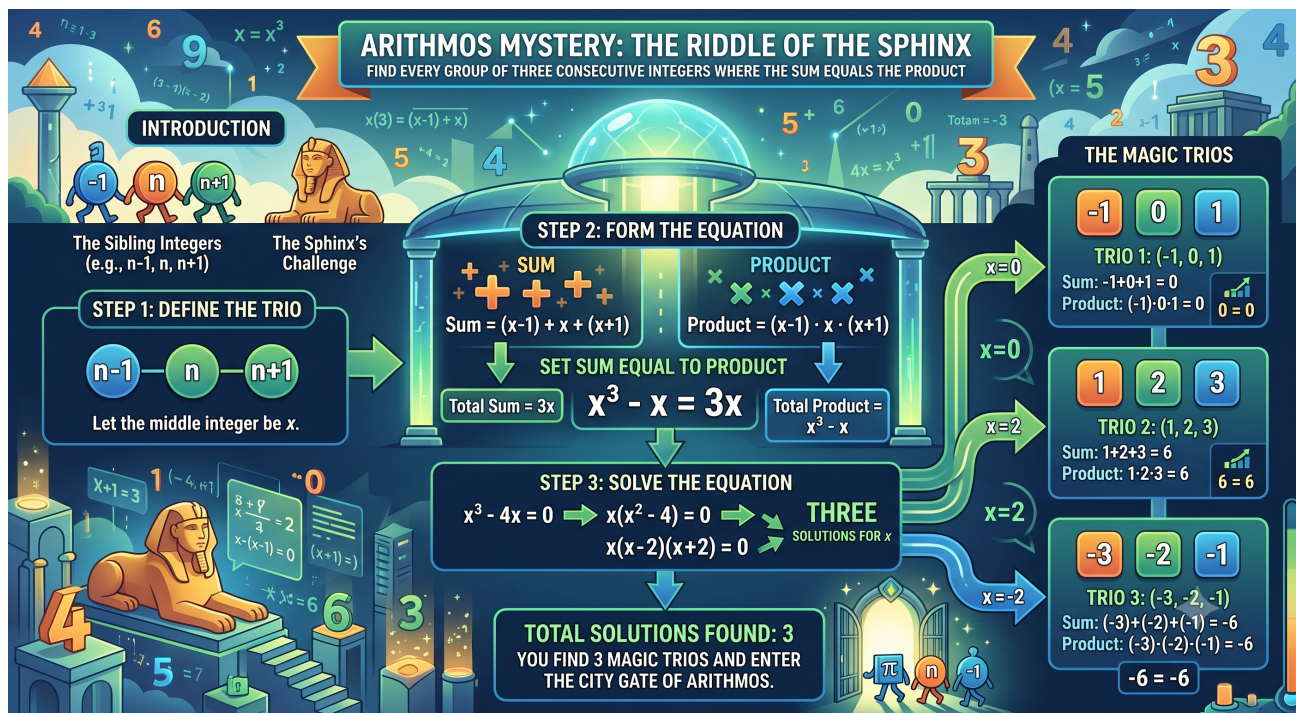
Q.13.



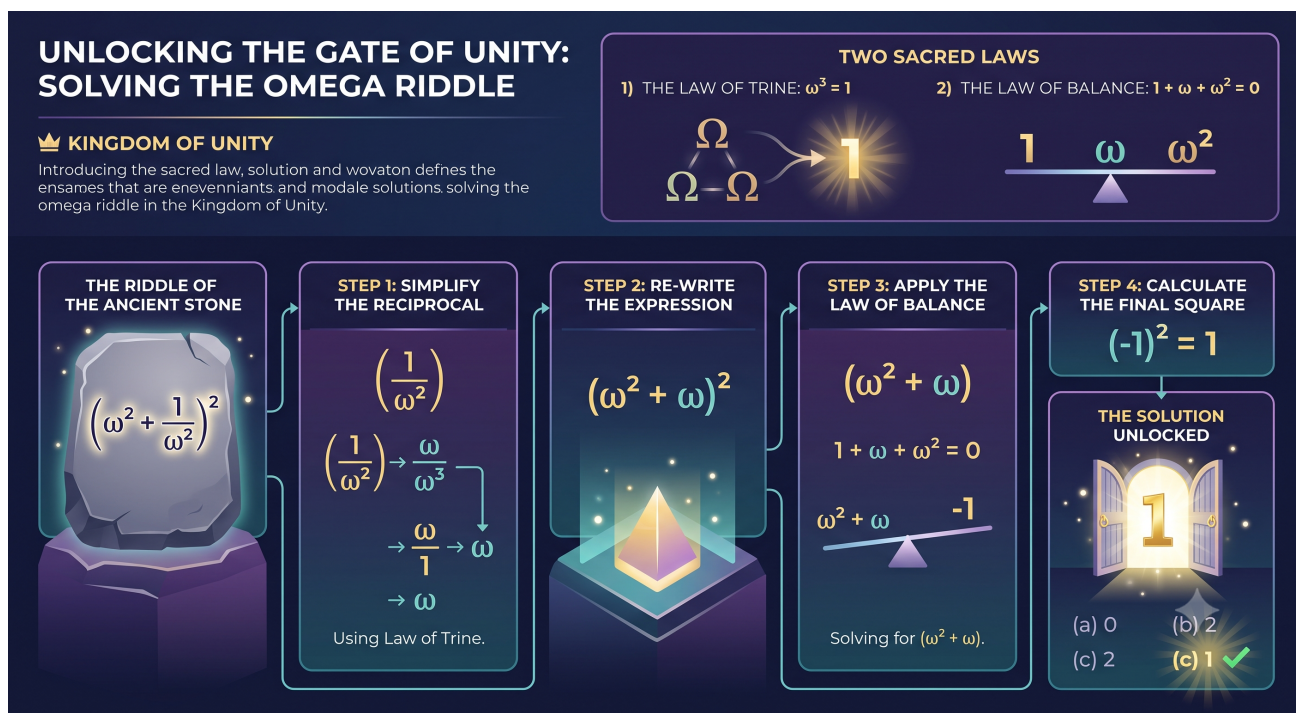
Q.14.



Q.15.



Q.16.



Q.17.

SOLUTION: THE WIZARD'S GLOWING CRYSTAL POWER LEVEL

STEP 1: THE INITIAL CONDITION

Great Oak
POWER LEVEL (x_1)

$n=1, x_1=2$

STEP 2: DIRECT VALUE SUBSTITUTION FOLLOW THE RECURRENCE RULE

START $\rightarrow x_1 = 2$

$$x_2 = \frac{x_1}{1+x_1} = \frac{2}{1+2} = \frac{2}{3}$$

$$x_3 = \frac{x_2}{1+x_2} = \frac{2/3}{1+2/3} = \frac{2/3}{5/3} = \frac{2}{5}$$

... and so on ...

STEP 3: COMPARE & FIND THE PATTERN

STEP (n)	FORMULA (x_n)	RESULT
1	x_1	2
2	x_2	$\frac{2}{3}$
3	x_3	$\frac{2}{5}$
4	x_4	$\frac{2}{7}$

GENERAL PATTERN: $x_n = \frac{\text{CONSTANT 2}}{\text{ODD NUMBERS } (2n-1)}$

STEP 4: CALCULATE THE 100TH STEP

$$x_{100} = \frac{2}{2(100)-1} = \frac{2}{200-1} = \frac{2}{199}$$

FINAL POWER LEVEL AT $n = 100$ IS $\frac{2}{199}$

CORRECT ANSWER: (b) $\frac{2}{199}$

Q.18.

INFOGRAPHIC: Dudeney's Spider and Fly Problem (Various 2D Unfolding Cases)

30' (L) x 12' (W) x 12' (H) - A: Back Wall, Middle, 1 ft from ceiling B: Opposite Wall, Middle, 1 ft from floor

INTUITIVE 2D UNFOLDING CASES (PATH DISTANCE > 40 FT)

Case 1: Across Ceiling (BW $\rightarrow$ Ceiling $\rightarrow$ FW)

Spider 30ft
11ft
z=12
Ceiling
11ft
B
Front Wall
Total dX = 11+30+11=52
dY=0
Path = $\sqrt{52^2 + 0^2} = 52.00$ ft

Case 2: Across Floor (BW $\rightarrow$ Floor $\rightarrow$ FW)

Back Wall 30ft
11ft
z=0
Floor
1ft
B
Front Wall
Total dX = 11+30+11=42
dY=0
Path = $\sqrt{42^2 + 0^2} = 42.00$ ft

Case 3: Across Side Wall (BW $\rightarrow$ SideWall $\rightarrow$ FW)

6ft 30ft 6ft
y=0
Side Wall
Total dX = 6+30+6=42
dY=0
Path = $\sqrt{42^2 + 0^2} = 42.00$ ft

THE OPTIMAL SOLUTION: THE 5-FACE CROSSING (DUDENEY'S TRICK)

Step 1: Unrolling Path

BW $\rightarrow$ SideWall1 $\rightarrow$ Ceiling $\rightarrow$ SideWall2 $\rightarrow$ FW

Step 2: Dudeney's Net Unfolding

y=12
z=12 ceiling unrolled line
width across ceiling
z=12 ceiling unrolled line of SideWall2
y=12
down height unrolled carefully...
A (6,11)
B (6,1)
FW

Step 3: Finding the Right Triangle

Height = 24 ft (traverses height sequence z unrolled across faces sequence)
Base = 32 ft (traverses length faces sequence)

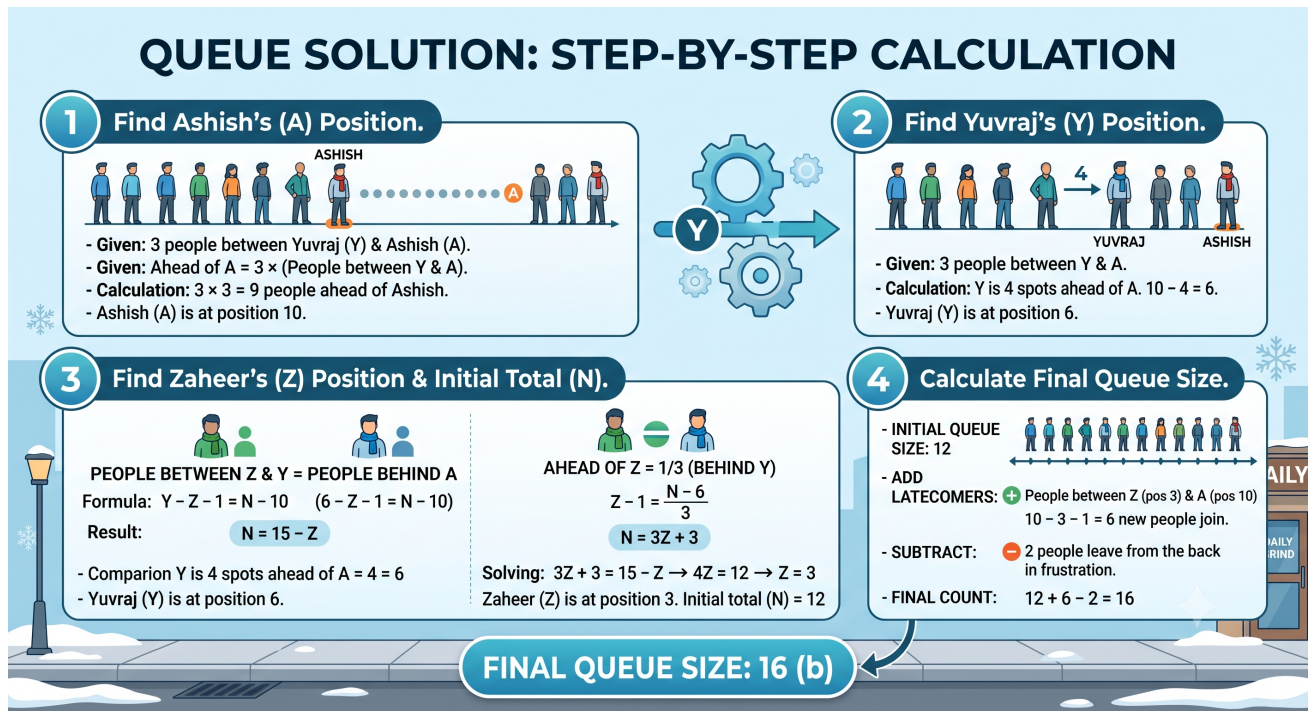
Calculation:

Shortest Path = $\sqrt{32^2 + 24^2}$
 Path = $\sqrt{1024 + 576} = \sqrt{1600}$
 Path = 40.00 ft

THE CORRECT SHORTEST PATH ACROSS THE REAL 3D SURFACE IS 40 FEET.

'Across Ceiling (unrolled width path)' 52 ft
40 ft
'Across 5 Faces'
'Across Floor/Ceiling' 42 ft
'Across Ceiling (unrolled width path)' 42 ft

Q.19.


FINAL QUEUE SIZE: 16 (b)

Q.20.

