

Section – A

1. (D)

Let the other root be r , $\sqrt{3} + r = -\frac{a}{2}$ which is rational

$$\Rightarrow r = -\frac{a}{2} - \sqrt{3}.$$

Product of roots, $\sqrt{3}\left(-\frac{a}{2} - \sqrt{3}\right) = \frac{b}{2}$ which is rational.

$$\sqrt{3}\left(-\frac{a}{2} - \sqrt{3}\right) \text{ is rational } \Rightarrow a = 0$$

$$b = 2\sqrt{3}(-\sqrt{3}) = -6$$

2. (A)

$$4a^2 = a^2 + 4b^2 \Rightarrow (a - 2b)^2 = 0 \Rightarrow a = 2b$$

$$\text{From } a + 2b = 10, a + a = 10 \Rightarrow a = 5.$$

3. (D)

Nothing can be said about degree.

$\therefore x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$ & both quadratics have non-real roots, $x^4 + x^2 + 1$ can be factored but has no real roots.

4. (C)

Remainder has degree < 2 . Let it be $ax + b$

$$b(x) = (x^2 - 5x + 6)q(x) + ax + b$$

$$\text{Put } x = 3 \rightarrow -7 = 3a + b$$

$$\text{Put } x = 2 \rightarrow -5 = 2a + b$$

$$\text{Solving, } a = -2 \text{ \& } b = -1$$

$$\therefore \text{ Remainder is } -2x - 1$$

5. (A)

$$x^2 + 3x + 1 = 0 \Rightarrow x^2 + 1 = -3x \Rightarrow x + \frac{1}{x} = -3$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 9 - 2 = 7$$

6. (B)
 $\frac{\lambda+5}{2} = \frac{\lambda+2}{3\lambda+8} \neq \frac{1}{2}$
 $(\lambda+5)(3\lambda+8) = 2(\lambda+2)$
 $3\lambda^2 + 2^1\lambda + 36 = 0$
 $\lambda = -3, -4$
 But $\lambda = -4$ make the ratio $\frac{1}{2}$.
 So only $\lambda = -3$
7. (C)
 $\therefore a < 0, a \neq 0 \text{ \& } c \neq 0$
 Product of zeros is $\frac{c}{a} = \frac{ac}{a^2} < 0$
 \therefore product is negative, they have opposite sign.
8. (C)
 $30x^2 - 3\sqrt{5}x - 2\sqrt{5}x + 1$
 $= 3\sqrt{5}x(2\sqrt{5}x - 1) - 1(2\sqrt{5}x - 1)$
 $\therefore a = 3\sqrt{5} \text{ \& } b = 2\sqrt{5}$
9. (A)
 Adding the two, $2(3^x) = 54 \Rightarrow x = 3$
 Subtracting the two, $2(2^y) = 16 \Rightarrow y = 3$
10. (A)
 $x^2 + y^2 + 4x + 6y = 10 \quad \dots (1)$
 $2x + 3y = 5 \quad \dots (2)$
 $(1) - 2 \times (2) \rightarrow x^2 + y^2 = 0$
 This happens only when $x = y = 0$
 But it does not satisfy (2)
 Hence no solution.
11. (A)
 Let present age of father & son be x & y years.
 $x = 4y$
 $x + 5 = 3(y + 5)$
 Solving, $4y + 5 = 3y + 15 \Rightarrow y = 10, x = 40$
12. (C)
 $p(x) = (x^2 - 4x + 3)q(x) + 3x$
 By remainder theorem, we want value of
 $p(1) = (1^2 - 4 \times 1 + 3)q(1) + 3 \times 1 = 0 \times q(1) + 3 = 3$

13. (C)
We have replace x with $x - k$
So new zeros will increase by k
 $\therefore k + k = 2k$ is a zero

14. (B)
 $\frac{x}{x^2 + y^2} = \frac{1}{5}$
 $\frac{y}{x^2 + y^2} = \frac{2}{5}$

$$\text{Square \& add } \Rightarrow \frac{x^2 + y^2}{(x^2 + y^2)^2} = \frac{1^2 + 2^2}{5^2}$$

$$\frac{1}{x^2 + y^2} = \frac{1}{5} \Rightarrow x^2 + y^2 = 5$$

$$\text{So } \frac{x}{5} = \frac{1}{5} \Rightarrow x = 1 \text{ \& } \frac{y}{5} = \frac{2}{5} \Rightarrow y = 2$$

15. (B)
 $\frac{ab}{a-b} = -3 \Rightarrow \frac{a-b}{ab} = -\frac{1}{3} \Rightarrow \frac{1}{b} - \frac{1}{a} = -\frac{1}{3}$

$$\text{Similarly } \frac{1}{b} + \frac{1}{a} = -\frac{1}{2}$$

$$\text{Subtracting, } -\frac{2}{a} = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6} \Rightarrow a = -12$$

16. (D)
 $9ax^2 + 3bx + c = a(3x)^2 + b(3x) + c$
So we replaced x with 3x. hence roots will get divided by 3.

17. (C)
Let the numbers be x & y
 $x + y = 76 \quad \dots (1)$
 $\frac{100-25}{100}x + \frac{100+25}{100}y = 71$
 $\Rightarrow 3x + 5y = 284 \quad \dots (2)$
 $4 \times (1) - (2) \rightarrow x - y = 304 - 284 = 20$

18. (B)
Let the present ages of son & father be x & 4x years
 $(x-6)(4x-6) = 52$
 $4x^2 - 30x - 16 = 0$
 $2x^2 - 15x - 8 = 0$
 $(2x+1)(x-8) = 0$
So their ages are 8 & $4 \times 8 = 32$ years

19. (C)

$$(x+y)^2 = x^2 + y^2 + 2xy = 36 + 2 \times 14 = 64$$

$$\Rightarrow x+y = \pm 8$$

$$\begin{aligned} \text{When } x+y=8 \rightarrow x^3+y^3 &= (x+y)^3 - 3xy(x+y) \\ &= 8^3 - 3 \times 14 \times 8 \\ &= 176 \end{aligned}$$

$$\begin{aligned} \text{When } x+y=-8 \rightarrow x^3+y^3 &= (-8)^3 - 3 \times 14(-8) \\ &= -176 \end{aligned}$$

20. (C)

$$9ax^2 + 9bx + 9c = 9(ax^2 + bx + c)$$

New polynomial has same zeroes as $ax^2 + bx + c$

Section – B: Challenge Yourself

21. (B)

$$\text{Observe } (10+3\sqrt{11})(10-3\sqrt{11}) = 1$$

$$\text{Let } (10+3\sqrt{11})^x = t \Rightarrow (10-3\sqrt{11})^x = \frac{1}{t}$$

$$\text{Equation becomes } t + \frac{1}{t} = 20 \Rightarrow t^2 - 20t + 1 = 0$$

$$t = \frac{20 \pm \sqrt{400-4}}{2} = 10 \pm 3\sqrt{11}$$

$$\therefore (10+3\sqrt{11})^x = 10+3\sqrt{11} \text{ or } (10+3\sqrt{11})^x = 10-3\sqrt{11}$$

$$\Rightarrow x = 1 \text{ or } -1$$

2. (C)

Square & add:
$$\frac{(x+y)^2 + (x-y)^2}{(x^2+y^2)^2} = p^2 + q^2$$

$$\frac{2(x^2+y^2)}{(x^2+y^2)^2} = p^2 + q^2$$

$$\frac{2}{x^2+y^2} = p^2 + q^2$$

$$x^2 + y^2 = \frac{2}{p^2 + q^2}$$

$$P = \frac{x+y}{x^2+y^2} = (x+y) \left(\frac{p^2+q^2}{2} \right) \Rightarrow x+y = \frac{2p}{p^2+q^2}$$

Similarly
$$x-y = \frac{2q}{p^2+q^2}$$

Adding,
$$2x = \frac{2p+2q}{p^2+q^2} \Rightarrow x = \frac{p+q}{p^2+q^2}$$

Subtracting,
$$2y = \frac{2p-2q}{p^2+q^2} \Rightarrow y = \frac{p-q}{p^2+q^2}$$

23. (C)

Let the roots be α, β

$$\alpha + \beta = 2m, \alpha\beta = 8m + 4$$

Given $|\alpha - \beta| = 2 \Rightarrow (\alpha - \beta)^2 = 4$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 4$$

$$\Rightarrow 4m^2 - 4(8m + 4) = 4$$

$$\Rightarrow m^2 - 8m - 5 = 0$$

Sum of roots of this quadratic is 8

24. (D)

Using long division, final remainder is

$$(2k-9)x + (k^2 - 8k + 10)$$

\therefore this equals $x + a$ we must have

$$2k - 9 = 1 \Rightarrow k = 5$$

$$\& a = k^2 - 8k + 10 = 5^2 - 8 \times 5 + 10 = -5$$

25. (D)

Let $2^x = t \Rightarrow 4^x = t^2$

Equation becomes $t^2 - t + 16 = 0$

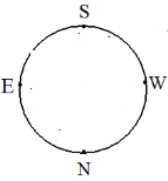
This has 2 values of $t \Rightarrow$ 2 values of x , say α & β

$$\therefore 2^\alpha \& 2^\beta \text{ are roots of } t^2 - t + 16 = 0$$

Product of roots, $2^\alpha \cdot 2^\beta = \frac{16}{1} \Rightarrow 2^{\alpha+\beta} = 16$

$$\therefore \alpha + \beta = 4$$

Section – C: Logical Reasoning

26. (A)
Every alternate number has sum of digits not divisible by 2
(10000, ~~10001~~, 10002, ~~10003~~)
Total 5-digit number are 90000
 \therefore Answer is $\frac{90000}{2} = 4500$
27. (D)
March 11, 1970 (exactly 56 years age) will be same day: Wednesday .February 28 (11 days ago) would be Saturday
28. (D)
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29. (D)
The big cube is dimension 4.
The inside cube of dimension $4 - 2 = 2$ will
Not be affected
So required number of cubes is $2^3 = 8$
30. (A)
Bell A rings every 10 seconds, bell B rings every 15 seconds
 \therefore LCM(10,15) = 30, they will ring together at 30th second. Before that, will A will ring 3 times
(0th, 10th, 20th second)