

Section – A

1. (D)

Let other root be r .

Sum of roots, $r + \sqrt{2} = -\frac{a}{2} \rightarrow$ rational.

So, $r = P - \sqrt{2}$ where P is rational.

Product of roots, $(P - \sqrt{2})\sqrt{2} = \frac{b}{2} \rightarrow$ rational.

Now, $(P - \sqrt{2})\sqrt{2} = \sqrt{2}P - 2$ which is rational only when $P = 0$.

$$\therefore r = -\sqrt{2}$$

$$-\frac{a}{2} = r + \sqrt{2} = 0 \Rightarrow a = 0$$

$$\frac{b}{2} = (-\sqrt{2})\sqrt{2} = -2 \Rightarrow b = -4.$$

$$\therefore a - b = 4.$$

2. (A)

$$\frac{ab}{a^2 + b^2} = \frac{1}{2} \Rightarrow a^2 - 2ab + b^2 = 0 \Rightarrow (a - b)^2 = 0$$

$$\Rightarrow a = b$$

$$a + b = 10 \Rightarrow 2a = 10 \Rightarrow a = 5$$

3. (B)

If n is even then clearly $n^4(n^4 + 15)$ is divisible by 16

Let $n = 2m - 1$ (odd).

$$n^4 + 15 = (n^4 - 1) + 16$$

$$= (n^2 + 1)(n + 1)(n - 1) + 16$$

$$= (4m^2 - 4m + 2)(2m)(2m - 2) + 16$$

$$= 8(2m^2 - 2m + 1)\underbrace{m(m - 1)}_{\text{divisible by 2}} + 16$$

$$\Rightarrow n^4 + 15 \text{ is divisible by 16.}$$

4. (C)

Remainder has degree < 2

Let it be $ax + b$.

$$p(x) = (x^2 + 5x + 6)q(x) + ax + b$$

Also, $p(-2) = 3$, $p(-3) = 5$ by remainder theorem.

$$p(-2)=3 \Rightarrow oq(x)-2a+b=3 \Rightarrow b-2a=3$$

$$p(-3)=5 \Rightarrow oq(x)-3a+b=5 \Rightarrow b-3a=5$$

Solving gives $a = -2$, $b = -1$.

5. (A)

$$x^2 + 1 = 4x \Rightarrow x + \frac{1}{x} = 4$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\ &= 4^3 - 3 \times 4 = 52 \end{aligned}$$

6. (B)

$$\frac{\lambda+3}{2} = \frac{\lambda}{3\lambda+2} \neq \frac{1}{2}$$

Solving first quality, $(\lambda+3)(3\lambda+2) = 2\lambda$

$$3\lambda^2 + 9\lambda + 6 = 0$$

$$\lambda = -1, -2$$

When $\lambda = -2$, first two ratios equal the third ratio.

7. (C)

$$4891 = 4757 + 134$$

So, check if 4757 & 134 have common factor.

$\therefore 67$ divides both, both 4757 & 4891 are divisible by 67.

8. (D)

$$\begin{aligned} 6x^2 - 7\sqrt{3}x + 5 &= 6x^2 - 2\sqrt{3}x - 5\sqrt{3}x + 5 \\ &= 2\sqrt{3}x(\sqrt{3}x - 1) - 5(\sqrt{3}x - 1) \\ &= (2\sqrt{3}x - 5)(\sqrt{3}x - 1) \end{aligned}$$

$$\therefore a = -1, b = 2\sqrt{3}, c = -5$$

9. (B)

$$\begin{aligned} 9^{\frac{x+2}{4}} &= 3^{\frac{x+2}{2}} = 3^{\frac{x}{2}+1} = 3 \times \left(3^{\frac{x}{2}}\right)^2 \\ &= 3 \times 64^{\frac{1}{2}} = 24 \end{aligned}$$

10. (C)

$$b+c=-a, c+a=-b, a+b=-c$$

$$\begin{aligned} \frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab} &= \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \\ &= \frac{a^3+b^3+c^3}{abc} \quad \{a^3+b^3+c^3=3abc\} \\ &= \frac{3abc}{abc} = 3 \end{aligned}$$

11. (A)
Let present ages of father be x , son be y in years.
Given $x = 4y$, $x + 5 = 3(y + 5)$
Solving, $4y + 5 = 3y + 15 \Rightarrow y = 10 \Rightarrow x = 40$

12. (C)
 $p(x) = (x^2 - 3x + 2)q(x) + 2x + 1$
Put $x = 1 \rightarrow p(1) = (1^2 - 3 \times 1 + 2)q(1) + 2 \times 1 + 1$
 $= 0 \times q(1) + 3$
 $\therefore p(1) = 3$, use remainder theorem.

13. (D)
 $ak^2 + bk + c = 0$
Clearly putting $x = 0$ in given polynomial gives 0.

14. (B)
Let the number be N .
 $N = 299Q + 184 = 13 \times 23Q + 13 \times 14 + 2$
 $= 13(23Q + 14) + 2$

15. (B)
 $\frac{a+b}{ab} = \frac{1}{3}, \frac{a-b}{ab} = \frac{1}{2}$
 $\frac{1}{b} + \frac{1}{a} = \frac{1}{3}, \frac{1}{b} - \frac{1}{a} = \frac{1}{2}$
Subtracting the two, $\frac{2}{a} = \frac{1}{3} - \frac{1}{2}$
 $\Rightarrow a = -12$

16. (D)
 $4ax^2 + 2bx + c = a(2x)^2 + b(2x) + c$
So, now we must have $2x = \alpha$ & $2x = \beta$.

17. (A)
Let the number be x, y .
 $x + y = 76, \frac{4}{5}x = \frac{11}{10}y \Rightarrow x = \frac{11}{8}y$
 $\frac{11}{8}y + y = 76 \Rightarrow \frac{19}{8}y = 76 \Rightarrow y = 32$
 $x = 76 - y = 44$

18. (B)
Let present age of father be x , son be y is years.
 $x = 4y, (x - 6)(y - 6) = 52$
Solving, $(4y - 6)(y - 6) = 52$
 $\Rightarrow 4y^2 - 30y - 16 = 0$

$$\Rightarrow y = 8, -\frac{1}{2} \left\{ -\frac{1}{2} \text{ rejected} \right\}$$

$$\Rightarrow x = 32$$

19. (C)

$$(x+y)^2 = (x^2 + y^2) + 2xy = 25 + 2 \times 12 = 49$$

$$\Rightarrow x+y = \pm 7$$

$$x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

$$\text{When } x+y = 7 \Rightarrow 7^3 - 3 \times 12 \times 7 = 91$$

$$\text{When } x+y = -7 \Rightarrow (-7)^3 - 3 \times 12(-7) = -91$$

20. (C)

$$4ax^2 + 4bx + 4c = 4(ax^2 + bx + c)$$

Multiplying a polynomial by some constant does not change zeros.

Section – B

21. (B)

$$\text{Observe } (7+4\sqrt{3})(7-4\sqrt{3}) = 7^2 - (4\sqrt{3})^2 = 1$$

So, $7+4\sqrt{3}$ & $7-4\sqrt{3}$ are reciprocals.

$$\text{Let } (7+4\sqrt{3})^x = t \Rightarrow (7-4\sqrt{3})^x = \frac{1}{t}$$

$$\text{Equation is } t + \frac{1}{t} = 14 \Rightarrow t^2 - 14t + 1 = 0$$

$$t = \frac{14 \pm \sqrt{196-4}}{2} = 7 \pm 4\sqrt{3}$$

$$(7+4\sqrt{3})^x = 7+4\sqrt{3} \quad \text{OR} \quad (7+4\sqrt{3})^x = 7-4\sqrt{3}$$

$$\Rightarrow x=1 \quad \text{OR} \quad x=-1$$

22. (B)

When n is even, $n^4 + 4^n$ is even so composite.

Let $n = 2m-1$ (odd)

$$n^4 + 4^n = (n^2)^2 + 2 \cdot (2^n) \cdot n^2 + (2^n)^2 - 2 \cdot 2^n \cdot n^2$$

$$= (n^2 + 2^n)^2 - 2 \cdot 2^n \cdot n^2$$

$$= (n^2 + 2^n)^2 - 2^{2m} \cdot n^2$$

$$= (n^2 + 2^n)^2 - (n \cdot 2^m)^2$$

$$= (n^2 + 2^n - n \cdot 2^m)(n^2 + 2^n + n \cdot 2^m)$$

$n^4 + 4^n$ is factorised, hence it is composite.

23. (C)
 Let roots be α, β .
 $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
 $4^2 = m^2 - 4(2m+1)$
 $\Rightarrow m^2 - 8m - 20 = 0$
 $\Rightarrow m = 10, -2$

24. (A)

$$\frac{2}{\sqrt{2} + \sqrt{3} + \sqrt{5}} = \frac{2(\sqrt{2} + \sqrt{3} - \sqrt{5})}{(\sqrt{2} + \sqrt{3} + \sqrt{5})(\sqrt{2} + \sqrt{3} - \sqrt{5})}$$

$$= \frac{2(\sqrt{2} + \sqrt{3} - \sqrt{5})}{(\sqrt{2} + \sqrt{3})^2 - (\sqrt{5})^2} = \frac{2(\sqrt{2} + \sqrt{3} - \sqrt{5})}{2 + 3 + 2\sqrt{6} - 5}$$

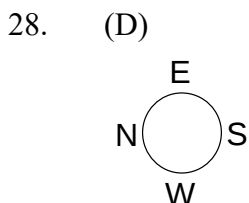
$$= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{\sqrt{6}} = \sqrt{\frac{1}{3}} + \sqrt{\frac{1}{2}} - \sqrt{\frac{5}{6}}$$

25. (C)
 Let $6^x = t \Rightarrow t^2 = 36^x$
 Equation is $\frac{t^2}{18} - t + 2 = 0 \Rightarrow t^2 - 18t + 36 = 0$
 Let x_1, x_2 be roots of original equation.
 $\Rightarrow 6^{x_1}, 6^{x_2}$ are roots of new equation.
 Product of roots, $6^{x_1} \cdot 6^{x_2} = 36 \Rightarrow 6^{x_1+x_2} = 36$
 $\Rightarrow x_1 + x_2 = 2$.

Section – C

26. (A)
 There are 90000 5-digit numbers.
 Observe that every alternate number has sum of digits even (eg. 10000 **x**, 10001 **✓**, 10002 **x**, 10003 **✓**,.....)
 So half of 90000, 45000 is the answer.

27. (A)
 January 31, 1970 (exactly 56 years age) will be same day : Saturday
 30 days ago it will be Thursday.



29. (D)
Father of father $\rightarrow$ grandfather
Granddaughter $\rightarrow$ same generation
Husband $\rightarrow$ same generation but cannot be brother cousin.

30. (C)
Bell A rings every 12 seconds.
Bell B rings every 15 seconds.
 $\text{LCM}(12, 15) = 60$
At 60th second, both ring together.
So bell A rings at 0th, 12th, 24th, 36th, 48th
Second $\Rightarrow$ 5 times.