

Section – A

1. (B)
Put $x = -1 - 2y + 4z$ in $-x + 5y + z = 5$ and $2x - 3y - z = -2$
Then solve two equations for y and z .
We get, $y = 1$ and $z = 1$
 $\therefore x = 1$
 $\therefore (\alpha, \beta, \gamma) = (1, 1, 1)$
2. (C)
 $x^2 + 7x + 3 = (x - \alpha)(x - \beta)$
 $\therefore \alpha + \beta = -7$ and $\alpha\beta = 3$
$$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^3} = \frac{(-7)^3 - 3 \times 3(-7)}{3^3} = \frac{-280}{27}$$
3. (A)
Clearly $\alpha = 2 - \sqrt{3}$ and $\beta = 1 + \sqrt{3}$
 $\therefore \alpha + \beta = 3$ and $\alpha\beta = \sqrt{3} - 1$
 $\therefore$ Required equation: $x^2 - 3x + \sqrt{3} - 1 = 0$
4. (D)
 $P(x) = (x^2 - 16)Q(x) + ax + b$
By remainder theorem,
 $P(4) = 3$ and $P(-4) = 7$
 $\therefore 4a + b = 3$ and $-4a + b = 7$
 $\therefore a = -\frac{1}{2}$ and $b = 5$
 $\therefore$ Remainder $= -\frac{x}{2} + 5$
5. (B)
For real and equal roots, $D = 0$.
i.e. $b^2 = 4ac$
 $\therefore (-2k)^2 = 4(k)(3)$
 $\therefore k^2 - 3k = 0$
 $\therefore k = 0, 3$

6. (D)

$$\text{Let } 2x + 3y = \frac{1}{u} \text{ and } 3x - y = \frac{1}{v}.$$

$$\therefore 13u + 6v = 3 \text{ and } 39u - 12v = -1$$

$$\text{On solving, we get, } u = \frac{1}{13} \text{ and } v = \frac{1}{3}.$$

$$\therefore 2x + 3y = 13 \text{ and } 3x - y = 3$$

On solving, we get $x = 2$ and $y = 3$.

$$\therefore (\alpha, \beta) = (2, 3)$$

$$\therefore \alpha + \beta = 5$$

7. (A)

$$\text{Let } P(x) = 2x^3 - 4x^2 + x + 3$$

By remainder theorem,

$$\text{Remainder} = P\left(\frac{-1}{2}\right)$$

$$= 2\left(-\frac{1}{2}\right)^3 - 4 \times \left(-\frac{1}{2}\right)^2 - \frac{1}{2} + 3$$

$$= -\frac{1}{4} - 1 - \frac{1}{2} + 3$$

$$= \frac{5}{4}$$

8. (C)

Let α, β and γ are roots of given polynomial.

$$\therefore 2x^3 + 3x^2 - 5x - 6 = k(x - \alpha)(x - \beta)(x - \gamma)$$

$$\therefore 2x^3 + 3x^2 - 5x - 6 = kx^3 - k(\alpha + \beta + \gamma)x^2 + k(\alpha\beta + \beta\gamma + \gamma\alpha)x - k\alpha\beta\gamma$$

$$\therefore k = 2 \text{ and } k\alpha\beta\gamma = 6$$

$$\therefore \alpha\beta\gamma = 3$$

$\therefore$ Given that product of two zeros (say α and β) is 3.

$$\text{i.e., } \alpha\beta = 3$$

$$\therefore \gamma = 1.$$

9. (D)

$f(x)$ intersect x -axis four times.

10. (A)

For no solution,

$$\frac{2}{k} = \frac{3}{4} \neq \frac{5}{7}$$

$$\therefore k = \frac{8}{3}$$

11. (C)

$$\begin{array}{r}
 x^2 + 2x + 3 \\
 x^2 + 5 \overline{) x^4 + 2x^3 + 8x^2 + 12x + 18} \\
 \underline{-x^4 + 5x^2} \\
 2x^3 + 3x^2 \\
 \underline{-2x^3 + 10x} \\
 3x^2 + 2x \\
 \underline{-3x^2 + 15} \\
 2x + 3
 \end{array}$$

$$\therefore p = 2 \text{ and } q = 3$$

12. (B)

$$x^2 - 8x + k = (x - \alpha)(x - \beta)$$

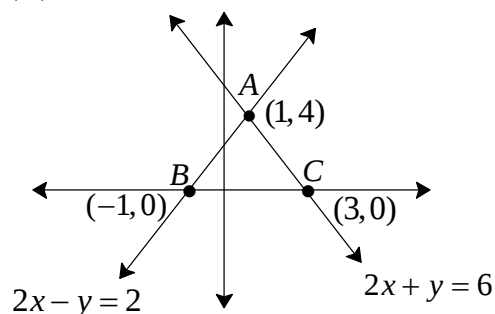
$$\therefore \alpha + \beta = 8 \text{ and } \alpha\beta = k$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$$

$$\therefore 2\alpha\beta = 8^2 - 40 = 24$$

$$\therefore \alpha\beta = 12$$

13. (D)



$$\text{Area of } \triangle ABC = \frac{1}{2} \times 4 \times 4 = 8 \text{ sq. units}$$

14. (D)

Theoretical

15. (A)

$$|5x + 7| = \frac{3}{2}$$

$$\Rightarrow 5x + 7 = \frac{3}{2} \text{ or } 5x + 7 = -\frac{3}{2}$$

$$\Rightarrow x = -\frac{11}{10} \text{ or } x = -\frac{17}{10}$$

$$\Rightarrow -\frac{11}{10} + \left(-\frac{17}{10}\right) = -\frac{28}{10} = -\frac{14}{5}$$

16. (C)

$$\text{Let } 2^x = t$$

$$\therefore t^2 - 10t + 16 = 0$$

$$\therefore (t - 8)(t - 2) = 0$$

$$\begin{aligned}\therefore t &= 8 \text{ and } t = 2 \\ \therefore 2^x &= 8 \text{ and } 2^x = 2 \\ \therefore x &= 3 \text{ and } x = 1 \\ \therefore \text{Product of roots} &= 3 \times 1 = 3\end{aligned}$$

17. (A)

$$x = 2y - 9$$

$$\therefore y^2 = 4(2y - 9) + 2$$

$$\therefore y^2 - 8y + 34 = 0$$

$$D = (-8)^2 - 4 \times 34 = 64 - 136 < 0$$

No real value of y .

$\therefore$ Given equations doesn't intersect.

18. (B)

Let α be one root of given equation.

$$\therefore \frac{1}{\alpha} \text{ is other root}$$

$$\therefore 2x^2 - 4x + 9k = a(x - \alpha)\left(x - \frac{1}{\alpha}\right)$$

$$\therefore a = 2 \text{ and } 9k = a$$

$$\therefore k = \frac{2}{9}$$

19. (A)

Let $P(x) = x^2 + ax + 3$ and $q(x) = x^3 + bx^2 + x + 1$.

By remainder theorem and given condition,

$$P(-1) = q(-1).$$

$$\therefore (-1)^2 + a(-1) + 3 = (-1)^3 + b(-1)^2 + (-1) + 1$$

$$\therefore 1 - a + 3 = -1 + b - 1 + 1$$

$$\therefore a + b = 5$$

20. (D)

$$x^2 - 18x + 77 = (x - 7)(x - 11) \leq 0$$

$$\begin{array}{ccccccc} & + & & - & & + & \\ & \bullet & & & & \bullet & \\ \hline & 7 & & & & 11 & \end{array}$$

$$\therefore x \in [7, 11]$$

Section – B

21. (C)

Let $t = (\sqrt{3} + \sqrt{2})^x$.

$$(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = 1$$

$$\therefore (\sqrt{3} - \sqrt{2})^x = \frac{1}{t}$$

$$\therefore t + \frac{1}{t} = 10$$

$$\therefore t^2 - 10t + 1 = 0$$

$$\therefore t = \frac{10 \pm \sqrt{100 - 4}}{2} = 5 \pm 2\sqrt{6}$$

$$\therefore (\sqrt{3} + \sqrt{2})^x = 5 + 2\sqrt{6} \quad \text{or} \quad (\sqrt{3} + \sqrt{2})^x = 5 - 2\sqrt{6}$$

$$\therefore x = 2 \quad \text{or} \quad x = -2$$

$$\therefore S = \{-2, 2\}$$

22. (B)

Let the cost of full and half ticket be ₹ x and ₹ $\frac{x}{2}$ and reservation charge be ₹ y per ticket.

$$\therefore x + y = 2530 \quad \text{and} \quad x + y + \frac{x}{2} + y = 3810$$

$$\therefore 3x + 4y = 7620$$

On solving for x and y , we get

$$x = 2500 \quad \text{and} \quad y = 30.$$

23. (D)

$$\text{Let } x + \frac{1}{x} = t$$

$$\therefore 3(t^2 - 2) - 2t + 5 = 0$$

$$\therefore 3t^2 - 2t - 1 = 0$$

$$\therefore (3t + 1)(t - 1) = 0$$

$$\therefore t = -\frac{1}{3} \quad \text{or} \quad t = 1$$

$$\therefore x + \frac{1}{x} = -\frac{1}{3} \quad \text{or} \quad x + \frac{1}{x} = 1$$

$$\text{But } x + \frac{1}{x} \in (-\infty - 2] \cup [2, \infty)$$

$\therefore$ No real value of x .

24. (A)

Let given fraction is $\frac{x}{y}$.

$$\therefore x + y = 2x + 4 \quad \text{and} \quad \frac{x+3}{y+3} = \frac{2}{3}$$

$$\therefore x - y = -4 \quad \text{and} \quad 3x - 2y = -3$$

$$\therefore x = 5 \quad \text{and} \quad y = 9$$

$$\therefore \frac{x}{y} = \frac{5}{9} \Rightarrow p = 5 \quad \text{and} \quad q = 9.$$

$$\therefore pq = 45$$

25. (A)

$$\text{Let } a = \sqrt{5 + \sqrt{5 + \sqrt{5 + \dots + \infty}}}$$

$$\therefore a = \sqrt{5 + a}$$

$$\therefore a^2 - a - 5 = 0$$

$$\therefore a = \frac{1 \pm \sqrt{1+20}}{2} = \frac{1 \pm \sqrt{21}}{2}$$

$$\therefore a > 0, a = \frac{1 + \sqrt{21}}{2}$$

Section – C

26. (C)

27. (C)

28. (A)

29. (D)

30. (D)