VECTORS

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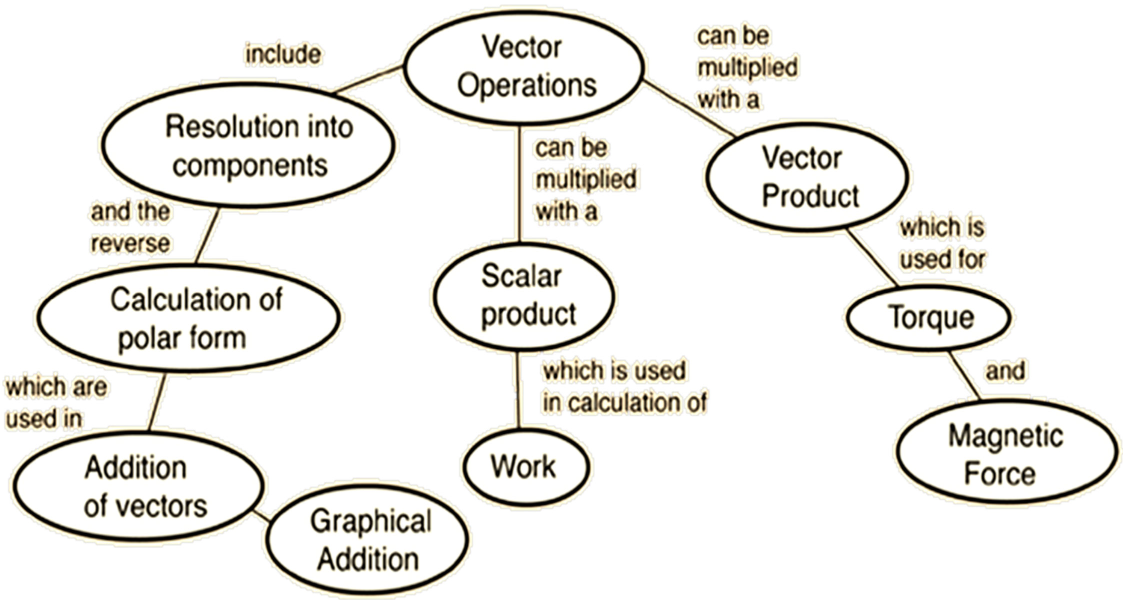
This chapter includes:

- Physical Quantities
- Types of Vectors
- Addition of Vectors
- Subtraction of Vectors
- Resolution of Vectors
- Multiplication in Vectors

VECTORS



Syllabus:

Scalar and vector quantities; Position and displacement vectors, general vectors and notation; Equality of vectors, multiplication of vectors by a real number; Addition and subtraction of vectors; Unit vector, Resolution of a vector in a plane into rectangular components, Multiplication of vectors-scalar and vector products; vectors in three dimensions (elementary idea only).

INTRODUCTION

PHYSICAL QUANTITIES

A **physical quantity** isa physical property of aphenomenon, body or substance that can be quantified by measurement by the measuring instrument.e.g., Length, Temperature, Velocity, Momentum etc.

They are classified into two parts usually.

1. Scalar quantity
2. Vector quantity

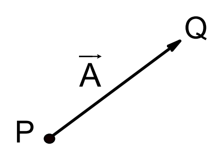
Scalar Quantity

A physical quantity which can be described completely by its magnitude only and does not require a direction is known as a scalar quantity. It obeys the ordinary rules of algebra. e.g.: distance, mass, time, speed, density, volume, temperature, current etc.

Vector Quantity:

A physical quantity which requires magnitude and a particular direction and obeying laws of vector algebra, known as vector quantity. eg: displacement, velocity, acceleration, force etc.

A vector is represented by putting an arrow (bar) over it. The length of the line drawn in a convenient scale represents the magnitude of the vector. The direction of the vector quantity is depicted by placing an arrow at the end of the line. Its length is proportional to its magnitude, with respect to a suitably chosen scale.



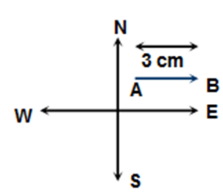
$\vec{A}$ is a vector and $\vec{A}=\overrightarrow{PQ}$

Magnitude of $\vec{A}=|\vec{A}|$ or A

Remark :	1. A physical quantity which possesses both magnitude and direction but does not add up according to vector rules is not a vector quantity. As, Electric current has magnitude as well as direction, but does not follow laws of vector addition. Hence it is not a vector. 2. There also exist some such pairs of physical quantities which have same units and dimensions but one is a scalar quantity and other one is a vector quantity. e.g. speed and velocity, etc.

Remark :	1. Scalar quantity may be negative e.g. charge, electric current, potential energy, work etc. 2. Scalar quantity are direction independent e.g pressure, electric current, surface tension etc. 3. Vector quantities are direction dependent e.g. force, velocity and displacement.

Example: If 1 cm length is equal to 20 km/hr, then vector $\overrightarrow{AB}$ represents 60 km/hr due east.

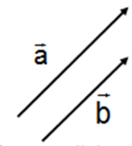


The point A is called initial point or tail and point B is called terminal point or head.

Types of Vectors

1. Parallel Vectors:

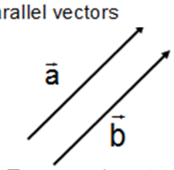
If two vectors have the same direction, they are **parallel,** no matter where they are located in space. Angle between the vectors will be zero. i.e., $\vec{a} \parallel \vec{b}$



Two parallel vectors

2. Equal Vectors:

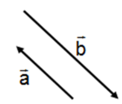
Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal when they have equal magnitudes and same direction. No matter where they are located in space. i.e., $\vec{a} = \vec{b}$



Two equal vectors

3. Anti – Parallel Vectors:

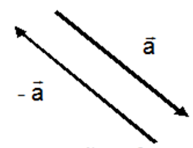
When two vectors $\vec{a}$ and $\vec{b}$ have opposite directions, whether their magnitudes are the same or not, we say that they are anti – parallel vectors



Two anti-parallel vectors

4. Opposite Vectors:

The **opposite of a vector** is defined as a vector having the same magnitude as the original vector but the opposite direction. It is also said Negative of a vector.

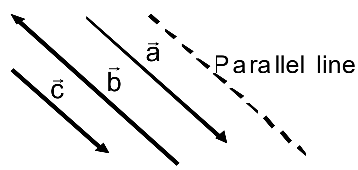


The opposite of a vector

5. Collinear Vectors:

When the vectors under consideration are along the same line are said to be collinear vectors. Angle between the vectors may be zero or 180°.

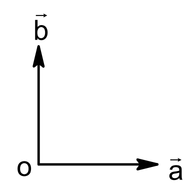
$\vec{a}, \vec{b}$ and $\vec{c}$ are collinear vectors.



Collinear vectors

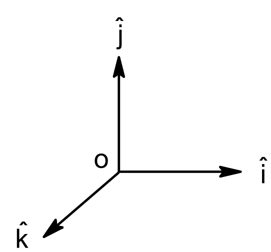
6. Normal Vectors:

If two vectors are perpendicular to each other then they are normal vectors. No matter where they are located in space. Angle between the vectors will be $90°$ i.e., $\vec{a} \perp \vec{b}$



7. Unit Vector:

A vector having magnitude equal to unity with no units. It is represented by $\hat{a}$. To find the unit vector in the direction of $\vec{a}$, we divide the given vector by its magnitude. i.e., $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$, or $\vec{a} = |a|\hat{a}$ or $\vec{a} = a\hat{a}$, where $|\vec{a}|$ or a is the magnitude of the vector $\vec{a}$. Unit vector is basically used to indicate the direction.



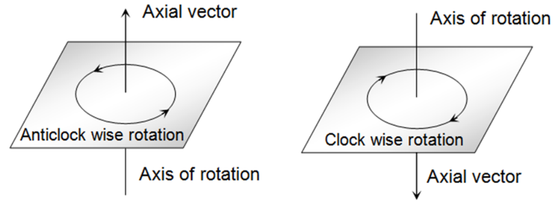
Remark :	
	1. $\hat{i}$ is a unit vector towards positive x–axis.
	2. $\hat{j}$ is a unit vector towards positive y–axis.
	3. $\hat{k}$ is a unit vector towards positive z–axis.

8. Zero Vector $(\vec{0})$:

A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector. It is also known as Null vector. It cannot represent graphically. It is used for the definition of Subtraction of vectors and Cross Multiplication of Collinear vectors..

9. Axial Vectors:

These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are example of physical quantities of this type.



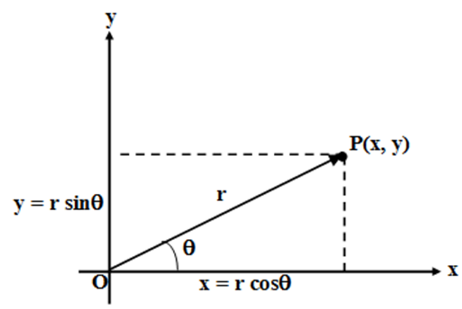
10. Coplanar Vector: Three (or more) vectors are called coplanar vector if they lie in the same plane. Two (free) vectors are always coplanar.

How to Locate Position Vectors in Space?

Position Vectors in space are designated relative to coordinate systems. A position vector also is a Euclidean Vector which represents the position of a point *P* in space in relation to a reference origin *O*. i.e. If initiation point of a vector is origin, then it is said to be a position vectors or location vectors. The **Cartesian coordinate** system is a particularly convenient coordinate system in which position is designated by distances (x, y, z) along three perpendicular axes that intersect at a point called the origin.

1. For Two Dimension:

In a **two dimension coordinate** system position vector in a plane are designated by a length r from the origin, and a usually measured from the positive x – axis. From simple trigonometry we see that the relationships between the polar coordinates and the Cartesian coordinates are $x = r\cos\theta$ & $y = r\sin\theta$ and position vector of OP can be written as



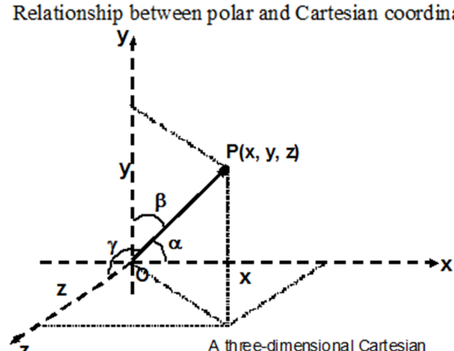
Relationship between polar and Cartesian coordinates

2. For Three Dimension :

In a **three dimension coordinate** system position vector in a plane are designated by a length r from the origin, and an angle α, β, γ usually measured from the positive x – axis, y–axis and z–axis. From simple trigonometry we see that the relationships between the polar coordinates and the Cartesian coordinates are

$x = r\cos\alpha$, $y = r\sin\beta$ and $z = r\cos\gamma$

$x^2 + y^2 + z^2 = r^2$



A three-dimensional Cartesian coordinate system

i.e., $\cos\alpha = \dfrac{x}{\sqrt{x^2+y^2+z^2}}$ (direction cosine of positive x–axis)

$\cos\beta = \dfrac{y}{\sqrt{x^2+y^2+z^2}}$ (direction cosine of positive y–axis)

$\cos\gamma = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$ (direction cosine of positive z–axis)

And position vector of OP can be written as $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

(Note: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$)

The unit vector in the direction of $\vec{A}$ is $\hat{n} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{|\vec{A}|}$

$$\hat{n} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}.$$

Addition of Vectors

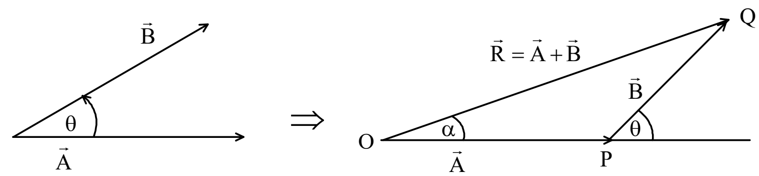
The addition of vectors can be done by following two methods:

(i) Geometrical method (ii)Analytical method

(i) Geometrical Method:

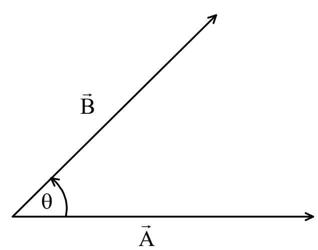
Triangle Law: If two non–zero vectors can be represent by the two sides of a triangle taken in same order, then their resultant is represented by third side of the triangle taken in the opposite order. Consider two vectors $\vec{A}$ and $\vec{B}$ at an angle θ between them.

Finding $\vec{A} + \vec{B}$: First draw vector $\vec{A}\left(\overrightarrow{OP}\right)$ in the given direction. Then draw vector $\vec{B}\left(\overrightarrow{PQ}\right)$;starting from the head of the vector $\vec{A}$. Then close the triangle. $\vec{R}\left(\overrightarrow{OQ}\right)$ will be their resultant

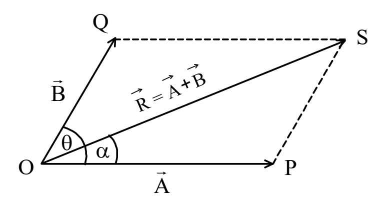


(a) (b)

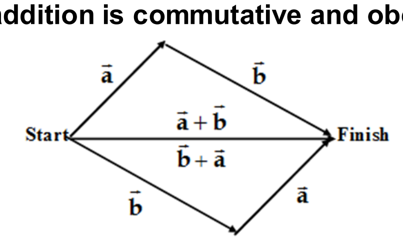
Parallelogram Law: If two non–zero vectors can be represented by the two adjacent sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram passing though the point of intersection of the vectors. Suppose two vectors $\vec{A}$ and $\vec{B}$ as shown in fig.



Finding: $\vec{A} + \vec{B}$: Draw vectors $\vec{A}\left(\overrightarrow{OP}\right)$ and $\vec{B}\left(\overrightarrow{OQ}\right)$ starting from a common point O in the given direction. Then complete parallelogram. The diagonal $\overrightarrow{OS}$ will represent their resultant.

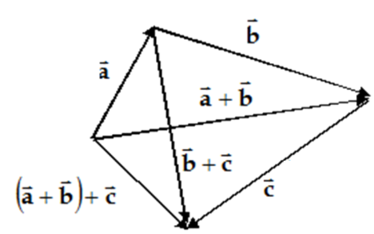


Vector addition is commutative and obeys the associative law.



Vector addition is commutative

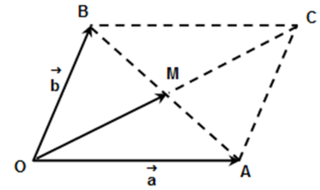
$\vec{a} + \vec{b} = \vec{b} + \vec{a}$



Vector addition is associative

$\left(\vec{a} + \vec{b}\right) + \vec{c} = \vec{a} + \left(\vec{b} + \vec{c}\right)$

Illustration 1. If the position vector of point A and B are $\vec{a}$ and $\vec{b}$ respectively. Find the position vector of middle point of AB.



Solution: $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$

$\overrightarrow{OA} + \overrightarrow{OB} = 2\overrightarrow{OM}$

$\vec{a} + \vec{b} = 2\overrightarrow{OM}$

$\overrightarrow{OM} = \frac{1}{2}(\vec{a} + \vec{b})$

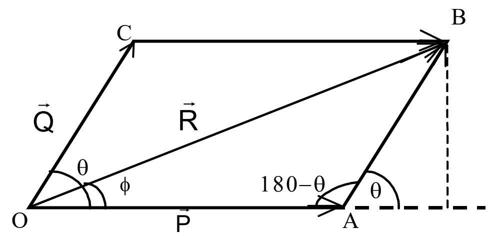
Analytical method (Parallelogram law of addition of vectors):

If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant will be represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.

We get, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\phi = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

where ϕ is the angle that the resultant makes with $\vec{P}$



(i) $\theta = 0^o$

$\vec{P}$ and $\vec{Q}$ are in the same direction i.e. they are parallel $\cos 0^o = 1$

$\therefore |\vec{R}| = |\vec{P}| + |\vec{Q}|$ & $\phi = 0^o$

(ii) $\theta = 180^o$, $\vec{P}$ and $\vec{Q}$ are in opposite direction i.e. they are antiparallel $\cos 180^o = -1$

$\therefore |\vec{R}| = |\vec{P}| \sim |\vec{Q}|$ and $\vec{R}$ is in the direction of the larger vector.

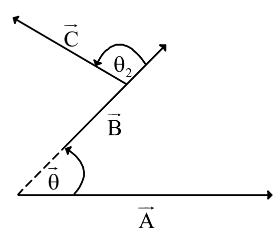
(iii) $\theta = 90^o$, $\cos 90^0 = 0$

$\vec{P}$ and $\vec{Q}$ are perpendicular to each other

$\therefore |\vec{R}| = (|\vec{P}|^2 + |\vec{Q}|^2)^{1/2}$ & $\phi = \tan^{-1}(Q/P)$

Polygon law of addition of vectors:

If a number of vectors are represented by the sides of an open polygon taken in the same order, then their resultant is represented by the closing side of the polygon taken in opposite order. Here in the fig $\vec{R}$(closing side of polygon) represents the resultant of vectors $\vec{A}, \vec{B}$ and $\vec{C}$



$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

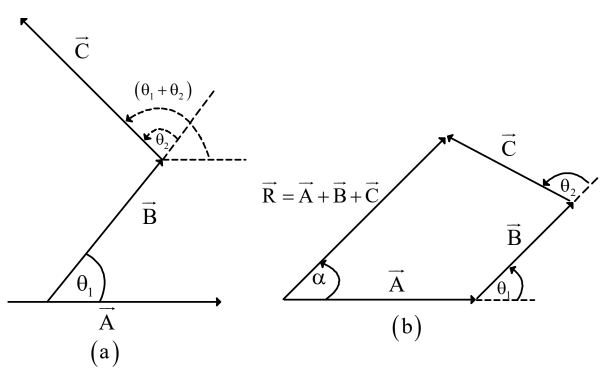
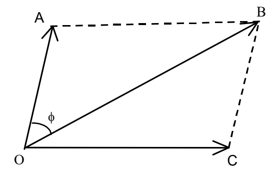


Illustration 2. Two forces of 60N and 80N acting at an angle of 90^0 with each other pull an object. What single pull would replace the given forces?

Solution: Two forces are drawn from a common origin O, making an angle of 90^0. OA and OC represent the forces 60N and 80N respectively. The diagonal OB represents the resultant R.



$\therefore R^2 = 60^2 + 80^2 + 2.60.80 \cos 90^0$

$= 3600 + 6400 + 0 = 10000$

$\therefore R = 100N$

Angle ϕ is given, $\tan\phi = \frac{60 \sin 90°}{80 + 60 \cos 90°}$

Which gives, $\phi = 37^0$

Illustration 3. The resultant of two vectors 3P and 2P is R. If the first vector is doubled, the resultant vector also becomes double. Find the angle between the vectors.

Solution: Let θ be the angle between vectors, then

$R^2 = (3P)^2 + (2P)^2 + 2(3P)(2P)\cos\theta = 13P^2 + 12P^2\cos\theta$ ...(1)

Also $(2R)^2 = (6P)^2 + (2P)^2 + 2(6P)(2P)\cos\theta$

$R^2 = 10P^2 + 6P^2\cos\theta$ ...(2)

From (1) and (2)

$\cos\theta = -\frac{1}{2}$

$\therefore \theta = 120°$

Illustration 4. The sum of magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. The resultant is at 90° with the force of smaller magnitude. What are the magnitudes of individual forces?

Solution: Let $\vec{A}$ and $\vec{B}$ be two forces

$|\vec{A}| + |\vec{B}| = 18 \Rightarrow |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}| = 234$

$R = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta} = 12$

$\Rightarrow \quad |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta = 144$

Hence $2|\vec{A}||\vec{B}|(1 - \cos\theta) = 180$ ...(1)

Now $\tan 90° = \frac{|\vec{B}|\sin\theta}{|\vec{A}| + |\vec{B}| + \cos\theta}$

$\Rightarrow \quad |\vec{A}| + |\vec{B}|\cos\theta = 0$

or $\quad (18 - |\vec{B}|) + |\vec{B}|\cos\theta = 0$

$|\vec{B}|(1 - \cos\theta) = 18$ ...(2)

(1) and (2) give $|\vec{A}| = 5$, $|\vec{B}| = 13$

IN–CHAPTER EXERCISE

(i) Is it possible that the resultant of two equal forces is equal to one of the forces?

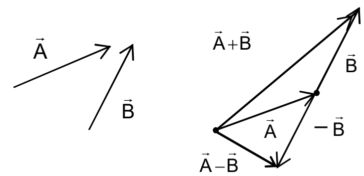
(ii) If a vector has zero magnitude is it meaningful to call it a vector?

(iii) Can three vectors, not in one plane, give a zero resultant? Can four vectors do?

Subtraction of Vectors:

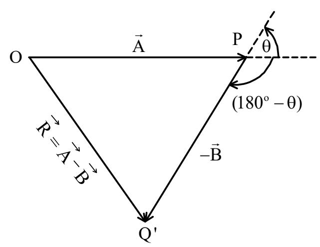
When a vector B is reversed in direction, then the reversed vector is written as $-\vec{B}$ then

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Subtraction of Vectors by Triangle Law :

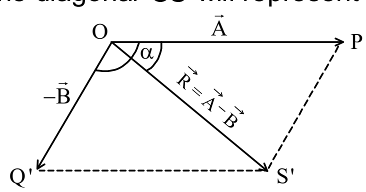
Finding $\vec{A}-\vec{B}$: First draw vector $\vec{A}(\overrightarrow{OP})$ in the given direction. Then draw vector $\vec{B}(\overrightarrow{PQ})$ starting from head of the vector $\vec{A}$. Then close the triangle, $\vec{R}(\overrightarrow{OQ})$ be their resultant



Subtraction of Vectors by Parallelogram Law :

Finding $\vec{A}-\vec{B}$: Draw vectors $\vec{A}(\overrightarrow{OP})$ and $-\vec{B}(\overrightarrow{OQ})$ starting from a common point O.

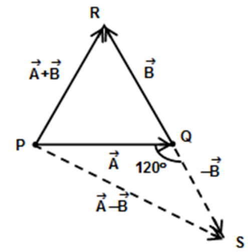
Then complete the parallelogram. The diagonal $\overrightarrow{OS}$ will represent their resultant.



Remark :	$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ The subtraction of $\vec{B}$ from $\vec{A}$ means addition of $\vec{B}$ to $\vec{A}$ with an angle $(180^0 - \theta)$ $\therefore R = \sqrt{A^2 + B^2 + 2AB\cos(180^0 - \theta)} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$ and $\tan\alpha = \dfrac{B\sin(180^0-\theta)}{A + B\cos(180^0 - \theta)} = \dfrac{B\sin\theta}{A - B\cos\theta}$

Illustration 5. If the sum of two unit vectors $\vec{A}$ and $\vec{B}$ is also equal to a unit vector, find the magnitude of the vector $\vec{A} - \vec{B}$.

Solution: Given that $|\vec{A}| = |\vec{B}| = |\vec{A}+\vec{B}| = 1$



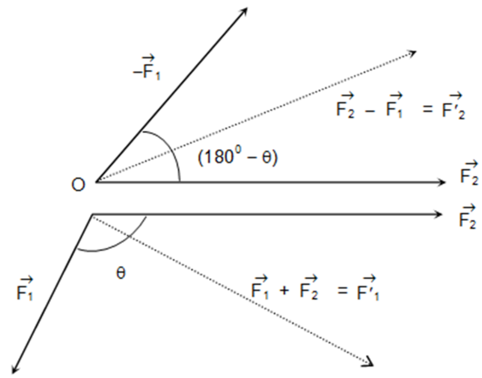
Hence the angle between $\vec{A}$ and $\vec{B}$ is 120°

Now $|\vec{PS}|^2 = |\vec{A}|^2 + |-\vec{B}|^2 + 2|\vec{A}||-\vec{B}|\cos 120°$

$= 1 + 1 + 2\times1\times(-1)\left(-\frac{1}{2}\right) = 3 \Rightarrow PS = \sqrt{3}$.

Illustration 6. Two forces of unequal magnitudes simultaneously act on a particle making an angle $\theta = 150^0$ with each other. If one of them is reversed, the acceleration of the particle is doubled. Calculate the ratio of the magnitude of the forces.

Solution: Let the two forces be forces $\vec{F}_1$ and $\vec{F}_2$. The resultant of these forces. is $\vec{F}_1' = \vec{F}_1 + \vec{F}_2$. Then, $|\vec{F}_1'| = |\vec{F}_1 + \vec{F}_2|$



Using parallelogram law of vector addition, we get,

$\vec{F}_1' = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

If the direction of $\vec{F}_1$ is reversed, new resultant force $\vec{F}_2' = (-\vec{F}_1) + \vec{F}_2$. Using parallelogram law of vectors, the magnitude of new resultant is

$\vec{F}_2' = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\theta}$

Since force is directly proportional to acceleration, $\frac{|\vec{F}'_1|}{|\vec{F}'_2|} = \frac{a_2}{a_1}$

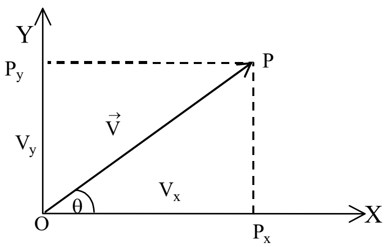
Substituting $|\vec{F}_1'|$, $|\vec{F}_2'|$ and $a_2/a_1 = 2/1$, we have, $\frac{\sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\theta}}{\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}} = 2$

Substituting $\theta = 150°$ and solving the quadratic equation, we have $\frac{F_1}{F_2} = \frac{5 \pm \sqrt{13}}{2\sqrt{3}}$

Resolution of Vectors

If $\vec{P} + \vec{Q} = \vec{R}$, the resultant, then conversely $\vec{R} = \vec{P} + \vec{Q}$ i.e. the vector $\vec{R}$ can be split up so that the vector sum of the split parts equals the original vector $\vec{R}$. If the split parts are mutually perpendicular then they are known as components of $\vec{R}$ and this process is known as resolution. The orthogonal component of any vector along another direction which is at an angular separation θ is the product of the magnitude of the vector and cosine of the angle between them (θ). Therefore the component of $\vec{A}$ is $A\cos\theta$.

Note: In physics, resolution gives unique and mutually independent components only if the resolved components are mutually perpendicular to each other. Such a resolution is known as rectangular or orthogonal resolution and the components are called rectangular or orthogonal components.



O – the origin, OP – the given vector $\vec{V}$

PP_x – perpendicular to X axis.

PP_y – Perpendicular to Y axis.

$$\overrightarrow{OP_x} + \overrightarrow{P_xP} = \overrightarrow{OP} = \vec{V}$$

$$\vec{V} = \vec{V_x} + \vec{V_y}$$

$V_x = V\cos\theta$ & $V_y = V\sin\theta$

Unit vector along the direction of $\vec{A}$ is $\hat{A} = \vec{A}/A$, Where A is magnitude of $\vec{A}$. $\hat{i}, \hat{j}, \hat{k}$, are the unit vectors along positive direction of X, Y and Z axis respectively, then the rectangular resolution of vector $\vec{A}$ can be represented.

$\vec{A} = A_x\hat{i} + A_Y\hat{j} + A_Z\hat{k}$ where A_X, A_Y, A_Z are the magnitudes of X, Y and Z components of $\vec{A}$. The magnitude of vector $\vec{A}$ is given by $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

Illustration 8: A force of 30 N is acting at an angle of 60^0 with the y–axis. Determine the components of the forces along x and y–axes.

Solution : $F_x = F\sin 60^0$

$$= \frac{30 \times \sqrt{3}}{2} = 15\sqrt{3}\,N$$

$$F_y = F\cos 60^0 = 30 \times \frac{1}{2} = 15\text{ N}$$

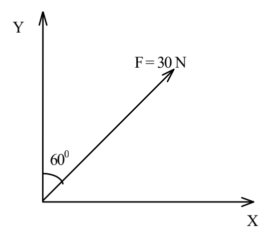
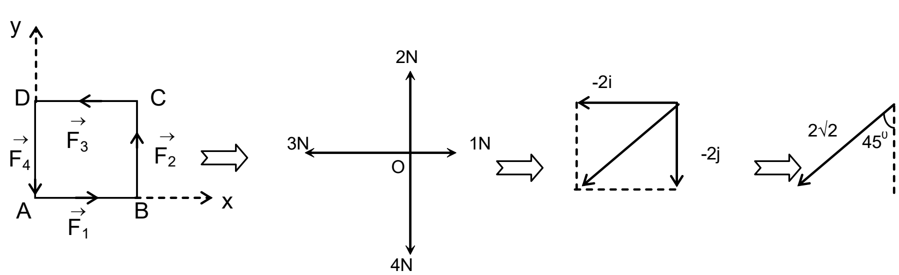


Illustration 9. *Four forces act along the sides of a smooth square frame ABCD in the order A → B, B → C, C → D and D → A. If the magnitude of the forces are F_1, F_2, F_3 and F_4 respectively, find the resultant force acting on the frame. Assume F_1 = 1 N, F_2 = 2N, F_3 = 3N and F_4 = 4N.*

Solution: Let us consider x – y coordinate system. The resultant of all the forces is $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$;

After bringing the tails of all the vectors to a point O and substituting $\vec{F}_1 = 1\hat{i}\,N$, $\vec{F}_2 = 2\hat{j}$ N $\vec{F}_3 - 3\hat{i}\,N$ and $\vec{F}_4 = -4\hat{j}\,N$

We have, $\vec{F} = -2(\hat{i} + \hat{j})N$



Hence $|\vec{F}| = 2\sqrt{2}$ is directed in 3[rd] quadrant as shown in the figure.

Illustration 13. Find the net displacement of a particle from its starting point if it undergoes three successive displacements given $\vec{S}_1$ = 20 m, 45^0 West of North, $\vec{S}_2$ = 15 m, 30^0 North of East; $\vec{S}_3$ =20m, due South.

Solution: Let us set our axial system such that x–axis is along West–East and y–axis along South–North.

$\Rightarrow$ $\vec{S_1} = 20\cos 45^0(-\hat{i}) + 20\sin 45^0(\hat{j})$

and $\vec{S_2} = 15\cos 30^0(\hat{i}) + 15\sin 30^0(\hat{j})$

$\vec{S_3} = 0(\hat{i}) + 20(-\hat{j})$

$$\vec{S} = \vec{S_1} + \vec{S_2} + \vec{S_3}$$

$$= \left(-\frac{20}{\sqrt{2}} + \frac{15\sqrt{3}}{2} + 0\right)\hat{i} + \left(\frac{20}{\sqrt{2}} + \frac{15}{2} - 20\right)\hat{j}$$

$$= -1.15\hat{i} + 1.64\hat{j} = S_x\hat{i} + S_y\hat{j}$$

$$|\vec{S}| = \sqrt{S_x^2 + S_Y^2} = \sqrt{(-1.15)^2 + (1.64)^2} = 2\text{m}$$

Direction $\theta = \tan^{-1}\frac{1.15}{1.64} = 35^0$ West of North

IN–CHAPTER EXERCISE

(i) Calculate the angle between a two dyne and a three dyne force so that their sum is four dyne.

(ii) Resultant of two forces which have equal magnitudes and which act at right angles to each other is 1414 dyne. Calculate the magnitude of each forces.

(iii) Find the direction cosines of $5\hat{i} + 2\hat{j} + 4\hat{k}$

(iv) Given: $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = 3\hat{j} + 4\hat{k}$, calculate the magnitude of the resultant.

(v) One of the rectangular components of an acceleration of 8 m/s^2 is 4 m/s^2, calculate the other component.

(vi) Find the unit vector in the direction of $3\hat{i} + 4\hat{j} - \hat{k}$

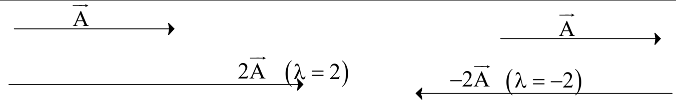
MULTIPLICATION IN VECTORS

1. Scalar multiple of a vector :

When a vector is multiplied by a scalar λ, we get a new vector which is γ times the vector $\vec{A}$ i.e. $\lambda\vec{A}$. The direction of resulting vector is that of $\vec{A}$.

If γ has negative value, then we get a vector whose direction is opposite of $\vec{A}$. The unit of resulting vector is the multiplied units of λ and $\vec{A}$. For example, when mass is multiplied with velocity,

we get momentum. The unit of momentum is obtained by multiplying units of mass and velocity.



Similarly, we can have vector $\vec{A}$ divided by a scalar λ. The resulting vector becomes $\frac{\vec{A}}{\lambda}$.

The magnitude of the new vector becomes $\frac{1}{\lambda}$ that of $\vec{A}$ and direction is same as that of $\vec{A}$.

2. Multiplication Of A Vector By Another Vector

(a) Scalar Product or Dot Product: $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$ where θ is the angle between the two vectors, when placed tail to tail.

- $\vec{A}.\vec{B}$ will be positive when angle between the vectors are acute.
- $\vec{A}.\vec{B}$ will be negative when angle between the vectors are obtuse.
- $\vec{A}.\vec{B}$ will be zero when angle between the vectors are 90^0.

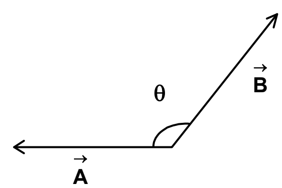
Scalar Product or Dot Product In Unit Vectors Notation:

Let there be two vectors given by

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}.\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right).\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$

$= A_xB_x + A_yB_y + A_zB_z$



(in the same line)

Some properties of Dot product

- It is commutative, i.e. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$
- It is distributive over addition and subtraction i.e. $\vec{a}.(\vec{b} \pm \vec{c}) = \vec{a}.\vec{b} \pm \vec{a}.\vec{c}$
- If $\vec{a}$ and $\vec{b}$ are perpendicular $\vec{a}.\vec{b} = 0$
- If $\vec{a}$ and $\vec{b}$ are parallel then $\vec{a}.\vec{b} = ab$
- Square of a vector is defined as $\vec{a}.\vec{a} = a^2$ (scalar) and not $\vec{a} \times \vec{a}$

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = (1)(1)\cos 0° = 1$

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = (1)(1)\cos 90° = 0$

- **Angle between two vectors:**

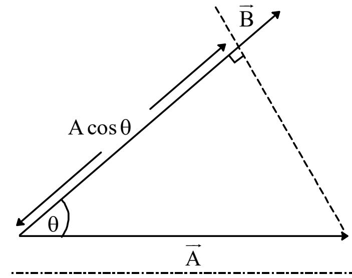
As we know $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

$$\Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \frac{A_xB_x + A_yB_y + A_zB_z}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)\left(\sqrt{B_x^2 + B_Y^2 + B_Z^2}\right)}$$

➢ **Component or projection of one vector along other vector:**

(a) Component of vector $\vec{A}$ along vector $\vec{B}$ is given by

$$A\cos\theta\hat{B} = \left(\frac{AB\cos\theta}{B}\right)\hat{B} = \left(\frac{\vec{A}.\vec{B}}{B}\right)\hat{B}$$



(b) Component of vector $\vec{B}$ along vector $\vec{A}$; is given by $B\cos\theta\hat{A} = \left(\frac{AB\cos\theta}{A}\right)\hat{A} = \left(\frac{\vec{A}.\vec{B}}{A}\right)\hat{A}$

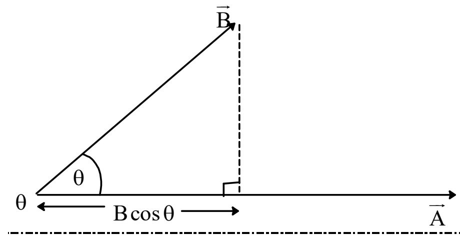


Illustration 13: Two constant forces $\vec{F}_1 = (2\hat{i} + 3\hat{j} + 3\hat{k})N$ and $\vec{F}_2 = (5\hat{i} - 6\hat{j} - 2\hat{k})N$ act together on a particle during its displacement from the position $(20\hat{i} + 15\hat{j})$ m to $8\hat{k}$ m. calculate the work done. If work done by a force $\vec{F}$, for a displacement $\Delta\vec{r}$ is given by $W = \vec{F}\cdot\Delta\vec{r}$

Solution: Total work done W = $\vec{F}_1\cdot\Delta\vec{r} + \vec{F}_2\cdot\Delta\vec{r}$ = $(\vec{F}_1 + \vec{F}_2)\cdot\Delta\vec{r}$

$= [(2\hat{i} + 3\hat{j} + 3\hat{k}) + (5\hat{i} - 6\hat{j} - 2\hat{k})].[8\hat{k} - (20\hat{i} + 15\hat{j})]$

$= (7\hat{i} - 3\hat{j} + \hat{k})\cdot(-20\hat{i} - 15\hat{j} + 8\hat{k})$

$= (7)(-20) + (-3)(-15) + (1)(8) = -87J$

Illustration 14. Show that if $|\vec{A}| = |\vec{B}|$, then the sum and difference of vectors $\vec{A}$ and $\vec{B}$ are at right angles.

Solution: $(\vec{A} + \vec{B})\cdot(\vec{A} - \vec{B}) = |\vec{A}|^2 - \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} - |\vec{B}|^2$

$= 0$ $(\because |\vec{A}| = |\vec{B}|)$

Illustration 15. If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} + 2\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

Solution: We have $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab}$ Where θ is the angle between $\vec{a}$ and $\vec{b}$

Now $\vec{a}$. $\vec{b}$ = $a_x b_x + a_y b_y + a_z b_z$

$= 2 \times 4 + 3 \times 3 + 4 \times 2 = 25$

Also, $a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$

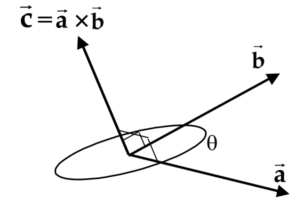
$b = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$

Thus $\cos\theta = \frac{25}{29}$ or $\theta = \cos^{-1}\left(\frac{25}{29}\right)$

(b) Vector Product or Cross product

The **vector (or cross)** product of two vectors $\vec{a}$ and $\vec{b}$ is written as $\vec{a}\times\vec{b}$ and is a vector $\vec{c}$ whose magnitude c is given by ab sinθ

in which θ is the smaller of the angles between the direction of $\vec{a}$ and $\vec{b}$. The direction of $\vec{c}$ is perpendicular to the plane defined by $\vec{a}$ and $\vec{b}$ and is given by right hand rule.



Cross product of vectors $\vec{a}$ and $\vec{b}$

$\vec{c} = \vec{a}\times\vec{b}$ (read as $\vec{a}$ cross $\vec{b}$)

$|\vec{c}| = |\vec{a}\times\vec{b}|$ = ab sin θ

And $\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|$ sin θ $\hat{n}$

where $\hat{n}$ is the unit vector

The vector $\vec{c}$ is directed perpendicular to the plane formed by $\vec{a}$ and $\vec{b}$. The direction of vector $\vec{c}$ may be obtained by using the **Right Hand Thumb Rule**. Stretch all the fingers and thumb of your right hand such that they are perpendicular to each other. Now align your hand such that its plane is perpendicular to the plane formed by vectors $\vec{a}$ and $\vec{b}$. Align the stretched fingers along the direction of the vector written first in order i.e., $\vec{a}$ in this case. Curl the fingers of your hand towards the second vector (i.e. vector $\vec{b}$) through the smaller angle. Then, the direction of the thumb gives the direction of the cross product.

Properties of Cross Product

- It is not commutative $\vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$. In fact $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$.
- It is distributive over addition and subtraction

  $\vec{a}\times(\vec{b}\pm\vec{c}) = \vec{a}\times\vec{b}\pm\vec{a}\times\vec{c}$

- If $\vec{a} \parallel \vec{b}$, then $\vec{a}\times\vec{b} = 0$

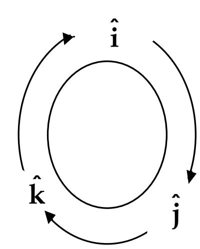
  and if $\vec{a}\perp\vec{b}$, $|\vec{a}\times\vec{b}|$ = ab

- If $\hat{i}$, $\hat{j}$, $\hat{k}$ be the unit vectors along in the positive directions of x, y, and z axes then

  $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

  $\hat{i}\times\hat{j} = \hat{k}$; $\hat{j}\times\hat{k} = \hat{i}$; $\hat{k}\times\hat{i} = \hat{j}$ (maintain cyclic order)

  $\hat{i}\times\hat{j} = -\hat{j}\times\hat{i}$

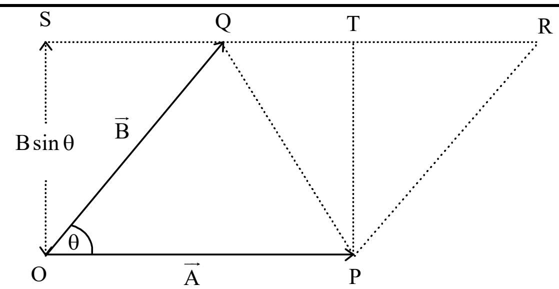


- The cross product may also be expressed by the determinant

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

where $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$



Suppose two vectors $\vec{A}$ and $\vec{B}$ are represented by the sides OP and OQ of a parallelogram, as shown in figure. The magnitude of vector product $\vec{A}\times\vec{B}$ is

$|\vec{A}\times\vec{B}|$ = AB $\sin\theta$

= A $(B\sin\theta)$

= area of rectangle OPTS

= area of parallelogram OPRQ

Thus the magnitude of the vector product of two vectors is equal to the area of the parallelogram (OPRQ) formed by the two vectors as its adjacent sides.

= area of triangle OPQ.

Area of triangle OPQ = $\frac{1}{2}$ (area of parallelogram OPRQ).

or $= \frac{1}{2}|\vec{A}\times\vec{B}|$.

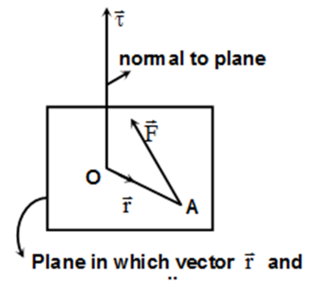
➢ **Moment Of A Force About A Point:**

Consider a force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ acting at point A whose position vector is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then moment of force (i.e, torque) about point O is given by

$\vec{\tau}_0 = \vec{r}\times\vec{F}$

$= (yF_z - zF_y)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}$

Physically $\vec{\tau}$ represents the tendency of the force $\vec{F}$ to rotate the body (on which it acts) about an axis which passes through O and perpendicular to the plane containing the force $\vec{F}$ and position vector $\vec{r}$.



➢ Moment Of Force About A Line:

Consider a force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ acting at a point a whose position vector is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Then moment of $\vec{F}$ about point O is given by

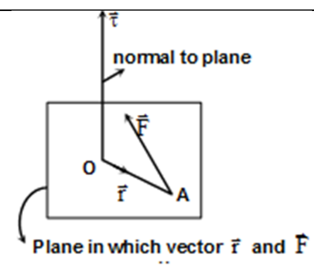
$\vec{\tau}_0 = \vec{r}\times\vec{F}$

Then the component of $\vec{\tau}$ in the direction of vector $\overline{OL}$ (i.e., $\overline{OB}$) is called the moment of force about a line and is given by $\vec{\tau}_L = \overline{OB}$

$= \left(\frac{\vec{\tau}_0.\overline{OL}}{|\overline{OL}|}\right)\widehat{OL} = (\vec{\tau}_0\cdot\widehat{OL})\widehat{OL}$

where $\widehat{OL}$ is unit vector along $\overline{OL}$. In particular if line OL coincides with the x–axis, then $\vec{\tau}_L = (yF_z - zF_y)\hat{i}$

but this is the component of $\overrightarrow{\tau_o}$ along x–axis



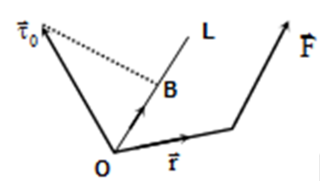


Illustration 16. If $\vec{a} = 2\hat{i} + 3\hat{i}$; $\vec{b} = 4\hat{i} + 2\hat{j}$ then, Find $\vec{d} = \vec{a} \times \vec{b}$

Solution $\vec{d} = (2\hat{i} + 3\hat{j}) \times (4\hat{i} + 2\hat{j})$

$\vec{d} = (2)(4)(\hat{i} \times \hat{i}) + (2)(2)(\hat{i} \times \hat{j}) + (3)(4)(\hat{j} \times \hat{i}) + (3)(2)(\hat{j} \times \hat{j})$

Since $\hat{i} \times \hat{i} = 0$; $\hat{j} \times \hat{j} = 0$; $\hat{i} \times \hat{j} = \hat{k}$; $\hat{j} \times \hat{i} = -\hat{k}$

$\vec{d} = 4\hat{k} - 12\hat{k} = -8\hat{k}$

Illustration 17. Find the cross product of two vectors $\vec{A}$ and $\vec{B}$ which are(i) parallel and equal (ii) anti-parallel and negative

Solution: If $\vec{A}$ is parallel (or equal) to $\vec{B}$, $\theta = 0^0$.

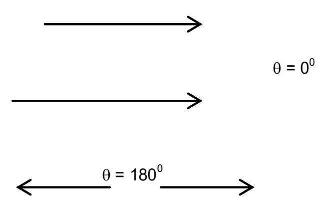
Hence $|\vec{A} \times \vec{B}| = AB \sin 0^0 = 0$

If $\vec{A}$ is ant–l parallel (or negative) to $\vec{B}$, $\theta = 180^0$. Hence

$|\vec{A} \times \vec{B}| = AB \sin 180^0 = 0$

If $\vec{A}$ is perpendicular to $\vec{B}$, $\theta = 90^0$

Then $|\vec{C}| = |\vec{A} \times \vec{B}| = AB \sin 90^0 = AB$ and C($= \vec{A} \times \vec{B}$) is directed outward obeying righth and thumb rule. $\vec{A} \times \vec{B}$ must be perpendicular to the plane containing $\vec{A}$ and $\vec{B}$



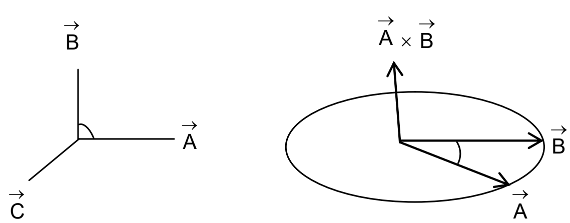


Illustration 18. If the magnitudes of the dot product and cross product of two vectors are equal, find the angle between the two vectors.

Solution: $|\vec{A} \times \vec{B}| = |\vec{A} \cdot \vec{B}|$

A.B $\sin\theta$ = A.B $\cos\theta \Rightarrow \tan\theta = 1$

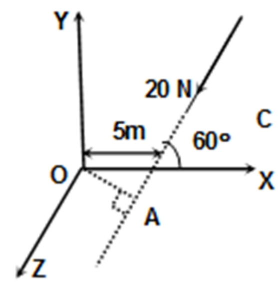
$\Rightarrow \theta = 45^0$.

Illustration 19. A force vector $10\hat{i} + 25\hat{j} + 35\hat{k}$ passes through a point (2, 5, 7). Prove that force passes through the origin.

Solution: If the force passes through the origin, its moment about the origin will be zero.

Moment of force (Torque) = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & 7 \\ 10 & 25 & 35 \end{vmatrix} = 0$

Illustration 20. Determine the torque of the20 N force (in XY–plane) about point O as shown in figure.



Solution: $\vec{\tau} = (-\hat{k})(\text{force})(\text{force arm})$

$= (-\hat{k})(20)(OA)$

$= (-\hat{k})\ (20)\ (5 \cos 30°)$

$= -86.6\ \hat{k}\,Nm$

Illustration 24. Two vectors in which one has magnitude twice that of the other, act on a particle. Find the angle between them, if their resultant is perpendicular to the first vector.

Solution: Let the vector $\vec{B}$ has magnitude twice that of $\vec{A}$; $|\vec{B}| = 2|\vec{A}|$

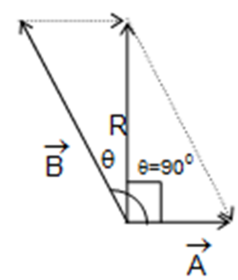
Since $\vec{R} \perp \vec{A}$, $\phi = 90^0$,

substituting $\phi = 90^0$ in $\phi = \tan^{-1}\dfrac{|\vec{B}|\sin\theta}{|\vec{A}|+|\vec{B}|\cos\theta}$

we have $|\vec{A}| + |\vec{B}|\cos\theta = 0$. This gives

$\theta = \cos^{-1}\left(-\dfrac{|\vec{A}|}{|\vec{B}|}\right)$; substituting $|\vec{A}| = \dfrac{|\vec{B}|}{2}$

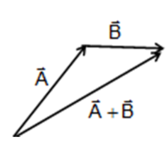
we have $\theta = 120^0$.



KEY CONCEPT

1. Scalar quantities are quantities with magnitudes only and combine with the usual rules of arithmetic e.g. speed, mass and temperature.
2. Vector quantities have magnitude as well as direction and combine according to the rules of vector addition. e.g. velocity and acceleration.
3. $\vec{B} = \lambda\vec{A}$

   Where λ is a real number. Magnitude of B is λ time the magnitude of A and direction is same as that of A. (If λ is positive).
4. Graphically, two vectors $\vec{A}$ and $\vec{B}$ are added by placing the tail of $\vec{B}$ at the head of $\vec{A}$. The vector sum $\vec{A}+\vec{B}$ then extends from the tail of $\vec{A}$ to the head of $\vec{B}$



5. Vector addition is

   $\vec{A}+\vec{B}=\vec{B}+\vec{A}$ (Commutative)

   $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$ (Associative law)
6. A vector with zero magnitude is called null vector and

   $\vec{A}+\vec{0}=\vec{A}$

   $\lambda\vec{0}=\vec{0}$

7. Subtraction of vectors
$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

8. Unit vectors describe directions in space. A unit vector has a magnitude of one. The unit vectors $\hat{i},\hat{j},\hat{k}$ are vectors of unit magnitude and points in the direction of the x, y and z axes, respectively, in a right – handed coordinate system.

10. Vector $\vec{A}$ can be expressed as $\vec{A}=A_x\hat{i}+A_y\hat{j}$ having magnitude $=\sqrt{A_x^2+A_y^2}$ and angle θ with the x – axis = $\tan^{-1}\frac{A_y}{A_x}$.

11. Scalar product of two vectors, $C=\vec{A}\cdot\vec{B}=AB\cos\phi$, where ϕ is the angle between two vectors and scalar product of two vectors is a scalar quantity. Scalar products obey the commutative and the distributive laws.

12. Cross – product of two vectors $\vec{A}$ and $\vec{B}$ is a vector quantity. $\vec{C}=\vec{A}\times\vec{B}=AB\sin\phi\hat{n}$ and its direction is given by right hand rule, $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ (non – commutative)

ANSWER TO INCHAPTER EXERCISES

1. (i) Yes (take angle between vectors as 120°)
   (ii) Yes
   (iii) No, yes

2. (i) $\cos^{-1}\left(\frac{1}{4}\right)$
   (ii) 1000 dyne
   (iii) $\frac{5}{3\sqrt{5}},\frac{2}{3\sqrt{5}},\frac{4}{3\sqrt{5}}$
   (iv) $\sqrt{74}$
   (v) $\pm4\sqrt{3}$
   (vi) $\frac{3\hat{i}+4\hat{j}-\hat{k}}{\sqrt{26}}$

EXERCISE

LEVEL –I

VECTORS AND PROPERTIES OF VECTORS

1. Identify the vector quantity
(a) time (b) work
(c) heat (d) angular momentum

2. A vector is not changed if
(a) it is rotated through an arbitrary angle (b) it is multiplied by an arbitrary scalar
(c) it is cross multiplied by a unit vector (d) it is slid parallel to itself

3. The magnitude of a given vector with end points (4, –4, 0) and (–2, –2, 0) must be
(a) 6 (b) $5\sqrt{6}$ (c) 4 (d) $2\sqrt{10}$

4. Find the value of c if $\vec{A} = 0.4\hat{i} + 0.3\hat{j} + c\hat{k}$ is a unit vector.
(a) 0.5 (b) √0.75 (c) 1 (d) none of these.

5. Let $\vec{A} = 5\hat{i} - 4\hat{j}$ and $\vec{B} = -7.5\hat{i} + 6\hat{j}$. Then
(a) $\frac{\vec{B}}{\vec{A}} = -1.5$ (b) $\frac{\vec{B}}{\vec{A}} = 1.5$
(c) $\frac{\vec{B}}{\vec{A}} = -\sqrt{\frac{92.25}{39}}$ (d) $\frac{\vec{B}}{\vec{A}}$ is undefined

6. If a particle moves from point P (2,3,5) to point Q (3,4,5). Its displacement vector be
(a) $\hat{i} + \hat{j} + 10\hat{k}$ (b) $\hat{i} + \hat{j} + 5\hat{k}$ (c) $\hat{i} + \hat{j}$ (d) $2\hat{i} + 4\hat{j} + 6\hat{k}$

7. If $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ the direction cosines of the vector $\vec{A}$ are
(a) $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}$ and $\frac{-5}{\sqrt{45}}$ (b) $\frac{1}{\sqrt{45}}, \frac{2}{\sqrt{45}}$ and $\frac{3}{\sqrt{45}}$
(c) $\frac{4}{\sqrt{45}}$, 0 and $\frac{4}{\sqrt{45}}$ (d) $\frac{3}{\sqrt{45}}, \frac{2}{\sqrt{45}}$ and $\frac{5}{\sqrt{45}}$

8. If two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $-4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are parallel to each other then value of λ be
(a) 0 (b) –2 (c) 2 (d) –1

9. The position vectors of points A, B, C and D are $A = 3\hat{i} + 4\hat{j} + 5\hat{k}$, $B = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $C = 7\hat{i} + 9\hat{j} + 3\hat{k}$ and $D = 4\hat{i} + 6\hat{j}$ then the displacement vectors AB and CD are
(a) Perpendicular (b) Parallel
(c) Antiparallel (d) Inclined at an angle of 60°

10. A force vector applied on a mass is represented as $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ and accelerates with $1\,m/s^2$. What will be the mass of the body in kg.
(a) $10\sqrt{2}$ (b) 20 (c) $2\sqrt{10}$ (d) 10

RESOLUTION AND ADDITION OF VECTORS

11. A particle is acted upon by two forces of 3 N and 4 N simultaneously. Which of the following is most correct?
(a) The resultant of these forces is 7 N.
(b) The resultant of these forces is 1 N.
(c) The resultant of these forces in 4 N.
(d) The resultant of these forces lies between 1 N and 7 N

12. If $\vec{A} = \vec{B} + \vec{C}$, and the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units respectively, the angle between $\vec{A}$ and $\vec{C}$ is
(a) $\cos^{-1}(3/5)$ (b) $\cos^{-1}(4/5)$ (c) $\pi/2$ (d) $\sin^{-1}(3/4)$

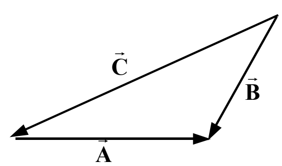
13. Let the angle between two nonzero vectors $\vec{A}$ and $\vec{B}$ be 120° and their resultant be $\vec{C}$,
(a) $|\vec{C}|$ must be equal to $|\vec{A} - \vec{B}|$ (b) $|\vec{C}|$ must be less than $|\vec{A} - \vec{B}|$
(c) $|\vec{C}|$ must be greater than $|\vec{A} - \vec{B}|$ (d) $|\vec{C}|$ may be equal to $|\vec{A} - \vec{B}|$

14. At what angle two forces 2F and $\sqrt{2}$ F should act, so that the resultant force is $F\sqrt{10}$?
(a) 45° (b) 60° (c) 120° (d) 90°

15. If two vectors $\vec{a}$ and $\vec{b}$ in the same plane, are inclined at an angle θ, and then resultant plane $\vec{a} + \vec{b}$ subtends an angle α with the direction of $\vec{a}$ then $\tan\alpha$ is equal to
(a) $\frac{b}{a}$ (b) $\frac{a + b\cos\theta}{b\sin\theta}$ (c) $\frac{b\sin\theta}{a + b\cos\theta}$ (d) $\frac{-b\sin\theta}{a + b\cos\theta}$

16. If the vector $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ can be represented by the sides of a right angled triangle, then
(a) $\vec{A} \perp \vec{B}$ (b) $\vec{B} \perp \vec{C}$ (c) $\vec{C} \perp \vec{A}$ (d) $\vec{A} + \vec{B} = \vec{C}$

17. For the given figure,



(a) $\vec{A} + \vec{B} = \vec{C}$ (b) $\vec{B} + \vec{C} = \vec{A}$ (c) $\vec{C} + \vec{A} = \vec{B}$ (d) $\vec{A} + \vec{B} + \vec{C} = 0$

18. If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j} + \hat{k}$, then the unit vector along $(\vec{A} + \vec{B})$ is
(a) $\frac{3\hat{i} + 7\hat{j} + \hat{k}}{\sqrt{59}}$ (b) $\frac{2\hat{i} + 3\hat{j}}{\sqrt{59}}$ (c) $\frac{\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{18}}$ (d) $\frac{2\hat{i} + 3\hat{j}}{\sqrt{13}}$

19. The vector that must be added to the vector $\hat{i} - 3\hat{j} + 2\hat{k}$ and $3\hat{i} + 6\hat{j} - 7\hat{k}$ so that the resultant vector is a unit vector along the y–axis is
(a) $4\hat{i} + 2\hat{j} + 5\hat{k}$ (b) $-4\hat{i} - 2\hat{j} + 5\hat{k}$ (c) $3\hat{i} + 4\hat{j} + 5\hat{k}$ (d) Null vector

20. A hall has the dimensions $10m \times 12m \times 14m$. A fly starting at one corner ends up at a diametrically opposite corner. What is the magnitude of its displacement
(a) 17 m (b) 26 m (c) 36 m (d) 21 m

21. Five equal forces of 10 N each are applied at one point and all are lying in one plane. If the angles between them are equal, the resultant force will be

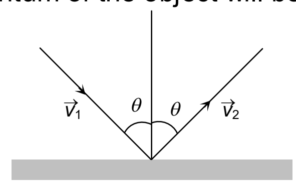
(a) Zero (b) 10 N (c) 20 N (d) $10\sqrt{2}$N

22. A boy walks uniformally along the sides of a rectangular park of size 400 m× 300 m, starting from one corner to the other corner diagonally opposite. Which of the following statement is incorrect
(a) He has travelled a distance of 700 m
(b) His displacement is 700 m
(c) His displacement is 500 m
(d) His velocity is not uniform throughout the walk

23. The unit vector parallel to the resultant of the vectors $\vec{A} = 4\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} - 8\hat{k}$ is
(a) $\frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (b) $\frac{1}{7}(3\hat{i} + 6\hat{j} + 2\hat{k})$
(c) $\frac{1}{49}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (d) $\frac{1}{49}(3\hat{i} - 6\hat{j} + 2\hat{k})$

24. $\vec{A} = 2\hat{i} + \hat{j}, B = 3\hat{j} - \hat{k}$ and $\vec{C} = 6\hat{i} - 2\hat{k}$.
Value of $\vec{A} - 2\vec{B} + 3\vec{C}$ would be
(a) $20\hat{i} + 5\hat{j} + 4\hat{k}$ (b) $20\hat{i} - 5\hat{j} - 4\hat{k}$ (c) $4\hat{i} + 5\hat{j} + 20\hat{k}$ (d) $5\hat{i} + 4\hat{j} + 10\hat{k}$

25. An object of m kg with speed of v m/s strikes a wall at an angle θ and rebounds at the same speed and same angle. The magnitude of the change in momentum of the object will be
(a) $2mv\cos\theta$
(b) $2mv\sin\theta$
(c) 0
(d) $2mv$



26. A particle is simultaneously acted by two forces equal to 4 N and 3 N. The net force on the particle is
(a) 7 N (b) 5 N
(c) 1 N (d) Between 1 N and 7 N

27. If the resultant of the two forces is zero, the two forces must be
(a) Different both in magnitude and direction
(b) Mutually perpendicular to one another
(c) Possess extremely small magnitude
(d) Point in opposite directions

28. If $|\vec{A} - \vec{B}| = |\vec{A}| = |\vec{B}|$, the angle between $\vec{A}$ and $\vec{B}$ is
(a) 60° (b) 0° (c) 120° (d) 90°

29. Magnitude of vector which comes on addition of two vectors, $6\hat{i} + 7\hat{j}$ and $3\hat{i} + 4\hat{j}$ is
(a) $\sqrt{136}$ (b) $\sqrt{13.2}$ (c) $\sqrt{202}$ (d) $\sqrt{160}$

30. The three vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form
(a) An equilateral triangle (b) Isosceles triangle
(c) A right angled triangle (d) No triangle

31. If vectors P, Q and R have magnitude 5, 12 and 13 units and $\vec{P}+\vec{Q}=\vec{R}$, the angle between Q and R is

(a) $\cos^{-1}\frac{5}{12}$ (b) $\cos^{-1}\frac{5}{13}$ (c) $\cos^{-1}\frac{12}{13}$ (d) $\cos^{-1}\frac{7}{13}$

32. Two forces, F_1 and F_2 are acting on a body. One force is double that of the other force and the resultant is equal to the greater force. Then the angle between the two forces is

(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(-1/2)$ (c) $\cos^{-1}(-1/4)$ (d) $\cos^{-1}(1/4)$

33. The vectors $5i+8j$ and $2i+7j$ are added. The magnitude of the sum of these vector is

(a) $\sqrt{274}$ (b) 38 (c) 238 (d) 560

34. Can the resultant of 2 vectors be zero

(a) Yes, when the 2 vectors are same in magnitude and direction
(b) No
(c) Yes, when the 2 vectors are same in magnitude but opposite in sense
(d) Yes, when the 2 vectors are same in magnitude making an angle of $\frac{2\pi}{3}$ with each other

DOT PRODUCT

35. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}$ = 0 and $\vec{A}.\vec{C}$ = 0. The vector $\vec{A}$ is parallel to

(a) $\vec{B}$ (b) $\vec{C}$ (c) $\vec{B}.\vec{C}$ (d) $\vec{B}\times\vec{C}$

36. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}=0$ and $\vec{A}.\vec{C}=0$. The value of $\vec{A}.(\vec{B}+\vec{C})$ is

(a) 0 (b) 1 (c) 2 (d) none

37. Given that $\vec{A}+\vec{B}=\vec{C}$ and that $\vec{C}$ is $\perp$ to $\vec{A}$. Further if $|\vec{A}|=|\vec{C}|$, then what is the angle between $\vec{A}$ and $\vec{B}$

(a) $\frac{\pi}{4}$ radian (b) $\frac{\pi}{2}$ radian (c) $\frac{3\pi}{4}$ radian (d) π radian

38. A body, acted upon by a force of 50 N is displaced through a distance 10 meter in a direction making an angle of 60° with the force. The work done by the force be

(a) 200 J (b) 100 J (c) 300 (d) 250 J

39. If for two vector $\vec{A}$ and $\vec{B}$, sum $(\vec{A}+\vec{B})$ is perpendicular to the difference $(\vec{A}-\vec{B})$. The ratio of their magnitude is

(a) 1 (b) 2 (c) 3 (d) None of these

40. Consider a vector $\vec{F}=4\hat{i}-3\hat{j}$. Another vector that is perpendicular to $\vec{F}$ is

(a) $4\hat{i}+3\hat{j}$ (b) $6\hat{i}$ (c) $7\hat{k}$ (d) $3\hat{i}-4\hat{j}$

41. A body, constrained to move in the Y–direction is subjected to a force given by $\vec{F}=(-2\hat{i}+15\hat{j}+6\hat{k})$N. What is the work done by this force in moving the body a distance 10 m along the Y–axis

(a) 20 J (b) 150 J (c) 160 J (d) 190 J

42. Let $\vec{A}=\hat{i}A\cos\theta+\hat{j}A\sin\theta$ be any vector. Another vector $\vec{B}$ which is normal to A is

(a) $\hat{i}B\cos\theta+jB\sin\theta$ (b) $\hat{i}B\sin\theta+jB\cos\theta$
(c) $\hat{i}B\sin\theta-jB\cos\theta$ (d) $\hat{i}B\cos\theta-jB\sin\theta$

43. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$, then projection of A on B will be :

(a) $\frac{3}{\sqrt{13}}$ (b) $\frac{3}{\sqrt{26}}$ (c) $\sqrt{\frac{3}{26}}$ (d) $\sqrt{\frac{3}{13}}$

44. $a_1\hat{i} + a_2\hat{j}$ is as unit vector perpendicular to $4\hat{i} - 3\hat{j}$ if :

(a) $a_1 = 0,6$, $a_2 = 0.8$ (b) $a_1 = 0.3$, $a_2 = 0.4$
(c) $a_1 = 0.8$, $a_2 = 0.6$ (d) $a_1 = 0.4$, $a_2 = 0.3$

CROSS PRODUCT

45. A vector $\vec{A}$ points vertically upwards and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is

(a) along west (b) along east
(c) zero (d) vertically downward

46. $\hat{i} \times (\hat{j} \times \hat{k})$ is

(a) $\hat{i} + \hat{j} + \hat{k}$ (b) $\hat{i} + \hat{j} + \hat{k}$ (c) zero vector (d) unit vector.

47. Which of the following statement is correct?

(a) A vector having zero length can have a unique direction.
(b) If $\vec{A} \times \vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
(c) If $\vec{A}.\vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
(d) The vector $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ are mutually perpendicular.

48. In none of the vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are zero and if $\vec{A} \times \vec{B} = 0$ and $\vec{B} \times \vec{C} = 0$, the value of $\vec{A} \times \vec{C}$ is

(a) unity (b) zero (c) B^2 (d) AC $\cos\theta$

49. If $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ then value of $|\vec{A} \times \vec{B}|$ will be

(a) $8\sqrt{2}$ (b) $8\sqrt{3}$ (c) $8\sqrt{5}$ (d) $5\sqrt{8}$

50. If $\vec{A} \times \vec{B} = \vec{C}$, then which of the following statements is wrong

(a) $\vec{C} \perp \vec{A}$ (b) $\vec{C} \perp \vec{B}$ (c) $\vec{C} \perp (\vec{A} + \vec{B})$ (d) $\vec{C} \perp (\vec{A} \times \vec{B})$

51. A vector $\vec{F}_1$ is along the positive X–axis. If its vector product with another vector $\vec{F}_2$ is zero then $\vec{F}_2$ could be

(a) $4\hat{j}$ (b) $-(\hat{i} + \hat{j})$ (c) $(\hat{j} + \hat{k})$ (d) $(-4\hat{i})$

52. If for two vectors $\vec{A}$ and $\vec{B}$, $\vec{A} \times \vec{B} = 0$, the vectors

(a) Are perpendicular to each other (b) Are parallel to each other
(c) Act at an angle of 60° (d) Act at an angle of 30°

53. What is the unit vector perpendicular to the following vectors $2\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 3\hat{j} + 2\hat{k}$

(a) $\frac{\hat{i} + 10\hat{j} - 18\hat{k}}{5\sqrt{17}}$ (b) $\frac{\hat{i} - 10\hat{j} + 18\hat{k}}{5\sqrt{17}}$ (c) $\frac{\hat{i} - 10\hat{j} - 18\hat{k}}{5\sqrt{17}}$ (d) $\frac{\hat{i} + 10\hat{j} + 18\hat{k}}{5\sqrt{17}}$

54. The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega}$ is the angular velocity and $\vec{r}$ is the radius vector. The angular velocity of a body is $\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$ and the radius vector $\vec{r} = 4\hat{j} - 3\hat{k}$, then $|\vec{v}|$ is

(a) $\sqrt{29}$ units (b) $\sqrt{31}$ units (c) $\sqrt{37}$ units (d) $\sqrt{41}$ units

LEVEL –II

VECTORS AND PROPERTIES OF VECTORS

1. A vector $\vec{A}$ has magnitude A and $\hat{A}$ is unit vector in the direction of $\vec{A}$, then which of the following are INCORRECT

(a) $\vec{A}\cdot\hat{A} = A$ (b) $\hat{A}=\dfrac{\vec{A}}{A}$ (c) $\vec{A}\cdot\vec{A}=A^2$ (d) $A = \dfrac{\vec{A}}{\hat{A}}$

2. A vector will change when

(a) Vector is translated parallel to itself (b) The frame of reference is translated
(c) the frame of reference is rotated (d) vector is rotated

3. A vector is represented by $3\hat{i}+\hat{j}+2\hat{k}$. Its length in XY plane is

(a) 2 (b) $\sqrt{14}$ (c) $\sqrt{10}$ (d) $\sqrt{5}$

4. What happens, when we multiply a vector by (–2)?

(a) Direction reverses and unit charges.
(b) Direction reverses and magnitude is doubled
(c) Direction remains unchanged and unit changes
(d) None of the above

5. If angle between $\vec{a}$ and $\vec{b}$ is $\pi/3$ then angle between $-3\vec{a}$ and $-2\vec{b}$ is :

(a) $\pi/3$ (b) $2\pi/3$ (c) $\pi/2$ (d) π

6. $\vec{A}=0.6\hat{i}+0.7\hat{j}+c\hat{k}$ is a unit vector then c = ?

(a) 0.15 (b) $\sqrt{0.15}$ (c) $\sqrt{0.75}$ (d) $\sqrt{0.85}$

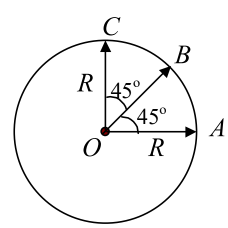
7. If two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $-4\hat{i}-6\hat{j}+\lambda\hat{k}$ are parallel to each other then value of λ will be :

(a) 1 (b) –1 (c) 2 (d) –2

RESOLUTION AND ADDITION OF VECTOR

8. The resultant of three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ as shown in the figure is

(a) $R\sqrt{2}$ (b) R
(c) $R(1+\sqrt{2})$ (d) $R(\sqrt{2}-1)$



9. If $\vec{a}+\vec{b}=\vec{c}$ and $c=\sqrt{a^2+b^2}$. The angle between $\vec{a}$ and $\vec{b}$ is

(a) 0° (b) 30° (c) 60° (d) 90°

10. It is given that $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ = 0 then which of the following statement(s) are false

(a) The magnitude of $\vec{a}+c$ equals the magnitude of $\vec{b}+\vec{d}$
(b) Each of $\vec{a},\vec{b},\vec{c},d$ must be a unit vector
(c) $\vec{b}+\vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear.
(d) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b},\vec{c}$ and $\vec{d}$.

11. Vector $\vec{A}$ makes equal angles with x, y and z axis. Value of its components (in terms of magnitude of $\vec{A}$) will be

(a) $\frac{A}{\sqrt{3}}$ (b) $\frac{A}{\sqrt{2}}$ (c) $\sqrt{3}A$ (d) $\frac{\sqrt{3}}{A}$

12. Unit vector parallel to the resultant of vectors $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 8\hat{i} + 8\hat{j}$ will be

(a) $\frac{24\hat{i}+5\hat{j}}{13}$ (b) $\frac{12\hat{i}+5\hat{j}}{13}$ (c) $\frac{6\hat{i}+5\hat{j}}{13}$ (d) None of these

13. The component of vector $A = 2\hat{i} + 3\hat{j}$ along the vector $\hat{i} + \hat{j}$ is

(a) $\frac{5}{\sqrt{2}}$ (b) $10\sqrt{2}$ (c) $5\sqrt{2}$ (d) 5

14. If the sum of two unit vectors is a unit vector, then magnitude of difference is

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $1/\sqrt{2}$ (d) $\sqrt{5}$

15. Maximum and minimum magnitudes of the resultant of two vectors of magnitudes P and Q are in the ratio $3:1$. Which of the following relations is true

(a) $P = 2Q$ (b) $P = Q$ (c) $PQ = 1$ (d) None of these

16. Two forces 3N and 2 N are at an angle θ such that the resultant is R. The first force is now increased to 6N and the resultant become 2R. The value of θ is

(a) 30° (b) 60° (c) 90° (d) 120°

17. The sum of the magnitudes of two forces acting at point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the, magnitudes of forces

(a) 12, 5 (b) 14, 4 (c) 5, 13 (d) 10, 8

18. If a vector $\vec{P}$ making angles α, β, and γ respectively with the X, Y and Z axes respectively. Then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(a) 0 (b) 1 (c) 2 (d) 3

19. The length of second's hand in watch is 1 cm. The change in velocity of its tip in 15 seconds is

(a) Zero (b) $\frac{\pi}{30\sqrt{2}}$ cm / sec (c) $\frac{\pi}{30}$ cm / sec (d) $\frac{\pi\sqrt{2}}{30}$ cm / sec

DOT PRODUCT

20. If $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$ and V_2 is finite, then

(a) V_1 is parallel to V_2 (b) $\vec{V}_1 = \vec{V}_2$

(c) V_1 and V_2 are mutually perpendicular (d) $|\vec{V}_1| = |\vec{V}_2|$

21. A particle moves in the x–y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $(\vec{P})$ at anytime t is $P_x = 2\cos t, p_y = 2\sin t$. The angle θ between $\vec{F}$ and $\vec{P}$ at a given time t. will be

(a) $\theta = 0°$ (b) $\theta = 30°$ (c) $\theta = 90°$ (d) $\theta = 180°$

22. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes a displacement $\vec{S} = -4\hat{i} + 2\hat{j} - 3\hat{k}$ in its own direction. If the work done is 6J, then the value of c will be

(a) 12 (b) 6 (c) 1 (d) 0

23. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?

(a) $6\hat{i} - 2\hat{j} + 3\hat{k}$ (b) $6\hat{i} - 2\hat{j} + 8\hat{k}$

(c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (d) $-18\hat{i} - 13\hat{j} + 2\hat{k}$

24. Two constant forces $F_1 = 2\hat{i} - 3\hat{j} + 3\hat{k}$ (N) and $F_2 = \hat{i} + \hat{j} - 2\hat{k}(N)$ act on a body and displace if from position $r_1 = \hat{i} + 2\hat{j} - 2\hat{k}$ to position $r_2 = 7\hat{i} + 10\hat{j} + 5\hat{k}$ (m). What is the net work done ?

(a) 9 joule (b) 41 joule (c) –3 joule (4) None of these

25. The vector having a magnitude of 10 and perpendicular to the vector $3\hat{i} - 4\hat{j}$ is:

(a) $4\hat{i} + 3\hat{j}$ (b) $5\sqrt{2}\hat{i} - 5\sqrt{2}\hat{j}$ (c) $8\hat{i} + 6\hat{j}$ (4) $8\hat{i} - 6\hat{j}$

26. The projection of vectors $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ along the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$ is :

(a) $\frac{19}{9}$ (b) $\frac{38}{9}$ (c) $\frac{8}{9}$ (d) $\frac{4}{9}$

27. A particle moves with a velocity $6\hat{i} - 4\hat{j} + 3\hat{k}$ m/s under the influence of a constant force $\vec{F} = 20\hat{i} + 15\hat{j} - 5\hat{k}N$. The instantaneous power applied to the particle is :

(a) 35 J/s (b) 45 J/s (c) 25 J/s (d) 195 J/s

CROSS PRODUCT

28. If $\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$, then $\vec{A}$ can be parallel to

(a) $\vec{B}$ (b) $\vec{C}$ (c) $\vec{B} \times \vec{C}$ (d) $\vec{B} + \vec{C}$

29. If $\vec{A} \times \vec{B} = \vec{C}$, $\vec{B} \times \vec{C} = \vec{A}$ and $\vec{C} \times \vec{A} = \vec{B}$ then

(a) $\vec{A}, \vec{B}, \vec{C}$ are coplanar

(b) $\vec{A} + \vec{B} + \vec{C}$ cannot be equal to zero

(c) angle between $\vec{A}$ and $\vec{B}$ may be less than 90°

(d) none of these

30. Which of the following expression is equal to zero.

(a) $\vec{A} \times \vec{A}$ (b) $\vec{A} \cdot (\vec{A} \times \vec{B})$ (c) $\vec{B} \times [\vec{A} \times (\vec{A} \times \vec{B})]$ (d) All are correct

31. The torque of the force $\vec{F} = (2\hat{i} - 3\hat{j} + 4\hat{k})N$ acting at the point $\vec{r} = (3\hat{i} + 2\hat{j} + 3\hat{k})m$ about the origin be

(a) $6\hat{i} - 6\hat{j} + 12\hat{k}$ (b) $17\hat{i} - 6\hat{j} - 13\hat{k}$ (c) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (d) $-17\hat{i} + 6\hat{j} + 13\hat{k}$

32. The diagonals of a parallelogram are $2\hat{i}$ and $2\hat{j}$. What is the area of the parallelogram

(a) 0.5 units (b) 1 unit (c) 2 units (d) 4 units

33. Two adjacent sides of a parallelogram are represented by the two vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. What is the area of parallelogram

(a) 8 (b) $8\sqrt{3}$ (c) $3\sqrt{8}$ (d) 192

34. The position of a particle is given by $\vec{r} = (\vec{i} + 2\vec{j} - \vec{k})$ momentum $\vec{P} = (3\vec{i} + 4\vec{j} - 2\vec{k})$. The angular momentum is perpendicular to
(a) x–axis
(b) y–axis
(c) z–axis
(d) Line at equal angles to all the three axes

35. $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, then the vector perpendicular to both the vectors $\vec{A}$ and $\vec{B}$ is :
(a) $11\hat{i} - 6\hat{j} - \hat{k}$
(b) $11\hat{i} + 6\hat{j} - \hat{k}$
(c) $11\hat{i} - 6\hat{j} + \hat{k}$
(d) $-11\hat{i} - 6\hat{j} + \hat{k}$

36. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?
(a) $6\hat{i} - 2\hat{j} + 3\hat{k}$
(b) $6\hat{i} - 2\hat{j} + 8\hat{k}$
(c) $4\hat{i} - 13\hat{j} + 6\hat{k}$
(d) $-18\hat{i} - 13\hat{j} + 2\hat{k}$

37. The area of a parallelogram whose diagonals are $\vec{P} = 2\hat{i} + 3\hat{j}$ and $\vec{Q} = \hat{i} + 4\hat{j}$ is:
(a) 5 square units
(b) 10 square units
(c) 20 square units
(d) 2.5 square units

ASSERTION & REASON

(a) Statement–1 is true, Statement –2 is true, Statement –2 is a correct explanation forstatement–1.
(b) Statement–1 is true, Statement –2 is true, Statement –2 is not a correct explanation for statement–1.
(c) Statement–1 is true, Statement –2 is false.
(d) Statement–1 is false, Statement –2 is False.

1. Assertion : $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$.
Reason : $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$ lie in the plane containing $\vec{A}$ and $\vec{B}$, but $\vec{A}\times\vec{B}$ lies perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

2. Assertion : Angle between $\hat{i}+\hat{j}$ and $\hat{i}$ is 45°
Reason : $\hat{i}+\hat{j}$ is equally inclined to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is 90°

3. Assertion : If θ be the angle between $\vec{A}$ and $\vec{B}$, then $\tan\theta = \frac{\vec{A}\times\vec{B}}{\vec{A}.\vec{B}}$
Reason : $\vec{A}\times\vec{B}$ is perpendicular to $\vec{A}.\vec{B}$

4. Assertion : If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is 90°
Reason : $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

5. Assertion : Vector product of two vectors is an axial vector
Reason : If $\vec{v}$ = instantaneous velocity, $\vec{r}$ = radius vector and $\vec{\omega}$ = angular velocity, then $\vec{\omega}=\vec{v}\times\vec{r}$.

6. Assertion : Vector addition of two vectors $\vec{A}$ and $\vec{B}$ is commutative.
Reason : $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

7. Assertion : $\vec{A}.\vec{B}=\vec{B}.\vec{A}$
Reason : Dot product of two vectors is commutative.

8. Assertion : $\vec{\tau}=\vec{r}\times\vec{F}$ and $\vec{\tau}\neq\vec{F}\times\vec{r}$
Reason : Cross product of vectors is commutative.

9. Assertion : A physical quantity cannot be called as a vector if its magnitude is zero.
Reason : A vector has both, magnitude and direction.

10. Assertion : The sum of two vectors can be zero.
Reason : The vector cancel each other, when they are equal and opposite.

11. Assertion : Two vectors are said to be like vectors if they have same direction but different magnitude.
Reason : Vector quantities do not have specific direction.

12. Assertion : The scalar product of two vectors can be zero.
Reason : If two vectors are perpendicular to each other, their scalar product will be zero.

13. Assertion : Multiplying any vector by an scalar is a meaningful operations.
Reason : In uniform motion speed remains constant.

14. Assertion : A null vector is a vector whose magnitude is zero and direction is arbitrary.
Reason : A null vector does not exist.

15. Assertion : If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that one of the vector $\vec{A}$ and $\vec{B}$ must be a null vector.
Reason : Null vector is a vector with zero magnitude.

16. Assertion : The cross product of a vector with itself is a null vector.
Reason : The cross–product of two vectors results in a vector quantity.

17. Assertion : The minimum number of non coplanar vectors whose sum can be zero, is four.
Reason : The resultant of two vectors of unequal magnitude can be zero.

18. Assertion : If $\vec{A}.\vec{B} = \vec{B}.\vec{C}$, then $\vec{A}$ may not always be equal to $\vec{C}$
Reason : The dot product of two vectors involves cosine of the angle between the two vectors.

19. Assertion : Any physical quantity that has magnitude and direction is a vector
Reason : All the vector physical quantities obey the commutative law of addition.

20. Assertion : Unit vector has a unit magnitude as well as direction
Reason : Unit vector is a vector so it must have a magnitude and direction

PREVIOUS YEAR QUESTIONS

1. The vector $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. The positive value of a is **[AFMC 2000; AIIMS 2002]**
(a) 3 (b) 4 (c) 9 (d) 13

2. When $\vec{A}.\vec{B} = -|A||B|$, then **[Orissa JEE 2003]**
(a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other (b) $\vec{A}$ and $\vec{B}$ act in the same direction
(c) $\vec{A}$ and $\vec{B}$ act in the opposite direction (d) $\vec{A}$ and $\vec{B}$ can act in any direction

3. With respect to a rectangular cartesian coordinate system, three vectors are expressed as $\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -\hat{k}$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors, along the X, Y and Z–axis respectively. The unit vectors $\hat{r}$ along the direction of sum of these vector is **[Kerala CET (Engg.) 2003]**
(a) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (b) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j} - \hat{k})$ (c) $\hat{r} = \frac{1}{3}(\hat{i} - \hat{j} + \hat{k})$ (d) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j} + \hat{k})$

4. A person goes 10 km north and 20 km east. What will be displacement from initial point **[AFMC 1994, 2003]**
(a) 22.36 km (b) 2 km (c) 5 km (d) 20 km

5. y component of velocity is 20 and x component of velocity is 10. The direction of motion of the body with the horizontal at this instant is **[Manipal 2003]**
(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(1/2)$ (c) 45° (d) 0°

6. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces **[CBSE AIPMT 2003]**
(a) are not equal to each other in magnitude
(b) cannot be predicted

(c) are equal to each other (d) are equal to each other in magnitude

7. If $|A \times B| = \sqrt{3}A.B$, then the value of $|A+B|$ is **[CBSE AIPMT 2004]**

(a) $\left(A^2+B^2+AB\right)^{1/2}$ (b) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$

(c) A + B (d) $\left(A^2+B^2+\sqrt{3}AB\right)^{1/2}$

8. Two equal forces (P each) act at a point inclined to each other at an angle of 120°. The magnitude of their resultant is **[Karnataka CET 2004]**

(a) P / 2 (b) P / 4 (c) P (d) 2P

9. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$ then the angle between A and B is **[AIEEE 2004]**

(a) $\pi/2$ (b) $\pi/3$ (c) π (d) $\pi/4$

10. Minimum number of unequal vectors which can give zero resultant are **[AFMC 2005]**

(a) two (b) three
(c) four (d) more than four

11. Which of the following is incorrect? **[Haryana PMT 2005]**

(a) $a\cdot(b+c)=b\cdot a+a\cdot c$ (b) $a\times(b+c)=(a\times c)+(a\times b)$

(c) $a\times(b\cdot c)=(a\times b)\cdot(a\times c)$ (d) $(b\cdot c)a=a(c\cdot b)$

12. If the angle between the vectors A and B is θ, the value of the product $(\mathbf{B}\times\mathbf{A})\times\mathbf{A}$ is equal to **[CBSE AIPMT 2005]**

(a) $BA^2\cos\theta$ (b) $BA^2\sin\theta$ (c) $BA^2\sin\theta\cos\theta$ (d) zero

13. The vectors from origin to the points A and B are $\mathbf{A} = 3\hat{i}-6\hat{j}+2\hat{k}$ and $\mathbf{B} = 2\hat{i}+\hat{j}-2\hat{k}$ respectively. The area of the triangle OAB is **[CBSE AIPMT 2005]**

(a) $\frac{5}{2}\sqrt{17}$ (b) $\frac{2}{5}\sqrt{17}$ (c) $\frac{3}{5}\sqrt{17}$ (d) $\frac{5}{3}\sqrt{17}$

14. The position vector of a particle is
$\mathbf{r}=(a\cos\omega t)\hat{i}+(a\sin\omega t)\hat{j}$
The velocity vector of the particle is **[Punjab PMET 2005]**

(a) Parallel to position vector (b) Perpendicular to position vector
(c) Directed towards the origin (d) Directed away from the origin

15. Angle (in rad) made by the vector $\sqrt{3}\,\hat{i}+\hat{j}$ with the X–axis **[EAMCET 2005]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

16. A particle moves with a velocity $(5\hat{i}-3\hat{j}+6\hat{k})\,ms^{-1}$ under the influence of a constant force $\mathbf{F}=10\hat{i}+10\hat{j}+20\hat{k}$ N. The instantaneous power applied to the particle is **[Manipal 2005]**

(a) 200 Js^{-1} (b) 40 Js^{-1} (c) 140 Js^{-1} (d) 170 Js^{-1}

17. If $|\mathbf{A}\times\mathbf{B}| = \sqrt{3}\,\mathbf{A}\cdot\mathbf{B}$, then the value of $|A+B|$ is **[CBSE AIPMT 2005]**

(a) $\left(A^2+B^2+AB\right)^{1/2}$ (b) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$

(c) A + B (d) $\left(A^2+B^2+\sqrt{3}AB\right)^{1/2}$

18. A river is flowing from west to east with a speed of 5 m min^{-1}. A man can swim in still water with a velocity 10 m min^{-1}. In which direction should the man swim so as to take the shortest possible path to go to the south? **[BHU 2005]**

(a) 30° east of south (b) 60° east of south

(c) 60° west of south (d) 30° west of south

19. Of the vectors given below, the parallel vectors are, **[EMACET 2006]**

$A = 6\hat{i} + 8\hat{j}$

$B = 210\hat{i} + 280\hat{k}$

$C = 0.3\hat{i} + 0.4\hat{j}$

$D = 3.6\hat{i} + 6\hat{j} + 4.8\hat{k}$

(a) A and B (b) A and C (c) A and D (d) C and D

20. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$, then projection of A on B will be

[Punjab PMET 2007]

(a) $\frac{3}{\sqrt{13}}$ (b) $\frac{3}{\sqrt{26}}$ (c) $\sqrt{\frac{3}{26}}$ (d) $\sqrt{\frac{3}{13}}$

21. Given two vectors A = $-\hat{i} + 2\hat{j} - 3\hat{k}$ and $B = 4\hat{i} - 2\hat{j} + 6\hat{k}$. The angle made by (A + B) with Z – axis is **[EAMCET 2007]**

(a) 30° (b) 45° (c) 60° (d) 90°

22. The sides of a parallelogram represented by vectors $p = 5\hat{i} - 4\hat{j} + 3\hat{k}$ and $q = 3\hat{i} + 2\hat{j} - \hat{k}$. Then the area of the parallelogram is **[Kerala CEE 2007]**

(a) $\sqrt{684}$ unit (b) $\sqrt{72}$ unit (c) $\sqrt{191}$ unit (d) 72 unit

23. A particle moves from position $3\hat{i} + 2\hat{j} + 6\hat{k}$ to $14\hat{i} + 13\hat{j} + 9\hat{k}$ due to uniform force of $4\hat{i} + \hat{j} + 3\hat{k}N$. Find the work done, if the displacement is in metre. **[J & K CET 2007]**

(a) 16 J (b) 64 J (c) 32 J (d) 48 J

24. A and B are two vectors and θ is the angle between them. If $|A \times B| = \sqrt{3}(A.B)$, then the value of θ is **[CBSE AIPMT 2007]**

(a) 60° (b) 45° (c) 30° (d) 90°

25. Vector which is perpendicular to $a\cos\theta\hat{i} + b\sin\theta\hat{j}$ is

(a) $b\sin\theta\hat{i} - a\cos\theta\hat{j}$ (b) $\frac{1}{a}\sin\theta\hat{i} - \frac{1}{b}\cos\theta\hat{j}$

(c) $5\hat{k}$ (d) All of these

26. The value of P so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar should be

[AMU 2008]

(a) 16 (b) – 4 (c) 4 (d) – 8

27. Three forces A = $\left(\hat{i}+\hat{j}+\hat{k}\right)$, B = $\left(2\hat{i}-\hat{j}+3\hat{k}\right)$ and C acting on a body to keep it in equilibrium. The C is **[EAMCET 2008]**
(a) $-\left(3\hat{i}+4\hat{k}\right)$ (b) $-\left(4\hat{i}+3\hat{k}\right)$ (c) $3\hat{i}+4\hat{j}$ (d) $2\hat{i}-3\hat{k}$

28. A is a vector with magnitude A, then the unit vector a in the direction of vector A is **[J & K CET 2008]**
(a) AA (b) A ·A (c) A × A (d) $\frac{\mathbf{A}}{A}$

29. Two forces of 12N and 8N act upon a body. The resultant force on the body has a maximum value of **[Manipal 2008]**
(a) 4N (b) zero (c) 20N (d) 8N

30. Given two vectors A = $-4\hat{i}+4\hat{j}+2\hat{k}$ and B = $2\hat{i}-\hat{j}-\hat{k}$. The angle made by (A + B) with $\hat{i}+2\hat{j}-4\hat{k}$ is **[Punjab PMET 2008]**
(a) 30° (b) 45° (c) 60° (d) 90°

31. The condition under which vectors (a + b) and (a – b) should be at right angles to each other is **[AMU 2008]**
(a) $a \neq b$ (b) $a \cdot b = 0$ (c) $|a| = |b|$ (d) $a \cdot b = 1$

32. A car travels 6 km towards north at an angle of 45° to the east and then travels distance of 4 km towards north at an angle 135° to east. How far is the point from the starting point? What angle does the straight line joining its initial and final positions make with the cast? **[AIIMS 2008]**
(a) $\sqrt{50}$ km and $\tan^{-1}$ (b) 10 km and $\tan^{-1}\left(\sqrt{5}\right)$
(c) $\sqrt{52}$ km and $\tan^{-1}(5)$ (d) $\sqrt{52}$ km and $\tan^{-1}\left(\sqrt{5}\right)$

33. A train of 150 m length is going towards north direction at a speed of 10 ms^{-1}. A parrot files at a speed of 5 ms^{-1} towards south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to **[BHU 2008]**
(a) 12 s (b) 8 s (c) 15 s (d) 10 s

34. Rain is falling vertically downwards with a velocity of 4 km^{-1}. A man walks in the rain with a velocity 3 km^{-1}. The raindrops will fall on the man with velocity of **[BHU 2008]**
(a) 1 kmh^{-1} (b) 3 kmh^{-1} (c) 4 kmh^{-1} (d) 5 kmh^{-1}

35. Police is chasing the thief 50 m ahead. In 10 s distance between them reduces by 6m. What is distance between them in 25 s? **[DUMET 2008]**
(a) 10 m (b) 25 m (c) 35 m (d) 20 m

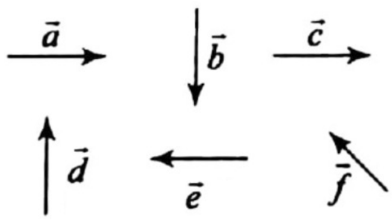
36. A man is walking due east in the rate of 2 kmh^{-1}. The rain appears to him to come down vertically at the rate of 2 kmh^{-1}. The actual velocity and direction of rainfall with the vertical respectively are **[EAMCET 2008]**
(a) $2\sqrt{2}\,kmh^{-1}, 45°$ (b) $\frac{1}{\sqrt{2}}kmh^{-1}, 90°$ (c) $2\,kmh^{-1}, 0°$ (d) $1\,kmh^{-1}, 90°$

37. A proton in a cyclotron changes its velocity from $30 kms^{-1}$ north to $40 kms^{-1}$ east in 20s. What is the magnitude of average acceleration during this time? **[Manipal 2008]**
(a) $2.5 kms^{-2}$ (b) $12.5 kms^{-2}$ (c) $22.5 kms^{-2}$ (d) $32.5 kms^{-2}$

38. If a_1 and a_2 are two non–collinear unit vector unit and if $|a_1 + a_2| = \sqrt{3}$, then the value of $(a_1 - a_2).(2a_1 + a_2)$ is **[Punjab PMET 2009]**
(a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) 1

39. There are N coplanar vectors each of magnitude V. Each vector is inclined to the preceding vector at angle $\frac{2\pi}{N}$. What is the magnitude of their resultant? **[AIIMS 2009]**
(a) $\frac{V}{N}$ (b) V (c) Zero (d) $\frac{N}{V}$

40. Six vectors $\vec{a}$ through $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true ? **[AIPMT – 2010]**



(a) $\vec{b}+\vec{c}=\vec{f}$ (b) $\vec{d}+\vec{c}=\vec{f}$ (c) $\vec{d}+\vec{e}=\vec{f}$ (d) $\vec{b}+\vec{e}=\vec{f}$

41. For any two vectors A and B, if A · B = |A × B|, the magnitude of C = A + B is equal to **[Haryana PMT 2010]**
(a) $\sqrt{A^2+B^2}$ (b) A + B
(c) $\sqrt{A^2+B^2+\frac{AB}{\sqrt{2}}}$ (d) $\sqrt{A^2+B^2+\sqrt{2}AB}$

42. If $\mathbf{a}+\mathbf{b}=\mathbf{c}$ and a + b = c, then the angle included between **a** and **b** is **[WB JEE 2010]**
(a) 90° (b) 180° (c) 120° (d) zero

43. Three equal masses of 1 kg each are placed at the vertices of an equilateral triangle PQR and a mass of 2 kg is placed at the centroid O of the triangle which is at a distance of $\sqrt{2}$ m from each of the vertices of the triangle. The force, in newton, acting on the mass of 2 kg is **[AMU 2010]**
(a) 2 (b) 1 (c) 1 (d) zero

44. The value of λ for which the two vectors a = $5\hat{i}+\lambda\hat{j}+\hat{k}$ and b = $\hat{i}-2\hat{j}+\hat{k}$ are perpendicular to each other is **[WB JEE 2010]**
(a) 2 (b) – 2 (c) 3 (d) – 3

45. At what angle must the two forces (x + y) and (x – y) act so that the resultant may be $\sqrt{x^2+y^2}$? **[JCECE 2010]**

(a) $\cos^{-1}\left[\frac{x^2+y^2}{2(x^2-y^2)}\right]$ (b) $\cos^{-1}\left[-\frac{2(x^2-y^2)}{x^2+y^2}\right]$

(c) $\cos^{-1}\left[-\frac{(x^2+y^2)}{(x^2-y^2)}\right]$ (d) $\cos^{-1}\left[-\frac{(x^2-y^2)}{(x^2+y^2)}\right]$

46. Find the torque of a force F = $-3\hat{i}+2\hat{j}+\hat{k}$ acting at the point r = $8\hat{i}+2j+3\hat{k}$ **[AMU 2010]**

(a) $14\hat{i}-38\hat{j}+16\hat{k}$ (b) $4\hat{i}+4\hat{j}+6\hat{k}$ (c) $-14\hat{i}+38\hat{j}-16\hat{k}$ (d) $-4\hat{i}-17\hat{j}+22\hat{k}$

47. A variable force, given by the two dimensional vector F = $(3x^2\hat{i}+4\hat{j})$, acts on a particle. The force is in newton and x is in metre. What is the change in the kinetic energy of the particles as it moves from the point with coordinates (2, 3) to (3, 0)? (The coordinates are in metres). **[AMU 2010]**

(a) – 7J (b) zero (c) + 7Jj (d) 19 J

48. A motorboat covers a given distance in 6h moving downstream on a river. It cover the same distance in same distance in 10h moving upstream. The time it takes to cover the same distance in still water is **[KCET 2010]**

(a) 9.5 h (b) 7.5 h (c) 6.5 h (d) 8 h

49. If vectors $\hat{i}-3\hat{j}+5\hat{k}$ and $\hat{i}-3\hat{j}-a\hat{k}$ are equal vectors, then the value of a is **[Kerala CEE 2011]**

(a) –5 (b) 2 (c) – 3 (d) 4

50. Given A = $2\hat{i}+3\hat{j}$ and B = $\hat{i}+\hat{j}$. The component of vector A along vector B is **[Kerala CEE 2011]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{5}{\sqrt{2}}$ (d) $\frac{7}{\sqrt{2}}$

51. Which of the following is correct relation between an arbitrary vector **A** and null vector **O**? **[DUMET 2011]**

(a) A + O + A × O = A (b) A + O + A × O ≠ A
(c) A + O + A × O = O (d) None of the above

52. If vectors $A=\cos\omega\hat{i}+\sin\omega\hat{j}$ and $B=\cos\frac{\omega t}{2}\hat{i}+\sin\frac{\omega t}{2}\hat{j}$ are functions of time, then the value of t at which they are orthogonal to each other, is **[CBSE AIPMT 2015]**

(a) $t=\frac{\pi}{4\omega}$ (b) $t=\frac{\pi}{2\omega}$ (c) $t=\frac{\pi}{\omega}$ (d) t = 0

53. A particle moves from a point $(-2\hat{i}+5\hat{j})$ to $(4\hat{j}+3\hat{k})$ when a force of $(4\hat{i}+3\hat{j})$N is applied. How much work has been done by the force? **[NEET 2016]**

(a) 8 J (b) 11 J (c) 5 J (d) 2 J

EXERCISE KEY

LEVEL –I

1.	(d)	2.	(d)	3.	(d)	4.	(b)	5.	(d)
6.	(c)	7.	(a)	8.	(b)	9.	(c)	10.	(a)
11.	(d)	12.	(a)	13.	(c)	14.	(a)	15.	(c)
16.	(c)	17.	(c)	18.	(a)	19.	(b)	20.	(d)
21.	(a)	22.	(b)	23.	(a)	24.	(b)	25.	(a)
26.	(d)	27.	(d)	28.	(a)	29.	(c)	30.	(c)
31.	(c)	32.	(c)	33.	(a)	34.	(c)	35.	(d)
36.	(a)	37.	(c)	38.	(d)	39.	(a)	40.	(c)
41.	(b)	42.	(c)	43.	(b)	44.	(a)	45.	(a)
46.	(c)	47.	(d)	48.	(b)	49.	(b)	50.	(d)
51.	(d)	52.	(b)	53.	(c)	54.	(a)		

LEVEL –II

1.	(d)	2.	(d)	3.	(c)	4.	(b)	5.	(a)
6.	(b)	7.	(c)	8.	(c)	9.	(d)	10.	(b)
11.	(a)	12.	(b)	13.	(a)	14.	(b)	15.	(a)
16.	(d)	17.	(c)	18.	(c)	19.	(d)	20.	(c)
21.	(c)	22.	(a)	23.	(d)	24.	(a)	25.	(c)
26.	(a)	27.	(b)	28.	(c)	29.	(b)	30.	(d)
31.	(b)	32.	(d)	33.	(b)	34.	(a)	35.	(a)
36.	(d)	37.	(a)						

ASSERTION & REASON

1.	(a)	2.	(a)	3.	(d)	4.	(b)	5.	(c)
6.	(b)	7.	(a)	8.	(c)	9.	(d)	10.	(a)
11.	(c)	12.	(a)	3.	(b)	14.	(c)	15.	(b)
16.	(b)	17.	(c)	18.	(a)	19.	(d)	20.	(a)

PREVIOUS YEARS QUESTIONS

1. (a)	2. (c)	3. (a)	4. (a)	5. (a)
6. (d)	7. (a)	8. (c)	9. (c)	10. (b)
11. (c)	12. (b)	13. (a)	14. (b)	15. (a)
16. (c)	17. (a)	18. (a)	19. (b)	20. (c)
21. (b)	22. (a)	23. (b)	24. (a)	25. (a)
26. (b)	27. (a)	28. (d)	29. (c)	30. (d)
31. (c)	32. (c)	33. (d)	34. (d)	35. (c)
36. (a)	37. (a)	38. (c)	39. (c)	40. (b)
41. (d)	42. (d)	43. (d)	44. (c)	45. (a)
46. (d)	47. (c)	48. (b)	49. (a)	50. (c)
51. (a)	52. (c)	53. (c)		