UNIT, DIMENSIONS, MEASUREMENT, ERROR ANALYSIS & CALCULUS

INDEX

This chapter includes:

- Physical Quantities
- Units and Measurements
- Dimensional Analysis
- Application of Dimensional Analysis
- Significant Figures
- Errors
- Differential Calculus
- Integral Calculus

Symbol ★ in Exercise denotes: Class Work

Symbol ☞ in Exercise denotes: Home Work

UNIT, DIMENSIONS, MEASUREMENT, ERROR ANALYSIS & CALCULUS

1. WHAT IS PHYSICS?

"Physics is a fundamental science concerned with understanding the natural phenomena that occurs in our universe".

Natural phenomena such as flow of water, heating of objects, sound of waterfall, rainbow, thunderbolt, energy coming from nucleus … etc., which are very exciting to us and a number of events taking place around us which are very useful in our life. All events in nature take place according to some basic laws and revealing these laws of nature from the observed events is physics.

WHAT IS PHYSICS?

What do you see? That depends not only on what you are looking at, but also on what you are looking for. If you adopt the most fundamental way of looking at things, you see matter interacting with other matter wherever you turn your attention. Billiard balls collide with other billiard balls, electrons with atoms, galaxies with galaxies. Turbines spin, electric light filaments glow, television sets receive signals from distant transmitters, spacecraft follow specific orbits, and your heart pumps blood through your body. As you explore more and more widely over the great theater of the universe, you find variety that seems to have no limit. But, beginning about four centuries ago, investigators using the methods of Physics have achieved a long series of successes — many of them spectacular —in interpreting an ever-increasing wealth of observed phenomena in terms of a tightly organized, relatively small set of basic principles. These principles are inferred more or less directly from the phenomena themselves. This two-way process – interpretation in terms of principles and inference from phenomena — is a hall-mark of physics. It has been widely applied to other sciences as well.

"The scientist does not study physics because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living"

Henri Poincare

The main objective of physics is to use the limited number of fundamental laws that governs natural phenomena to develop theories that can predict the results of future experiments.

The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiments.

Between 1600 and 1900, three broad areas were developed. What is called classical physics.

(i) **Classical Mechanics** deals with the study of the motion of particles and fluids.

(ii) **Thermodynamics** deals with the study of temperature, heat transfer and properties of aggregations of many particles.

(iii) **Electromagnetism** deals with electricity, magnetism, electromagnetic wave, and optics.

These three areas explain all the physical phenomena with which we are familiar.

But by 1905 it became apparent that classical ideas failed to explain several phenomena. Then some new theories were developed in what is called Modern Physics.

Three important theories in Modern Physics are

(i) **Special Relativity:** A theory of the behavior of particles moving at high speeds. It led to a radical revision of our ideas of space, time and energy.

(ii) **Quantum Mechanics:** A theory of submicroscopic world of the atom. It also required a profound upheaval in our vision of how nature operates.

(iii) **General Relativity:** A theory that relates the force of gravity to the geometrical properties of space.

It is also useful in the study of other subjects like biotechnology, geophysics, geology etc.

1.1 SCOPES AND EXCITEMENT

The scope of physics is very wide. We can understand the scope of physics by looking at its various sub-discipline. It covers a very wide variety of natural phenomena. It deals with the phenomena from microscopic level to macroscopic level. The microscopic domain includes atomic, molecular and nuclear phenomena. The macroscopic domain includes phenomena at laboratory, terrestrial and astronomical scales.

For example forces we encounter in nature are nuclear forces, chemical forces and forces exerted by ropes, springs, fluids, electric charge, magnets, the earth and the Sun. Their ranges and relative strength can be summarized as shown below in the table.

Forces	Relative Strength	Range
Strong	1	10^{-15} m
Electromagnetic	10^{-2}	Infinite
Weak	10^{-6}	10^{-17} m
Gravitational	10^{-38}	Infinite

Similarly the range of distance we study in physics vary from 10^{-14} m (size of nuclear) to 10^{+25} m (size of universe)

The range of masses includes in study of physics varies from 10^{-30} kg (mass of electron) to 10^{55} kg (mass of universe)

The range of time interval varies from 10^{-22} s (time taken by a light to cross a nuclear distance) to 10^{18} s. (life time of sun).

So we can say scope of physics is really wide.

The study of physics is very exciting in many ways.

EXCITEMENT OF PHYSICS CAN BE SEEN IN EVERY FIELD.

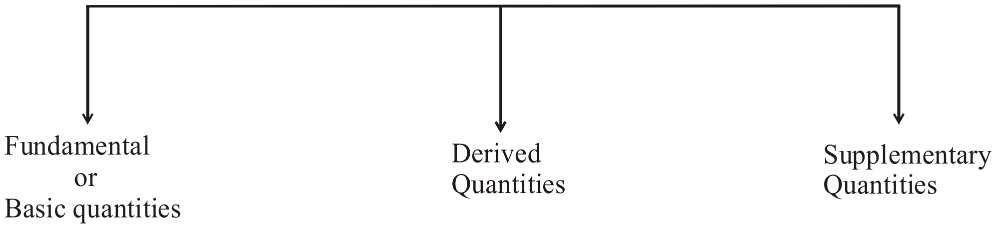
- Advancement of technology has upgraded the entire scenario of entertainment, starting from a radio to the most advanced cyber park.
- Communication system has been also deepened its root by bridging distant areas closer.
- Advances in health science, which has enabled operations without surgery.
- Telescopes & satellite have broken the limits of knowledge of the undiscovered universe.
- Exploring the sources of energy from the unexplored sources.
- Made possible the reach of man beyond the earth, towards the cosmos.
- Use of Robots in a hazardous places is highly beneficial.

2. PHYSICS QUANTITIES

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class X we have studied many physical quantities

eg. length, velocity, acceleration, force, time, pressure, mass, density etc.

Physical quantities are of three types



All the quantities by means of which we describe the laws of nature and which can be measured are called physical quantities. Examples are Mass, length, time, force, velocity, acceleration etc.

'Beauty and intelligence' are not physical quantities, because they can't be measured.

2.1 FUNDAMENTAL PHYSICAL QUANTITY

It is an elementary physical quantity, which doesn't require any other physical quantity to express it. It means it cannot be resolved further in terms of any other physical quantity. It is also known as basic physical quantity.

In mechanics length, mass and time are only three basic physical quantities. Other basic physical quantities are electric current, temperature, luminous intensity and amount of substance.

2.2. DERIVED PHYSICAL QUANTITY

All those physical quantities, which can be derived from the combination of two or more fundamental quantities or can be expressed in terms of basic physical quantities, are called derived physical quantities.

Examples: Velocity, density, force, energy etc.

Velocity can be expressed in terms of distance and time

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

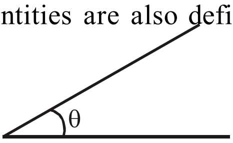
Density can be expressed in terms of mass and length

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Length}^3}$$

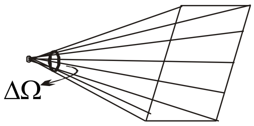
2.3 SUPPLEMENTARY QUANTITIES :

Besides seven fundamental quantities two supplementary quantities are also defined. They are

Plane angle (The angle between two lines)



Solid angle (The angle made by an area at a point)



3. UNITS AND MEASURMENT

We know that the results of physics are based on experimental observation and quantitative measurement, so it is also known as quantitative science.

Now to express the magnitude of any physical quantity we need a unit.

Suppose we say that the distance between two stations *A* and *B* is 500. Without a unit, we are unable to understand how far *B* is from *A*.

But when we say that distance is 500 m or 500 km then we get a picture of how far *B* is from *A,* since we know the standard length of meter. Similarly, when we say that the speed of an automobile is 20, unless we also specify the units meter per second or kilometer per hour it is meaningless. When we want to know the amount of fuel needed for a long trip it is difficult to arrive at an answer without knowing the unit of speed.

So we can say that all physical quantity needs a unit.

The need of unit felt long long ago in *A.D.* 1120. The king of England decided the unit of length, which was yard. The yard was the distance from the tip of his nose to the end of outstretched arm.

Similarly, the original standard for the foot adopted by the French was the length of royal foot of king Louis XIV. This standard prevailed until 1799, until the legal standard in France became the meter.

In this way we see that different standards were decided in different periods of time and in different parts of the world. Due to this reason again a major problem was faced which can be understood from this example. Suppose a visitor from another place is talking about a length of 8 "glitches" and we don't know the meaning of the unit glitch. So this talk is meaningless for us. Then the need of a standard unit was felt which should be universal.

In 1960, an international committee was established to set a standard of unit for physical quantities. This system is called International System (SI) of units.

3.1 UNITS

To measure a physical quantity we require a standard of that physical quantity. This standard is called unit of that physical quantity. It can be defined in this way.
"Measurement of any physical quantity involves comparison with a certain basic internationally accepted reference standard called unit.
The value of any physical quantity must be expressed in terms of some standard or unit. For example, we might specify the distance between two posts in meters or in feet. Such units are necessary for us to compare measurements and also to distinguish between different physical quantities.
Measure of physical quantity = Numerical value × Unit
Length of a rod = 8 m
where 8 is numerical value and m (metre) is unit of length.

3.2 FUNDAMENTAL AND DERIVED UNITS

There are a large number of physical quantities and every quantity needs a unit.

However, all the quantities are not independent. For example, if a unit of length is defined, a unit of area or volume is automatically obtained. Thus, we can define a set of fundamental quantities and all other physical quantities, which can be expressed in terms of fundamental quantities, are derived quantities.

Fundamental Units: The units of fundamental physical quantities are called fundamental units.

In mechanics, unit of length, mass and time are m (meter), kg (kilogram) and s (second), respectively, which are called fundamental units.

Derived Units: The units of all other physical quantities, which can be obtained from fundamental units, are called derived units.

For example units of velocity, density and force are m/s, kg/m^3, $kg\ m/s^2$ respectively.

Who decides the units In 1960, an international committee established a set of standards for these fundamental quantities. This system is called the International System (SI) of units. In this system units of length, mass and time are the meter, kilogram, and second, respectively. Other standard basic SI units established by the committee are for temperature (the kelvin), electric current (the ampere), Luminous intensity (the candela) and for the amount of substance (the mole)

These seven units are the basic/fundamental SI units as shown below in tables with symbol.

S.No.	Fundamental Physical quantity	Units	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	S
4.	Electric current	ampere	A
5.	Luminous intensity	canada	cd
6.	Temperature	kelvin	K
7.	Amount of Substance	mole	mol

However definition of units is under constant review and is changed from time to time. A body named "General conference on Height and Measures' holds the meeting to decide any changes in units.

OTHER INTERNATIONAL SYSTEMS ARE

(1) **The F.P.S. System** is the British Engineering system of units. In this system unit of length is foot, of mass is pound and of time is second.

(2) **The C.G.S. system** is the Gaussian system. In this system units of length, mass and time are centimetre, gm and second respectively.

(3) **The M.K.S. System** in this system the units of length, mass and time are metre, kilogram and second respectively.

3.3 DEFINITIONS OF FUNDAMENTAL UNIT

Length: To measure the distance between any two points in the space we use the term length and to specify the magnitude we use a unit of length.

Most common unit of length is metre.

In 1799 legal standard of length became metre, which was defined as one ten-millionth the distance from the equator to the North Pole.

In 1960, the length of metre was defined as the distance between two lines on a specific platinum iridium bar stored under controlled condition.

Recently the metre was defined as 1650,763.73 wavelength of orange red light emitted from a krypton-86. However in October 1983, the metre was redefined as the distance traveled by light in vaccum during a time of 1/299792458 second.

Mass: The mass is a basic property of matter. The basic S.I. unit of mass is kilogram, which is defined as the mass of a specific platinum-indium alloy cylinder kept at the international Bureau of Heights and Measures at Severs, France. This mass standard was established in 1887, and there has been no change since that time because platinum-indium is an unusually stable alloy. A duplicate is kept at the National Institute of Standards and Technology in Gaithersburg.

Time: The concept of time is very old. Before 1960, the standard of time was defined in terms of mean solar day. The mean solar second was the basic unit of time, which was defined as $\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right)$ of a mean solar day.

In 1967, the definition of second was modified using the characteristic frequency of a particular kind of cesium atom as the "reference clock".

The basic SI unit of time, the second, is defined as 91926631770 periods of the radiation from cesium – 133 atoms.

SI BASE QUANTITIES AND UNITS

Base Quantity	SI Unit		
	Name	Symbol	Definition
Length	metre	m	The metre is the length of the path traveled by light in vacuum during a time interval of 1/299,792,458 of a second. (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1989)
Time	second	s	The second is the duration of 9,192,631, 770 periods of the radiation corresponding to the transition between the two-hyperfine levels of the ground state of the cesium-133 atom. (1967)
Electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. (1948)
Thermo dynamic temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979)

SOME SI DERIVED UNITS EXPRESSED IN SI BASE UNITS

Physical quantity	SI Unit	
	Name	Symbol
Area	Square metre	m^2
Volume	cubic metre	m^3
Angular acceleration	radian per second square	rad/s^2 or $rad\ s^{-2}$
Wave number	per metre	m^{-1}
Density, mass density	kilogram per cubic metre	kg/m^3 or $kg\ m^{-3}$
current density	ampere per square metre	A/m^2 or Am^{-2}
Magnetic field strength, magnetic intensity, magnetic moment density	ampere per metre	A/m or Am^{-1}
Concentration (of amount of substance)	mole per cubic metre	mol/m^3 or $mol\ m^{-3}$
Specific volume	cubic metre per kilogram	m^3/kg or $m^3\ kg^{-1}$
Luminance, intensity of illumination	candela per square metre	cd/m^2 or $cd\ m^{-2}$
Kinematic viscosity	square metre per second	m^2/s or $m^2\ s^{-1}$
Radius of gyration	metre	m
Linear/superficial/volume expansivities	per kelvin	K^{-1}

SI DERIVED UNITS WITH SPECIAL NAMES

Physical quantity	**SI Unit**			
	Name	**Symbol**	**Expression in terms of other units**	**Expression in terms of SI base Units**
Energy, work quantity of Heat	joule	J	Nm	kg m^2 s^{-2} or kg m^2/s^2
Power, radiant flux	watt	W	J/s or Js^{-1}	kg m^2 s^{-3} or kg m^2/s^3
Quantity of electricity, Electric charge	coulomb	C	—	A-s
Electric potential, Potential difference, Electromotive force	volt	V	W/A or WA^{-1}	Kg m^2s^{-3} A^{-1} or kg m^2/s^3 A
Capacitance	farad	F	C/V	A^2s^4 kg^{-1} m^{-2}
Electric resistance	ohm	Ω	V/A	kg m^2 s^{-3} A^{-2}
Conductance	siemens	S	A/V	m^{-2} kg^{-1} s^3 A^2
magnetic flux	weber	Wb	Vs or J/A	kg m^2 s^{-2} A^{-1}
Magnetic field, magnetic flux density, magnetic induction	tesla	T	Wb/m^2	kg s^{-2} A^{-1}
Inductance	henry	H	Wb/A	kg m^2 s^{-2} A^{-2}
Luminous flux, luminous Power	lumen	lm	—	cd /sr
Illuminance	lux	lx	lm/m^2	m^{-2} cd sr^{-1}
Activity of a radio nuclide/radioactive source	becquerel	Bq		s^{-1}

3.4 CONVERSION OF UNITS

In making calculations, it is essential that all the physical quantities used should be consistent in units. If we want to know the total amount of fuel consumed by an automobile during a long trip, we cannot simply add one purchase made in litres to another made in gallons. So it is necessary to convert different units of a physical quantity to a desired unit.

Suppose we wish to convert 5 miles per hour (mi/h) into meter per second (m/s).

Given that 1 mile = 1.6 km

$$5.0\frac{\text{mile}}{\text{hour}} = \frac{5\times 1.6\,\text{km}}{1\times 60\,\text{minutes}} = \frac{8}{60}\frac{\text{km}}{\text{minutes}} = \frac{8\times 1000}{60\times 60}\frac{\text{meter}}{\text{second}} = \frac{20}{9}\frac{\text{meter}}{\text{second}} = 2.22\,\text{ms}^{-1}$$

3.5 PREFIXES AND MULTIPLICATION FACTORS

The magnitude of physical quantities vary over a wide range as discussed in scope of physics, so standard prefixes for certain power of 10 was decided by CGPM, which are shown below in table.

The most commonly used prefixes in CBSE exam are given below in tabular form.

Power of 10	Prefix	Symbol
+ 18	Exa	E
+ 15	Peta	P
+ 12	Tera	T
+ 9	Giga	G
+ 6	Mega	M
+ 3	Kilo	K
– 2	centi	c
– 3	mili	m
– 6	micro	μ
– 9	nano	n
-12	pico	p
– 15	femto	f
– 18	atto	a

CONVERSION TABLE

Length

1 in. = 2.54 cm

1 m = 39.37 in. = 3.281 ft

1 ft = 0.3048 m

12 in. = 1 ft

3 ft = 1 yd

1 yd = 0.9144 m

1 km = 0.621 mi

1 mi = 1.609 km

1 mi = 5280 ft

1 Å = 10^{-10} m

1 μm = 10^{-6} m I= 10^4 Å

1 light-year = 9.461×10^{15} m

Area

1 $m^2 = 10^4\ cm^2 = 10.76\ ft^2$

1 $ft^2 = 0.0929\ m^2 = 144\ in.^2$

1 $in.^2 = 6.452\ cm^2$

Force

1 N = 10^5 dyne = 0.2248 lb

1 1b = 4.448 N

1 dyne = 10^{-5} N = 2.248×10^{-6} lb

Velocity

1 mi/h = 1.47 ft/s = 0.447 m/s = 1.61 km/h

1 m/s = 100 cm/s = 3.28 ft/s

1 mi/min = 60 mi/h = 88 ft/s

Acceleration

1 $m/s^2 = 3.28\ ft/s^2 = 100\ cm/s^2$

1 $ft/s^2 = 0.3048\ m/s^2 = 30.48\ cm/s^2$

Pressure

1 bar = $10^5\ N/m^2 = 14.50\ lb/in.^2$

1 atm = 760 mm Hg = 76.0 cm Hg

1 atm = $14.7\ lb/in.^2 = 1.013 \times 10^5\ N/m^2$

1 Pa = $1\ N/m^2 = 1.45 \times 10^{-4}\ lb/in.^2$

Time

1 year = 365 days = 3.16×10^7 s

1 day = 24 h = 1.44×10^3 min = 8.64×10^4 s

Volume

$1\ m^3 = 10^6\ cm^3 = 6.102 \times 10^4\ in.^3$

$1\ ft^3 = 1728\ in.^3 = 2.83 \times 10^{-2}\ m^3$

1 liter $= 1000\ cm^3 = 1.0576$ qt $= 0.0353\ ft^3$

$1\ ft^3 = 7.481$ gal $= 28.32$ liters $= 2.832 \times 10^{-2}\ m^3$

1 gal $= 3.786$ liters $= 281\ in.^3$

Mass

1000 kg = 1 t (metric ton)

1 slug = 14.59 kg

$1\ u = 1.66 \times 10^{-27}$ kg

Energy

$1\ J = 0.738$ ft. lb $= 10^7$ ergs

1 cal = 4.186 J

1 Btu $= 252$ cal $= 1.054 \times 10^3$ J

$1\ eV = 1.6 \times 10^{-19}$ J

931.5 MeV is equivalent to 1 u

$1\ kWh = 3.60 \times 10^6$ J

Power

1 hp = 550 ft. lb/s = 0.746 kW

1 W = 1J/s = 0.738 ft.lb/s

1 Btu/h = 0.293 W

Illustration 1 : Write the unit of following derived quantities is SI system.

(i) Velocity (ii) Acceleration (iii)Force (iv) Energy

***Solution*:**

(i) $\text{Velocity} = \dfrac{\text{displacement}}{\text{time}} \therefore$ Its unit is ms^{-1}

(ii) $\text{Acceleration} = \dfrac{\text{change in velocity}}{\text{time}} \therefore$ Its unit is $\dfrac{m/s}{s} = ms^{-2}$

(iii) Force = mass × acceleration $\therefore$ Its unit is kg ms^{-2}

(iv) Energy = Force × displacement $\therefore$ Its unit is $kg\,ms^{-2} \times m = kg\,m^2s^{-2}$

Illustration 2 : The SI unit of force is newton such that $1N = 1kgms^{-2}$. In C.G.S. system, force is expressed in dyne. How many dyne of force is equivalent to a force of 5 N?

***Solution*:**

Let 1 N = *n* dynes

$$\Rightarrow \quad \left(\frac{1\,kg\,m}{s^2}\right) = n\left(\frac{g-cm}{s^2}\right)$$

$$\text{or} \quad \frac{1000g \times (100)cm}{s^2} = n\left(\frac{g-cm}{s^2}\right)$$

$$\Rightarrow \quad n = 10^5$$

$$\therefore \quad 5\ N = 5 \times 10^5 \text{ dyne.}$$

Illustration 3 : The mass of solid cube is 856 g, and each edge has a length of 5.35 cm. Determine the density ρ of the cube in basic SI units.

Solution: Since 1 $g = 10^{-3}$ kg and 1 cm = 10^{-2} m, the mass, m, and volume, V, in basic SI units are given by

$$m = 856\ g \times 10^{-3}\ \text{kg/g} = 0.856\ \text{kg}$$

$$V = L^3 = (5.35\ \text{cm} \times 10^{-2}\ \text{m/cm})^3$$

$$= (5.35)^3 \times 10^{-6}\ \text{m}^3 = 1.53 \times 10^{-4}\ \text{m}^3$$

Therefore $\rho = \frac{m}{V} = \frac{0.856\,kg}{1.53 \times 10^{-4} m^3} = 5.59 \times 10^3\ \text{kg/m}^3$

4. DIMENSIONAL ANALYSIS

The word **dimension** has a special meaning in Physics. It usually denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or furlongs, it is a distance. We say its dimension is **length.**

The symbols we use to express the dimensions are only for fundamental physical quantities so we use seven symbols for seven fundamental quantities, which are denoted by capital letters with square brackets. Remaining all derived physical quantities are expressed in terms of dimensions of fundamental quantities.

"The dimensions of a physical quantities are the powers to which the base quantities are raised to represent that quantity"

Dimension, of fundamental quantities are

Length	[L]
Mass	[M]
Time	[T]
Electric current	[A]
Temperature	[K]
Luminous Intensity	[cd]
Amount of substance	[mol]

Volume occupied by an object = Length × breadth × height

Dimensions of volume are $[L] \times [L] \times [L] = [L^3]$

As the volume is independent of mass and time. It is said to be zero dimension in mass [M°], zero dimension in time [T°] and three dimensions in length. All dimensions are written together in this way $[M^o L^3 T^\circ]$

$$\text{Force} = \text{Mass} \times \text{acceleration} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}$$

So the dimensions of force are $= \frac{[M]\,[L]}{[T^2]} = [MLT^{-2}]$

This equation of dimensions is also known as dimensional equation.

Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

- Density $= \frac{\text{mass}}{\text{volume}}$

$$[\text{Density}] = \frac{[\text{mass}]}{[\text{volume}]} = \frac{M}{L^3} = [M^1L^{-3}]$$

- Velocity $(v) = \dfrac{\text{displacement}}{\text{time}}$

$$[v] = \frac{[\text{Displacement}]}{[\text{time}]} = \frac{L}{T} = [M^0L^1T^{-1}]$$

- Acceleration $(a) = \dfrac{dv}{dt}$

$$[a] = \frac{dV}{dt} \quad \begin{matrix}\text{Small change in velocity}\\ \text{Small change in time}\end{matrix} = \frac{LT^{-1}}{T} = LT^{-2}$$

- Momentum $(P) = mv$

$$[P] = [M]\,[v] = [M]\,[LT^{-1}] = [M^1L^1T^{-1}]$$

- Force $(F) = ma$

$$[F] = [m]\,[a]$$
$$= [M]\,[LT^{-2}] = [M^1L^1T^{-2}]$$

- Work or Energy = force × displacement

$$[\text{Work}] = [\text{force}]\,[\text{displacement}] = [M^1L^1T^{-2}]\,[L] = [M^1L^2T^{-2}]$$

- Power $= \dfrac{\text{work}}{\text{time}}$

$$[\text{Power}] = \frac{[\text{work}]}{[\text{time}]} = \frac{M^1L^2T^{-2}}{T} = [M^1L^2T^{-3}]$$

- Pressure $= \dfrac{\text{Force}}{\text{Area}}$

$$[\text{Pressure}] = \frac{[\text{Force}]}{[\text{Area}]} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$$

4.1 DIMENSIONS OF ANGULAR QUANTITIES

- Angle (θ) (Angular displacement) $\theta = \dfrac{\text{Arc}}{\text{radius}}$

$$[\theta] = \frac{[\text{Arc}]}{[\text{radius}]} = \frac{L}{L} = [M^0L^0T^0] \text{ (Dimensionless)}$$

- Angular velocity $(\omega) = \dfrac{\theta}{t}$

$$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0L^0T^{-1}]$$

- Angular acceleration $(\alpha) = \dfrac{d\omega}{dt}$

$$[\alpha] = \frac{[d\omega]}{[dt]} = \frac{M^0L^0T^{-1}}{T} = [M^0L^0T^{-2}]$$

- Torque = Force × Arm length

$$[\text{Torque}] = [\text{force}] \times [\text{arm length}] = [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$$

4.2 DIMENSIONS OF PHYSICAL CONSTANTS

l **Gravitational Constant :**

If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is,

Gravitational force $F_g = \frac{Gm_1m_2}{r^2}$

where G is a constant called Gravitational constant

$$[F_g] = \frac{[G][m_1][m_2]}{[r^2]}$$

$$[M^1L^1T^{-2}] = \frac{[G][M][M]}{[L^2]}$$

$$[G] = M^{-1}L^3T^{-2}$$

- **Specific heat capacity :**

To increase the temperature of a body by ΔT, Heat required is $Q = ms\,\Delta T$

Here s is called specific heat capacity.

$$[Q] = [m]\,[s]\,[\Delta T]$$

Here Q is heat : A kind of energy so $[Q] = M^1L^2T^{-2}$

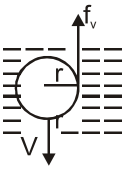
$$[M^1L^2T^{-2}] = [M]\,[S]\,[K]$$

$$[s] = [M^0L^2T^{-2}K^{-1}]$$

- **Coefficient of viscosity :**

If any spherical ball of radius r moves with velocity v in a viscous Liquid, then viscous force acting on it is given by

$$F_v = 6\pi\eta rv$$



Here h is coefficient of viscosity

$$[F_v] = [6\pi]\,[\eta]\,[r]\,[v]$$

$$M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}]$$

$$[\eta] = M^1L^{-1}T^{-1}$$

- **Planck's constant :**

If light of frequency ' ν ' is falling, energy of a photon is given by

$E = h\nu$ Here h = Planck's constant

$$[E] = [h]\,[\nu]$$

$$\nu = \text{frequency} = \frac{1}{\text{Time Period}} \Rightarrow [\nu] = \frac{1}{[\text{Time Period}]} = \left[\frac{1}{T}\right]$$

so $M^1L^2T^{-2} = [h]\,[T^{-1}]$

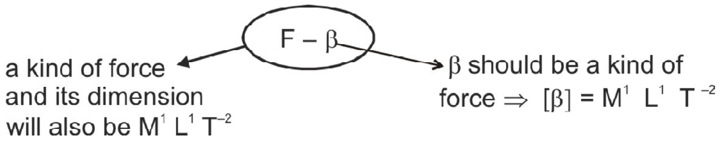
$$[h] = M^1L^2T^{-1}$$

4.3 SOME SPECIAL FEATURES OF DIMENSIONS

- Suppose in any formula, (L + a) term is coming (where L is length). As length can be added only with a length, so a should also be a kind of length.

So $[a] = [L]$

- Similarly consider a term (F – β) where F is force. A force can be added/substracted with a force only and give rise to a third force. So β should be a kind of force and its result (F – β) should also be a kind of force.



RULE NO. 1 :

One quantity can be added / substracted with a similar quantity only and give rise to the similar quantity.

Illustraton 4 : Find dimension formula for $[\alpha]$ and $[\beta]$ (where t = time, F = force, v = velocity, x = distance)

$$\frac{\alpha}{t^2} = Fv + \frac{\beta}{x^2}$$

Solution : Since $[Fv] = M^1L^2T^{-3}$,

so $\left[\frac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$$\frac{[\beta]}{[x^2]} = M^1 L^2T^{-3} \Rightarrow \quad [\beta] = M^1L^4T^{-3}$$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$

so $\frac{[\alpha]}{[t^2]} = M^1L^2T^{-3}$

$$[\alpha] = M^1L^2T^{-1}$$

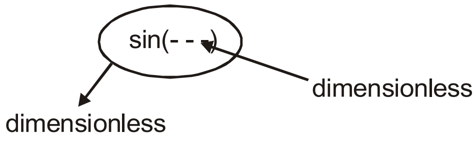
RULE NO. 2 :

CONSIDER A TERM SIN(θ)

Here 'θ' is dimensionless and sinθ i.e. $\left(\frac{\text{Perpendicular}}{\text{Hypoteneous}}\right)$ is also dimensionless.

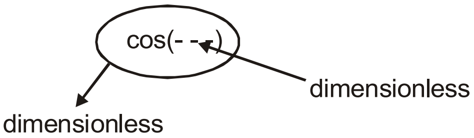
⇒ Whatever comes in sin(......) is dimensionless and entire [sin (.......)] is also dimensionless.

⇒



SIMILARLY :

⇒



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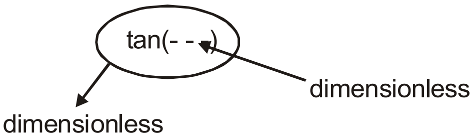

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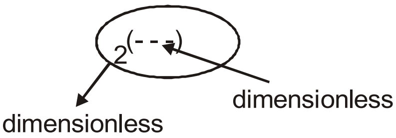

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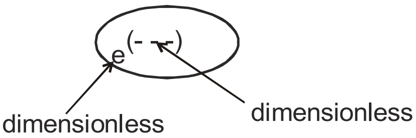

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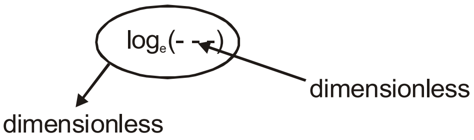
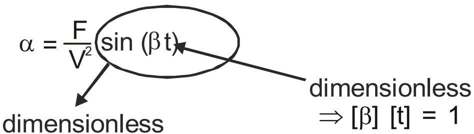


illustration 5 : $a = \frac{F}{V^2}$ sin (bt) (here V = velocity, F = force, t = time)

Find the dimension of α and β

Solution :



$\Rightarrow [\beta]\ [t] = 1$

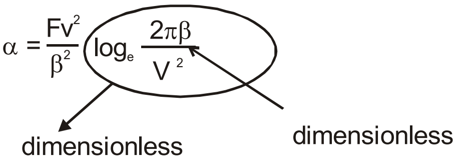
$[\beta] = [T^{-1}]$

So $[\alpha] = \frac{[F]}{[V^2]} = \frac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$

illustration 6 : $\alpha = \frac{FV^2}{\beta^2} \log_e\left(\frac{2\pi\beta}{V^2}\right)$ where F = force , V = velocity

Find the dimension of α and β

Solution :



$\Rightarrow [\alpha] = \frac{[F][V^2]}{[\beta^2]} \qquad \Rightarrow \frac{[2\pi][\beta]}{[V^2]} = 1$

$\Rightarrow \frac{[1][\beta]}{L^2T^{-2}} = 1 \qquad \Rightarrow [\beta] = L^2T^{-2}$

$\Rightarrow [\alpha] = \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[L^2T^{-2}]^2} \qquad \Rightarrow [\alpha] = M^1L^{-1}T^0$

Dimensions of commonly used Physical Quantities

S.No.	Physical Quantity (Mechanics)	SI Units	Dimensional formula
1.	Velocity = displacement/time	m/s	M^0LT^{-1}
2.	Acceleration = velocity/time	m/s^2	M^0LT^{-2}
3.	Force = mass × acceleration	$kg\text{-}m/s^2$ = newton or N	MLT^{-2}
4.	Work = force × displacement	$kg\text{-}m^2/s^2$ = N-m = joule or	ML^2T^{-2}
5.	Energy	J	ML^2T^{-2}
6.	Torque = force × perpendicular distance	N-m	ML^2T^{-2}
7.	Power = work/time	J/s or watt	ML^2T^{-3}
8.	Momentum = mass × velocity	Kg-m/s	MLT^{-1}
9.	Impulse = force × time	Kg-m/s or N-s	MLT^{-1}
10.	Angle = arc/radius	radian or rad	$M^0L^0T^0$
11.	Strain = $\frac{\Delta L}{L}$ or $\frac{\Delta V}{V}$	no units	
12.	Stress = force/area	N/m^2	$ML^{-1}T^{-2}$
13.	Pressure = force/area	N/m^2	$ML^{-1}T^{-2}$
14.	Modulus of elasticity = stress/strain	N/m^2	$ML^{-1}T^{-2}$
15.	Frequency = 1/ time period	per sec or hertz (Hz)	$M^0L^0T^{-1}$
16.	Angular velocity = angle/time	rad/s	$M^0L^0T^{-1}$
17.	Moment of inertia = (mass) (distance) ^{2}	$kg\text{-}m^2$	ML^2T^0
18.	Surface tension = force/length	N/m	ML^0T^{-2}
19.	Gravitational constant	$N\text{-}m^2/kg^2$	$M^{-1}L^3T^{-2}$
20.	Thermodynamic temperature	kelvin (K)	$M^0L^0T^0K$
21.	Heat	joule	ML^2T^{-2}
22.	Specific heat	$Jkg^{-1}K^{-1}$	$M^0L^2T^{-2}K^{-1}$
23.	Latent heat	$J\ kg^{-1}$	$M^0L^2T^{-2}$
24.	Universal gas constant	$J\ mol^{-1}\ K^{-1}$	$ML^2T^{-2}K^{-1}mol^{-1}$
25.	Boltzmann's constant	JK^{-1}	$ML^2T^{-2}K^{-1}$
26.	Stefan's constant	$Js^{-1}m^{-2}\ K^{-4}$	$MT^{-3}K^{-4}$
27.	Planck's constant	Js	ML^2T^{-1}
28.	Solar constant	$J\ m^{-2}\ s^{-1}$	ML^0T^{-3}
29.	Thermal conductivity	$Js^{-1}m^{-1}\ K^{-1}$	$MLT^{-3}K^{-1}$
30.	Thermal resistance	$Kscal^{-1}$	$M^{-1}L^{-2}T^3K$
31.	Enthalpy	cal	ML^2T^{-2}
32.	Entropy	$cal\ K^{-1}$	$ML^2T^{-2}K^{-1}$

5. APPLICATION OF DIMENSIONAL ANALYSIS

(I) IN CONVERSION OF UNITS FROM ONE SYSTEM TO OTHER

This is based on the fact that the product of the numerical value and its corresponding unit is constant.

Numerical value × Unit = constant

So when the unit will change, numerical value will also change.

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2 in SI units. We can convert it in C.G.S. system in this way.

The dimensional formula of G is $[M^{-1}L^3T^{-2}]$

Let n_1 represents magnitude in S.I. unit and n_2 in C.G.S. unit

$$n_1[M_1^{-1}L_1^{-2}] = n_2[M_1^{-1}L_2^3T_2^{-2}]$$

$$\Rightarrow \qquad n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^3\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 6.67 \times 10^{-11}\left[\frac{1kg}{1gm}\right]^{-1}\left[\frac{1m}{1cm}\right]^3\left[\frac{1s}{1s}\right]^{-2} = 6.67\times10^{-11}\left[\frac{1000gm}{1gm}\right]^{-1}\left[\frac{100cm}{1cm}\right]^3\left[\frac{1s}{1s}\right]^{-2}$$

$$= 6.67 \times 10^{-11} \times 10^{-3} \times 10^{6} = 6.67 \times 10^{-8} \text{ dyne cm}^2/g^2$$

illustration 7 : In a new system of units, unit of mass is taken as 50 kg, unit of length is taken as 100 m and unit of time is 1 minute. What will be the weight of a body in this system, if in SI system, its weight is 10 N.

***Solution*:** Let the weight of the body in new system is X units

$$\Rightarrow \qquad 10 \text{ N} = X \text{ units}$$

Let M_1, L_1, T_1 and M_2, L_2, T_2 be the symbols for mass, length and time in the two system respectively, then

$$10[M_1 L_1 T_1^{-2}] = X[M_2 L_2 T_2^{-2}]$$

$$M_1 = 1 \text{ kg}, M_2 = 50 \text{ kg}, L_1 = 1 \text{ m}, L_2 = 100 \text{ m}, T_1 = 1 \text{ s}, T_2 = 60 \text{ s}$$

$$\Rightarrow \qquad X = \frac{10}{50}\times\frac{1}{100}\times\left(\frac{1}{60}\right)^{-2} = \frac{36000}{5000} = 7.2 \text{units}$$

$$\therefore \qquad 10 \text{ N} = 7.2 \text{ units.}$$

(II) TO CHECK THE DIMENSIONAL CORRECTNESS OF A GIVEN PHYSICAL RELATION

This is based on the principle of homogeneity. According to this principle the dimensions of each term on both sides of an equation must be the same.

If the dimensions of each term on both sides of equation are same then it is called dimensionally correct equation. But it may or may not be physically correct. If an equation is physically correct then it will be also dimensionally correct.

Time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Dimension of left hand side = [T]

Dimension of right hand side = $\sqrt{\frac{[L]}{[LT^{-2}]}} = \sqrt{[T^2]} = [T]$

In the above equation, the dimensions of both sides are same. So the given formula is dimensionally correct.

(III) TO ESTABLISH THE RELATION AMONG VARIOUS PHYSICAL QUANTITIES

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors.

It is used as a research tool to derive new relations.

Stoke's law has been developed using this principle.

This law explains the motion of a spherical ball in a viscous liquid, where viscous force due to liquid opposes the motion.

It is found experimentally that viscous force depends on radius (r) of the sphere, velocity of the sphere (v) and the viscosity η of the liquid

$$F \propto (\eta r v)$$

$$F = k\,\eta^x r^y v^z$$

Where k is is dimensionless constant. To be a correct equation. It should be dimensionally correct

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^x\,[L]^y\,[LT^{-1}]^z$$

$$[MLT^{-2}] = [M^x L^{-x+y+z} T^{-x-z}]$$

equating the power of similar quantities

$$x = 1,\ - x + y + z = 1 \text{ and } - x - z = -2 \quad \Rightarrow \quad x = y = z = 1$$

$$F = k\eta r\, v$$

From experiment $k = 6\,\pi \;\Rightarrow F = 6\,\pi\eta r v$

(IV) TO FIND DIMENSIONS OF PHYSICAL CONSTANTS OR CO-EFFICIENT.

Suppose there is a physical equation with a constant, we can find out the dimensions of that constant by substuting the dimensions of all other quantities.

Example: $F = \frac{Gm_1m_2}{r^2}$

Where G is universal gravitational constant

$$G = \frac{Fr^2}{m_1m_2} = \frac{[MLT^{-2}]\,[L^2]}{[M^2]} = [\,M^{-1}L^3\,T^{-2}\,]$$

LIMTATIONS OF DIMENSIONAL ANALYSIS

The method of dimensions has the following limitations:

(i) by this method the value of dimensionless constant cannot be calculated.

(ii) by this method the equation containing trigonometric, exponential and logarithmic terms cannot be analyzed.

(iii) if a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalizing the powers of M, L and T.

(iv) it doesn't tell whether the quantity is vector or scalar.

Dimensions of physical quantities in heat

S.No.	Thermal Quantity	SI Units	Dimensional formula
1.	Thermodynamic temperature	kelvin (K)	$[M^0L^0T^0K]$
2.	Heat	joule	$[ML^2T^{-2}]$
3.	Specific heat	$Jkg^{-1}K^{-1}$	$[M^0L^2T^{-2}K^{-1}]$
4.	Latent heat	$J\,kg^{-1}$	$[M^0L^2T^{-2}]$
5.	Universal gas constant	$J\,mol^{-1}\,K^{-1}$	$[ML^2T^{-2}K^{-1}mol^{-1}]$
6.	Boltzmann's constant	JK^{-1}	$[ML^2T^{-2}K^{-1}]$
7.	Stefan's constant	$Js^{-1}m^{-2}\,K^{-4}$	$[MT^{-3}K^{-4}]$
8.	Planck's constant	Js	$[ML^2T^{-1}]$
9.	Solar constant	$J\,m^{-2}\,s^{-1}$	$[ML^0T^{-3}]$
10.	Thermal conductivity	$Js^{-1}m^{-1}\,K^{-1}$	$[MLT^{-2}]$
11.	Thermal resistance	$Ks^{-1}cal^{-1}$	$[M^{-1}L^{-1}T^3K]$
12.	Enthalpy	cal	$[ML^2T^{-2}]$
13.	Entropy	$cal\,K^{-1}$	$[ML^2T^{-2}K^{-1}]$

Illustration 8 : Show that the expression $v = v_0 + at$ is dimensionally correct, where v and v_0 represent speeds, a is acceleration, and t is a time interval.

Solution: For the speed dimensions will be

$$[v] = [v_0] = \text{Length/Time} = [L/T] = [LT^{-1}]$$

Dimension for acceleration will be $[LT^{-2}]$

So the dimension of $[at] = [LT^{-2}]\,[T] = [LT^{-1}]$

Therefore the expression is dimensionally correct.

Illustration 9 : Natural frequency (f) of closed pipe depends on

(1) Length of the tube (l) (2) Density of air (ρ) (3) Pressure of air (P)

Find the formula for frequency (f) using dimensional analysis.

Solution:

So we can say that f = (some number) $(l)^a\,(\rho)^b\,(P)^c$

$$\left[M^0L^0T^{-1}\right]=\left[M^0L^1T^0\right]^a\left[ML^{-3}T^0\right]^b\left[ML^{-1}T^{-2}\right]^c$$

$$\left[M^0L^0T^{-1}\right]=M^{b+c}L^{a-3b-c}T^{-2c}$$

Equating powers of M, L and T

$\Rightarrow$ $b + c = 0$

$a - 3b - c = 0$

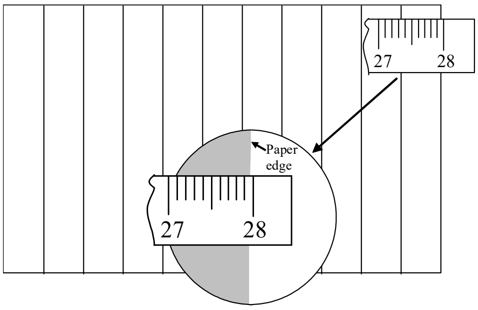
$-2c = -1$

$$\Rightarrow \quad a=-1,\ b=-\frac{1}{2},\ c=\frac{1}{2}$$

So $$f = (\text{some number})\ \frac{1}{l}\sqrt{\frac{P}{\rho}}$$

6. SIGNIFICANT FIGURES

Suppose you want to measure the length of a sheet of paper with an ordinary scale. Place the zero mark of the scale exactly at one end of the sheet and read the mark at other end. You may obtain a doughtful digit. It means that exact reading is not possible. It can be understood from the figure shown below.



The end of the sheet lies between 27.9 and 28.0 cm. Then you can estimate the distance between 27.9 cm and end of the sheet in this way. You mentally divide the 1 small division into 10 equal parts and guess on which part the edge is falling. You may notedown the reading as 27.96 cm. In this reading the digits 2, 7 and 9 are certain but 6 is doubtful. All these digits including doubtful digit are called significant digits. The rightmost or doubtful digit is called the least significant digit and the leftmost digit is called the most significant digit. We can define in this way the reliable digits plus the 1st uncertain digit are called significant digits or significant figure.

We can say, significant figures indicate the precision of measurement and depends on the least count of the measuring instrument.

6.1 THE RULES FOR DETERMINING THE NUMBER OF SIGNIFICANT FIGURES ARE AS FOLLOWS

(i) All the non-zero digits are significant.

Example: 156.78 contains five significant figure.

(ii) All the zeros between two non-zero digits are significant no matter where the decimal point is

Example: 108.006 contains six significant figures.

(iii) If the number is less than 1, the zeros on the right of decimal point but to the left of 1st non-zero digit are not significant.

Example: In $\underline{0}.\underline{00}2308$ the under lined zeros are not significant.

(iv) All the zeros to the right of the last non-zero digit. (The terminal trailing zeros) in a number without a decimal point are not significant.

Thus 123 m = 12300 cm = 123000 mm has three significant figures. The trailing zeros are not significant.

But if these are observed from a measurement, then they are significant.

(v) The trailing zeros in a number with a decimal point are significant.

The number 3.500 or 0.06900 have four significant figure.

Now note that choice of change of different units does not change the number of significant digits or figures in measurement.

☞ *The length 2.308* cm *has four significant figures, but in different units, the same value can be written as 0.02308* m *or 23.08* mm*. All these number have the same number of significant figures. It shows that location of decimal point does not matter in determining the number of significant figures.*

When there are zeros at the right end of the number, then there may be some confusion.

☞ *If length is 500* mm *and we don't know least count of the measuring instrument, then we can't be sure that last digits (zeros) are significant or not.*

Scientific Notation: If the scale had marking only at each meter, then the digit 5 can be obtained by eye approximation. So only 5 is significant figure, but if the markings at centimeters, then only 5,0 of the reading will be significant. If the scale used have marking in millimeters, all three digits 6,0,0 are significant. To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement in scientific notation, every number should express as $a \times 10^b$ where a is between 1 and 10 and b is any +ve or –ve power of 10, and decimal is placed after the first digit.

Now the confusion mention above can be removed.

$4.700 \text{ m} = 4.700 \times 10^2 \text{ cm} = 4.700 \times 10^3 \text{ m}$

Here power of 10 is irrelevant to the determination of significant figures.

6.2 SIGNIFICANT FIGURE IN ALGEBRAIC OPERATION

To know the number of significant figures after an algebric operation (Addition, subtraction, multiplication and division) certain rules can be followed which are as follows.

(i) In multiplication or division, the number of significant digits in the final result should be equal to the number of significant digits in the quantity, which has the minimum number of significant digits.

Example: If mass of an object measured is 4.237 gm (four significant figure) and its volume is measured to be 2.57cm^3 (three significant figure), then its density = 1.6486 gm/cm^3. But it should be up to three significant digits. Density = 1.65 gm/cm^3

(ii) In addition or subtraction the final result should retain as many decimal places as are there in the number with the least decimal place.

Example: Suppose we have to find out the sum of the numbers 436.32 gm, 227.2 g and 0.301 gm by arithmetic addition.

$$\begin{array}{r} 436.32 \\ 227.2 \\ +\ \ 0.301 \\ \hline 663.821 \\ \hline \end{array}$$

But the least precise measurement (227.2) gm is correct to only one decimal place. So final should be rounded off to one decimal place.

So sum will be 663.8 gm

Similarly in subtraction we follow the same rule.

6.3 ROUNDING OFF THE UNCERTAIN DIGIT: (LEAST SIGNIFICANT DIGIT):

The least significant digit is rounded according the rules given below.

(i) If the digit next to the least significant (Uncertain) digit is more than 5, the digit to be rounded is increased by 1.

(ii) If the digit next to the one rounded is less than 5, the digit to be rounded is left unchanged.

(iii) If the digit next to the one rounded is 5, then the digit to be rounded is increased by 1 if it is odd and left unchanged if it is even.

☞ *The insignificant digits are dropped from the result if they appear after the decimal point. Zero replaces them if they appear to the left of the decimal point.*

Example: Suppose we have to round of up to three significant digits to 15462.

In 15462, third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore increased by 6, The digits to be dropped should be replaced by zeros, because they appear to the left of decimal point thus 15462 becomes 15500 on rounding to three significant figure.

Illustration 10 : Round of the following numbers to three significant digits

(a) 14.745 (b) 14.750

Solution: (a) The third significant digit in 14.745 is 7. The number next to it is less than 5. So 14.745 becomes 14.7 on rounding to three significant digits.

(b) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

Illustration 11 : A cube has side of length $1.2\times10^{-2}\,\text{m}$. Calculate its volume.

Solution: Volume = $(\text{Side})^3 = (1.2\times10^{-2})^3 = 1.728\times10^{-6}\ \text{m}^3$

As the volume cannot have more than two decimal digits, hence volume should be reported as $1.7 \times 10^{-6}\ \text{m}^3$.

7. ERRORS

The measured value of a physical quantity is usually different from its true value. The result of any experimentally measured value contains some uncertainty. This uncertainty is called error.

In general, the errors in measurements can be broadly classified into two categories as

(a) Systematic errors and

(b) Random errors

(c) Accidental errors

Systematic Errors: These are the errors occurring due to the instruments used, imperfection in experimental technique or carelessness of the person performing the experiment. These errors tends to move in one direction only.

Random Errors: The random errors occur irregularly and therefore random in magnitude and sign. These errors tends to move in both directions. They may arise due to unpredictable fluctuation while performing the experiment. These errors can be minimized by taking a large number of observations for the same physical quantity.

Accidental Errors: Some time the measured value is to much small or to much larger then their average value. This is called accidental error. These values are dropped before calculating the average value.

Instrumental errors: These errors depend on the instruments used for measurement. Mainly following two types of errors occur due to instruments.

(i) **Zero error:** It is a type of systematic error. It occurs due to improper positioning of zero mark of the instrument.

(ii) **Least count error:** This is the error associated with the resolution of the instrument. Least count has been discussed in details in the earlier section.

For example to measure the length of an object its one end is placed at the zero mark of the scale and other end is found to come at 10th millimeter mark from the start. This implies that the length of the object is 10 mm. But, if one end of the object is placed at 1st mm mark instead of zero mark, the measurement will come out be 11 mm. So a zero error of 1 mm is introduced in the measurement.

ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR

Let a physical quantity is measured in an instrument. Various readings taken are x_1, x_2, x_3 ...., x_n. The true value of the measurement is taken as the mean of above readings.

$$\therefore \quad x = \frac{x_1 + x_2 + x_3 + \ldots.. + x_n}{n}$$

Absolute Error: The magnitude of the difference between the true value of the quantity and the individual measured value is called absolute error of the measurement.

Therefore, $\Delta x_1 = |x_1 - x|$ is the absolute error in measurement of x_1.

Mean absolute Error: The arithmetic mean of all the absolute errors is taken as the final or mean absolute error in the value of physical quantity.

$$\Delta x = \frac{\Delta x_1 + \Delta x_2 + \Delta x_3 + ... + \Delta x_n}{n}$$

The final answer or result of the experiment is reported as $x \pm \Delta x$. This implies that any measurement of physical quantity x is likely to fall between $(x - \Delta x)$ and $(x + \Delta x)$.

Relative error: Relative error is the ratio of the mean absolute error to mean value of quantity measured.

Relative error $= \frac{\Delta x}{x}$ It represents the accuracy in measurement i.e., smaller the relative error more is the accuracy.

Percentage error: When the relative error is expressed as percent, it is called the percentage error. Percentage error $= \frac{\Delta x}{x} \times 100\%$

For example in an experiment, the time period of oscillations of a simple pendulum is measured and the readings are 3.63 s, 3.56 s, 3.42 s, 3.71 s and 3.80 s.

The true value is $x = \frac{3.63 + 3.56 + 3.42 + 3.71 + 3.80}{5} = 3.62\,s$.

The absolute errors in the measurements are

$$\Delta x_1 = |x_1 - x| = 0.01\,s$$

$$\Delta x_2 = |x_2 - x| = 0.06\,s$$

$$\Delta x_3 = |x_3 - x| = 0.20\,s$$

$$\Delta x_4 = |x_4 - x| = 0.09\,s$$

$$\Delta x_5 = |x_5 - x| = 0.18\,s$$

Mean absolute error $\Delta x = \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5} = 0.11\,s$.

From the above calculations, we can infer following two points:

(i) The time period of oscillations of the pendulum is (3.62 ± 0.11) s.

(ii) As the absolute error is 0.11 s, there is already an error in the first digit after decimal. So there is no point in reporting to the answer to two digits after decimal. Thus, a better way to write the result is (3.6 ± 0.1) s.

Combination of errors: In many cases, a physical quantity depends on certain other measurable physical quantities. The accuracy or error in the measurement of the physical quantity depends on the errors in the quantities on which it depends. In calculations, errors can combine in different manners.

(a) **Error of a sum or difference:** Let a quantity Z depends on two quantities A and B as $Z = A + B$.

$$\therefore \quad Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$

Here DA and DB are absolute errors in the measurements of A and B. The above equation can be written as

$$Z \pm \Delta Z = (A + B) \pm (\Delta A + \Delta B)$$

$$\therefore \quad \Delta Z = \Delta A + \Delta B \text{ (i.e., absolute errors are added up)}$$

Note: The above result is applicable for $Y = A - B$ also. That is the maximum possible error in Y is $\Delta Y = \Delta A + \Delta B \cdot$

Illustration 12: The original length of a wire is (153.7 ± 0.6) cm. It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.

Solution : Elongation $\delta = l' - l = 155.3 - 153.7 = 1.6$ cm

Error in elongation $\Delta\delta = \Delta l' + \Delta l$

$= (0.6 + 0.2) = 0.8$ cm

So elongation = (1.6 ± 0.8) cm.

(b) **Error of a product or division:** Let $Z = A \cdot B$

$\therefore \quad Z \pm \Delta Z = (A \pm \Delta A) \cdot (B \pm \Delta B)$

$Z \pm \Delta Z = A \cdot B \pm A \cdot \Delta B \pm B \cdot \Delta A \pm \Delta A \cdot \Delta B$

$\Rightarrow \quad \Delta Z = A \cdot \Delta B + B \cdot \Delta A + \Delta A \cdot \Delta B$

When ΔA and ΔB are small, $\Delta A \cdot \Delta B$ can be neglected. So, we have,

$$\Delta Z = A \cdot \Delta B + B \cdot \Delta A \rightarrow \frac{\Delta Z}{Z} = \frac{A\Delta B}{Z} + \frac{B\Delta A}{Z} = \frac{\Delta B}{B} + \frac{\Delta A}{A}$$

or $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$ (i.e., relative errors are being added)

Note: Same result is applicable for division of two quantities.

That is if $Y = \dfrac{A}{B}$, the $\dfrac{\Delta Y}{Y} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

Alternate Method: $Z = A \cdot B$ or $\log Z = \log A + \log B$

Differentiating both sides w.r.t. Z, we get

$$\frac{1}{Z} = \frac{1}{A}\frac{dA}{dZ} + \frac{1}{B}\frac{dB}{dZ}$$

or $\dfrac{dZ}{Z} = \dfrac{dA}{A} + \dfrac{dB}{B}$ $\left(\text{This is same as } \dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}\right)$

Illustratiln 13: The mass of a container measured by a beam balance is 2.4 kg. Two small objects of masses 30.12 g and 30.15 g are added to the container. Calculate
(a) the total mass of the container and
(b) difference in masses of the pieces to correct significant figures?

Solution : **(a)** Total mass of the container $= 2.4 + \dfrac{30.12}{1000} + \dfrac{30.15}{1000}$

$= 2.4 + 0.03012 + 0.03015$

$= 2.46027$

= 2.5 kg (final result should contain one digit after decimal).

(b) Difference $= 30.15 - 30.12$

= 0.03 g (final result contains 2 digits after decimal).

Illustration 14: Area of a rectangle is measured by measuring its sides. The sides are found to be (2.0 ± 0.1) cm and (3.00 ± 0.01) cm. Find its area within proper error limits.

Solution : Area = side × side

$\therefore \quad A = 2.0 \times 3.00 = 6.0\ \text{cm}^2$ (Result must contain 2 significant digits)

As $\quad A = L \times B$

$\therefore \quad \dfrac{\Delta A}{A} = \dfrac{\Delta L}{L} + \dfrac{\Delta B}{B}$

or $$\Delta A = A\left(\frac{\Delta L}{L}+\frac{\Delta B}{B}\right) = 6.0\times\left(\frac{0.1}{2.0}+\frac{0.01}{3.00}\right) = 0.32\ \text{cm}^2$$

So, final result is, $A = (6.0 \pm 0.32)\ \text{cm}^2$

The result should be reported as $A = (6.0 \pm 0.3)$

Illustration 15: The length of an object is measured as 1.525 m. Find the error, relative error and percentage error.

Solution : As nothing is mentioned about the measuring instrument by which reading is taken, we shall assume that last digit of value 1.525 is uncertain. The value can lie between 1.524m – 1.526m.

$\therefore$ Absolute error = 0.001 m

Relative error $= \frac{0.001}{1.525} = 0.00065$

Percentage error = 0.065%

Note: If significant figures are considered in the above example, relative error would be 0.001 and percentage error would be 0.1%.

Illustration 16: If velocity of light in vacuum $(2.998\times10^8\ \text{ms}^{-1})$, acceleration due to gravity $(9.81\,\text{ms}^{-2})$ and density of mercury (13600 kg m^{-3}) be adopted as the fundamental units, then what will be the corresponding unit of mass.

Solution : Let the dependence of mass (M) on velocity (v), acceleration due to gravity (g) & density(ρ) be as $\text{M} = \text{v}^x\text{g}^y\rho^z$

Substituting the dimensions of v, g and r in R.H.S.

$$ML^0T^0 = [M^0LT^{-1}]^x[M^0LT^{-2}]^y[ML^{-3}]^z$$

or $$M^1L^0T^0 = M^zL^{x+y-3z}T^{-x-2y}$$

Comparing the dimensions on both sides, we get

$x = 6$, $y = -3$ and $z = 1$

Therefore, mass $[\text{M}] = \text{v}^6\text{g}^{-3}\rho$

Hence, unit of mass $= \frac{(2.998\times10^8)^6}{(9.81)^3}\times13600 = 1.05\times10^{52}\ \text{kg}$.

Illustration 17: A body moves in a straight line such that half of its journey is covered with velocity $V_1 = (15.0\pm0.5)\ \text{m/s}$ and the next half with velocity $V_2 = (30.0\pm0.1)\ \text{m/s}$. Find average velocity of the body.

Solution: It is given that $v_{av} = \frac{\text{Total displacement}}{\text{Total time}}$

$$v_{av} = \frac{2d}{\frac{d}{v_1}+\frac{d}{v_2}}$$

$$\Rightarrow \frac{1}{v_1}+\frac{1}{v_2} = \frac{2}{v_{av}}$$

$$\Rightarrow v_{av} = \frac{2\times15.0\times30.0}{45.0}$$

$$v_{av} = 20.0\ \text{m/s}$$

Now $\frac{dv_1}{v_1^2}+\frac{dv_2}{v_2^2}=\frac{2dv_{av}}{v_{av}^2}$

$$\frac{0.5}{(15.0)^2}+\frac{0.1}{(30.0)^2}=\frac{2dv_{av}}{(20.0)^2}$$

$$dv_{av}=\frac{1}{2}\left[\frac{0.5}{(15.0)^2}\times(20.0)^2+\frac{0.1}{(30.0)^2}\times(20.0)^2\right]$$

$$dv_{av}=0.5$$

So average velocity of the body is = (20.0 ± 0.5) m/s.

Operation	Formula	Maximum Absolute Error	Maximum Relative Error	Maximum Percentage Error
Sum	$A+B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A+B}$	$\left(\frac{\Delta A+\Delta B}{A+B}\right)\times 100$
Difference	$A-B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A-B}$	$\left(\frac{\Delta A+\Delta B}{A-B}\right)\times 100$
Multiplication	$A\times B$	$A\Delta B+B\Delta A$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A+A\Delta B}{B^2}$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A}\times 100$

8. DIFFERENTIAL CALCULUS

The purpose of differential calculus is to study the nature (i.e., increase or decrease) and the amount of variation in a quantity when another quantity (on which first quantity depends) varies independently.

Quantity: Anything which can be measured is called a quantity.

Constant: A quantity, whose value remains unchanged during mathematical operations, is called a constant quantity. The integers, fractions like π, e, etc are all constants.

Variable: A quantity which can have any numerical value between certain specified limit is called as variable.

Function: A quantity y is called a function of a variable x, if corresponding to any given value of x, there exists a single definite value of y. The phrase 'y is function of x' is represented as $y=f(x)$

For example, consider that y is a function of the variable x which is given by

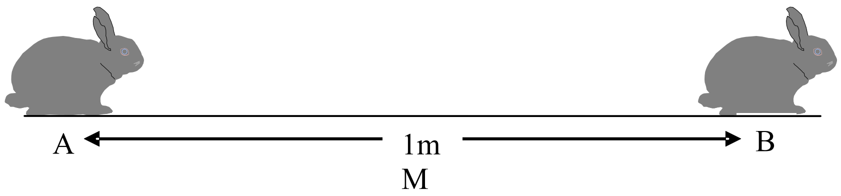
$$y=3x^2+7x+2$$

If x = 1, then $y=3(1)^2+7(1)+2=12$

and when $x=2, y=3(2)^2+7(2)+2=28$

Therefore, when the value of variable x is changed, the value of the function y also changes but corresponding to each value of x, we get a single definite value of y. Hence, $y=3x^2+7x+2$ represents a function of x.

MEANING OF LIMIT



Rabbit A wants to reach rabbit B, which sits stationary. A jumps half the distance remaining between them every second. How soon does the rabbit A reach its goal ?

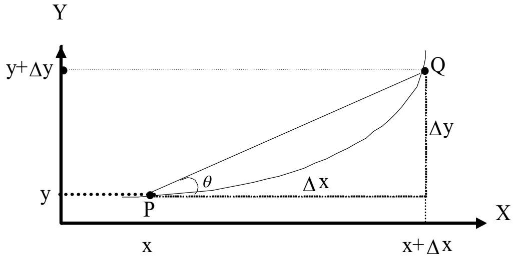
Never!

- The gap remaining between them becomes infinitesimal (i.e. very very small) after a long time.
- It is not a number, that can be said. It is smaller than the smallest positive number, that you can say.

The gap $\Delta x \to 0$ or it is dx. (dx denotes a very very small change in x.) and is represented as $\lim\limits_{\Delta x \to 0}$ (read as limit of delta x tends to zero)

- Rabbit A tends to rabbit B.
- Rabbit A's limit is rabbit B.

SLOPE OF SECANT



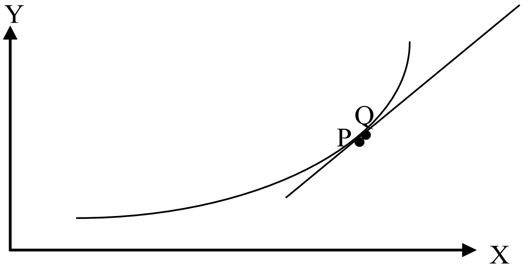
Consider a curve where the variation of a function y = f(x) is plotted with respect to variable x. Let P and Q be two points on the curve. The segment PQ is called as secant.

The slope of the secant: $\text{slope} = \tan\theta = \dfrac{\Delta y}{\Delta x}$

(Slope is defined as tan of angle between line and positive x -axis taken counter - clockwise)

GEOMETRIC MEANING OF DERIVATIVE

- Let P go closer to Q. When the gap between P and Q becomes infinitesimal (very very small), the secant can be approximated as tangent



Hence, $\frac{dy}{dx}$ (or $f'(x)$) at any point represents the rate of change of y (or $f(x)$) with respect to x at that point and is also known as derivative of y w.r.t. x.

PHYSICAL MEANING OF $\frac{dy}{dx}$

1. The ratio of small change in the function 'y' and the variable '*x*' is called the average rate of change of y. w.r.t. '*x*'. For example, if a body covers a small distance Δs in small time Δt, then average velocity of the body,

   the body, $v_{av} = \frac{\Delta s}{\Delta t}$ Also, if the velocity of a body changes by a small amount Δv in small time Δt,

   then average acceleration of the body, $a_{av} = \frac{\Delta v}{\Delta t}$
2. The differentiation of a function w.r.t. a variable implies the instantaneous rate of change of the function w.r.t. that variable.

   Thus, instantaneous velocity of the body, $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

   and instantaneous acceleration of the body, $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

Illustration 18: Find $\frac{dy}{dx}$ where $y = x^2$

Solution:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} \text{ (By definition)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2\Delta x \cdot x + (\Delta x)^2 - x^2}{\Delta x};$$

Here $(\Delta x)^2$ can be neglected as Δx itself is very small

$$= \lim_{\Delta x \to 0} \frac{2\Delta x \cdot x}{\Delta x} = 2x \text{ so, } \frac{d(x^2)}{dx} = 2x$$

$\frac{dy}{dx}$ OF FUNCTIONS AND THEIR PROPERTIES

We have found the derivative of x^2 with respect to x. Like wise we can also find derivatives of other functions.Some standard derivatives are as given in the table

y	dy/dx
x^n	nx^{n-1}
sinx	cosx
cosx	-sinx
tanx	sec^2x
cotx	$-cosec^2x$
secx	secx.tanx
cosecx	-cosecx.cotx
a^x	$a^x.\mathit{l}na$
*l*nx	*1*/x

MATHEMATICAL OPERATIONS FOR DERIVATIVES :

$$\frac{dK}{dx} = 0 ,$$

$$\frac{dKu}{dx} = K\frac{du}{dx}$$

$$\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

(Here, K is a constant and u and v are functions of x.)

Illustration19 : If $y = x^5$, then find $\frac{dy}{dx}$.

Solution: Given $y = x^5$

Differentiating both sides w.r.t x, using $\frac{dx^n}{dx} = nx^{n-1}$ $\frac{dy}{dx} = \frac{d}{dx}[x^5] = 5x^{5-1} = 5x^4$

Illustration20: If $y = x^2 + 5x^{3/2} + \frac{2}{x}$, then find $\frac{dy}{dx}$.

Solution: Differentiating both sides w.r.t. x, $\frac{dy}{dx} = \frac{d}{dx}\left[x^2 + 5x^{3/2} + \frac{2}{x}\right]$

$$= 2x + 5\left(\frac{3}{2}\right)x^{1/2} + 2(-1)x^{-2}$$

$$= 2x + \frac{15}{2}x^{1/2} - \frac{2}{x^2}$$

Illustration21 : If $y = e^x$ ln x, then find $\frac{dy}{dx}$.

Solution: Here, $u = e^x, v = \ln x$. Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[e^x \ln x]$$

Using product rule,

$$\frac{dy}{dx} = \frac{e^x d}{dx}[\ln x] + \ln x\frac{d}{dx}[e^x]$$

$$=e^x\frac{1}{x}+(\ln x)e^x$$

$$=\frac{e^x}{x}+e^x\ln x$$

Illustration 22: If $y=\frac{(x^2+2x)}{(3x-4)}$, then find $\frac{dy}{dx}$.

Solution: $u(x)=x^2+2x, v(x)=3x-4.$

Using quotient rule, $\frac{dy}{dx}=\frac{(3x-4)\frac{d(x^2+2x)}{dx}-(x^2+2x)\frac{d(3x-4)}{dx}}{(3x-4)^2}$

$$=\frac{(3x-4)(2x+2)-(x^2+2x)3}{(3x-4)^2}$$

$$=\frac{3x^2-8x-8}{(3x-4)^2}$$

CHAIN RULE

Suppose we have a function given by $y=g(h(x))$. Then its derivative w.r.t. x is given by

$$\frac{dy}{dx}=\frac{d(g(h(x)))}{d(h(x))}\cdot\frac{d(h(x))}{dx}$$

Illustration23: Find the derivative of $y=\ln(\sin^2 x)$

Solution. Here $f(x)=\ln(x)$ $g(x)=\sin^2 x$ $h(x)=\sin x$

$$\frac{dy}{dx}\cdot=\frac{d\log(\sin^2 x)}{d(\sin^2 x)}\cdot\frac{d(\sin x)^2}{d(\sin x)}\cdot\frac{d\sin x}{dx}=\frac{1}{\sin^2 x}\cdot 2\sin x\cdot\cos x=2\cot x.$$

Illustration24 : Find the derivative of $y=\sin^2(\ln x)$.

Solution $2\sin(\ln x)\cdot\cos(\ln x)\cdot\frac{1}{x}$.

Next, suppose we want to find the rate of change of volume ($V=\frac{4}{3}\pi r^3$) of a sphere w.r.t. time (i.e. $\frac{dV}{dt}$)

when the rate of change of radius w.r.t. time (i.e. $\frac{dr}{dt}$) is given. Then we can write

$$\frac{dV}{dt}=\frac{dV/dr}{dt/dr}=\frac{dV}{dr}.\frac{dr}{dt}=\frac{d(\frac{4}{3}\pi r^3)}{dr}.\frac{dr}{dt}=4\pi r^2.\frac{dr}{dt}$$

Illustration25 : The rate of change of radius of a sphere with respect to time is given as 4cm/s. Find the rate of change of volume when the radius is 50cm.

Solution: Given rate of change of radius w.r.t. time, $\frac{dr}{dt}=4\text{cm/s}$

Then, $\frac{dV}{dt}=4\pi r^2.\frac{dr}{dt}=4\pi\left(50^2\right)(4)\ =\ 40000\pi\text{cm}^3/\text{s}$

Illustration 26 : The radius of a cone is increasing at a rate of 2 cm/s. If the apex angle does not change, find the rate of change in height when R = 5 cm, H = 8 cm

Solution : If the apex angle does not change, for any R and H,

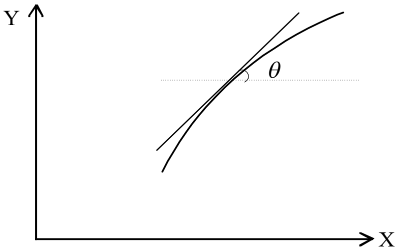
we have $R = H\ \tan\theta$

$$\Rightarrow \frac{dR}{dt}=\frac{dH}{dt}\cdot\tan\theta$$

$$\Rightarrow\ 2=\frac{dH}{dt}\cdot\frac{5}{8}\quad\Rightarrow\ \frac{dH}{dt}=\frac{16}{5}\text{cm/s}$$

Rate of change of height is $\frac{16}{5}$ cm/s.

INCREASING AND DECREASING FUNCTIONS

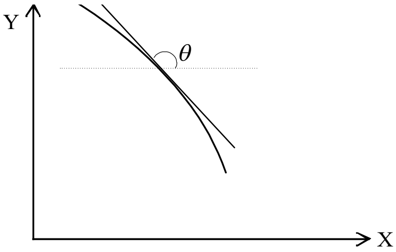


Increasing function

$0<\theta<\frac{\pi}{2}\Rightarrow\ \tan\theta>0\Rightarrow\frac{dy}{dx}>0$

Slope of tangent $>\ 0$

Rate of change of y w.r.t. x is positive, therefore the curve is increasing



Decreasing function

$\frac{\pi}{2}<\theta<\pi\ \ hence\ \tan\theta<0\Rightarrow\frac{dy}{dx}<0$

Slope of tangent <0

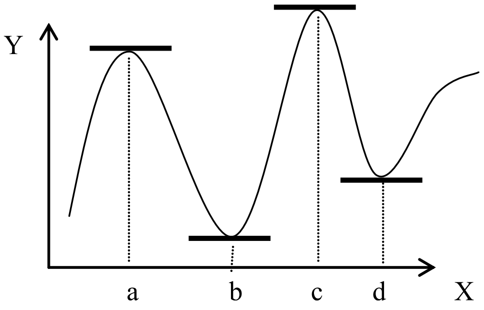
Rate of change of y w.r.t. x is negative, therefore the curve is decreasing

NOTE:

A function may not be continuously increasing or decreasing during the entire range (e.g. sin x, cos x).

Then we say function is increasing in the range where $\frac{dy}{dx}>0$ and decreasing where $\frac{dy}{dx}<0$.

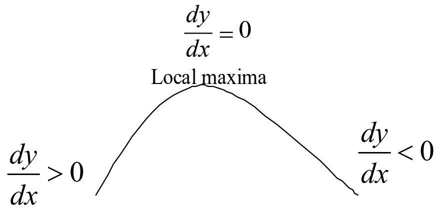
LOCAL MAXIMA AND LOCAL MINIMA

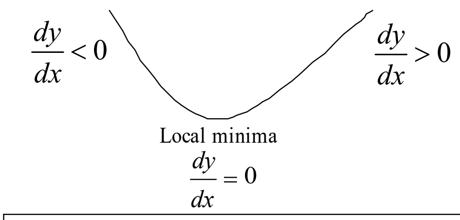


- Function is↑ for x<a,b<x<c,x > d
- Function is↓ for a<x<b, c<x<d
- Peaks(local maxima) at x = a , c
- Valleys(local minima) at x = b , d

For local maxima or minima, slope of the curve where it lies must be zero. So, $\frac{dy}{dx} = 0$

CRITERION FOR LOCAL MAXIMA AND LOCAL MINIMA





Condition for local maxima
- Slope changes from +ve to -ve
- Rate of change of slope w.r.t. x is -ve
- dy/dx is ↓
- $\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dx} \langle 0$

  $\Rightarrow \frac{d^2y}{dx^2} < 0$

Condition for local minima
- Slope changes from -ve to +ve
- Rate of change of slope w.r.t. x is +ve
- dy/dx is ↑
- $\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dx} \rangle 0$

  $\Rightarrow \frac{d^2y}{dx^2} > 0$

Note 1: When we talk of local maxima / minima, we are talking of x; while when we say local maximum value or local minimum value, we are talking of y (= f (x)) [corresponding to that particular local maxima or minima]

Note 2: Local maxima/minima does not mean that the function has the highest / lowest value at that point. It only means that the function was increasing before and decreasing after that point in case of local maxima and vice-versa in case of minima.

INCHAPTER EXERCISE - I

1. When x tends to y, is x equal to y ? What is (x-y) equal to ?

2. Which is smaller, infinitesimal or $10^{-100000000000000000000000000000000000000000000000000000000}$?

3. What is 100000000000 x (infinitesimal)?

4. What is 100000000000 x $(10^{-100000000000})$?

5. What is (1/infinity) ?

Differentiate the following functions w.r.t. x

6. $(4x^2 - 7x + 5).\sec x$

7. $x^4(5\sin x - 3\cos x)$

8. $\frac{\sqrt{x}+1}{\sqrt{x}-1}$

9. $\frac{x\ln x}{e^x}$

10. Find the points of local maxima or local minima for the following:

(i) $y = \frac{x^3}{3} - x$ (ii) $y = \sin x,\ x\varepsilon(0,\ 2\pi)$

11. Find the local maximum and local minimum value for the following:

(i) $y = x^3 - 3x + 10$ (ii) $y = \frac{x^2}{2} + \frac{1}{x}$

ANSWER KEY

1. No, dx 2. infinitesimal 3. infinitesimal 4. 1

5. 0 6. $(8x-7)\sec x + (4x^2 - 7x + 5)\sec x.\tan x$

7. $x^4(5\cos x + 3\sin x) + 4x^3(5\sin x - 3\cos x)$

8. $\frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$ 9. $\frac{1+\ln x - x\ln x}{e^x}$ 10. (i) $x = \pm 1$ (ii) $x = \frac{\pi}{2}, \frac{3\pi}{2}$

11. (i) Local minima at x = 1, value = 8

Local maxima at x = -1, value = 12

(ii) Local minima at x = 1 , value = 3/2

9. INTEGRAL CALCULUS

In integral calculus, the differential coefficient of a function is given. We are required to find the function. Thus, integration is the reverse of differentiation.

'$\int$ ' sign is used for integration. If I is integration of f(x) with respect to x then $I = \int f(x)\,dx$ and it is read as integration of f(x) w.r.t. x is I

For example, let us proceed to obtain intergral of x^n w.r.t. x. We already know that

$$\frac{d}{dx}(x^{n+1}) = (n+1)x^n$$

Since the process of integration is the reverse process of differentiation,

$$\int (n+1)x^n dx = x^{n+1} \text{ or } \int x^n dx = \frac{x^{n+1}}{n+1}$$

The above formula holds for all values of n, except $n = -1$.

It is because, for $n = -1$, $\int x^n dx = \int x^{-1}dx = \int \frac{1}{x}dx$ $\because \frac{d}{dx}(\log_e x) = \frac{1}{x}$ $\therefore \int \frac{1}{x}dx = \log_e x$

Similarly, the formula for integration of some other functions can be obtained if we know the differential coefficients of various functions

BASIC INTEGRATION FORMULAS

1. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x dx = e^x + C$

5. $\int \sin xdx = -\cos x + C$

6. $\int \cos xdx = \sin x + C$

7. $\int \tan xdx = \ln|\sec x| + C$

8. $\int \cot xdx = -\ln|\text{cosec}x| + C$

9. $\int \sec xdx = \ln|\sec x + \tan x| + C$

10. $\int \text{cosec}dx = -\ln|\text{cosec}x + \cot x| + C$

11. $\int \sec^2 xdx = \tan x + C$

12. $\int \text{cosec}^2 xdx = -\cot x + C$

13. $\int \sec x \tan xdx = \sec x + C$

14. $\int \text{cosec}x \cot xdx = -\text{cosec}x + C$

Illustration 27: Find the following integrals:

1. $\int 6.e^x dx$ **2.** $\int \frac{5}{x}dx$

3. $\int 3\cos xdx$ **4.** $\int [x^3 + 2x^2 + 3x - 4]dx$

Solutions:

1. $\int 6.e^x dx = 6\int e^x dx = 6e^x + c$

2. $\int \frac{5}{x}dx = 5.\int \frac{1}{x}dx = 5.\ln|x| + c$

3. $\int 3\cos xdx = 3\int \cos xdx = 3\sin x + c$

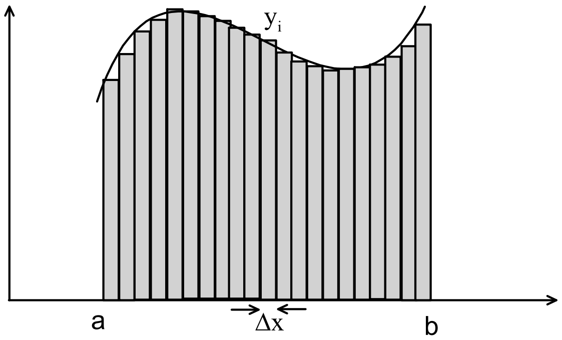
4. $\int [x^3 + 2x^2 + 3x - 4]dx = \int x^3 dx + \int 2x^2 dx + \int 3xdx - \int 4dx$

$$= \int x^3 dx + 2\int x^2 dx + 3\int xdx - 4\int dx = \frac{x^4}{4} + c_1 + \frac{2x^3}{3} + c_2 + 3\frac{x^2}{2} + c_3 - 4x + c_4$$

$$= \frac{x^4}{4} + \frac{2x^3}{3} + \frac{3x^2}{2} - 4x + c$$

DEFINITE INTEGRAL

Consider the curve as shown. The area under the curve (the area bounded by the curve and the x-axis) can be found by dividing this area into infinitesimal areas and adding them up.



Consider this area to be divided into n parts, where each part can be assumed as a rectangle if n is very large. The length of each such part at $x = x_i$ will be equal to $y_i = f(x_i)$ while the breadth will be equal to Δx where $\Delta x = \frac{b-a}{n}$

Area of each rectangle = $A_i = y_i.\Delta x$

The total area will be the sum of all these areas and will be given by

$$A = \sum_{i=1}^{i=n} y_i \Delta x$$

If $\Delta x \to 0$, the same area is represented by $\int_{x=a}^{x=b} y\,dx$ or $\int_{x=a}^{x=b} f(x)\,dx$

This integral is known as definite integral of the curve y = f(x) between x = a to x = b, where a and b are known as the lower and upper limits of the integral respectively.

FUNDAMENTAL THEOREM OF INTEGRAL CALCULUS:

If $\int f(x)dx = F(x) + C$, then $\int f(x)dx = F(x) + C, \int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

Illustration 28 : Integrate $\int_0^{\pi/2} (\sin x + \cos x)dx$

Solution: $\int_0^{\pi/2} (\sin x + \cos x)dx = \int_0^{\pi/2} \sin x dx + \int_0^{\pi/2} \cos x dx$

$$= \left[-\cos x\right]_0^{\pi/2} + \left[\sin x\right]_0^{\pi/2} = -\left[0-1\right] + \left[1-0\right]$$

$$= 1 + 1 = 2$$

Illustration 29 : Integrate $\int_0^1 (x^{3/2}+2e^x)dx$

Solution: $$\int_0^1 (x^{3/2}+2e^x)dx = \int_0^1 x^{3/2}dx + 2\int_0^1 e^x dx = \left[\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_0^1 + \left[2e^x\right]_0^1$$

$$= \left[\frac{x^{5/2}}{5/2}\right]_0^1 + \left[2e^x\right]_0^1 = \left[\frac{1}{5/2}-0\right] + \left[2e^1 - 2e^o\right] = \frac{2}{5} + 2e - 2$$

APPLICATION OF CALCULUS IN KINEMATICS

The problems in kinematics can be solved using the differential and integral calculus, in addition to the already known equations which are given as under

$$v = \frac{dx}{dt} \;;$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\times\frac{dx}{dt} = v\frac{dv}{dx} \;;\; a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

(Where 'x' is displacement, 'v' is velocity and 'a' is acceleration)

Similarly, for circular motion

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\times\frac{d\theta}{dt} = \frac{\omega d\omega}{d\theta} \qquad \text{or} \quad \alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$$

(Where 'θ' is angular displacement, 'ω' is angular velocity and 'α' is angular acceleration)

Also by the definition of integral,

$$\Delta x = \int v dt \qquad \Delta v = \int a dt$$

$$\Delta\theta = \int \omega dt \qquad \Delta\omega = \int \alpha dt$$

NOTE:

$\frac{dx}{dt}$ is called as instantaneous velocity, it is the velocity in small time dt. Average velocity over a period of time Δt can be given as $v_{avg} = < v > = \frac{\Delta x}{\Delta t}$.

Illustration 30: The speed of a particle is given by $v = (3t^2 + t + 2) m/s$. Find the (a) distance covered by it in first two seconds, (b) acceleration at time t = 1s.

Solution. (a) for calculating distance,

$$v = 3t^2 + t + 2$$

Also we have $v = \frac{ds}{dt} \Rightarrow \frac{ds}{dt} = 3t^2 + t + 2 \Rightarrow ds = \left(3t^2 + t + 2\right)dt$

Integrating both the sides, we have $\int_{s=0}^{s=s} ds = \int_{t=0}^{t=2} (3t^2 + t + 2)dt$

Limits are put s = 0 to s = s and t = 0 to t = 2

$$\therefore \quad [s]_0^s = \left[t^3 + \frac{t^2}{2} + 2t\right]_0^2$$

$$[s - 0] = [8 + 2 + 4] - [0 + 0 + 0]$$

$$s = 14 \text{ m}$$

$\therefore$ Distance covered in first two seconds is 14 m.

(b) For calculating acceleration, $V = 3t^2 + t + 2$

Also we have $a = \frac{dv}{dt}$

$$\therefore \quad a = \frac{d(3t^2 + t + 2)}{dt} = 6t + 1$$

at t = 1s, acceleration = 6(1) + 1 = 7 m/s^2

INCHAPTER EXERCISE - II

Find the following Integrals :

1. $\int \left[x^5 + \frac{2}{x^2} - \frac{1}{x} - \frac{4}{\sqrt{x}} + 10\right] dx$

2. $\int \left[7e^x + 4\sin x - \frac{9}{x^3} + e\right] dx$

3. $\int_1^5 (3 + 2t)\, dt$

4. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\sin x - \cos x)\, dx$

5. If $s = (2t + 4t^2)m$, find the values of 'v' and 'a' at $t = 0s$, 2s and 10s.

6. If $v = 3t^2$ m/s, find the values of 's' and 'a' at $t = 0s$, 2s, & 10s.(assume all quantities to be zero at the start)

7. If $v = u + at$, derive $s = ut + 0.5at^2$. (where a is a constant)

8. Use $\frac{dv}{ds}\frac{ds}{dt} = a$, to prove $v^2 = u^2 + 2as$. (where a is a constant)

ANSWER KEY

1. $\frac{x^6}{6} - \frac{2}{x} - \ln x - 8\sqrt{x} + 10x + c$

2. $7e^x - 4\cos x + \frac{9}{2x^2} + ex + c$

3. 36

4. $\frac{3}{\sqrt{2}} - 1$

5. $v = 2m/s, 18m/s, 82m/s\,;\ a = 8m/s^2\ , 8m/s^2,\ 8m/s^2$

6. $s = 0m, 8m, 1000m\,;\ a = 0m/s^2, 12m/s^2, 60m/s^2$

Subjective Questions

Differentiate the given functions with respect to 'x' . (a, b, c are to be treated as constants)

1. $y = (x^2 - 3x + 3)(x^2 + 2x - 1);$

2. $y = \dfrac{x+1}{x-1}$

3. $y = \dfrac{x}{x^2 + 1}$

4. $y = \dfrac{ax+b}{cx+d}$

5. $z = \dfrac{x^2+1}{3\left(x^2-1\right)} + \left(x^2-1\right)\left(1-x\right)$

6. $y = \dfrac{1-x^3}{1+x^3}.$

7. $y = \dfrac{2}{x^3 - 1}.$

8. $y = \dfrac{x^2 - x + 1}{a^3 - 3}$

9. $y = \dfrac{1-x^3}{\sqrt{\pi}}.$

10. $y = \sin x + \cos x$

11. $y = \text{x} \sin x + \cos x$

12. $y = \cos^2 x.$

13. $y = 3\sin^2 x - \sin^3 x.$

14. $y = x \ln x$

15. $y = \ln^2 x.$

16. $y = \ln x^2$

17 $\int \sqrt{x}dx$

18. $\int \sqrt[m]{x^n}\, dx.$

19. $\int \dfrac{dx}{x^2}$

20. $\int \dfrac{dx}{2\sqrt{x}}.$

Level - I

DIMENSIONAL ANALYSIS

1. The velocity ‘v’ of a particle is given in terms of time ‘t’ where $v = at + \frac{b}{t+c}$. The dimensions of a, b, c are

(a) L^2 ; T ; LT^{-2}
(b) LT^2 ; LT ; L
(c) LT^{-2} ; L ; T
(d) L ; LT ; T^2

2. In a particular system, the unit of length, mass and time are chosen to be 10cm, 10 g and 0.1s respectively. The unit of force in this system will be equivalent to

(a) 0.1 N
(b) 1 N
(c) 10 N
(d) 100 N

3. The time dependence of a physical quantity ‘P’ is given by $P = P_0 \exp(-\alpha t^2)$, where α is a constant and ‘t’ is time. The constant α is

(a) dimensionless
(b) has dimension of T^{-2}
(c) has dimensions of P
(d) has dimension of T^2

4. Which of the following sets can enter into the list of fundamental quantities in any system of units?

(a) Length, time and velocity
(b) Length, mass and velocity
(c) Mass, time and velocity
(d) Length, mass and time

5. In the standard equation $S_{nth} = u + \frac{a}{2}[2n-1]$, what dimensions do you view for S_{nth}?

(a) $M^o L^1 T^o$
(b) $M^o L^{-1} T^1$
(c) $M^o L^1 T^{-1}$
(d) $M^o L^o T^1$

6. If units of length, mass and force are chosen as fundamental units, the dimensions of time would be:

(a) $M^{1/2} L^{-1/2} F^{1/2}$
(b) $M^{1/2} L^{1/2} F^{1/2}$
(c) $M^{1/2} L^{1/2} F^{-1/2}$
(d) $M^1 L^{-1/2} F^{-1/2}$

7. The dimensions of Intensity are:

(a) $L^0 M^1 T^{-3}$
(b) $L^1 M^2 T^{-2}$
(c) $M^1 L^2 T^{-2}$
(d) $M^2 L^2 T^{-3}$

8. Given : Force = $\frac{\alpha}{\text{Density} + \beta^3}$. What are the dimensions of α, β ?

(a) $ML^{-2}T^{-2}, ML^{-1/3}$
(b) $M^2L^4T^{-2}, M^{1/3}L^{-1}$
(c) $M^2L^{-2}T^{-2}, M^{1/3}L^{-1}$
(d) $M^2L^{-2}T^{-2}, ML^{-3}$

9. Which one of the following has the dimensions of pressure?

(a) $\frac{ML}{T^2}$
(b) $\frac{M}{L^2T^2}$
(c) $\frac{M}{LT^2}$
(d) $\frac{M}{LT}$

10. Which one of the following quantities has not been expressed in proper units?

(a) Coefficient of Elasticity : N/m^2 (b) Surface Tension : N/m

(c) Energy : kg m/s (d) Pressure : N/m^2

11. Dimensions of Gravitational constant are

(a) $M^{-1} L^3 T^{-2}$ (b) $M^{-2} L^3 T^{-1}$

(c) $M^3 L^{-1} T^{-2}$ (d) $M^{-1} L^2 T^{-3}$

12. The dimensional formula of Electric flux is

(a) $[M^1 L^2 T^{-2} A^{-1}]$ (b) $[M^1 L^0 T^{-2} A^{-2}]$

(c) $[M^0 L^{-2} T^{-2} A^{-2}]$ (d) $[M^1 L^3 T^{-3} A^{-1}]$

13. The unit of Impulse is the same as that of

(a) Energy (b) Force

(c) Angular momentum (d) Linear momentum

14. If force (F) is given by $F = Pt^{-1} + \alpha t$, where t is time. The unit of P is same as that of

(a) velocity (b) displacement

(c) acceleration (d) momentum

15. Lux is the unit of

(a) intensity of illumination (b) luminous efficiency

(c) luminous flux (d) luminous intensity

16. A calorie is a unit of heat energy and equals 4.2 J. Suppose we use a system of units in which the unit of mass is α kg. The unit of length is β metre and unit of time is γ second. In this new system 1 calorie is equal to

(a) $\alpha^{-1}\beta^{-2}\gamma^{2}$ (b) $4.2\alpha\beta^{2}\gamma^{-2}$

(c) $\alpha\beta^{2}\gamma^{-2}$ (d) $4.2\alpha^{-1}\beta^{-2}\gamma^{2}$

17. CGS unit of Coefficient of Viscosity is Poise. If one poise is equal to x in a system of units in which unit of mass is 2 kg, unit of length 2m and that of time 4sec, x is equal to

(a) 0.40 (b) 0.62

(c) 1.86 (d) 3.92

18. The dimension of $\frac{h}{e}$ is the same as of (here h is Planck's constant and e is electronic charges)

(a) voltage (b) magnetic flux

(c) current (d) angular momentum

19. The dimensions of $B^2 L^2 C$ is same as that of (where B is magnetic field, L is length and C is capacitance)

(a) mass (b) length

(c) time (d) force

20. Which of the options has two quantities that have different dimensions
(a) Reynold's number and coefficient of friction
(b) Curie and frequency of light
(c) Latent heat and gravitation constant
(d) Planck's constant and angular momentum

ERROR ANALYSIS AND SIGNIFICANT FIGURE

21. The number of significant figures in 3400 is
(a) 3 (b) 4
(c) 2 (d) 1

22. The length and breadth of a metal sheet are 3.124 m and 3.002 m respectively. The area of this sheet up to four correct significant figures is
(a) 9.37 m^2 (b) 9.378 m^2
(c) 9.3782 m^2 (d) 9.378248 m^2

23. The breadth of a thin rectangular sheet is measured as 10.1 cm. The uncertainty in the measurement is
(a) ± 1% (b) ± 0.5 %
(c) ± 0.1 % (d) ± 5%

24. The least count of a stop watch is $\frac{1}{5}$ s. The time of 20 oscillations of a pendulum is measured as 25s. The percentage error in the measurement of time will be
(a) 0.1% (b) 0.8%
(c) 1.8% (d) 8%

25. The radius of a circle is stated a 2.12 cm. Its area should be written as
(a) 14 cm^2 (b) 14.1 cm^2
(c) 14.11 cm^2 (d) 14.1124 cm^2.

26. The density of a cube is found by measuring its mass and the length of its side. If the maximum errors in the measurement of mass and length are 0.3% and 0.2% respectively, the maximum error in the measurement of density is
(a) 0.3% (b) 0.5%
(c) 0.9% (d) 1.1%

27. The kinetic energy of a particle depends on the square of speed of the particle. If error in measurement of speed is 40%, the error in the measurement of kinetic energy will be
(a) 40% (b) 80%
(c) 96% (d) 20%

Level - II

DIMENSIONAL ANALYSIS

1. A unitless quantity

(a) Never has a nonzero dimension
(b) Always has a nonzero dimension
(c) May have a nonzero dimension
(d) Does not exist.

2. Which of the following can be used as a set of fundamental quantities in place of mass, time & length?

(a) length, velocity, time
(b) momentum, mass, velocity
(c) force, mass, velocity
(d) momentum, time, frequency

3. If we change unit of a physical quantity then

(a) its dimension changes
(b) its dimension remain same
(c) it may change or may not change
(d) its magnitude changes

4. Energy due to position of a particle is given by, $U = \frac{\alpha\sqrt{y}}{y+\beta}$, where α and β are constants, y is distance. The dimensions of $(\alpha \times \beta)$ are

(a) $[M^0LT^0]$
(b) $[M^{1/2}L^{3/2}T^{-2}]$
(c) $[M^0L^{-7/2}T^0]$
(d) $[ML^{7/2}T^{-2}]$

5. Which of the following equations can be dimensionally correct.(Symbol have their usual notations).

(a) $t = 2\pi\sqrt{\frac{ml^2}{E}}$
(b) $l = \frac{F}{m^2(f_1^2 - f_2^2)}$
(c) $t = \sqrt{\frac{ml^2}{6F\sin\theta}}$
(d) $a = \left(\frac{lv^2}{Gm}\right)^2$

6. Which of the following group have different dimension?

(a) Potential difference, EMF, Voltage
(b) Pressure, Stress, Young's modulus
(c) Heat, Energy, Work-done
(d) Dipole moment, Electric flux, Electric field

7. Kinetic energy (K) depends upon momentum (p) and mass (m) of a body.
If $K \alpha p^a m^b$ then find the value of 'a' and 'b'.

(a) a = 1; b = 1
(b) a = 2; b = –1
(c) a = 2; b = 1
(d) a = 1; b = 2

8. Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross–sectional area of the stream (A). The expression of the force should be

(a) ρAv
(b) ρAv^2
(c) $\rho^2 Av$
(d) $\rho A^2 v$

9. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:

(a) FT^2
(b) $F^{-1}A^2T^{-1}$
(c) FA^2T
(d) AT^2

CALCULUS

From problems 10 to 14, differentiate the following w.r.t. x

10. $\frac{1}{\sqrt{x}}$

(a) $x^{-\frac{3}{2}}$
(b) $-\frac{1}{2}x^{-3/2}$
(c) $\frac{1}{\sqrt{x}}$
(d) $\frac{1}{2}x^{\frac{1}{2}}$

11. $\frac{1}{(ax+b)^2}$

(a) $-2a\,(ax+b)^{-3}$
(b) $-2a(ax+b)^{-1}$
(c) $a(ax+b)^{-1}$
(d) $(ax+b)^{-1}$

12. $x^3+\frac{1}{x^3}+8$

(a) $3x^{-2}+8$
(b) $3x^2+3x^3$
(c) $3x^2-3x^{-4}$
(d) $3x^{-2}+3x^2$

13. $\sin x^3$

(a) $3.\cos x^3$
(b) $\cos^3 x$
(c) $\cos^3 x.3x^2$
(d) $3x^2\cos x^3$

14. $\sqrt{4x^3-5}$

(a) $(4x^2-5)^{\frac{-1}{2}}$
(b) $\sqrt{4x^3-5}$
(c) $6x^2(4x-5)^{\frac{1}{2}}$
(d) $6x^2(4x^3-5)^{-1/2}$

15. $\frac{d}{dx}\sin(\ln x)$:

(a) $\cos(\ln x)$
(b) $\ln(\cos x)$
(c) $x\cos(\ln x)$
(d) $\frac{\cos(\ln x)}{x}$

16. $\frac{d}{dx}\sqrt{2x^2+1}$

(a) $2x(2x^2+1)^{1/2}$
(b) $2x(2x^2+1)^{-1/2}$
(c) $(2x^2+1)^{1/2}$
(d) $(2x^2+1)^{-1/2}$

17. $\dfrac{d}{dx}e^{\sqrt{2x}}$

(a) $\dfrac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (b) $\sqrt{2x}\ \mathrm{e}^{\sqrt{2x}}$

(c) $e^{\sqrt{2x}}$ (d) $2^{(2x)^{-1/2}}$

18. $\dfrac{d}{dx}(x^4 - 2\sin x + 3\cos x)$

(a) $4x^3 - 2\cos x + 3\sin x$ (b) $3x^2 + 2\cos x + 3\sin x$

(c) $4x^3 + 2\cos x - 3\sin x$ (d) $4x^3 - 2\cos x - 3\sin x$

19. $\dfrac{d}{dx}\left(x^2 \sin x \cdot nx\right)$

(a) $2x\sin \mathrm{x}\ nx + \mathrm{x}^2\cos x\ nx + nx^2 \sin x$ (b) $x^2\sin x\ .\ nx + 2x\cos x\ nx + x\sin x$

(c) $2x.\ \sin x\ nx + x^2\cos x\ nx + \sin x$ (d) None of these

20. $\dfrac{d\left(\dfrac{x^2+1}{x+1}\right)}{dx}$

(a) $\dfrac{x^2+2x-1}{(x+1)^2}$ (b) $\dfrac{x^2-2x+1}{(x+1)^2}$

(c) $\dfrac{x^2+2x-1}{x+1}$ (d) $\dfrac{x^2+2x+1}{(x+1)^2}$

21. If $V = \dfrac{4}{3}\pi r^3$, find $\dfrac{dV}{dr}$

(a) $\dfrac{4}{3}\pi r^2$ (b) $4\pi r^3$

(c) $4\pi r$ (d) $4\pi r^2$

22. $xy = c^2$, then $\dfrac{dy}{dx}$

(a) $\dfrac{x}{y}$ (b) $\dfrac{y}{x}$

(c) $-\dfrac{x}{y}$ (d) $-\dfrac{y}{x}$

23. $x = at^2$; $y = 2at$, then $\dfrac{dy}{dx}$ =

(a) t (b) $\dfrac{1}{t}$

(c) 1 (d) None of these

24. $\int \sqrt[5]{x}\, dx$

(a) $\dfrac{5}{6}x^{6/5} + C$ (b) $\dfrac{6}{5}x^2 + C$

(c) $\dfrac{6}{3}x^3 + C$ (d) None of these

25. $\int \dfrac{1}{(ax+b)^2} dx$

(a) $(ax+b)^{-1} + C$ (b) $(ax+b)^3 + C$

(c) $-\dfrac{1}{a}\left(\dfrac{1}{ax+b}\right) + C$ (d) None of these

26. $\int \sin x . \cos x \; dx$

(a) $-\dfrac{\cos 2x}{4} + C$ (b) $\dfrac{\sin 2x}{4} + C$

(c) $\cos 2x + C$ (d) None of these

27. $\int \dfrac{x}{x^2 + a^2} dx$

(a) $(x^2 + a^2)^{\frac{1}{2}} + C$ (b) $\dfrac{1}{2}\log_e(x^2 + a^2) + C$

(c) $\log_e\left(x^2 + a^2\right) + C$ (d) None of these

28. $\int_{-\pi/2}^{\pi/2} \cos x\, dx$

(a) 0 (b) 1

(c) 2 (d) None of these

29. $\int_0^{\pi/2} \sqrt{1+\cos x}\, dx$

(a) 2 (b) 1

(c) 0 (d) None of these

30. $\int (1-x)\sqrt{x}\, dx$:

(a) $\frac{2}{3}x^{3/2}+\frac{2}{5}x^{5/2}+C$ (b) $-\frac{2}{3}x^{3/2}+\frac{2}{5}x^{5/2}+C$

(c) $-\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}+C$ (d) $\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}+C$

31. $\int_0^{\pi/2} (\sin x+\cos x)dx$

(a) 2 (b) 1

(c) 3 (d) 4

ASSERTION & REASON

Choose any one of the following four responses :

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) If assertion is true but reason is false.

(d) If the assertion and reason both are false.

1. Assertion : '*Light year*' and 'Wavelength' both measure distance.
Reason : Both have dimensions of time.

2. Assertion : Light year and year, both measure time.
Reason : Because light year is the time that light takes to reach the earth from the sun.

3. Assertion : Force cannot be added to pressure.
Reason : Because their dimensions are different.

4. Assertion : Linear mass density has the dimensions of $[M^1L^{-1}T^0]$.
Reason : Because density is always mass per unit volume.

5. Assertion : Rate of flow of a liquid represents velocity of flow.
Reason : The dimensions of rate of flow are $[M^0L^1T^{-1}]$.

6. Assertion : Dimesion of Rydberg constant 'R' is L^{-1}
Reason : It follows from Bohr's formula $\bar{v} = R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$, where the symbols have their usual meaning.

7. Assertion : Parallex method cannot be used for measuring distances of stars more than 100 light years away.

Reason : Because parallex angle reduces so much that it cannot be measured accurately.

8. Assertion : Number of significant figures in 0.005 is one and that in 0.500 is three.

Reason : This is because zeros are not significant.

9. Assertion : Out of three measurements $l = 0.7\ m;\ l = 0.70\ m$ and $l = 0.700\ m$, the last one is most accurate.

Reason : In every measurement, only the last significant digit is not accurately known.

10. Assertion : Mass, length and time are fundamental physical quantities.

Reason : They are independent of each other.

11. Assertion : Density is a derived physical quantity.

Reason : Density cannot be derived from the fundamental physical quantities.

12. Assertion : Now a days a standard *metre* is defined as in terms of the wavelength of light.

Reason : Light has no relation with length.

13. Assertion : Radar is used to detect an aeroplane in the sky

Reason : Radar works on the principle of reflection of waves.

14. Assertion : Surface tension and surface energy have the same dimensions.

Reason : Because both have the same S.I. unit

15. Assertion : In $y = A\sin(\omega t - kx)$, $(\omega t - kx)$ is dimensionless.

Reason : Because dimension of $\omega = [M^0L^0T]$.

16. Assertion : Radian is the unit of distance.

Reason : One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

17. Assertion : A.U. is much bigger than Å.

Reason : A.U. stands for astronomical unit and Å stands from *Angstrom*.

18. Assertion : When we change the unit of measurement of a quantity, its numerical value changes.

Reason : Smaller the unit of measurement smaller is its numerical value.

19. Assertion : Dimensional constants are the quantities whose value are constant.

Reason : Dimensional constants are dimensionless.

20. Assertion : The time period of a pendulum is given by the formula, $T = 2\pi\sqrt{g/l}$.

Reason : According to the principle of homogeneity of dimensions, only that formula is correct in which the dimensions of L.H.S. is equal to dimensions of R.H.S.

21. Assertion : In the relation $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$, where symbols have standard meaning, *m* represent linear mass density.

Reason : The frequency has the dimensions of inverse of time.

22. Assertion : The graph between *'P'* and *'Q'* is straight line, when *'P/Q'* is constant.

Reason : The straight line graph may mean that *'P'* is proportional to *'Q'* or *'P'* is equal to constant multiplied by *'Q'*.

23. Assertion : Avogadro number is the number of atoms in one gram mole.

Reason : Avogadro number is a dimensionless constant.

24. Assertion : *L/R* and *CR* both have same dimensions.

Reason : *L/R* and *CR* both have dimension of time.

25. Assertion : The quantity $(1/\sqrt{\mu_0\varepsilon_0})$ is dimensionally equal to velocity and numerically equal to velocity of light.

Reason : μ_0 is permeability of free space and ε_0 is the permittivity of free space.

Previous Year's Questions

UNITS

1. The SI unit of activity of a radioactive sample is **[J&K CET 2011]**
(a) Curie (b) Rutherford
(c) Becquerel (d) Millicurie

2. SI unit of power is **[J&K CET 2011]**
(a) Joule (b) Erg
(c) Newton (d) Watt

3. The SI unit of thermal conductivity is **[J&K CET 2011]**
(a) $Js m^{-1} K^{-1}$ (b) $W^{-1} m^{-1} K^{-1}$
(c) $Wm^{-1} K^{-1}$ (d) $Wm^{-2} K^{-2}$

4. The unit of magnetic moment is **[Guj CET 2010]**
(a) TJ^{-1} (b) JT^{-1}
(c) Am^{-2} (d) Am^{-1}

5. Unit of electrical conductivity is **[UP CPMT 2010]**
(a) ohm (b) siemen
(c) m/mho (d) mho/m

6. Which one of the following quantities has not been expressed in proper units? **[Kerala CEE 2009]**
(a) Torque : Newton metre
(b) Stress : Newton metre^{-2}
(c) Modulus of elasticity : Newton metre^{-2}
(d) Power : Newton metre second^{-1}
(e) Surface tension : Newton metre^{-2}

7. The unit of specific conductivity is **[Manipal 2009]**
(a) Ω-cm^{-1} (b) Ω-cm^{-2}
(c) Ω^{-1}-cm (d) Ω^{-1}cm^{-1}

8. The unit of thermal conductance is **[AMU 2008]**
(a) WK^{-1} (b) JK^{-1}
(c) WK (d) JK

9. Match the following columns **[Kerala CEE 2008]**

	Column I		Column II
(A)	Capacitance	(i)	volt (ampere)$^{-1}$
(B)	Magnetic induction	(ii)	volt-sec (ampere)$^{-1}$
(C)	Inductance	(iii)	newton (ampere)$^{-1}$ (metre)$^{-1}$
(D)	Resistance	(iv)	coulomb^2 (joule)$^{-1}$

	A	B	C	D
(a)	(ii)	(iii)	(iv)	(i)
(b)	(iv)	(iii)	(ii)	(i)

(c) (iii) (iv) (i) (ii)
(d) (iv) (i) (ii) (iii)
(e) (ii) (iv) (i) (iii)

10. If 'muscle times speed equals power', what is the ratio of the SI unit and the CGS unit of muscle? **[Manipal 2008]**

(a) 10^5
(b) 10^3
(c) 10^7
(d) 10^{-5}

11. The unit of universal gas constant is **[Manipal 2008]**

(a) watt/K
(b) dyne/°C
(c) erg/K
(d) newton/°R

12. Parsec is the unit of **[BHU 2007]**

(a) time
(b) distance
(c) frequency
(d) angular acceleration

13. Light year is used to measure **[DUMET 2007]**

(a) distance between stars
(b) distance between atoms
(c) revolution time of earth around sun
(d) none of the above

14. The unit of permittivity of free space, ε_0, is **[Manipal 2007]**

(a) coulomb/newton-metre
(b) newton-metre^2/coulomb^2
(c) coluomb^2/newton-metre^2
(d) coulomb^2/(newton-metre)^2

15. Given that $y = A\sin\left[\left(\frac{2\pi}{\lambda}(ct - x)\right)\right]$ where y and x are measured in metre. Which of the following statements is ture? **[AFMC 2006]**

(a) The unit of λ is same as that of x and A
(b) The unit of λ is same as that of x but not of A
(c) The unit of c is same is as that of $\frac{2\pi}{\lambda}$
(d) The unit of (ct – x) is same as that of $\frac{2\pi}{\lambda}$

16. The magnitude of any physical quantity **[Punjab PMET 2006]**

(a) depends on the method of measurement
(b) does not depend on the method of measurement
(c) is more in SI system than in CGS system
(d) directly proportional to fundamental unit of mass,

17. The volume of a cube in m^3 is equal to the surface area of the cube in m^2. The volume of the cube is **[DUMET 2006]**

(a) 64 m^3
(b) 216 m^3
(c) 512 m^3
(d) 195 m^3

18. The unit of Stefan's constant is **[KCET 2006]**

(a) $Wm^{-2} K^{-1}$
(b) $Wm\ k^{-4}$
(c) $Wm^{-2}\ K^{-4}$
(d) $Nm^{-2}\ K^{-4}$

19. Which one of the following is not a derived unit? **[Kerala CEE 2006]**

(a) Frequency
(b) Planck's constant
(c) Gravitational constant
(d) Charge
(e) Electric current

20. What is SI unit of electric field intensity? **[Guj CET 2006]**

(a) cm (b) Vm^{-1}

(c) Am^{-1} (d) NA

21. If the magnetic flux is represented in weber, then the unit of magnetic induction will be

(a) $\frac{Wb}{m^2}$ (b) Wb × m **[Guj CET 2006]**

(c) $Wb \times m^2$ (d) $\frac{Wb}{m}$

22. Light year is a unit of **[MP PMT 1989; CPMT 1991; AFMC 1991,2005]**

(a) Time (b) Mass

(c) Distance (d) Energy

23. *Joule-second* is the unit of **[CPMT 1990; CBSE PMT 1993; BVP 2003]**

(a) Work (b) Momentum

(c) Pressure (d) Angular momentum

24. Temperature can be expressed as a derived quantity in terms of any of the following **[MP PET 1993; UPSEAT 2001]**

(a) Length and mass (b) Mass and time

(c) Length, mass and time (d) None of these

25. In which of the following systems of unit, *Weber* is the unit of magnetic flux **[SCRA 1991; CBSE PMT 1993; DPMT 2005]**

(a) CGS (b) MKS

(c) SI (d) None of these

26. Which of the following is not a unit of time **[UPSEAT 2001]**

(a) *Leap year* (b) *Micro second*

(c) *Lunar month* (d) *Light year*

27. Which one of the following is not a unit of young's modulus **[KCET 2005]**

(a) Nm^{-1} (b) Nm^{-2}

(c) $Dyne\ cm^{-2}$ (d) Mega Pascal

28. In C.G.S. system the magnitutde of the force is 100 *dynes*. In another system where the fundamental physical quantities are kilogram, *metre* and minute, the magnitude of the force is **[EAMCET 2001]**

(a) 0.036 (b) 0.36

(c) 3.6 (d) 36

29. The unit of L/R is (where L = inductance and R = resistance) **[Orissa JEE 2002]**

(a) sec (b) sec^{-1}

(c) *Volt* (d) *Ampere*

30. Which is different from others by units **[Orissa JEE 2002]**

(a) Phase difference (b) Mechanical equivalent

(c) Loudness of sound (d) Poisson's ratio

31. *Faraday* is the unit of **[AFMC 2003]**

(a) Charge (b) emf

(c) Mass (d) Energy

32. *Candela* is the unit of **[UPSEAT 1999; CPMT 2003]**

(a) Electric intensity (b) Luminous intensity

(c) Sound intensity (d) None of these

33. The unit of reactance is **[MP PET 2003]**

(a) *Ohm* (b) *Volt*

(c) *Mho* (d) *Newton*

34. The unit of Planck's constant is **[RPMT 1999; MP PET 2003; Pb. PMT 2004]**

(a) *Joule* (b) *Joule*/s

(c) *Joule*/*m* (d) *Joule*-s

35. Number of base SI units is **[MP PET 2003]**

(a) 4 (b) 7

(c) 3 (d) 5

36. Which does not has the same unit as others **[Orissa PMT 2004]**

(a) *Watt-sec* (b) *Kilowatt-hour*

(c) *eV* (d) *J-sec*

37. Unit of surface tension is **[Orissa PMT 2004]**

(a) Nm^{-1} (b) Nm^{-2}

(c) N^2m^{-1} (d) Nm^{-3}

38. Which of the following system of units is not based on units of mass, length and time alone

(a) SI (b) MKS **[Kerala PMT 2004]**

(c) FPS (d) CGS

39. The unit of the coefficient of viscosity in S.I. system is **[J & K CET 2004]**

(a) $m/kg\text{-}s$ (b) $m\text{-}s/kg^2$

(c) $kg/m\text{-}s^2$ (d) $kg/m\text{-}s$

40. The unit of Young's modulus is **[Pb. PET 2001]**

(a) Nm^2 (b) Nm^{-2}

(c) Nm (d) Nm^{-1}

41. One femtometer is equivalent to **[DCE 2004]**

(a) $10^{15}\,m$ (b) $10^{-15}\,m$

(c) $10^{-12}\,m$ (d) $10^{12}\,m$

42. Which of the following pairs is wrong **[AFMC 2003]**

(a) Pressure-Baromter (b) Relative density-Pyrometer

(c) Temperature-Thermometer (d) Earthquake-Seismograph

DIMENSION

43. If force (F), velocity (V) and time (T) are taken as fundamental units, then the dimensions of mass are **[AIPMT 2014]**

(1) $[FV^{-1}T^{-1}]$ (2) $[FV^{-1}\,T]$

(3) $[F\,V\,T^{-1}]$ (4) $[F\,V\,T^{-2}]$

44. The dimensions of $(\mu_0\varepsilon_0)^{-1/2}$ are **[CBSE AIPMT 2012]**

(a) $[L^{-1}\,T]$ (b) $[LT^{-1}]$

(c) $[L^{-1/2}\,T^{1/2}$ (d) $[L^{1/2}\,T^{-1/2}]$

45. Surface tension has the same dimensions as that of **[Kerala CEE 2011]**

(a) coefficient of viscosity (b) impluse

(c) momentum (d) spring constant

(e) frequency

46. From the dimensional consideration which of the following equation is correct? **[Har PMT 2010]**

(a) $T = 2\pi\sqrt{\dfrac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\dfrac{GM}{R^3}}$

(c) $T = 2\pi\sqrt{\dfrac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\dfrac{R^2}{GM}}$

47. Dimensions of capacitance is **[Manipal 2010]**

(a) $\left[M^{-1}L^{-2}T^{4}A^{2}\right]$ (b) $\left[MLT^{-3}A^{-1}\right]$

(c) $\left[ML^{2}T^{-3}A^{-1}\right]$ (d) $\left[M^{-1}L^{-2}T^{3}A^{-1}\right]$

48. The relation $p = \frac{\alpha}{\beta} e^{\frac{-\alpha Z}{k\theta}}$, where p is pressure, Z is distance, k is Boltzmann constant and θ is temperature. The dimensional formula of β will be **[AFMC 2010]**

(a) $\left[M^{0}L^{2}T^{0}\right]$ (b) $\left[ML^{2}T\right]$

(c) $\left[ML^{0}T^{-1}\right]$ (d) $\left[M^{0}L^{2}T^{-1}\right]$

49. The dimensions of electromotive force in terms of current A is **[BVP 2010]**

(a) $\left[ML^{-2}A^{-2}\right]$ (b) $\left[ML^{2}T^{-2}A^{-2}\right]$

(c) $\left[ML^{2}T^{-2}A^{-2}\right]$ (d) $\left[ML^{2}T^{-3}A^{-1}\right]$.

50. If $p = \frac{RT}{V-b} e^{-\alpha V/RT}$, then dimensional formula of α is same as that of **[UP CPMT 2010]**

(a) p (b) R

(c) T (d) V

51. Velocity v is given by $v = at^2 + bt + c$, where t is time. What is the dimensions of a, b and c respectively? **[UP CPMT 2010]**

(a) $\left[LT^{-3}\right],\left[LT^{-2}\right]and\left[LT^{-1}\right]$ (b) $\left[LT^{-1}\right],\left[LT^{-2}\right]$ and $\left[LT^{-3}\right]$

(c) $\left[LT^{-2}\right],\left[LT^{-3}\right]$ and $\left[LT^{-1}\right]$ (d) $\left[LT^{-1}\right],\left[LT^{-3}\right]$ and $\left[LT^{-2}\right]$.

52. If E, M, L and G denote energy, mass angular momentum and gravitational constant respectively, then the quantity $(E^2L^2 / M^5 G^2)$ has the dimensions of **[AMU 2010]**

(a) angle (b) length

(c) mass (d) None of these.

53. The physical quantity having the dimensions $[M^{-1}L^{-3}T^{3}A^{2}]$ is **[AFMC - 2008]**

(a) resistance (b) resistivity

(c) electrical conductivity (d) electromotive force.

54. The speed of light c, gravitational constant G and Planck's constant h are taken as fundamental units in a system. The dimensions of time in this new system should be

(a) $\left[G^{1/2}h^{1/2}c^{-5/2}\right]$ (b) $\left[G^{-1/2}h^{1/2}c^{1/2}\right]$ **[AIIMS - 2008]**

(c) $\left[G^{1/2}h^{1/2}c^{-3/2}\right]$ (d) $\left[G^{1/2}h^{1/2}c^{1/2}\right]$.

55. Dimensions of resistance in an electrical circuit, in terms of dimension of mass M, of length L, of time T and of current A, would be **[UP CPMT - 2008]**

(a) $\left[ML^{-2}T^{-3}A^{-1}\right]$ (b) $\left[ML^{2}T^{-2}\right]$

(c) $\left[ML^{2}T^{-1}A^{-1}\right]$ (d) $\left[ML^{2}T^{-3}A^{-2}\right]$.

56. Given that the displacement of an oscillating particle is given by y = A sin (Bx + Ct + D). The dimensional formula for (ABCD) is **[AMU- 2008]**

(a) $\left[M^0L^{-1}T^0\right]$ (b) $\left[M^0L^0T^{-1}\right]$

(c) $\left[M^0L^{-1}T^{-1}\right]$ (d) $\left[M^0L^0T^0\right]$.

57. If p represents radiation pressure, c represents speed of light and Q represents radiation energy striking a unit area per second then non-zero integers x, y and z such that $p^xQ^yc^z$ is dimensionless, are **[BHU- 2008]**

(a) x = 1, y = 1, z = – 1 (b) x = 1, y = – 1, z = 1

(c) x = – 1, y = 0, z = 1 (d) x = 1, y = 1, z = 1.

58. Given that T stands for time period and ℓ stands for the length of simple pendulum. If g is the acceleration due to gravity, then which of the following statements about the relation $T^2 = \ell / g$ is correct? **[MP PMT- 2008]**

(a) It is correct both dimensionally as well as numerically

(b) It is neither dimensionally correct nor numerically

(c) It is dimensionally correct but not numerically

(d) it is numerically correct but not dimensionally.

59. The physical quantities not having same dimensions are **[RPMT – 2008]**

(a) torque and work (b) momentum and Planck's constant

(c) stress and Young's modulus (d) speed and $(\mu_0\varepsilon_0)^{-1/2}$

60. The ratio of the dimensions of Planck's constant and that of the moment of inertia is the dimensions of

(a) frequency (b) velocity **[AFMC – 2007]**

(c) angular momentum (d) time

61. The speed v of ripples on the surface of water depends on surface tension σ, density ρ and wavelength λ. the square of speed v is proportional to **[AIIMS – 2007]**

(a) $\frac{\sigma}{\rho\lambda}$ (b) $\frac{\rho}{\sigma\lambda}$

(c) $\frac{\lambda}{\sigma\rho}$ (d) $\rho\lambda\sigma$

62. Using mass M, length L, time T and current A as fundamental quantities, the dimensions of permeability is **[UP CPMT – 2007]**

(a) $[M^{-1}LT^{-2}A]$ (b) $[ML^{-2}T^{-2}A^{-1}]$

(c) $[MLT^{-2}A^{-2}]$ (d) $[MLT^{-1}A^{-1}]$

63. The positionof the particle moving along Y-axis is given as $y = At^2 - Bt^3$, where y is measured in metre and t in second. Then the dimensions of B is **[AMU – 2007]**

(a) $[LT^{-2}]$ (b) $[LT^{-1}]$

(c) $[LT^{-3}]$ (d) $[MLT^{-2}]$

64. Which of the following units denotes the dimensions $[ML^2 / Q^2]$, where Q denotes the electric charge? **[BHU 2007]**

(a) Wb/m^2 (b) Henry (H)

(c) H/m^2 (d) Weber (Wb)

65. If L and R are respectively the inductance and resistance, then the dimensions of $\frac{L}{R}$ will be

[CPMT 1986; CBSE PMT 1988; Roorkee 1995; MP PET/PMT 1998; DCE 2002]

(a) $M^0L^0T^{-1}$ (b) M^0LT^0

(c) M^0L^0T (d) Cannot be represented in terms of M, L and T

66. Dimensional formula for latent heat is **[MNR 1987; CPMT 1978, 86; IIT 1983, 89; RPET 2002]**

(a) $M^0L^2T^{-2}$ (b) MLT^{-2}

(c) ML^2T^{-2} (d) ML^2T^{-1}

67. Dimensional formula for volume elasticity is **[MP PMT 1991, 2002; CPMT 1991; MNR 1986]**

(a) $M^1L^{-2}T^{-2}$ (b) $M^1L^{-3}T^{-2}$

(c) $M^1L^2T^{-2}$ (d) $M^1L^{-1}T^{-2}$

68. The dimensions of universal gravitational constant are

[AIIMS 2000; RPET 2001; Pb. PMT 2002, 03; UPSEAT 1999; BCECE 2003, 05;]

(a) $M^{-2}L^2T^{-2}$ (b) $M^{-1}L^3T^{-2}$

(c) $ML^{-1}T^{-2}$ (d) ML^2T^{-2}

69. The dimensional formula for Planck's constant (h) is

[AFMC 2003; RPMT 1999; Kerala PMT 2002]

(a) $ML^{-2}T^{-3}$ (b) ML^2T^{-2}

(c) ML^2T^{-1} (d) $ML^{-2}T^{-2}$

70. The dimensional formula for impulse is same as the dimensional formula for

[CPMT 1982, 83; CBSE PMT 1993; UPSEAT 2001]

(a) Momentum (b) Force

(c) Rate of change of momentum (d) Torque

71. A small steel ball of radius r is allowed to fall under gravity through a column of a viscous liquid of coefficient of viscosity η. After some time the velocity of the ball attains a constant value known as terminal velocity v_T. The terminal velocity depends on (i) the mass of the ball m, (ii) η, (iii) r and (iv) acceleration due to gravity g. Which of the following relations is dimensionally correct

[CPMT 1992; CBSE PMT 1992; NCERT 1983; MP PMT 2001]

(a) $v_T \propto \frac{mg}{\eta r}$ (b) $v_T \propto \frac{\eta r}{mg}$

(c) $v_T \propto \eta rmg$ (d) $v_T \propto \frac{mgr}{\eta}$

72. The quantity $X = \frac{\varepsilon_0 LV}{t}$: ε_0 is the permittivity of free space, L is length, V is potential difference and t is time. The dimensions of X are same as that of **[IIT 2001]**

(a) Resistance (b) Charge (c) Voltage (d) Current

73. Dimensional formula of magnetic flux is

[DCE 1993; IIT 1982; CBSE PMT 1989, 99; DPMT 2001; Kerala PMT 2005]

(a) $ML^2T^{-2}A^{-1}$ (b) $ML^0T^{-2}A^{-2}$

(c) $M^0L^{-2}T^{-2}A^{-3}$ (d) $ML^2T^{-2}A^3$

74. Inductance L can be dimensionally represented as

[DPMT 1999; KCET 2004; J&K CET 2005]

(a) $ML^2T^{-2}A^{-2}$ (b) $ML^2T^{-4}A^{-3}$

(c) $ML^{-2}T^{-2}A^{-2}$ (d) $ML^2T^4A^3$

75. Dimensional formula for torque is **[BHU 1995, 2001; RPMT 1999; RPET 2003; DCE 1999, 2000; DCE 2004]**

(a) L^2MT^{-2}
(b) $L^{-1}MT^{-2}$
(c) L^2MT^{-3}
(d) LMT^{-2}

76. Dimensions of coefficient of viscosity are **[AIIMS 1993; DCE 1999; AIEEE 2004; DPMT 2004]**

(a) ML^2T^{-2}
(b) ML^2T^{-1}
(c) $ML^{-1}T^{-1}$
(d) MLT

77. $ML^{-1}T^{-2}$ represents **[EAMCET (Med.) 1995; Pb. PMT 2001]**

(a) Stress
(b) Young's Modulus
(c) Pressure
(d) All the above three quantities

78. Dimensions of magnetic field intensity is **[RPMT 1997; EAMCET (Med.) 2000; MP PET 2003]**

(a) $[M^0L^{-1}T^0A^1]$
(b) $[MLT^{-1}A^{-1}]$
(c) $[ML^0T^{-2}A^{-1}]$
(d) $[MLT^{-2}A]$

79. Dimensions of luminous flux are **[UPSEAT 2001]**

(a) ML^2T^{-2}
(b) ML^2T^{-3}
(c) ML^2T^{-1}
(d) MLT^{-2}

80. A physcial quantity x depends on quantities y and z as follows: $x = Ay + B\tan Cz$, where A, B and C are constants. Which of the following do not have the same dimensions **[AMU (Engg.) 2001]**

(a) x and B
(b) C and z^{-1}
(c) y and B/A
(d) x and A

81. Which of the following pair does not have similar dimensions **[AIIMS 2001]**

(a) Stress and pressure
(b) Angle and strain
(c) Tension and surface tension
(d) Planck's constant and angular momentum

82. Out of the following which pair of quantities do not have same dimensions **[RPET 2001]**

(a) Planck's constant and angular momentum
(b) Work and energy
(c) Pressure and Young's modulus
(d) Torque & moment of inertia

83. Identify the pair which has different dimensions **[KCET 2001]**

(a) Planck's constant and angular momentum
(b) Impulse and linear momentum
(c) Angular momentum and frequency
(d) Pressure and Young's modulus

84. The dimensional formula $M^0L^2T^{-2}$ stands for **[KCET 2001]**

(a) Torque
(b) Angular momentum
(c) Latent heat
(d) Coefficient of thermal conductivity

85. Which of the following represents the dimensions of *Farad* **[AMU (Med.) 2002]**

(a) $M^{-1}L^{-2}T^4A^2$
(b) $ML^2T^2A^{-2}$
(c) $ML^2T^2A^{-1}$
(d) $MT^{-2}A^{-1}$

86. If L, C and R denote the inductance, capacitance and resistance respectively, the dimensional formula for C^2LR is **[UPSEAT 2002]**

(a) $[ML^{-2}T^{-1}I^0]$
(b) $[M^0L^0T^3I^0]$
(c) $[M^{-1}L^{-2}T^6I^2]$
(d) $[M^0L^0T^2I^0]$

87. If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then the dimensions of mass in new system is **[UPSEAT 2002]**

(a) $c^{1/2}G^{1/2}h^{1/2}$
(b) $c^{1/2}G^{1/2}h^{-1/2}$
(c) $c^{1/2}G^{-1/2}h^{1/2}$
(d) $c^{-1/2}G^{1/2}h^{1/2}$

88. Dimensions of charge are **[DPMT 2002]**

(a) $M^0L^0T^{-1}A^{-1}$ (b) $MLTA^{-1}$

(c) $T^{-1}A$ (d) TA

89. According to Newton, the viscous force acting between liquid layers of area A and velocity gradient $\Delta v/\Delta z$ is given by $F=-\eta A\frac{\Delta v}{\Delta z}$ where η is constant called coefficient of viscosity. The dimension of η are **[JIPMER 2001, 02]**

(a) $[ML^2T^{-2}]$ (b) $[ML^{-1}T^{-1}]$

(c) $[ML^{-2}T^{-2}]$ (d) $[M^0L^0T^0]$

90. Identify the pair whose dimensions are equal **[AIEEE 2002]**

(a) Torque and work (b) Stress and energy

(c) Force and stress (d) Force and work

91. The dimensions of pressure is equal to **[AIEEE 2002]**

(a) Force per unit volume (b) Energy per unit volume

(c) Force (d) Energy

92. Which of the two have same dimensions **[AIEEE 2002]**

(a) Force and strain (b) Force and stress

(c) Angular velocity and frequency (d) Energy and strain

93. An object is moving through the liquid. The viscous damping force acting on it is proportional to the velocity. Then dimension of constant of proportionality is **[Orissa JEE 2002]**

(a) $ML^{-1}T^{-1}$ (b) MLT^{-1}

(c) M^0LT^{-1} (d) ML^0T^{-1}

94. The dimensions of emf in MKS is **[CPMT 2002]**

(a) $ML^{-1}T^{-2}Q^{-2}$ (b) $ML^2T^{-2}Q^{-2}$

(c) $MLT^{-2}Q^{-1}$ (d) $ML^2T^{-2}Q^{-1}$

95. Which of the following quantities is dimensionless **[MP PET 2002]**

(a) Gravitational constant (b) Planck's constant

(c) Power of a convex lens (d) None

96. The dimensional formula for Boltzmann's constant is **[MP PET 2002; Pb. PET 2001]**

(a) $[ML^2T^{-2}\theta^{-1}]$ (b) $[ML^2T^{-2}]$

(c) $[ML^0T^{-2}\theta^{-1}]$ (d) $[ML^{-2}T^{-1}\theta^{-1}]$

97. The dimensions of K in the equation $W=\frac{1}{2}Kx^2$ is **[Orissa JEE 2003]**

(a) $M^1L^0T^{-2}$ (b) $M^0L^1T^{-1}$

(c) $M^1L^1T^{-2}$ (d) $M^1L^0T^{-1}$

98. The physical quantities not having same dimensions are **[AIEEE 2003]**

(a) Speed and $(\mu_0\varepsilon_0)^{-1/2}$ (b) Torque and work

(c) Momentum and Planck's constant (d) Stress and Young's modules

99. Dimension of Resistance is **[AFMC 2003; AIIMS 2005]**

(a) ML^2T^{-1} (b) $ML^2T^{-3}A^{-2}$

(c) $ML^{-1}T^{-2}$ (d) None of these

100. The dimensional formula of relative density is **[CPMT 2003]**

(a) ML^{-3} (b) LT^{-1}

(c) MLT^{-2} (d) Dimensionless

101. The dimensional formula for young's modulus is **[BHU 2003; CPMT 2004]**

(a) $ML^{-1}T^{-2}$ (b) M^0LT^{-2}

(c) MLT^{-2} (d) ML^2T^{-2}

102. Time is the function of density (ρ), length (a) and surface tension (T). Then its value is **[BHU 2003]**

(a) $k\rho^{1/2}a^{3/2}/\sqrt{T}$
(b) $k\rho^{3/2}a^{3/2}/\sqrt{T}$
(c) $k\rho^{1/2}a^{3/2}/T^{3/4}$
(d) $k\rho^{1/2}a^{1/2}/T^{3/2}$

103. The dimensions of electric potential are **[UPSEAT 2003]**

(a) $[ML^2T^{-2}Q^{-1}]$
(b) $[MLT^{-2}Q^{-1}]$
(c) $[ML^2T^{-1}Q]$
(d) $[ML^2T^{-2}Q]$

104. Dimensions of potential energy are **[MP PET 2003]**

(a) MLT^{-1}
(b) ML^2T^{-2}
(c) $ML^{-1}T^{-2}$
(d) $ML^{-1}T^{-1}$

105. The dimension of $\frac{R}{L}$ are **[MP PET 2003]**

(a) T^2
(b) T
(c) T^{-1}
(d) T^{-2}

106. The dimensions of shear modulus are **[MP PMT 2004]**

(a) MLT^{-1}
(b) ML^2T^{-2}
(c) $ML^{-1}T^{-2}$
(d) MLT^{-2}

107. Pressure gradient has the same dimension as that of **[AFMC 2004]**

(a) Velocity gradient
(b) Potential gradient
(c) Energy gradient
(d) None of these

108. In the relation $y = a\cos(\omega t - kx)$, the dimensional formula for k is **[BHU 2004]**

(a) $[M^0L^{-1}T^{-1}]$
(b) $[M^0LT^{-1}]$
(c) $[M^0L^{-1}T^0]$
(d) $[M^0LT]$

109. Position of a body with acceleration 'a' is given by $x = Ka^m t^n$, here t is time. Find dimension of m and n. **[Orissa JEE 2005]**

(a) $m = 1$, $n = 1$
(b) $m = 1, n = 2$
(c) $m = 2, n = 1$
(d) $m = 2, n = 2$

110. "Pascal-Second" has dimension of **[AFMC 2005]**

(a) Force
(b) Energy
(c) Pressure
(d) Coefficient of viscosity

111. In a system of units if force (F), acceleration (A) and time (T) are taken as fundamental units then the dimensional formula of energy is **[BHU 2005]**

(a) FA^2T
(b) FAT^2
(c) F^2AT
(d) FAT

112. The ratio of the dimension of Planck's constant and that of moment of inertia is the dimension of **[CBSE PMT 2005]**

(a) Frequency
(b) Velocity
(c) Angular momentum
(d) Time

113. Which of the following group have different dimension **[IIT JEE 2005]**

(a) Potential difference, EMF, voltage
(b) Pressure, stress, young's modulus
(c) Heat, energy, work-done
(d) Dipole moment, electric flux, electric field

114. Out of following four dimensional quantities, which one quantity is to be called a dimensional constant **[KCET 2005]**

(a) Acceleration due to gravity
(b) Surface tension of water
(c) Weight of a standard kilogram mass
(d) The velocity of light in vacuum

115. Density of a liquid in CGS system is 0.625 g/cm^3. What is its magnitude in SI system **[J&K CET 2005]**

(a) 0.625
(b) 0.0625
(c) 0.00625
(d) 625

ERRORS OF MEASUREMENT

116. In an experiment four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows : **[NEET 2013]**

$P = \frac{a^3b^2}{cd}$ % error in P is:

(a) 4% (b) 14%

(c) 10% (d) 7%

117. A stone is dropped from a height h. It hits the ground with a certain momentum P. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by **[AIPMT2012]**

(a) 100% (b) 68%

(c) 41% (d) 200%

118. If the momentum of an electron is changed by P, then the de Broglie wavelength associated with it changes by 0.5%. The initial momentum of electron will be **[AIPMT2012]**

(a) 100P (b) 200P

(c) 400P (d) P/200

119. At constant temperature, the volume of a gas is to be decreased by 4%. The pressure must be increased by **[BVP 2010]**

(a) 4% (b) 4.16%

(c) 8% (d) 3.86%

120. Choose the incorrect statement out of the following. **[AMU 2010]**

(a) Every measurement by any measuring instrument has some errors

(b) Every calculated physical quantity that is based on measured values has some error

(c) A measurement can be more accuracy but less precision and *vice versa.*

(d) The percentage error is different from relative error

121. By what percentage should the pressure of a given mass of a gas be increased, so as to decrease its volume by 10% at a constant temperature? **[AIIMS 2009]**

(a) 5% (b) 7.2%

(c) 12.5% (d) 11.1%

122. Percentage error in the measurement of mass and speed are 2% and 3% respectively. The error in the estimation of kinetic energy obtained by measuring mass and speed will be **[AIIMS 2009]**

(a) 12% (b) 10%

(c) 2% (d) 8%

123. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be **[CBSE AIPMT 2008]**

(a) 4% (b) 6%

(c) 8% (d) 2%

124. A: The error in the measurement of radius of the sphere is 0.3%. The permissible error in its surface areas is 0.6%. **[AIIMS 2008]**

R: The permissible error is calculated by the formula $\frac{\Delta A}{A} = \frac{4\Delta r}{r}$.

125. The period of oscillation of a simple pendulum in the experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. The average absolute error is **[AMU 2008]**

(a) 0.1 s (b) 0.11 s

(c) 0.01 s (d) 1.0 s

126. A wire has a mass (0.3 ± 0.003) g, radius (0.5 ± 0.005) mm and length (0.6 ± 0.006) cm. The maximum percentage error in the measurement of its density **[AMU 2007]**

(a) 1 (b) 2

(c) 3 (d) 4

127. The length, breadth and thickness of a block are given by ℓ = 12 cm, b = 6 cm and t = 2.45 cm. The volume of the block according to the idea of significant figures should be **[MP PMT 2007]**

(a) 1×10^3 cm^3 (b) 2×10^2 cm^3

(c) 1.764×10^2 cm^3 (d) None of these

128. Error in the measurement of radius of a sphere is 1%. The error in the calculated value of its volume is

(a) 1% (b) 3% **[AFMC 2005]**

(c) 5% (d) 7%

129. If $L = 2.331\,cm, B = 2.1\,cm$, then $L + B =$ **[DCE 2003]**

(a) 4.431 cm (b) 4.43 cm

(c) 4.4 cm (d) 4 cm

130. The number of significant figures in all the given numbers 25.12, 2009, 4.156 and 1.217×10^{-4} is

(a) 1 (b) 2 **[Pb. PET 2003]**

(c) 3 (d) 4

131. If the length of rod A is 3.25 ± 0.01 cm and that of B is 4.19 ± 0.01 cm then the rod B is longer than rod A by **[J&K CET 2005]**

(a) 0.94 ± 0.00 cm (b) 0.94 ± 0.01 cm

(c) 0.94 ± 0.02 cm (d) 0.94 ± 0.005 cm

132. A physical quantity is given by $X = M^a L^b T^c$. The percentage error in measurement of M, L and T are α, β and γ respectively. Then maximum percentage error in the quantity X is **[Orissa JEE 2005]**

(a) $a\alpha + b\beta + c\gamma$ (b) $a\alpha + b\beta - c\gamma$

(c) $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}$ (d) None of these

QUESTIONS ASKED IN 2014 TO 2021

1. If force (F), velocity(V) and time (T) are taken as fundamental units, the dimensions of mass are

(a) $[FVT^{-1}]$ (b) $[FVT^{-2}]$ **[AIPMT 2014]**

(c) $[FV^{-1}T^{-1}]$ (d) $[FV^{-1}T]$

2. If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities, the dimensional formula of surface tension will be :

(a) $[E^{-2}\ V^{-1}\ T^{-3}]$ (b) $[E\ V^{-2}\ T^{-1}]$ **[AIPMT 2015]**

(c) $[E\ V^{-1}\ T^{-2}]$ (d) $[E\ V^{-2}\ T^{-2}]$

3. Planck's constant (h), speed of light in vacuum (c) and Newton's gravitation constant (G) are three fundamental constants. Which of the following combinations of the has the dimension of length?

[NEET 2016 - Phase II]

(a) $\frac{\sqrt{hG}}{c^{3/2}}$ (b) $\frac{\sqrt{hG}}{c^{5/2}}$ (c) $\sqrt{\frac{hc}{G}}$ (d) $\sqrt{\frac{Gc}{h^{3/2}}}$

4. A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and ε is charge]: **[NEET 2017]**

(a) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ (b) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ (c) $\frac{1}{c^2}\left[\frac{e^2}{G4\pi\varepsilon_0}\right]^{1/2}$ (d) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$

5. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of -0.004 cm, the Correct diameter of the ball is **[NEET 2018]**
(a) 0.053cm (b) 0.525 cm (c) 0.521 cm (d) 0.529 cm

6. The unit of thermal conductivity is : **[NEET (National) 2019]**
(a) $Jm^{-1} K^{-1}$ (b) $W m K^{-1}$ (c) $W m^{-1} K^{-1}$ (d) $J m K^{-1}$

7. The angle of 1' (minute of arc) in radian is nearly equal to **[NEET (Oct) 2020]**
(a) 2.91×10^{-4} rad (b) 4.85×10^{-4} rad (c) 4.80×10^{-6} rad (d) 1.75×10^{-2} rad

8. Time intervals measured by a clock give the following readings
1.25s, 1.24s, 1.27 s, 1.21 s and 1.28 s.
What is the percentage relative error of the observations ? **[NEET (Oct) 2020]**
(a) 2% (b) 4% (c) 16% (d) 1.6%

9. A screw gauge has least count of 0.01mm and there are 50 divisions in its circular scale. The pitch of the scew gauge is **[NEET (Sep.) 2020]**
(a) 0.25 mm (b) 0.5 mm (c) 1.0 mm (d) 0.01 mm

10. Taking into account of the significant figures, what is the value of 9.99 m - 0.0099 m? **[NEET (sep.) 2020]**
(a) 9.98 m (b) 9.980 m (c) 9.9 m (d) 9.9801 m

11. Dimensions of stress are **[NEET (sep.) 2020]**
(a) $[ML^2T^{-2}]$ (b) $[ML^0T^{-2}]$ (c) $[ML^{-1}T^{-2}]$ (d) $[MLT^{-2}]$

12. A screw gauge gives the following readings when used to measure the diameter of a wire main scale reading when used to measure the diameter of a wire main scale reading : 0 mm circular scale reading : 52 divisions given that, 1mm on main scale corresponds to 100 divisions on the circular scale. the diameter of the wire form the above data is **[NEET 2021]**
(a) 0.52 cm (b) 0.026 cm (c) 0.26 cm (d) 0.052 cm

13. If force [F], acceleration [a] and time [T] are chosen as the fundamental physical quantities. Find the dimensions of energy. **[NEET 2021]**
(a) [F] [a] [T] (b) $[F]\,[a]\,[T^2]$ (c) $[F]\,[a]\,[T^{-1}]$ (d) $[F][a^{-1}][T]$

14. If E and G respectively denote energy and gravitational constant then $\frac{E}{G}$ has the dimensions of **[NEET 2021]**
(a) $[M^2][L^{-1}][T^0]$ (b) $[M][L^{-1}][T^{-1}]$ (c) $[M][L^0][T^0]$ (d) $[M^2][L^{-2}][T^{-1}]$

EXERCISE KEY

SUBJECTIVE TYPE QUESTION

1. $4x^3-3x^2-8x+9$
2. $-\frac{2}{(x-1)^2}$
3. $\frac{1-x^2}{(1+x^2)^2}$
4. $\frac{ad-bc}{(cx+d)^2}$
5. $-\frac{4x}{3(x^2-1)^2}+1+2x-3x^2$
6. $-\frac{6x^2}{(x^3+1)^2}$
7. $-\frac{6x^2}{(x^3-1)^2}$
8. $\frac{2x-1}{a^3-3}$
9. $-\frac{3x^2}{\sqrt{\pi}}$
10. $\cos x-\sin x$
11. $x\cos x$
12. $-\sin 2x$
13. $\frac{3}{2}\sin 2x(2-\sin x)$
14. $1+\ln x$
15. $\frac{2\ln x}{x}$
16. $\frac{2}{x}$
17. $\frac{2}{3}\sqrt{x^3}+c$
18. $\frac{mx^{\frac{n}{m}+1}}{n+m}+c$
19. $c-\frac{1}{x}$
20. $\sqrt{x}+c$

Level - I

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(d)	**5.**	(c)
6.	(c)	**7.**	(a)	**8.**	(c)	**9.**	(c)	**10.**	(c)
11.	(a)	**12.**	(d)	**13.**	(d)	**14.**	(d)	**15.**	(a)
16.	(b)	**17.**	(a)	**18.**	(b)	**19.**	(a)	**20.**	(c)
21.	(c)	**22.**	(b)	**23.**	(a)	**24.**	(b)	**25.**	**(b)**
26.	**(c)**	**27.**	**(c)**						

Level - II

1.	**(a)**	**2.**	**(c)**	**3.**	**(b)**	**4.**	**(d)**	**5.**	**(a)**
6.	**(d)**	**7.**	**(b)**	**8.**	**(b)**	**9.**	**(d)**	**10.**	**(b)**
11.	**(a)**	**12.**	**(c)**	**13.**	**(d)**	**14.**	**(d)**	**15.**	**(d)**
16.	**(b)**	**17.**	**(a)**	**18.**	**(d)**	**19.**	**(a)**	**20.**	**(a)**
21.	**(d)**	**22.**	**(d)**	**23.**	**(b)**	**24.**	**(a)**	**25.**	**(c)**
26.	**(a)**	**27.**	**(b)**	**28.**	**(c)**	**29.**	**(a)**	**30.**	**(d)**
31.	**(a)**								

Assertion & Reason

1	**(c)**	**2**	**(d)**	**3**	**(a)**	**4**	**(c)**	**5**	**(d)**
6	**(a)**	**7**	**(a)**	**8**	**(c)**	**9**	**(b)**	**10**	**(a)**
11	**(c)**	**12**	**(c)**	**13**	**(a)**	**14**	**(c)**	**15**	**(c)**
16	**(e)**	**17**	**(b)**	**18**	**(c)**	**19**	**(c)**	**20**	**(e)**
21	**(b)**	**22**	**(a)**	**23**	**(b)**	**24**	**(a)**	**25**	**(b)**

Previous Year's Questions

UNITS

1.	(c)	2.	(d)	3.	(c)	4.	(b)	5.	(d)
6.	(e)	7.	(d)	8.	(a)	9.	(b)	10.	(a)
11.	(c)	12.	(b)	13.	(a)	14.	(c)	15.	(a)
16.	(b)	17.	(b)	18.	(c)	19.	(e)	20.	(b)
21.	(a)	22.	(c)	23.	(d)	24.	(d)	25.	(c)
26.	(d)	27.	(a)	28.	(c)	29.	(a)	30.	(d)
31.	(a)	32.	(b)	33.	(a)	34.	(d)	35.	(b)
36.	(d)	37.	(a)	38.	(a)	39.	(d)	40.	(b)
41.	(b)	42.	(b)						

DIMENSIONS

43.	(b)	44.	(b)	45.	(d)	46	(a)	47.	(a)
48.	(a)	49.	(d)	50.	(a)	51.	(a)	52.	(d)
53.	(c)	54.	(b)	55.	(d)	56.	(b)	57.	(b)
58.	(c)	59.	(b)	60.	(a)	61.	(a)	62.	(c)
63.	(c)	64.	(b)	65.	(c)	66	(a)	67.	(d)
68.	(b)	69.	(c)	70.	(a)	71.	(a)	72.	(d)
73.	(a)	74.	(a)	75.	(a)	76.	(c)	77.	(d)
78.	(c)	79.	(b)	80.	(d)	81.	(c)	82.	(d)
83.	(c)	84.	(c)	85.	(a)	86.	(b)	87.	(c)
88.	(d)	89.	(b)	90.	(a)	91.	(b)	92.	(c)
93.	(d)	94.	(d)	95.	(d)	96.	(a)	97.	(a)
98.	(c)	99.	(b)	100.	(d)	101.	(a)	102.	(a)
103.	(a)	104.	(b)	105.	(c)	106.	(c)	107.	(d)
108.	(c)	109.	(b)	110.	(d)	111.	(b)	112.	(a)
113.	(d)	114.	(d)	115.	(d)				

ERROR ANALYSIS

116.	(b)	117.	(c)	118.	(b)	119.	(b)	120.	(d)
121.	(d)	122.	(d)	123.	(b)	124.	(c)	125.	(b)
126.	(d)	127.	(b)	128.	(b)	129.	(c)	130.	(d)
131.	(c)	132.	(a)						

QUESTIONS ASKED IN 2014 TO 2021

1.	(d)	2.	(d)	3.	(a)	4.	(a)	5.	(a)
6.	(c)	7.	(a)	8.	(d)	9.	(b)	10.	(a)
11.	(c)	12.	(d)	13.	(b)	14.	(a)		