
KINEMATICS

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This chapter includes:

- Motion in One Dimension
- Projectile Motion
- Relative Velocity

KINEMATICS

1. Kinematics

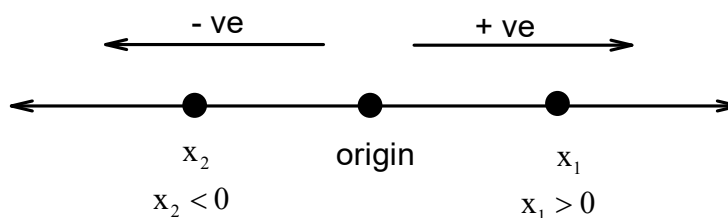
- Kinematics is the study of motion of physical bodies without going into the cause of the motion.
- Kinematics deals with physical quantities like distance, displacement, speed, velocity, acceleration etc.

1.1 Motion and Rest

- Motion is a combined property of the object under study and observer.
- If the position of the object under study changes with time, as seen by the observer, the object is said to be in motion from the frame of reference of the observer.
- Rest and motion of an object under study depend on the frame of reference of the observer. For eg. A book kept on a table may be at rest for all students sitting in the class. But the same book will be in motion, as seen by an observer on a moving bus. Thus absolute rest and absolute motion are meaningless.
- If position of the object does not change with time, as seen by observer, object is said to be at rest from the frame of reference of the observer.
- In most cases, if attributes of motion of an object are given without specifying the frame of the observer, it is to be assumed that the object under consideration is being observed by an observer who is at rest with respect to the earth.

1.2 Position

- For a particle moving along a straight line, position of the particle can be specified with only one coordinate. A coordinate system is chosen by choosing some reference point as the origin. The origin is assigned the number zero. Most situations can be analyzed by setting up an appropriate coordinate system. In order to do so, following are the essential requirements:
- Choice of origin
- Choice of coordinate axis
- Choice of positive direction of axis. All these parameters constitute a reference frame. In any physics problem, the reference frame must be specified.
- In the figure below, point O is the chosen origin, X – axis is the chosen coordinate axis and rightward direction is chosen as the positive direction.



- Similarly, if the motion of a particle is 2 – dimensional or 3 – dimensional, the coordinate axis will comprise of x, y and z axis and position will include x, y, and z coordinates.

1.3 Kinematic Parameters

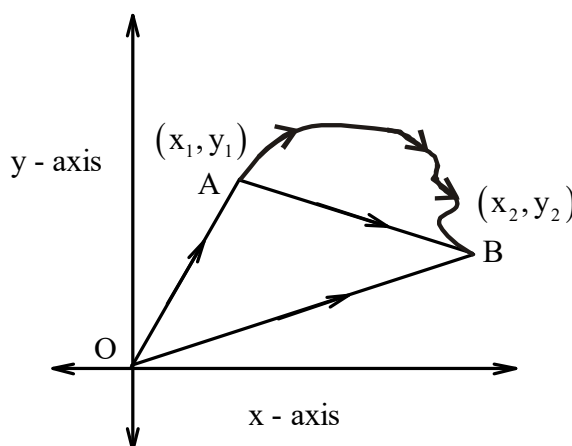
There are five kinematic parameters.

1.3.1 Distance

1. The total length of actual path traversed by the body between initial and final positions is called distance. It is a scalar quantity. Its S.I. unit is meter (m).
2. It has no direction and is always positive.
3. Distance covered by particle never decreases.

1.3.2 Displacement :

- Displacement is a vector quantity. It is the change in position vector.
- Suppose a particle travels from point A to point B as shown in the fig below along a zig – zag path, in a finite time interval.

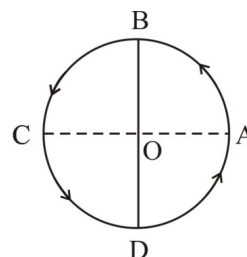


Coordinates of A are (x_1, y_1) and that of B are (x_2, y_2) . Position vector of A is $\vec{r}_A = x_1\hat{i} + y_1\hat{j}$, position vector of B is $\vec{r}_B = x_2\hat{i} + y_2\hat{j}$. Distance will be equal to the total length of the actual path covered by the particle. Displacement will be $\vec{S} = \vec{r}_B - \vec{r}_A = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

Note:

- The distance covered will always be greater than or equal to the magnitude of the displacement.
- Displacement and distance are equal in magnitude in case the particle is travelling along a straight line without change in direction.
- SI unit of distance and displacement is meters.
- In simple language, displacement can be said to be the shortest line joining the initial and final positions of a body in motion, irrespective of path followed and it is directed from initial position to final position.
- Change in position vector is displacement and change in displacement vector is also displacement.

Illustration 1. As shown in fig, a particle moves along a circular path of radius r . It starts from point A and moves anticlockwise. Find the magnitude of displacement and distance travelled by the particle as it
 (i) moves from A to B (ii) moves from A to C
 (iii) moves from A to D
 (iv) completes one revolution



Solution: (i) From A to B. Distance covered $= \frac{1}{4} \times (2\pi r)$

$$= \frac{\pi r}{2}$$

$$\text{Displacement } |\vec{AB}| = \sqrt{OA^2 + OB^2} = r\sqrt{2}$$

(ii) From A to C Distance covered $= \frac{1}{2} \times 2\pi r$

$$= \pi r$$

$$\text{Displacement} = |\vec{AC}| = 2r$$

(iii) From A to D, Distance covered = $\frac{3}{4} \times 2\pi r = \frac{3\pi r}{2}$

Displacement = $|\overline{AD}| = r\sqrt{2}$

(iv) From A to A, distance covered = $2\pi r$

Displacement = 0, because the initial position coincides with final position.

1.3.3 Speed

- The distance travelled per unit time is called speed. It is a scalar quantity and its S.I. unit is m/s.

Average speed

- Average speed = $\frac{\text{Total Distance travelled}}{\text{Total time taken}}$, We define average speed of a particle as the ratio of the total distance travelled to the total time taken.

Instantaneous speed

- Speed of a particle at a particular instant is called instantaneous speed.
- Instantaneous speed is given by, $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$. Where s is the distance travelled by the particle.
- The speedometer of a vehicle indicates the instantaneous speed. The speedometer reading on a crowded city road continuously changes, indicating instantaneous speed is continuously changing.

1.3.4 Velocity

- Velocity is defined as rate of change of displacement with time. Velocity is a vector quantity. SI unit of velocity is m/s.

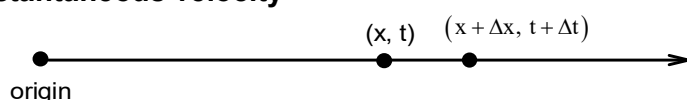
Average velocity

Average velocity = $\frac{\text{Total Displacement}}{\text{Total time}}$, Average velocity is defined as the ratio of the total displacement covered to the total time taken.

Note:

- Just as distance is always greater than or equal to the magnitude of displacement, average speed is greater than or equal to the magnitude of average velocity. Average speed and the magnitude of average velocity are equal when particle is travelling in a straight line without change in direction.

Instantaneous velocity



- Suppose a particle moves from position x at time t to position $x + \Delta x$ at time $t + \Delta t$. Then, the average velocity of the particle over time interval Δt is $\frac{\Delta x}{\Delta t}$.

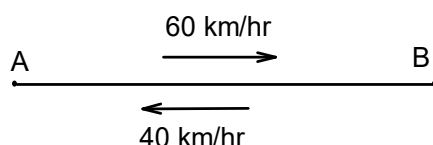
Making Δt infinitely small, $\frac{\Delta x}{\Delta t}$ gives the velocity of the particle at instant t and can be written

as $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$, where v is the instantaneous velocity of the particle at time instant t.

Note: The magnitudes of instantaneous velocity and instantaneous speed are always equal.

Illustration 2. A particle travels from point A to B in a straight line with uniform speed of 60 km/hr. It immediately returns back from B to A with uniform speed of 40 km/hr. Find average velocity and average speed of particle over the whole journey.

Solution:



Let $AB = x$

Displacement = 0. \therefore Average velocity = $\frac{\text{Total Displacement}}{\text{Total time}}$

\therefore Average velocity = 0

Total distance = $2x$

$t_{AB} = \frac{x}{60}$, $t_{BA} = \frac{x}{40}$, Total time = $t_{AB} + t_{BA}$

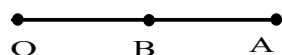
Total time = $\frac{x}{60} + \frac{x}{40}$

Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{2x}{\frac{x}{60} + \frac{x}{40}}$

\therefore Average speed = 48 km/hr.

Illustration 3. An athlete runs 150 m in 15 seconds, then turns around and jogs 50 m back towards starting point in 25 s. What is his average speed and average velocity for total time? Assume he travels along the same straight line always.

Solution:



O is the origin and $OA = 150$ m, $AB = 50$ m

Total distance travelled = $OA + AB$

$= 150 + 50 = 200$ m

Total time taken = 40 s.

Average speed = $\frac{200\text{m}}{40\text{s}} = 5\text{m/s}$

Total displacement = 100 m (OB)

Average velocity = $\frac{100\text{m}}{40\text{s}} = 2.5\text{m/s}$

Illustration 4. A body covers one – third of its journey with speed V_0 . The remaining portion of the distance was covered with velocity V_1 , for half the remaining time and with velocity V_2 for the other half of the remaining time. Assuming the body always travels in a straight line, find the average velocity of body over the whole journey.

Solution: Let x be the total distance and T be the total time. We require average velocity

$$= \frac{x}{T}$$

Now time required to cover $1/3^{\text{rd}}$ of total distance = $\frac{x}{3V_0}$

As per data

$$\frac{2x}{3} = \frac{1}{2} \times \left[T - \frac{x}{3V_0} \right] \times V_1 + \frac{1}{2} \times \left[T - \frac{x}{3V_0} \right] \times V_2$$

$$x \times \left[\frac{2}{3} + \frac{V_1}{6V_0} + \frac{V_2}{6V_0} \right] = T \times \left[\frac{V_1}{2} + \frac{V_2}{2} \right]$$

$$\therefore \text{Average velocity} = \frac{x}{T} = \frac{3V_0(V_1 + V_2)}{(V_1 + V_2 + 4V_0)}$$

1.3.5 Acceleration

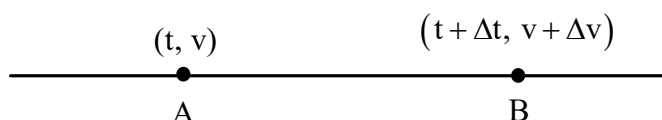
- Acceleration is defined as the rate of change of velocity with time.
- Acceleration is a vector quantity.
- SI unit of acceleration is m/s^2 .

Average acceleration :

- Average acceleration is defined as the ratio of change in velocity over a time interval to the time interval.
- If a particle moving along a straight line has velocity V_1 at an instant t_1 and velocity V_2 at instant t_2 ($t_2 > t_1$), then average acceleration during time interval $(t_2 - t_1)$ is given by

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1}$$

Instantaneous acceleration



Suppose a particle moving in a straight line has velocity v at time t at position A and velocity $v + \Delta v$ at time $t + \Delta t$ at position B, the average acceleration over time interval Δt can be

written as $a = \frac{\Delta v}{\Delta t}$.

- To get instantaneous acceleration at time t ,

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Uniform and variable acceleration:

If the change in velocity of the particle is equal in equal intervals of time, then the acceleration of the body is said to be **uniform**. Neither direction, nor magnitude changes with respect to time.

If change in velocity is different in equal intervals of time, then the acceleration of the particle is known as **variable**. If either direction or magnitude or both magnitude and direction of acceleration changes with respect to time, then acceleration is **variable**.

1.4 Uniform motion

- Motion of a body in a straight line with uniform velocity is called uniform motion.

$$\bullet \frac{ds}{dt} = v, \text{ but } v \text{ is constant in uniform motion.}$$

$$\therefore \int ds = \int v dt = v \int dt = vt$$

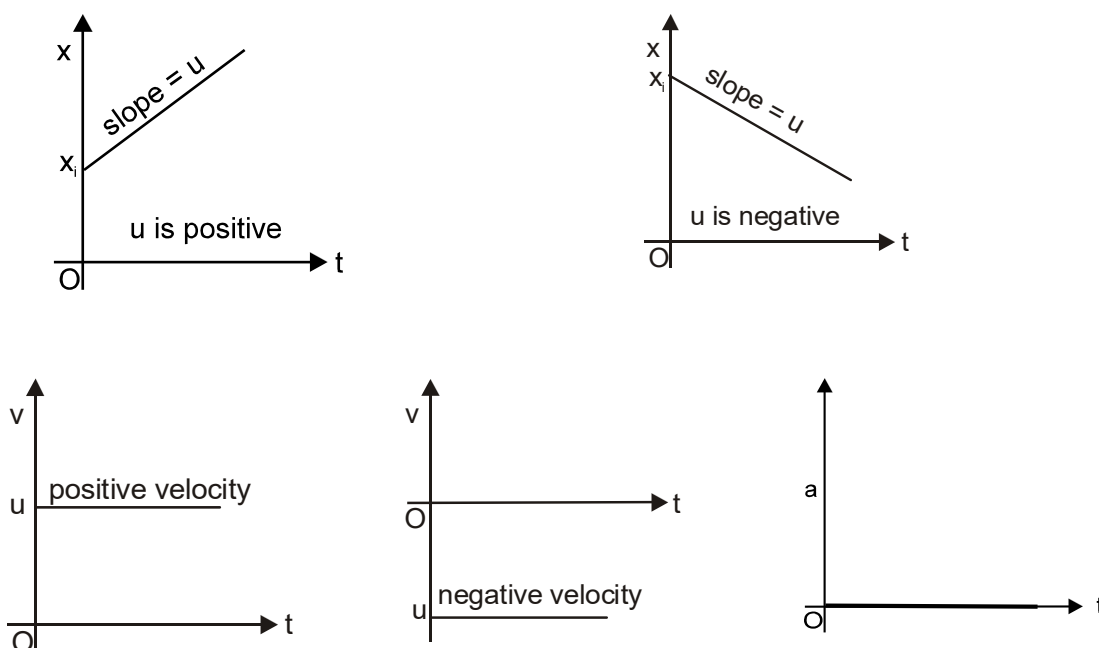
$\therefore s = vt$, where s is the displacement, t is the time interval, v is uniform velocity.

- Uniform motion can also be said to be motion in which equal displacements are covered in equal intervals of time, however small the time intervals may be.

x-t, v-t and a-t curve for uniform motion

- $x-t$ graph is a straight line of slope u through x_i
- as velocity is constant, $v-t$ graph is a horizontal line

- a-t graph coincides with time axis because $a = 0$ at all time instants



IN-CHAPTER EXERCISE - 1

1. A car travels first half distance between two places with uniform speed of 60 km / hr. What should be its uniform speed for the second half of the distance so that its average speed over the entire journey becomes 90 km / hr.
2. A body covers one – third of its journey with speed u , next third with speed v and the last third with speed w . Calculate the average speed of the body over the entire journey.
3. A 10 hour trip is made at an average speed of 40 km / hr. If during the first half of the distance, average speed of the bus was 30 km / hr, what was the average speed for the second half of trip?
4. A body, moving in a straight line, covers a certain distance in the following four separate and independent ways.
 - (a) half the time is covered with speed V_1 and the other half of the time with speed V_2 .
 - (b) half the distance is covered with speed V_1 and next half distance is covered with speed V_2 .
 - (c) $1/3^{\text{rd}}$ of the distance is covered with speed V_1 and for the remaining distance, the first $1/4^{\text{th}}$ of the remaining time is covered with speed V_2 and $3/4^{\text{th}}$ of the remaining time is covered with speed V_3 .
 - (d) $1/5^{\text{th}}$ of the time is covered with speed V_1 and in the remaining time, $3/4^{\text{th}}$ of remaining distance is covered with speed V_2 and $1/4^{\text{th}}$ of the remaining distance is covered with speed V_3 .

In each of the above four cases find average speed over the entire journey.
5. A steamer travelling in a river, moves from P to Q (downstream) in 2.5 hours and from Q to P (upstream) in 5 hours. If due to monsoon, speed of the river flow gets doubled, find the new time taken by the same steamer to go from
 - (a) P to Q (downstream)
 - (b) Q to P (upstream)

6. A particle is moving along a circle such that it completes one revolution in 40 seconds. In 2 minutes 20 seconds, the ratio $\frac{|\text{displacement}|}{\text{dis tance}}$ is
 (a) 0 (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{1}{11}$
7. Consider the motion of the tip of the second hand of a clock. In one minute (R be the length of second hand), its
 (a) Displacement is $2\pi R$ (b) Distance covered is $2R$
 (c) Displacement is zero (d) Distance covered is zero
8. The position of a body moving along x-axis at time t is given by $x = (t^2 - 4t + 6)$ m. The distance travelled by body in time interval $t = 0$ to $t = 3$ s is
 (a) 5 m (b) 7 m (c) 4 m (d) 3 m
9. A particle moves along x-axis with speed 6 m/s for the first half distance of a journey and the second half distance with a speed 3 m/s. The average speed in the total journey is
 (a) 5 m/s (b) 4.5 m/s (c) 4 m/s (d) 2 m/s
10. A car moves with speed 60 km/h for 1 hour in east direction and with same speed for 30 min in south direction. The displacement of car from initial position is
 (a) 60 km (b) $30\sqrt{3}$ km (c) $30\sqrt{5}$ km (d) $60\sqrt{2}$ km
11. A person travels along a straight road for the first $\frac{t}{3}$ time with a speed v_1 and for next $\frac{2t}{3}$ time with a speed v_2 . Then the mean speed v is given by
 (a) $v = \frac{v_1 + 2v_2}{3}$ (b) $\frac{1}{v} = \frac{1}{3v_1} + \frac{2}{3v_2}$ (c) $v = \frac{1}{3}\sqrt{2v_1v_2}$ (d) $v = \sqrt{\frac{3v_2}{2v_1}}$
12. A body in one dimensional motion has zero speed at an instant. At that instant, it must have
 (a) Zero velocity (b) Zero acceleration
 (c) Non-zero velocity (d) Non-zero acceleration
13. If a particle is moving along straight line with increasing speed, then
 (a) Its acceleration is negative (b) Its acceleration may be decreasing
 (c) Its acceleration is positive (d) Both (b) & (c)
14. If average velocity of particle moving on a straight line is zero in a time interval, then
 (a) Acceleration of particle may be zero
 (b) Velocity of particle must be zero at an instant
 (c) Velocity of particle may be never zero in the interval
 (d) Average speed of particle may be zero in the interval
15. A particle travels half of the distance of a straight journey with a speed 6 m/s. The remaining part of the distance is covered with speed 2 m/s for half of the time of remaining journey and with speed 4 m/s for the other half of time.
 The average speed of the particle is
 (a) 3 m/s (b) 4 m/s (c) $3/4$ m/s (d) 5 m/s
16. If magnitude of average speed and average velocity over an interval of time are same, then
 (a) Particle must move with zero acceleration
 (b) Particle must move with uniform acceleration
 (c) Particle must be at rest

(d) Particle must move in a straight line without turning back

ANSWER KEY (IN-CHAPTER EXERCISE – 1)

1. 180 km / hr

2.
$$\frac{3 u v w}{u v + v w + u w}$$

3. 60 km / hr

4. (a) $\frac{V_1 + V_2}{2}$ (b) $\frac{2V_1 V_2}{V_1 + V_2}$ (c) $\frac{3 V_1 V_2 + 9 V_1 V_3}{8 V_1 + V_2 + 3 V_3}$ (d) $\frac{16 V_2 V_3 + 3 V_1 V_3 + V_1 V_2}{15 V_3 + 5 V_2}$

5. (a) 2 hours

(b) 10 hours

6. (d)

7. (c)

8. (a)

9. (c)

10. (c)

11. (a)

12. (a)

13. (b)

14. (b)

15. (b)

16. (d)

1.5 Uniformly accelerated motion.

- A particle moving in a straight line is said to be uniformly accelerated, if it makes equal changes in velocities in equal intervals of time, however small the time interval may be.
- In uniformly accelerated motion, instantaneous acceleration is constant and is independent of time.
- **Derivation of equations for uniformly accelerated motion.**

$$(a) \quad a = \frac{dv}{dt}$$

$$\int_{v=u}^{v=v} dv = a \int_{t=0}^{t=t} dt$$

$$[v]_u^v = a[t]_0^t$$

$$\therefore v = u + at$$

... Eq. (I)

$$(b) \quad v = \frac{ds}{dt}$$

From Eq. (I) $v = u + at$

$$\frac{ds}{dt} = u + at$$

$$\int_{s=0}^{s=s} ds = \int_{t=0}^{t=t} (u + at) dt$$

$$\int_{s=0}^{s=s} ds = \int_{t=0}^{t=t} u dt + \int_{t=0}^{t=t} at dt$$

$$s = ut + \frac{1}{2}at^2$$

... Eq. (II)

(c) Let at $t = 0$, $x = x_0$

$t = t$, $x = x$

$$v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = u + at$$

$$\int_{x=x_0}^{x=x} dx = \int_{t=0}^{t=t} (u + at) dt$$

$$x = x_0 + ut + \frac{1}{2}at^2$$

$$\text{Or} \quad x - x_0 = ut + \frac{1}{2}at^2 \quad \dots \text{Eq. (III)}$$

(Here $s = x - x_0$ = displacement or change in position)

$$(d) \quad a = v \left(\frac{dv}{ds} \right)$$

$$\int_{v=u}^{v=v} v dv = a \int_{s=0}^s ds$$

$$v^2 = u^2 + 2as$$

... Eq. (IV)

$$(e) \quad a = v \left(\frac{dv}{dx} \right)$$

$$\int_{v=u}^{v=v} v dv = a \int_{x=x_0}^{x=x} dx$$

$$v^2 = u^2 + 2a(x - x_0) \quad \dots \text{Eq. (V)}$$

(f) Eliminating t between Eq. (I) and Eq. (II), we get

$$s = \frac{(u + v)}{2} \times t \quad \dots \text{Eq. (VI)}$$

(g) Displacement in n^{th} second for a uniformly accelerated body.

Let total displacement of body in n seconds be S_n .

$$\therefore S_n = un + \frac{1}{2}an^2$$

Let total displacement of body in $(n - 1)$ seconds be S_{n-1} .

$$\therefore S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

Then displacement of body in n^{th} second will be $S_{n^{\text{th}}} = S_n - S_{n-1}$

$$S_{n^{\text{th}}} = un + \frac{1}{2}an^2 - \left[u(n-1) + \frac{1}{2}a(n-1)^2 \right]$$

$$S_{n^{\text{th}}} = u + a \left[n - \frac{1}{2} \right] \quad \dots \text{Eq. VII}$$

- Equations for uniformly accelerated motion in vector form.

$$\vec{v} = \vec{u} + \vec{a}t \quad ; \quad \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad ; \quad \vec{v} \cdot \vec{v} = \vec{u} \cdot \vec{u} + 2\vec{a} \cdot \vec{s} \quad ; \quad \vec{s} = \frac{(\vec{u} + \vec{v})}{2} \times t$$

Illustration 6. An automobile manufacturer claims that his sports car will accelerate from rest to a speed of 42 m/s in 8 seconds. Assuming that acceleration is constant.

(a) Determine acceleration of car

(b) Distance travelled by car in 8 s.

(c) Distance travelled by car in 8^{th} second.

Solution:

$$(a) \quad v = u + at \quad \therefore \quad a = \frac{v - u}{t} = \frac{42 - 0}{8} = 5.25 \text{ m/s}^2$$

$$(b) \quad \text{Distance travelled in 8 s} \quad s = 0 \times t + \frac{1}{2} \times 5.25 \times 8^2 = 168 \text{ m}$$

$$(c) \quad S_{n^{\text{th}}} = u + a \left(n - \frac{1}{2} \right) = 0 + 5.25 \left[8 - \frac{1}{2} \right] = 39.375 \text{ m}$$

Illustration 7. A particle starts from rest and under uniform acceleration covers a distance x in t sec. Find the distance it will cover in the next t sec.

$$\text{Solution:} \quad s = ut + \frac{1}{2}at^2$$

$$x = \frac{1}{2}at^2 \quad \dots (I)$$

Now if it travels additional distance y in next t sec, total distance travelled in $2t$ s will be

$$x + y = 2at^2 \quad \dots (II)$$

Dividing Eq. II by Eq. I

$$\frac{x+y}{x} = 4 \quad \therefore y = 3x$$

Illustration 8. Two particles P and Q starts simultaneously from point A with velocities 15 m/s and 20 m/s respectively. They move in the same direction with different uniform accelerations. When P overtakes Q at B, velocity of P is 30 m/s. Find velocity of Q at B.

Solution:

$$\text{For P} \quad s = \left(\frac{15+30}{2} \right) \times t \quad \dots (I)$$

$$\text{For Q} \quad s = \left(\frac{20+v}{2} \right) \times t \quad \dots (II).$$

Equating Eq. I and Eq. II

$$V = 25 \text{ m/s}$$

Illustration 9. A particle moving with uniform acceleration in a straight line covers 3 m in the 8th second and 5 m in the 16th second of its motion. What is the displacement of the particle from beginning of 6th second to the end of 15th second?

Solution:

$$S_{n^{\text{th}}} = u + a \left(n - \frac{1}{2} \right)$$

$$3 = u + a \left[8 - \frac{1}{2} \right] \quad \dots \text{Eq. I}$$

$$5 = u + a \left[16 - \frac{1}{2} \right] \quad \dots \text{Eq. II}$$

$$\text{From Eq. I and Eq. II,} \quad a = \frac{1}{4} \text{ m/s}^2$$

$$\text{Putting } a = \frac{1}{4} \text{ m/s}^2 \quad \text{in Eq. I}$$

$$\text{We get} \quad u = \frac{9}{8} \text{ m/s}$$

Now, velocity at end of 5 s (velocity at beginning of 6th second), $v_1 = u + 5a$.

$$\text{Velocity after 15s} = v_2 = u + 15a. \quad S = \left(\frac{v_1 + v_2}{2} \right) \times t = \left(\frac{2u + 20a}{2} \right) \times 10 = 36.25 \text{ m}$$

IN-CHAPTER EXERCISE - 2

1. A race car accelerates on a straight road from rest to 180 km/hr in 25s. Assuming uniform acceleration, find the distance covered in this time.
2. A car moving along a straight highway with 126 km/hr speed is brought to rest in 200 m. What is the uniform retardation of the car and how long does it take for the car to stop.
3. A driver takes 0.2s reaction time to apply brakes after he sees the need for it. If he is driving the car at a speed of 54 km/hr, and the brakes cause uniform retardation of 6 m/s^2 , find the distance travelled by the car after he sees the need to apply brakes.
4. A body moving with uniform acceleration, covers 20 m in the 7th second and 24 m in the 9th second of its motion. Find the distance it will cover in the 15th second of its motion.

5. An object is moving along the x – axis with uniform acceleration of 4 m/s^2 . At time $t = 0$, $x = 5 \text{ m}$ and $v = 3 \text{ m/s}$.
 - (a) What will be the velocity and the position of the object at time $t = 2\text{s}$.
 - (b) What will be the position of the object when it has velocity of 5 m/s .
6. A particle move with velocity v_1 for time t_1 and v_2 for time t_2 along a straight line. The magnitude of its average acceleration is
 - (a) $\frac{v_2 - v_1}{t_1 - t_2}$
 - (b) $\frac{v_2 - v_1}{t_1 + t_2}$
 - (c) $\frac{v_2 - v_1}{t_2 - t_1}$
 - (d) $\frac{v_1 + v_2}{t_1 - t_2}$
7. A particle starts moving with acceleration 2 m/s^2 . Distance travelled by it in 5th half second is
 - (a) 1.25 m
 - (b) 2.25 m
 - (c) 6.25 m
 - (d) 30.25 m
8. The two ends of a train moving with constant acceleration pass a certain point with velocities u and $3u$. The velocity with which the middle point of the train passes the same point is
 - (a) $2u$
 - (b) $\frac{3}{2}u$
 - (c) $\sqrt{5}u$
 - (d) $\sqrt{10}u$
9. A train starts from rest from a station with acceleration 0.2 m/s^2 on a straight track and then comes to rest after attaining maximum speed on another station due to retardation 0.4 m/s^2 . If total time spent is half an hour, then distance between two stations is [Neglect length of train]
 - (a) 216 km
 - (b) 512 km
 - (c) 728
 - (d) 1296 km
10. A car travelling at a speed of 30 km/h is brought to rest in a distance of 8 m by applying brakes. If the same car is moving at a speed of 60 km/h then it can be brought to rest with same brakes in
 - (a) 64 m
 - (b) 32 m
 - (c) 16 m
 - (d) 4 m
11. A car moving with speed v on a straight road can be stopped with in distance d on applying brakes. If same car is moving with speed $3v$ and brakes provide half retardation, then car will stop after travelling distance
 - (a) $6d$
 - (b) $3d$
 - (c) $9d$
 - (d) $18d$

ANSWER KEY (IN CHAPTER EXERCISE – 2)

1. 625 m 2. 3.06 m/s^2 , 11.43 s 3. 21.75 m 4. 36 m

5. (a) 11 m/s and 19 m (b) 7 m

6. (b)

7. (b)

8. (c)

9. (a)

10. (b)

11. (d)

1.6 Motion under gravity

- A body thrown vertically upwards or vertically downwards or dropped from a height will move in a straight vertical line.
- If air resistance is ignored, the body will be subjected to acceleration due to gravitational force exerted by the earth, which is denoted by g . The value of g on the earth is 9.8 m/s^2 in the downward direction.
- We shall take upward direction as positive & down direction as negative, as our convention.

1.6.1 Motion of a particle projected vertically upward from the ground.

- Consider a particle projected vertically upward from the ground with velocity u .
- Taking upward direction positive $u = u$, $a = -g$

\therefore At any time t , velocity $v = u - g t$ and displacement $s = ut - \frac{1}{2}gt^2$

- To find time of ascent (t_a), apply $v = u + at$ between the point of projection and the highest point.

$$v = 0, \quad u = u, \quad a = -g.$$

$$\therefore \quad t_a = \frac{u}{g}$$

- To find total time of flight (T), apply $s = ut + \frac{1}{2}at^2$ between the point of projection and the time instant when the particle is again at point of projection $s = 0$, $u = u$, $a = -g$

$$\therefore \quad T = \frac{2u}{g}$$

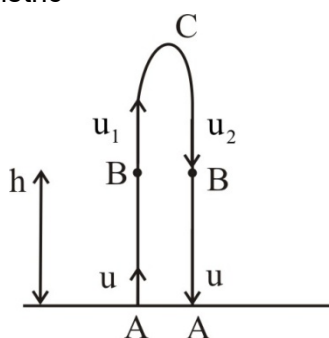
- Time of descent (t_d) between the highest point and back to the point of projection is

$$\text{also } \frac{u}{g} \therefore t_d = \frac{u}{g}$$

- For maximum height attained (h_{\max}) apply $v^2 = u^2 + 2as$ between the point of projection and the topmost point, $v = 0$, $u = u$, $a = -g$

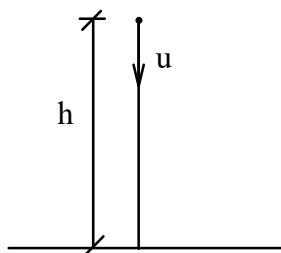
$$\therefore \quad h_{\max} = \frac{u^2}{2g}$$

- The particle will return back to the point of projection with same speed as the speed of projection but in the opposite direction.
- Motion under gravity is symmetric



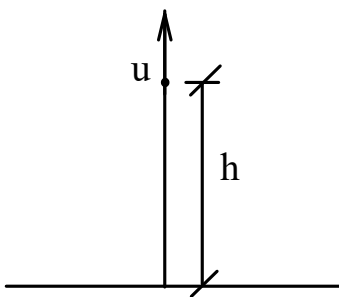
Consider a particle projected from A. B is a point at height h and C is the topmost point. In the above case speeds u_1 and u_2 are equal, $t_{BC} = t_{CB}$, $t_{AB} = t_{BA}$.

1.6.2 Motion of a particle projected downwards from height h above surface of earth



Suppose a particle is projected downwards from height h above the surface of the earth with speed u . To find the time taken by it to strike the surface of the earth, taking upward direction as positive, $u = -u$, $a = -g$, $s = -h$, apply $s = ut + \frac{1}{2}at^2$, solve the quadratic and get the positive value of t .

1.6.3 Motion of a particle projected vertically upwards from height h above surface of earth.



$-h = ut - \frac{1}{2}gt^2$, solve the quadratic and get the positive value of t for which the particle strikes the surface.

1.6.4 Motion of a particle dropped from a height h above surface of earth.

Solve using $v^2 = u^2 + 2as$ and $s = ut + \frac{1}{2}at^2$, taking $u = 0$,

Velocity with which it strikes the surface will be $\sqrt{2gh}$ and the time

it will take to strike the surface will be $\sqrt{\frac{2h}{g}}$

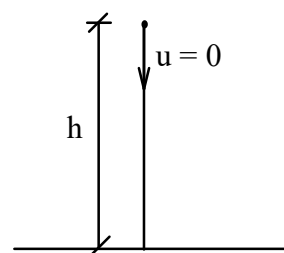


Illustration 11. A particle is projected upwards from top of a tower of height 40m with a speed of 10m/s. Find time it will take to strike ground ($g = 10\text{m/s}^2$)

Solution: Take upward direction positive and apply $s = ut + \frac{1}{2}at^2$

$$s = -40, u = 10, a = -10$$

$$\therefore -40 = 10t - \frac{1}{2} \times 10 \times t^2$$

$$\therefore t = 4\text{s or } t = -2\text{s}$$

$$\therefore t = 4\text{s}$$

Illustration 12. A stone A is dropped from rest from height h above ground. A second stone B is simultaneously thrown vertically up with velocity v . Find v so that B meets A midway between their initial positions.

Solution: Time of travel of each stone = t

$$\text{Distance travelled by each stone} = \frac{h}{2}$$

$$\text{For stone A, } \frac{h}{2} = \frac{1}{2}gt^2 \quad t = \sqrt{\frac{h}{g}}$$

$$\text{For stone B, } \frac{h}{2} = vt - \frac{1}{2}gt^2 \quad \text{Put } t = \sqrt{\frac{h}{g}}, \quad \therefore v = \sqrt{gh}$$

Illustration 13. A body is dropped from rest from height h . It covers $\frac{9h}{25}$ distance in last second of fall. Find h ($g = 10 \text{ m/s}^2$).

Solution: $h = \frac{1}{2}gt^2$

$$\left(1 - \frac{9}{25}\right)h = \frac{1}{2}g(t-1)^2$$

$$\therefore t = 5 \text{ sec s, } h = 125 \text{ m}$$

Illustration 14. A balloon starts from ground with 1.25 m/s^2 acceleration. After 8 s, a stone is released from balloon. Find time in which stone will strike ground. Find distance covered by stone and its displacement from point from where it was released. Find also height of balloon when stone strikes ground. $g = 10 \text{ m/s}^2$.

Solution: For balloon, $s = 0 \times 8 + \frac{1}{2} \times 1.25 \times 8^2 = 40 \text{ m}$

$$v = 0 + 1.25 \times 8 = 10 \text{ m/s}$$

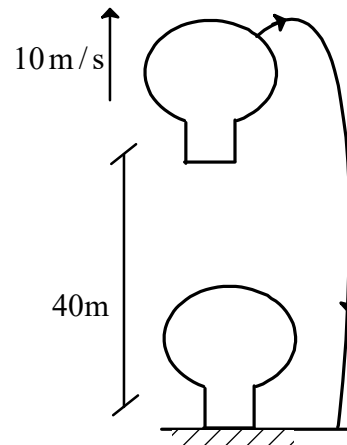
$$\text{For stone, } -40 = 10 \times t - \frac{1}{2} \times 10 \times t^2$$

$$t = -2 \text{ or } 4 \text{ s} \quad \therefore t = 4 \text{ s.}$$

$$\text{Distance covered by stone} = 40 + \frac{2 \times 10^2}{2 \times 10} = 50 \text{ m}$$

$$\text{Height of balloon} = \text{Displacement of balloon in 12 s.}$$

$$s = \frac{1}{2} \times 1.25 \times 12^2 = 90 \text{ m}$$



IN-CHAPTER EXERCISE - 3

1. A body is thrown vertically upwards from the ground with velocity of 98 m/s. Calculate
 - (i) The maximum height reached
 - (ii) time taken to reach the highest point.
 - (iii) velocity at height of 196m from the point of projection.
 - (iv) velocity with which it returns to the ground.
 - (v) Time taken to reach the ground ($g = 9.8 \text{ m/s}^2$)
2. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower strikes the ground in 6s. Find the height of the tower. ($g = 9.8 \text{ m/s}^2$)
3. A ball thrown vertically up is caught by the person who threw the ball after 4s. How high did it go and with what velocity was it thrown? How far was it below the highest point, 3s after it was thrown? ($g = 9.8 \text{ m/s}^2$)
4. A balloon ascending at constant velocity of 9.8m/s is at a height of 39.2m above ground when a food packet is dropped from the balloon. After how much time and with what velocity does the packet reach the ground. ($g = 9.8 \text{ m/s}^2$)
5. Two bodies A and B are thrown simultaneously. A is projected vertically upwards with 20 m/s speed from the ground and B is projected vertically downwards from a height of 40 m with the same speed and along the same line of motion. At what point do the 2 bodies collide? ($g = 9.8 \text{ m/s}^2$)
6. A stone falls from a cliff and travels 24.5 m in the last second of its motion before it reaches the ground at the foot of a cliff. Find the height of the cliff. ($g = 9.8 \text{ m/s}^2$)
7. A stone is dropped from the top of a tall tower and after one second another stone is dropped from a balcony 20m below the top. If both stones reach the ground at the same instant, calculate the height of the tower. ($g = 10 \text{ m/s}^2$)
8. A stone is dropped into a well of 45 m depth. Sound of splash due to the stone striking the water surface in the well is heard after 3.125s. Find the velocity of sound in air ($g = 10 \text{ m/s}^2$)
9. Acceleration due to gravity on a planet is $\frac{1}{5}$ that on earth. If it is safe to jump from a height of 2m on earth, what will be corresponding safe height on this planet? (Assume safety depends on the velocity of reaching ground)
10. A stone is dropped from a point 0.4m above the top of a 0.5m high window. Find the time taken by the stone to pass the span of the window. ($g = 9.8 \text{ m/s}^2$)
11. A particle is thrown with any velocity vertically upward, the distance travelled by the particle in first second of its decent is

(a) g	(b) $\frac{g}{2}$	(c) $\frac{g}{4}$	(d) Cannot be calculated
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12. A body is thrown vertically upwards and takes 5 seconds to reach maximum height. The distance travelled by the body will be same in
 (a) 1st and 10th second (b) 2nd and 8th second
 (c) 4th and 6th second (d) Both (b) & (c)
13. A ball dropped from the top of tower falls first half height of tower in 10 s. The total time spend by ball in air is [Take $g = 10 \text{ m/s}^2$]
 (a) 14.14 s (b) 15.25 s (c) 12.36 s (d) 17.36 s
14. A ball is projected vertically upwards. Its speed at half of maximum height is 20 m/s. The maximum height attained by it is [Take $g = 10 \text{ ms}^2$]
 (a) 35 m (b) 15 m (c) 25 m (d) 40 m
15. A stone dropped from the top of a tower is found to travel of the height of the tower during the last second of its fall. The time of fall is
 (a) 2 s (b) 3 s (c) 4 s (d) 5 s
16. A stone thrown upward with a speed u from the top of a tower reaches the ground with a velocity $4u$. The height of the tower is
 (a) $\frac{15u^2}{2g}$ (b) $\frac{7u^2}{2g}$ (c) $\frac{16u^2}{g}$ (d) Zero

ANSWER KEY (IN CHAPTER EXERCISE – 3)

- | | | | |
|-----|--|-----|--------------|
| 1. | (i) 490 m (ii) 10 s (iii) 75.9 m/s (iv) 98 m/s (v) 20s | 2. | 58.8m |
| 3. | 19.6m, 19.6 m/s , 4.9m | 4. | 4s, 29.4 m/s |
| 5. | 15.1m from ground after 1s | 6. | 44.1m |
| 7. | 31.25m | 8. | 360m/s |
| 9. | 10m | 10. | 0.14s |
| 11. | (b) | 12. | (a) |
| 13. | (a) | 14. | (d) |
| 15. | (b) | 16. | (a) |

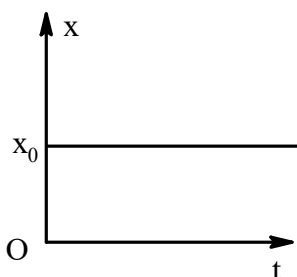
1.7 Graphs describing motion of a particle along a straight line.

1.7.1 Position versus time graph (x – t graph)

- Slope of tangent drawn at any point on the curve of the $x - t$ graph gives instantaneous velocity and magnitude of slope gives instantaneous speed

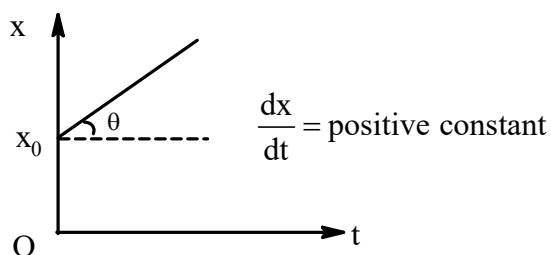
Description of motion using $x - t$ graph.

(A) Particle at rest

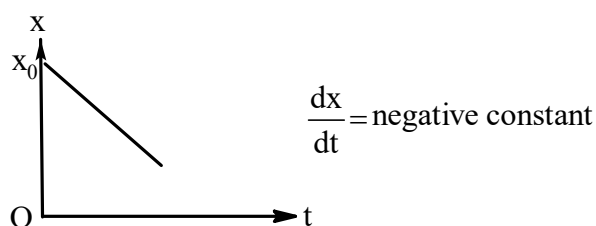


$$\frac{dx}{dt} = 0, \text{ Particle is at rest}$$

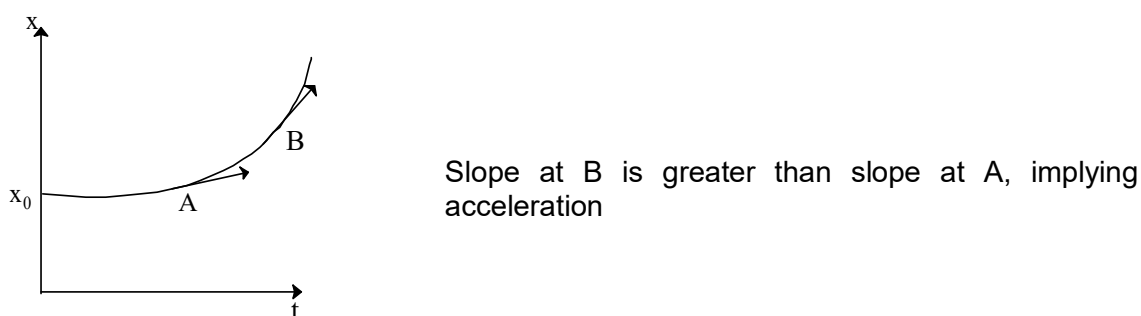
(B) Particle moving with uniform velocity in the positive direction.



(C) Particle moving with uniform velocity in negative direction.

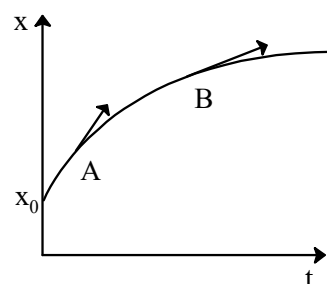


(D) Particle moving with uniform acceleration.

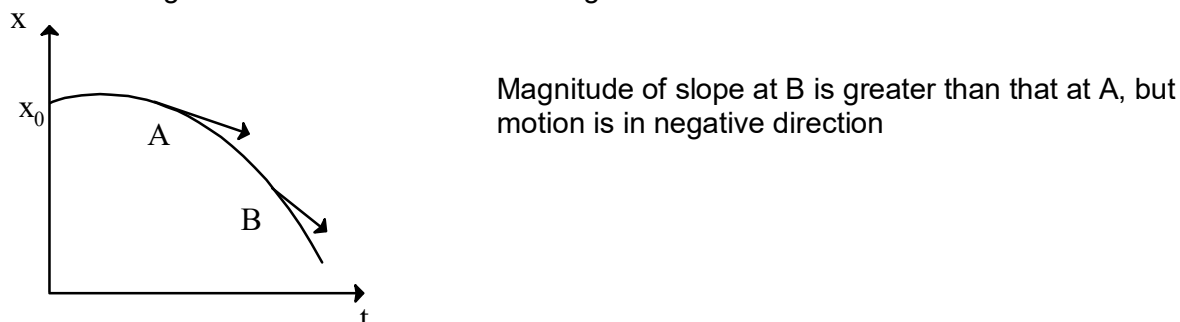


(E) Particle moving with uniform retardation.

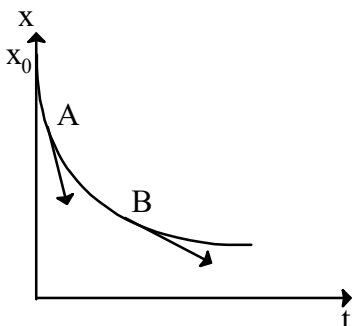
Slope at B is less than slope at A, implying retardation.



(F) Particle moving with uniform acceleration in negative direction

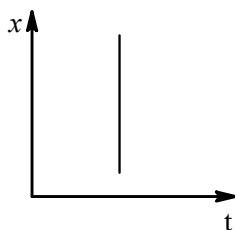


(G) Particle moving with uniform retardation in negative direction.



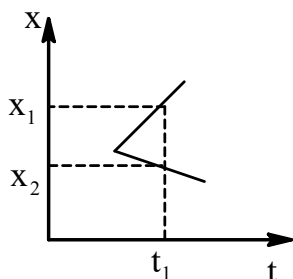
Slope at B is less than that of A. Motion is in the negative direction.

(H) The following x-t graphs are not possible.



Slope is infinite, implying infinite velocity, which is not possible

Particle is at two positions x_1 and x_2 at a given time instant t_1 which is not possible.



1.7.2 Velocity versus time graph

- $\frac{dv}{dt} = a$, Slope of $v - t$ graph gives instantaneous acceleration

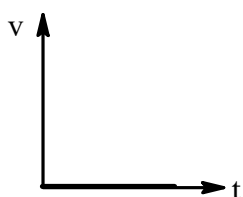
- $\frac{ds}{dt} = v$, $\therefore \int ds = \int v dt$ $\therefore \int v dt = s$,

Area under $v - t$ graph gives displacement.

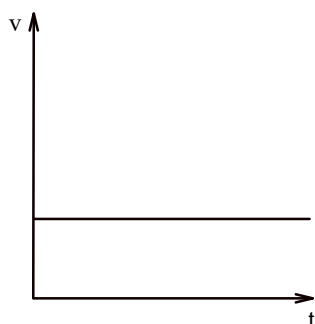
- Thus $v - t$ graph gives us instantaneous velocity, instantaneous acceleration as well as displacement covered. Hence $v - t$ graphs can be effectively utilized in solving problems.

Description of motion with v – t graph

(A) Particle at rest

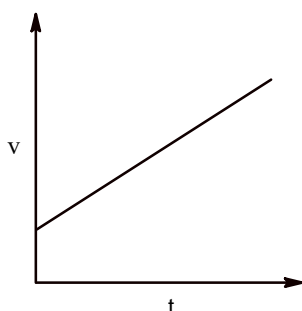


(B) Particle moving with uniform velocity



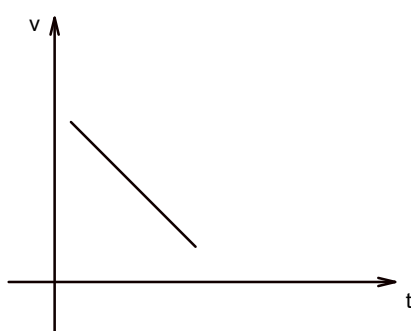
$$\frac{dv}{dt} = 0, \text{ acceleration is zero.}$$

(C) Particle moving with uniform acceleration



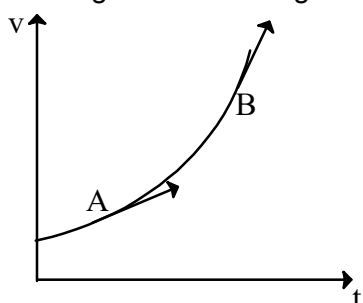
Slope of above graph is constant and positive, implying uniform acceleration.

(D) Particle moving with uniform retardation



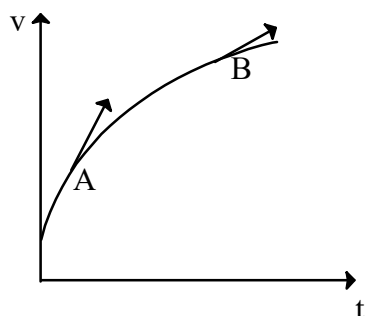
Slope of above graph is constant and negative, implying uniform retardation.

(E) Particle moving with increasing acceleration.



Slope at A is less, slope at B is more, implying increasing acceleration.

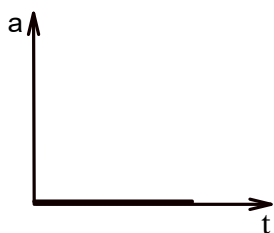
(F) Particle moving with decreasing acceleration.



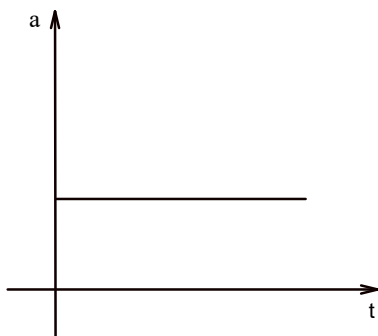
Slope at B is less than that at A implying decreasing acceleration.

1.7.3 Acceleration time graph

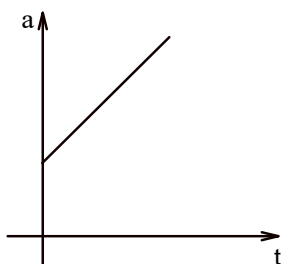
(A) Particle with zero acceleration. (rest or uniform velocity)



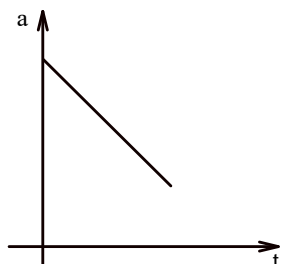
(B) Particle moving with uniform acceleration



(C) Particle moving with increasing acceleration



(D) Particle moving with decreasing acceleration



1.7.4 Area under various graphs

$$\bullet \quad \frac{dv}{dt} = a \quad \therefore \int_u^v dv = \int_0^t a dt \quad \therefore v - u = \int a dt$$

\therefore Area under $a - t$ graph gives change in velocity.

$$\bullet \quad a = v \left(\frac{dv}{ds} \right) \quad \therefore \int a ds = \int_u^v v dv$$

$$\therefore \int a ds = \frac{v^2 - u^2}{2}$$

$$\text{Area under } a - s \text{ graph} = \frac{v^2 - u^2}{2}$$

Where, v is instantaneous velocity and u is initial velocity.

$$\bullet \quad \frac{ds}{dt} = v$$

$$\int ds = \int v dt \quad \therefore s = \int v dt$$

\therefore Area under $v - t$ graph gives displacement.

Illustration 16. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate of β to come to rest. If the total time elapsed is t second, then calculate;

- The maximum velocity attained by the car, and
- The total displacement travelled by the car in terms of α, β and t .

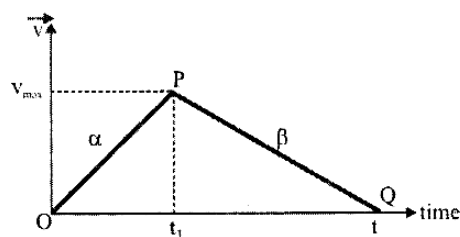
Solution: Let v_{\max} be the maximum velocity attained and t_1 be the time at which maximum velocity will occur. The velocity vs time graph can be drawn as follows:

$$\text{The slope of line OP, } \alpha = \frac{v_{\max}}{t_1}$$

$$\Rightarrow v_{\max} = \alpha t_1 \quad \dots (i)$$

$$\text{The slope of line PQ, } \beta = \frac{v_{\max}}{t - t_1}$$

$$v_{\max} = \beta(t - t_1) \quad \dots (ii)$$



From equations (i) and (ii), we get $\alpha t_1 = \beta(t - t_1)$

$$\text{which gives } t_1 = \frac{\beta t}{\alpha + \beta} \quad \dots (iii)$$

Substituting value of t_1 in equation (i), we get

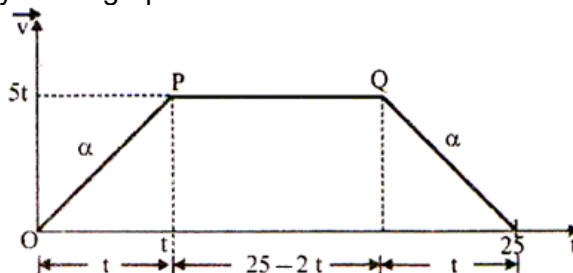
$$(i) \quad v_{\max} = \frac{\alpha \beta t}{\alpha + \beta} \quad \text{Ans.}$$

$$(ii) \quad \text{Total displacement } s = \text{area of } v - t \text{ graph}$$

$$= \frac{1}{2} \times v_{\max} \times t = \frac{1}{2} \times \frac{\alpha \beta t}{\alpha + \beta} \times t = \frac{\alpha \beta t^2}{2(\alpha + \beta)} \quad \text{Ans.}$$

Illustration 17. A car starts moving rectilinearly, first with acceleration $\alpha = 5 \text{ m/s}^2$ (the initial velocity is equal to zero), then uniformly, and finally, decelerating at the same rate α comes to a stop. The total time of motion equals $\tau = 25 \text{ s}$. The average velocity during that time is equal to $\langle v \rangle = 72 \text{ km/h}$. How long does the car move uniformly?

Solution: Let t be the time duration for which car accelerates or decelerates. The maximum velocity attained in this duration is $5t$. The time during which car move uniformly $= 25 - 2t$. The velocity - time graph of the motion of car is drawn as follows;



Given the average velocity in whole time of motion $v_{av} = \frac{72 \times 5}{18} = 20 \text{ m/s}$

The average velocity from the graph can be obtained as

$$v_{av} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{\text{Area of } v-t \text{ graph}}{\text{Total time}}$$

$$\therefore 20 = \frac{\frac{1}{2} \times [t + t + (25 - 2t)] \times 5t}{25}$$

$$= \frac{\frac{1}{2} \times [50 - 2t] \times 5t}{25}$$

$$\begin{aligned} \text{or } 200 &= 50t - 2t^2 \\ \text{or } t^2 - 25t + 100 &= 0 \\ (t - 20)(t - 5) &= 0 \\ t &= 5 \text{ s or } 25 \text{ s} \end{aligned}$$

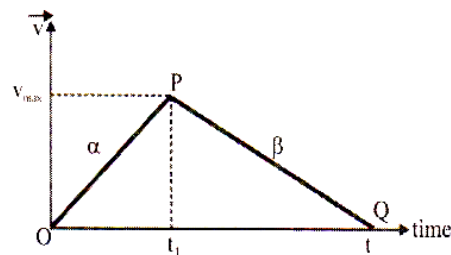
But $t = 25$ is not possible

$$\therefore t = 5 \text{ s}$$

The time for which car moves uniformly $= 25 - 2t = 25 - 2 \times 5 = 15 \text{ s}$ Ans.

Illustration 18. The distance (s) between two stations is to be covered in minimum time. The maximum value of acceleration or retardation of a car can not exceed α and β respectively. Find the time of motion.

Solution: To cover the distance in minimum time the car must get the maximum possible acceleration α and then retard to maximum possible value β . Let t_1 be the time up to which car accelerates and t is the required time of motion. The velocity-time graph of motion of car can be drawn as follows:



$$\text{We have already calculated that } s = \frac{\alpha \beta t^2}{2(\alpha + \beta)}$$

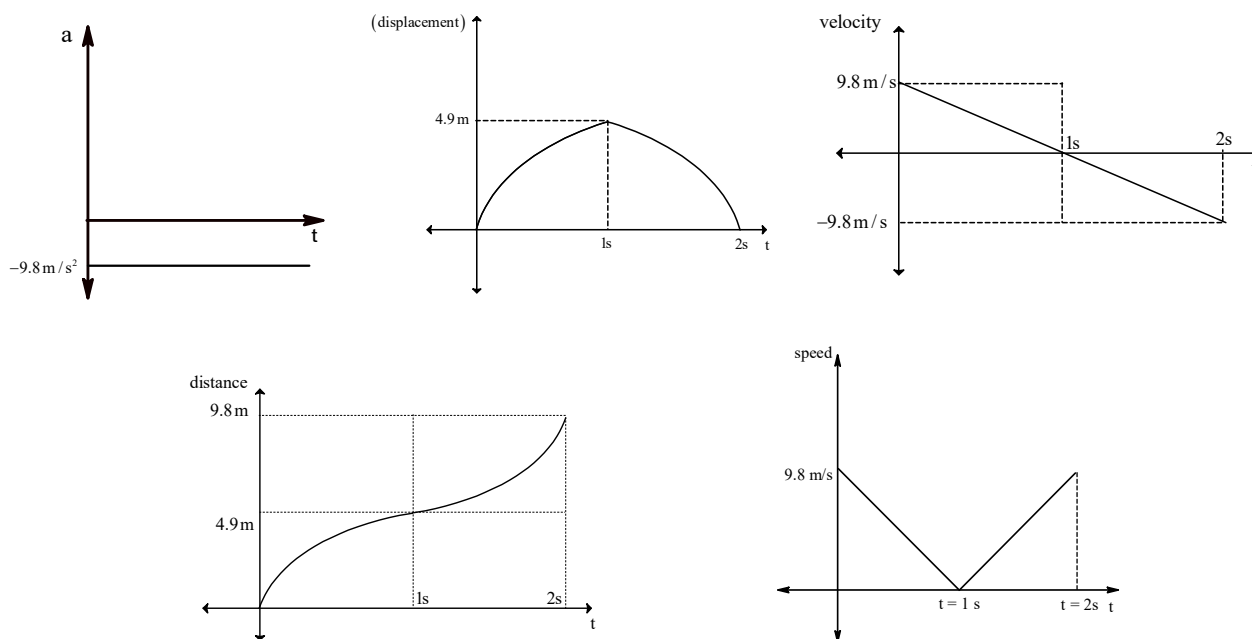
(Refer illustration 4)

$$\text{Solved above equation for } t, \text{ we have } t = \sqrt{\frac{2s(\alpha + \beta)}{\alpha \beta}} = \sqrt{2s \left(\frac{1}{\alpha} + \frac{1}{\beta} \right)} \quad \text{Ans.}$$

Illustration 19. A body is thrown vertically upwards with 9.8 m/s. Take origin as point of projection and upward direction positive, plot displacement–time, distance–time, velocity–time, speed–time, acceleration–time graph. ($g = 9.8 \text{ m/s}^2$)

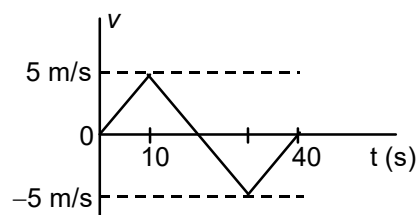
Solution : Time of ascent = time of descent = $\frac{u}{g} = 1\text{s}$

$$h_{\max} = \frac{u^2}{2g} = \frac{9.8^2}{2 \times 9.8} = 4.9\text{m}$$

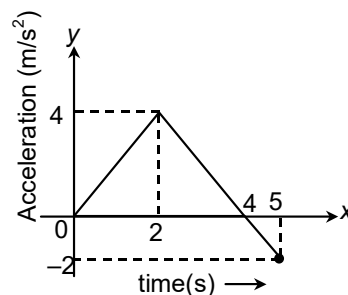


IN-CHAPTER EXERCISE - 4

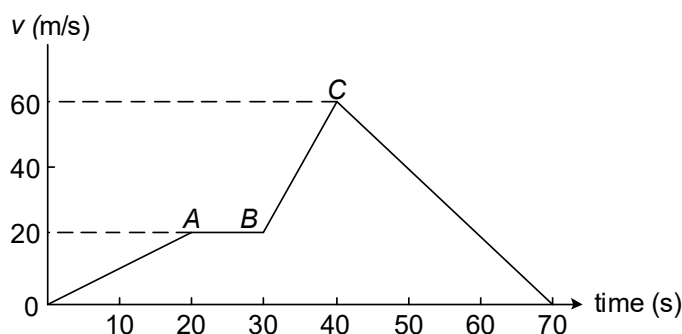
- The velocity-time plot is shown in figure. Find the average speed in time interval $t = 0$ to $t = 40\text{s}$.



- Figure shows the graph of acceleration of particle as a function of time. Find the maximum speed of the particle (particle starts from rest).

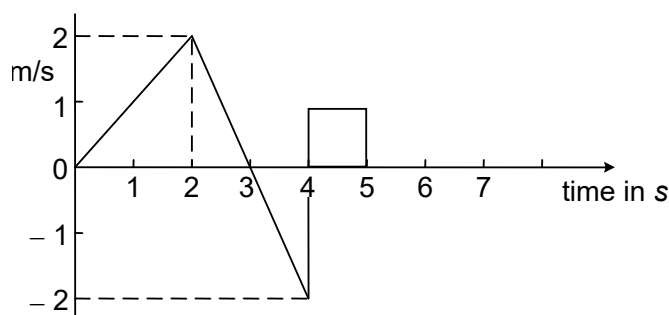


3. The velocity-time graph of a body is given below.



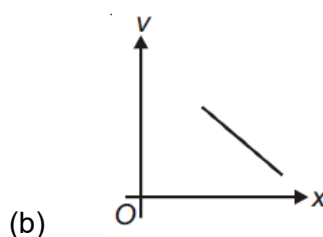
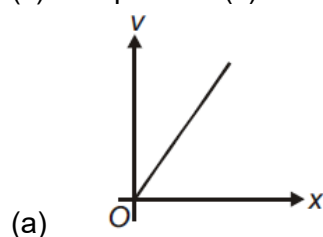
Find the maximum acceleration in m/s^2 .

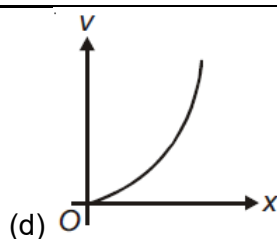
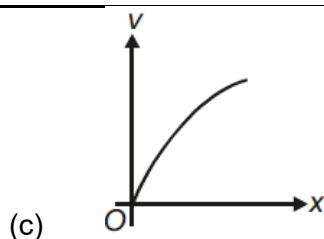
4. The velocity-time graph of a body moving along a straight line is as follows:



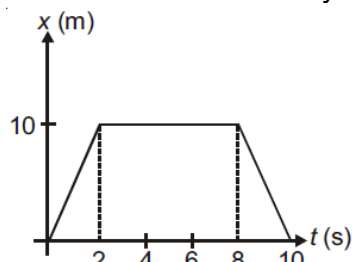
What is the displacement of the body in 5 s.

5. A hundred meter sprinter increases his speed from rest uniformly, at the rate of 1 m/s^2 upto three quarters of the total run and covers the last quarter with uniform speed. How much time is taken to cover the first half and second half of the run.
6. A motor car starts from rest & accelerates uniformly for 10 s to a velocity of 20 m/s. It then runs at a constant speed & finally is brought to rest in 40m with constant retardation. Total distance covered is 640m. Find value of acceleration, retardation & total time taken.
7. A train starting from rest accelerates uniformly for 100s, runs at a constant speed for 5 minutes & then comes to rest with uniform retardation in next 150 seconds. During motion, it covers a distance of 4.25 km. Find (i) the constant speed (ii) acceleration (iii) retardation
8. For a body moving with uniform acceleration along straight line, the variation of its velocity (v) with position (x) is best represented by



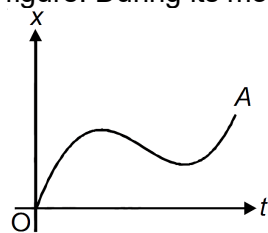


9. The position-time graph for a particle moving along a straight line is shown in figure. The total distance travelled by it in time $t = 0$ to $t = 10$ s is



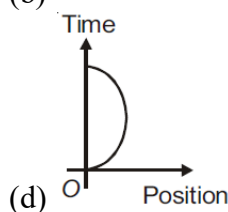
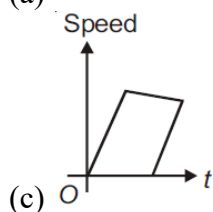
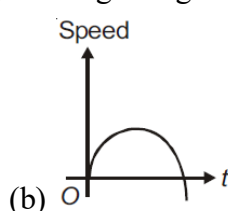
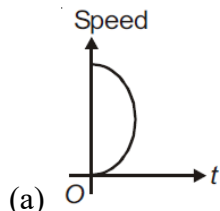
- (a) zero (b) 10 m (c) 20 m (d) 80 m

10. The position-time graph for a body moving along a straight line between O and A is shown in figure. During its motion between O and A, how many times body comes to rest?

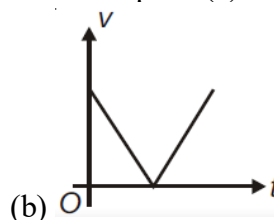
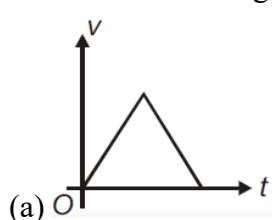


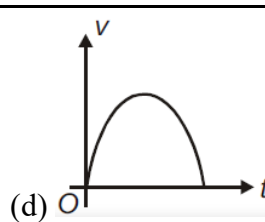
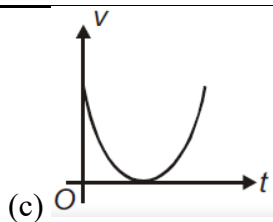
- (a) Zero (b) 1 time (c) 2 times (d) 3 times

11. Which one of the following graph for a body moving along a straight line is possible?

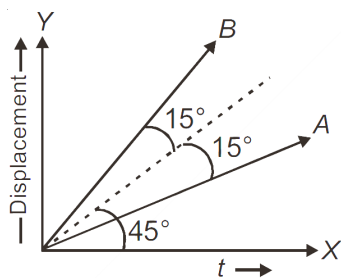


12. A body is projected vertically upward from ground. If we neglect the effect of air, then which one of the following is the best representation of variation of speed (v) with time (t)?



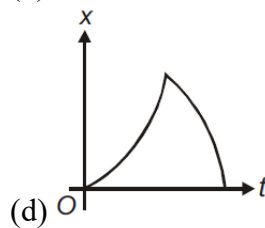
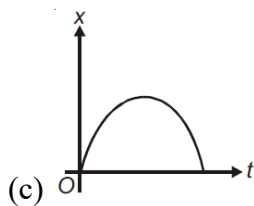
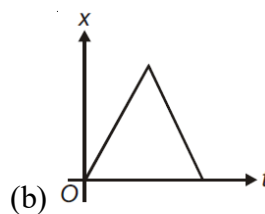
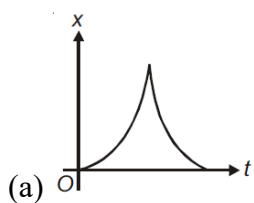
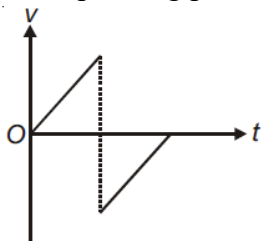


13. The displacement-time graph for two particles A and B is as follows. The ratio $\frac{V_A}{V_B}$ is

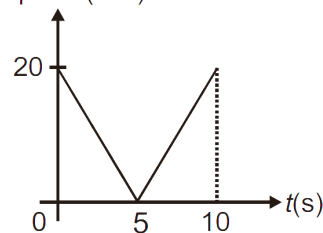


- (a) 1 : 2 (b) $1 : \sqrt{3}$ (c) $\sqrt{3} : 1$ (d) 1 : 3

14. The velocity (v)-time (t) graph for a particle moving along x -axis is shown in the figure. The corresponding position (x)-time (t) is best represented by



15. The speed-time graph for a body moving along a straight line is shown in figure. The average acceleration of body may be speed (m/s)



- (a) 0 (b) 4 m/s^2 (c) -4 m/s^2 (d) all of these

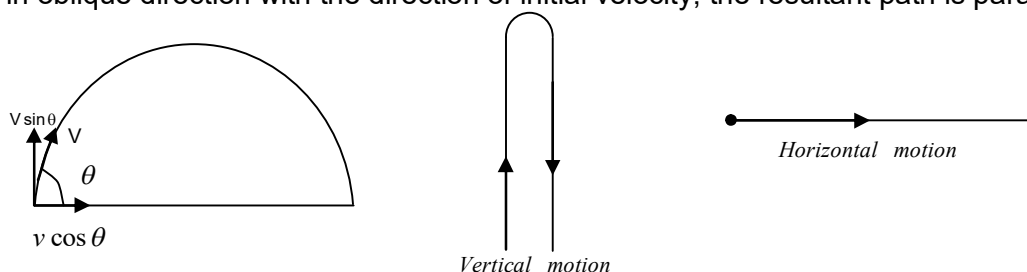
ANSWERS

- | | | | | |
|---|----------|-----------------------|--|-----------------|
| 1. 2.5 m/s | 2. 8 m/s | 3. 4 m/s ² | 4. 3 m | 5. 10 s, 4.24 s |
| 6. 2 m/s ² , 5 m/s ² , 39 s | | | 7. 10 m/s, 0.1 m/s ² , 0.067 m/s ² | |
| 8. (c) | 9. (c) | 10. (c) | 11. (d) | |
| 12. (b) | 13. (d) | 14. (a) | 15. (d) | |

1.8 Projectile Motion

1.8.1 Basic concept

- (I) Any particle which is thrown into space or air such that it moves under the influence of an external force (e.g. gravity, electric forces etc.) is called a projectile. The motion of such a particle is referred to as projectile motion.
- (II) It is an example of two dimensional motion with constant acceleration.
- (III) If the force acting on the projectile is constant, then acceleration is constant. When the force is in oblique direction with the direction of initial velocity, the resultant path is parabolic.



Parabolic motion = vertical motion + horizontal motion

- (IV) Projectile motion can be considered to be two simultaneous motions in mutually perpendicular directions which are completely independent of each other i.e. horizontal motion and vertical motion.

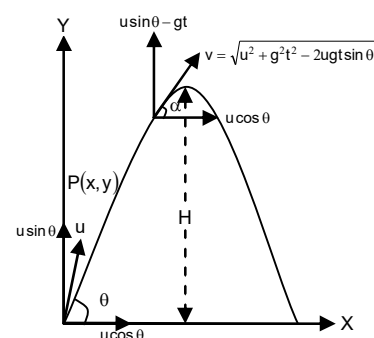
1.8.2 Ground to ground projectile

Consider a projectile thrown from horizontal ground with a velocity u making an angle θ with the horizontal. Take the point of projection as origin O and the path of the projectile in the first quadrant of xy - plane, as shown in the figure. The initial velocity u is resolved in the horizontal and vertical directions i.e.

$$u_x = u \cos \theta \quad u_y = u \sin \theta$$

Since gravity is the only force acting on the projectile in vertically downward direction, (ignoring air resistance)

$$a_x = 0, a_y = -g$$



Analyzing the motion of the projectile in horizontal and vertical directions:

Horizontal direction

(a) Initial velocity $u_x = u \cos \theta$

(b) Acceleration $a_x = 0$

(c) Velocity after time t , $v_x = u \cos \theta$

Vertical direction

(a) Initial velocity $u_y = u \sin \theta$

(b) Acceleration $a_y = -g$

(c) Velocity after time t , $v_y = u \sin \theta - gt$

- The position vector of the projectile after time t is $\vec{r} = x\hat{i} + y\hat{j} = (u\cos\theta t)\hat{i} + \left(u\sin\theta t - \frac{1}{2}gt^2\right)\hat{j}$;
- Velocity after time t is $\vec{v} = v_x\hat{i} + v_y\hat{j} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$;
- Acceleration is constant, $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$
- Trajectory equation:** The path traced by the projectile is called the trajectory of the projectile.

For displacement in the horizontal direction, $x = u_x \cdot t$

$$x = u \cos\theta \cdot t \quad \dots (1)$$

For displacement in the vertical direction, $y = u_y \cdot t - \frac{1}{2}gt^2$

$$y = u \sin\theta t - \frac{1}{2}gt^2 \quad \dots (2)$$

Substituting the value of t from eqn. (1) into eqn. (2), we get

$$y = u \sin\theta \cdot \frac{x}{u \cos\theta} - \frac{1}{2}g \cdot \left(\frac{x}{u \cos\theta}\right)^2$$

$$\Rightarrow y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta} \Rightarrow y = x \tan\theta \left[1 - \frac{x}{R}\right] \text{ (R is the horizontal range covered by the projectile)}$$

The equation of trajectory of the projectile is that of a parabola because the projectile covers a parabolic path.

- Time of flight:** The displacement along vertical direction is zero for ground to ground projectile.

$$(u\sin\theta)T - \frac{1}{2}gT^2 = 0 \quad T = \frac{2u\sin\theta}{g}$$

- Horizontal range:** The horizontal displacement of the projectile from the point of projection to the point it strikes the ground is called the horizontal range of the projectile.

$$R = u_x \cdot T$$

$$R = u \cos\theta \cdot \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

- Maximum height:**

Applying $v^2 = u^2 + 2as$ in the vertical direction between the point of projection and the topmost point, we get $0^2 = u^2 \sin^2\theta - 2gH$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

- Resultant velocity, at any instant t :**

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|\vec{v}| = \sqrt{u^2 \cos^2\theta + (u\sin\theta - gt)^2} \quad \& \quad \tan\alpha = v_y / v_x = \frac{u\sin\theta - gt}{u\cos\theta}$$

α is the angle made by the velocity vector of the projectile with the horizontal at any time instant t .

• **General result:**

(i) For maximum range $\theta = 45^\circ$ $R_{\max} = \frac{u^2}{g}$

In this situation $H_{\max} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g}$, $H_{\max} = \frac{R_{\max}}{4}$

(ii) We get the same range for two angles of projection α and $(90 - \alpha)$. But in each of the two cases, maximum height attained by the particle is different.

$$R = \frac{2u^2 \sin \alpha \cos \alpha}{g} = \frac{2u^2 \sin(90 - \alpha) \cos(90 - \alpha)}{g}$$

(iii) If $R = H$ i.e. $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$, $\frac{2u^2 \sin \theta \cos \theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$
 $\therefore \tan \theta = 4$; $\theta = \tan^{-1} 4$

(iv) Range can also be expressed as $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u \sin \theta \cdot u \cos \theta}{g} = \frac{2u_x u_y}{g}$

Illustration 20: A body is projected with a velocity of 30 ms^{-1} at an angle of 30° with the vertical. Find the maximum height, time of flight and the horizontal range.

Solution: Here 30 ms^{-1} , Angle of projection, $\theta = 90^\circ - 30^\circ = 60^\circ$

Maximum height, $H = \frac{u^2 \sin^2 \theta}{2g} = \frac{30^2 \sin^2 60^\circ}{2 \times 9.8} = 34.44 \text{ m}$

Time of flight, $T = \frac{2u \sin \theta}{g} = \frac{2 \times 30 \sin 60^\circ}{9.8} = 5.3 \text{ s}$

Horizontal range, $R = \frac{u^2 \sin 2\theta}{g} = \frac{30^2 \sin 120^\circ}{9.8} = \frac{30^2 \sin 60^\circ}{9.8} = 79.53 \text{ m}$.

Illustration 21: Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range.

Solution: For $\theta = 45^\circ$, the horizontal range is maximum and is given by, $R_{\max} = \frac{u^2}{g}$

Maximum height attained, $H_{\max} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g} = \frac{R_{\max}}{4}$ or $R_{\max} = 4H_{\max}$

Illustration 22: Show that a given gun will shoot the bullet three times as high when elevated at an angle of 60° as when fired at angle of 30° but will carry the same distance on a horizontal plane.

Solution: The vertical height attained by a projectile is given by,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

When $\theta = 60^\circ$, $H_1 = \frac{u^2 \sin^2 60^\circ}{2g} = \frac{u^2}{2g} \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{3u^2}{8g}$

When $\theta = 30^\circ$, $H_2 = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{u^2}{2g} \left(\frac{1}{2} \right)^2 = \frac{u^2}{8g}$

$\therefore H_1 : H_2 = \frac{3u^2}{8g} \times \frac{8g}{u^2} = 3 : 1$

Thus the same gun will shoot three times as high when elevated at an angle of 60° as when fired at an angle of 30° .

$$\text{Horizontal range of a projectile, } R = \frac{u^2 \sin 2\theta}{g}$$

$$\text{When } \theta = 60^\circ, \quad R_1 = \frac{u^2 \sin 120^\circ}{g} = \frac{\sqrt{3}u^2}{2g}$$

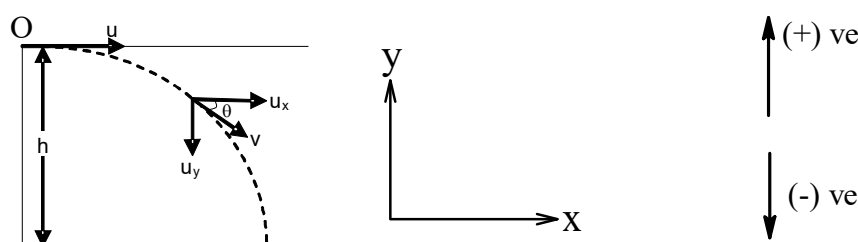
$$\text{When } \theta = 30^\circ, \quad R_2 = \frac{u^2 \sin 60^\circ}{g} = \frac{\sqrt{3}u^2}{2g}$$

Thus $R_1 = R_2$, i.e. the horizontal distance covered will be same in both cases.

1.8.3 Projectile thrown horizontally from an elevated point

Consider a projectile thrown from point O at some height h from the ground with a velocity u, in the horizontal direction.

Analyzing the projectile motion along the horizontal and vertical directions



Horizontal direction

(i) Initial velocity $u_x = u$

(ii) Acceleration $a_x = 0$

Vertical direction

(i) Initial velocity $u_y = 0$

(ii) Acceleration $a_y = -g$

Trajectory Equation: The path traced by the projectile is called the trajectory.

$$\text{After time } t, \quad x = ut \quad \dots (1)$$

$$y = -\frac{1}{2}gt^2 \quad \dots (2)$$

From eqn. (1) $t = x/u$

$$\text{Put value of } t \text{ in eqn. (2)} \quad y = -\frac{1}{2}g\left(\frac{x}{u}\right)^2, \quad y = -\frac{gx^2}{2u^2}$$

This is the equation of trajectory

Velocity at a general point P (x, y) after time t

Here horizontal velocity of the projectile after time t, $v_x = u$

Velocity of projectile in vertical direction after time t

$$v_y = 0 - gt = -gt$$

$$\therefore \quad \vec{v} = v_x \hat{i} + v_y \hat{j} = u\hat{i} - gt\hat{j}$$

Displacement: The displacement of the particle after time t, is expressed by

$$S = x\hat{i} + y\hat{j}, \text{ where } x = ut, \quad y = -\frac{1}{2}gt^2, \quad \therefore s = ut\hat{i} - \frac{1}{2}gt^2\hat{j}$$

Time of flight: Time taken by the projectile to strike the ground, is the time of flight.

Apply $S = ut + \frac{1}{2}at^2$, along vertical direction

$u = 0$ in vertical direction, $y = -h$

$$-h = -\frac{1}{2}gt^2 \quad t = \sqrt{\frac{2h}{g}}$$

Horizontal range: Distance covered by the projectile along the horizontal direction between the point of projection to the point where it strikes the ground.

$$R = u_x t = u \sqrt{\frac{2h}{g}}$$

Illustration 23: A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a 490m hill. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground.

Solution: (i) The projectile is fired from the top O of a hill with velocity $u = 98\text{ms}^{-1}$ along the horizontal. It reaches the target P in time t . The initial velocity in the downward direction = 0
vertical distance $OA = y = 490\text{m}$.

As, $y = \frac{1}{2}gt^2$
 $\therefore 490 = \frac{1}{2} \times 9.8 t^2 \quad \text{or} \quad t = \sqrt{100} = 10 \text{ s}$

(ii) Distance of the target from the hill is given by,

$$AP = x = u \times t = 98 \times 10 = 980\text{m}.$$

(iii) The horizontal and vertical components of velocity v of the projectile at point P are

$$v_x = u = 98 \text{ ms}^{-1} \quad v_y = u_x + gt = 0 + 9.8 \times 10 = 98\text{ms}^{-1} \text{ (downwards)}$$

$$\therefore V = \sqrt{V_x^2 + V_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2} = 138.59\text{ms}^{-1}$$

Now if the resultant velocity v makes angle β with the horizontal, then

$$\tan \beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \quad \therefore \beta = 45^\circ$$

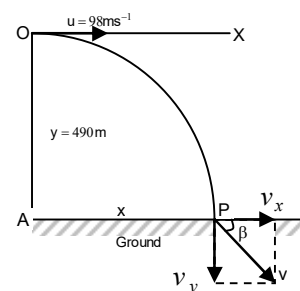


Illustration 24. A ball rolls off top of a stairway with a horizontal velocity $u \text{ m/s}$. If the steps are $h \text{ m}$ high and $b \text{ meters}$ wide, the ball will just hit the edge of the n^{th} step, Find n

Solution: If the ball hits the n^{th} step, the horizontal and vertical distances traversed are nb and nh respectively. Let t be the time taken by the

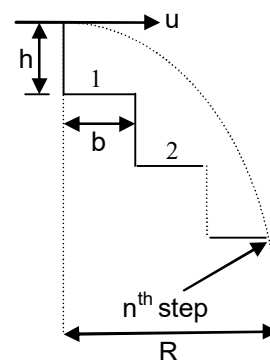
ball for these horizontal and vertical displacement. Then velocity along horizontal direction remains constant = u
initial vertical velocity is zero.

$$\therefore nb = ut \quad \dots (1)$$

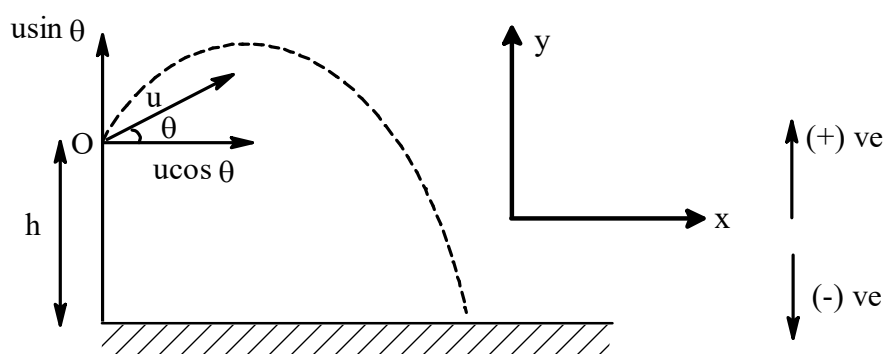
$$nh = 0 + (1/2)gt^2 \quad \dots (2)$$

From (1) & (2), we get by eliminating t ,

$$nh = (1/2)g(nb/u)^2 \quad n = \frac{2hu^2}{gb^2}$$



1.8.4 Projectile fired from the top of a tower in upward inclined direction.



Consider a projectile thrown from point O at height h from the ground with speed u at an angle θ with the horizontal in the upward direction.

Horizontal direction

(i) Initial velocity $u_x = u \cos \theta$

(ii) Acceleration $a_x = 0$

Vertical direction

(i) Initial velocity $u_y = u \sin \theta$

(ii) Acceleration $a_y = -g$

- Equation of trajectory

At any time t

$$y = u \sin \theta t - \frac{1}{2} g t^2 \quad \dots (1)$$

$$x = u \cos \theta \times t \quad \dots (2)$$

Eliminating t between (1) & (2)

$$y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}$$

- Velocity at any time t

At any time t

$$V_x = u \cos \theta \quad V_y = u \sin \theta - g t$$

$$\vec{V} = V_x \hat{i} + V_y \hat{j} = u \cos \theta \hat{i} + (u \sin \theta - g t) \hat{j}$$

- Displacement at any time t

At any time t

$$y = u \sin \theta t - \frac{1}{2} g t^2 \quad x = u \cos \theta t$$

$$\vec{S} = x \hat{i} + y \hat{j} = (u \cos \theta t) \hat{i} + \left(u \sin \theta t - \frac{1}{2} g t^2 \right) \hat{j}$$

- Time of flight

$$y = u \sin \theta t - \frac{1}{2} g t^2 \quad (\text{for displacement in vertical direction})$$

$$y = -h, \quad -h = u \sin \theta t - \frac{1}{2} g t^2$$

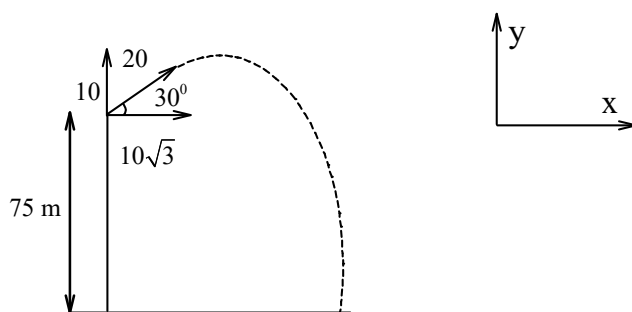
Solving the above quadratic and accepting the positive value of t, we get the time of flight.

- Horizontal range** – Distance covered by projectile along horizontal direction between point of projection and the point where it strikes the ground.

$$R = u_x t = u \cos \theta \times \text{time of flight.}$$

Illustration 25. From top of cliff 75 m high, a body is projected with 20m/s velocity at angle 30° with horizontal in upward inclined direction. Determine time of flight, horizontal range covered & velocity with which it strikes the ground ($g = 10\text{m/s}^2$)

Solution:



For vertical motion

$$-75 = 10t - \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 5\text{s or } t = -3\text{s}$$

\therefore time of flight = 5s

Horizontal range $R = 10\sqrt{3} \times 5 = 50\sqrt{3}\text{m}$

$$v_x = 10\sqrt{3}, v_y = 10 - 10 \times 5 = -40$$

$$\therefore \text{Velocity of striking ground} = v_x \hat{i} + v_y \hat{j} = 10\sqrt{3} \hat{i} - 40 \hat{j}$$

IN-CHAPTER EXERCISE - 5

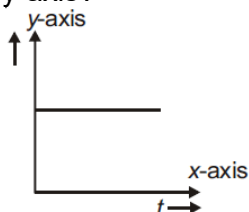
1. A body is thrown horizontally from the top of a tower & strikes the ground after 3 seconds at angle of 45° with horizontal. Find the height of tower & the speed with which the body was projected ($g = 9.8\text{m/s}^2$)
2. A bomb is dropped from an aeroplane when it is directly above a target at height of 1000 m. The aeroplane is moving horizontally with a speed of 500km/hr. By how much distance will the bomb miss the target ($g = 9.8\text{m/s}^2$)
3. A body is projected horizontally from the top of a cliff with velocity of 9.8 m/s. Find time elapsed before the magnitudes of horizontal & vertical components of the velocities of the body become equal. ($g = 9.8\text{m/s}^2$)
4. A body is projected from the ground with velocity of 30m/s, making an angle 30° with the vertical. Find maximum height attained, time of flight & horizontal range covered by the body. ($g = 9.8\text{m/s}^2$)
5. The ceiling of a long hall is 25m high. What is the horizontal distance that a ball thrown from a point on the ground with a speed of 40m/s will cover, given that the ball just scraps the ceiling. ($g = 9.8\text{m/s}^2$)
6. A projectile fired from horizontal ground has a range of 50m & reaches a maximum height of 10m. Calculate the angle with the horizontal, at which the projectile is fired. ($g = 10\text{m/s}^2$)
7. A ball is thrown at angle θ with the horizontal & another ball is thrown at an angle $(90 - \theta)$ with the horizontal from the same point on horizontal ground with the same speed of 39.2 m/s. The second ball attains a maximum height of 50m more as compared to the maximum height attained by the first ball. Find the maximum height, attained by each of the ball. ($g = 9.8\text{m/s}^2$)
8. A ball is kicked from horizontal ground at an angle 30° with vertical. If the horizontal component of velocity is 19.6 m/s, find the maximum height attained and the horizontal

range covered by the ball. ($g=9.8\text{m/s}^2$)

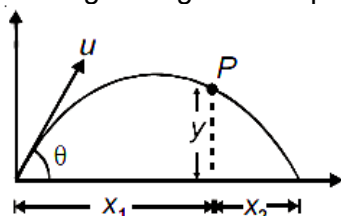
9. A particle of mass 100g is fired from a point on horizontal ground with a speed of 20m/s making angle 30° with horizontal. Find the magnitude of change in momentum of the particle between the highest point of its trajectory and the point of projection.
10. For a projectile launched from a point on horizontal ground, the speed when it is at the greatest height is $\sqrt{\frac{2}{5}}$ times the speed when it is at half of its greatest height. Determine the angle of projection.
11. Two projectiles are projected at angles $\left(\frac{\pi}{4} + \theta\right)$ and $\left(\frac{\pi}{4} - \theta\right)$ with the horizontal, where $\theta < \frac{\pi}{4}$, with same speed.

The ratio of horizontal ranges described by them is

- (a) $\tan \theta : 1$ (b) $1 : \tan^2 \theta$ (c) $1 : 1$ (d) $1 : \sqrt{3}$
12. In the graph shown in figure, which quantity associated with projectile motion is plotted along y-axis?



- (a) Kinetic energy (b) Momentum (c) Horizontal velocity (d) None of these
13. The equation of a projectile is $y = ax - bx^2$. Its horizontal range is
- (a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $a + b$ (d) $b - a$
14. In the given figure for a projectile



- (a) $y = \left[\frac{x_1 x_2}{x_1 - x_2} \right]$ (b) $y = \left[\frac{x_1 x_2}{x_1 + x_2} \right] \tan \theta$
- (c) $y = \left[\frac{2x_1 x_2}{x_1 + x_2} \right] \cos \theta$ (d) $y = \left[\frac{2x_1 x_2}{x_1 + x_2} \right] \tan \theta$
15. A ball is projected from ground at an angle 45° with horizontal from distance d_1 from the foot of a pole and just after touching the top of pole it falls on ground at distance d_2 from pole on other side, the height of pole is
- (a) $2\sqrt{d_1 d_2}$ (b) $\frac{d_1 + d_2}{4}$ (c) $\frac{2d_1 d_2}{d_1 + d_2}$ (d) $\frac{d_1 d_2}{d_1 + d_2}$
16. A ball is thrown at an angle θ with the horizontal. Its horizontal range is equal to its maximum height. This is possible only when the value of $\tan \theta$ is
- (a) 4 (b) 2 (c) 1 (d) 0.5

ANSWER KEY

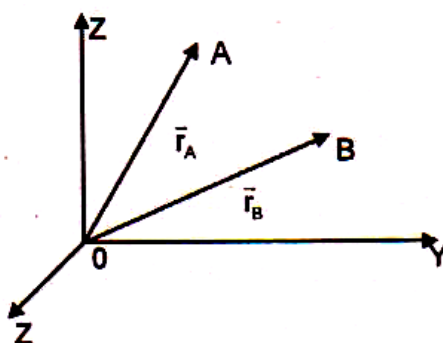
1:	44.1m, 29.4m/s	2:	1984.13 m	3:	1 s
4:	34.44 m, 5.3s, 79.53 m	5:	150.7 m	6:	$\tan^{-1}(0.8)$
7:	14.2 m, 64.2 m	8:	58.8m, 135.8m	9:	1kg.m/s
10:	60°	11:	(c)	12:	(c)
13:	(a)	14:	(b)		
15:	(d)	16:	(a)		

1.9 Relative Velocity

Let two particles A & B be placed at two points as shown in the figure. The position vectors of the particles with respect to the inertial reference frame are \vec{r}_A & \vec{r}_B respectively. The relative separation between the particles is given by $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$. Differentiating both sides w.r.t. time, we obtain.

$$\frac{d\vec{r}_{BA}}{dt} = \frac{d\vec{r}_B}{dt} - \frac{d\vec{r}_A}{dt} \Rightarrow \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

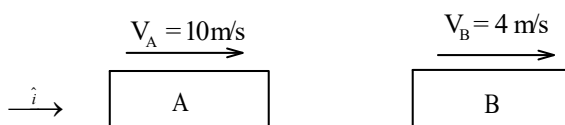
\vec{r}_{BA} is the position of B with respect to A & \vec{v}_{BA} is the velocity of B relative to A.



1.9.1 Relative Motion in One Dimension

- Consider 2 cars A and B moving along same line in the same direction with constant velocity

$$V_A = 10\text{m/s and } V_B = 4\text{m/s}$$



- Observer on A cannot register motion of A. The observer on A notices car B to be moving towards him with speed $10 - 4 = 6\text{ m/s}$. This is velocity of B with respect to A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = \text{velocity of B with respect to A}$$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = 4\hat{i} - 10\hat{i} = -6\hat{i} \text{ (Rightward positive)}$$

This means observer on A will register that B is moving with 6m/s in negative x direction or towards the left, i.e B is approaching him with 6m/s.

- Similarly observer on B cannot register motion of B. Observer on B notices car A to approach him with $(10 - 4) = 6\text{ m/s}$. This is velocity of A with respect to B.

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = 10\hat{i} - 4\hat{i} = 6\hat{i} \text{ (Rightward positive)}$$

This means observer on B will register that A is moving with 6m/s in positive x-direction or towards the right i.e A is approaching B with 6m/s

Illustration 26: An elevator, in which a man is standing, is moving upward with a constant acceleration of 1 m/s^2 . At some instant when speed of elevator is 10 m/s , the man drops a coin from a height of 2 m . Find the time taken by the coin to reach the floor. ($g = 9.8 \text{ m/s}^2$)

Solution: Analyse the motion of coin with respect to the observer standing in the elevator. As the coin releases from rest inside elevator, its velocity with respect to ground is equal to the velocity of elevator, 10 m/s .

$$\therefore \vec{V}_{\text{coin.elevator}} = \vec{V}_{\text{coin.ground}} - \vec{V}_{\text{elevator.ground}}$$

$$\text{or } \vec{V}_{\text{CE}} = \vec{V}_{\text{CG}} - \vec{V}_{\text{EG}}$$

$$= 10 - 10 = 0$$

To find acceleration of coin with respect to the observer in the elevator:

$$\vec{a}_{\text{CE}} = \vec{a}_{\text{CG}} - \vec{a}_{\text{EG}} = -g - a$$

Now using second equation for relative motion

$$\vec{s}_{\text{CE}} = \vec{u}_{\text{CE}} t + \frac{1}{2} \vec{a}_{\text{CE}} t^2$$

$$2 = 0 + \frac{1}{2}(g + a)t^2 \text{ or } 2 = \frac{1}{2}(9.8 + 1)t^2$$

$$\text{or } t = \sqrt{\frac{2 \times 2}{10.8}} = 0.61 \text{ s}$$

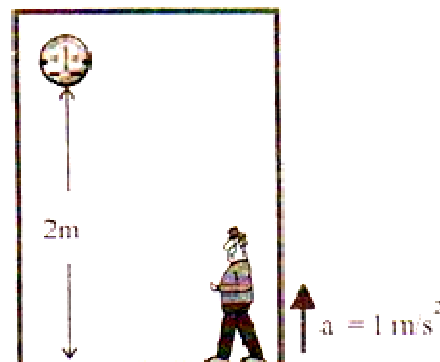


Illustration 27: The engineer of a train moving at a speed v_1 sights a freight train a distance 'd' ahead of him on the same track moving in the same direction with a slower speed v_2 . He puts on the brakes and gives his train a constant deceleration α . Find the minimum value of d at which brakes are applied so as to avoid collision.

Solution: Collision will be avoided if speed of train v_1 become equal to v_2 in traveling a relative distance d. Therefore final relative speed of trains become zero. The initial relative speed $v_{12} = v_1 - v_2$. By third equation of motion, we have

$$v_{12}^2 = u_{12}^2 - 2a_{12}s \quad 0 = (v_1 - v_2)^2 - 2\alpha d \quad \text{or } d = \frac{(v_1 - v_2)^2}{2\alpha}$$

$$\text{The collision can be avoided if } d \geq \frac{(v_1 - v_2)^2}{2\alpha}$$

Illustration 28: Two parallel rail tracks run north south. Train A moves north with a speed of 54 km/h and train B moves south with a speed of 90 km/h . What is the

(i) relative velocity of B with respect to A?

(ii) relative velocity of ground with respect to B?

(iii) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km/h with respect to the train A) as observed by a man standing on the ground?

Solution: (i) Let $v_A = 54 \text{ km/h}$ then $v_B = -90 \text{ km/h}$ (Taking Northward positive)

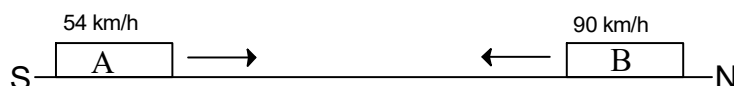
\therefore Relative velocity of B with respect to A

$$\vec{v}_{\text{BA}} = -90 - 54 = -144 \text{ km/h} \quad \text{Ans.}$$

The velocity of train B with respect to train A appears 144 km/h along south.

(ii) Relative velocity of ground with respect to train B

$$v_g - v_B = 0 + 90 = 90 \text{ km/hr}$$



To train B, the ground appears to move with a speed of 90 km/h along North

(iii) The velocity of monkey with respect to train A

$$= V_{\text{monkey, A}} = V_{\text{monkey, ground}} - V_{\text{A, ground}}$$

$$\therefore V_{\text{monkey, ground}} = V_{\text{monkey, A}} + V_{\text{A, ground}} = -18 + 54 = 36 \text{ km/h Ans.}$$

To an observer on ground, monkey appears to travel with a speed of 36 km/hr towards North.

Illustration 29: Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 km/h in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

Solution:

Let speed of each bus = v km/h

The distance between the nearest buses plying on either side = vT km (i)

For buses going from town A to B.

Relative speed of bus in the direction of motion of man, = (v - 20)

Buses plying in this direction go past the cyclist after every 18 min. Therefore

$$\text{separation between the buses} = (v - 20) \times \frac{18}{60}$$

$$\text{From equation (i), } (v - 20) \times \frac{18}{60} = vT \quad \dots (ii)$$

For buses coming from B to A:

The relative velocity of bus with respect to man = (v + 20)

Buses coming from town B go past the cyclist after every 6 min therefore

$$(v + 20) \times \frac{6}{60} = vT \quad \dots (iii)$$

Solving equations (ii) and (iii), we get v = 40 km/h and T = $\frac{3}{20}$ h Ans.

IN-CHAPTER EXERCISE - 6

- Car A moving at 10m/s on a straight road, is ahead of car B moving in the same direction at 6m/s. Find velocity of A relative to B & velocity of B relative to A
- A man is 9m behind the door of a train when it starts from rest with a uniform acceleration of 2m/s^2 . The man runs at constant speed. How far does he have to run & after how much time does he get into train, assuming that the man and the train have the same velocity when the man boards the train? What is the speed with which the man was running?
- A boat covers certain distance between two spots in a river taking t_1 hrs going downstream and t_2 hrs going upstream. What time will be taken by boat to cover same distance in still water?
 (1) $\frac{t_1 + t_2}{2}$ (b) $2(t_2 - t_1)$ (c) $\frac{2t_1 t_2}{t_1 + t_2}$ (d) $\sqrt{t_1 t_2}$
- A train of 150 m length is going towards North at a speed of 10 m/s. A bird is flying at 5 m/s parallel to the track towards South. The time taken by the bird to cross the train is
 (a) 10s (b) 15s (c) 30s (d) 12 s
- Two cars are moving in the same direction with a speed of 30 km/h. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. The speed of the third car is

- (a) 30 km/h (b) 25 km/h (c) 40 km/h (d) 45 km/h
6. Two trains each of length 100 m moving parallel towards each other at speed 72 km/h and 36 km/h respectively. In how much time will they cross each other?
(a) 4.5 s (b) 6.67 s (c) 3.5 s (d) 7.25 s
7. A body is dropped from a certain height h (h is very large) and second body is thrown downward with velocity of 5 m/s simultaneously. What will be difference in heights of the two bodies after 3 s?
(a) 5 m (b) 10 m (c) 15 m (d) 20 m
8. Two bodies starts moving from same point along a straight line with velocities $v_1 = 6$ m/s and $v_2 = 10$ m/s, simultaneously. After what time their separation becomes 40 m?
(a) 6 s (b) 8 s (c) 12 s (d) 10 s
9. Ball A is thrown up vertically with speed 10 m/s. At the same instant another ball B is released from rest at height h . At time t , the speed of A relative to B is
(a) 10 m/s (b) $10 - 2gt$ (c) $\sqrt{10^2 - 2gh}$ (d) $10 - gt$

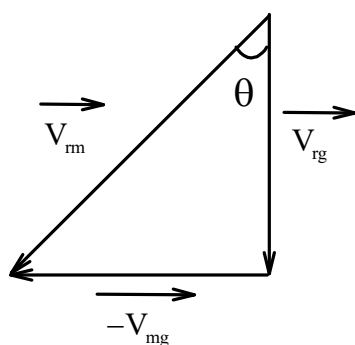
ANSWER KEY (IN-CHAPTER EXERCISE - 6)

- | | | |
|-------------------|--------------------|--------|
| 1. 4 m/s, - 4 m/s | 2. 18m, 3s, 6 m/s. | |
| 3. (C) | 4. (a) | 5. (d) |
| 6. (b) | 7. (c) | 8. (d) |
| 9. (a) | | |

1.9.2 Relative Motion in Two Dimension

Rain – Man Problems

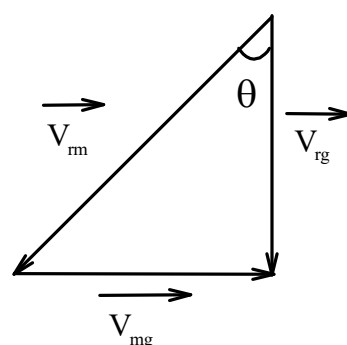
- \vec{V}_{rm} = velocity of rain w.r.t. man = $\vec{V}_{rg} + (-\vec{V}_{mg})$
 \vec{V}_{rg} = velocity of rain w.r.t. ground, \vec{V}_{mg} = velocity of man w.r.t. ground.
- Suppose rain is falling vertically with respect to the ground & man is walking on a horizontal road towards right.



$$\vec{V}_{rm} = \vec{V}_{rg} + (-\vec{V}_{mg})$$

θ is the angle made by \vec{V}_{rm} with the vertical

$$\tan \theta = \frac{V_{mg}}{V_{rg}}$$



$$\vec{V}_{rm} + \vec{V}_{mg} = \vec{V}_{rg}$$

In case wind is blowing

$$\vec{V}_{rg} = \vec{V}_{rw} + \vec{V}_{wg}$$

\vec{V}_{rg} = velocity of rain w.r.t. ground.

\vec{V}_{rw} = velocity of rain w.r.t. wind.

\vec{V}_{wg} = velocity of wind w.r.t. ground.

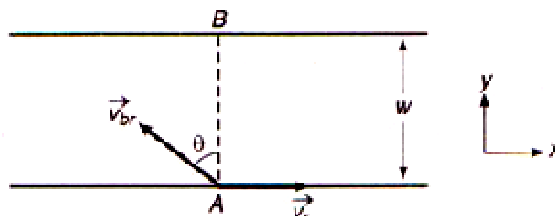
River – Boat Problems

In river-boat problems we come across the following three terms:

\vec{v}_r = absolute velocity of river

\vec{v}_{br} = velocity of boat with respect to river or velocity of boat in still water and

\vec{v}_b = absolute velocity of boat.



Here, it is important to note that \vec{v}_{br} is the velocity of boat with which it is steered and \vec{v}_b is the actual velocity of boatman relative to ground. Further, $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases. A boat starts from point A on one bank of a river with velocity \vec{v}_{br} in the direction shown in Figure. River is flowing along positive x-direction with velocity \vec{v}_r . Width of the river is w. Then $\vec{v}_b = \vec{v}_r + \vec{v}_{br}$

Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br} \sin \theta$

and $v_{by} = v_{ry} + v_{bry} = 0 + v_{br} \cos \theta = v_{br} \cos \theta$

now, time taken by the boat to cross the river is:

$$t = \frac{w}{v_{by}} = \frac{w}{v_{br} \cos \theta} \quad \text{or} \quad t = \frac{w}{v_{br} \cos \theta} \quad \dots (i)$$

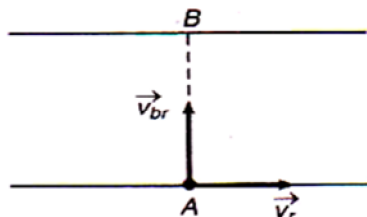
Further, displacement along x-axis when he reaches on the other bank (also called drift) is

$$x = v_{bx} t = (v_r - v_{br} \sin \theta) \frac{w}{v_{br} \cos \theta} \quad \text{or} \quad x = (v_r - v_{br} \sin \theta) \frac{w}{v_{br} \cos \theta} \quad \dots (ii)$$

Two special cases are:

(i) **Condition when the boatman crosses the river in shortest interval of time**

From Eq. (i) we can see that time (t) will be minimum when $\theta = 0^\circ$, i.e., the boatman should steer his boat perpendicular to the river current.



Also, $t_{\min} = \frac{w}{v_{br}}$ as $\cos \theta = 1$

(ii) **Condition when the boatman wants to reach point B, i.e., at a point just opposite from, the point he started.** In this case, the drift (x) should be zero.

$$\therefore x = 0 \quad \text{from eq. (ii)} \quad \text{or} \quad (v_r - v_{br} \sin \theta) \frac{w}{v_{br} \cos \theta} = 0 \quad \text{or} \quad v_r = v_{br} \sin \theta$$

$$\text{or} \quad \sin \theta = \frac{v_r}{v_{br}} \quad \text{or} \quad \theta = \sin^{-1} \left(\frac{v_r}{v_{br}} \right)$$

Hence, to reach point B, the boatman should row at an angle $\theta = \sin^{-1} \left(\frac{v_r}{v_{br}} \right)$

Further, since $\sin \theta \leq 1$, so, if $v_r \geq v_{br}$, the boatman can never reach at point B. If $v_r = v_{br}$, then $\theta = 90^\circ$, $v_b = 0$. If $v_r > v_{br}$, then $\sin \theta$ needs to be greater than 1, which is not possible

(iii) **Shortest path**

Path length travelled by the boatman when he reaches the opposite shore is $s = \sqrt{w^2 + x^2}$
Here, w = width of river is constant. So for s to be minimum modulus of x (drift) should be minimum. Now two cases are possible.

When $v_r < v_{br}$: In this case $x = 0$, when $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ or $s_{\min} = w$ at $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$.

This is zero drift condition.

When $v_r > v_{br}$: In this case x is minimum, where $\frac{dx}{d\theta} = 0$

Note that in this case zero drift is not possible. So we can only minimize drift.

$$\text{or } \frac{d}{d\theta} \left\{ \frac{w}{v_{br} \cos \theta} (v_r - v_{br} \sin \theta) \right\} = 0 \quad \text{or } -v_{br} \cos^2 \theta - (v_r - v_{br} \sin \theta)(-\sin \theta) = 0$$

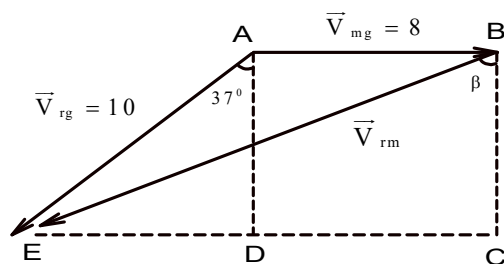
$$\text{or } -v_{br} + v_r \sin \theta = 0 \quad \text{or } \theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$$

Now, at this angle, we can find x_{\min}

$$\text{Put } \sin \theta = \frac{v_{br}}{v_r} \text{ in formula for } x, \text{ we get } x_{\min} = \frac{w}{v_{br}} \sqrt{v_r^2 - v_{br}^2}$$

$$S_{\min} = \sqrt{w^2 + x_{\min}^2} = \frac{w v_r}{v_{br}}$$

Illustration 30: Rain pouring at 37° with vertical has 10m/s speed. A person runs against rain with 8m/s & sees rain making angle β with vertical. Find β .



$$\vec{V}_{mg} + \vec{V}_{rm} = \vec{V}_{rg}$$

$$\text{From fig } \tan \beta = \frac{CE}{BC}$$

$$BC = AD = 10 \cos 37^\circ = 8$$

$$CE = CD + DE = 8 + 10 \sin 37^\circ = 14$$

$$\therefore \tan \beta = \frac{14}{8} = \frac{7}{4},$$

$$\therefore \beta = \tan^{-1} \frac{7}{4}$$

Illustration 31: A swimmer starts from point A on the bank of 200 m wide river, crosses the river to reach opposite bank. He returns to the point B on the original bank such that $AB = 300\text{m}$ in the downstream direction. Find the magnitude and the direction of the velocity of the swimmer relative to the bank if his velocity w.r.t.to river is always perpendicular to the bank.

Solution: Time to cross the river = 5 min . The displacement perpendicular to flow = 200 m

$$\therefore \text{Velocity of swimmer } v_{sy} = \frac{200}{5 \times 60} = \frac{2}{3} \text{ m/s}$$

The displacement covered in the direction of flow = 300 m in 10 min .

\therefore The velocity of river flow

The velocity of swimmer in the direction of flow $v_{sx} = \frac{300}{10 \times 60} = \frac{1}{2} \text{ m/s}$

His velocity with respect to bank $v_s = \sqrt{v_{sy}^2 + v_{sx}^2} = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)^2}$
 $= \frac{5}{6} \text{ m/s} \quad = 3 \text{ km/h}$ **Ans.**

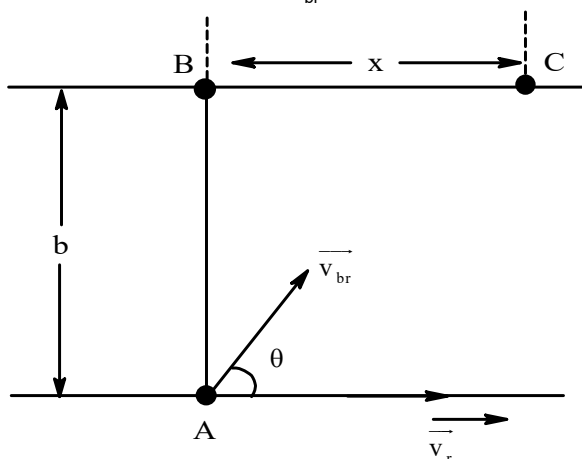
The velocity \vec{v}_s makes an angle θ with the bank, then $\tan \theta = \frac{v_{sy}}{v_{sx}} = \frac{2/3}{1/2}$
 $= \frac{4}{3}$ or $\theta = \tan^{-1}(4/3)$ **Ans.**

Illustration 32: A boat moves relative to water with a velocity which is $\eta = 2.0$ times less than the river flow velocity. At what angle to the stream direction must the boat move to minimize drifting? Find the minimum drift achieved.

Solution: Suppose velocity of river flow $v_r = v$, then velocity of boat relative to water $v_{br} = \frac{v}{2}$.

Let boat moves at an angle θ with the direction of stream, then

Time to cross the stream $t = \frac{b}{v_{br} \sin \theta}$, where $v_{br} \sin \theta$ is the velocity of boat along AB.



The velocity of boat along the direction of flow, $v_{bx} = (v_r + v_{br} \cos \theta)$ and drift in the direction of flow, $x = v_{bx} \times t$

$$= (v_r + v_{br} \cos \theta) \times \frac{b}{v_{br} \sin \theta} = \left(v + \frac{v}{2} \cos \theta \right) \times \left(\frac{b}{\frac{v}{2} \sin \theta} \right) = b \left(\frac{2 + \cos \theta}{\sin \theta} \right)$$

Where b is the width of the river.

For the drift to be minimum, $\frac{dx}{d\theta} = 0$ or $\frac{d}{d\theta} \left[b \left(\frac{2 + \cos \theta}{\sin \theta} \right) \right] = 0$

$$\text{or} \quad \sin \theta \times (-\sin \theta) - (2 + \cos \theta) \times (\cos \theta) = 0 \quad \text{or} \quad \sin^2 \theta + 2 \cos \theta + \cos^2 \theta = 0$$

$$\text{or} \quad 2 \cos \theta = -(\sin^2 \theta + \cos^2 \theta) \quad \text{or} \quad 2 \cos \theta = -1 \quad \therefore \cos \theta = -\frac{1}{2} \quad \text{or} \quad \theta = 120^\circ$$

Hence to minimize drifting boat should move at an angle 120° with the direction of stream.

$$\text{Thus } x_{\min} = b \left(\frac{2 + \cos 120^\circ}{\sin 120^\circ} \right) = b \left(\frac{2 - 1/2}{\sqrt{3}/2} \right) = b\sqrt{3} \quad \text{Ans.}$$

Illustration 33: Two ships A and B are 10 km apart on a line running south to north. Ship A further north is streaming west at 20 km/hr and ship B streaming north at 20 km/hr. What is their distance of closest approach and how long do they take to reach it?

Solution: Consider a rectangular coordinate axes system with y axis from south to north and x –axis from west to east. The positions of the ships A and B are given by the co-ordinates (0 , 10 km) and (0 , 0) respectively. Velocity of ship

$$A = \vec{V}_A = (-20 \text{ km/hr}) \hat{i}$$

$$\text{Velocity of ship B} = \vec{V}_B = (20 \text{ km/hr}) \hat{j}$$

$$\text{Relative velocity of ship B w.r.t. ship A} = \vec{V}_{BA}$$

$$\vec{V}_B - \vec{V}_A = (20 \text{ km/hr}) \hat{j} + (20 \text{ km/hr}) \hat{i}$$

Angle made by \vec{V}_{BA} with x axis is

$$\tan^{-1} \left(\frac{V_y}{V_x} \right) = \tan^{-1} \left(\frac{20}{20} \right) = \frac{\pi}{4}$$

$\therefore \vec{V}_{BA}$ is in north east direction. Hence relative to A , B is moving in north east direction

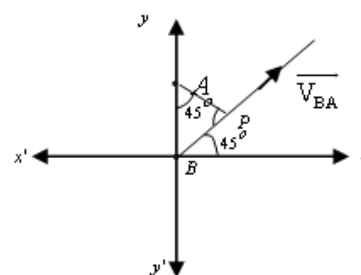
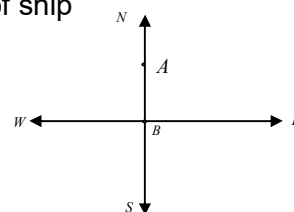
Since the perpendicular distance is the shortest distance , the minimum distance between A and B is AP as shown in above figure.

$\triangle BAP$ is an isosceles right angled triangle with right angle at P.

$$\text{From } \triangle BAP, AP = BA \cos 45^\circ = \frac{10}{\sqrt{2}} \text{ km} = 5\sqrt{2} \text{ km}$$

Time taken to attain the minimum distance

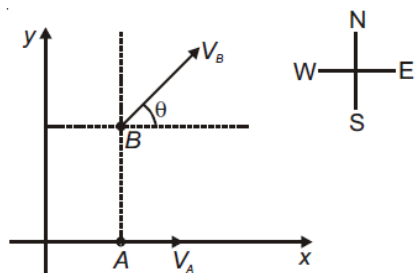
$$= \frac{BP}{|\vec{V}_{BA}|} = \frac{BA \sin 45^\circ}{|\vec{V}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} = \frac{1}{4} \text{ hr} = 0.25 \text{ hr}$$



IN-CHAPTER EXERCISE - 7

- In the absence of wind, rain is falling vertically with a speed of 4m/s. After some time, wind starts blowing with 3 m/s speed in North to South direction. Find the angle with the vertical that a man standing on horizontal ground must hold his umbrella to keep himself dry.
- A man standing on a horizontal road has to hold his umbrella at 30° to the vertical to keep rain away. When he walks on the road at 10km/hr , he has to hold the umbrella vertically to keep himself dry. Find the speed of the rain drops, with respect to the ground.
- For a man running on horizontal road at 8km/hr , rain appears to fall vertically. He increases his speed to 12km/hr . Now he finds rain hitting him at 30° with the vertical. Find the speed of the rain drops with respect to the ground.
- When a man walks with a speed V on a horizontal road, rain appears to fall vertically. When he doubles his speed, rain appears to fall at 45° to the vertical. Find the direction of rain as it appears to the man when he triples his speed.
- Rain is falling vertically with respect to the ground. For a man walking on a horizontal road at a speed of 5m/s, rain appears to fall at 45° to the vertical. When the man reduces his speed, rain appears to fall at 30° to the vertical. Find the reduced speed of the man.
- A man swims across a river & reaches a point directly opposite in time t_1 . He swims an equal distance downstream & covers distance in time t_2 . If speed of man in still water is V and speed of river water is U, find $\frac{t_1}{t_2}$.

7. An aeroplane has to go from point A to another point B, 500km away due 30° east of North. Wind is blowing due north at speed of 20m/s. Air speed of plane (speed of plane in still air) is 150 m/s. Find the direction in which pilot should head plane w.r.t line AB to reach point B
8. A river 400 m wide is flowing with a speed of 2m/s. A boat is sailing with 10m/s speed w.r.t. the river water in direction such that the boat reaches the opposite bank in minimum time.
(a) Find the time taken by the boat to reach the opposite bank.
(b) Find the drift suffered by the boat.
9. A swimmer wishes to cross a 500 m wide river flowing at 5km/hr. His speed in still water is 3km/hr
(a) If he heads in direction making angle θ with flow, find time he takes to cross river.
(b) Find shortest possible time to cross.
10. Consider a river 500 m wide. Speed of river water is 5 km/hr. Swimmer's speed in still water is 3 km/hr. If swimmer wants to reach a point directly opposite the point from where he started to swim, find minimum distance he has to walk.
11. Ram moves in east direction at a speed of 6 m/s and Shyam moves 30° east of north at a speed of 6 m/s. The magnitude of their relative velocity is
(a) 3 m/s (b) 6 m/s (c) $6\sqrt{3}$ m/s (d) $6\sqrt{2}$ m/s
12. A train is running at a constant speed of 90 km/h on a straight track. A person standing at the top of a boggy moves in the direction of motion of the train such that he covers 1 meters on the train each second. The speed of the person with respect to ground is
(a) 25 m/s (b) 91 km/h (c) 26 km/h (d) 26 m/s
13. Figure shows two ships moving in x-y plane with velocities V_A and V_B . The ships move such that B always remains north of A. The ratio $\frac{V_A}{V_B}$ is equal to



- (a) $\cos \theta$ (b) $\sin \theta$ (c) $\sec \theta$ (d) $\operatorname{cosec} \theta$
14. A person, reaches a point directly opposite on the other bank of a flowing river, while swimming at a speed of 5 m/s at an angle of 120° with the flow. The speed of the flow must be
(a) 2.5 m/s (b) 3 m/s (c) 4 m/s (d) 1.5 m/s
15. A car with a vertical windshield moves in a rain storm at a speed of 40 km/hr. The rain drops fall vertically with constant speed of 20 m/s. The angle at which rain drops strike the windshield is
(a) $\tan^{-1} \frac{5}{9}$ (b) $\tan^{-1} \frac{9}{5}$ (c) $\tan^{-1} \frac{3}{2}$ (d) $\tan^{-1} \frac{2}{3}$
16. Two particles A and B start moving with velocities 20 m/s and $30\sqrt{2}$ m/s along x-axis and at an angle 45° with x-axis respectively in xy-plane from origin. The relative velocity of B w.r.t. A
(a) $(10\hat{i} + 30\hat{j})$ m/s (b) $(30\hat{i} + 10\hat{j})$ m/s (c) $(30\hat{i} - 20\sqrt{2}\hat{j})$ m/s (d) $(30\sqrt{2}\hat{i} + 10\sqrt{2}\hat{j})$ m/s

ANSWERS KEY (IN-CHAPTER EXERCISE - 7)

1. 37°
2. 20 km/hr
3. $4\sqrt{7}$ km/hr
4. $\tan^{-1} 2$ with vertical
5. $\frac{5}{\sqrt{3}}$ m/s
6. $\sqrt{\frac{V+U}{V-U}}$
7. $\sin^{-1}\left(\frac{1}{15}\right)$
8. 40 s, 80 m
9. (a) $\frac{10}{\sin \theta}$ minutes
- (b) 10 minutes when $\theta = 90^\circ$
10. $\frac{2}{3}$ km
11. (b)
12. (d)
13. (a)
14. (a)
15. (a)
16. (a)

SUBJECTIVE TYPE

Example 1:

A car starts from rest and moves with a constant acceleration of 2.0 m/s^2 for 30 seconds. The brakes are then applied and the car comes to rest in another 60 seconds. Find

- total distance covered by the car.
- Maximum speed attained by the car
- Find shortest distance from initial point to the point when its speed is half of maximum speed.

Solution :

Final velocity at A $v_A = 2 \times t_1 = 2 \times 30 = 60 \text{ m/sec}$.

For AB, Let the retardation be 'b'

$$\therefore 0 = v_A + bt$$

$$\therefore b = -\frac{v_A}{t} = -\frac{60}{60} = -1 \text{ m/s}^2$$

(a) Total distance = OA + AB

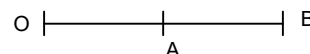
$$OB = \frac{1}{2}at_1^2 + (v_A t_2 - \frac{1}{2}bt_2^2)$$

$$= \left(\frac{1}{2} \times 2 \times 30 \times 30 + 60 \times 60 \times 60 - \frac{1}{2} \times 1 \times 60 \times 60\right) = 900 + 3600 - 1800 = 2700 \text{ m}.$$

(b) Maximum speed $v_A = 60 \text{ m/s}$.

(c) $v^2 = 2 \times a \times s$

$$s = \frac{(v_A / 2)^2}{2 \times a} = \frac{30 \times 30}{2 \times 2} = 225 \text{ m}.$$



Example 2:

A farmer has to go 500 m due north, 400 m due east and 200 m due south to reach his field. If he takes 20 minutes to reach the field,

- what distance he has to walk to reach the field ?
- what is his displacement from his house to the field ?
- what is the average speed of farmer during the walk ?
- what is the average velocity of farmer during the walk ?

Solution :

(a) Distance = $500 + 400 + 200 = 1100 \text{ m}$

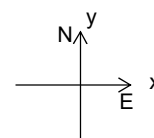
(b) Displacement = $500(\hat{j}) + 400(\hat{i}) + 200(-\hat{j}) = 300\hat{j} + 400\hat{i}$

$$\text{Magnitude of displacement} = \sqrt{(400)^2 + (300)^2} = 500 \text{ m}$$

(c) Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{1100}{20 \times 60} = \frac{11}{12} \text{ m/s}$

(d) Average velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{500}{20 \times 60} = \frac{5}{12} \text{ m/s}$

$$\theta = \tan^{-1}\left(\frac{300}{400}\right) = 37^\circ \text{ due North of East.}$$



Example 3:

A body is projected up such that its position vector varies with time as $\vec{r} = 6t\hat{i} + (8t - 5t^2)\hat{j}$. Find the (a) initial velocity (b) time of flight

Solution :

(a) The position of the body at any time t is given as $\vec{r} = 6t\hat{i} + (8t - 5t^2)\hat{j}$. When $t = 0$, $r = 0$. That means the body is projected from the origin of the coordinate system. Differentiating both sides w.r.t. time 't', we obtain

$$\frac{d\vec{r}}{dt} = 6\hat{i} + (8-10t)\hat{j} \Rightarrow \vec{v} = 6\hat{i} + (8-10t)\hat{j}.$$

Putting $t = 0$, we obtain the initial velocity (velocity of projection) given as

$$(\vec{v})_{t=0} = \vec{v}_0 = 6\hat{i} + 8\hat{j} \Rightarrow v_0 = 10 \text{ m/sec};$$

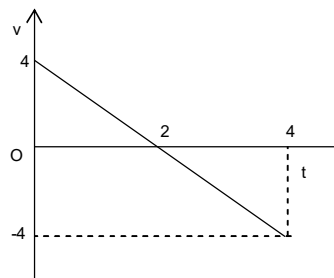
$$(b) \text{ The time of flight } T = \frac{2v_0 \sin \theta_0}{g} \Rightarrow T = \frac{2(v_y)_0}{g} \text{ where } (v_y)_0 = 8$$

$$\Rightarrow T = \frac{2 \times 8}{10} = 1.6 \text{ sec.}$$

Example 4:

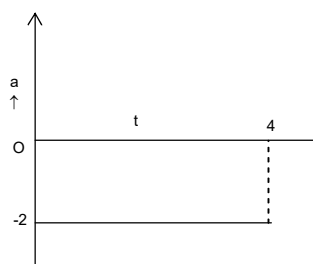
A particle starts from origin at $t = 0$ along +ve x axis. Its velocity-time graph is shown in the figure. Draw

- a, t graph
- x, t graph.

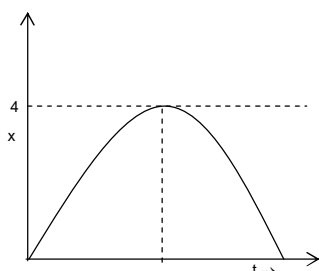


Solution :

- Velocity is decreasing
so, $a = -4/2 = -2$



-



Example 5:

A stone 'A' is dropped from the top of a tower 20 m high. Simultaneously another stone 'B' is thrown up from the bottom of the tower so that it can reach just on the top of the tower. What is the distance of the stones from the ground while they pass each other?

Solution :

Let t be the time when they pass one another

$$\text{For stone B, } y = v_B t + \frac{1}{2}(-g)t^2 \quad \dots (i)$$

$$\text{For stone A, } H - y = \frac{1}{2}gt^2 \quad \dots (ii)$$

From (i) and (ii),

$$H = v_B t \quad \dots (iii)$$

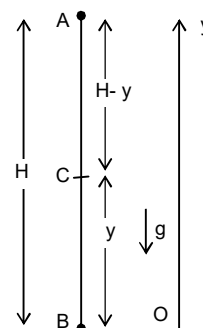
Stone B can reach just on the top of tower. We can calculate the velocity of stone B,

$$v_f^2 = v_i^2 + 2a_y y$$

$$u_f = 0, \text{ for } y_{\max} = H = 20 \text{ m}$$

$$v_i = v_B; \quad a_y = -g; \quad v_B = 20 \text{ m/s}$$

$$\text{From (iii) } t = \frac{20 \text{ m}}{20 \text{ m/s}} = 1 \text{ sec.}$$



From equation (i), the required distance (BC) from ground = $20 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 15 \text{ m}$

Example 6:

The position of a particle moving along the x-axis depends on the time as $x = at^2 - bt^3$ where $a = 3.0 \text{ m/s}^2$ and $b = 1.0 \text{ m/s}^3$ respectively.

- At what time does the particle reach its maximum positive x-position ?
- What total path length does the particle cover in the first 2.0 sec?
- Does the particle cover equal path length in the opposite direction in the subsequent 2.0 sec ? If not, explain why?
- Find the total path length covered in the first 4.0 sec.
- Find the displacement during the first 4.0 sec.
- What is the particles speed and acceleration at the end of first 3.0 sec.?

Solution :

Here, the position (x) is time dependent as $x = at^2 - bt^3 = 3t^2 - t^3$

$$\text{Instantaneous velocity } v = \frac{dx}{dt} = 6t - 3t^2 \quad \dots (i)$$

$$\text{And acceleration } a = \frac{dv}{dt} = 6 - 6t \quad \dots (ii)$$

Note that the acceleration is not uniform (like gravity) but time dependent.

(a) At the maximum positive x-position the particle comes to momentary rest ($v = 0$) and then moves in the negative x-direction with non uniform acceleration.

From equation (i), $v = 0 = 6t - 3t^2$

\therefore Required time = 2.0 seconds.

(b) The x-coordinate of the particle increases from zero to $(x)_{\max}$ during the first 2 seconds.

$$\therefore \text{ path length } = (x)_{t=2s} = (3t^2 - t^3)_{t=2s} = 4.0 \text{ m}$$

(c) For $t > 2s$, the particle moves in the backward direction with time dependent acceleration. Hence, subsequent motion is not repeated (as we have seen in free fall) where the acceleration $|\vec{g}|$ remains constant. Hence, the path length for the subsequent 2 seconds will be different.

(d) Position at $t = 2 \text{ sec.}$ is

$$(x)_{t=2} = 4.0 \text{ m and}$$

$$(x)_{t=4} = (3t^2 - t^3)_{t=4} = 48 - 64 = -16 \text{ m.}$$

Hence, the path length during the first 4.0 is

$$OA + AO + OC$$

$$= 4 + 4 + 16 = 24.0 \text{ m}$$

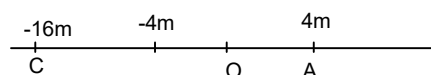
(e) Displacement during the first 4.0 sec is -16.0 m.

(f) Speed at the end of 3.0 sec is

$$(v)_{t=3} = (6t - 3t^2)_{t=3s} = -9.0 \text{ m/s.}$$

Negative sign indicates that motion is along the negative x-direction.

$$\text{Acceleration } (a)_{t=3s} = (6 - 6t)_{t=3s} = -12 \text{ m/s}^2.$$



Example 7:

A man can row a boat with a speed of 4 km/hr in still water. He is crossing a river where the speed of current is 2 km/hr.

(a) In what direction will his boat be headed if he wants to reach a point on the other bank, directly opposite to starting point?

(b) If width of the river is 4 km how long will it take him to cross the river, with the condition in part 'a' ?

- (c) In what direction should he head the boat if he wants to cross the river in shortest time?
 (d) How long will it take him to row 2 kms up the stream and then back to his starting point?

Solution :

B is a point directly opposite to the starting point A.
 Let the man heads the boat in a direction making an angle θ with the line AB.

Here $\vec{v}_w = 2\hat{i}$

$$\vec{v}_{bw} = -4\sin\theta\hat{i} + 4\cos\theta\hat{j}$$

$$\therefore \vec{v}_{\text{absolute}} = \vec{v}_{bw} + \vec{v}_w$$

$$= (2 - 4\sin\theta)\hat{i} + 4\cos\theta\hat{j}$$

$$\Rightarrow v_{bx} = 2 - 4\sin\theta \text{ and } v_{by} = 4\cos\theta$$

(a) For directly opposite point $v_{bx} = 0$

$$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30^\circ \Rightarrow \theta = 30^\circ$$

Hence, to reach the point directly opposite to starting point he should head the boat an angle $\beta = (90^\circ + 30^\circ) = 120^\circ$ with the river flow.

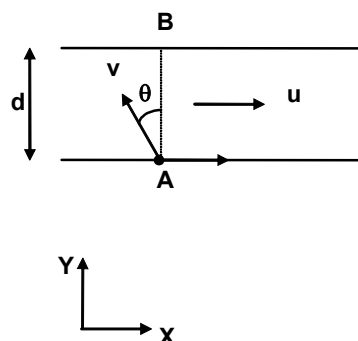
$$(b) t = \frac{y}{v_{by}} = \frac{d}{4\cos\theta} = \frac{4}{4\cos 30^\circ} = \frac{2}{\sqrt{3}} \text{ hr.}$$

(c) For t to be minimum $\cos\theta = 1$

$$\Rightarrow \theta = 0^\circ$$

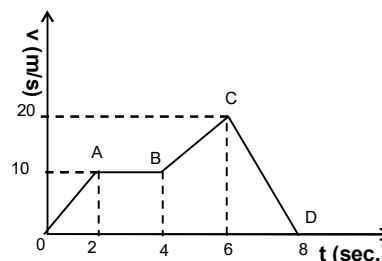
$$\Rightarrow t_{\min} = \frac{4}{4\cos 0} = 1 \text{ hr.}$$

$$(d) T = \frac{2}{(4-2)} \text{ hr} + \frac{2}{(4+2)} \text{ hr} = \left(1 + \frac{1}{3}\right) \text{ hr} = \frac{4}{3} \text{ hr.}$$

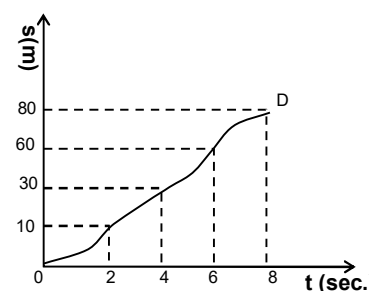


Example 8:

Velocity–time graph of a particle moving in a straight line is shown in figure. Plot the corresponding displacement–time graph of a particle if at $t = 0$ displacement $s = 0$.



Solution :



Example 9:

A particle moves in a circle of radius 20 cm at a speed given by $v = 1 + t + t^2$ m/s where t is time in s. Find (a) the initial tangential and normal acceleration. (b) the angle covered by the radius in first 2 s.

Solution :

$$(a) \text{ Tangent acceleration } a_t = \frac{dv}{dt} = 2t + 1$$

$$\text{Normal acceleration } a_n = \frac{v^2}{R}$$

$$\therefore (a_t)_{t=0} = 1 \text{ m/s}^2$$

$$(a_n)_{t=0} = \frac{v_0^2}{R} = \frac{1}{(0.2)} = 5 \text{ m/s}^2$$

$$(b) v = R \frac{d\theta}{dt}$$

$$R d\theta = (1+t+t^2) dt \quad \therefore R \int_0^{\theta'} d\theta = \int_0^2 (1+t+t^2) dt$$

$$\theta = 33.3 \text{ rad}$$

EXERCISE

LEVEL – I

DISTANCE AND DISPLACEMENT, SPEED AND VELOCITY, AVG. SPEED AND AVG. VELOCITY





1. A particle moves in the direction of east for 2 s with velocity of 15 ms^{-1} . Then it moves towards north for 8s with a velocity of 5 ms^{-1} . The average velocity of the particle is
(a) 1 ms^{-1} (b) 5 ms^{-1} (c) 7 ms^{-1} (d) 10 ms^{-1}
2. An aeroplane moves 400 m towards north, 300 m towards west and then 1200 m vertically upwards. Then its displacement from the initial position is
(a) 1300 m (b) 1400 m (c) 1500 m (d) 1600 m
3. An athlete completes one round of a circular track of radius R in 40 sec. What will be his displacement at the end of 2 min. 20 sec.
(a) zero (b) $2R$ (c) $2\pi R$ (d) $7\pi R$
4. A bus traveling the first one-third distance at a speed of 10 km/h, the next one-fourth at 20 km/h and the remaining at 40 km/h. The average speed of the bus is nearly
(a) 9 km/h (b) 16 km/h (c) 18 km/h (d) 48 km/h
5. An insect crawls a distance of 4m along north in 10 seconds and then a distance of 3m along east in 5 seconds. The average velocity of the insect is:
(a) $7/15 \text{ m/s}$ (b) $1/5 \text{ m/s}$ (c) $5/15 \text{ m/s}$ (d) $12/15 \text{ m/s}$
6. A projectile is thrown with an initial velocity of $[a\hat{i} + b\hat{j}] \text{ m/s}$. If the range of projectile is twice the maximum height reached by it, then
(a) $b = a/2$ (b) $b = a$ (c) $b = 2a$ (d) $b = 4a$
7. A hall has the dimensions $10\text{m} \times 10\text{m} \times 10\text{m}$. A fly starting at one corner ends up at a diagonally opposite corner. The magnitude of its displacement is nearly :
(a) $5\sqrt{2}\text{m}$ (b) $10\sqrt{3}\text{m}$ (c) $20\sqrt{3}\text{m}$ (d) $30\sqrt{3}\text{m}$
8. A particle starts from the origin, goes along y-axis to the point (0, 30m) and then returns along the same line to the point (0, -30m). The distance and displacement of the particle during the trip is :
(a) 0, 0 (b) 90m, -30m (c) -30m, 90m (d) 0, -30m
9. A train moves with a speed of 30 km/h in the 1st 15 minutes, with another speed of 40 km/h in the next 15 minutes and then with a speed of 60 km/h in the last 30 minutes. The average speed of the train for this journey is :
(a) 60 km/h (b) 47.5 km/h (c) 45 km/h (d) 50 km/h

AVG. ACCELERATION AND APPLICATION OF CALCULAS

10. The position of a particle moving along x-axis is given by $x = 10t - 2t^2$. Then the time (t) at which it will momentarily come to rest is
(a) 0 (b) 2.5 s (c) 5 s (d) 10 s

11. At any instant, the velocity and acceleration of a particle moving along a straight line are v and a . The speed of the particle is increasing if
 (a) $v > 0, a > 0$ (b) $v < 0, a > 0$ (c) $v > 0, a < 0$ (d) $v > 0, a = 0$
12. If v is the velocity of a body moving along x -axis, then acceleration of body is
 (a) $\frac{dv}{dx}$ (b) $v \frac{dv}{dx}$ (c) $x \frac{du}{dx}$ (d) $v \frac{dx}{dv}$
13. If a body is moving with constant speed, then its acceleration
 (a) Must be zero (b) May be variable (c) May be uniform (d) Both (b) & (c)
14. The initial velocity of a particle is u (at $t = 0$) and the acceleration a is given by $\alpha t^{3/2}$. Which of the following relations is valid?
 (a) $v = u + \alpha t^{3/2}$ (b) $v = u + \frac{3\alpha t^3}{2}$ (c) $v = u + \frac{2}{5} \alpha t^{5/2}$ (d) $v = u + \alpha t^{5/2}$
15. The position x of particle moving along x -axis varies with time t as $x = A \sin(\omega t)$ where A and ω are positive constants. The acceleration a of particle varies with its position (x) as
 (a) $a = Ax$ (b) $a = -\omega^2 x$ (c) $a = A \omega x$ (d) $a = \omega^2 x A$
16. A body is moving with variable acceleration (a) along a straight line. The average acceleration of body in time interval t_1 to t_2 is
 (a) $\frac{a[t_2 + t_1]}{2}$ (b) $\frac{a[t_2 - t_1]}{2}$ (c) $\frac{\int_{t_1}^{t_2} a dt}{t_2 + t_1}$ (d) $\frac{\int_{t_1}^{t_2} a dt}{t_2 - t_1}$
17. The position of a particle moving along x -axis given by $x = (-2t^3 + 3t^2 + 5)\text{m}$. The acceleration of particle at the instant its velocity becomes zero is
 (a) 12 m/s^2 (b) -12 m/s^2 (c) -6 m/s^2 (d) Zero
18. A body is projected vertically upward direction from the surface of earth. If upward direction is taken as positive, then acceleration of body during its upward and downward journey are respectively
 (a) Positive, negative (b) Negative, negative
 (c) Positive, positive (d) Negative, positive

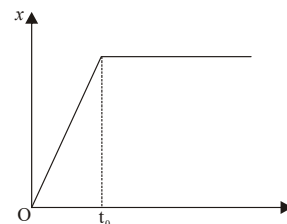
CONSTANT ACCELERATION AND FREE FALL

19. Shown here are the velocity and acceleration vectors for an object in several different types of motion. In which case is the object slowing down and turning to the left?
 (a)  (b)  (c)  (d) 
20. A body is projected vertically upwards with a velocity v . It returns to the point from which it was projected after some time. The average velocity and speed for the total time of flight are respectively.
 (a) $\frac{v}{2}, v$ (b) $0, \frac{v}{2}$ (c) $0, 0$ (d) $\frac{v}{2}, 0$

21. A body is released from the top of a tower of height h metre, it takes t second to reach the ground. Where is the body at the time $\frac{t}{2}$?
 (a) at $h/2$ meters from the ground
 (b) at $h/4$ m from the ground
 (c) depends on the mass and velocity of body
 (d) at $3h/4$ m from the ground
22. A particle is projected vertically upwards and it reaches the maximum height H in time T seconds. The height of the particle at any time t will be
 (a) $g(t - T)^2$ (b) $H - \frac{1}{2}g(t - T)^2$ (c) $\frac{1}{2}g(t - T)^2$ (d) $H - g(t - T)$
23. A body starts from rest with uniform acceleration. If its velocity after n seconds is v , then its displacement in the last two seconds is
 (a) $\frac{2v(n-1)}{n}$ (b) $\frac{v(n-1)}{n}$ (c) $\frac{v(n+1)}{n}$ (d) $\frac{2v(2n+1)}{n}$
24. A particle start moving from rest state along a straight line under the action of a constant force and travel distance x in first 5 seconds. The distance travelled by it in next five seconds will be
 (a) x (b) $2x$ (c) $3x$ (d) $4x$
25. A body is projected vertically upward with speed 40 m/s. The distance travelled by body in the last second of upward journey is [take $g = 9.8 \text{ m/s}^2$ and neglect effect of air resistance]
 (a) 4.9 m (b) 9.8 m (c) 12.4 m (d) 19.6 m
26. A ball is dropped from a bridge of 122.5 metre above a river. After the ball has been falling for two seconds, a second ball is thrown straight down after it. Initial velocity of second ball so that both hit the water at the same time is
 (a) 49 m/s (b) 55.5 m/s (c) 26.1 m/s (d) 9.8 m/s
27. A balloon starts rising from ground from rest with an upward acceleration 2 m/s^2 . Just after 1 s, a stone is dropped from it. The time taken by stone to strike the ground is nearly
 (a) 0.3 s (b) 0.7 s (c) 1 s (d) 1.4 s
28. A boy throws balls into air at regular interval of 2 second. The next ball is thrown when the velocity of first ball is zero. How high do the ball rise above his hand? [Take $g = 9.8 \text{ m/s}^2$]
 (a) 4.9 m (b) 9.8 m (c) 19.6 m (d) 29.4 m
29. A ball projected from ground vertically upward is at same height at time t_1 and t_2 . The speed of projection of ball is [Neglect the effect of air resistance]
 (a) $g[t_2 - t_1]$ (b) $\frac{g[t_1 + t_2]}{2}$ (c) $\frac{g[t_2 - t_1]}{2}$ (d) $g[t_1 + t_2]$
30. A ball is dropped from a height h above ground. Neglect the air resistance, its velocity (v) varies with its height above the ground as
 (a) $\sqrt{2g(h-y)}$ (b) $\sqrt{2gh}$ (c) $\sqrt{2gy}$ (d) $\sqrt{2g(h+y)}$

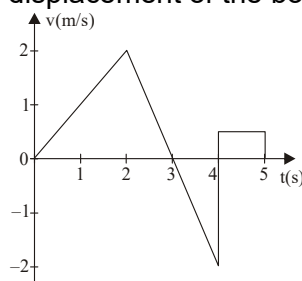
GRAPHICAL ANALYSIS

31. Figure shows the displacement (x)-time (t) graph of a particle moving on the X-axis.
- The particle is at rest
 - The particle is continuously going along X-direction
 - The velocity of particle increases upto time t_0 and then becomes constant
 - The particle moves at a constant velocity up to a time t_0 , and then stops.



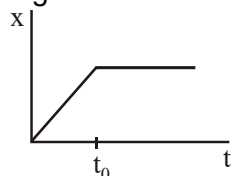
32. The displacement-time graph for two particles A and B are straight lines inclined at angles of 30° and 60° with the time axis. The ratio of velocities of $V_A : V_B$ is
- 1 : 2
 - 1 : $\sqrt{3}$
 - $\sqrt{3} : 1$
 - 1 : 3

33. The velocity versus time graph of a body in a straight line is as shown in Figure. The displacement of the body in five seconds is



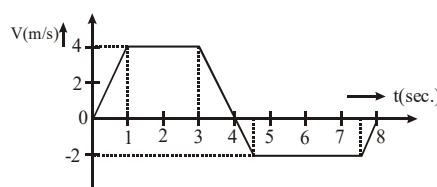
- 2 m
- 2.5 m
- 4 m
- 5 m

34. Figure shows the displacement-time graph of a particle moving on the X-axis.



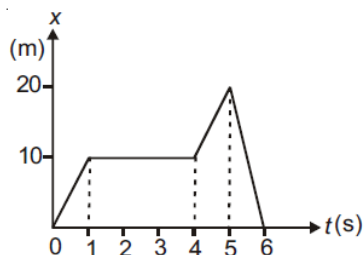
- the particle is continuously going in positive x direction
- the particle is at rest
- the velocity increases up to a time t_0 , and then becomes constant
- the particle moves at a constant velocity up to a time t_0 , and then stops.

35. The velocity - time graph of a linear motion is shown in figure. The displacement from the origin after 8 sec. is :



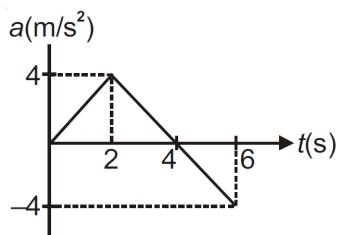
- 5 m
- 16 m
- 8 m
- 6 m.

36. Figure shows the graph of x -coordinate of a particle moving along x -axis as a function of time. Average velocity during $t = 0$ to 6 s and instantaneous velocity at $t = 3$ s respectively, will be



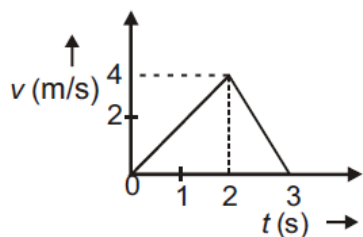
- (a) 10 m/s, 0 (b) 60 m/s, 0 (c) 0, 0 (d) 0, 10 m/s

37. For the acceleration-time (a - t) graph shown in figure, the change in velocity of particle from $t = 0$ to $t = 6$ s is



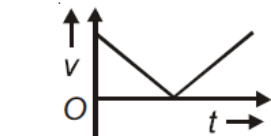
- (a) 10 m/s (b) 4 m/s (c) 12 m/s (d) 8 m/s

38. The velocity versus time graph of a body moving in a straight line is as shown in the figure below

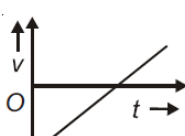


- (a) The distance covered by the body in 0 to 2 s is 8 m
 (b) The acceleration of the body in 0 to 2 s is 4 ms^{-2}
 (c) The acceleration of the body in 2 to 3 s is 4 ms^{-2}
 (d) The distance moved by the body during 0 to 3 s is 6 m

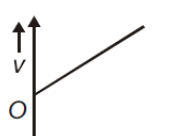
39. A particle moves along x -axis in such a way that its x -co-ordinate varies with time according to the equation $x = 4 - 2t + t^2$. The speed of the particle will vary with time as



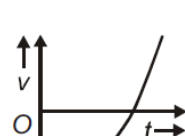
(a)



(b)

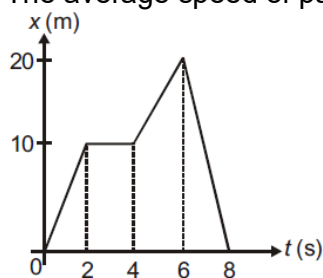


(c)



(d)

40. The position (x)-time (t) graph for a particle moving along a straight line is shown in figure. The average speed of particle in time interval $t = 0$ to $t = 8$ s is



- (a) Zero (b) 5 m/s (c) 7.5 m/s (d) 9.7 m/s
41. If R and H are the horizontal range and maximum height attained by a projectile, then its speed of projection is
- (a) $\sqrt{2gR + \frac{4R^2}{gH}}$ (b) $\sqrt{2gH + \frac{R^2g}{8H}}$ (c) $\sqrt{2gH + \frac{8H}{Rg}}$ (d) $\sqrt{2gH + \frac{R^2}{H}}$

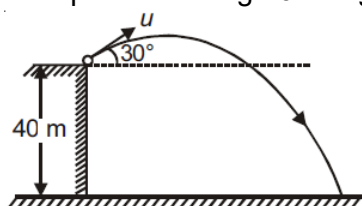
GROUND TO GROUND PROJECTION

42. Two projectiles A and B are projected with angle of projection 15° for the projectile A and 45° for the projectile B. If R_A and R_B be the horizontal range for the two projectiles, then :
- (a) $R_A < R_B$
 (b) $R_A = R_B$
 (c) $R_A > R_B$
 (d) the information is insufficient to decide the relation of R_A with R_B .
43. A projectile is thrown with an initial velocity of $[a\hat{i} + b\hat{j}]$ m/s. If the range of projectile is twice the maximum height reached by it, then
- (a) $b = a/2$ (b) $b = a$ (c) $b = 2a$ (d) $b = 4a$
44. The angle of projection, at which the horizontal range and maximum height of projectile are equal, is
- (a) 45° (b) $\theta = \tan^{-1}(0.25)$ (c) $\theta = \tan^{-1} 4$ (d) 60° .
45. A javelin thrown into air at an angle with the horizontal has a range of 200 m. If the time of flight is 5 second, then the horizontal component of velocity of the projectile at the highest point of the trajectory is
- (a) 40 m/s (b) 20 m/s (c) 9.8 m/s (d) 5 m/s
46. The position coordinates of a projectile projected from ground on a certain planet (with no atmosphere) are given by $y = (4t - 2t^2)$ m and $x = (3t)$ metre, where t is in second and point of projection is taken as origin. The angle of projection of projectile with vertical is
- (a) 30° (b) 37° (c) 45° (d) 60°
47. A particle is projected from ground with speed 80 m/s at an angle 30° with horizontal from ground. The magnitude of average velocity of particle in time interval $t = 2$ s to $t = 6$ s is [Take $g = 10 \text{ m/s}^2$]
- (a) $40\sqrt{2} \text{ m/s}$ (b) 40 m/s (c) Zero (d) $40\sqrt{3} \text{ m/s}$

48. Two objects are thrown up at angles of 45° and 60° respectively, with the horizontal. If both objects attain same vertical height, then the ratio of magnitude of velocities with which these are projected is
 (a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{\frac{3}{5}}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}$
49. For an object projected from ground with speed u horizontal range is two times the maximum height attained by it. The horizontal range of object is
 (a) $\frac{2u^2}{3g}$ (b) $\frac{3u^2}{4g}$ (c) $\frac{3u^2}{2g}$ (d) $\frac{4u^2}{5g}$
50. The velocity at the maximum height of a projectile is $\frac{\sqrt{3}}{2}$ times its initial velocity of projection (u). Its range on the horizontal plane is
 (a) $\frac{\sqrt{3}u^2}{2g}$ (b) $\frac{3u^2}{2g}$ (c) $\frac{3u^2}{g}$ (d) $\frac{u^2}{2g}$
51. A projectile is thrown into space so as to have a maximum possible horizontal range of 400 metres. Taking the point of projection as the origin, the co-ordinates of the point where the velocity of the projectile is minimum are
 (a) (400, 100) (b) (200, 100) (c) (400, 200) (d) (200, 200)
52. If the time of flight of a bullet over a horizontal range R is T , then the angle of projection with horizontal is
 (a) $\tan^{-1}\left(\frac{gT^2}{2R}\right)$ (b) $\tan^{-1}\left(\frac{2R^2}{gT}\right)$ (c) $\tan^{-1}\left(\frac{2R}{g^2T}\right)$ (d) $\tan^{-1}\left(\frac{2R}{gT}\right)$
53. When a particle is projected at some angle to the horizontal, it has a range R and time of flight t_1 . If the same particle is projected with the same speed at some other angle to have the same range, its time of flight is t_2 , then
 (a) $t_1 + t_2 = \frac{2R}{g}$ (b) $t_1 - t_2 = \frac{R}{g}$ (c) $t_1 t_2 = \frac{2R}{g}$ (d) $t_1 t_2 = \frac{R}{g}$
54. A projectile is thrown with velocity v at an angle θ with horizontal. When the projectile is at a height equal to half of the maximum height, the vertical component of the velocity of projectile is
 (a) $v \sin \theta \times 3$ (b) $\frac{v \sin \theta}{3}$ (c) $\frac{v \sin \theta}{\sqrt{2}}$ (d) $\frac{v \sin \theta}{\sqrt{3}}$

PROJECTILE FROM A HEIGHT

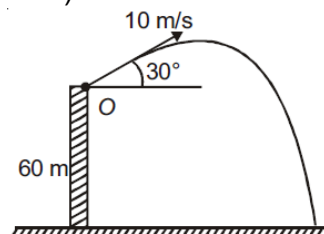
55. Figure shows a projectile thrown with speed $u = 20$ m/s at an angle 30° with horizontal from the top of a building 40 m high. Then the horizontal range of projectile is



- (a) $20\sqrt{3}$ m (b) $40\sqrt{3}$ m (c) 40 m (d) 20 m

56. Two paper screens A and B are separated by distance 100 m. A bullet penetrates A and B , at points P and Q respectively, where Q is 10 cm below P . If bullet is travelling horizontally at the time of hitting A , the velocity of bullet at A is nearly
(a) 100 m/s (b) 200 m/s (c) 600 m/s (d) 700 m/s

57. A ball is projected from a point O as shown in figure. It will strike the ground after ($g = 10 \text{ m/s}^2$)



- (a) 4 s (b) 3 s (c) 2 s (d) 5 s

RELATIVE MOTION IN ONE DIMENSION

58. A body A of mass 4 kg is dropped from a height of 100 m. Another body B of mass 2 kg is dropped from a height of 50 m at the same time. Then :
(a) Both the bodies reach the ground simultaneously.
(b) A takes nearly 0.7^{th} of time required by B
(c) B takes nearly 0.7^{th} of time required by A
(d) A takes double the time required by B .
59. Two particles are moving with velocities v_1 and v_2 . Their relative velocity is the maximum when the angle between their velocities is :
(a) zero (b) $\pi/4$ (c) $\pi/2$ (d) π
60. A body falls freely from a height of 50 m. Simultaneously, another body is thrown from the surface of earth with a certain initial velocity. The two bodies meet at a height of 10 m. What is the initial velocity of the second body ?
(a) 4.9 m/s (b) 9.8 m/s (c) 17.5 m/s (d) 19.6 m/s
61. A body is projected vertically upward with speed 10 m/s and other at same time with same speed in downward direction from the top of a tower. The magnitude of acceleration of first body w.r.t. second is {take $g = 10 \text{ m/s}^2$ }
(a) Zero (b) 10 m/s^2 (c) 5 m/s^2 (d) 20 m/s^2
62. Two balls are projected upward simultaneously with speeds 40 m/s and 60 m/s. Relative position (x) of second ball w.r.t. first ball at time $t = 5 \text{ s}$ is [Neglect air resistance].
(a) 20 m (b) 80 m (c) 100 m (d) 120 m

RELATIVE MOTION IN TWO DIMENSION

63. A swimmer wishes to reach directly opposite bank of a river, flowing with velocity 8 m/s. The swimmer can swim 10 m/s in still water. The width of the river is 480 m. Time taken by him to do so:
(a) 60 sec (b) 48 sec (c) 80 sec (d) none of these.
64. A man who can swim at the rate of 2 km/h crosses a river to an exactly opposite point on the other bank by swimming in a direction of 120° to the flow of the water in the river. The velocity of the water current in km/h is :
(a) 1 (b) 2 (c) $1/2$ (d) $3/2$

65. A boat covers certain distance between two spots in a river taking t_1 hrs going downstream and t_2 hrs going upstream. What time will be taken by boat to cover same distance in still water?
- (a) $\frac{t_1+t_2}{2}$ (b) $2(t_2-t_1)$ (c) $\frac{2t_1t_2}{t_1+t_2}$ (d) $\sqrt{t_1t_2}$
66. A train of 150 m length is going towards North at a speed of 10 m/s. A bird is flying at 5 m/s parallel to the track towards South. The time taken by the bird to cross the train is
- (a) 10 s (b) 15 s (c) 30 s (d) 12 s
67. Two cars are moving in the same direction with a speed of 30 km/h. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. The speed of the third car is
- (a) 30 km/h (b) 25 km/h (c) 40 km/h (d) 45 km/h
68. Four persons P , Q , R and S are initially at the four corners of a square of side d . Each person now moves with a constant speed v in such a way that P always moves directly towards Q , Q towards R , R towards S , and S towards P . The four persons will meet after time
- (a) $\frac{d}{2v}$ (b) $\frac{d}{v}$ (c) $\frac{3d}{2v}$ (d) They will never meet
69. Two trains each of length 100 m moving parallel towards each other at speed 72 km/h and 36 km/h respectively. In how much time will they cross each other?
- (a) 4.5 s (b) 6.67 s (c) 3.5 s (d) 7.25 s
70. A ball is dropped from the top of a building of height 80 m. At same instant another ball is thrown upwards with speed 50 m/s from the bottom of the building. The time at which balls will meet is
- (a) 1.6 s (b) 5 s (c) 8 s (d) 10 s
71. A shell is fired vertically upwards with a velocity v_1 from a trolley moving horizontally with velocity v_2 . A person on the ground observes the motion of the shell as a parabola, whose horizontal range is
- (a) $\frac{2v_1^2v_2}{g}$ (b) $\frac{2v_1^2}{g}$ (c) $\frac{2v_2^2}{g}$ (d) $\frac{2v_1v_2}{g}$
72. Out of the two cars A and B , car A is moving towards east with a velocity of 10 m/s whereas B is moving towards north with a velocity 20 m/s, then velocity of A w.r.t. B is (nearly)
- (a) 30 m/s (b) 10 m/s (c) 22 m/s (d) 42 m/s
73. A man moves in an open field such that after moving 10 m on a straight line, he makes a sharp turn of 60° to his left. The total displacement just at the start of 8th turn is equal to
- (a) 12 m (b) 15 m (c) 17.32 m (d) 14.14 m

LEVEL – II

DISTANCE AND DISPLACEMENT, SPEED AND VELOCITY, AVG. SPEED AND AVG. VELOCITY

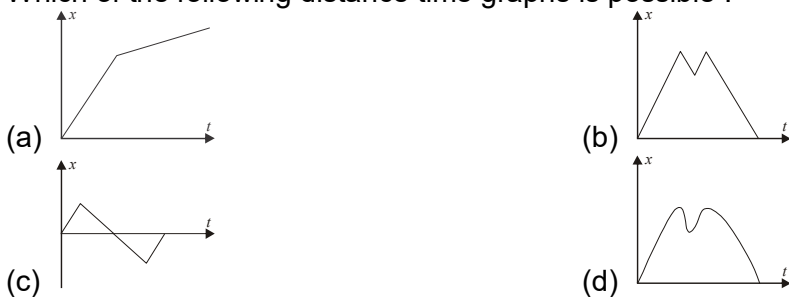
- A particle moving in a straight line covers half the distance with speed of 3m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is
(a) 4.0 m/s (b) 5.0 m/s (c) 5.5 m/s (d) 4.8 m/s
- The table shows the distance covered in successive seconds by a body accelerated uniformly from rest

Time interval (s)	I	II	III	IV
Distance (cm)	2	6	10	14

What is the speed of the body after 4 sec?
(a) 4 cm/sec (b) 8 cm/sec (c) 14 cm/sec (d) 16 cm/sec
- A body moves in a straight line along x-axis, its distance from the origin is given by the equation $x = 8t - 3t^2$. The average velocity in the interval from $t = 0$ to $t = 4$ is
(a) 2 m/s (b) -16 m/s (c) -4 m/s (d) 5 m/s
- The height y and distance x along the horizontal for a body projected in the vertical plane are given by $y = 8t - 5t^2$ and $x = 6t$. The initial velocity of the projected body is
(a) 8 m/s (b) 9 m/s (c) 10 m/s (d) $\frac{10}{3}$ m/s
- Choose the correct statement from the following:
(a) The magnitude of velocity of a particle is equal to its speed.
(b) The magnitude of the average velocity in an interval is equal to its average speed in that interval.
(c) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.
(d) It is possible to have a situation in which the instantaneous speed of particle is always zero but the average speed is not zero.
- The location of a particle has changed. What can we say about the displacement and the distance covered by the particle.
(a) Neither can be zero (b) One may be zero
(c) Both may be zero (d) One is +ve, other is -ve and vice versa
- A train moving with a constant speed along a straight track takes a bend in a curve with the same speed. Due to this :
(a) its velocity is changed in magnitude (b) its velocity is not changed
(c) its speed only is changed (d) its velocity is changed.
- A particle projected from origin moves in x-y plane with a velocity $\vec{v} = 3\hat{i} + 6x\hat{j}$, where \hat{i} and \hat{j} are the unit vectors along x and y axis. Find the equation of path followed by the particle
(a) $y = x^2$ (b) $y = \frac{1}{x^2}$ (c) $y = 2x^2$ (d) $y = \frac{1}{x}$

AVG. ACCELERATION AND APPLICATION OF CALCULAS

9. Choose the wrong statement .
 (a) Zero velocity of a particle does not necessarily mean that its acceleration is zero.
 (b) Zero acceleration of a particle does not necessarily mean that its velocity is zero.
 (c) If speed of a particle is constant, its acceleration must be zero.
 (d) none of these.
10. The initial velocity of a particle moving along x-axis is u (at $t = 0$ and $x = 0$) and its acceleration a is given by $a = kx$. Which of the following equation is correct between its velocity (v) and position (x)?
 (a) $v^2 - u^2 = 2kx$ (b) $v^2 = u^2 + 2kx^2$ (c) $v^2 = u^2 + kx^2$ (d) $v^2 + u^2 = 2kx$
11. A particle moving with a constant acceleration from A to B in the straight line AB has velocities u and v at A and B respectively. If C is the mid-point of AB then the velocity of particle while passing C will be
 (a) $\sqrt{\frac{v^2 + u^2}{2}}$ (b) $\frac{v+u}{2}$ (c) $\frac{v-u}{2}$ (d) $\frac{\left(\frac{1}{v} + \frac{1}{u}\right)}{2}$
12. If a body travels half of its total path in the last second of its fall from rest then the total time of its fall will be
 (a) 1.82 s (b) 3.4 s (c) 3.41 (d) 3.73
13. The displacement of a particle undergoing rectilinear motion along the x-axis is given by $x = 2t^3 - 21t^2 + 60t + 6$. The acceleration of the particle when its velocity is zero will be.
 (a) 36 m/sec² (b) 9 m/sec² (c) -9m/sec² (d) -18m/sec²
14. A particle, initially at rest, starts moving in a straight line with an acceleration $a = 6t + 4$ m/s². The distance covered by it in 3 s is
 (a) 15 m (b) 30 m (c) 45 m (d) 60 m
15. The acceleration 'a' in m/s², of a particle is given by $a = 3t^2 + 2t + 2$ where 't' is the time. If the particle starts out with a velocity $v = 2$ m/s at $t = 0$, then the velocity at the end of 2 s is
 (a) 12 m/s (b) 14 m/s (c) 16 m/s (d) 18 m/s
16. A balloon is ascending at the rate of 12 m/s. When it is at a height of 65 m from the ground, a packet is dropped from it. The packet will reach the ground after time (take $g = 10$ m/s²)
 (a) 10 s (b) 2.5 s (c) 5 s (d) 7.5 s
17. A body falls from a certain height. Two seconds later another body falls from the same height. How long after the beginning of motion of the first body is the distance between the bodies twice the distance at the moment the second body starts to fall?
 (a) 3 s (b) 10 s (c) 15 s (d) 20
18. The velocity v of body moving along a straight line varies with time t as $v = 2t^2 e^{-t}$, where v is in m/s and t is in second. The acceleration of body is zero at $t =$
 (a) 0 (b) 2s (c) 3 (d) Both (a) & (b)
19. The velocity of a body depends on time according to the equation $v = \frac{t^2}{10} + 20$. The body is undergoing
 (a) Uniform acceleration (b) Uniform retardation
 (c) Non-uniform acceleration (d) Zero acceleration

20. A body starts from origin and moves along x -axis so that its position at any instant is $x = 4t^2 - 12t$ where t is in second and v in m/s. What is the acceleration of particle?
 (a) 4 m/s^2 (b) 8 m/s^2 (c) 24 m/s^2 (d) 0 m/s^2
21. The relation between position (x) and time (t) are given below for a particle moving along a straight line. Which of the following equation represents uniformly accelerated motion? [where α and β are positive constants]
 (a) $\beta x = \alpha t + \alpha\beta$ (b) $\alpha x = \beta + t$ (c) $xt = \alpha\beta$ (d) $\alpha t = \sqrt{\beta + x}$
22. Which of the following distance time graphs is possible :

23. If magnitude of average speed and average velocity over a time interval are same, then
 (a) The particle must move with zero acceleration
 (b) The particle must move with non-zero acceleration
 (c) The particle must be at rest
 (d) The particle must move in a straight line without turning back
24. When the velocity of body is variable, then
 (a) Its speed may be constant
 (b) Its acceleration may be constant
 (c) Its average acceleration may be constant
 (d) All of these
25. An object is moving with variable speed, then
 (a) Its velocity may be zero
 (b) Its velocity must be variable
 (c) Its acceleration may be zero
 (d) Its velocity may be constant
26. If the displacement of a particle varies with time as $\sqrt{x} = t + 7$, then
 (a) Velocity of the particle is inversely proportional to t
 (b) Velocity of the particle is proportional to t^2
 (c) Velocity of the particle is proportional to \sqrt{t}
 (d) The particle moves with constant acceleration
27. A particle moves in a straight line and its position x at time t is given by $x^2 = 2 + t$. Its acceleration is given by
 (a) $-\frac{2}{x^3}$ (b) $-\frac{1}{4x^3}$ (c) $-\frac{1}{4x^2}$ (d) $\frac{1}{x^2}$

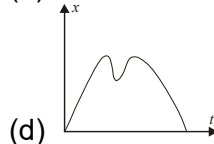
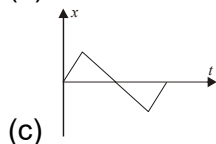
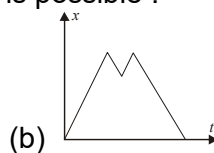
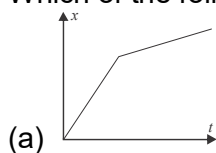
CONSTANT ACCELERATION AND FREE FALL

28. A, B, C and D are points in a vertical line such that $AB = BC = CD$. If a body falls from rest at A, then the times of descent through AB, BC and CD are in the ratio :
 (a) $1 : \sqrt{2} : \sqrt{3}$ (b) $\sqrt{2} : \sqrt{3} : 1$
 (c) $\sqrt{3} : 1 : \sqrt{2}$ (d) $1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

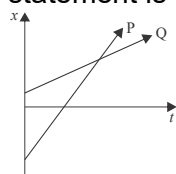
29. A ball is thrown upward with speed 10 m/s from the top of the tower reaches the ground with a speed 20 m/s. The height of the tower is [Take $g = 10 \text{ m/s}^2$]
 (a) 10 m (b) 15 m (c) 20 m (d) 25 m
30. A body thrown vertically up with initial velocity 52 m/s from the ground passes twice a point at h height above at an interval of 10 s. The height h is ($g = 10 \text{ m/s}^2$)
 (a) 22 m (b) 10.2 m (c) 11.2 m (d) 15 m
31. When a particle is thrown vertically upwards, its velocity at one third of its maximum height is $10\sqrt{2} \text{ m/s}$. The maximum height attained by it is
 (a) 20.2 m (b) 30 m (c) 15 m (d) 12.8 m
32. A particle has a velocity u towards east at $t = 0$. Its acceleration is towards west and is constant. Let x_A and x_B be the magnitude of displacement in the first 10 seconds and the next 10 seconds :
 (a) $x_A < x_B$
 (b) $x_A = x_B$
 (c) $x_A > x_B$
 (d) the information is insufficient to decide the relation of x_A and x_B .
33. A balloon rises from rest with a constant acceleration $g/8$. A stone is released from it when it has risen to a height h . The time taken by the stone to reach the ground is
 (a) $\sqrt{\frac{h}{g}}$ (b) $\sqrt{\frac{2h}{g}}$ (c) $2\sqrt{\frac{h}{g}}$ (d) $4\sqrt{\frac{h}{g}}$

GRAPHICAL ANALYSIS

34. Which of the following distance time graphs is possible :

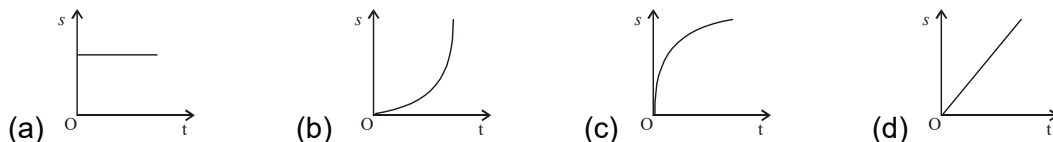
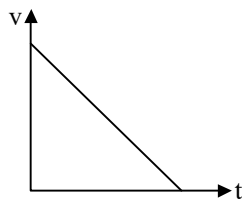


35. Figure shows the time-displacement curve of the particles P and Q. Which of the following statement is correct?

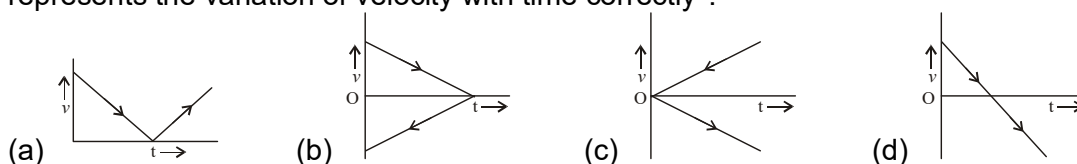


- (a) Both P and Q move with uniform equal speed
 (b) P is accelerated Q is retarded
 (c) Both P and Q move with uniform speeds but the speed of P is more than the speed of Q
 (d) Both P and Q move with uniform speeds but the speed of Q is more than the speed of P.

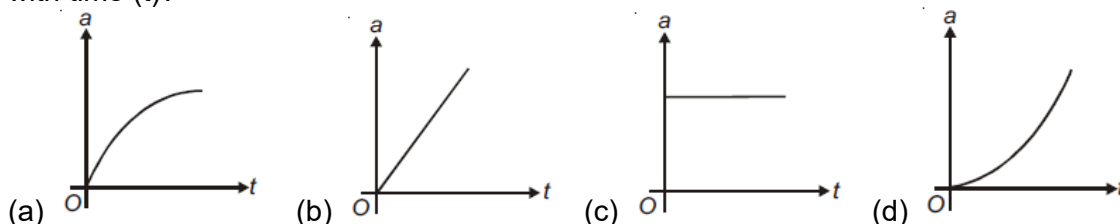
36. A ball is thrown vertically upwards and its velocity v varies with time t as shown in the figure. Which of the graphs (a), (b), (c), (d) shows the correct curve of displacement versus time?



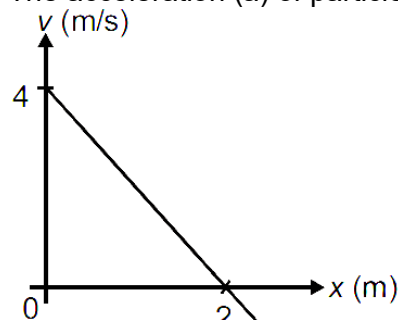
37. A body is thrown vertically upwards with a velocity u . Which of the following graphs represents the variation of velocity with time correctly?



38. The velocity v of a particle moving along x -axis varies with its position (x) as $v = \alpha\sqrt{x}$; where α is a constant. Which of the following graph represents the variation of its acceleration (a) with time (t)?

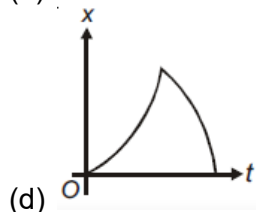
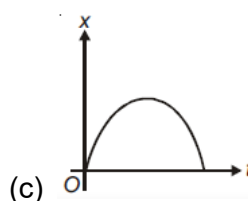
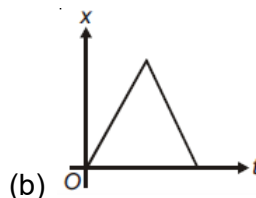
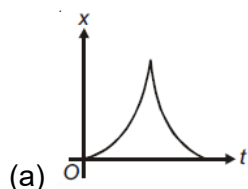
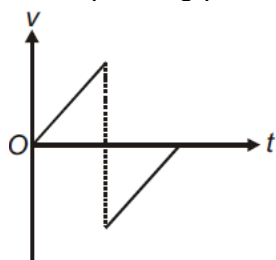


39. The velocity (v) of a particle moving along x -axis varies with its position x as shown in figure. The acceleration (a) of particle varies with position (x) as

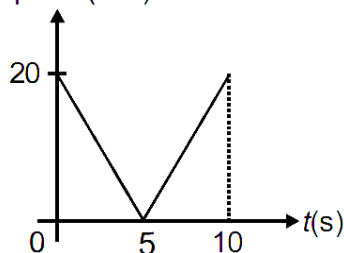


- (a) $a^2 = x + 3$ (b) $a = 2x^2 + 4$ (c) $2a = 3x + 5$ (d) $a = 4x - 8$

40. The velocity (v)-time (t) graph for a particle moving along x -axis is shown in the figure. The corresponding position (x)-time (t) is best represented by

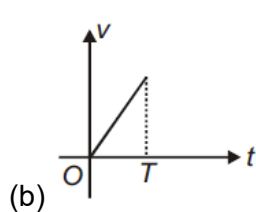
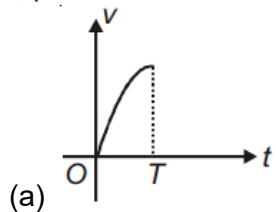
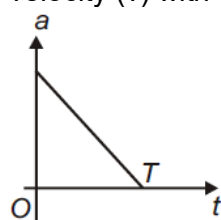


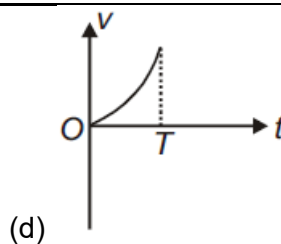
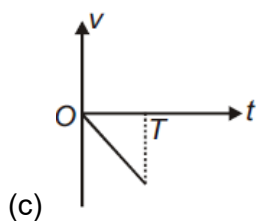
41. The speed-time graph for a body moving along a straight line is shown in figure. The average acceleration of body may be speed (m/s)



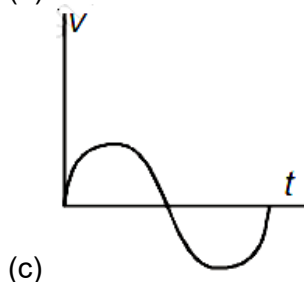
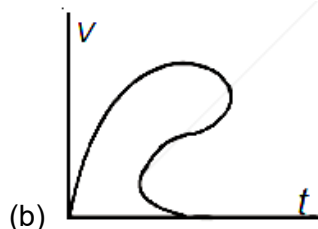
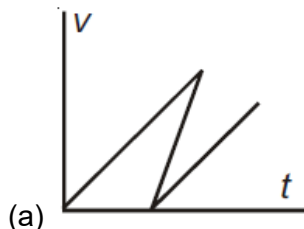
- (a) 0 (b) $4 m/s^2$ (c) $-4 m/s^2$ (d) All of these

42. The acceleration (a)-time (t) graph for a particle moving along a straight starting from rest is shown in figure. Which of the following graph is the best representation of variation of its velocity (v) with time (t)?



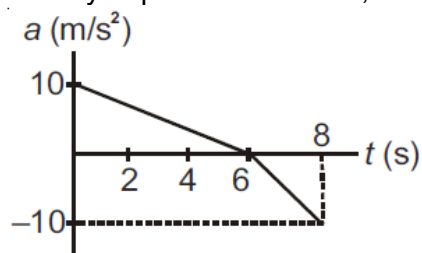


43. Which of the following speed-time ($v - t$) graphs is physically not possible?

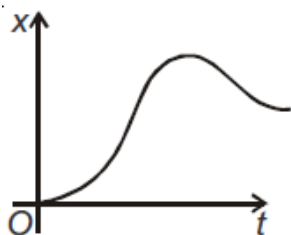


(d) All of these

44. The acceleration-time graph for a particle moving along x-axis is shown in figure. If the initial velocity of particle is -5 m/s , the velocity at $t = 8 \text{ s}$ is

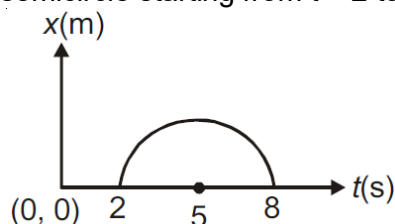


- (a) $+15 \text{ m/s}$ (b) $+20 \text{ m/s}$ (c) -15 m/s (d) -20 m/s
45. The displacement (x) - time (t) graph of a particle is shown in figure. Which of the following is correct?

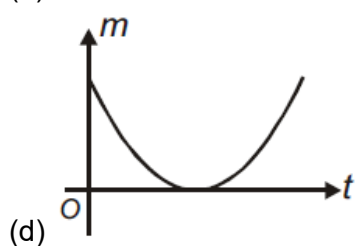
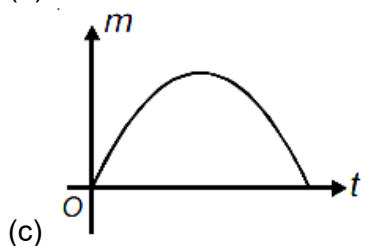
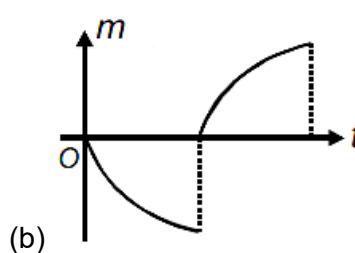
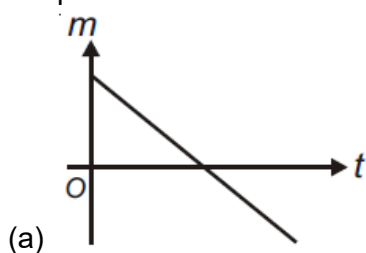


- (a) Particle starts with zero velocity and variable acceleration
 (b) Particle starts with non-zero velocity and variable acceleration
 (c) Particle starts with zero velocity and uniform acceleration
 (d) Particle starts with non-zero velocity and uniform acceleration

46. Position time graph of a particle moving along straight line is shown which is in the form of semicircle starting from $t = 2$ to $t = 8$ s. Select correct statement

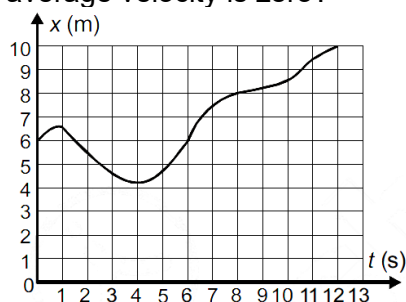


- (a) Velocity of particle between $t = 0$ to $t = 2$ s is positive
 (b) Velocity of particle is opposite to acceleration between $t = 2$ to $t = 5$ s
 (c) Velocity of particle is opposite to acceleration between $t = 5$ to $t = 8$ s
 (d) Acceleration of particle is positive between $t_1 = 2$ s to $t_2 = 5$ s while it is negative between $t_1 = 5$ s to $t_2 = 8$ s
47. A particle is projected at angle θ with horizontal from ground. The slope (m) of the trajectory of the particle varies with time (t) as



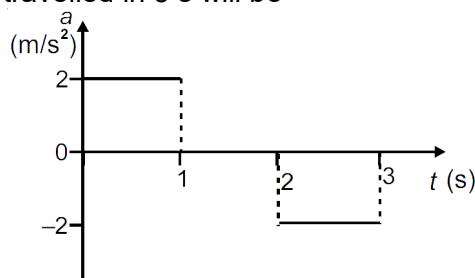
48. If H_1 and H_2 be the greatest heights of a projectile in two paths for a given value of range, then the horizontal range of projectile is given by
 (a) $\frac{H_1 + H_2}{2}$ (b) $\frac{H_1 + H_2}{4}$ (c) $4\sqrt{H_1 H_2}$ (d) $4[H_1 + H_2]$
49. A particle projected from ground moves at angle 45° with horizontal one second after projection and speed is minimum two seconds after the projection. The angle of projection of particle is [Neglect the effect of air resistance]
 (a) $\tan^{-1}(3)$ (b) $\tan^{-1}(2)$ (c) $\tan^{-1}(\sqrt{2})$ (d) $\tan^{-1}(4)$
50. A particle is projected with speed u at angle θ with horizontal from ground. If it is at same height from ground at time t_1 and t_2 , then its average velocity in time interval t_1 to t_2 is
 (a) Zero (b) $u \sin \theta$ (c) $u \cos \theta$ (d) $\frac{1}{2}[u \cos \theta]$

51. Position-time graph for a particle is shown in figure. Starting from $t = 0$, at what time t , the average velocity is zero?



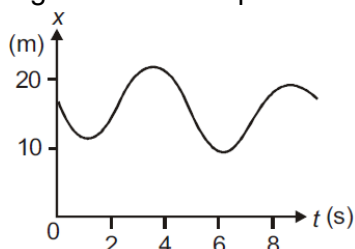
- (a) 1 s (b) 3 s (c) 6 s (d) 7 s

52. Acceleration-time graph for a particle is given in figure. If it starts motion at $t = 0$, distance travelled in 3 s will be



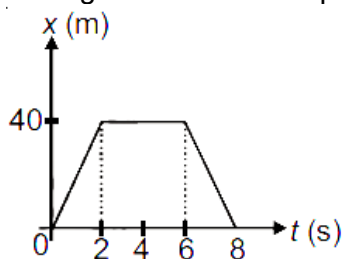
- (a) 4 m (b) 2 m (c) 0 (d) 6 m

53. Figure shows the position of a particle moving on the x-axis as a function of time



- (a) The particle has come to rest 4 times
(b) The velocity at $t = 8$ s is negative
(c) The velocity remains positive for $t = 2$ s to $t = 6$ s
(d) The particle moves with a constant velocity

54. The position (x) of a particle moving along x-axis varies with time (t) as shown in figure. The average acceleration of particle in time interval $t = 0$ to $t = 8$ s is

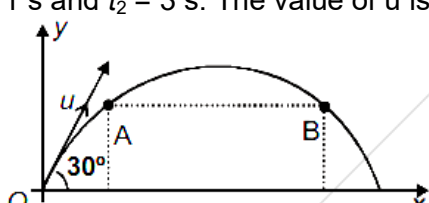


- (a) 3 m/s^2 (b) -5 m/s^2 (c) -4 m/s^2 (d) 2.5 m/s^2

PROJECTILE MOTION

55. The distance x and y along the horizontal plane of a projectile is given by $x = 6t$ and $y = 8t - 5t^2$ in metres where t is in seconds. The angle with the horizontal at which the projectile is projected is

- (a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$ (c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\cos^{-1}\left(\frac{3}{4}\right)$

56. Three particles, A, B, C are projected from the same point with same initial speeds making angles 30° , 45° and 60° respectively with the horizontal. Which of the following statement is correct?
 (a) A, B and C have unequal ranges
 (b) Ranges of A and C are equal and less than that of B
 (c) Ranges of A and C are equal and greater than that of B
 (d) A, B and C have equal ranges.
57. A projectile is projected at an angle with an initial velocity u . The time t , at which its horizontal velocity will equal the vertical velocity for the first time
 (a) $t = \frac{u}{g} (\cos \alpha - \sin \alpha)$ (b) $t = \frac{u}{g} (\cos \alpha + \sin \alpha)$
 (c) $t = \frac{u}{g} (\sin \alpha - \cos \alpha)$ (d) $t = \frac{u}{g} (\sin^2 \alpha - \cos^2 \alpha)$
58. What is the path followed by a moving body, on which a constant force acts in a direction other than initial velocity (i.e. excluding parallel and antiparallel direction)?
 (a) Straight line (b) Parabolic (c) Circular (d) Elliptical
59. Two stones are thrown with same speed u at different angles from ground in air. If both stones have same range and height attained by them are h_1 and h_2 , then $h_1 + h_2$ is equal to
 (a) $\frac{u^2}{g}$ (b) $\frac{u^2}{2g}$ (c) $\frac{u^2}{3g}$ (d) $\frac{u^2}{4g}$
60. A projectile is projected with speed u at an angle θ with the horizontal. The average velocity of the projectile between the instants it crosses the same level is
 (a) $u \cos \theta$ (b) $u \sin \theta$ (c) $u \cot \theta$ (d) $u \tan \theta$
61. A particle is thrown with a velocity of u m/s. It passes A and B as shown in figure at time $t_1 = 1$ s and $t_2 = 3$ s. The value of u is ($g = 10 \text{ m/s}^2$)
- 
- (a) 20 m/s (b) 10 m/s (c) 40 m/s (d) 5 m/s
62. Which one of the following statements is not true about the motion of a projectile?
 (a) The time of flight of a projectile is proportional to the speed with which it is projected at a given angle of projection
 (b) The horizontal range of a projectile is proportional to the square root of the speed with which it is projected
 (c) For a given speed of projection, the angle of projection for maximum range is 45°
 (d) At maximum height, the acceleration due to gravity is perpendicular to the velocity of the projectile
63. A projectile is thrown with speed 40 ms^{-1} at angle θ from horizontal. It is found that projectile is at same height at 1 s and 3 s. What is the angle of projection?
 (a) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (b) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (c) $\tan^{-1}(\sqrt{3})$ (d) $\tan^{-1}(\sqrt{2})$

64. Bullets are fired with velocity u at a fixed angle to the horizontal in all the directions. The maximum area covered by the bullets is
 (a) $\frac{\pi u^2}{g}$ (b) $\frac{\pi^2 u^2}{g}$ (c) $\frac{\pi^2 u^4}{g}$ (d) $\frac{\pi u^4}{g^2}$
65. A projectile is projected with kinetic energy K . If it has the maximum possible horizontal range, then its kinetic energy at the highest point will be
 (a) $0.25 K$ (b) $0.5 K$ (c) $0.75 K$ (d) K
66. A stone projected from ground with certain speed at an angle θ with horizontal attains maximum height h_1 . When it is projected with same speed at an angle θ with vertical attains height h_2 . The horizontal range of projectile is
 (a) $\frac{h_1 + h_2}{2}$ (b) $2h_1 h_2$ (c) $4\sqrt{h_1 h_2}$ (d) $h_1 + h_2$
67. From the top of a tower of height 40 m a ball is projected upwards with a speed of 20 m/sec at an angle of elevation of 30° . Then the ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (take $g = 10 \text{ m/sec}^2$)
 (a) $2 : 1$ (b) $3 : 1$ (c) $3 : 2$ (d) $4 : 1$
68. Two seconds after projection, a projectile is moving at 30° above the horizontal; after one more second it is moving horizontally. The initial speed of the projectile is ($g = 10 \text{ m/s}^2$)
 (a) $10\sqrt{3} \text{ m/s}$ (b) 20 m/s (c) 10 m/s (d) $20\sqrt{3} \text{ m/s}$

RELATIVE MOTION IN ONE DIMENSION

69. To a man walking at the rate of 3 km/h the rain appears to fall vertically. When he increases his speed to 6 km/h it appears to meet him at an angle of 45° with vertical from the front. The actual speed of rain is
 (a) 3 km/hr (b) 4 km/hr (c) $3\sqrt{2} \text{ km/h}$ (d) $2\sqrt{3} \text{ km/h}$
70. Two cars are moving in the same direction with a speed of 30 km/h. These are separated from each other by 5 km. Another third car moving in the opposite direction meets the two cars after an interval of 4 minutes. The speed of the third car is
 (a) 45 km/h. (b) 40 km/h (c) 35 km/h (d) 30 km/h.
71. A passenger sitting by the window of a train moving with a velocity of 72 km/h observes for 10 seconds that a train moving with a velocity of 32.4 km/h completely passes by it in 10 seconds. The length of the second train is
 (a) 110 m (b) 145 m (c) 210 m (d) 290 m
72. A body is thrown up in a lift with a velocity u relative to a lift and its time of flight is found to be t . The acceleration with which the lift is moving up will be.
 (a) $\frac{u - gt}{2}$ (b) $\frac{2u - gt}{t}$ (c) $\frac{u^2 - gt}{t}$ (d) $\frac{u^2}{gt}$
73. A parachutist drops freely from an aeroplane for 10 s before the parachute opens out. Then he descends with a net retardation of 2.5 m/s^2 . If he falls out of the plane at a height of 2495 m and $g = 10 \text{ m/s}^2$, hit velocity on reaching the ground will be :
 (a) 5 m/s (b) 10 m/s (c) 15 m/s (d) 20 m/s

74. A ball is dropped from an elevator moving upward with acceleration 'a' by a boy standing in it. The acceleration of ball with respect to [Take upward direction positive]
 (a) Boy is $-g$ (b) Boy is $-(g + a)$ (c) Ground is $-g$ (d) Both (b) & (c)
75. A stone is released from an elevator going up with acceleration 5 m/s^2 . The acceleration of the stone after the release is :
 (a) 5 ms^{-2} (b) 4.8 ms^{-2} upward (c) 4.8 down ward (d) 9.8 ms^{-2} down ward.
76. A ball 'A' is thrown up vertically with speed u . At the same instant another ball 'B' is released from rest from a height h . At time t , the velocity of A relative to B is :
 (a) u (b) $u - 2gt$ (c) $\sqrt{u^2 - 2gh}$ (d) $u - gt$
77. Two cars A and B are moving in same direction with velocities 30 m/s and 20 m/s . When car A is at a distance d behind the car B, the driver of the car A applies brakes producing uniform retardation of 2 m/s^2 . There will be no collision when
 (a) $d < 2.5 \text{ m}$ (b) $d > 125 \text{ m}$ (c) $d > 25 \text{ m}$ (d) $d < 125 \text{ m}$
78. A man can swim at a speed of 5 km/h w.r.t. water. He wants to cross a 1.5 km wide river flowing at 3 km/h . He keeps himself always at an angle of 60° with the flow direction while swimming. The time taken by him to cross the river will be :
 (a) 0.25 hr. (b) 0.35 hr. (c) 0.45 hr. (d) 0.55 hr.

ASSERTION & REASON

Each of the questions given below consists of two statements, an assertion (A) and reason (R). Select the number corresponding to the appropriate alternative as follows

- (A) If both assertion and reason are true and the reason is the correct explanation of the assertion.
 (B) If both assertion and reason are true but reason is not the correct explanation of the assertion.
 (C) If assertion is true but reason is false.
 (D) If the assertion and reason both are false.

1. **Assertion** : A body can have acceleration even if its velocity is zero at a given instant of time.
Reason : A body is momentarily at rest when it reverses its direction of motion.
2. **Assertion** : If the displacement of the body is zero, the distance covered by it may not be zero.
Reason : The displacement depends only on end points; the distance (path length) depends on the actual path.
3. **Assertion** : The magnitude of average velocity of the object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.
Reason : Path length (distance) is either equal or greater than the magnitude of displacement.
4. **Assertion** : When a body starting from rest is subjected to a uniform acceleration, it always moves in a straight line.
Reason : Straight line motion is the natural tendency of the body.
5. **Assertion** : The distance traversed, during equal intervals of time, by a body falling from rest is in ratio 1 : 3 : 5 : 7 : [Galileo's law of odd numbers]
Reason : A particle in one-dimensional motion with zero speed may have non-zero velocity.
6. **Assertion** : An object may have varying speed without having varying velocity.
Reason : If the velocity is zero at an instant, the acceleration should also be zero at that instant.
7. **Assertion** : In successive time intervals if the average velocities of a particle are equal then the particle must be moving with constant velocity.
Reason : When a particle moves with uniform velocity, the magnitude of its displacement always increases with time.
8. **Assertion** : In a free fall, the initial velocity of a body may or may not be zero.
Reason : A heavy body falls at a faster rate as compared to a lighter body.
9. **Assertion** : A body dropped from a given height and another body projected horizontally from the same height strike the ground simultaneously.
Reason : Because horizontal velocity has no effect in the vertical direction.
10. **Assertion** : In projectile motion, the vertical velocity of the particle is continuously decreased during its ascending motion.
Reason : In projectile motion downward constant acceleration is present in vertical direction.
11. **Assertion** : The relative velocity between any two bodies moving in opposite direction is equal to sum of the velocities of two bodies.
Reason : Sometimes relative velocity between two bodies is equal to difference in velocities of the two.

12. **Assertion** : The displacement-time graph of a body moving with uniform acceleration is a straight line.
 Reason : The displacement is proportional to time for uniformly accelerated motion.

13. **Assertion** : Velocity-time graph for an object in uniform motion along a straight path is a straight line parallel to the time axis.
 Reason : In uniform motion of an object velocity increases as the square of time elapsed.

14. **Assertion** : A body may be accelerated even when it is moving uniformly.
 Reason : When direction of motion of the body is changing then body may have acceleration.

15. **Assertion** : A body, whatever its motion is always at rest in a frame of reference which is fixed to the body itself.
 Reason : The relative velocity of a body with respect to itself is zero.

16. **Assertion** : Displacement of a body may be zero when distance travelled by it is not zero.
 Reason : The displacement is the longest distance between initial and final position.

17. **Assertion** : The equation of motion can be applied only if acceleration is along the direction of velocity and is constant.
 Reason : If the acceleration of a body is constant then its motion is known as uniform motion.

18. **Assertion** : Displacement of a body is vector sum of the area under velocity-time graph.
 Reason : Displacement is a vector quantity.

19. **Assertion** : The position-time graph of a body moving uniformly is a straight line parallel to position-axis.
 Reason : The slope of position-time graph in a uniform motion gives the speed of an object.

20. **Assertion** : The average speed of an object may be equal to arithmetic mean of individual speed.
 Reason : Average speed is equal to total distance travelled per total time taken.

21. **Assertion** : The speedometer of an automobile measure the average speed of the automobile.
 Reason : Average velocity is equal to total distance per unit time taken.

22. **Assertion** : The speed of a body can be negative.
 Reason : If the body is moving in the opposite direction of positive motion, then its speed is negative.

23. **Assertion** : The position-time graph of a uniform motion in one dimension of a body can have negative slope.
 Reason : When the speed of body decreases with time, the position-time graph of the moving body has negative slope.

24. **Assertion** : A positive acceleration of a body can be associated with a 'slowing down' of the body.
 Reason : Acceleration is a vector quantity.

25. **Assertion** : A negative acceleration of a body can be associated with a 'speeding up' of the body.
Reason : Increase in speed of a moving body is independent of its direction of motion.
26. **Assertion** : Two balls of different masses are thrown vertically upward with same speed. They will pass through their point of projection in the downward direction with the same speed.
Reason : The maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.
27. **Assertion** : The average velocity of the object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.
Reason : Velocity is a vector quantity and speed is a scalar quantity.
28. **Assertion** : Rocket in flight is not an illustration of projectile.
Reason : Rocket takes flight due to combustion of fuel and does not move under the gravity effect alone.
29. **Assertion** : The average speed of a body over a given interval of time is equal to the average velocity of the body in the same interval of time if a body moves in a straight line in one direction.
Reason : Because in this case distance travelled by a body is equal to the displacement of the body.
30. **Assertion** : Position-time graph of a stationary object is a straight line parallel to time axis.
Reason : For a stationary object, position does not change with time.
31. **Assertion** : Distance-time graph of the motion of a body having uniformly accelerated motion is a straight line inclined to the time axis.
Reason : Distance travelled by a body having uniformly accelerated motion is directly proportional to the square of the time taken.

Previous Year Questions

Kinematic Parameters

1. A particle is moving such that its position coordinates (x, y) are (2m, 3m) at time $t = 0$, (6m, 7m) at time $t = 2$ s and (13m, 14m) at time $t = 5$ s. Average velocity vector (\vec{V}_{av}) from $t = 0$ to $t = 5$ s is : **[AIPMT 2014]**
 (a) $2(\hat{i} + \hat{j})$ (b) $\frac{11}{5}(\hat{i} + \hat{j})$ (c) $\frac{1}{5}(13\hat{i} + 14\hat{j})$ (d) $\frac{7}{3}(\hat{i} + \hat{j})$
2. A body is moving with velocity 30 ms^{-1} towards east. After 10 s its velocity becomes 40 ms^{-1} towards north. The average acceleration of the body is **[CBSE AIPMT 2011]**
 (a) 7 ms^{-2} (b) $\sqrt{7} \text{ ms}^{-2}$ (c) 5 ms^{-2} (d) 1 ms^{-2}
3. A particle has a displacement of 12 m towards east and 5m towards north and finally 6 m vertically upwards. The sum of these displacement is **[BVP 2010]**
 (a) 12 m (b) 10.04 m (c) 14.31 m (d) None of these

4. A wheel completes 200 revolutions to cover the 9.5 km distance, then the diameter of the wheel is **[JCECE 2010]**
 (a) 1.5 m (b) 15 m (c) 7.5 cm (d) 7.5 m
5. The displacement of a particle, starting from rest (at $t = 0$) is given by $s = 6t^2 - t^3$. The time in seconds at which the particle will obtain zero velocity again is **[AIIMS 2009]**
 (a) 2 (b) 4 (c) 6 (d) 8
6. Which of the following can be zero, when a particle is in motion for some time? **[Manipal 2008]**
 (a) distance (b) displacement (c) speed (d) none of these
7. A car moves from X to Y with a uniform speed v_u and returns to X with a uniform speed v_d . The average speed for this round trip is **[CBSE AIPMT 2007]**
 (a) $\frac{2}{v_d + v_u}$ (b) $\sqrt{v_u v_d}$ (c) $\frac{2v_d v_u}{v_d + v_u}$ (d) $\frac{v_u + v_d}{2}$
8. A man goes 10m towards North, then 20m towards east then displacement is **[KCET 1999; JIPMER 1999; AFMC 2003]**
 (a) 22.5m (b) 25m (c) 25.5m (d) 30m
9. A wheel of radius 1 meter rolls forward half a revolution on a horizontal ground. The magnitude of the displacement of the point of the wheel initially in contact with the ground is **[BCECE 2005]**
 (a) 2π (b) $\sqrt{2}\pi$ (c) $\sqrt{\pi^2 + 4}$ (d) π
10. An aeroplane flies 400 m north and 300 m east and then flies 1200 m upwards then net displacement is **[AFMC 2004]**
 (a) 1200 m (b) 1300 m (c) 1400 m (d) 1500 m
11. An athlete completes one round of a circular track of radius R in 40 sec. What will be his displacement at the end of 2 min. 20 sec **[NCERT 1990; Kerala PMT 2004]**
 (a) Zero (b) $2R$ (c) $2\pi R$ (d) $7\pi R$
12. A particle is constrained to move on a straight line path. It returns to the starting point after 10 sec. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is false **[CBSE PMT 2000; AFMC 2001]**
 (a) Displacement of the particle is zero (b) Average speed of the particle is 3 m/s
 (c) Displacement of the particle is 30 m (d) Both (a) and (b)
13. A particle moves along a semicircle of radius 10m in 5 seconds. The average velocity of the particle is **[Kerala (Engg.) 2001]**
 (a) $2\pi \text{ ms}^{-1}$ (b) $4\pi \text{ ms}^{-1}$ (c) 2 ms^{-1} (d) 4 ms^{-1}

14. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. The average speed of the man over the interval of time 0 to 40 min. is equal to **[AMU (Med.) 2002]**
- (a) 5 km/h (b) $\frac{25}{4}$ km/h (c) $\frac{30}{4}$ km/h (d) $\frac{45}{8}$ km/h
15. A person travels along a straight road for the first half time with a velocity v_1 and the next half time with a velocity v_2 . The mean velocity V of the man is **[RPET 1999; BHU 2002]**
- (a) $\frac{2}{V} = \frac{1}{v_1} + \frac{1}{v_2}$ (b) $V = \frac{v_1 + v_2}{2}$ (c) $V = \sqrt{v_1 v_2}$ (d) $V = \sqrt{\frac{v_1}{v_2}}$
16. If a car covers $2/5^{\text{th}}$ of the total distance with v_1 speed and $3/5^{\text{th}}$ distance with v_2 then average speed is **[MP PMT 2003]**
- (a) $\frac{1}{2} \sqrt{v_1 v_2}$ (b) $\frac{v_1 + v_2}{2}$ (c) $\frac{2v_1 v_2}{v_1 + v_2}$ (d) $\frac{5v_1 v_2}{3v_1 + 2v_2}$
17. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion? **[AIEEE 2005]**
- (a) 1.5 cm (b) 1.0 cm (c) 3.0 cm (d) 2.0 cm
18. A car travels half the distance with constant velocity of 40 kmph and the remaining half with a constant velocity of 60 kmph. The average velocity of the car in kmph is **[Kerala PMT 2005]**
- (a) 40 (b) 45 (c) 48 (d) 50
19. A 100 m long train is moving with a uniform velocity of 45 km/hr. The time taken by the train to cross a bridge of length 1 km is **[BHU 2004]**
- (a) 58 s (b) 68 s (c) 78 s (d) 88 s
20. Which of the following is a one dimensional motion **[BHU 2000; CBSE PMT 2001]**
- (a) Landing of an aircraft (b) Earth revolving a round the sun
(c) Motion of wheels of a moving trains (d) Train running on a straight track
21. A 150 m long train is moving with a uniform velocity of 45 km/h. The time taken by the train to cross a bridge of length 850 meters is **[CBSE PMT 2001]**
- (a) 56 sec (b) 68 sec (c) 80 sec (d) 92 sec
22. The position vector of a particle \vec{R} function of time is given by:

$$\vec{R} = 4 \sin(2\pi t) \hat{i} + 4 \cos(2\pi t) \hat{j}$$
[AIPMT 2015]
 Where R is in meters, t is seconds and \hat{i} and \hat{j} denote unit vectors along x-and y-directions, respectively.
 Which one of the following statements is wrong for the motion of particle?
 (a) Magnitude of acceleration vectors is $\frac{v^2}{R}$, where v is the velocity of particle.

- (b) Magnitude of the velocity of particle is 8 meter/second
 (c) Path of the particle is a circle of radius 4 meter
 (d) Acceleration vector is along $-\vec{R}$

23. Two particles A and B, move with constant velocities \vec{v}_1 and \vec{v}_2 . At the initial moment their position vectors are \vec{r}_1 and \vec{r}_2 respectively. The condition for particles A and B for their collision is : **[AIPMT 2015]**

(a) $\vec{r}_1 \cdot \vec{v}_1 = \vec{r}_2 \cdot \vec{v}_2$ (b) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$ (c) $\vec{r}_1 - \vec{v}_1 = \vec{r}_2 - \vec{v}_2$ (d) $\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

24. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = at + bt^2$ and $x_Q(t) = ft - t^2$. At what time do the cars have the same velocity? **[NEET (Phase-2) 2016]**

(a) $\frac{a-f}{1+b}$ (b) $\frac{a+f}{2(b-1)}$ (c) $\frac{a+f}{2(1+b)}$ (d) $\frac{f-a}{2(1+b)}$

25. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1 s and 2 s is **[NEET-2016]**

(a) $\frac{A}{2} + \frac{B}{3}$ (b) $\frac{3}{2}A + 4B$ (c) $3A + 7B$ (d) $\frac{3}{2}A + \frac{7}{3}B$

26. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$, where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x , is given by **[AIPMT-2015]**

(a) $-2n\beta^2 e^{-4n+1}$ (b) $-2n\beta^2 x^{-2n-1}$ (c) $-2n\beta^2 x^{-4n-1}$ (d) $-2\beta^2 x^{-2n+1}$

27. A stone falls freely under gravity. It covers distances h_1 , h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1 , h_2 and h_3 is **[NEET-2013]**

(a) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$ (b) $h_2 = 3h_1$ and $h_3 = 3h_2$ (c) $h_1 = h_2 = h_3$ (d) $h_1 = 2h_2 = 3h_3$

28. The motion of a particle along a straight line is described by equation $x = 8 + 12t - t^3$ where x is in metre and t in second. The retardation of the particle when its velocity becomes zero, is **[AIPMT (Prelims)-2012]**

(a) 6 ms^{-2} (b) 12 ms^{-2} (c) 24 ms^{-2} (d) Zero

29. A boy standing at the top of a tower of 20 m height drops a stone. Assuming $g = 10 \text{ ms}^{-2}$, the velocity with which it hits the ground is **[AIPMT (Prelims)-2011]**

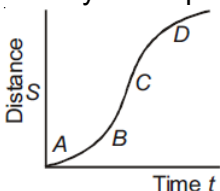
(a) 5.0 m/s (b) 10.0 m/s (c) 20.0 m/s (d) 40.0 m/s

30. A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2 . Its average speed during the complete journey is **[AIPMT (Mains)-2011]**

(a) $\frac{v_1^2 v_2^2}{v_1^2 + v_2^2}$ (b) $\frac{v_1 + v_2}{2}$ (c) $\frac{v_1 v_2}{v_1 + v_2}$ (d) $\frac{2v_1 v_2}{v_1 + v_2}$

31. A ball is dropped from a high rise platform at $t = 0$ starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v . The two balls meet at $t = 18 \text{ s}$. What is the value of v ? (Take $g = 10 \text{ m/s}^2$) **[AIPMT (Prelims)-2010]**

(a) 60 m/s (b) 75 m/s (c) 55 m/s (d) 40 m/s

32. A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to
 (a) (Velocity)^{3/2} (b) (Distance)² (c) (Distance)⁻² (d) (Velocity)^{2/3} **[AIPMT (Prelims)-2010]**
33. A bus is moving with a speed of 10 ms⁻¹ on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus?
 (a) 40 ms⁻¹ (b) 25 ms⁻¹ (c) 10 ms⁻¹ (d) 20 ms⁻¹ **[AIPMT (Prelims)-2009]**
34. A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2 , then
 (a) $S_2 = 3S_1$ (b) $S_2 = 4S_1$ (c) $S_2 = S_1$ (d) $S_2 = 2S_1$ **[AIPMT (Prelims)-2009]**
35. A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point
[AIPMT (Prelims)-2008]
- 
- (a) A (b) B (c) C (d) D
36. A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms⁻¹ to 20 ms⁻¹ while passing through a distance 135 m in t second. The value of t is
[AIPMT (Prelims)-2008]
- (a) 9 (b) 10 (c) 1.8 (d) 12
37. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ ms⁻², in the third second is
 (a) $\frac{19}{3}$ m (b) 6 m (c) 4 m (d) $\frac{10}{3}$ m
38. A particle moving along x-axis has acceleration f , at time t , given $f = f_0 \left(1 - \frac{t}{T}\right)$, where f_0 and T are constants. The particle at $t = 0$ has zero velocity. When $f = 0$, the particle's velocity (v_x) is
[AIPMT (Prelims)-2007]
- (a) $\frac{1}{2}f_0T$ (b) f_0T (c) $\frac{1}{2}f_0T^2$ (d) f_0T^2
39. The position x of a particle with respect to time t along x-axis is given by $x = 9t^2 - t^3$, where x is in metres and t in seconds. What will be the position of this particle when it achieves maximum speed along the positive x-direction?
[AIPMT (Prelims)-2007]
- (a) 24 m (b) 32 m (c) 54 m (d) 81 m
40. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest?
[AIPMT (Prelims)-2006]
- (a) 24 m (b) 40 m (c) 56 m (d) 16 m

41. Two bodies, A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is

[AIPMT (Prelims)-2006]

- (a) $\frac{5}{4}$ (b) $\frac{12}{5}$ (c) $\frac{5}{12}$ (d) $\frac{4}{5}$

42. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a , b , α and β are positive constants. The velocity of the particle will

[AIPMT (Prelims)-2005]

- (a) Go on decreasing with time (b) Be independent of β
(c) Drop to zero when α and β (d) Go on increasing with time

43. A ball is thrown vertically upward. It has a speed of 10 m/s when it has reached one half of its maximum height.

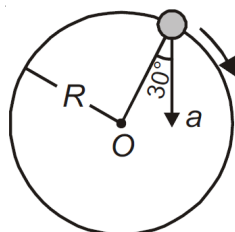
How high does the ball rise? (Taking $g = 10 \text{ m/s}^2$)

[AIPMT (Prelims)-2005]

- (a) 15 m (b) 10 m (c) 20 m (d) 5 m

44. In the given figure, $a = 15 \text{ m/s}^2$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5 \text{ m}$ at a given instant of time. The speed of the particle is

[NEET (Phase-2) 2016]



- (a) 4.5 m/s (b) 5.0 m/s (c) 5.7 m/s (d) 6.2 m/s

45. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is

[NEET-2016]

- (a) 180° (b) 0° (c) 90° (d) 45°

46. A particle moves so that its position vector is given by $\vec{r} = \cos \omega t \hat{x} + \sin \omega t \hat{y}$, where ω is a constant. Which of the following is true?

[NEET-2016]

- (a) Velocity is perpendicular to \vec{r} and acceleration is directed away from the origin
(b) Velocity and acceleration both are perpendicular to \vec{r}
(c) Velocity and acceleration both are parallel to \vec{r}
(d) Velocity is perpendicular to \vec{r} and acceleration is directed towards the origin

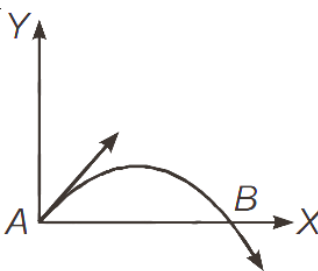
47. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to $8 \times 10^{-4} \text{ J}$ by the end of the second revolution after the beginning of the motion?

[NEET-2016]

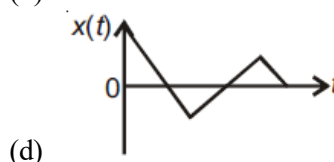
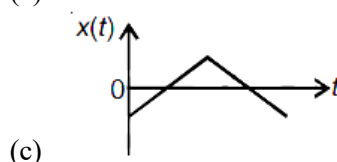
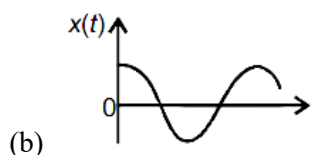
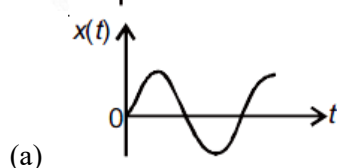
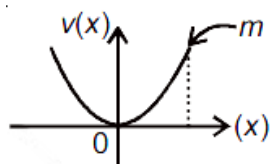
- (a) 0.2 m/s^2 (b) 0.1 m/s^2 (c) 0.15 m/s^2 (d) 0.18 m/s^2

48. The position vector of a particle \vec{R} as a function of time is given by $\vec{R} = 4 \sin(2\pi t) \hat{i} + 4 \cos(2\pi t) \hat{j}$, where R is in meters, t is in seconds and \hat{i} and \hat{j} denote unit vectors along x - and y -directions, respectively. Which one of the following statements is wrong for the motion of particle?

- (a) Path of the particle is a circle of radius 4 meter
(b) Acceleration vectors is along $-\vec{R}$
(c) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle
(d) Magnitude of the velocity of particle is 8 meter/second

49. Two particles A and B, move with constant velocity \vec{v}_1 and \vec{v}_2 . At the initial moment their position vectors are \vec{r}_1 and \vec{r}_2 respectively. The condition for particles A and B for their collision is
[Re-AIPMT-2015]
(a) $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$ (b) $\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$ (c) $\vec{r}_1 \cdot \vec{v}_1 = \vec{r}_2 \cdot \vec{v}_2$ (d) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$
50. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A, is moving northwards with a speed of 10 km h^{-1} . The time after which the distance between them becomes shortest, is
[AIPMT-2015]
(a) $10\sqrt{2} \text{ h}$ (b) 0h (c) 5h (d) $5\sqrt{2} \text{ h}$
51. A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is (Given $g = 9.8 \text{ ms}^{-2}$)
[AIPMT-2014]
(a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8
52. A particle is moving such that its position coordinates (x, y) are
(2 m, 3 m) at time $t = 0$,
(6 m, 7 m) at time $t = 2 \text{ s}$ and
(13 m, 14 m) at time $t = 5 \text{ s}$.
Average velocity vector (\vec{v}_{av}) from $t = 0$ to $t = 5 \text{ s}$ is
[AIPMT-2014]
(a) $\frac{1}{5}(13\hat{i} + 14\hat{j})$ (b) $\frac{7}{3}(\hat{i} + \hat{j})$ (c) $2(\hat{i} + \hat{j})$ (d) $\frac{11}{5}(\hat{i} + \hat{j})$
53. The velocity of a projectile at the initial point A is $(2\hat{i} + 3\hat{j}) \text{ m/s}$. its velocity (in m/s) at point B is
[NEET-2013]

(a) $-2\hat{i} + 3\hat{j}$ (b) $2\hat{i} - 3\hat{j}$ (c) $2\hat{i} + 3\hat{j}$ (d) $-2\hat{i} - 3\hat{j}$
54. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is
[AIPMT (Prelims)-2012]
(a) $\theta = \tan^{-1}(2)$ (b) $\theta = 45^\circ$ (c) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (d) $\theta = \tan^{-1}(4)$
55. A particle has initial velocity $(2\vec{i} + 3\vec{j})$ and acceleration $(0.3\vec{i} + 0.2\vec{j})$. The magnitude of velocity after 10s will be
[AIPMT (Prelims)-2012]
(a) 5 units (b) 9 units (c) $9\sqrt{2}$ units (d) $5\sqrt{2}$ units
56. A particle moves in a circle of radius 5 cm with constant speed and time period $0.2 \pi \text{ s}$. The acceleration of the particle is
[AIPMT (Prelims)-2011]
(a) 5 m/s^2 (b) 15 m/s^2 (c) 25 m/s^2 (d) 36 m/s^2

57. A body is moving with velocity 30 m/s towards east. After 10 s its velocity becomes 40 m/s towards north. The average acceleration of the body is **[AIPMT (Prelims)-2011]**
 (a) 5 m/s^2 (b) 1 m/s^2 (c) 7 m/s^2 (d) $\sqrt{7} \text{ m/s}^2$
58. A missile is fired for maximum range with an initial velocity of 20 m/s. If $g = 10 \text{ m/s}^2$, the range of the missile is **[AIPMT (Prelims)-2011]**
 (a) 20 m (b) 40 m (c) 50 m (d) 60 m
59. A particle of mass m is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small, which graph correctly depicts the position of the particle as a function of time? **[AIPMT (Prelims)-2011]**



60. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection is **[AIPMT (Mains)-2011]**
 (a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) 45° (c) 60° (d) $\tan^{-1}\frac{1}{2}$
61. The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is **[AIPMT (Mains)-2010]**
 (a) 60° (b) 15° (c) 30° (d) 45°
62. A particle moves in x-y plane according to rule $x = a \sin \omega t$ and $y = a \cos \omega t$. The particle follows **[AIPMT (Mains)-2010]**
 (a) An elliptical path
 (b) A circular path
 (c) A parabolic path
 (d) A straight line path inclined equally to x and y-axes
63. A particle has initial velocity $(3\hat{i} + 4\hat{j})$ and has acceleration $(0.4\hat{i} + 0.3\hat{j})$. Its speed after 10 s is **[AIPMT (Prelims)-2010]**
 (a) 7 units (b) $7\sqrt{2}$ units (c) 8.5 units (d) 10 units
64. A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. When the particle lands on the level ground the magnitude of the change in its momentum will be **[AIPMT (Prelims)-2008]**
 (a) Zero (b) $2mv$ (c) $\frac{mv}{\sqrt{2}}$ (d) $mv\sqrt{2}$

65. A particle starting from the origin $(0, 0)$ moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the x -axis an angle of **[AIPMT (Prelims)-2007]**
 (a) 0° (b) 30° (c) 45° (d) 60°
66. For angles of projection of a projectile at angles $(45^\circ - \theta)$ and $(45^\circ + \theta)$, the horizontal ranges described by the projectile are in the ratio of **[AIPMT (Prelims)-2006]**
 (a) $1 : 1$ (b) $2 : 3$ (c) $1 : 2$ (d) $2 : 1$
67. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular lap. The average velocity and average speed for each circular lap respectively is **[AIPMT (Prelims)-2006]**
 (a) 0, 0 (b) 0, 10 m/s (c) 10 m/s, 10 m/s (d) 10 m/s, 0
68. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of α is
 (a) -1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1
69. A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 s, what is the magnitude and direction of acceleration of the stone? **[AIPMT (Prelims)-2005]**
 (a) $\frac{\pi^2}{4} \text{ ms}^{-2}$ and direction along the radius towards the centre
 (b) $\pi^2 \text{ ms}^{-2}$ and direction along the radius away from centre
 (c) $\pi^2 \text{ ms}^{-2}$ and direction along the radius towards the centre
 (d) $\pi^2 \text{ ms}^{-2}$ and direction along the tangent to the circle

Relative Motion

1. A train is moving slowly on a straight track with a constant speed of 2 m/s. A passenger in the train starts walking at a steady speed of 2 m/s to the back of the train in the opposite direction of the motion of the train so to an observer standing on the platform directly in front of that passenger, the velocity of the passenger appears to be **[KCET 2010]**
 (a) 4 ms^{-1}
 (b) 2 ms^{-1}
 (c) 2 ms^{-1} in the opposite direction of the train
 (d) zero
2. Two boys are standing at the ends A and B of a ground where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1 . The boy at A starts running simultaneously with velocity v and catches the other boy in a time t , where t is **[CBSE PMT 2005]**
 (a) $a / \sqrt{v^2 + v_1^2}$ (b) $\sqrt{a^2 / (v^2 - v_1^2)}$
 (c) $a / (v - v_1)$ (d) $a / (v + v_1)$

3. From a balloon rising vertically upwards as 5 ms^{-1} stone is thrown up at 10 ms^{-1} relative to the balloon. Its velocity with respect to ground after 2 s is **[J&K CET 2006]**
 (a) zero (b) 5 ms^{-1} (c) 10 ms^{-1} (d) 20 ms^{-1}
4. Two cars A and B are travelling in the same direction with velocities v_1 and v_2 ($v_1 > v_2$). When the car A is at a distance d ahead of the car B, the driver of the car A applied the brake producing a uniform retardation a . There will be no collision when **[Pb. PET 2004]**
 (a) $d < \frac{(v_1 - v_2)^2}{2a}$ (b) $d < \frac{v_1^2 - v_2^2}{2a}$ (c) $d > \frac{(v_1 - v_2)^2}{2a}$ (d) $d > \frac{v_1^2 - v_2^2}{2a}$
5. A student is standing at a distance of 50 metres from the bus. As soon as the bus begins its motion with an acceleration of 1 ms^{-2} , the student starts running towards the bus with a uniform velocity u . Assuming the motion to be along a straight road, the minimum value of u , so that the student is able to catch the bus is **[KCET 2003]**
 (a) 5 ms^{-1} (b) 8 ms^{-1} (c) 10 ms^{-1} (d) 12 ms^{-1}
6. A body A moves with a uniform acceleration a and zero initial velocity. Another body B, starts from the same point moves in the same direction with a constant velocity v . The two bodies meet after a time t . The value of t is **[MP PET 2003]**
 (a) $\frac{2v}{a}$ (b) $\frac{v}{a}$ (c) $\frac{v}{2a}$ (d) $\sqrt{\frac{v}{2a}}$
7. A man is 45 m behind the bus when the bus start accelerating from rest with acceleration 2.5 m/s^2 . With what minimum velocity should the man start running to catch the bus ? **[J&K CET 2005]**
 (a) 12 m/s (b) 14 m/s (c) 15 m/s (d) 16 m/s
8. A boat is moving with velocity of $3\hat{i} + 4\hat{j}$ in river and water is moving with a velocity of $-3\hat{i} - 4\hat{j}$ with respect to ground. Relative velocity of boat with respect to water is: **[Pb. PET 2002]**
 (a) $-6\hat{i} - 8\hat{j}$ (b) $6\hat{i} + 8\hat{j}$ (c) $8\hat{i}$ (d) $6\hat{i}$
9. A boat moves with a speed of 5 km/h relative to water in a river flowing with a speed of 3 km/h and having a width of 1 km. The minimum time taken around a round trip is **[J&K CET 2005]**
 (a) 5 min (b) 60 min (c) 20 min (d) 30 min
10. A 120 m long train is moving in a direction with speed 20 m/s. A train B moving with 30 m/s in the opposite direction and 130 m long crosses the first train in a time **[CPMT 1996; Kerala PET 2002]**
 (a) 6 s (b) 36 s (c) 38 s (d) None of these

11. A police jeep is chasing with, velocity of 45 km/h a thief in another jeep moving with velocity 153 km/h . Police fires a bullet with muzzle velocity of 180 m/s . The velocity it will strike the car of the thief is **[BHU 2003; CPMT 2004]**
 (a) 150 m/s (b) 27 m/s (c) 450 m/s (d) 250 m/s
12. A boat is sent across a river with a velocity of 8 km/hr . If the resultant velocity of boat is 10 km/hr , then velocity of the river is : **[Pb. PET 2004]**
 (a) 10 km/hr (b) 8 km/hr (c) 6 km/hr (d) 4 km/hr
13. A river is flowing from W to E with a speed of 5 m/min . A man can swim in still water with a velocity 10 m/min . In which direction should the man swim so as to take the shortest possible path to go to the south. **[BHU 2005]**
 (a) 30° with downstream (b) 60° with downstream
 (c) 120° with downstream (d) South
14. An express train is moving with a velocity v_1 . Its driver finds another train is moving on the same track in the same direction with velocity v_2 . To escape collision, driver applies a retardation a on the train. the minimum time of escaping collision will be **[RPET 2002]**
 (a) $t = \frac{v_1 - v_2}{a}$ (b) $t_1 = \frac{v_1^2 - v_2^2}{2}$ (c) None (d) Both
15. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A, is moving Northwards with a speed of 10 km h^{-1} . The time after which the distance between them becomes shortest, is : **[AIPMT 2015]**
 (a) $10\sqrt{2}h$ (b) $0 h$ (c) $5h$ (d) $5\sqrt{2}h$

Motion Under Gravity

1. A ball thrown vertically upwards with an initial velocity of 1.4 m/s . The total displacement of the ball is **[Manipal 2010]**
 (a) 22.4 cm (b) zero (c) 44.8 m (d) 33.6 m
2. From the top of tower, a particle is thrown vertically downwards with a velocity of 10 m/s . The ratio of distances covered by it in the 3^{rd} and 2^{nd} second of its motion is (Take $g = 10 \text{ m/s}^2$) **[PMET 2010]**
 (a) $5 : 7$ (b) $7 : 5$ (c) $3 : 6$ (d) $6 : 3$
3. A stone falls freely from rest and the total distance covered by it in the last second of its motion equals the distance covered by it in the first three seconds of its motion. The stone remains in the air for **[WB JEE 2010]**
 (a) 6 s (b) 5 s (c) 7 s (d) 4 s

4. A body falls freely from the top of a tower. It covers 36% of the total height in the last second before striking the ground level, the height of the tower is **[Haryana PMT 2010]**
 (a) 50 m (b) 75 m (c) 100 m (d) 125 m
5. A body A is thrown up vertically from the ground with a velocity v_0 and another body B is simultaneously dropped from a height H. They meet at a height $\frac{H}{2}$, if v_0 is equal to **[AMU 2010]**
 (a) $\sqrt{2gH}$ (b) \sqrt{gH} (c) $\frac{1}{2}\sqrt{gH}$ (d) $\sqrt{\frac{2g}{H}}$
6. A stone is thrown vertically upwards. When the stone is at a height equal to half of its maximum height, its speed will be 10 m/s, then the maximum height attained by the stone is (Take $g = 10 \text{ m/s}^2$) **[PMET 2009]**
 (a) 5 m (b) 150 m (c) 20 m (d) 10 m
7. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2} . He reaches the ground with a speed of 3 ms^{-1} . At what height, did he bail out? **[AIIMS 2008]**
 (a) 91 m (b) 182 m (c) 293 m (d) 111 m
8. A body is projected vertically upwards. The times corresponding to height h while ascending and while descending are t_1 and t_2 respectively. Then the velocity of projection is (g is acceleration due to gravity) **[KCET 2008]**
 (a) $\frac{g\sqrt{t_1 t_2}}{4}$ (b) $\frac{gt_1 t_2}{t_1 + t_2}$ (c) $\frac{g\sqrt{t_1 t_2}}{2}$ (d) $\frac{g(t_1 + t_2)}{2}$
9. A man throws balls with the same speed vertically upwards one after the other at an interval of 2s. What should be the speed of the throw so that more than two balls are in the sky at any time? Given $g = 9.8 \text{ ms}^{-2}$ **[AFMC 2007]**
 (a) Any speed less than 19.6 ms^{-1} (b) Only with speed 19.6 ms^{-1}
 (c) More than 19.6 ms^{-1} (d) At least 9.8 ms^{-1}
10. Two spheres of same size, one of mass 2 kg and another of mass 4 kg, are dropped simultaneously from the top of Qutab Minar (height = 72 m). When they are 1 m above the ground, the two spheres have the same **[AIIMS 2006]**
 (a) momentum (b) kinetic energy (c) potential energy (d) acceleration
11. A ball is projected upwards from a height h above the surface of the earth with velocity v. The time at which the ball strikes the ground is **[Haryana PMT 2006]**
 (a) $\frac{v}{g} + \frac{2hg}{\sqrt{2}}$ (b) $\frac{v}{g} \left[1 - \sqrt{1 + \frac{2h}{g}} \right]$ (c) $\frac{v}{g} \left[1 + \sqrt{1 + \frac{2gh}{v^2}} \right]$ (d) $\frac{v}{g} \left[1 + \sqrt{v^2 + \frac{2g}{h}} \right]$

12. A body is released from the top of a tower of height h . It takes t sec to reach the ground. Where will be the ball after time $t/2$ sec **[NCERT 1981; MP PMT 2004]**
 (a) At $h/2$ from the ground
 (b) At $h/4$ from the ground
 (c) Depends upon mass and volume of the body
 (d) At $3h/4$ from the ground
13. A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity $3u$. The height of the tower is **[EAMCET 1983; RPET 2003]**
 (a) $3u^2/g$ (b) $4u^2/g$ (c) $6u^2/g$ (d) $9u^2/g$
14. A body falling from a high Minaret travels 40 meters in the last 2 seconds of its fall to ground. Height of Minaret in meters is (take $g = 10 \text{ m/s}^2$) **[MP PMT 2002]**
 (a) 60 (b) 45 (c) 80 (d) 50
15. A body falls from a height $h = 200 \text{ m}$ (at New Delhi). The ratio of distance travelled in each 2 sec during $t = 0$ to $t = 6$ second of the journey is **[BHU 2003; CPMT 2004]**
 (a) 1 : 4 : 9 (b) 1 : 2 : 4 (c) 1 : 3 : 5 (d) 1 : 2 : 3
16. A man drops a ball downside from the roof of a tower of height 400 meters. At the same time another ball is thrown upside with a velocity 50 meter/sec. from the surface of the tower, then they will meet at which height from the surface of the tower **[CPMT 2003]**
 (a) 100 meters (b) 320 meters (c) 80 meters (d) 240 meters
17. Two balls are dropped from heights h and $2h$ respectively from the earth surface. The ratio of time of these balls to reach the earth is **[CPMT 2003]**
 (a) $1:\sqrt{2}$ (b) $\sqrt{2}:1$ (c) 2 : 1 (d) 1 : 4
18. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B **[CBSE PMT 2003]**
 (a) $18m$ (b) $6m$ (c) $\frac{2}{3}m$ (d) $\frac{2}{9}m$
19. If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is ($g = 10 \text{ m/s}^2$) **[MP PMT 2003]**
 (a) 11.25 m (b) 16.2 m (c) 24.5 m (d) 7.62 m
20. A balloon is rising vertically up with a velocity of 29 ms^{-1} . A stone is dropped from it and it reaches the ground in 10 seconds. The height of the balloon when the stone was dropped from it is ($g = 9.8 \text{ ms}^{-2}$) **[KCET 2004]**
 (a) 100 m (b) 200 m (c) 400 m (d) 150 m

21. If a freely falling body travels in the last second a distance equal to the distance travelled by it in the first three second, the time of the travel is **[Pb. PMT 2004; MH CET 2003]**
 (a) 6 sec (b) 5 sec (c) 4 sec (d) 3 sec
22. A body is thrown vertically upwards with velocity u . The distance travelled by it in the fifth and the sixth seconds are equal. The velocity u is given by ($g = 9.8 \text{ m/s}^2$) **[UPSEAT 2004]**
 (a) 24.5 m/s (b) 49.0 m/s (c) 73.5 m/s (d) 98.0 m/s
23. A body, thrown upwards with some velocity reaches the maximum height of 50 m. Another body with double the mass thrown up with double the initial velocity will reach a maximum height of **[BHU 2004]**
 (a) 100 m (b) 200 m (c) 300 m (d) 400 m
24. From the top of a tower two stones, whose masses are in the ratio 1 : 2 are thrown one straight up with an initial speed u and the second straight down with the same speed u . Then, neglecting air resistance **[KCET 2005]**
 (a) The heavier stone hits the ground with a higher speed
 (b) The lighter stone hits the ground with a higher speed
 (c) Both the stones will have the same speed when they hit the ground.
 (d) The speed can't be determined with the given data.
25. When a ball is thrown up vertically with velocity V_0 , it reaches a maximum height of ' h '. If one wishes to triple the maximum height then the ball should be thrown with velocity **[AIIMS 2005]**
 (a) $\sqrt{3}V_0$ (b) $3V_0$ (c) $9V_0$ (d) $3/2V_0$
26. A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is **[CPMT 1983; Kerala PMT 2002]**
 (a) 4.9m (b) 9.8m (c) 19.6m (d) 24.5m
27. Time taken by an object falling from rest to cover the height of h_1 and h_2 is respectively t_1 and t_2 then the ratio of t_1 to t_2 is **[RPMT 1999; RPET 2002]**
 (a) $h_1 : h_2$ (b) $\sqrt{h_1} : \sqrt{h_2}$ (c) $h_1 : 2h_2$ (d) $2h_1 : h_2$
28. From the top of a tower, a particle is thrown vertically downwards with a velocity of 10 m/s. The ratio of the distances, covered by it in the 3rd and 2nd seconds of the motion is (Take $g = 10 \text{ m/s}^2$) **[AIIMS 2000; CBSE PMT 2002]**
 (a) 5 : 7 (b) 7 : 5 (c) 3 : 6 (d) 6 : 3

29. A stone dropped from a building of height h and it reaches after t seconds on earth. From the same building if two stones are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively, then

[CPMT 1997; KCET 2002]

- (a) $t = t_1 - t_2$ (b) $t = \frac{t_1 + t_2}{2}$ (c) $t = \sqrt{t_1 t_2}$ (d) $t = t_1^2 t_2^2$

30. If a ball is thrown vertically upwards with speed u , the distance covered during the last t seconds of its ascent is

[CBSE PMT 2003]

- (a) $\frac{1}{2}gt^2$ (b) $ut - \frac{1}{2}gt^2$ (c) $(u - gt)t$ (d) utg

Non-Uniform Motion

1. The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration of the particle will be zero at time t equal to

[CMC 2010]

- (a) $\frac{2a}{3b}$ (b) $\frac{1}{b}$ (c) $\frac{a}{3b}$ (d) c

2. A body starts from rest with an uniform acceleration. If its velocity after n second is v , then its displacement in the last 2 s is

[WB JEE 2010]

- (a) $\frac{2v(n+1)}{n}$ (b) $\frac{v(n+1)}{n}$ (c) $\frac{v(n-1)}{n}$ (d) $\frac{2v(n-1)}{n}$

3. A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1} while passing through a distance 135 m in t second. The value of t is

[CBSE AIPMT 2008]

- (a) 10 (b) 1.8 (c) 12 (d) 9

4. A car, starting from rest, accelerates at the rate f through a distance S , then continues at constant speed for time t and then decelerates as the rate $f/2$ to come to rest. If the total distance travelled is $15S$, then

[AIIMS 2008]

- (a) $S = ft$ (b) $S = \frac{1}{6}ft^2$ (c) $S = \frac{1}{72}ft^2$ (d) $S = \frac{1}{4}ft^2$

5. A particle moving in a straight line covers half the distance with speed of 3 ms^{-1} . The other half of the distance is covered in two equal time intervals with speed of 4.5 ms^{-1} and 7.5 ms^{-1} respectively. The average speed of the particle during this motion is

[Kerala CEE 2008]

- (a) 4.0 ms^{-1} (b) 5.0 ms^{-1} (c) 5.5 ms^{-1} (d) 4.8 ms^{-1}

6. A car moving with a speed of 50 kmh^{-1} , can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 kmh^{-1} , the minimum stopping distance is

[MP PMT 2008]

- (a) 12 m (b) 18 m (c) 24 m (d) 6 m

7. The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by **[RPMT 2008]**
 (a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$ (c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$
8. The position x of a particle with respect to time t along X-axis is given by $x = 9t^2 - t^3$ where x is in metre and t in second. What will be the position of this particle when it achieves maximum speed along the + X direction? **[CBSE AIPMT 2007]**
 (a) 32 m (b) 54 m (c) 81 m (d) 24 m
9. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle, makes with the X-axis an angle of **[AIPMT 2007]**
 (a) 30° (b) 45° (c) 60° (d) 0°
10. A particle moving along X-axis has acceleration f , at time t , given by $f = f_0 \left(1 - \frac{t}{T}\right)$, where f_0 and T are constant. The particle at $t = 0$ has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the maximum particle's velocity (v_x) is **[CBSE AIPMT 2007]**
 (a) $f_0 T$ (b) $f_0 T^2$ (c) $f_0 T^2$ (d) $\frac{1}{2} f_0 T$
11. The acceleration of a particle is increasing linearly with time t as bt . The particle starts from the origin with an initial velocity v_0 . The distance travelled by the particle in time t will be **[AMU 2007]**
 (a) $v_0 t + \frac{1}{3} bt^2$ (b) $v_0 t + \frac{1}{3} bt^3$ (c) $v_0 t + \frac{1}{6} bt^3$ (d) $v_0 t + \frac{1}{2} bt^2$
12. A ball is projected horizontally with a velocity of 4 ms^{-1} from the top of a tower. The velocity of the ball after 0.7s is (Take $g = 10 \text{ ms}^{-2}$) **[BHU 2007]**
 (a) 1 ms^{-1} (b) 10 ms^{-1} (c) 8 ms^{-1} (d) 3 ms^{-1}
13. When a bullet is fired a target, its velocity decreases by half after penetrating 30 cm into it. The additional thickness it will penetrate before coming to rest is **[Kereala CEE 2007]**
 (a) 30 cm (b) 40 cm (c) 10 cm (d) 50 cm
14. If acceleration of a particle at any time is given by $a = 2t + 5$
 Calculate the velocity after 5 s, if it starts from rest **[Haryana PMT 2007]**
 (a) 50 ms^{-1} (b) 25 ms^{-1}
 (c) 100 ms^{-1} (d) 75 ms^{-1}

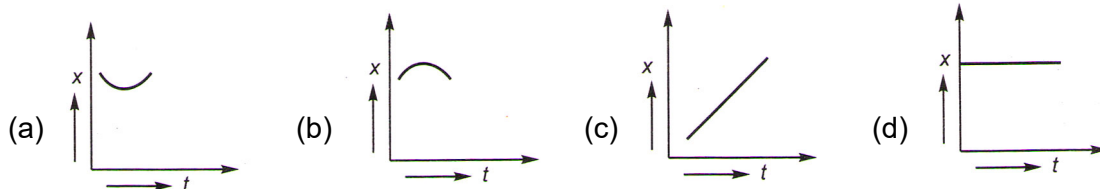
15. A particle moving with a uniform acceleration travels 24 m and 64 m in the first two consecutive interval of 4 s each. Its initial velocity will be
(a) 5 ms^{-1} (b) 3 ms^{-1} (c) 1 ms^{-1} (d) 4 ms^{-1}
16. A body is moving with uniform acceleration covers 200 m in the first 2 s and 220 m in the next 4 s. Find the velocity in ms^{-1} after 7 s. **[J & K CET 2007]**
(a) 10 (b) 15 (c) 20 (d) 30
17. The velocity of a particle at an instant is 10 ms^{-1} . After 3 s its velocity will become 16 ms^{-1} . The velocity at 2 s, before the given instant would have been **[AMU 2006]**
(a) 6 ms^{-1} (b) 4 ms^{-1} (c) 2 ms^{-1} (d) 1 ms^{-1}
18. A car accelerates from rest at constant rate for first 10 s and covers a distance x. It covers a distance y in next 10 s at the same acceleration. Which of the following is true? **[Punjab PMET 2006]**
(a) $x = 3y$ (b) $y = 3x$ (c) $x = y$ (d) $y = 2x$
19. A metro train starts from rest and in 5 s achieves a velocity of 108 kmh^{-1} . After that it moves with constant velocity for some time and comes to rest after travelling 45 m with uniform retardation. If total distance travelled is 395 m, find total time of travelling. **[DUMET 2006]**
(a) 12.2 s (b) 15.3 s (c) 9 s (d) 17.2 s
20. If relation between distance and time is $s = a + bt + ct^2$ find initial velocity and acceleration **[MP PMT 2006]**
(a) $b + 2ct, 2c$ (b) $b, 2c$ (c) $2c, b$ (d) $b + 2c, 2c$
21. The displacement of a particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively **[CPMT 1999, 2003]**
(a) $b, -4d$ (b) $-b, 2c$ (c) $b, 2c$ (d) $2c, -4d$
22. A particle starts from rest, accelerates at 2 m/s^2 for 10s and then goes for constant speed for 30s and then decelerates at 4 m/s^2 till it stops. What is the distance travelled by it **[AIIMS 2002; DCE 2003]**
(a) 750 m (b) 800 m (c) 700 m (d) 850 m
23. A car, moving with a speed of 50 km/hr , can be stopped by brakes after at least 6m. If the same car is moving at a speed of 100 km/hr , the minimum stopping distance is
(a) 6m (b) 12m (c) 18m (d) 24m
24. If a train travelling at 72 kmph is to be brought to rest in a distance of 200 metres, then its retardation should be **[SCRA 1998; MP PMT 2004]**
(a) 20 ms^{-2} (b) 10 ms^{-2} (c) 2 ms^{-2} (d) 1 ms^{-2}

25. The relation between time and distance is $t = \alpha x^2 + \beta x$, where α and β are constants. The retardation is **[NCERT 1982; AIEEE 2005]**
 (a) $2\alpha v^3$ (b) $2\beta v^3$ (c) $2\alpha\beta v^3$ (d) $2\beta^2 v^3$
26. If the velocity of a particle is given by $v = (180 - 16x)^{1/2}$ m/s, then its acceleration will be **[J & K CET 2004]**
 (a) Zero (b) 8 m/s^2 (c) -8 m/s^2 (d) 4 m/s^2
27. Starting from rest, acceleration of a particle is $a = 2(t - 1)$. The velocity of the particle at $t = 5$ s is **[RPET 2002]**
 (a) 15 m/sec (b) 25 m/sec (c) 5 m/sec (d) None of these
28. The displacement x of a particle varies with time t , $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will **[CBSE PMT 2005]**
 (a) Go on decreasing with time (b) Be independent of α and β
 (c) Drop to zero when $\alpha = \beta$ (d) Go on increasing with time
29. A particle moves along x-axis as $x = 4(t - 2) + a(t - 2)^2$. Which of the following is true? **[J&K CET 2005]**
 (a) The initial velocity of particle is 4 (b) The acceleration of particle is $2a$
 (c) The particle is at origin at $t = 0$ (d) None of these
30. What determines the nature of the path followed by the particle **[AFMC 2005]**
 (a) Speed (b) Velocity (c) Acceleration (d) None of these
31. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{2n}$, where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x , is given by : **[AIPMT 2015]**
 (a) $-2n\beta^2 e^{-4n+1}$ (b) $-2n\beta^2 x^{-2n-1}$ (c) $2n\beta^2 x^{4n-1}$ (d) $-2\beta^2 x^{-2n+1}$
32. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = at + bt^2$ and $x_Q(t) = ft - t^2$. At what time do the cars have the same velocity? **[NEET 2016]**
 (a) $\frac{a-f}{1+b}$ (b) $\frac{a+f}{2(b-1)}$ (c) $\frac{a+f}{2(1+b)}$ (d) $\frac{f-a}{2(1+b)}$
33. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1s and 2s is **[NEET 2016]**
 (a) $3A + 7B$ (b) $\frac{3}{2}A + \frac{7}{3}B$ (c) $\frac{A}{2} + \frac{B}{3}$ (d) $\frac{3}{2}A + 4B$

Graphical Questions

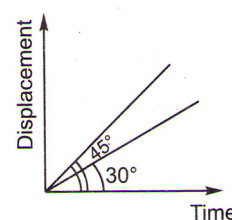
1. The area under velocity-time for a particle in a given interval of time represents **[Kerala CEE 2011]**
- (a) velocity (b) acceleration (c) work done (d) momentum
- (e) displacement

2. Position-time graph for motion with zero acceleration is **[J&K CET 2011]**



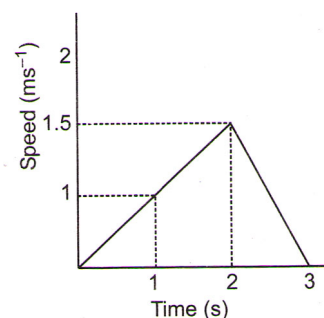
3. The displacement-time graph of two moving particles make angles of 30° and 45° with the X-axis. The ratio of their velocities is **[KCET 2011]**

- (a) $\sqrt{3} : 2$ (b) $1 : 1$
- (c) $1 : 2$ (d) $1 : \sqrt{3}$



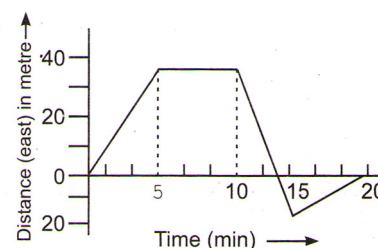
4. The speed-time graph of a particle moving along a solid curve is shown below. The distance traversed by the particle from $t = 0$ s to $t = 3$ s is **[DUMET 2011]**

- (a) $\frac{9}{2}$ m (b) $\frac{9}{4}$ m
- (c) $\frac{9}{3}$ m (d) $\frac{9}{5}$ m



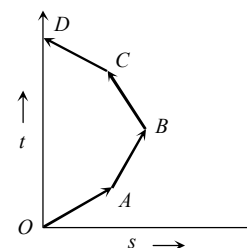
5. A boy begins to walk eastward along a street in front of his house and the graph of his displacement from home is shown in the following figure. His average speed for the whole time interval is equal to **[BHU 2006]**

- (a) 8 m min^{-1} (b) 6 m min^{-1}
- (c) $\frac{8}{3} \text{ m min}^{-1}$ (d) 2 m min^{-1}



6. Which of the following options is correct for the object having a straight line motion represented by the following graph **[DCE 2004]**

- (a) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity.
- (b) Velocity of the object increases uniformly
- (c) Average velocity is zero
- (d) The graph shown is impossible



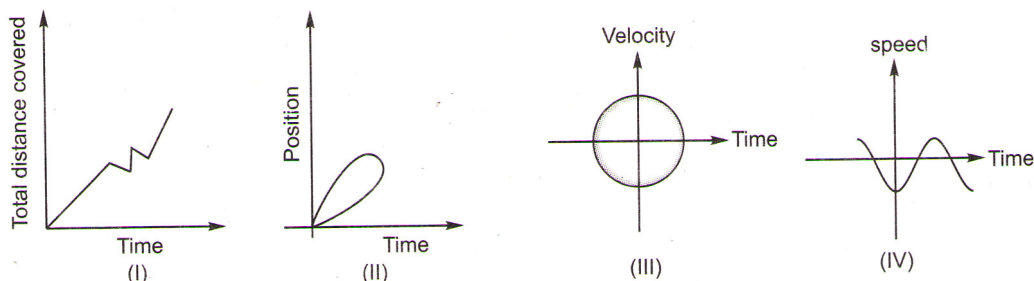
7. The displacement-time graph for two particles A and B are straight lines inclined at angles of 30° and 60° with the time axis. The ratio of velocities of $V_A : V_B$ is

[CPMT 1990; Pb. PET 2003]

- (a) 1:2 (b) $1:\sqrt{3}$ (c) $\sqrt{3}:1$ (d) 1:3

8. Which of the following graphs cannot possibly represent one dimensional motion of a particle?

[AMU 2010]



- (a) I and II (b) II and III (c) II and IV (d) All the above four

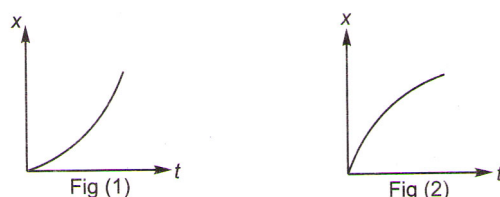
9. Area under the velocity-time curve over a given interval of time represents?

[Kerala CEE 2009]

- (a) Acceleration (b) Momentum (c) Velocity (d) Displacement
(e) Kinetic energy

10. Figures (a) and (b) show the displacement-time graphs of two particles moving along the x -axis. We can say that

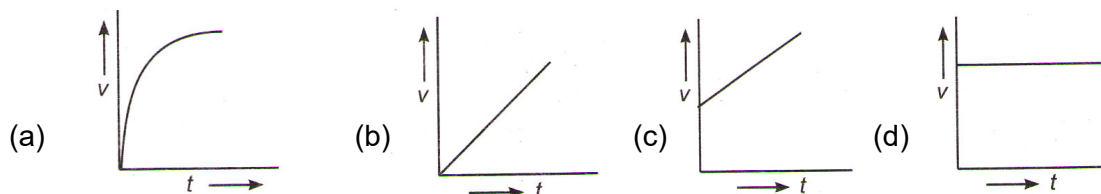
[UP CPMT, Punjab PMET 2009]



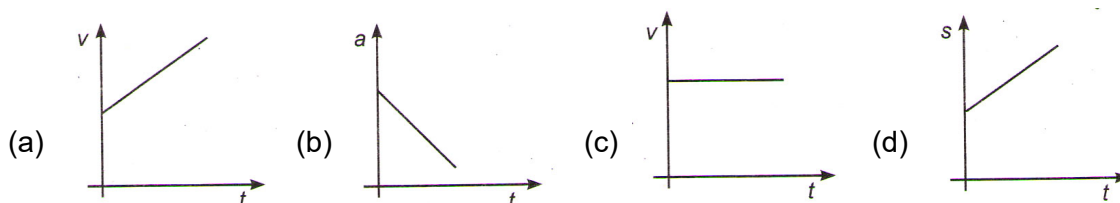
- (a) both the particles are having an uniformly accelerated motion
(b) both the particles are having an uniformly retarded motion
(c) particle (a) is having on uniformly accelerated motion which particle (b) is having an uniformly retarded motion
(d) particle (a) is having and uniformly retarded motion while particle (b) is having an uniformly accelerated motion

11. A body starts from rest and moves with uniform acceleration. Which of the following graphs represent its motion?

[J&K CET 2008]

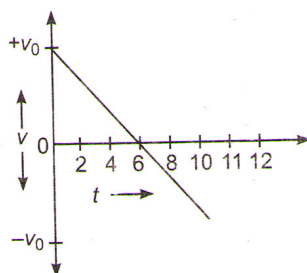


12. A body moves with uniform acceleration, then which of the following graphs is correct? **[DUMET 2007]**



13. Consider the given velocity-time graph

[Kerala CEE 2006]



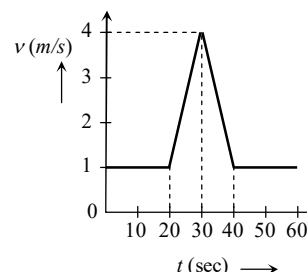
It represents the motion of

- (a) a projectile projected vertically upward, from a point
 (b) an electron in the hydrogen atom
 (c) a car with constant acceleration along a straight road
 (d) a bullet fired horizontally from the top of a tower
 (e) an object in the positive direction with decreasing speed
14. Velocity-time (v - t) graph for a moving object is shown in the figure. Total displacement of the object during the time interval when there is non-zero acceleration and retardation is

[Kerala PMT

2005]

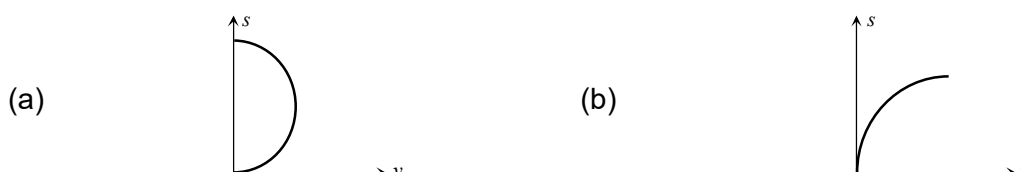
- (a) 60 m (b) 50 m
 (c) 30 m (d) 40 m



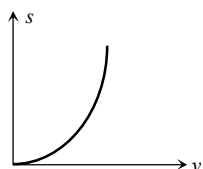
15. Velocity-time curve for a body projected vertically upwards is **[AIIMS 1999; BHU 2004]**
 (a) Parabola (b) Ellipse (c) Hyperbola (d) Straight line

16. An object is moving with a uniform acceleration which is parallel to its instantaneous direction of motion. The displacement (s) – velocity (v) graph of this object is

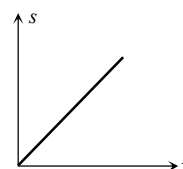
[SCRA 1998; Orissa PMT 2004]



(c)



(d)



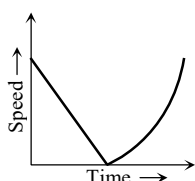
17. The area under acceleration-time graph gives

[Kerala PET 2005]

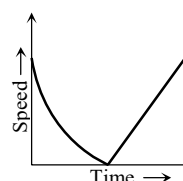
- (a) Distance travelled (b) Change in acceleration
(c) Force acting (d) Change in velocity

18. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its height if the air resistance is not ignored [AIIMS 2003]

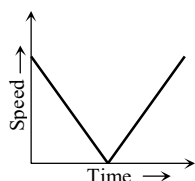
(a)



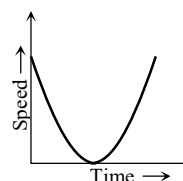
(b)



(c)



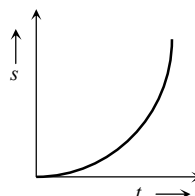
(d)



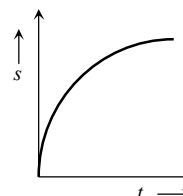
19. Which graph represents the uniform acceleration

[DCE 2003]

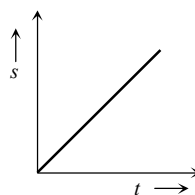
(a)



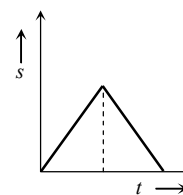
(b)



(c)



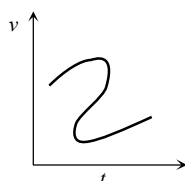
(d)



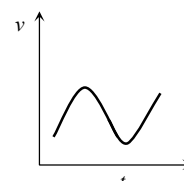
20. Which of the following velocity-time graphs shows a realistic situation for a body in motion

[AIIMS 2004]

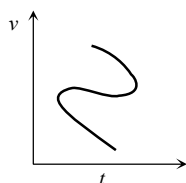
(a)



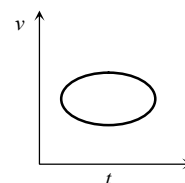
(b)



(c)

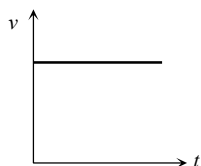


(d)

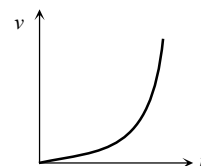


21. Which of the following velocity-time graphs represent uniform motion **[Kerala PMT 2004]**

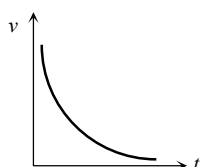
(a)



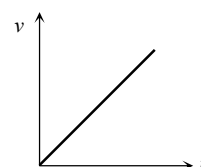
(b)



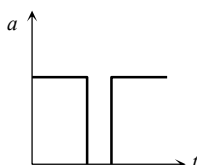
(c)



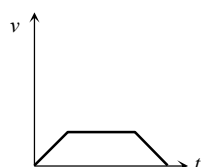
(d)



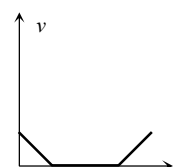
22. Acceleration-time graph of a body is shown. The corresponding velocity-time graph of the same body is **[DPMT 2004]**



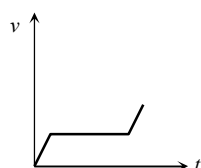
(a)



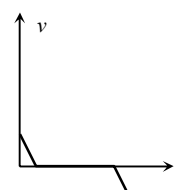
(b)



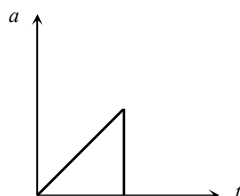
(c)



(d)

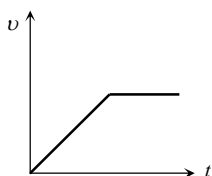


23. The acceleration-time graph of a body is shown below

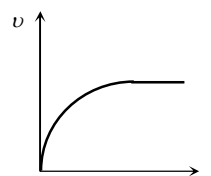


The most probable velocity-time graph of the body is

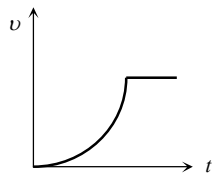
(a)



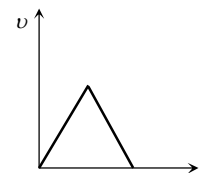
(b)



(c)

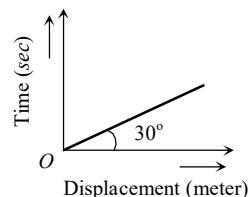


(d)



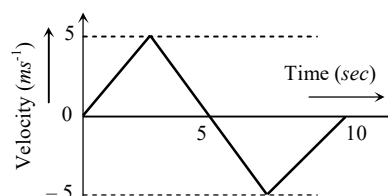
24. From the following displacement-time graph find out the velocity of a moving body

- (a) $\frac{1}{\sqrt{3}}$ m/s (b) 3 m/s
(c) $\sqrt{3}$ m/s (d) $\frac{1}{3}$



25. The $v-t$ plot of a moving object is shown in the figure. The average velocity of the object during the first 10 seconds is

- (a) 0 (b) 2.5 ms^{-1}
(c) 5 ms^{-1} (d) 2 ms^{-1}



Horizontal Projectile Motion

- A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following **[NCERT 1972; AFMC 1996; BHU 2000]**
(a) Straight path (b) Circular path (c) Parabolic path (d) Hyperbolic path
- An aeroplane is flying horizontally with a velocity of 600 km/h at a height of 1960 m. When it is vertically at a point A on the ground, a bomb is released from it. The bomb strikes the ground at point B. The distance AB is **[CPMT 1996; JIPMER 2001, 02]**
(a) 1200 m (b) 0.33 km (c) 3.33 km (d) 33 km
- A particle (a) is dropped from a height and another particle (b) is thrown in horizontal direction with speed of 5 m/sec from the same height. The correct statement is **[CBSE PMT 2002; Orissa JEE 2003]**
(a) Both particles will reach at ground simultaneously
(b) Both particles will reach at ground with same speed
(c) Particle (a) will reach at ground first with respect to particle (b)
(d) Particle (b) will reach at ground first with respect to particle (a)
- A particle moves in a plane with constant acceleration in a direction different from the initial velocity. The path of the particle will be **[MP PMT 2004; CPMT 1982]**
(a) A straight line (b) An arc of a circle (c) A parabola (d) An ellipse
- At the height 80 m, an aeroplane is moving with 150 m/s. A bomb is dropped from it so as to hit a target. At what distance from the target should the bomb be dropped (given $g = 10 \text{ m/s}^2$) **[BCECE 2004]**
(a) 605.3 m (b) 600 m (c) 80 m (d) 230 m

6. A bomber plane moves horizontally with a speed of 500 m/s and a bomb released from it, strikes the ground in 10 sec . Angle at which it strikes the ground will be ($g = 10 \text{ m/s}^2$)

[MH CET 2003]

- (a) $\tan^{-1}\left(\frac{1}{5}\right)$ (b) $\tan\left(\frac{1}{5}\right)$ (c) $\tan^{-1}(1)$ (d) $\tan^{-1}(5)$

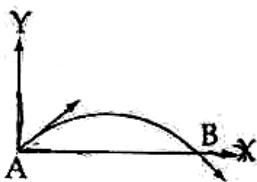
Oblique Projectile Motion

1. A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is: (given $g = 9.8 \text{ ms}^{-2}$)

[NEET 2014]

- (a) 16.3 (b) 110.8 (c) 3.5 (d) 5.9

2. The velocity of a projectile at the initial point A is $(2\hat{i} + 3\hat{j}) \text{ m/s}$. Its velocity (in m/s) at point B is :



[NEET 2013]

- (a) $2\hat{i} + 3\hat{j}$ (b) $-2\hat{i} - 3\hat{j}$ (c) $-2\hat{i} + 3\hat{j}$ (d) $2\hat{i} - 3\hat{j}$

3. A ball is projected with kinetic energy E at an angle of 45° to the horizontal. At the highest point during its flight, its kinetic energy will be

[MP PMT 1994; CBSE PMT 1997, 2001; AIEEE 2002; Pb. PMT 2004; Orissa PMT 2004]

- (a) Zero (b) $\frac{E}{2}$ (c) $\frac{E}{\sqrt{2}}$ (d) E

4. A ball thrown by one player reaches the other in 2 sec . the maximum height attained by the ball above the point of projection will be about

[Pb. PMT 2002]

- (a) 10 m (b) 7.5 m (c) 5 m (d) 2.5 m

5. In a projectile motion, velocity at maximum height is [AIEEE 2002]

- (a) $\frac{u \cos \theta}{2}$ (b) $u \cos \theta$ (c) $\frac{u \sin \theta}{2}$ (d) None of these

6. For a given velocity, a projectile has the same range R for two angles of projection if t_1 and t_2 are the times of flight in the two cases then [KCET 2003; AIEEE 2004]

- (a) $t_1 t_2 \propto R^2$ (b) $t_1 t_2 \propto R$ (c) $t_1 t_2 \propto \frac{1}{R}$ (d) $t_1 t_2 \propto \frac{1}{R^2}$

7. A cricketer can throw a ball to a maximum horizontal distance of 100 m . With the same effort, he throws the ball vertically upwards. The maximum height attained by the ball is

[UPSEAT 2002]

- (a) 100 m (b) 80 m (c) 60 m (d) 50 m

8. Neglecting the air resistance, the time of flight of a projectile is determined by
[J & K CET 2004]
(a) U_{vertical} (b) $U_{\text{horizontal}}$
(c) $U = U_{\text{vertical}}^2 + U_{\text{horizontal}}^2$ (d) $U = (U_{\text{vertical}}^2 + U_{\text{horizontal}}^2)^{1/2}$
9. A ball is thrown from a point with a speed v_0 at an angle of projection θ . From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection
[AIEEE 2004]
(a) Yes, 60° (b) Yes, 30° (c) No (d) Yes, 45°
10. A stone is thrown at an angle θ to the horizontal reaches a maximum height H . Then the time of flight of stone will be
[BCECE 2004]
(a) $\sqrt{\frac{2H}{g}}$ (b) $2\sqrt{\frac{2H}{g}}$ (c) $\frac{2\sqrt{2H\sin\theta}}{g}$ (d) $\frac{\sqrt{2H\sin\theta}}{g}$
11. The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projection will be
[J & K CET 2004; DPMT 2003]
(a) 45° (b) 60° (c) 90° (d) 30°
12. A ball is projected upwards from the top of tower with a velocity 50 ms^{-1} making an angle 30° with the horizontal. The height of tower is 70 m . After how many seconds from the instant of throwing will the ball reach the ground
[DPMT 2004]
(a) 2 s (b) 5 s (c) 7 s (d) 9 s
13. Two bodies are thrown up at angles of 45° and 60° , respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is
[DPMT 2005]
(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$
14. At what point of a projectile motion acceleration and velocity are perpendicular to each other
[Orissa JEE 2005]
(a) At the point of projection
(b) At the point of drop
(c) At the topmost point
(d) Any where in between the point of projection and topmost point
15. The maximum horizontal range of a projectile is 400 m . The maximum value of height attained by it will be
[AFMC 2005]
(a) 100 m (b) 200 m (c) 400 m (d) 800 m

Questions asked in 2017 to 2021

1. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1 . On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2 . The time taken by her to walk up on the moving escalator will be
[NEET 2017]
(a) $\frac{t_1 + t_2}{2}$ (b) $\frac{t_1 t_2}{t_2 - t_1}$ (c) $\frac{t_1 t_2}{t_2 + t_1}$ (d) $t_1 - t_2$

2. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in meters and t in seconds. The acceleration of the particle at $t = 2$ s is:
[NEET 2017]
(a) 0 (b) 5 m/s^2 (c) -4 m/s^2 (d) -8 m/s^2
3. A toy car with charge q moves on a frictionless horizontal plane surface under the influence of a uniform electric field E. Due to the force qE, its velocity increases from 0 to 6 m/s in one second. At that instant, the direction of the field is reversed. The car continues to move for two more seconds under the influence of this field. The average velocity and the average speed of the toy car between 0 to 3 seconds are respectively
[NEET 2018]
(a) 1 m/s, 3.5 m/s (b) 1 m/s, 3 m/s (c) 2 m/s, 4 m/s (d) 1.5 m/s, 3 m/s
4. When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be
[NEET (National) 2019]
(a) $\sqrt{2} : 1$ (b) $1 : \sqrt{3}$ (c) $1 : 2\sqrt{3}$ (d) $1 : \sqrt{2}$
5. A person sitting in the ground floor of a building notices through the window of height 1.5 m, a ball dropped from the roof of the building ion, crosses the window in 0.1 s. What is the velocity of the ball when it is at the topmost point of the window? ($g = 10 \text{ m/s}^2$)
[NEET (Oct.) 2020]
(a) 15.5 m/s (b) 14.5 m/s (c) 4.5 m/s (d) 20 m/s
6. A ball is thrown vertically downward with a velocity of 20 m/s from the top of a tower. It hits the ground after some time with a velocity of 80 m/s. The height of the tower is ($g = 10 \text{ m/s}^2$)
[NEET (Sep.) 2020]
(a) 340 m (b) 320 m (c) 300 m (d) 360 m
7. A car starts from rest and accelerates at 5 m/s^2 . At $t = 4$ s, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at $t = 6$ s? (take, $g = 10 \text{ m/s}^2$)
[NEET 2021]
(a) 20 m/s, 5 m/s^2 (b) 20 m/s, 0
(c) $20\sqrt{2} \text{ m/s}$, 0 (d) $20\sqrt{2} \text{ m/s}$, 10 m/s^2
8. A small block slides down on a smooth inclined plane, starting from rest at time $t = 0$. Let s, be the distance travelled by the block in the interval $t = n-1$ to $t = n$. Then, the ratio $\frac{s_n}{s_{n+1}}$ is
[NEET 2021]
(a) $\frac{2n-1}{2n}$ (b) $\frac{2n-1}{2n+1}$ (c) $\frac{2n+1}{2n-1}$ (d) $\frac{2n}{2n-1}$

9. A particle moving in a circle of radius R with a uniform speed takes a time T to complete one revolution. If this particle were projected with the same speed at an angle θ to the horizontal, the maximum height attained by it equals $4R$. The angle of projection θ is then given by **[NEET 2021]**

(a) $\theta = \cos^{-1} \left(\frac{gT^2}{\pi^2 R} \right)^{\frac{1}{2}}$

(b) $\theta = \cos^{-1} \left(\frac{\pi^2 R}{gT^2} \right)^{\frac{1}{2}}$

(c) $\theta = \sin^{-1} \left(\frac{\pi^2 R}{gT^2} \right)^{\frac{1}{2}}$

(d) $\theta = \sin^{-1} \left(\frac{2gT^2}{\pi^2 R} \right)^{\frac{1}{2}}$

EXERCISE KEY

Level-I

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (a) | 3. (b) | 4. (c) | 5. (c) |
| 6. (c) | 7. (b) | 8. (b) | 9. (b) | 10. (b) |
| 11. (a) | 12. (b) | 13. (b) | 14. (c) | 15. (b) |
| 16. (d) | 17. (c) | 18. (b) | 19. (b) | 20. (b) |
| 21. (d) | 22. (b) | 23. (a) | 24. (c) | 25. (a) |
| 26. (c) | 27. (b) | 28. (c) | 29. (b) | 30. (a) |
| 31. (d) | 32. (d) | 33. (b) | 34. (d) | 35. (a) |
| 36. (c) | 37. (b) | 38. (d) | 39. (a) | 40. (b) |
| 41. (b) | 42. (d) | 43. (c) | 44. (c) | 45. (a) |
| 46. (b) | 47. (d) | 48. (d) | 49. (d) | 50. (a) |
| 51. (b) | 52. (a) | 53. (c) | 54. (c) | 55. (b) |
| 56. (d) | 57. (a) | 58. (c) | 59. (d) | 60. (c) |
| 61. (a) | 62. (c) | 63. (c) | 64. (a) | 65. (c) |
| 66. (a) | 67. (d) | 68. (b) | 69. (b) | 70. (a) |
| 71. (d) | 72. (c) | 73. (c) | | |

Level-II

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (a) | 2. (d) | 3. (c) | 4. (c) | 5. (a) |
| 6. (a) | 7. (d) | 8. (a) | 9. (c) | 10. (c) |
| 11. (a) | 12. (c) | 13. (d) | 14. (c) | 15. (d) |
| 16. (c) | 17. (a) | 18. (d) | 19. (c) | 20. (b) |
| 21. (d) | 22. (a) | 23. (d) | 24. (d) | 25. (b) |
| 26. (d) | 27. (b) | 28. (d) | 29. (b) | 30. (b) |
| 31. (c) | 32. (d) | 33. (c) | 34. (a) | 35. (c) |
| 36. (c) | 37. (d) | 38. (c) | 39. (d) | 40. (a) |
| 41. (d) | 42. (a) | 43. (d) | 44. (a) | 45. (a) |
| 46. (b) | 47. (a) | 48. (c) | 49. (b) | 50. (c) |
| 51. (c) | 52. (a) | 53. (a) | 54. (b) | 55. (b) |
| 56. (b) | 57. (c) | 58. (b) | 59. (b) | 60. (a) |
| 61. (c) | 62. (b) | 63. (b) | 64. (d) | 65. (b) |
| 66. (c) | 67. (a) | 68. (d) | 69. (c) | 70. (a) |
| 71. (d) | 72. (b) | 73. (a) | 74. (d) | 75. (c) |
| 76. (a) | 77. (c) | 78. (b) | | |

Assertion & Reason

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (a) | 2. (a) | 3. (a) | 4. (c) | 5. (c) |
| 6. (d) | 7. (d) | 8. (c) | 9. (a) | 10. (a) |
| 11. (b) | 12. (d) | 13. (c) | 14. (d) | 15. (a) |
| 16. (c) | 17. (d) | 18. (b) | 19. (d) | 20. (b) |
| 21. (d) | 22. (d) | 23. (b) | 24. (b) | 25. (b) |

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|---------|---------|---------|---------|---------|
| 26. (a) | 27. (b) | 28. (a) | 29. (a) | 30. (a) |
| 31. (d) | | | | |

Previous Years' Questions

Kinematic Parameters

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (c) | 3. (c) | 4. (a) | 5. (b) |
| 6. (b) | 7. (c) | 8. (a) | 9. (c) | 10. (b) |
| 11. (b) | 12. (c) | 13. (d) | 14. (d) | 15. (b) |
| 16. (d) | 17. (b) | 18. (c) | 19. (d) | 20. (d) |
| 21. (c) | 22. (b) | 23. (d) | 24. (d) | 25. (d) |
| 26. (c) | 27. (a) | 28. (b) | 29. (c) | 30. (d) |
| 31. (b) | 32. (a) | 33. (d) | 34. (b) | 35. (c) |
| 36. (a) | 37. (d) | 38. (a) | 39. (c) | 40. (d) |
| 41. (d) | 42. (d) | 43. (b) | 44. (c) | 45. (c) |
| 46. (d) | 47. (b) | 48. (d) | 49. (b) | 50. (c) |
| 51. (a) | 52. (d) | 53. (b) | 54. (d) | 55. (d) |
| 56. (a) | 57. (a) | 58. (b) | 59. (b) | 60. (d) |
| 61. (a) | 62. (b) | 63. (b) | 64. (d) | 65. (d) |
| 66. (a) | 67. (b) | 68. (b) | 69. (c) | |

Relative Motion

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (b) | 3. (b) | 4. (c) | 5. (c) |
| 6. (a) | 7. (c) | 8. (b) | 9. (d) | 10. (d) |
| 11. (a) | 12. (c) | 13. (c) | 14. (a) | 15. (c) |

Motion Under Gravity

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (b) | 3. (b) | 4. (d) | 5. (b) |
| 6. (d) | 7. (c) | 8. (d) | 9. (c) | 10. (d) |
| 11. (c) | 12. (d) | 13. (b) | 14. (b) | 15. (c) |
| 16. (c) | 17. (a) | 18. (a) | 19. (a) | 20. (b) |
| 21. (b) | 22. (b) | 23. (b) | 24. (c) | 25. (a) |
| 26. (d) | 27. (b) | 28. (b) | 29. (c) | 30. (a) |

Non-Uniform Motion

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (c) | 2. (d) | 3. (d) | 4. (c) | 5. (a) |
| 6. (c) | 7. (b) | 8. (b) | 9. (c) | 10. (d) |
| 11. (c) | 12. (c) | 13. (c) | 14. (a) | 15. (c) |
| 16. (a) | 17. (a) | 18. (b) | 19. (d) | 20. (b) |
| 21. (c) | 22. (a) | 23. (d) | 24. (d) | 25. (a) |
| 26. (c) | 27. (a) | 28. (d) | 29. (b) | 30. (d) |
| 31. (c) | 32. (d) | 33. (b) | | |

Graphical Questions

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (e) | 2. (c) | 3. (d) | 4. (b) | 5. (b) |
| 6. (c) | 7. (d) | 8. (d) | 9. (d) | 10. (c) |

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|---------|---------|---------|---------|---------|
| 11. (b) | 12. (a) | 13. (a) | 14. (b) | 15. (d) |
| 16. (c) | 17. (d) | 18. (c) | 19. (a) | 20. (b) |
| 21. (a) | 22. (c) | 23. (c) | 24. (c) | 25. (a) |

Horizontal Projectile Motion

- | | | | | |
|--------|--------|--------|--------|--------|
| 1. (c) | 2. (c) | 3. (a) | 4. (c) | 5. (a) |
| 6. (a) | | | | |

Oblique Projectile Motion

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (c) | 2. (d) | 3. (b) | 4. (c) | 5. (b) |
| 6. (b) | 7. (d) | 8. (a) | 9. (a) | 10. (b) |
| 11. (d) | 12. (c) | 13. (c) | 14. (c) | 15. (b) |

Questions asked in 2017 to 2021

- | | | | | |
|--------|--------|--------|--------|--------|
| 1. (c) | 2. (c) | 3. (b) | 4. (b) | 5. (d) |
| 6. (c) | 7. (d) | 8. (b) | 9. (d) | |