Section – A

1. (B)

Given: $x+\frac{1}{x}=7$

We need to find: $x^4+\frac{1}{x^4}$

Step 1. Computer $x^2+\frac{1}{x^2}$.

$$\left(x+\frac{1}{x}\right)^2=7^2 x^2+2+\frac{1}{x^2}=49,$$

So $x^2+\frac{1}{x^2}=49-2=47$

Step 2. Compute $x^4+\frac{1}{x^4}$.

$$\left(x^2+\frac{1}{x^2}\right)^2=47^2 \Rightarrow x^4+2+\frac{1}{x^4}=2209,$$

$$x^4+\frac{1}{x^4}=2209-2=2207.$$

$$x^4+\frac{1}{x^4}=2207$$

2. (D)

Simplify: $(a+b)^3+(a-b)^3+6a\left(a^2-b^2\right)$

Step 1: Expand the cubes.

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$

$$(a-b)^3=a^3-3a^2b+3ab^2-b^3$$

Step 2: Add them together.

$$(a+b)^3+(a-b)^3=\left(a^3+a^3\right)+\left(3a^2b-3a^2b\right)+\left(3ab^2+3ab^2\right)+\left(b^3-b^3\right)=2a^3+6ab^2$$

Step 3: Expand the third term.

$$6a\left(a^2-b^2\right)=6a^3-6ab^2$$

Step 4: Add all parts.

$$\left[(a+b)^3+(a-b)^3\right]+6a\left(a^2-b^2\right)=\left(2a^2+6ab^2\right)+\left(6a^3-6ab^2\right)$$

$$=\left(2a^3+6a^3\right)+\left(6ab^2-6ab^2\right)$$

$$=8a^3$$

3. (C)

Given: $\{12x+13y=29, 13x+12y=21$

We need to find $x+y$

Step 1: Add both equations.

$$(12x+13y)+(13x+12y)=29+21$$

$$25x+25y=50$$

Step 2: Simplify. $x+y=\frac{50}{25}=2$

$x+y=2$

4. (A)

Given the equation $x^2+px+q=0$ and p and q are its roots.

Sum of roots $=p+q=-p$

Product of roots $=pq=q$

From $pq=q$, if $q\neq 0$, we get $p=1$

Substitute $p=1$ in the sum of roots equation:

$1+q=-1$

$\Rightarrow q=-2$

Hence, $p=1, q=-2$

Therefore, the correct option is (A).

5. (C)

We are asked to find the lowest (minimum) value of x^2+4x+2

Step 1: Complete the square.

$x^2+4x+2=\left(x^2+4x+4\right)-4+2=\left(x+2\right)^2-2$

Step 2: The minimum value of $\left(x+2\right)^2$ is 0 when $x=-2$

Step 3: Therefore, the minimum value of the expression is $0-2=-2$

Hence,

Lowest value $=-2atx=-2$.

6. (A)

We are given the ration $a^2:b^2$.

Let the common quantity to be added to each terms be k.

Then the new ratio becomes $\left(a^2+k\right):\left(b^2+k\right)$

And this must be equal to $a:b$ $\frac{a^2+k}{b^2+k}=\frac{a}{b}$

Cross multiply: $b\left(a^2+k\right)=a\left(b^2+k\right)$

Expand $a^2b+bk=ab^2+ak$

Simplify:

$bk-ak=ab^2-a^2b$

$k\left(b-a\right)=ab\left(b-a\right)$

Since $a\neq b$, $k=ab$

Common quantity to be added is ab.

7. (C)

Let the present age of the son be x years. Then, the father's present age is 6x years.

Four years hence:

Son's age $x+4$, Father's age $=6x+4$

According to the question, $6x+4=4\left(x+4\right)$

Simplify: $6x+4=4x+16$

$6x-4x=16-4$

$$2x = 12$$
$$x = 6$$

Therefore, Son’s present age = 6 years

Father’s present age $= 6x = 36$ years

Son and Father are 6years and 36 years old respectively.

Hence, the correct option is (C).

8. (D)

Let the two consecutive integer roots be n and $n+1$ for some integer n.

For the quadratic $x^2 - bx + c = 0$ (here $a = 1$), by Vietes’s relations:

Sum of roots $= n + (n+1) = 2n + 1 = b$,

Product of roots $= n(n+1) = c$

The discriminant is $\Delta = b^2 - 4abc = b^2 - 4c$.

Substitute $b = 2n+1$ and $c = n(n+1)$:

$$\Delta = (2n+1)^2 - 4(n(n+1)) = (4n^2 + 4n + 1) - (4n^2 + 4n) = 1$$

Hence, $b^2 - 4c = 1$

Therefore, the correct option is (D).

9. (C)

Let the quadratic polynomial be $f(x)$.

When $f(x)$ is divided by $(x+2)(x-1)$,

let the remainder be a liner expression $R(x) = ax + b$.

The we can write:

$$f(x) = (x+2)(x-1)Q(x) + (ax+b)$$

Since $f(x)$ leaves remainder 1 when divided by $x+2$,

$f(-2) = 1$

and since it leaves remainder 4 when divided by $x-1$,

$f(1) = 4$

Now, using the expression for $f(s)$:

$f(-2) = a(-2) + b = -2a + b = 1$ (i)

$f(1) = a(1) + b = a + b = 4$ (ii)

Subtract equation (i) from (ii):

$$(a+b) - (-2a+b) = 4 - 1$$
$$3a = 3a \Rightarrow a = 1$$

Substitute a = 1 into (ii):

$$1 + b = 4b = 3$$

Hence, the remainder is $R(x) = ax + b = x + 3$

Remainder $= x + 3$

Therefore, the correct option is (C).

10. (C)

Possible rational roots are factors of the constant term -6 divided by factors of the leading coefficient 2:

$\pm 1, \pm 2, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

Test $x=-1$:

$2(-1)^3+3(-1)^2-5(-1)-6=-2+3+5-6=0$

So, $x=-1$ is a root.

Step 5: Divided by $(x+1)$ using synthetic or long division

$$\begin{array}{r|rrrr} -1 & 2 & 3 & -5 & -6 \\ & & -2 & -1 & 6 \\ \hline & 2 & 1 & -6 & 0 \end{array}$$

The quotient is $2x^2+x-6$.

Step 6: Factorize $2x^2+x-6$

$$2x^2+4x-3x-6=\left(2x^2+4x\right)-(3x+6)$$
$$=2x(x+2)-3(x+2)$$
$$=(x+2)(2x-3)$$

Step 7: Combine all factors.

$$2x^3+3x^2-5x-6=(x+1)(x+2)(2x-3)$$

Factors are $(x+1)(x+2)(2x-3)$

Hence, the correct option is (C)

11. (D)

Let the numerator be x and the denominator be y.

From the first condition: $y=x+2$ (i)

From the second condition: $\frac{x+5}{y}=\frac{x}{y}+1$

Simplify $x+5=x+y$

$y=5$

Substitute y = 5 in equation (i): $5=x+2 \Rightarrow x=3$

Therefore, the fraction is: $\frac{x}{y}=\frac{3}{5}$

Hence, the correct option is (D)

12. (A)

Since 4 is a root of $x^2+px+12=0$, substitute $x=4$:

$4^2+p\cdot 4+12=0 \quad \Rightarrow \quad 16+4p+12=0$

$4p+28=0 \quad \Rightarrow \quad p=-7$

For the equation $x^2+px+q=0$ to have equal roots, its discriminant must be zero:

$\Delta=p^2-4q=0 \quad \Rightarrow q=\frac{p^2}{4}$.

Substitute $p=-7$: $q=\frac{(-7)^2}{4}=\frac{49}{4}$.

Hence the correct option is (A).

13. (C)

Let the zeroes of the polynomial be α and $-\alpha$.
Step 1: Use relationships between coefficients and roots.
Sum of zeroes $=\alpha+(-\alpha)=0$

Product of zeroes $=\alpha\times(-\alpha)=-\alpha^2$

For a quadratic polynomial x^3+ax+b:
Sum of zeroes = – a , Product of zeroes = b
From the given condition:

$$-a=0\Rightarrow a=0,$$
$$b=-\alpha^2$$

Step 2: Write the polynomial.

$$x^2+ax+b=x^2-\alpha^2=(x+\alpha)(x-\alpha).$$

Step 3: Analyze the results. – The polynomial can be expressed as a product of two linear factors, so option (a) is correct. –since $b=-\alpha^2<0,$ the constant term is negative, so option (b) is also correct.
Hence, both(a) and (b) are correct.
Correct option: (c)

14. (A)

Given that the roots are $a-b, a,$ and $a+b$.

Step 1: Sum of roots $(a-b)+a+(a+b)=3a=3$

$\Rightarrow a=1$

Step 2: Sum of product of roots taken two at a time for a cubic $x^3-3x^2+x+1=0$, the coefficient comparison gives
Sum of products of roots two at a time = 1.

So, $(a-b)a+a(a+b)+(a-b)(a+b)=1$

Simplify: $a^2-ab+a^3+ab+a^2-b^2=1$

$3a^2-b^2=1$

Substitute $a=1$: $3(1)^2-b^2=1\Rightarrow 3-b^2=1$

$b^2=2\Rightarrow b=\pm\sqrt{2}$

Step 3: Find $a+b$

$a+b=1\pm\sqrt{2}$

$a+b=1\pm\sqrt{2}$

Correct option (A) $1+\sqrt{2}$ or equivalently $1\pm\sqrt{2}$

15. (A)

Given equation : $x^2-3|x|+2=0$

Let $y=|x|$, where $y\ge 0$ $\Rightarrow y^2-3y+2=0$

$\Rightarrow(y-1)(y-2)=0$

$\Rightarrow y=1$ or $y=2$

Since $|x| = y$: $|x| = 1 \Rightarrow x = \pm 1$

$|x| = 2 \Rightarrow x = \pm 2$

Hence, the roots are $x = -2, -1, 1, 2$

Total number of roots = 4

16. (A)

Since the given equation is an identity in x, all coefficients of powers of x must be zero.

Coefficient of $x^3 : a^2 - 1 = 0$, Coefficient of $x^2 : 0$, Coefficient $x : a - 1 = 0$

Constant term: $a^2 - 4a + 3 = 0$

From $a - 1 = 0$, we get $a = 1$.

Now check for $a = 1$: $a^2 - 1 = 0$ and $a^2 - 4a + 3 = 0$

Both are satisfied $a = 1$

17. (B)

Factorization: $f(x) = (x-1)(x+2)(x^2+1)$

Thus $f(x)$ has two linear factors(x − 1) and (x + 2).

Answer: (B) 2

18. (B)

For a pair linear equations

$$a_1x + b_1y = c_1, a_2x + b_2y = c_2,$$

A unique solution exists if and only if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}.$$

Here, $a_1 = k,\ b_1 = 2,\ a_2 = 3,\ b_2 = 1.$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \neq \frac{2}{1} \Rightarrow k \neq 6$$

For unique solution, $k \neq 6$

19. (D)

$3x + 4y = 14$

Rewriting in slope-intercept from:

$$y = -\frac{3}{4}x + \frac{14}{4}$$

$$\Rightarrow y = -\frac{3}{4}x + \frac{7}{2}$$

This is a straight line with slope $-\frac{3}{4}$ and y-intercept $\frac{7}{2}$

Since the intercept is not zero, the line does not pass through the origin.

It is also not parallel to either the x-axis or the y-axis (because both x and y have nonzero coefficients).

Hence, the correct answer is (D) None of these

20. (A)

Note that $81 = 3^4$. Hence $3^{x+y} = 3^4 \Rightarrow x + y = 4$ and

$81^{x-y} = \left(3^4\right)^{x-y} = 3^{4(x-y)} = 3^1 \Rightarrow 4(x-y) = 1 \Rightarrow x - y = \frac{1}{4}$.

Solve the system $\left\{ x + y = 4,\ x - y = \frac{1}{4}. \right.$

Adding gives $2x = 4 + \frac{1}{4} = \frac{17}{4} \Rightarrow x = \frac{17}{8}$.

Then $y = 4 - x = 4 - \frac{17}{8} = \frac{15}{8}$

Now compute $x^2 + y^2 = \left(\frac{17}{8}\right)^2 + \left(\frac{15}{8}\right)^2 = \frac{289 + 225}{64} = \frac{514}{64} = \frac{257}{32}$.

$$x^2 + y^2 = \frac{257}{32}$$

Answer (A) $\frac{257}{32}$

Section – B

21. (B)

Observe that $S(x)$ is a polynomial in x of degree at most 2. Evaluate S at the three points $x = -a, -b, -c$:

$$x = -a:\ S(-a) = \frac{(-a+b)(-a+c)}{(b-a)(c-a)} + \frac{0\cdot(-a+c)}{\underbrace{(a-b)(c-b)}_{=0}} + \frac{0\cdot(-a+b)}{\underbrace{(c-a)(c-b)}_{=0}} = \frac{(b-a)(c-a)}{(b-a)(c-a)} = 1$$

Similarly, $S(-b) = 1,\ \ S(-c) = 1$

So S(x) (a polynomial of degree ≤ 2) takes the value 1 at three distinct points.

Here $S(x) \equiv 1$ for all x.

22. (A)

Set $a = 5 + 2\sqrt{6}$, $t = x^2 - 3$

Note that $a(1/a) = 1$ since

$$\left(5 + 2\sqrt{6}\right)\left(5 - 2\sqrt{6}\right) = 25 - 24 = 1,$$

So $5 - 2\sqrt{6} = a^{-1}$. The equation becomes

$$a^t + a^{-t} = 10$$

Put $u = a^t$. Then

$$u + \frac{1}{u} = 10 \Rightarrow u^2 - 10u + 1 = 0$$

Solving,

$$u = \frac{10 \pm \sqrt{100 - 4}}{2} = \frac{10 \pm \sqrt{96}}{2} = \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

Thus $u = a$ or $u = a^{-1}$, i.e.

$a^t = a$ or $a^t = a^{-1}$

Since $a > 0$ and not equal to 1, this gives

$$t = 1 \text{ or } t = -1$$

Recall $t = x^2 - 3$. Hence

$$x^2 - 3 = 1,\ x^2 = 4,\ x = \pm 2,$$

Or $$x^2 - 3 = -1,\ x^2 = 2,\ x = \pm\sqrt{2}$$

The roots are $2, -2, \sqrt{2} - \sqrt{2}$. Their sum is

$$2 + (-2) + \sqrt{2} + \left(-\sqrt{2}\right) = 0$$

23. (D)

For the quadratic polynomial $p(S) = 3S^2 + 6S + 4$, we know:

$$\alpha + \beta = -\frac{6}{3} = -2, \qquad \alpha\beta = \frac{4}{3}$$

Now compute each term of E:

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(-2)^2 - 2\left(\frac{4}{3}\right)}{\frac{4}{3}} = \frac{4 - \frac{8}{3}}{\frac{4}{3}} = \frac{\frac{4}{3}}{\frac{4}{3}} = 1.$$

Next,

$$2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = 2 \cdot \frac{\alpha + \beta}{\alpha\beta} = 2 \cdot \frac{-2}{\frac{4}{3}} 2 \cdot \left(-\frac{3}{2}\right) = -3$$

And $$3\alpha\beta = 3 \times \frac{4}{3} = 4$$

Hence, $$E = 1 + (-3) + 4 = 2$$

24. (B)

Let the speed of Ritu in still water be x km/h, and the speed of the current be y km/h.
Then:

Down stream speed = x + y, Up stream speed = x – y.

From the given data:

Down stream : 20 = 2(x + y) Up stream :4 = 2(x – y)

Simplify: x + y = 10, x – y = 2

Adding both equations:

$$2x = 12 \Rightarrow x = 6$$

Substituting x = 6 in x + y =10:

$$6 + y = 10 \Rightarrow y = 4$$

Speed in still water = 6 km/h, Speed of current = 4 km/h.

25. (B)

Let the roots of $ax^2 + bx + c = 0$ be α and β.

Then the new equation is formed by **decreasing each root by 1**, so the roots are $\alpha - 1$ and $\beta - 1$.

Hence the new equation is obtained by replacing x with x + 1 in the original:

$$a(x+1)^2 + b(x+1) + c = 0$$

Expand: $a\left(x^2+2x+1\right)+bx+b+c=0$,

$\Rightarrow ax^2+\left(2a+b\right)x+\left(a+b+c\right)=0$

We are told this equation equals:

$2x^2+8x+2=0$

Thus, by comparing coefficients:

$\{a=2, 2a+b=8, a+b+c=2$

Substitute a = 2 into the other equations:

$2\left(2\right)+b=8 \Rightarrow b=4$,

$2+4+c=2 \Rightarrow c=-4$

Now check the relationship:

$b=4,\;\; c=-4 \Rightarrow b=-c$

Section – C

26. (B)

Compare letters of "LOGIC" and "MPHJD" (using their positions in the English alphabet):

$L\left(12\right)\to M\left(13\right), O\left(15\right)\to P\left(16\right), G\left(7\right)\to H\left(8\right), I\left(9\right)\to J\left(10\right), C\left(3\right)\to D\left(4\right).$

Each letter is shifted forward by +1. Apply the same rule to REASON:

$R\to S,\; E\to F,\; A\to B,\; S\to T,\; O\to P,\; N\to O.$

Thus $\text{REASON } 7\to \text{SFBTPO}$.

27. (B)

Observe the n-th term pattern:

2 = 1 · 2, 6 = 2 · 3, 12 = 3 · 4, 20 = 4 · 5, 30 = 5 · 6.

So the k-th term is k(k + 1) with k = 1, 2, 3, ... The next term corresponds to k = 6

$6\cdot 7 \;=\; 42.$

28. (B)

Place A at the origin (0, 0). Take North as +y and East as + x.

A (0,0) 5 km North B (0,5),

B (0,5) right (East) 8 km C (8,5),

C (8,5) right (South) 10 km D(8,5-10) = (8,-5),

D (8,-5) left (East) 4 km E(8+4,-5) = (12,-5).

Thus the displacement from A to E is the vector (12, −5). The shortest distance AE is

$AE=\sqrt{12^2+\left(-5\right)^2}=\sqrt{144+25}=\sqrt{169}=13\,\text{km}$

Since E is 12 km East and 5 km South of A, the direction from A to E is

South–East.

29. (C)

All persons are facing the center, so:

Immediate right = clockwise direction.

1. Start with B. Since A is to the immediate right of B, we place A one position clockwise from B.
2. D is second to the right of A. From A, count two positions clockwise
   $\to$ position for D fixed.
3. C is to the immediate left of D. Facing the center, left means one position anti-clockwise from D. Hence C sits immediately before D (in anti-clockwise direction).

4. E is not an immediate neighbor of B. This ensures E must occupy the remaining seat not adjacent to B.

After placing all members step by step, the final clockwise order around the table is:
B, A, E, C, D.
Now, between C and A (going clockwise or anti-clockwise), the person seated is E.

30. (A)
Look for a simple pattern. Compute squares:
$4^2 + 3^2 = 16 + 9 = 25$,
$6^2 + 2^2 = 36 + 4 = 40$
$5^2 + 4^2 = 25 + 16 = 41$
So the rule is $a * b = a^2 + b^2$. Therefore
$7 * 3 = 7^2 + 3^2 = 49 + 9 = 58$