Ace of Pace

Sample Paper (Engineering)

Grade X moving to XI

Solutions

Q.1 (a)

Let $P(x)=6x^4-2x^2+7x+10$

Since $1-2x$ divides $P(x)$

By remainder theorem,

$$\text{Remainder}=P\left(\frac{1}{2}\right)=6\left(\frac{1}{2}\right)^4-2\left(\frac{1}{2}\right)^2+7\left(\frac{1}{2}\right)+10$$

$$=\frac{6}{16}-\frac{2}{4}+\frac{7}{2}+10$$

$$=\frac{107}{8}$$

Q.2 (b)

Let $f(x)=Kx^2+(K-2)x+4$

Given, $f(1)=0$

$K(1)^2+(K-2)(1)+4=0$

$\therefore\quad K+K-2+4=0$

$\therefore\quad 2K=-2$

$\therefore\quad K=-1$

Q.3 (d)

For $4x^2+1$, all the exponents of x are non-negative integers.

Q.4 (b)

$x^2+bx+c=(x-\alpha)(x-\beta)$

$x^2+bx+c=x^2-(\alpha+\beta)x+\alpha\beta$

By comparing

$\alpha+\beta=-b$

$\alpha\beta=c$

$\alpha+\beta+\alpha\beta=-b+c=c-b$

Q.5 (a)

Use $a^3-b^3=(a-b)(a^2+ab+b^2)$

Q.6 (c)

$x-\frac{1}{x}=1$, square it

$\left(x-\frac{1}{x}\right)^2=1^2,\ x^2+\frac{1}{x^2}-2=1$

$x^2+\frac{1}{x^2}=3$

Q.7 (b)

$x+4y=14 \quad \times 7$

$7x-3y=5 \quad \times 1$

$7x+28y=98$

$-7x\mp 3y=-5$

$31y=93$

$y=3$

$x=14-4y=14-4\times(3)=14-12$

$x=2$

$(x,y)=(2,3)$

Q.8 (a)

Theoretical

Q.9 (d)

$\frac{12}{x}+\frac{3}{y}=3$

$\frac{12}{6}+\frac{3}{y}=3$

$2+\frac{3}{y}=3$

$\frac{3}{y}=1$

$\therefore y=3$

Q.10 (d)

$x^2-2x-35=0$

$(x-7)(x+5)=0$

$\therefore\ x=7$ or $x=-5$

Sum of square of roots

$=(7)^2+(-5)^2$

$=49+25$

$=74$

Q.11 (b)

$2x^2 - 3x + 1 = 0$

$a = 2, b = -3, c = 1$

$D = b^2 - 4ac$

$= (-3)^2 - 4\times 2\times 1 = 9 - 8 = 1$

Q.12 (c)

$2^x + 2^{1-x} = 3$

$2^x + \frac{2}{2^x} = 3$

Let $t = 2^x$

$t + \frac{2}{t} = 3$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$\therefore\ t = 2\ \&\ t = 1$

$\therefore\ 2^x = 2\ \&\ 2^x = 1$

$\therefore\ x = 1\ \&\ x = 0$

Q.13 (b)

$\alpha + \beta = 5,\ \alpha\beta = 3$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{25-6}{3} = \frac{19}{3}$

$\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

So, equation is $x^2 - \frac{19}{3}x + 1 = 0$

$3x^2 - 19x + 3 = 0$

Q.14 (c)

The two equations are : 2o + 3b + 4a = 15 and 3o + 2b + a = 10

Adding the two equations, we get

5o + 5b + 5a = 25

$\Rightarrow$ o + b + a = 5

$\therefore 3o + 3b + 3a = 15$

Q.15 (c)

$p(x) = x + \frac{1}{x}$ has degree of x a negative integer.

Q.16 (b)

$x^2-2x+1=x+k$

$x^2-3x+1-k=0$

$\Delta>0$

$k>-5/4$

Q.17 (a)

Taking

$\Rightarrow(x+1)^2>(5x-1)$

$\Rightarrow x^2-3x+2>0$

$\Rightarrow(x-1)(x-2)>0$

$x<1$ or $x>2$ ….. (i)

Taking

$\Rightarrow(x+1)^2<(7x-3)$

$\Rightarrow x^2-5x+4<0$

$\Rightarrow(x-1)(x-4)<0$

$\Rightarrow 1<x<4$ …..(ii)

Combining (i) and (ii) we get $2<x<4$

Hence, x will only take one integer i.e. 3

So, option (A) is correct.

Q.18 (b)

$|3x-5|=\frac{17}{2}$

$3x-5=\pm\frac{17}{2}$

$3x-5=\frac{17}{2}$ $\quad$ $3x-5=-\frac{17}{2}$

$3x=\frac{17}{2}+5$ $\quad$ $3x=-\frac{17}{2}+5$

$3x=\frac{27}{2}$ $\quad$ $3x=-\frac{7}{2}$

$x=\frac{9}{2}$ $\quad$ $x=-\frac{7}{6}$

Sum of all $x=\frac{9}{2}-\frac{7}{6}=\frac{27}{6}-\frac{7}{6}=\frac{20}{6}=\frac{10}{3}$

Q.19 (a)

Theoretical

Q.20 (b)

$f(x)=3x^2-5x+6k$

Let α be one zero of $f(x)$.

Then $\frac{1}{\alpha}$ is other zero of $f(x)$.

By Vieta's formula

$\alpha \times \frac{1}{\alpha} = \frac{6k}{3}$

$1 = 2k$

$k = \frac{1}{2}$

Q.21 (c)

Let α be a root, then $-\alpha$ is also root.

$p - 1 = 0 \Rightarrow p = 1$

Q.22 (a)

Let $t = \sqrt{30 + \sqrt{30 + \sqrt{30 + ...\infty}}}$

$t = \sqrt{30 + t}$

$t^2 = 30 + t$

$(t-6)(t+5) = 0$

$t = 6$ and $t = -5$

Since square root is always positive.

$\sqrt{30 + \sqrt{30 + \sqrt{30 + ...\infty}}} = 6$

Q.23 (d)

Let *x* students planned for a picnic and per person food cost is *y*.

$\therefore \quad xy = 400$

$\therefore \quad (x-10)(y+20) = 400$

$\therefore \quad xy = xy + 20x - 10y - 200$

$\therefore \quad 20x - 10y = 200$

$\therefore \quad 2x - y = 20$

$\therefore \quad y = 2x - 20$

$\therefore \quad x(2x-20) = 400 \qquad ..... (\because xy = 400)$

$\therefore \quad 2x^2 - 20x - 400 = 0$

$\therefore \quad x^2 - 10x - 200 = 0$

$\therefore \quad (x-20)(x+10) = 0$

$\therefore \quad x = 20$ and $x = -10$

Q.24 (a)

$f(x) = a(x^2 - (\alpha+\beta)x + \alpha\beta)$

$f(x+1) = a((x+1)^2 - (\alpha+\beta)(x+1) + \alpha\beta)$

$= a(x^2 + (2 - (\alpha+\beta)x + (1 - (\alpha+\beta) + \alpha\beta)$

$= 0$

$\gamma + \delta = -(2 - (\alpha+\beta)) = 3$

$\alpha + \beta = 5$

$\gamma\delta = 1 - (\alpha+\beta) + \alpha\beta = 2$

$\alpha\beta = 6$

So, $f(x) = a(x^2 - 5x + 6)$

Q.25 (c)

$\left|x^2 - 3\right| = |2x|$

$(x^2 - 3)^2 = (2x)^2$

$x^4 - 6x^2 + 9 = 4x^2$

$x^4 - 10x^2 + 9 = 0$

$(x^2 - 1)(x^2 - 3) = 0$

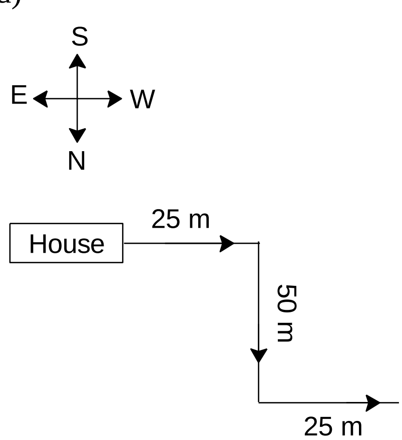
$x = \pm 1,\ x \pm 3$

Sum is 0.

Q.26 (b)

XYZ is a three-digit number.
So, we can write this as : 100*X* + 10*Y* + *Z*
XYZ + *YZX* + *ZXY*
= 100*X* + 10*Y* + *Z* + 100*Y* + 10*Z* + *X* +100*Z* + 10*X* + *Y*
= 111*X* + 111*Y* + 111*Z*
= 111 (*X* + *Y* + *Z*)
Hence, *XYZ* + *YZX* + *ZXY* is divisible by *X* + *Y* + *Z* and 111.
Also 111 is divisible by 3 and 37.
Since *X* + *Y* + *Z* is not divisible by 3.
XYZ + *YZX* + *ZXY* is not divisible by 9.

Q.27 (d)



By looking at the diagram we can easily say that the direction of man from the starting point is North-West.

Q.28 (a)

2, 6, 12, 20, 30, *X*, 56

$2 = 2^2 - 2$

$6 = 3^2 - 3$

$12 = 4^2 - 4$

$20 = 5^2 - 5$

$30 = 6^2 - 6$

$X = 7^2 - 7 = 49 - 7 = 42$

$56 = 8^2 - 8$

Q.29 (b)

$\otimes + 1\otimes + 5\otimes + \otimes\otimes + \otimes 1 = 1\otimes\otimes$

Let $\otimes = x$ where $x \in \{1, 2, ..., 9\}$

$x + 1x + 5x + xx + x1 = 1xx$

$x + (10 + x) + (50 + x) + (10x + x) + (10x + 1) = 100 + 10x + x$

$24x + 61 = 100 + 11x$

$13x = 39$

$x = 3$

$\otimes = 3$

Q.30 (d)

G H I 7 8 9	$789 \times 2 = 1578$
D E F 4 5 6	$456 \times 2 = 912$
A B C 1 2 3	$123 \times 2 = 246$