PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / HYDERABAD / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / PATNA / BARAMATI

IIT – JEE: 2027 **MAJOR TEST - 1** **DATE: 27/07/25**

ANSWER KEY

PAPER – 2 (CODE: 22)

MATHEMATICS

Q. No.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	A	C	AB	AB	BCD	3	0	2
Q. No.	11	12	13	14	15	16	17			
Ans.	1	8	98	8	4	2	5			

PHYSICS

Q. No.	18	19	20	21	22	23	24	25	26	27
Ans.	D	A	D	A	AC	ABC	ABCD	60	0	30
Q. No.	28	29	30	31	32	33	34			
Ans.	13	37	3	37	5	60	45			

CHEMISTRY

Q. No.	35	36	37	38	39	40	41	42	43	44
Ans.	B	C	C	A	ACD	ABCD	ABCD	9	2	4
Q. No.	45	46	47	48	49	50	51			
Ans.	3	2	10	40	45	5.25 - 5.30	4			

PART (A) : MATHEMATICS

1. (B)

2. (B)

$f(-1) = 1 + 2a - 3 + a = 3a - 2$

$\hat{f}(2) = 4 + 6 - 4a + a = 10 - 3a$

Case I

$\hat{f}(-1) f(2) < 0 \Rightarrow (3a-2)(10-3a) < 0$

$\Rightarrow \quad a \in \left(-\infty, \frac{2}{3}\right) \cup \left(\frac{10}{3}, \infty\right)$

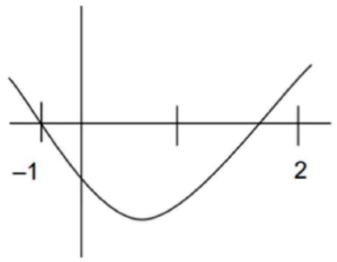
Case II

$f(-1) = 0 \Rightarrow a = \frac{2}{3}$

$f(2) = 10 - 3a = 10 - 2 = 8 > 0$

Also, x- coordinate of vertex lies in $(-1, 2)$

$\Rightarrow$ Exactly one root lies between –1 and 2 for this value $a\left(= \frac{2}{3}\right)$



Case III

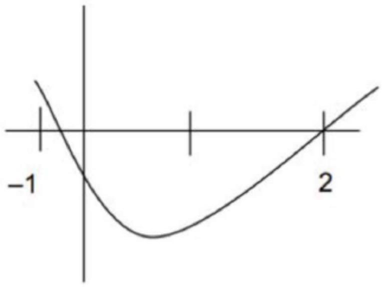
$f(2) = 0a = \frac{10}{3}$

$f(-1) = 3a - 2 - 3\left(\frac{10}{3}\right) - 2 = 8 > 0$

X – coordinate of vertex $= \frac{2a-3}{2} = \frac{11}{6} < 2$

$\Rightarrow$ Exactly one root lies between – 1 and 2 for this value of $a\left(\frac{10}{3}\right)$

$\therefore \quad a \in \left(-\infty, \frac{2}{3}\right] \cup \left[\frac{10}{3}, \infty\right)$



3. (A)

$$\tan\frac{\pi}{28}\tan\frac{2\pi}{28}...\tan\frac{13\pi}{28}$$

$$\left(\tan\frac{\pi}{28}\cot\frac{\pi}{28}\right)\left(\tan\frac{2\pi}{28}\cot\frac{2\pi}{28}\right)...\tan\frac{\pi}{4}=1$$

4. (C)

$$32\sin\frac{A}{2}\sin\frac{5A}{2}=16[\cos 2A-\cos 3A]$$

$$=16\left[\left(2\cos^2 A-1\right)-\left(4\cos^3 A-3\cos A\right)\right]$$

$$=16\left[\left(2\frac{9}{16}-1\right)-\left(4.\frac{27}{64}-3.\frac{3}{4}\right)\right]$$

$$=2-16\left(\frac{27}{16}-\frac{36}{16}\right)=2+9=11$$

5. (AB)

(A) 1 Radian = 57.16 (approx.)

$\sin 1>\sin 5^{\circ}$

(B) $\cos 3$ is $^{-}$ve so LHS is $^{-}$ve and RHS is $^{+}$ve

(C) $\tan 1+\tan 4$ is $^{+}$v $\tan 2$ $^{-}$ve

(D) $\tan 3>\tan 1^{\circ}$ $^{-}$ve $^{+}$ve

6. (A, B)

Let the common root be y. Then $y^2+py+q=0$ and $y^2+\alpha y+\beta=0$

On solving by cross multiplication, we have

$$\frac{y^2}{p\beta-q\alpha}=\frac{y}{q-\beta}=\frac{1}{\alpha-p}$$

$$\therefore\quad y=\frac{q-\beta}{\alpha-p}\ \text{and}\ \frac{y^2}{y}=y=\frac{p\beta-q\alpha}{q-\beta}$$

7. (BCD)

$$x=\frac{1-\sin\phi}{\cos\phi}\ \ \&\ \ y=\frac{(1+\cos\phi)}{\sin\phi}$$

$$x=\frac{1-\cos\left(90^{\circ}-\phi\right)}{\sin\left(90^{\circ}-\phi\right)}\ \ \&\ \ y=\frac{2\cos^2\frac{\phi}{2}}{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}$$

$$x=\frac{2\sin^2\left(45^{\circ}-\frac{\phi}{2}\right)}{2\sin\left(45^{\circ}-\frac{\phi}{2}\right)\cos\left(45^{\circ}-\frac{\phi}{2}\right)}\ \ \&\ \ y=\frac{1}{\tan\frac{\phi}{2}}$$

$$x=\tan\left(45^{\circ}-\frac{\phi}{2}\right)\qquad x=\frac{\left(1-\frac{1}{y}\right)}{\left(1+\frac{1}{y}\right)}\Rightarrow x=\frac{(y-1)}{(y+1)}$$

8. (3)

Given $2\left(\sin\frac{A+B}{2}\cos\frac{A-B}{2}+\sin\frac{C+D}{2}\cdot\cos\frac{C-D}{2}\right)=2\times 2$

$\Rightarrow \quad \sin A+\sin B+\sin C+\sin D=4$

$\Rightarrow \quad \sin A=\sin B=\sin C=\sin D=1$

$\Rightarrow \quad \angle A=\angle B=\angle C=\angle D=90^\circ$

$\therefore \quad \sum\cos\frac{A}{2}\cos\frac{B}{2}=\sum\cos^2 45^\circ=6\left(\frac{1}{\sqrt{2}}\right)^2=3$

9. (0)

If $x^2+\lambda x+1=0; \lambda\in(-2,2)$ then both roots are imaginary….

Let roots of this equation be α and β so $\alpha+\beta=-\lambda$ and $\beta\alpha=1$

Now cubic $4x^3+3x+2c=0$ has roots α,β,y with $\alpha\beta y=-\frac{2c}{4}=-\frac{c}{2}$ and $\alpha+\beta+y=0$

Hence $\lambda+\frac{c}{2}=0$

10. (2)

Let α,β be the roots of the equation

$(a+1)x^2+(2a+3)x+(3a+4)=0$. Then,

$\alpha+\beta=-1\Rightarrow-\left(\frac{2a+3}{a+1}\right)=-1\Rightarrow a=-2$

$\therefore \quad$ Product of the roots $=\frac{3a+4}{a+1}=\frac{-6+4}{-2+1}=2$

11. (1)

$M=\cot\alpha\cdot\cot(2\alpha).........\cot(2n-2)\alpha\cdot\cot(2n-1)\alpha$

$\left\{\begin{array}{l}\because\ 4n\alpha=\pi\\ \therefore\ 2n\alpha=\frac{\pi}{2}\end{array}\right\}$

$\therefore \quad M=\cot\alpha\cdot\cot(2\alpha).....\cot\left(\frac{\pi}{2}-2\alpha\right)\cdot\cot\left(\frac{\pi}{2}-\alpha\right)$

$M=\cot\alpha.\cot(2\alpha).....\tan(2\alpha)\cdot\tan(\alpha)$

$M=1$

$\therefore \quad M^{100}=1^{100}$

$\Rightarrow \quad M^{100}=1$

12. (8)

Use the formula $\cot\theta-\tan\theta=2\cot 2\theta$ and we know that

$(1+\tan\theta)\left(1+\tan\left(45^\circ-\theta\right)\right)=2$

$\left(2+\cot 41.5^\circ-\cot 48.5^\circ\right)\left(2+\cot 26^\circ-\cot 64^\circ\right)$

$=\left(2+2\cot 83^\circ\right)\left(2+2\cot 52^\circ\right)$

$=4\left(1+\tan 7^\circ\right)\left(1+\tan 38^\circ\right)=8$

13. (98)

$$x^2-4\lambda x+5=0 \begin{cases}\alpha\\ \beta\end{cases}$$

$$x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+\left(7+3\lambda\sqrt{3}\right)=0 \begin{cases}\alpha\\ \gamma\end{cases}$$

$\alpha+\beta=4\lambda$

$\alpha+\gamma=3\sqrt{2}+2\sqrt{3}$

$\beta+\gamma=3\sqrt{2}$ $\quad$ $\alpha\gamma=7+3\lambda\sqrt{3}$

$\therefore$ $\alpha=2\lambda+\sqrt{3}$ $\quad$ $\alpha\beta=5$

$\beta=2\lambda-\sqrt{3}$ $\quad$ $4\lambda^2=8\Rightarrow\lambda=\sqrt{2}$

$\therefore$ $(\alpha+2\beta+\gamma)^2=\left(4\lambda+3\sqrt{2}\right)^2=\left(7\sqrt{2}\right)^2=98$

14. (8)

$f(6)=8\left(\cos^6\theta-\sin^6\theta\right)$

$=8\left(\cos^2\theta-\sin^2\theta\right)\left(\cos^4\theta+\sin^4\theta+\sin^2\theta\cos^2\theta\right)$

$=8\cos2\theta\left(1-\dfrac{\sin^2 2\theta}{4}\right)$

$=8\cos2\theta\left(1-\dfrac{1-\cos4\theta}{8}\right)$

$=\cos2\theta(8-1+\cos4\theta)$

$=7\cos2\theta+\cos4\theta\cos2\theta$

$=7\cos2\theta+\dfrac{\cos6\theta+\cos2\theta}{2}$

$=\dfrac{15}{2}\cos2\theta+\dfrac{1}{2}\cos6\theta$

$a+b=8$

15. (4)

$\dfrac{g(6)}{2}+3\sin^2 2\theta$

$=\dfrac{8\left(\cos^6\theta+\sin^6\theta\right)}{2}+3\sin^2 2\theta \Rightarrow 4\left(1-\dfrac{3}{4}\sin^2 2\theta\right)+3\sin^2 2\theta$

16. (2)

$x^3-x^2(1+\sin\theta+\cos\theta)+x(\sin\theta+\cos\theta+\sin\theta\cos\theta)-\sin\theta\cos\theta=0$

$\Rightarrow$ Roots are $1,\sin\theta,\cos\theta$

$\alpha^2+\beta^2+\gamma^2=1+\sin^2\theta+\cos^2\theta=2$

17. (5)

$x^3-x^2(1+\sin\theta+\cos\theta)+x(\sin\theta+\cos\theta+\sin\theta\cos\theta)-\sin\theta\cos\theta=0$

$\Rightarrow$ Roots are $1,\ \sin\theta,\cos\theta$

For at lest 2 to be equal

$\sin\theta=1$ or $\cos\theta=1$ or $\sin\theta=\cos\theta$ or $\sin\theta=\cos\theta=1$

$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\cos\theta = 1 \Rightarrow \theta = 0, 2\pi$

$\sin\theta = \cos\theta \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\sin\theta = \cos\theta = 1$

18. (D)

$\vec{\mathbf{v}}_{\text{ball, train}} = \vec{\mathbf{v}}_{\text{ball}} - \vec{\mathbf{v}}_{\text{train}}$

or $\vec{\mathbf{v}}_{\text{ball}} = \vec{\mathbf{v}}_{\text{ball, train}} + \vec{\mathbf{v}}_{\text{train}}$

Since ball moves in vertical direction its *x*-component of velocity must vanish

$\upsilon_{bx} = \upsilon\cos 60° - \upsilon_T = 0$ ...(1)

and $\upsilon_{by} = \upsilon\sin 60°$ ...(2)

From eqn. (1),

$$\upsilon = \frac{\upsilon_T}{\cos 60°} = 20\,\text{m/s}$$

and $\upsilon_{by} = (20)\sin 60°$

$$= \frac{20\sqrt{3}}{2}\,\text{m/s}$$

Maximum height reached by ball

$$= \frac{(\upsilon_{by})^2}{2g}$$

$$= \frac{\left(20\sqrt{3}/2\right)^2}{2\times 9.8}$$

= 15.30 m

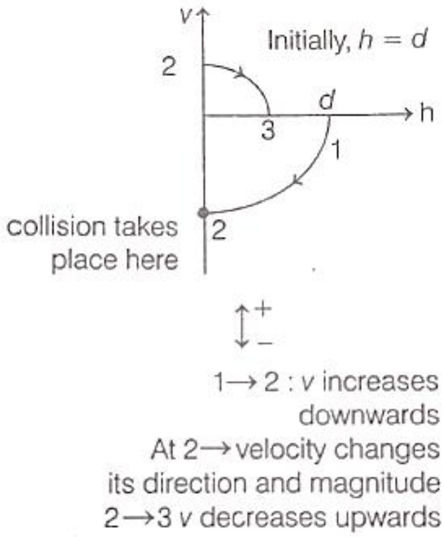
19. (A)

For uniformly
accelerated / decelerated motion

$$v^2 = u^2 \pm 2gh$$

i.e. *v*-*h* graph will be a parabola (because equation is quadratic).

Initially velocity is downwards (–ve) and then after collision, it reverses its direction with lesser magnitude, i.e. velocity is upwards (+ve). Graph (a) satisfies both these conditions.



20. (D)

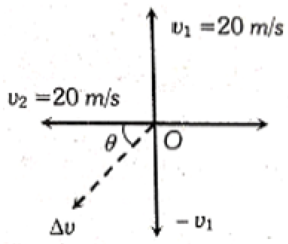
From figure

$\vec{\upsilon}_1 = 20\hat{j}$ and $\vec{\upsilon}_2 = -20\hat{j}$

$\Delta\vec{\upsilon} = \vec{\upsilon}_2 - \vec{\upsilon}_1 = -20(\hat{i} + \hat{j})$

$|\Delta\vec{\upsilon}| = 20\sqrt{2}$ and direction

$\theta = \tan^{-1}(1) = 45°$ i.e. S–W



21. (A)

Here, $\vec{P} + b\vec{R} = \vec{S} \Rightarrow \vec{R} = \dfrac{\vec{S} - \vec{P}}{b}$

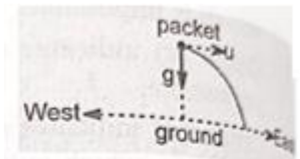
Also $\vec{R} = \vec{Q} - \vec{P}$

$\therefore \dfrac{\vec{S} - \vec{P}}{b} = \vec{Q} - \vec{P} \Rightarrow \vec{S} - \vec{P} = b\vec{Q} - b\vec{P}$

$\therefore \vec{S} = b\vec{Q} + (1-b)\vec{P}$

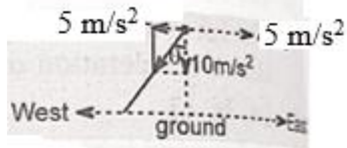
22. (A, C)

If the velocity of aeroplane is u m/s when the packet is dropped then path of packet is parabolic with respect to ground as shown in figure.



With respect to aeroplane the initial velocity of the packet is zero and acceleration is as shown in figure.

$$\theta = \tan^{-1}\frac{5}{10} = \tan^{-1}\frac{1}{2} \text{ west of vertical.}$$



23. (A, B, C)

$$a = \frac{dv}{dt} = A - Bv$$

→ max. possible velocity is terminal velocity (*i.e.*, when $a = 0$)

⇒ $A - Bv = 0$

→ initial acc. is when $t = 0, u = 0$

$\therefore$ $a = A - 0 = A \text{ m/s}^2$

→ $\dfrac{dv}{dt} = A - Bv \Rightarrow \int_0^v \dfrac{dv}{A - Bv} = \int_0^t dt$

⇒ $\dfrac{1}{B}\ln\dfrac{A - Bv}{A} = -t \Rightarrow 1 - \dfrac{B}{A}v = e^{-Bt}$

⇒ $\dfrac{A}{B}\left(1 - e^{-Bt}\right) = v$

24. (A, B, C, D)

They will always lie on a regular octagon but side of octagon will keep decreasing.

Velocity of approach of two adjacent particle is $v - v\cos\left(\dfrac{2\pi}{8}\right)$

$$v_{\text{app}} = v\left(1 - \frac{1}{\sqrt{2}}\right)$$

Finally all will meet at the centroid,

$$t=\frac{a}{v_{app}}=\frac{10}{1-\frac{1}{\sqrt{2}}}=\frac{10\sqrt{2}}{\sqrt{2}-1}\text{s}$$

25. (60)

At highest point, $v_1=u\cos\theta$ at $y=\frac{h}{2}\text{max}, v_y^2=u^2\sin^2\theta-2g\frac{h_{max}}{2}$

$$h_{max}=\frac{u^2\sin^2\theta}{2g}$$

$$\therefore\ v_y^2=u^2\sin^2\theta-\frac{2g}{2}\frac{u^2\sin^2\theta}{2g}$$

$$\therefore\ v_y=\frac{u\sin\theta}{\sqrt{2}}$$

Resultant velocity at $y=\frac{h_{max}}{2}$

$$v_2=\sqrt{v_x^2+v_y^2}=\sqrt{u^2\cos^2\theta+\frac{u^2\sin^2\theta}{2}}$$

$$\frac{v_1}{v_2}=\sqrt{\frac{2}{5}}=\frac{u\cos\theta}{\sqrt{u^2\cos^2\theta+\frac{u^2\sin^2\theta}{2}}}$$

$$\therefore\ \frac{2}{5}=\frac{u^2\cos^2\theta}{u^2\cos^2\theta+u^2\sin^2\theta}$$

$$\therefore\ \tan^2\theta=3$$

$$\therefore\ \tan\theta=\sqrt{3}$$

$$\theta=60^\circ$$

26. (0)

Let angle between the two vectors be θ

$\therefore\quad 3^2=7^2+4^2+2\times7\times4\cos\theta$

$\cos\theta=-1$

$\Rightarrow\quad \theta=180^\circ$

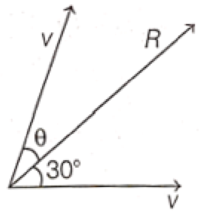
$\therefore$ Cross product will be zero.

27. (30)

As per question, magnitude of velocity of the swimmer is same as that of the river.

So resultant would pass exactly midway through them

$\therefore$ Angle $\theta=30^\circ$



28. (13)

i.e., $2=(\upsilon\cos\theta-13)t$

and $3=\upsilon\sin\theta t$

On eliminating υ from eqns. (1) and (2), we get

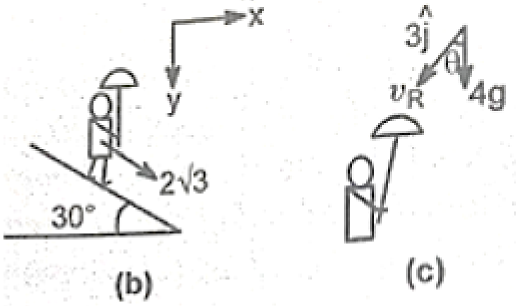
$$t=\frac{1}{13}\left(\frac{3}{\tan\theta}-2\right)=\frac{2}{13}$$

29. (37)

$$\vec{\mathbf{v}}_{R/M}=x\hat{\mathbf{j}}=\vec{\mathbf{v}}_R-\vec{\mathbf{v}}_M$$

$$\vec{\mathbf{v}}_M=2\sqrt{3}\left[\cos 30\,\hat{\mathbf{i}}+\sin 30\,\hat{\mathbf{j}}\right]=3\hat{\mathbf{i}}+\sqrt{3}\,\hat{\mathbf{j}}$$

$$\Rightarrow\quad \vec{\mathbf{v}}_R=-3\hat{\mathbf{i}}+\left(x-\sqrt{3}\right)\hat{\mathbf{j}}$$



$$5=\sqrt{3^2+\left(x-\sqrt{3}\right)^2}$$

$$\Rightarrow\quad 16=\left(x-\sqrt{3}\right)^2$$

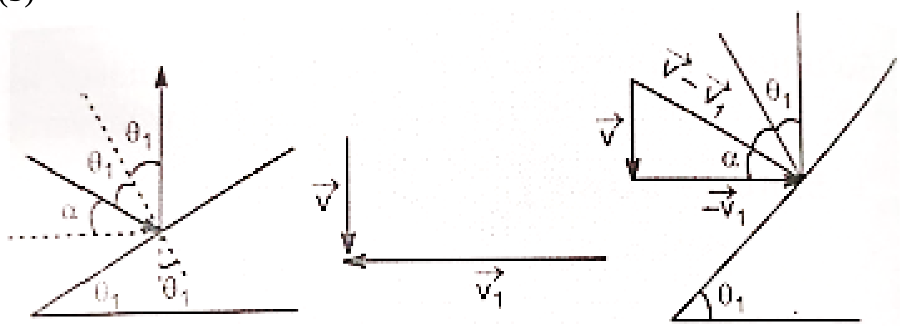
$$\Rightarrow\quad 4+\sqrt{3}=x$$

$$\vec{\mathbf{v}}_R=-3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$$

$$\tan\theta=3/4$$

$$\Rightarrow\quad \theta=37^\circ$$

30. (3)



From figure,

$$\alpha+2\theta_1=\frac{\pi}{2}$$

and $\tan\alpha=\dfrac{\upsilon}{\upsilon_1}$

Hence $\tan\alpha=\tan\left(\dfrac{\pi}{2}-2\theta_1\right)$

$$=\cot 2\theta_1$$

or $\dfrac{\upsilon}{\upsilon_1}=\cot 2\theta_1$

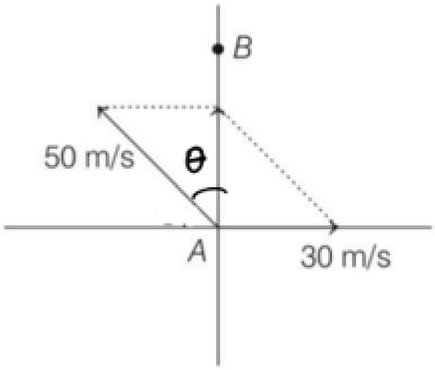
Similarly for second car,

$$\frac{\upsilon}{\upsilon_2}=\cot 2\theta_2$$

Solution of Que. 31 & 32:

31. (37)

32. (5)



$50\sin\theta = 30$

$\theta = 37°$ with north

Time taken $= \dfrac{720}{50\cos\theta \times \dfrac{18}{5}} = 5\text{ h}$

33. (60)

Range of stone as observed from ground

$$R = (10\cos\theta + 10)\frac{2\times 10\times \sin\theta}{g}$$

$$= (\cos\theta + 1)20\sin\theta$$

$$\frac{dR}{d\theta} = 20\sin\theta \times (-\sin\theta) + 20\cos\theta(\cos\theta + 1)$$

$$= 20\cos^2\theta - 20\sin^2\theta + 20\cos\theta = 0$$

$\Rightarrow \quad 2\cos^2\theta + \cos\theta - 1 = 0$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2} \Rightarrow \theta = 60°$

$$\left(\frac{d^2R}{dQ^2}\right)_{\theta=60°} = -\text{ve}$$

$\Rightarrow$ maximum R at $\theta = 60°$

34. (45)

w.r.t boat it is just an simple projectile motion

$\therefore$ Range is maximum for $\theta = 45°$

PART (C) : CHEMISTRY

35. (B)

36. (C)

$\sqrt{\upsilon} = aZ - ab \qquad ab = 1, \quad a = \tan 45° = 1$

$\sqrt{\upsilon} = 1(51) - 1 = 50$

$\therefore\ \upsilon = (50)^2 = 2500\ s^{-1}$

37. (C)
38. (A)
39. (A, C, D)
40. (A, B, C, D)
41. (A, B, C, D)
42. (9)
43. (2)

44. (4)

$2s^2 2p^6 = 8$ electrons total out of which will have $m = -\frac{1}{2}$

45. (3)
46. (2)

47. (10)

	1s	2s	2p	3s	3p	4s
$n + \ell$ value	1 + 0 = 1	2 + 0 = 2	2 + 1 = 3	3 + 0 = 3	3 + 1 = 4	4 + 0 = 4
No. of orbitals	1	1	3	1	3	1

Total no. of orbitals = 10

48. (40)

10 different radiations means 5 energy levels are involved.

Let's take $n_1 = n$

Then $n_2 = n + 4$

$$-0.544 = -13.6 \times \left(\frac{1}{5}\right)^2 = -13.6 \times \frac{z^2}{(n+4)^2} \qquad ...(1)$$

$$-0.85 = -13.4\left(\frac{1}{4}\right)^2 = -13.6\frac{z^2}{n^2} \qquad ...(2)$$

Solving (1) and (2)

We get to n = 16 and z = 4

Now, $n_1 = 16$, $n_2 = n + 4 = 20$, z = 4

$z + n_1 + n_2 = 4 + 16 + 20 = 40$

49. (45)

$$\left(E_{photon}\right)_{\text{emitted from } He^+} = 13.6 \times 4\left\{\frac{1}{1} - \frac{1}{4}\right\} = 40.8\ eV$$

Now this photon is being used to ionize H-atom.

$$K.E._{\text{ejected } e^-} = E_{\text{photon}} - I.E$$

$$= 40.8 - 13.6 = 27.2 \text{ eV}$$

$$\lambda_{\text{de-Broglie}} = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{27.2}}\text{Å} = x$$

Now, $8.16x^2 = 45$

50. (5.25 – 5.30)

$$\Delta x.\Delta p \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{h}{4\pi\Delta x}$$

$$\Delta p = \frac{6.6\times10^{-34}}{4\times3.14\times10^{-10}}$$

51. (4)

$$\Delta x_A \cdot m_A \Delta v_A = \Delta X_B \cdot \Delta V_B \qquad \Delta V_B = \Delta V_A$$

$$\frac{\Delta X_A}{\Delta X_B} = \frac{N_B}{M_A} = \frac{1.0\times10^{-31}}{1.0\times10^{-27}} = 10^{-4}$$