Ace of Pace

Sample Paper (Engineering)

Grade X moving to XI

Solutions

Q.1 (a)

Let $P(x)=6x^4-2x^2+7x+10$

Since $1-2x$ divides $P(x)$

By remainder theorem,

Remainder $=P\left(\frac{1}{2}\right)=6\left(\frac{1}{2}\right)^4-2\left(\frac{1}{2}\right)^2+7\left(\frac{1}{2}\right)+10$

$=\frac{6}{16}-\frac{2}{4}+\frac{7}{2}+10$

$=\frac{107}{8}$

Q.2 (b)

Let $f(x)=Kx^2+(K-2)x+4$

Given, $f(1)=0$

$K(1)^2+(K-2)(1)+4=0$

$\therefore\ K+K-2+4=0$

$\therefore\ 2K=-2$

$\therefore\ K=-1$

Q.3 (d)

For $4x^2+1$, all the exponents of x are non-negative integers.

Q.4 (b)

$x^2+bx+c=(x-\alpha)(x-\beta)$

$x^2+bx+c=x^2-(\alpha+\beta)x+\alpha\beta$

By comparing

$\alpha+\beta=-b$

$\alpha\beta=c$

$\alpha+\beta+\alpha\beta=-b+c=c-b$

Q.5 (a)

Use $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

Q.6 (c)

$x-\frac{1}{x}=1$, square it

$\left(x-\frac{1}{x}\right)^2=1^2,\ x^2+\frac{1}{x^2}-2=1$

$x^2+\frac{1}{x^2}=3$

Q.7 (b)

$x+4y=14 \quad \times 7$

$7x-3y=5 \quad \times 1$

$7x+28y=98$

$-7x\mp 3y=-5$

$31y=93$

$y=3$

$x=14-4y=14-4\times(3)=14-12$

$x=2$

$(x, y)=(2, 3)$

Q.8 (a)

Theoretical

Q.9 (d)

$\frac{12}{x}+\frac{3}{y}=3$

$\frac{12}{6}+\frac{3}{y}=3$

$2+\frac{3}{y}=3$

$\frac{3}{y}=1$

$\therefore y=3$

Q.10 (d)

$x^2-2x-35=0$

$(x-7)(x+5)=0$

$\therefore\ x=7$ or $x=-5$

Sum of square of roots

$=(7)^2+(-5)^2$

$=49+25$

$=74$

Q.11 (b)

$2x^2 - 3x + 1 = 0$

$a = 2, b = -3, c = 1$

$D = b^2 - 4ac$

$= (-3)^2 - 4 \times 2 \times 1 = 9 - 8 = 1$

Q.12 (c)

$2^x + 2^{1-x} = 3$

$2^x + \frac{2}{2^x} = 3$

Let $t = 2^x$

$t + \frac{2}{t} = 3$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$\therefore\ t = 2\ \&\ t = 1$

$\therefore\ 2^x = 2\ \&\ 2^x = 1$

$\therefore\ x = 1\ \&\ x = 0$

Q.13 (a)

$t_4 = -1$

$\therefore\ a + 3d = -1$ .....(1)

$t_9 = -16$

$\therefore\ a + 8d = -16$ .....(2)

On solving (1) & (2), we get

$d = -3, a = 8$

Q.14 (b)

$t_3 = \frac{1}{12}$

$\therefore\ ar^2 = \frac{1}{12}$ .....(1)

$t_6 = -\frac{1}{96}$

$\therefore\ ar^5 = -\frac{1}{96}$ .....(2)

On solving (1) & (2), we get

$r = -\frac{1}{2},\ a = \frac{1}{3}$

Q.15 (c)

p, q, r are in A.P.

$\therefore\ 2q = p + r$

x, y, z are in G.P.

$\therefore\ \frac{y}{x} = \frac{z}{y}$

$x^{q-r} y^{r-p} z^{p-q}$

$= \frac{x^q}{x^r} \cdot \frac{y^r}{y^p} \cdot \frac{z^p}{z^q}$

$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^r \left(\frac{z}{y}\right)^p$

$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^r \left(\frac{y}{x}\right)^p$ ......$\left(\because \frac{z}{y} = \frac{y}{x}\right)$

$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^{p+r}$

$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^{2q}$ ......$(\because 2q = p + r)$

$= \frac{x^q}{z^q} \cdot \frac{y^{2q}}{x^{2q}}$

$= \frac{\left(y^2\right)^q}{\left(xz\right)^q}$

$= \frac{\left(y^2\right)^q}{\left(y^2\right)^q}$ ......$(\because\ y^2 = xz)$

= 1

Q.16 (d)

$2(k+7) = (2k+3) + (3k-4)$ ...(Property of A.P.)

$\therefore\ 2k + 14 = 2k + 3 + 3k - 4$

$\therefore\ 3k = 15$

$\therefore\ k = 5$

Q.17 (a)

$a = 7,\ r = 2$

$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

$S_9 = 7\left(\frac{2^9 - 1}{2 - 1}\right)$

$= 7(512 - 1)$

$= 7 \times 511$ $\therefore\ S_9 = 3577$

Q.18 (b)

$|3x-5|=\frac{17}{2}$

$3x-5=\pm\frac{17}{2}$

$3x-5=\frac{17}{2}$ $\quad$ $3x-5=-\frac{17}{2}$

$3x=\frac{17}{2}+5$ $\quad$ $3x=-\frac{17}{2}+5$

$3x=\frac{27}{2}$ $\quad$ $3x=-\frac{7}{2}$

$x=\frac{9}{2}$ $\quad$ $x=-\frac{7}{6}$

Sum of all $x=\frac{9}{2}-\frac{7}{6}=\frac{27}{6}-\frac{7}{6}=\frac{20}{6}=\frac{10}{3}$

Q.19 (a)

Theoretical

Q.20 (b)

$f(x)=3x^2-5x+6k$

Let α be one zero of $f(x)$.

Then $\frac{1}{\alpha}$ is other zero of $f(x)$.

By Vieta's formula

$\alpha\times\frac{1}{\alpha}=\frac{6k}{3}$

$1=2k$

$k=\frac{1}{2}$

Q.21 (b)

Let α and β are roots of required quadratic equation

$x^2-(\alpha+\beta)x+\alpha\beta=0$

Given: $\frac{\alpha+\beta}{2}=15$ and $\sqrt{\alpha\beta}=12$

$\therefore$ $\alpha+\beta=30$ and $\alpha\beta=144$

$\therefore$ $x^2-30x+144=0$

Q.22 (a)

Let $t=\sqrt{30+\sqrt{30+\sqrt{30+.....\infty}}}$

$t=\sqrt{30+t}$

$t^2=30+t$

$(t-6)(t+5)=0$

$t=6$ and $t=-5$

Since square root is always positive.

$\sqrt{30+\sqrt{30+\sqrt{30+.....\infty}}}=6$

Q.23 (d)

Let x students planned for a picnic and per person food cost is y.

$\therefore\quad xy=400$

$\therefore\quad (x-10)(y+20)=400$

$\therefore\quad xy=xy+20x-10y-200$

$\therefore\quad 20x-10y=200$

$\therefore\quad 2x-y=20$

$\therefore\quad y=2x-20$

$\therefore\quad x(2x-20)=400$ ….. ($\because xy=400$)

$\therefore\quad 2x^2-20x-400=0$

$\therefore\quad x^2-10x-200=0$

$\therefore\quad (x-20)(x+10)=0$

$\therefore\quad x=20$ and $x=-10$

Q.24 (a)

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

Given, $S_p=S_q$

$\frac{p}{2}\left[2a+(p-1)d\right]=\frac{q}{2}\left[2a+(q-1)d\right]$

$\therefore\ 2ap+p(p-1)d=2aq+q(q-1)d$

$\therefore\ 2ap-2aq=q(q-1)d-p(p-1)d$

$\therefore\ 2a(p-q)=d\left[q(q-1)-p(p-1)\right]$

$\therefore\ 2a(p-q)=d\left[q^2-q-p^2+p\right]$

$\therefore\ 2a(p-q)=d\left[(q-p)(q+p)-(q-p)\right]$

$\therefore\ 2a(p-q)=d(q-p)(q+p-1)$

Since $p\neq q, 2a=-d(q+p-1)$

$\therefore\ 2a+d(p+q-1)=0$

$S_{p+q}=\frac{p+q}{2}\left[2a+(p+q-1)d\right]=\frac{p+q}{2}\times 0$

$\therefore\ S_{p+q}=0$

Q.25 (b)

The principal amount invested for every = 1000

The amount at the end of 100 years for the principal amount, P $= A = P\left(1+\frac{r}{100}\right)^n$

Where P = Principal amount = 1000/-
r = Rate of interest = 12%
N = Number of years = 10

The amount invested in the first year will earn an interested for 10 years and the amount in the second year will earn an interest for 9 years and so on. Let us make a chart to find the amounts at the end of 10^{th} year.

Year	Amount Invested	Interest calculated for Years	Amount of the end of 10^{th} year
1^{st}	1000	10	$1000\left(1+\frac{12}{100}\right)^{10}=\left(\frac{112}{100}\right)^{10}\times 10^3$
2^{nd}	1000	9	$1000\left(1+\frac{12}{100}\right)^{9}=\left(\frac{112}{100}\right)^{9}\times 10^3$
3^{rd}	1000	8	$1000\left(1+\frac{12}{100}\right)^{8}=\left(\frac{112}{100}\right)^{8}\times 10^3$
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The earning of the man for every years investment at the end of 10^{th} year are $10^3\left(\frac{112}{100}\right)^{10}, 10^3\left(\frac{112}{100}\right)^{9}$ …….

The sum of all these earnings will give the total amount earned by him after 10 years

$$=10^3\left(\frac{112}{100}\right)^{10}+10^9+10^3\times\left(\frac{112}{100}\right)^{8}+\ldots\ldots\ldots$$

The above series forms a geometric progression with first term $=a=10^3\left(\frac{112}{100}\right)$

Common ratio $=r=\left(\frac{100}{112}\right)$

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

$$\therefore S_{10}=\frac{10^3\left(\frac{112}{100}\right)^{10}\left[\left(\frac{100}{112}\right)^{10}-1\right]}{\left(\frac{100}{112}-1\right)}=19652.5\text{Rs. (approx)}$$

The total amount saved by the man at the rate of 12% for Rs. 1000 per year for 10 years is 10,000 for which he gets a return of Rs. 19,652.5

Q.26 (b)

XYZ is a three-digit number.

So, we can write this as : $100X + 10Y + Z$

$XYZ + YZX + ZXY$

$= 100X + 10Y + Z + 100Y + 10Z + X + 100Z + 10X + Y$

$= 111X + 111Y + 111Z$

$= 111 (X + Y + Z)$

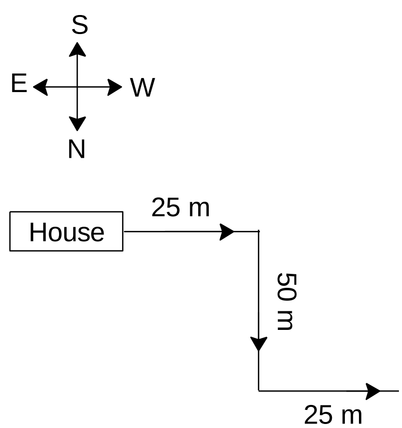
Hence, $XYZ + YZX + ZXY$ is divisible by $X + Y + Z$ and 111.

Also 111 is divisible by 3 and 37.

Since $X + Y + Z$ is not divisible by 3.

$XYZ + YZX + ZXY$ is not divisible by 9.

Q.27 (d)



By looking at the diagram we can easily say that the direction of man from the starting point is North-West.

Q.28 (a)

2, 6, 12, 20, 30, *X*, 56

$2 = 2^2 - 2$

$6 = 3^2 - 3$

$12 = 4^2 - 4$

$20 = 5^2 - 5$

$30 = 6^2 - 6$

$X = 7^2 - 7 = 49 - 7 = 42$

$56 = 8^2 - 8$

Q.29 (b)

$\otimes + 1\otimes + 5\otimes + \otimes\otimes + \otimes 1 = 1\otimes\otimes$

Let $\otimes = x$ where $x \in \{1, 2, ..., 9\}$

$x + 1x + 5x + xx + x1 = 1xx$

$x + (10 + x) + (50 + x) + (10x + x) + (10x + 1) = 100 + 10x + x$

$24x + 61 = 100 + 11x$

$13x = 39$

$x = 3$

$\otimes = 3$

Q.30 (d)

G H I 7 8 9	$789 \times 2 = 1578$
D E F 4 5 6	$456 \times 2 = 912$
A B C 1 2 3	$123 \times 2 = 246$