ACE OF PACE

Grade X moving to XI

ENGINEERING **SOLUTIONS** **DATE: 24/08/2025**

Section – A

1. (A)

Taking $a = 10^x$, $a + \frac{4}{a} = \frac{81}{2}$

This would give the quadratic equation:

$2a^2 - 81a + 8 = 0$

We want to find the sum of possible values of x,

Let the value of x be x_1 and x_2 these would correspond to log a_1, and log a_2

The sum of log a_1 + log a_2 would log $(a_1 \times a_2)$

From the quadratic equation we got above, we can see that the product of the possible value

$a_1 a_2 = \frac{8}{2} = 4$

Therefore, the sum of values of x would be log (4) which be $2\log_{10} 2$

Therefore, option A is the correct answer.

2. (C)

Let the initial number of chocolates be 64x.

First child gets $32x + 1$ and $32x - 1$ are left.

2nd child gets $16x + 1/2$ and $16x - 3/2$ are left

3rd child gets $8x + 1/4$ and $8x - 7/4$ are left

4th child gets $4x + 1/8$ and $4x - 15/8$ are left

5th child gets $2x + 1/16$ and $2x - 31/16$ are left

Given, $2x - 31/16 = 0 \Rightarrow 2x = 31/16 \Rightarrow x = 31/32$

$\therefore$ Initially the Gentleman has 64x i.e

$64 * 31/32 = 62$ Chocolates

3. (D)

Given a and b are the distinct roots of the equation $2x^2 - 6x + k = 0$

$\Rightarrow a + b = (-6/2) = 3$ (Sum of the roots)

$\Rightarrow ab = k/2$ (Product of the roots)

Now, $(a+b)$ and ab are the roots of the quadratic equation $x^2 + px + p = 0$

$\Rightarrow a + b + ab = -p \Rightarrow 3 + k/2 = -p$ … (1)

$\Rightarrow (a+b)(ab) = p \Rightarrow 3(k/2) = p$ … (2)

$3 + \frac{k}{2} = -\frac{3k}{2} \Rightarrow 2k = -3 \Rightarrow k = -\frac{3}{2}$

$p = \frac{3k}{2} = \frac{3}{2}\left(-\frac{3}{2}\right) = -\frac{9}{4}$

$\Rightarrow 8(k-p)=8\left(-\frac{3}{2}+\frac{9}{4}\right)=-12+18=6$

4. (A)

$f(x)=\frac{x^2+2x-15}{x^2-7x-18}<0$

$\frac{(x+5)(x-3)}{(x-9)(x+2)}<0$

We have four inflection points $-5,-2,3$ and 9.

For, $x<-5$ all four terms $(x+5),(x-3),(x-9)$, $(x+2)$ will be negative. Hence, the overall expression will be positive. Similarly, when $x>9$, all four terms will be positive. When x belongs to $(-2,3)$, two terms are negative and two are positive. Hence, the overall expression is positive again, we are left with the range $(-5,-2)$ and (3, 9) where the expression will be negative.

5. (A)

There are two critical points for the inequality to consider: $x=-5$ and $x=3/2$

Region I: x is greater than 3/2 in this scenario, both the terms would be positive; cross- multiplying, we get the relation $2x-3<x+5$ giving the boundary $x\le 8$, hence giving us the valid range as

$\frac{3}{2}<x\le 8$

Region II: $-5<x<\frac{3}{2}$

In this case, the right-hand side will be a negative value, and hence, the sign would change when multiplying, giving the inequality $2x-3\ge x+5$

Which will give $x>8$, which is out of bounds for this region another way is to put a value in the inequality; by putting $x=0$, we could see that the inequality does not hold in this region

Region III: x less than -5

In this scenario both the terms are negative, essentially giving us the same boundary as region 1; we take the lower bounds, giving us that x has to be less than 5

Therefore, for the given inequality to hold true $x<-5$ or $\frac{3}{2}x\le 8$

Hence, option A is the correct answer.

6. (D)

Option $4:\frac{1}{p}-3$

For the given A.P

$a=1/p, d=((1-p)/p)-(1/p)=-1$

4th term for the $A.P=a+(4-1)d=a+3d=1/p-3$

Hence, the correct options is 4

7. (A)

Option 1:36

The numbers from 9 to 59 divisible by 3 are: 9, 12, 15, and 57.

This is an Arithmetic Progression with:

* First term (a) = 9

* Common difference (d) = 3
* Last term $(a_n) = 57$

First, find the total number of terms (n) in this AP:

$\Rightarrow 57 = 9 + (n-1) \times 3$

$\Rightarrow 57 - 9 = (n-1) \times 3$

$\Rightarrow 48 = (n-1) \times 3$

$\Rightarrow 48/3 = n-1$

$\Rightarrow 16 = n-1$

$\Rightarrow n = 16+1$

$\Rightarrow n = 17$

The numbers are arranged in descending order. We need then 10^{th} number from the bottom.

The 10^{th} number from the bottom is the same as the 10^{th} number if the list were arranged in ascending order (which is our original AP) find the 10^{th} term (a_{10}) of the ascending AP:

$\Rightarrow a_{10} = a + (10-1) \times d$

$\Rightarrow a_{10} = 9 + (9) \times 3$

$\Rightarrow a_{10} = 9 + 27$

$\Rightarrow a_{10} = 36$

$\therefore$ The number at the tenth place from the bottom is 36.

8. (D)

In the given AP,

First term $a_1 = 6$

Last term $a_n = 216$

Common difference = 7

Now, to find the number of term, $n = ((a_n - a_1)/d) + 1$

$n = (216-6)7+1$

$n = 210/7+1$

$n = 30+1$

$n = 31$

So, middle term is $(n+1)/2$

$= (31+1)/2$

$= 16$

Now to calculate the middle term

$a_{16} = a_1 + 15 \times d$

$a_{16} = 6 + 15 \times 7$

$a_{16} = 6 + 105 = 111$

So the middle term of the given AP is 111

9. (B)

Let $f(x) = x^4 - ax^3 + bx^2 - cx + 8$

$f(1) = 4$

$1 - a + b - c + 8 = 4$

$-a + b - c = -5$ ...Eqn. (1)

$f(-1) = 3$

$1 + a + b + c + 8 = 3$

$a + b + c = -6$ …Eqn. (2)

Adding Equation (1) and (2) we have

$2b = -11$ or $b = -5.5$

Hence the answer is –5.5

10. (C)

$a = 3 + 2\sqrt{2}$

$\frac{1}{a} = \frac{1}{3+2\sqrt{2}}$

$\Rightarrow \frac{1}{a} = \frac{3-2\sqrt{2}}{\left(3+2\sqrt{2}\right)\times\left(3-2\sqrt{2}\right)}$

$\Rightarrow \frac{1}{a} = \frac{3-2\sqrt{2}}{3^2 - \left(2\sqrt{2}\right)^2}$

$\Rightarrow \frac{1}{a} = \frac{3-2\sqrt{2}}{9-8}$

$\Rightarrow \frac{1}{a} = 3 - 2\sqrt{2}$

Now,

$a + \frac{1}{a} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$

$\Rightarrow a + \frac{1}{a} = 6$

$\frac{a^6 - a^4 - a^2 + 1}{a^3}$

$\Rightarrow \left(a^3 + \frac{1}{a^3}\right) - \left(a + \frac{1}{a}\right)$

$\Rightarrow \left\{\left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)\right\} - a + \frac{1}{a}$

$\Rightarrow \left(6^3 - 3\times 6\right) - 6$

$\Rightarrow 216 - 18 - 6$

$\Rightarrow 192$

$\therefore$ The required value of $\frac{a^6 - a^4 - a^2 + 1}{a^3}$ is 192.

11. (A)

$\left(Ax\right)^3 + \left(By\right)^3 + CxyD = \left(5x^2 + 2y^2\right)\left(\sqrt{5}x + \sqrt{2}y\right)$

$\Rightarrow \left(Ax\right)^3 + \left(By\right)^3 + CxyD = 5\sqrt{5}x^3 + 2\sqrt{2}y^3 + 5\sqrt{2}x^2y + 2\sqrt{5}xy^2$

$\Rightarrow \left(Ax\right)^3 + \left(By\right)^3 + CxyD = \left(\sqrt{5}x\right)^3 + \left(\sqrt{2}y\right)^3 + \sqrt{10}xy\left(\sqrt{5}x + \sqrt{2}y\right)$

On comparing, $A = \sqrt{5}, B = \sqrt{2}, C = \sqrt{10}$

$\Rightarrow\ AB = C$

$\therefore$ The relation between A, B, and C is AB = C

12. (B)

The equations can be represented as:

$8x + 240y = 400 \qquad ...(i)$

$6x + 200y = 320 \qquad ...(ii)$

Solving these equations:

Subtract equation 2 from equation 1,

$\Rightarrow\ (8x - 6x) + (240y - 200y) = (400 - 320)$

$\Rightarrow\ 2x + 40y = 80$

$\Rightarrow\ x + 20y = 40$

$\Rightarrow\ x = 40 - 20y$

Substituting x = 40 – 20y in Equation 2,

$\Rightarrow\ 6(40 - 20y) + 200y = 320$

$\Rightarrow\ 240 - 120y + 200y = 320$

$\Rightarrow\ 240 + 80y = 320$

$\Rightarrow\ 80y = 80$

$\Rightarrow\ y = 1$

Substituting y = 1 in equation 1,

$\Rightarrow\ 8x + 240 = 400$

$\Rightarrow\ 8x = 160$

$\Rightarrow\ x = 20$

$\therefore$ The monthly electricity bill for a house with m rooms and consuming n units is 20m + n.

13. (A)

$x - y = \frac{x + y}{7} = \frac{xy}{6}$,

$\Rightarrow\ x - y = \frac{x + y}{7} = \frac{xy}{6} = K$

$\Rightarrow\ x - y = K;\ x + y = 7K;\ xy = 6K$

$\Rightarrow\ (x + y)^2 - (x - y)^2 = (7K)^2 - K^2$

$\Rightarrow\ x^2 + y^2 + 2xy - x^2 - y^2 + 2xy = 49K^2 - K^2$

$\Rightarrow\ 4xy = 48K^2$

Putting value of xy

$\Rightarrow\ 4 \times 6K = 48K^2$

$\Rightarrow\ 24K = 48K^2$

$\Rightarrow\ K = 0.5$

Now,

$xy = 6 \times 0.5$

$xy = 3$

Thus, the value of xy is 3

14. (D)

Let my current age = x years and my cousin's age = y years.

Three-fifths of my current age is the same as five-sixths of that of one of my cousins',

$\Rightarrow \frac{3x}{5}=\frac{5y}{6}$

$\Rightarrow 18x=25y$

My age ten years ago will be his age four years hence,

$\Rightarrow x-10=y+4$

$\Rightarrow y=x-14$,

$\Rightarrow 18x=25(x-14)$

$\Rightarrow 18x=25x-350$

$\Rightarrow 7x=350$

$\therefore x=50$ years

15. (C)

We know that $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

In $x^3+8y^3+z^3-6xyz$, $a=x$, $b=2y$ and $c=z$

By using the above equation, we get

$x^3+8y^3+z^3-6xyz=(x+2y+z)\left(x^2+(2y)^2+z^2-x(2y)-(2y)(z)-zx\right)$

$=(x+2y+z)(x^2+4y^2+z^2-2xy-2yz-zx)$.

16. (A)

Let $y=x^2$. The given polynomial can be written as a quadratic equation in y:

$y^2-10y+22=0$

Using the quadratic formula to find the roots for y, where a = 1, b = –10, c = 22

$$y=\frac{-(-10)\pm\sqrt{(-10)^2-4\times1\times22}}{2\times1}$$

$$\Rightarrow y=\frac{10\pm\sqrt{100-88}}{2}$$

$$\Rightarrow y=\frac{10\pm\sqrt{12}}{2}$$

$$\Rightarrow y=\frac{10\pm2\sqrt{3}}{2}$$

$\Rightarrow y=5\pm\sqrt{3}$

So, the two roots for y are:

$y_1=5+\sqrt{3}$

$y_2=5-\sqrt{3}$

Therefore, the quadratic in y can be factored as:

$(y-y_1)(y-y_2)=\left(y-(5+\sqrt{3})\right)\left(y-(5-\sqrt{3})\right)$

Substitute back $y=x^2$:

$\left(x^2-\left(5+\sqrt{3}\right)\right)\left(x^2-\left(5-\sqrt{3}\right)\right)$

$\therefore$ The factorization of the polynomial x^4-10x^2+22 into a product of two quadratic polynomials is $\left(x^2-\left(5+\sqrt{3}\right)\right)\left(x^2-\left(5-\sqrt{3}\right)\right)$.

17. (A)

Let $2^x=3^y=6^{-z}=k$

From this:

$2^x=k$

$\Rightarrow\ 2=k^{1/x}$ (Equation 1)

$3^y=k$

$\Rightarrow\ 3=k^{1/y}$ (Equation 2)

$6^{-z}=k$

$\Rightarrow\ 6=k^{-1/z}$ (Equation 3)

We know that $6=2\times 3$

Substitute values from Equation 1, Equation 2, and Equation 3:

$\Rightarrow\ k^{-1/z}=k^{1/x}\times k^{1/y}$

$\Rightarrow\ k^{-1/z}=k^{(1/x)+(1/y)}$

Equating the exponents (since bases are same):

$\Rightarrow\ -\frac{1}{z}=\frac{1}{x}+\frac{1}{y}$

$\Rightarrow\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$\therefore\ \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to 0.

18. (B)

$(5x-3y)^2=(5x+3y)^2-4\times 5x.3y$

$\Rightarrow\ (5x-3y)^2=(5x+3y)^2-60xy$

$\Rightarrow\ (5x-3y)^2=15^2-60\times 3=225-180=45$

$(5x-3y)=\sqrt{45}$

$\Rightarrow\ 5x-3y=3\sqrt{5}$

$\therefore$ The correct option is (B).

19. (A)

Let boat speed = 8x, stream speed = 5x

$\Rightarrow$ Upstream speed = 8x – 5x = 3x

$\Rightarrow\ 33.2\div 3x=1.383$

$\Rightarrow\ 3x=33.2\div 1.383=24$

$\Rightarrow\ x=8$

$\Rightarrow$ Boat speed = 8x = 64 km/h

$\Rightarrow$ Stream speed = 5x = 40 km/h

$\Rightarrow$ Upstream speed = 64 – 40 = 24 km/h

$\Rightarrow$ Downstream speed = 64 + 40 = 104 km/h

Time for 47.1 km upstream $= 47.1 \div 24 = 1.9625$ hours

Time for 55.9 km downstream $= 55.9 \div 104 = 0.5375$ hours

Total time = 1.9625 + 0.5375 = 2.5 hours

$\therefore$ The correct answer is 2.5 hours.

20. (B)

Let $\sqrt{6+\sqrt{6+\sqrt{6+........\infty}}}$

$x = \sqrt{6+x} \Rightarrow x^2 = 6 + x \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$

$\Rightarrow x = 3$ or $x = -2$

But, the given expression is positive.

Hence, the value of the given expression is 3.

Section – B

21. (A)

$2x^2 + kx + 5 = 0$ has no real roots so $D < O$

$k^2 - 40 < 0$

$\left(k - \sqrt{40}\right)\left(k + \sqrt{40}\right) < 0$

$k \in \left(-\sqrt{40}, \sqrt{40}\right)$

$x^2 + (k-5)x + 1 = 0$ has two district real roots so $D > O$

$(k-5)^2 - 4 > 0$

$k^2 - 10k + 21 > 0$

$(k-3)(k-7) > 0$

$k \in (-\infty, 3) \cup (7, \infty)$

Therefore possible value of K are $-6, -5, -4, -3, -2, -1, 0, 1, 2$ In 9 total 9 integer value of K are possible.

22. (D)

Let $a_1, d_1; a_2, d_2$ be first term and common difference of two A.P.'s respectively.

Given: $\dfrac{S_n \text{ of Ist AP}}{S_n \text{ of IInd AP}} = \dfrac{7n+1}{4n+27}$

$$\Rightarrow \frac{\frac{n}{2}\left[2a_1 + (n-1)d_1\right]}{\frac{n}{2}\left[2a_2 + (n-1)d_2\right]} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{7n+1}{4n+27} \quad .....(1)$$

For mth term, we have

$$\frac{t_m \text{ of Ist AP}}{t_m \text{ of IInd AP}} = \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2} \quad .....(2)$$

Compare LHS of (10 with RHS of (2)

Put $\frac{n-1}{2} = m-1$

$\Rightarrow \quad n-1 = 2m-2$

$\Rightarrow \quad n = 2m-1$

Replace n by 2m – 1 in (1) we get

$$\frac{a_1+(m-1)d_1}{a_2+(m-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

$\therefore$ Required rate is (14n – 6) : (8m + 23)

23. (B)

The price of petrol (in Rupees per litre) on m^{th} day of the year is 80 + 0.1m where m = 1, 2, 3 ….100 and thereafter remains constant.

On 100^{th} day price $= 80 + 0.1 \times 100 = \text{Rs. } 90$ thereafter remains constant

On the other hand, the price of diesel (in Rupees per litre) on n^{th} day of 2021 is 69 + 0.15n for any n.

For equal prices there are two cases

Case (i): before 100^{th} day

80 + 0.1m = 69 + 0.15m (where m = n prices become equal)

m = 220 days

Thus it is not possible as m < 100 days

Case (ii): $m \geq 100$

As the price will remain constant

69 + 0.15m = 90

m = 140 days

Thus, the date will be 20^{th} May

Hence, option (B) is correct

24. (B)

$a_n - 6a_{n-1} - 2a_{n-2} = 0$

$a_{10} - 6a_9 - 2a_8 = 0$

$a_{10} - 2a_8 = 6a_9$

$$\frac{a_{10} - 2a_8}{a_9} = 6$$

25. (B)

Let roots of the quadratic equation are α, β.

Given, $\lambda = \frac{\alpha}{\beta}$ and $\lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$

$$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = 1 \quad .....(1)$$

The quadratic equation is, $3m^2x^2 + m(m-4)x + 2 = 0$

$\therefore \quad \alpha+\beta=\frac{m(4-m)}{3m^2}=\frac{4-m}{3m}$ and $\alpha\beta=\frac{2}{3m^2}$

Put these values in eq. (1),

$$\frac{\left(\frac{4-m}{3m}\right)^2}{\frac{2}{3m^2}}=3$$

$\Rightarrow \quad (m-4)^2=18$

$\Rightarrow \quad m=4\pm\sqrt{18}$

Therefore, least value is $4-\sqrt{18}=4-3\sqrt{2}$

Section – C

26. (C)

$x=2^{12(7+4\sqrt{3})}$

$x^{\frac{7}{2}}=2^{42(7+4\sqrt{3})}$

$x^{2\sqrt{3}}=2^{24\sqrt{3}(7+4\sqrt{3})}$

$\frac{x^{\frac{7}{2}}}{x^{2\sqrt{3}}}=2^{(7+4\sqrt{3})(42-24\sqrt{3})}=2^{(7+4\sqrt{3})(7-4\sqrt{3})6}=2^6=64$

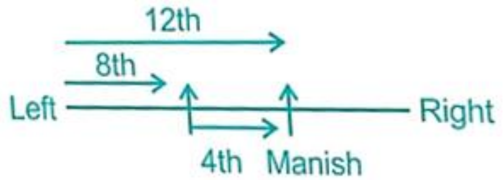
Hence C is correct answer.

27. (C)

The logic followed here is:

* In a row of 21 men
* Manish was shifted by four places towards the right, now he becomes 12th from the left end

On combining the statements, we get



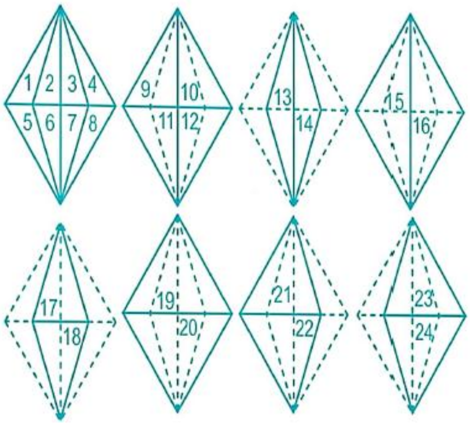
Earlier position from the left end = 12th from the left end – between shifted four places towards the right

$=12-4$

$=8$th

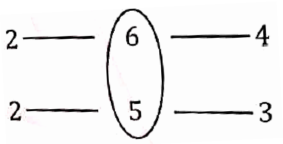
Position from right end : 21 – 8 + 1 = 14th

28. (D)



Hence there are "24" triangles in the figure

29. (A)



3 at the top

30. (B)

Pair of opposite letters