Work, Power & Energy

JEE Main Exercise

1. (b)

$W = \int F\,dx$

$W = \int_0^d (a + bx)\,dx$

$= \left[ax + \dfrac{bx^2}{2}\right]_0^d = ad + \dfrac{db^2}{2}$

2. (b)

$W = \Delta K$

$W = K_2 - K_1$

$W = \dfrac{1}{2} \times 2 \times 0^2 - \dfrac{1}{2} \times 2 \times (20)^2$

$W = -400\text{ J}$

3. (a)

$W = \mathbf{F} \cdot \mathbf{s}$

$W = F\,s\cos\theta$

$W = FR$

4. (a)

$v = a\sqrt{s}$

$\dfrac{ds}{dt} = a\sqrt{s}$

$\Rightarrow \int_0^s \dfrac{ds}{\sqrt{s}} = a\int_0^t dt$

$\Rightarrow 2\sqrt{s} = at$

$\Rightarrow s = \dfrac{1}{4}a^2t^2$

$W = \Delta K = \dfrac{1}{2}m\left(a\sqrt{s}\right)^2 - \dfrac{1}{2}m(0)^2$

$= \dfrac{1}{2}ma^2s - 0 = \dfrac{1}{2}ma^2\left(\dfrac{1}{4}a^2t^2\right)$

$= \dfrac{1}{8}ma^4t^2$

5. (a)

Applying work-energy theorem,

$$W_{mg} + W_N + W_{\text{friction}} + W_{\text{spring}} = \Delta K$$

$$0 + 0 - (\mu mg)x + \frac{1}{2}K\left(0^2 - x^2\right) = 0 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad -50x - 50x^2 = -\frac{1}{2} \times 50 \times 2^2$$

$$\Rightarrow \quad x^2 + x - 2 = 0$$

$$\Rightarrow \quad x = 1\text{m}$$

6. (b)

From A to B, applying work-energy theorem,

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad -mg(R-h) + 0 = 0 - \frac{1}{2}mu^2$$

$$u = \sqrt{2g(R-h)}$$

7. (d)

$U = 2x + 5y - xy$

$$\mathbf{F} = \left(-\frac{\partial U}{\partial x}\right)\hat{\mathbf{i}} + \left(-\frac{\partial U}{\partial y}\right)\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{F} = (-2+y)\hat{\mathbf{i}} + (-5+x)\hat{\mathbf{j}}$$

At (2, –2)

$$\mathbf{F} = -4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$$

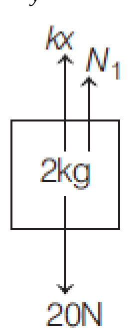
$$\Rightarrow \quad \mathbf{a} = -4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$$

$$\Rightarrow \quad a = 5\ \text{m/s}^2$$

8. (b)

For 2 kg block

$$\sum F_y = 0$$



$$\Rightarrow \quad Kx + N_1 = 20$$

When 2 kg leaves contact $(N_1 = 0)$

$$\Rightarrow \quad Kx = 20 \Rightarrow x = 0.5\ \text{m}$$

Applying work-energy theorem for 5 kg block

$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +5\times10\times0.5+\frac{1}{2}\times40\times\left(0^2-(0.5)^2\right)$$

$$=\frac{1}{2}\times5\times v^2-0$$

$$\Rightarrow \quad v = 2\sqrt{2}\text{ m/s}$$

9. (a)

Applying work-energy theorem for (2 kg + 1 kg) system

$$W_{mg} + W_T = \Delta K_{\text{system}}$$

$$\Rightarrow \quad (+2\times10\times0.6-1\times10\times0.6)+0=\left(\frac{1}{2}\times2\times v^2+\frac{1}{2}\times1\times v^2\right)-(0+0)$$

$$\Rightarrow \quad v = 2\text{ m/s}$$

10. (c)

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad v = x\sqrt{\frac{k}{m}} = 0.05\sqrt{\frac{600}{15\times10^{-3}}}$$

$$\Rightarrow \quad v = 10\text{ m/s}$$

$$R_{\max} = \frac{v^2}{g} = \frac{(10)^2}{10} = 10\text{ m}$$

11. (c)

$$P = \frac{3t^2}{2}$$

$$\Rightarrow \quad \frac{dK}{dt} = \frac{3t^2}{2}$$

$$\Rightarrow \quad \int_0^K dK = \int_0^t \frac{3t^2}{2}dt$$

$$\Rightarrow \quad K = \frac{t^3}{2}$$

At $t = 2$,

$$K = \frac{2^3}{2} = \frac{1}{2}\times2\times v^2$$

$$\Rightarrow \quad v = 2\text{ m/s}$$

12. (d)

$$U_1 = \frac{1}{2}kx^2$$

$$U_2 = \frac{1}{2}K(x+y)^2$$

$$W_{\text{ext}} = \Delta U + \Delta K = \left(\frac{1}{2}k\left(x+y\right)^2 - \frac{1}{2}kx^2\right) + 0$$

$$= \frac{1}{2}ky\left(2x+y\right)$$

13. (b)

$\mathbf{v} = 2\hat{\mathbf{i}} + 4t\,\hat{\mathbf{j}}$

$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 4\hat{\mathbf{j}}$

$P = \mathbf{F}\cdot\mathbf{v} = \left(ma\right)\cdot\mathbf{v} = 2\left(16t\right)$

At $t = 5$, $P = 160\,\text{W}$

14. (a)

$W = \Delta K$

$$W = \frac{1}{2}m\left(v^2 - u^2\right)$$

From rest to speed v,

$$W = \frac{1}{2}m\left(v^2 - 0^2\right) = \frac{1}{2}mv^2$$

From v to $2v$,

$$W = \frac{1}{2}m\left(\left(2v\right)^2 - v^2\right) = \frac{3}{2}mv^2$$

15. (d)

$K_{\text{man}} = \frac{1}{2}k_{\text{boy}}$

$$\Rightarrow \quad \frac{1}{2}mv_{\text{man}}^2 = \frac{1}{2}\left(\frac{1}{2}\frac{m}{2}v_{\text{boy}}^2\right)$$

$$\Rightarrow \quad v_{\text{man}} = \frac{v_{\text{boy}}}{2}$$

Now, $\frac{1}{2}m\left(v_{\text{man}}+1\right)^2 = \frac{1}{2}\left(\frac{m}{2}\right)v_{\text{boy}}^2$

$$\Rightarrow \quad \left(V_{\text{man}}+1\right)^2 = \frac{\left(2v_{\text{man}}\right)^2}{2}$$

$$\Rightarrow \quad V_{\text{man}}^2 + 1 + 2v_{\text{man}} = 2v_{\text{man}}^2$$

$$\Rightarrow \quad v_{\text{man}}^2 - 2v_{\text{man}} - 1 = 0$$

$$\Rightarrow \quad v_{\text{man}} = \frac{2+\sqrt{8}}{2} = 1+\sqrt{2}$$

16. (a)

$P = Fv =$ constant

$\Rightarrow \quad mav =$ constant

$\Rightarrow \quad m\frac{dv}{dt}v =$ constant

$\Rightarrow \quad \int v\,dv \propto \int dt$

$\Rightarrow \quad v^2 \propto t$

$\Rightarrow \quad v \propto \sqrt{t}$

$\frac{ds}{dt} \propto \sqrt{t}$

$\Rightarrow \quad \int ds \propto \int \sqrt{t}\,dt$

$\Rightarrow \quad s \propto t^{3/2}$

$\frac{s}{v} = \frac{t^{3/2}}{t^{1/2}} = t$

17. (c)

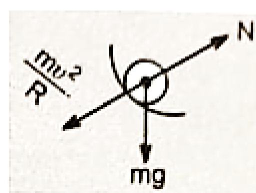
Speed of the block is maximum when acceleration of block is zero.

So, $mg\cos\theta = kx$

$\Rightarrow \quad x = \frac{mg\cos\theta}{k}$

18. (b)

By energy conservation, we have



$\frac{1}{2}mv^2 = mgR\sin\theta$

$\Rightarrow \quad \frac{mv^2}{R} = 2mg\sin\theta$

$N = \frac{mv^2}{R} + mg\sin\theta$

$N = \frac{mv^2}{R} + mg\sin\theta$

19. (d)

$K = \frac{p^2}{2m}$

$\Rightarrow \quad \log_e K = \log_e\left(\frac{p^2}{2m}\right)$

$\Rightarrow \quad \log_e K = \log_e p^2 - \log_e 2m$

$\Rightarrow \quad \log_e K = 2\log_e p - \log_e 2m$

$\Rightarrow \quad y = 2x - \log_e 2m$

20. (a)

When the object is lowered very slowly to its equilibrium position, then at equilibrium position

$Kx = mg$

$\Rightarrow \quad K(0.1) = 0.5 \times 10$

$\Rightarrow \quad K = 50 \text{ N/m}$

Applying work-energy theorem,

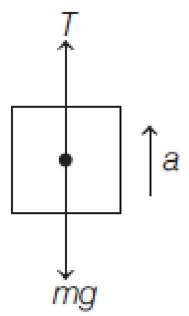
$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +0.5 \times 10 \times 0.1 + \frac{1}{2} \times 50\left(0^2 - (0.1)^2\right)$$

$$= \frac{1}{2} \times 0.5v^2 - 0$$

$\Rightarrow \quad v = 1 \text{ m/s}$

21. (a)



$$\sum F_y = ma_y$$

$\Rightarrow \quad T - mg = ma$

$\Rightarrow \quad T = m(g + a)$

$$W_T = Fs \cos\theta$$

$$= m(g+a)\frac{1}{2}at^2 \cos 0°$$

$$= \frac{m}{2}(g+a)at^2$$

22. (c)

$P = Fv$ = constant

mav = constant

$\Rightarrow \quad \frac{dv}{dt}v = \text{constant}$

$\Rightarrow \quad \int v\,dv \propto \int dt$

$\Rightarrow \quad v^2 \propto t$

$\Rightarrow \quad v \propto \sqrt{t}$

Now, $v = \frac{dx}{dt} \propto \sqrt{t}$

$\Rightarrow \quad \int dx \propto \int \sqrt{t}\,dt$

$\Rightarrow \quad x \propto t^{3/2}$

23. (d)

Applying work-energy theorem

$$W_{mg} + W_N + +W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +mg\left(\frac{3R}{2}\right)+0+\frac{1}{2}k\left(\left(\frac{R}{2}\right)^2-0^2\right)$$
$$=\frac{1}{2}mv^2-0$$
$$\Rightarrow \quad v=\sqrt{4gR}$$

24. (b)

Lets say maximum elongation in the spring is x.

$$W_{mg}+W_N+W_F+W_{\text{spring}}=\Delta K$$
$$\Rightarrow \quad 0+0+Fx+\frac{1}{2}k\left(0^2-x^2\right)=0-0$$
$$\Rightarrow \quad x=\frac{2F}{k}$$
$$W_F=Fx=F\left(\frac{2F}{k}\right)=\frac{2F^2}{k}$$

25. (a)

$$W_{\text{friction}}=\int -\mu mg\cos\theta dx=\int_0^5 -\mu_0\, x\, mg\cos\theta\, dx$$
$$=\frac{-\mu_0 mg\cos\theta\, s^2}{2}$$

Applying work-energy theorem

$$W_{mg}+W_N+W_{\text{friction}}=\Delta K$$
$$\Rightarrow \quad (mg\sin\theta)s+0-\frac{(\mu_0 mg\cos\theta)s^2}{2}=0-0$$
$$\Rightarrow \quad s=\frac{2\tan\theta}{\mu_0}$$

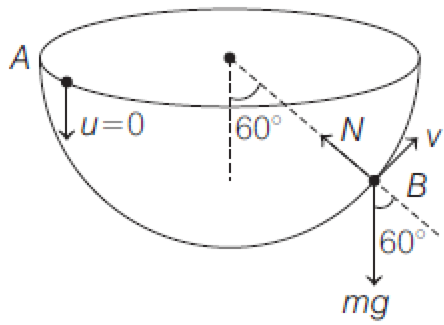
26. (c)

Using work-energy theorem between A and B.

$$W_{mg}+W_N=\Delta K$$
$$\Rightarrow \quad +mgR\cos 60°+0=\frac{1}{2}mv^2-0$$
$$\Rightarrow \quad v=\sqrt{gR}$$

Equation for centripetal force at B,

$$N-mg\cos 60°=\frac{mv_B^2}{R}$$

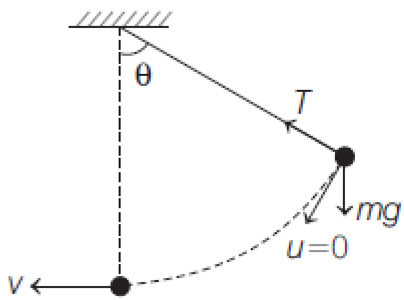


$N = 1.5\,mg$

27. (d)

In extreme position,

$$a_{\text{centripetal}} = 0$$



$$a_{\text{tantential}} = g\sin\theta$$

$$a_1 = \sqrt{a_c^2 + a_T^2} = g\sin\theta$$

Using work-energy theorem,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mgl\left(1-\cos\theta\right)+0 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{2gl\left(1-\cos\theta\right)}$$

$$a_{\text{centriperal}} = \frac{v^2}{l} = 2g\left(1-\cos\theta\right)$$

$$a_{\text{tantential}} = 0$$

$$a_2 = \sqrt{a_c^2 + a_T^2} = 2g\left(1-\cos\theta\right)$$

$$\because \quad a_1 = a_2$$

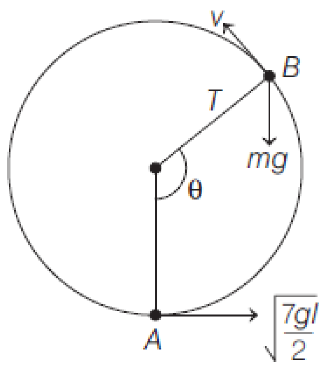
$$\Rightarrow \quad g\sin\theta = 2g\left(1-\cos\theta\right)$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

28. (c)

Using work-energy theorem between A and B,

$$W_{mg} + W_T = \Delta K$$



$$-mgl\left(1+\cos\left(180°-\theta\right)\right)+0=\frac{1}{2}mv^2-\frac{1}{2}m\left(\sqrt{\frac{7gl}{2}}\right)^2$$

$$v^2=\frac{7gl}{2}-2gl\left(1-\cos\theta\right) \qquad \text{...(i)}$$

At *B*,

$$mg\cos\left(180°-\theta\right)+T=\frac{mv^2}{l}$$

$$\Rightarrow \quad -mg\cos\theta+0=\frac{m}{l}\left(\frac{7gl}{2}-2gl\left(1-\cos\theta\right)\right)$$

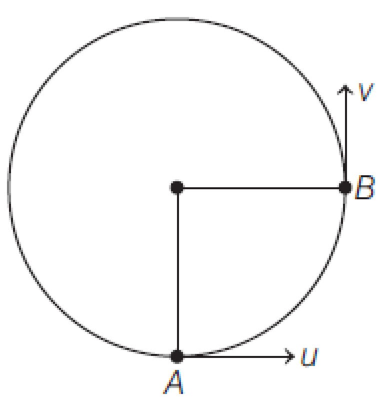
$$\Rightarrow \quad -2\cos\theta=7-4\left(1-\cos\theta\right)$$

$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$

$$\theta=120°$$

29. (d)

$$\mathbf{v}_A=u\hat{\mathbf{i}}$$



Using work-energy theorem between *A* and *B*,

$$W_{mg}+W_T=\Delta T$$

$$\Rightarrow \quad -mgL+0=\frac{1}{2}mv^2-\frac{1}{2}mu^2$$

$$\Rightarrow \quad v=\sqrt{u^2-2gL}$$

$$\Rightarrow \quad \mathbf{v}_B=\sqrt{u^2-2gL}\,\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{v}_B - \mathbf{v}_A = \sqrt{u^2 - 2gL}\,\hat{\mathbf{j}} - u\,\hat{\mathbf{i}}$$

$$|\mathbf{v}_B - \mathbf{v}_A| = \sqrt{\left(\sqrt{u^2 - 2gL}\right)^2 + u^2} = \sqrt{2\left(u^2 - gL\right)}$$

30. (a)

Using work - energy theorem between A and B,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mgl\cos\theta + 0 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{2gl\cos\theta}$$

At *B*,

Centripetal acceleration $= \dfrac{v^2}{l} = \dfrac{\left(\sqrt{2gl\cos\theta}\right)^2}{l} = 2g\cos\theta$

Tangential acceleration $= \sqrt{a_c^2 + a_T^2}$

$$= \sqrt{(2g\cos\theta)^2 + (g\sin\theta)^2}$$

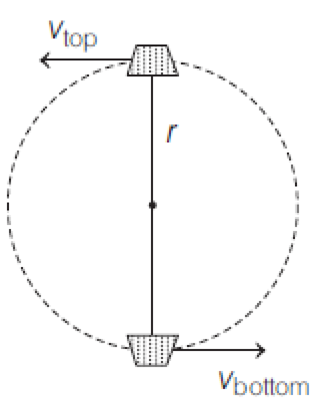
$$= g\left(\sqrt{4\cos^2\theta + \sin^2\theta}\right)$$

$$= g\sqrt{3\cos^2\theta + 1}$$

31. (a)

At the top most point,

$$T + mg = \frac{mv_{\text{top}}^2}{r}$$



For v_{top} to be minimum

$$T = 0$$

$$\Rightarrow \quad v_{\text{top}} = \sqrt{rg}$$

Using work - energy theorem,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mg(2r) + 0 = \frac{1}{2}mv_{\text{bottom}}^2 - \frac{1}{2}m\left(\sqrt{rg}\right)^2$$

$$v_{\text{bottom}} = \sqrt{5rg} \text{ and } T - mg = \frac{mv_{\text{bottom}}^2}{r}$$

$$\Rightarrow \quad T - mg = \frac{m}{r}\left(\sqrt{5rg}\right)^2$$

$$\Rightarrow \quad T = 6\,mg$$

32. (a)

Using work-energy theorem between A and B,

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mg\left(2R\right) + 0 = \frac{1}{2}mv_2^2 - 0$$

$$\Rightarrow \quad v_2 = \sqrt{4gR}$$

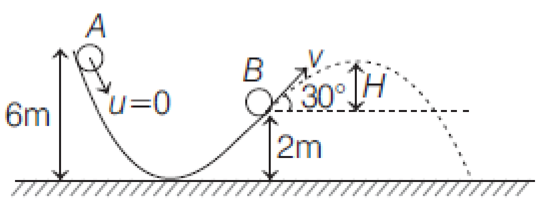
At point 2,

$$N + mg = \frac{mv_2^2}{R}$$

$$\Rightarrow \quad N + mg = 4mg$$

$$\Rightarrow \quad N = 3mg$$

33. (3)



Between A and B

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mg\left(6-2\right) + 0 = \frac{1}{2}mv^2 - 0$$

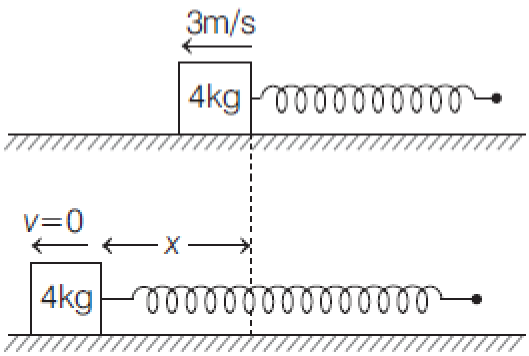
$$\Rightarrow \quad v = \sqrt{8g}$$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{\left(\sqrt{8g}\right)^2 \sin^2 30°}{2g} = 1\text{ m}$$

$$H_{\max} = 2 + 1 = 3\text{ m}$$

34. (6)

With respect to free end, velocity of block is 1 − (−2) = 3 m/s left.



Elongation in the spring will be maximum when velocity of block with respect to free end is zero.

$$W_{mg} + W_N + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad 0 + 0 + \frac{1}{2} \times \frac{100}{10^{-2}} \left(0^2 - x^2\right) = 0 - \frac{1}{2} \times 4 \times 3^2$$

$$x = 0.06 \text{ m} = 6 \text{ cm}$$

35. (8)

$$P = 640 - 16v - 8v^2$$

$$\Rightarrow \quad Fv = 640 - 16v - 8v^2$$

For velocity to be maximum, $a = 0$

$$\Rightarrow \quad F = 0$$

$$\Rightarrow \quad 0 = 640 - 16v - 8v^2$$

$$\Rightarrow \quad v^2 + 2v - 80 = 0$$

$$\Rightarrow \quad v = 8 \text{ m/s}$$

36. (2)

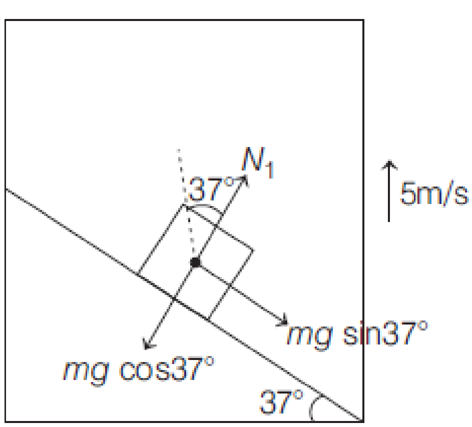
Using work-energy theorem between A and B

$$W_{mg} + W_T + W_F = \Delta K$$

$$\Rightarrow \quad -mgl\left(1 - \cos 37°\right) + 0 + \left(\frac{mg}{2}\right)\left(l \sin 37°\right)$$

$$= \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 \Rightarrow v_B = 2 \text{ m/s}$$

37. (320)



$$\sum F_y = 0$$

$\Rightarrow \quad N_1 - mg\cos 37° = 0$

$\Rightarrow \quad N_1 = mg\cos 37°$

$$W_{N_1} = Fs\cos\theta$$

$$= (mg\cos 37°)(vt)\cos 37° = 320 \text{ J}$$

38. (2)

Using work-energy theorem for block,

$$W_{mg} + W_N + W_{\text{friction}} + W_{\text{spring}} = \Delta K$$

$\Rightarrow \quad +mg\sin 37° \times 0.5 + 0 - \frac{1}{8}(mg\cos 37°)(0.5) + \frac{1}{2}\times 8\left(0^2 - (0.5)^2\right)$

$$= \frac{1}{2}mv^2 - 0$$

$\Rightarrow \quad v = 2 \text{ m/s}$

39. (1.5)

Using work-energy theorem for block,

$$W_{mg} + W_{\text{spring 1}} + W_{\text{spring 2}} + \Delta K$$

$\Rightarrow \quad +2\times 10\times(1+x) + \frac{1}{2}\times 24\left(0^2 - (1+x)^2\right) + \frac{1}{2}\times 24\left(0^2 - x^2\right) = 0 - 0$

$\Rightarrow \quad 20 + 20x - 12 - 12x^2 - 24x - 12x^2 = 0$

$\Rightarrow \quad 24x^2 + 4x - 8 = 0$

$\Rightarrow \quad 6x^2 + x - 2 = 0$

$\Rightarrow \quad 6x^2 + 4x - 3x - 2 = 0$

$\Rightarrow \quad 2x(3x+2) - (3x+2) = 0$

$\Rightarrow \quad x = 0.5$

So, maximum extension in upper spring

$$= 1 + 0.5 = 1.50 \text{ m}$$

40. (730)

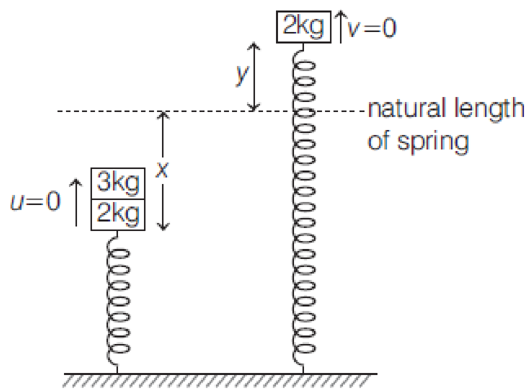
Using work-energy theorem for the object

$$W_{mg} + W_{\text{air}} = \Delta K$$

$\Rightarrow \quad +5\times 9.8\times 20 + W_{\text{air}} = \frac{1}{2}\times 5\times 10^2 - 0$

$$W_{\text{air}} = -730 \text{ J}$$

41. (1.50)



Initially lets take compression to be x.

$$\sum F_y = 0$$

$$\Rightarrow \quad 30 + 20 - 40x = 0$$

$$\Rightarrow \quad x = 1.25\ \text{m}$$

After removing 3 kg block, lets take maximum elongation in the spring to be y.

Using work-energy theorem for 2 kg block,

$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad -2\times 10(1.25 + y) + \frac{1}{2}\times 40\left[(1.25)^2 - y^2\right]$$

$$= 0 - 0$$

$$\Rightarrow \quad y = 0.25\ \text{m}$$

So, the maximum height reached by 2 kg block

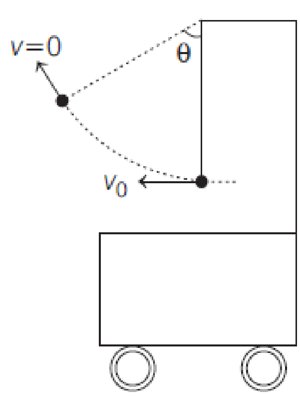
= 1.25 + 0.25

= 1.5 m

42. (7)

Using work - energy theorem,

$$W_{mg} + W_T = \Delta T$$



$$\Rightarrow \quad -mgl(1 - \cos 60°) + 0$$

$$= 0 - \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad v_0 = \sqrt{gl} = \sqrt{9.8\times 5} = 7\ \text{m/s}$$

SECTION – I

1. (B)

By work-energy theorem

$W = \Delta K$

$\Rightarrow W = \frac{1}{2}m\left(v_f^2 - v_i^2\right) \Rightarrow W = \frac{1}{2}\times 0.5\times\left(16^2 - 4^2\right)$

$\Rightarrow W = \frac{1}{4}\times 240 \Rightarrow W = 60J$

2. (D)

By work − energy theorem

$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}\times 0.5\times\left(b^2.4^5\right)$

$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{4^2}\times 4^5 = 16J$

3. (B)

From work-energy theorem,

$W_{Porter} + W_{mg} = \Delta K.E = 0 \quad (\because \text{velocity constant})$

Or, $W_{Porter} = -W_{mg} = -mgh$

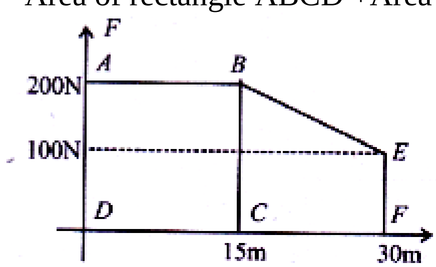
$\therefore W_{Porter} = -80\times 9.8\times\frac{80}{100} = -627.2J$

4. (D)

The given situation can be drawn graphically as shown in figure.

Work done = Area under F-x graph

= Area of rectangle ABCD +Area of trapezium BCFE



$W = (200\times 15) + \frac{1}{2}(100+200)\times 15 = 3000 + 2250$

$\Rightarrow W = 5250J$

5. (C)

Work done, $W = \int \vec{F}.\vec{ds} = \left(-x\hat{i} + y\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right)$

$\Rightarrow W = -\int_1^0 xdx + \int_0^1 ydy = \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1J$

6. (D)

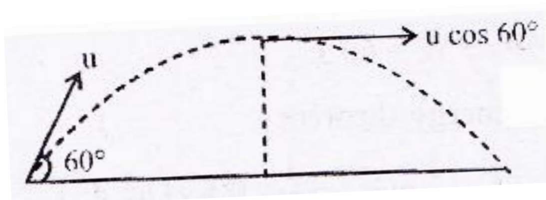
Here, $N - mg = ma = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

N-normal reaction

Now, work done by normal reaction 'N' on block in time, $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}g/2t^2\right)$

Or, $W = \frac{3mg^2t^2}{8}$

7. (C)



At maximum height, we only have horizontal component of velocity . So, Velocity $v = u\cos 60^\circ = \frac{u}{2}$

$\therefore$ K.E. at top most point $= \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{E}{4}$

8. (C)

Momentum of a body is increased by $P' = P + \frac{20}{100}P = 1.2P$

Percentage change in $KE = \frac{K' - K}{K} \times 100$

$$= \left(\frac{\frac{P'^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}}\right) \times 100 = \left[(1.2)^2 - 1\right] \times 100 = 44\%$$

9. (A)

Using $mv = \sqrt{2mk} \Rightarrow v = \frac{1}{m}\sqrt{2mk}$

So, $u = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 90} = 30\,m/s$

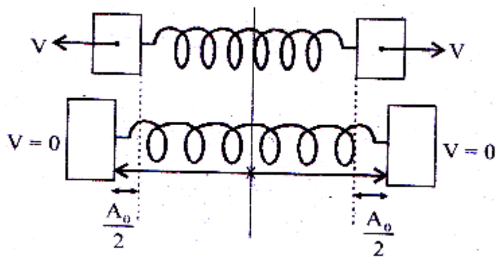
$v = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 40} = 20\,m/s$

$a = \frac{20 - 30}{1} = -10\,m/s^2$ So, $s = \frac{u^2}{2a} = 45\,m$

10. (B)

Given, spring constant of spring , $K = 2\,Nm^{-1}$

Mass of block, $m = 250\,g = \frac{250}{1000}g = \frac{1}{4}kg$



Using energy conservation

$$\frac{1}{2}mv^2 \times 2 = \frac{1}{2}kx^2 \Rightarrow \frac{1}{4}v^2 = \frac{1}{2} \times 2 \times x^2$$

$$\therefore x = \frac{v}{2}$$

11. (B)

Kinetic energy, K.E. $\frac{p^2}{2m}$

$$\frac{K.E_1}{K.E_2} = \left(\frac{P_1}{P_2}\right)^2 \times \left(\frac{m_2}{m_1}\right) = \left(\frac{1}{2}\right)^2 \times \frac{8}{5} = \frac{2}{5}$$

12. (D)

By law of conservation of mechanical energy $\Delta k = -\Delta U$

$$\Rightarrow k_f - k_i = U_i - U_f \Rightarrow k_f = mgy - mg[y - y_0]$$

$$[\because k_i = 0, U_i = mgy \text{ and } U_f = mg(y - y_0)]$$

$$\Rightarrow k_f = mgy_0$$

13. (A)

At maximum height , $\nu = 0$

$$\Rightarrow m\nu = 0 \Rightarrow P = 0$$

14. (C)

By work-energy theorem,

$$\Delta k = W_{\text{all forces}} = \int \vec{F}.d\vec{r}$$

$$= \int \left(4x\hat{i} + 3y^2\hat{j}\right).\left(dx\hat{i} + dy\,\hat{j}\right) = \int_1^2 4x\,dx + \int_2^3 3y^2 dy$$

$$= 4\left[\frac{x^2}{2}\right]_1^2 + 3\left[\frac{y^3}{3}\right]_2^3 = 2\left[2^2 - 1^2\right] + \left[3^3 - 2^3\right]$$

$$= 6 + 19 = 25J$$

15. (A)

Work done by air friction= Final kinetic energy. Initial potential energy $W_{air-friction} = \frac{1}{2}mv^2 - mgh$

$$= \frac{1}{2}m\left(0.8\sqrt{gh}\right)^2 - mgh$$

$$W_{air-friction} = \frac{64}{2} mgh - mgh = -0.68\, mgh$$

16. (C)

We know area under F-x graph gives the work done by the body

$$\therefore W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2 = 2.5 + 4 = 6.5 J$$

Using work energy theorem,

$w = \Delta KE$ = work done $\therefore \Delta K.E. = 6.5 J$

17. (C)

$l_1 + l_2 = l$ and $l_1 = n l_2$ $\quad \therefore l_1 = \frac{nl}{n+1}$ and $l_2 = \frac{l}{n+1}$

As $k \propto \frac{1}{l}$, $\quad \therefore \frac{k_1}{k_2} = \frac{l/(n+1)}{(nl)/(n+1)} = \frac{1}{n}$

18. (E)

Velocity of 1 kg block just before it collides with 3 kg block $= \sqrt{2gh} = \sqrt{2000}\, m/s$

Using principle of conservation of linear momentum just after collision, we get

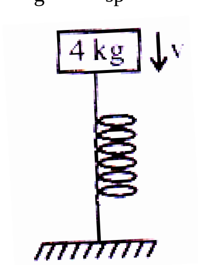
$$1 \times \sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4} m/s$$

Initial compression of spring

$$1.25 \times 10^6 x_0 = 30 \Rightarrow x_0 \approx 0$$

Using work energy theorem,

$$W_g + W_{sp} = \Delta KE$$



$$\Rightarrow 40 \times x + \frac{1}{2} \times 1.25 \times 10^6 \left(0^2 - x^2\right) = 0 - \frac{1}{2} \times 4 \times v^2$$

Solving $x \approx 2cm$

19. (A)

$$W = u_f - u_i = 0 - \left(-\frac{mg}{n} \times \frac{L}{2n}\right) = \frac{MgL}{2n^2}$$

20. (C)

$$m\nu = (m+M)V'$$

Or $\nu = \frac{m\nu}{m+M} = \frac{m\nu}{m+4m} = \frac{\nu}{5}$

Using conservation of ME, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+4m)\left(\frac{v}{5}\right)^2 + mgh \text{ or } h = \frac{2}{5}\frac{v^2}{g}$$

21. (D)

When force 'F' is applied, initially $F > F_s$. As F_s will be increase, suppose after x distance $F = F_s$ and there is equilibrium. At this moment block has maximum velocity.

$$F = kx \Rightarrow x = \frac{F}{k}$$

From work-energy theorem,

$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0 \Rightarrow F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}mv^2 \text{ or, } v_{max} = \frac{F}{\sqrt{mk}}$$

22. (D)

Position, $x = 3t^2 + 5$

$$\therefore \text{ Velocity, } v = \frac{dx}{dt} \Rightarrow v = \frac{d(3t^2+5)}{dt}$$

$\Rightarrow v = 6t + 0$

At $t = 0$ $\quad$ $v = 0$

And, at $t = 5\,\text{sec}$ $\quad$ $v = 30\,m/s$

According to work-energy theorem , $w = \Delta KE$

Or $W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900J$

23. (C)

$$F = -\frac{\partial u}{\partial r}\hat{r} = \frac{K}{r^3}\hat{r}$$

Since particle is moving in circular path

$$F = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2}$$

$$\therefore K.E. = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

Total energy $= P.E. + K.E.$

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{zero} \left(\because P.E. = -\frac{K}{2r^2} \text{ given}\right)$$

24. (B)

As the particles moving in circular orbits. So $\frac{mv^2}{r} = \frac{16}{r} + r^2$

Kinetic energy, $KE_0 = \frac{1}{2}mv^2 = \frac{1}{2}\left[16 + r^4\right]$

For first particle, $r = 1, K_1 = \frac{1}{2}m(16+1)$

Similarly, for second particle, $r = 4, K_2 = \frac{1}{2}m(16+256)$

$$\therefore \frac{K_1}{k_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} \simeq \frac{17}{272} \simeq 6\times10^{-2}$$

25. (A)

Let V_f is the final speed of the body. From questions,

$$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \Rightarrow V_f = \frac{V_0}{2} = 5m/s$$

$$F = m\left(\frac{dV}{dt}\right) = -kV^2 \qquad \therefore \left(10^{-2}\right)\frac{dV}{dt} = -kV^2$$

$$\int_{10}^{5}\frac{dV}{V^2} = -100\,K\int_{0}^{10}dt$$

$$\frac{1}{5}-\frac{1}{10} = 100K(10) \qquad \text{or, } K = 10^{-4}\,kgm^{-1}$$

26. (A)

$$h \propto \frac{V^2}{2g}$$

$h \propto$ K.E.

As K.E. becomes half after every collision. So height will also become half.

So, total distance $= h + 2\left(\frac{h}{2}+\frac{h}{4}+.....\right)$

$$= h + 2h\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = 3h$$

27. (C)

Using, $F = ma = m\frac{dV}{dt}$

$$6t = 1.\frac{dV}{dt} \qquad [\because m = 1kg\,given]$$

$$\int_{0}^{v}dV = \int 6t\,dtV = 6\left[\frac{t^2}{2}\right]_0^1 = 3ms^{-1}\,[\because t = 1sec\,given]$$

From work energy theorem,

$$W = \Delta KE = \frac{1}{2}m\left(V^2-u^2\right) = \frac{1}{2}\times1\times9 = 4.5J$$

28. (A)

Work done by friction at $QR = \mu mgx$

In triangle, $\sin 30^\circ = \frac{1}{2} = \frac{2}{PQ} \Rightarrow PQ = 4m$

Work done by friction at $PQ = \mu mg \times \text{Cos}30^\circ \times 4$

$= \mu mg \times \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3}\mu mg$

Since work done by friction on parts PQ and QR are equal, $\mu mgx = 2\sqrt{3}\mu mg \Rightarrow x = 2\sqrt{3} \cong 3.5m$

Using work energy theorem $4mg\sin 30^\circ = 2\sqrt{3}\mu mg + \mu mgx$

$\Rightarrow 2 = 4\sqrt{3}\mu \Rightarrow \mu = 0.29$

29. (B)

$$n = \frac{W}{\text{input}} = \frac{mgh \times 1000}{\text{input}} = \frac{10 \times 9.8 \times 1 \times 1000}{\text{input}}$$

Input $= \frac{98000}{0.2} = 49 \times 10^4 J$

Fat used $= \frac{49 \times 10^4}{3.8 \times 10^7} = 12.89 \times 10^{-3} kg$

30. (B)

As we know, $dU = F.dr$

$$U = \int_0^r \alpha r^2 dr = \frac{\alpha r^3}{3} \qquad \text{.....(i)}$$

As, $\frac{mv^2}{r} = \alpha r^2 \Rightarrow m^2v^2 = m\alpha r^3$

Or, $2m(KE) = \frac{1}{2}\alpha r^3 \qquad \text{.....(ii)}$

Total energy $=$Potential energy + kinetic energy

Now, from equation (i) and (ii)

Total energy $= K.E. + P.E. = \frac{\alpha r^3}{3} + \frac{\alpha r^3}{2} = \frac{5}{6}\alpha r^3$

31. (A)

Let u be the initial velocity of the bullet of mass m. After passing through a plank of width x, its velocity decreases ti v.

$\therefore u - v = \frac{u}{n}$ or, $v = u - \frac{u}{n} = \frac{u(n-1)}{n}$

If F be the retarding force applied by each plank, then using work – energy theorem.

$$Fx = \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mu^2\frac{(n-1)^2}{n^2}$$

$$= \frac{1}{2}mu^2\left[\frac{1-(n-1)^2}{n^2}\right]$$

$$Fx = \frac{1}{2}mu^2\left(\frac{2n-1}{n^2}\right)$$

Let P be the number of planks required to stop the bullet. Total distance travelled by the bullet before coming to rest $= Px$

Using work-energy theorem again,

$$F(Px) = \frac{1}{2}mu^2 - 0$$

Or, $P(Fx) = P\left[\frac{1}{2}mu^2\frac{(2n-1)}{n^2}\right] = \frac{1}{2}mu^2$

$$\therefore P = \frac{n^2}{2n-1}$$

32. (A)

Given: $k_A = 300\,N/m, k_B = 400\,N/m$

Let when the combination of springs is compressed by force F. Spring A is compressed by x. Therefore compression in spring B

$x_B = (8.75 - x)cm$. In series force is same across both spring

So, $F = 300 \times x = 400(8.75 - x)$

Solving we get, $x = 5cm$

$x_B = 8.75 - 5 = 3.75cm$

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300 \times (5)^2}{400 \times (3.75)^2} = \frac{4}{3}$$

33. (D)

$$F_{thrust} = V_{rel}\frac{dm}{dt} = 5 \times 0.5 = 2.5N$$

So, Power $=$ Force $\times$ Velocity $= 2.5 \times 5 = 12.5$ watt

34. (B)

We know that

Power, $P = Fv$

But $F = max = m\frac{dv}{dt}$ $\quad \therefore P = mv\frac{dv}{dt} \Rightarrow P\,dt = mv\,dv$

Integrating both sides $\int_0^t P\,dt = m\int_0^v v\,dv$

$$P.t = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

Distance, $s = \int_0^t v\,dt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt = \sqrt{\frac{2P}{m}}.\frac{t^{3/2}}{3/2}$

$$\Rightarrow s = \sqrt{\frac{8P}{9m}}.t^{3/2} \Rightarrow s \propto t^{3/2}$$

So, graph (B) is correct.

35. (B)

Total force required to lift maximum load capacity against frictional force = 4000 N

$F_{total} = Mg + friction$

$= 2000\times10 + 4000 = 20,000 + 4000 = 4000N$

Using power, $P = F\times v$

$60\times746 = 24000\times v \Rightarrow 1.86m/s = 1.9m/s$

Hence speed pf the elevator at full load is close to $1.9\,ms^{-1}$

36. (B)

Centripetal acceleration $a_c = n^2Rt^2$

$a_c = \frac{v^2}{R} = n^2Rt^2$

$v^2 = n^2R^2t^2$

$v = nRt$

Here power is delivered by tangential force only because power by centripetal force is zero. $\left[\text{Since}\,\vec{F}_c \perp \vec{V}\right]$

$a_t = \frac{dv}{dt} = nR$

Power $= ma_t v = m\,nR\,nRt = Mn^2R^2t$

37. (C)

Power, $P = \frac{w}{t} = \frac{E}{t} = \text{constant}$ $\qquad \therefore \frac{\frac{1}{2}mv^2}{t} = \text{constant}$

From work-energy theorem, not work done =change in kinetic energy.

$\Rightarrow \frac{v^2}{t} = \text{constant}(k) \qquad \therefore kt^{1/2}\text{and}\,\frac{ds}{dt} = kt^{1/2}$

Or, $ds = kt^{1/2}dt$

By integrating, we get $\Rightarrow s = \frac{2kt^{3/2}}{3} + C \Rightarrow s \propto t^{3/2}$

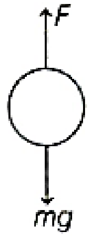
i.e., Distance moved $S \propto t^{3/2}$

38. (C)

Work done by a force is negative when work force is acting in opposite direction of displacement. So, work of gravitational force while lifting a bucket and due to air resistance on oscillating pendulum are negative.

39. (C)

Let’s two motor A and B



For motor A

$P_A = F_1 v_1$ ....(1)

Given M = 300 kg

$\therefore$ $F_1 = mg = 300$ N

d = 100 m; t = 5 min

t = 5 x 60 = 300 s

$\therefore$ $v_1 = \frac{d}{t} = \frac{100}{300} = \frac{1}{3} m/s$

$\therefore$ From Eq. (i) ,

$P_A = 3000 \times \frac{1}{3} = 100W$

For motor B

M = 50 kg $F_2 = mg$

$\Rightarrow 50 \times 10 = 500N$

$v_2 = \frac{100}{2 \times 60}$

$\Rightarrow$ $v_2 = \frac{5}{6} m/s$

$\therefore$ $P_B = F_2 v_2 = 500 \times \frac{5}{6} = \frac{2500}{6}$

$\therefore$ $\frac{P_A}{P_B} = \frac{1000}{2500} \Rightarrow \frac{P_A}{P_B} = \frac{12}{5}$ ....(ii)

But, $\frac{P_A}{P_B} = \frac{3\sqrt{x}}{\sqrt{x}+1}$

$\therefore$ $\frac{3\sqrt{x}}{\sqrt{x}+1} = \frac{12}{5}$ [from Eq. (ii)]

$15\sqrt{x} = 12\sqrt{x} + 12$

$\Rightarrow$ $3\sqrt{x} = 12$

$\sqrt{x} = 4 \Rightarrow x = 16$

40. (B)

For statement I

Work done = ΔKE

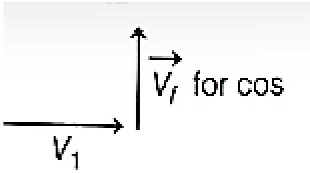
$= -FS = 0 - KE$

$\Rightarrow$ $KE = SF$

$\Rightarrow$ $S = \frac{KE}{F}$

Hence, both car and truck will cover same distance .

For Statement II



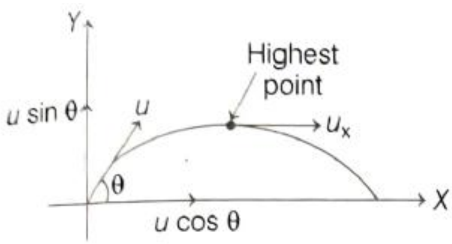
From the diagram, $\Delta V = V_f - V_i$

$|\Delta V| = \sqrt{2}V$

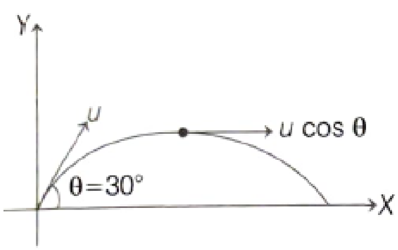
Thus the velocity is changing, therefore $a \neq 0$.
Hence, statement II is incorrect.

41. (C)
At highest point during projectile motion, the velocity of projectile in vertical direction becomes zero but it has horizontal component of velocity. Because of this, kinetic energy will not be equal to zero. Gravitational potential energy is maximum at highest point and equal to

$$mgH = mg\left(\frac{\mu^2 \sin^2\theta}{2g}\right)$$



42. (D)



Kinetic energy of particle at point of projection,
$K_1 = 1/2\ mu^2$
Kinetic energy at highest point,

$$K_2 = \frac{1}{2} mu^2 \cos^2\theta$$

$$= \left(\frac{1}{2} mu^2\right) \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} K_1$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{4}{3} \qquad [\because \theta = 30^o]$$

SECTION – II

1. (2)

Work done by A=Work done by B

$F_A d\cos 45^\circ = F_B d\cos 60^\circ$

$\Rightarrow F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2} \Rightarrow \frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow x = 2$

2. (450)

Given,

Force, $F = (5y + 20)\hat{j}$

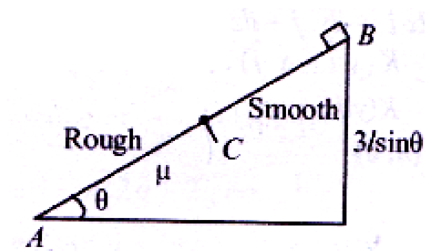
Work done, $E = \int F.dy$

$\Rightarrow W = \int_0^{10} (5y + 20)dy = \left[\frac{5y^2}{2} + 20y\right]_0^{10}$

$= \frac{5}{2} \times 100 + 20 \times 10 = 450J$

3. (3)

If $AC = l$ then according to question, $BC = 2l$ and $AB = 3l$.



Here, work done by all the forces is zero.

$W_{friction} + W_{mg} = 0$

$mg(3l)\sin\theta - \mu mg\cos\theta(l = 0)$

$\Rightarrow \mu mg\cos\theta l = 3mgl\sin\theta \Rightarrow \mu = 3\tan\theta = k\tan\theta$

$\therefore k = 3$

4. (24)

Using work-energy theorem, $W_{net} = (K_f - K_i)$

$\Rightarrow \frac{1}{2}Kx^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 = \frac{E}{4} - E$

$\Rightarrow \frac{1}{2}Kx^2 = \frac{3E}{4} \Rightarrow K = \frac{3E}{2x^2} \Rightarrow K = \frac{3E}{2\times\left(\frac{1}{4}\right)^2} = 24E$

So, value of spring constant of used spring is 24 times of kinetic energy

$\therefore n = 24$

5. (2)

Using energy conservation for plane AB

$\frac{1}{2}mu^2 = mgh$ (Here, u= initial velocity of block)

$\Rightarrow \frac{1}{2}\times m\times u^2 = m\times 10\times 10 \Rightarrow u = 10\sqrt{2}$

At point B

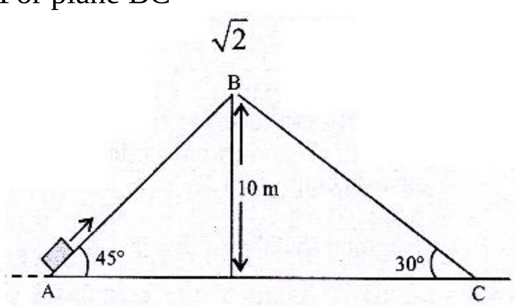
Acceleration, $a = -g\sin 45^\circ = \frac{-10}{\sqrt{2}}$

Using $v = u + at_1$

$\Rightarrow 0 = 10\sqrt{2} - \frac{10}{\sqrt{2}}t_1$

$\Rightarrow t_1 = 2\sec$

For plane BC



Using $s = ut_2 + \frac{1}{2}at_2^2$

$\Rightarrow \frac{10}{\sin 30^\circ} = \frac{1}{2}\left(10\sin 30^\circ\right)t_2^2 \quad \left(\because s = \frac{10}{\sin 30^\circ}\right)$

$\Rightarrow t_2 = 2\sqrt{2}$

So total time $T = t_1 + t_2 = 2\sqrt{2} + 2 = 2\left(\sqrt{2}+1\right)\sec$

6. (16)

Mass of engine − wagon system, $m = 40,000\,kg$ Velocity , $v = 72\times 5/18 = 20\,m/s$

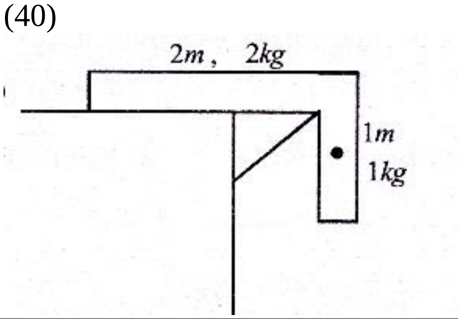
$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\times(40,000)\times(20)^2 = 8000000J$

As 90% of K.E. of system lost in friction, only 10% is transferred to spring.

$\therefore \frac{1}{2}Kx^2 = \frac{10}{100}\times 8000000 \Rightarrow \frac{1}{2}\times K\times 1\times 1 = 8\times 10^5$

$\Rightarrow K = 16\times 10^5\,N/m$

7. (40)



Loss in potential energy =gain in kinetic energy
Take zero potential energy at table, initial potential energy

$= -1\times 10\times \frac{1}{2} = -5\text{J}$

Final potential energy $= -3\times 10\times \frac{3}{2} = -45\text{J}$

Change in potential energy $= -5-(-45)\text{J} = 40\text{J}$

$\therefore k = 40$

8. (10)
By mechanical energy conservation,

$T.E_A = T.E_B$

$PE_A + KE_A = PE_B + KE_B$

$mg(10) + 0 = mg(5) + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2\times g\times 5} = 10\,m/s$

$\therefore x = 10$

9. (6)
Here kinetic energy of ball is equal to P.E. stored in spring i.e., $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$

$\Rightarrow \frac{1}{2}\times 4\times (10)^2 = \frac{1}{2}\times 100\times (\Delta x)^2 \Rightarrow \Delta x = 2m$

Therefore length of the compressed spring

$x = 8-2 = 6m$

10. (150)
From work energy theorem,

$W = F.s = \Delta KE = \frac{1}{2}mv^2$

Here $V^2 = 2gh$

$\therefore F.s = F\times \frac{2}{10} = \frac{1}{2}\times \frac{15}{100}\times 2\times 10\times 20$

$\therefore F = 150N$

11. (10)
Kinetic energy =change in potential energy of the particle.

$KE = mg\Delta h$

Given, $m = 1kg$.

$\Delta h = h_2 - h_1 = 2-1 = 1m$

$\therefore KE = 1\times 10\times 1 = 10J$

12. (18)
Given, Mass of the body, $m = 2kg$

Power delivered by engine, $P = 1J/s$

Time, t=9 seconds

Power, $P = Fv$

$\Rightarrow P = ma\nu \qquad [\because F = ma]$

$\Rightarrow m\dfrac{d\nu}{dt}\nu = P \qquad \left(\because a = \dfrac{d\nu}{dt}\right),$

$\Rightarrow \nu d\nu = \dfrac{P}{m}dt$

Integrating both sides we get

$\Rightarrow \int_0^{\nu} \nu d\nu = \dfrac{P}{m}\int_0^t dt \Rightarrow \dfrac{\nu^2}{2} = \dfrac{Pt}{m} \Rightarrow \nu = \left(\dfrac{2Pt}{m}\right)^{1/2}$

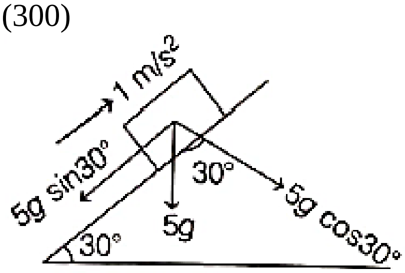
$\Rightarrow \dfrac{dx}{dt} = \sqrt{\dfrac{2P}{m}}t^{1/2} \qquad \left(\because \nu = \dfrac{dx}{dt}\right)$

$\Rightarrow \int_0^x dx = \sqrt{\dfrac{2P}{m}}\int_0^t t^{1/2}dt$

$\therefore$ Distance, $x = \sqrt{\dfrac{2P}{m}}\dfrac{t^{3/2}}{3/2} = \sqrt{\dfrac{2P}{m}} \times \dfrac{2}{3}t^{3/2}$

$\Rightarrow x = \sqrt{\dfrac{2\times 1}{2}} \times \dfrac{2}{3} \times 9^{3/2} = \dfrac{2}{3} \times 27 = 18m$

13. (300)



F at t = 10 sec

F – 5g sin 30° = 5a

$F - 5 \times 10\dfrac{1}{2} = 5 \times 1$

F = 30 N

$\therefore$ $\nu = u + at$

$\nu = 0 + 1(10)$; $\nu = 10$ m/s

$P = F\nu = 30 \times 10$

= 300 W

14. (48)

Given, max-load = 1400 kg

Speed of elevator (v) =3 ms^{-1}

Friction force = 2000 N

Force due to load = Ma = mg

= 1400 x 10= 1400 N = 14000 N

Maximum force = Force on elevator + Friction

= 14000 + 2000 = 16000 N

$P_{max} = F_{max} \times \nu$

$16000 \times 3 = 48000\ W = 48\,kW$

15. (4)

Given, m = 2 kg

We know that, P = F x v

$$P = m \times a \times \nu$$

$$\Rightarrow \int Pdt = \int m\nu d\nu$$

$$Pt = \frac{m\nu^2}{2}$$

$$\Rightarrow \nu = \sqrt{\frac{2pt}{m}} \Rightarrow \frac{ds}{dt} = \sqrt{\frac{2Pt}{m}}$$

$$\Rightarrow ds = \sqrt{\frac{2pt}{m}}.dt \Rightarrow \int ds = \int \sqrt{\frac{2Pt}{m}}\ dt$$

$$\Rightarrow s = \sqrt{\frac{2p}{m}}.\frac{t^{3/2}}{3/2} = \frac{2}{3}\sqrt{\frac{2P}{m}}.t^{3/2}$$

$$= \frac{2}{3}\sqrt{\frac{2P}{m}}.(4)^{3/2} \qquad (\because t = 4 \text{ s and } m = 2 \text{ kg})$$

$$= \frac{16}{3}\sqrt{P} = \frac{4^2}{3}\sqrt{P} = \frac{\alpha^2\sqrt{P}}{3} \quad \text{(given)}$$

$\therefore\ \alpha = 4$

16. (296)

Given, force acting on body,

$$F = t\hat{i} + 3t^2\hat{j}$$

Acceleration of body,

$$a = \frac{F}{m} = \left(t\hat{i} + 3t^2\hat{j}\right) m/s^2$$

Velocity of body at instant t,

$$\nu = u + at \qquad (u = 0)$$

$$\Rightarrow \nu = t^2\hat{i} + 3t^3\hat{j}\ \text{m/s}$$

Power developed by force, P = F . v

$$= (t\hat{i} + 3t^2\hat{j}).(t^2\hat{i} + 3t^3\hat{j}) = t^3 + 9t^5$$

At = t = 2 s, power is ;

$$P = 2^3 + 9 \times 2^5 = 8 + 288 = 296W$$

17. (2)

Given, a = –2x

$$\nu\frac{dv}{dx} = -2x$$

$$\Rightarrow \quad vdx = -2xdx$$

$$\int_{\nu_1}^{\nu_2} \nu d\nu = -2\int_0^x xdx$$

$$\Rightarrow \frac{\nu_2^2}{2} - \frac{\nu_1^2}{2} = -\frac{2x^2}{2}$$

$$\Rightarrow \quad KE = \frac{2\sin\theta}{k} = \frac{x}{k}\sin\theta$$

$\Rightarrow \quad \frac{mv_1^2}{2} - \frac{mv_2^2}{2} = mx^2$

$\because \quad$ Loss in KE = $mx^2 = \frac{10}{1000}x^2$

$= 10^{-2}x^2 = \left(\frac{10}{x}\right)^{-2}$

n = 2

18. (3)

Given, $E_1 = \frac{1}{2}mu^2 - 0$

$E_1 = \frac{1}{2}mu^2 = E$

$E_2 = \frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2$

$2mu^2 - \frac{1}{2}mu^2$

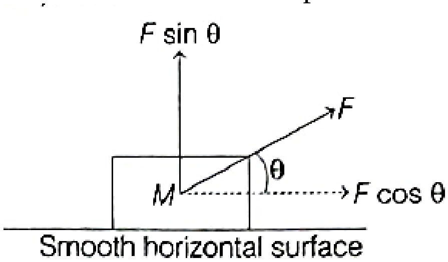
$E_2 = \frac{3}{2}mu^2 = 3\left(\frac{1}{2}mu^2\right) = 3E\ J$

Hence, value of n = 3

19. (2)

Given that, F = 2N

If a be the acceleration produced in object, then



F = cos θ = ma

$\Rightarrow \quad$ F cos kx = ma $\quad [\because \theta = kx]$

$\Rightarrow \quad F\cos kx = m\frac{dv}{dt} \quad \left[\because a = \frac{dv}{dt}\right]$

$\Rightarrow \quad 2\cos kx = m\frac{dv}{dt}[\because F = 2N]$

$\Rightarrow \quad 2\cos kx = m.\frac{dv}{dx}.\frac{dx}{dt}$

$\Rightarrow \quad 2\cos kx\,dx = mv\,dv$

On integrating both sides, we get

$\int 2\cos kx\,dx = \int mv\,dv$

$\Rightarrow \quad \frac{2\sin kx}{k} = \frac{mv^2}{2}$

$\Rightarrow \quad \dfrac{2\sin\theta}{k} = KE \qquad [\because \theta = kx]$

$\Rightarrow \quad KE = \dfrac{2\sin\theta}{k} = \dfrac{x}{k}\sin\theta \qquad$ [Given]

$\therefore \quad x = 2$

20. (7)
From third equation of motion
$V^2 = u^2 + 2as = 2^2 + 2 \times 2 \times 6$

$\Rightarrow \quad v^2 = 28$

Kinetic energy to lift , $K = \dfrac{1}{2}mv^2$

$$K = \frac{1}{2} \times 500 \times 28$$

$$K = 7 \times 10^3 \text{ J} \Rightarrow K = 7\text{kJ}$$

21. (375)
Let u and v be speeds, just before and after body strikes the ground.

Given, $\dfrac{u}{v} = \dfrac{4}{1}$

Loss in KE $\Rightarrow \Delta K = \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2}$

$$\Delta KE = 1 - \left(\frac{v}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\%\text{loss} = \frac{15}{16} \times 100 = \frac{375}{4}, x = 375$$

22. (32)
Given
$F = (2 + 3x)\hat{i}$

$$W = \int_0^4 F.dx = \int_0^4 (2+3x)\,dx$$

$$W = \left[2x + \frac{3x^2}{2}\right]_0^4 = [8 + 24]$$

W = 32 J

23. (30)
(30) Work done by variable force,

$$W = \int F dx = \int_2^4 5x\,dx = \left[5.\frac{x^2}{2}\right]_2^4$$

$$= \frac{5}{2}[4^2 - 2^2] = \frac{5}{2}[12]$$

= 30 J

24. (132)

Given that, B/F =(5+ $3y^2$) $\hat{j}$ N Displacement is also along Y-axis.

So, work done, W = $\int F.dy$

$$\%loss = \frac{15}{16}\times 100 = \frac{375}{4}, x = 3751 - \left(\frac{\nu}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$= \left[5y + \frac{3y^3}{3}\right]_2^5 = \left[5y + y^3\right]_2^5$$

$$= [5\times 5 + (5)^3] - [5\times 2 + (2)^3]$$

25. (40)

(40) Force,

$$F = (5\hat{i} + 2\hat{j} + 7k)\,N$$

Displacement vector,

$\Delta r = r_f - r_i$

$$= (5\hat{i} - 2\hat{j} + k) - (2\hat{i} + 3\hat{j} - 4k)$$

$$\Delta r = (3\hat{i} - 5\hat{j} + 5k)\,m$$

Work done, W = F . Δr

$$= (5\hat{i} + 2\hat{j} + 7k).(3\hat{i} - 5\hat{j} + 5k)$$

= (5) (3) + (2) (- 5) + (7) (5)

=15 – 10 + 35= 40J

Alternate solution

$$F = 5\hat{i} + 2\hat{j} + 7k$$

$$F = |F| = \sqrt{5^2 + 2^2 + 7^2} = \sqrt{78}$$

$$\Delta r = r_f - r_i = 3\hat{i} - 5\hat{j} + 5k$$

$$\Delta r = \sqrt{3^2 + (-5)^2 + 5^2} = \sqrt{59}$$

Angle between F and Δ r is given as,

$$\cos\theta = \frac{F.\Delta r}{|F||\Delta r|} = \frac{5\times 3 + 2(-5) + 7\times 5}{|\sqrt{78}|\sqrt{59}}$$

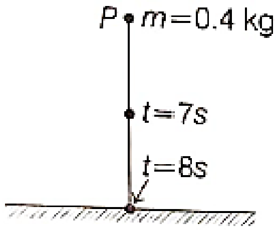
$$= \frac{40}{|\sqrt{78}|\sqrt{59}}$$

∴ Work done, W = $|F|\times|\Delta r|\cos\theta = \sqrt{78}\times\sqrt{59}\times\frac{40}{\sqrt{78}\times\sqrt{59}} = 40\,J$

26. (300)

Distance covered by mass in 8 s.

Using, $h = ut + \frac{1}{2}at^2$



$h_1 = 0\times 8 + \frac{1}{2}(10)\times 8^2 = 320\text{m}$

Distance covered by mass in 7 s is

$h_2 = \frac{1}{2}\times 10\times 7^2 = 245\text{m}$

Distance covered by mass in last 1 s

= 320 – 245 = 75 m

So, loss of PE in last second

= $mgh = 0.4\times 10\times 75$

= 300 J

Alternate solution

Distance covered by the body in last second (8^{th} second)

$h_8 = 0 + \frac{1}{2}g(2t-1)$

$= \frac{1}{2}\times 10(2\times 8-1) = 75\,\text{m}$

$\therefore$ Loss of PE in last second $= mgh_8$

$= 0.4\times 10\times 75 = 300\,\text{J}$

27. (784)

4.(784) Speed of bus (v) = 80 km/h

mass of bus (m) =500 kg

distance (x)=4 km =4 x 10^3 m

For a constant speed,

Work done by engine + Work done by

Friction = 0

Thus, $WD_{engine} = -WD_{friction} = (-\mu\, mg\ x)$

$= 0.04\times 500\times 9.8\times 4\times 10^3 = 784\text{kJ}$

28. (245)

By work energy theorem

$W_{friction} + W_{gravity} = \triangle K$

$\Rightarrow W_f + (10\times 0.3) = 0 - \frac{1}{2}(1)(22)^2$

$\Rightarrow W_f = -245\,\text{J}$