Trigonometric Ratio

EXERCISE - 1 [A]

1. (B)

π radian = 180 degree $\Rightarrow$ 1 radian = $\frac{180}{\pi}$ degree $\simeq 57.3$ degree, hence sin 1^c > sin 1^0.

2. (B)

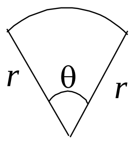
Length of arc of circle of radius r subtending θ at the center = rθ.

Hence $15 = r \times \frac{3}{4}$ or r = 20 cm

3. (D)

Perimeter $= r\theta + 2r = m \cdot r$

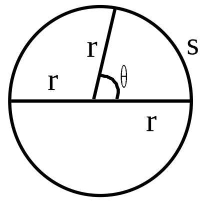
$\theta = (m-2)^c$



4. (C)

$\pi r = S + 2r$

$\Rightarrow S = (\pi - 2)r$



5. (B)

At half past 4, hour hand will be at $4\frac{1}{2}$ and minute hand will be at 6(for 30 minutes). In 12 hours, angle made by hour hand

$\frac{360}{12} \times \frac{9}{2} = 135^\circ$

In 60 minutes, angle made by minute hand

$= 360^\circ$

In 30 minutes, angle made by minute hand

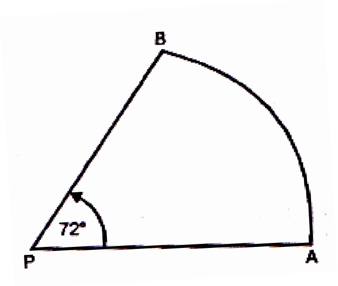
$= \frac{360^\circ}{60} \times 30 = 180^\circ$

$\therefore$ reqd. angle $= 180^\circ - 135^\circ = 45^\circ = \frac{\pi}{4}$

Hence (B) holds.

6. (B)

Let the post be at point P and let PA be the length of the rope in tight position. Suppose the horse moves along the arc AB so that $\angle APB = 72^\circ$ and arc $AB = 88m$. Let r be the length of the rope i.e $PA = r$ metres.



Now, $\theta = 72^\circ = \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$

And $s = 88m$ $\therefore \theta = \frac{\text{arc}}{\text{radius}}$

$\Rightarrow \frac{2\pi}{5} = \frac{88}{r} \Rightarrow r = 88 \times \frac{5}{2\pi} = 70m$

Hence (B) holds.

7. (C)

Since $180^\circ = \pi$ radians

$\therefore 1^\circ = \frac{\pi}{180}$ radians $\therefore p^\circ = \frac{p\pi}{180}$ radians

$\therefore \frac{p\pi}{180} = q$

$[\because p \text{ degrees} = q \text{ radians}]$

$\Rightarrow \frac{p}{180} = \frac{q}{\pi}$ i.e. $\frac{p}{90} = \frac{2q}{\pi}$

Hence (C) holds.

8. (B)

Clearly length of the wire

$=$ circumference of circle of radius 7

$= 2\pi(7) = 14\pi$

$\therefore l = 14\pi, r = 12cm$

Since $\theta = \frac{l}{r} = \frac{14\pi}{12} = \frac{7\pi}{6} = \frac{7}{6} \times 180^\circ = 210^\circ$

Hence angle subtended at the centre $= 210^\circ$

9. (A)

$\tan(90^0 - \theta) = \cot\theta \Rightarrow \tan 89^0 = \cot 1^0$, $\tan 88^0 = \cot 2^0$, $\tan 87^0 = \cot 3^0$, … etc.

Also, $\tan\theta \times \cot\theta = 1$, hence $\tan 1^0 \tan 2^0 \tan 3^0 \ldots \tan 89^0 = 1$

10. (A)

$\sin\theta - \cos\theta = 1 \Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta\cos\theta = 0$

11. (C)

$$\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}=\frac{5\tan\theta-3}{5\tan\theta+2}$$

$$5\tan\theta=4\Rightarrow\frac{5\tan\theta-3}{5\tan\theta+2}=\frac{4-3}{4+2}=\frac{1}{6}$$

12. (A)

Sin x + cosec x = 2 $\Rightarrow$ $\sin^2x-2\sin x+1=0$

$\Rightarrow$ sin x = 1 $\Rightarrow$ $\sin^n x+\text{cosec}^n x=2$

13. (C)

$$x\sin45^0\cos^2 60^0=\frac{\tan^2 60^0\text{cosec}30^0}{\sec45^0\cot^2 30^0}\Rightarrow x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)^2=\frac{\left(\sqrt{3}\right)^2(2)}{\left(\sqrt{2}\right)\left(\sqrt{3}\right)^2}\Rightarrow x=8$$

14. (A)

$$\tan^2 30+4\sin^2 45+\frac{1}{3}\cos^2 30=\frac{1}{3}+\frac{4}{2}+\frac{1}{3}\times\frac{3}{4}$$

$$=\frac{1}{3}+\frac{1}{4}+2$$

$$=2.\frac{7}{12}$$

15. (D)

$$\frac{\tan^2 60-2\tan^2 45+\sec^2 30}{3\sin^2 45-\sin90+\cos^2 60.\cos^3 0}=\frac{3-2+\frac{4}{3}}{3.\frac{1}{2}+\frac{1}{4}}$$

$$=\frac{1+\frac{4}{3}}{\frac{1}{2}\left(3+\frac{1}{2}\right)}=\frac{\frac{7}{3}}{\frac{1}{2}.\frac{7}{2}}=\frac{4}{3}$$

16. (C)

$$2P_6-3P_4+1$$

$$=2\left(1-3\sin^2 x\cos^2 x\right)-3\left[\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x\right]+1$$

$$=2-6\sin^2 x\cos^2 x-3\left(1-2\sin^2 x\cos^2 x\right)+1=0$$

17. (A)

$90^0<130^0<135^0$ hence sin A > 0 & cos A < 0 & $|\sin A|>|\cos A|$

$\Rightarrow\sin A+\cos A>0$

18. (A)

$$\cos24^0+\cos5^0+\cos175^0+\cos204^0+\cos300^0$$

$$=\cos24^0+\cos5^0+\cos\left(180^0-5^0\right)+\cos\left(180^0+24^0\right)+\cos\left(360^0-60^0\right)$$

$$= \cos 24^0 + \cos 5^0 - \cos 5^0 - \cos 24^0 + \cos 60^0$$

$$= \cos 60^0 = \frac{1}{2}$$

19. (C)

$\sin\theta < 0 \,\&\, \tan\theta > 0 \Rightarrow \theta$ lies in 3rd quadrant.

20. (C)

Given expression

$$= \frac{-\sin\left(660^\circ\right)\tan\left(1050^\circ\right)\sec\left(420^\circ\right)}{\cos\left(180^\circ + 45^\circ\right)\operatorname{cosec}\left(360^\circ - 45^\circ\right)\cos\left(360^\circ + 150^\circ\right)}$$

$$= \frac{-\sin\left(7\times 90 + 30^\circ\right)\tan\left(3.360^\circ - 30^\circ\right)\sec\left(360^\circ + 60^\circ\right)}{\left(-\cos 45^\circ\right) - \operatorname{cos\,ec}\left(45^\circ\right)\cos 150^\circ}$$

$$= \frac{\cos\left(30^\circ\right)\left(-\tan 30^\circ\right)\sec 60^\circ}{-\left(\cos 45^\circ\right)\left(-\operatorname{cosec} 45^\circ\right)\left(-\cos 30^\circ\right)}$$

$$= \frac{\cos 30^\circ \tan 30^\circ \sec 60^\circ}{\cos 45^\circ \operatorname{cosec} 45^\circ \cos 30^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}}.2}{\frac{1}{\sqrt{2}}.\frac{\sqrt{2}}{1}} = \frac{2}{\sqrt{3}}$$

Hence (C) holds

21. (C)

Given expression

$$= \frac{\left(-\sin\theta\right)\left(\sec\theta\right)\left(-\tan\theta\right)}{\left(\sin\theta\right)\left(-\sec\theta\right)\tan\theta} = -1$$

Hence (C) holds.

22. (B)

Since $\sec\theta$ i.e $\cos\theta$ is negative.

$\therefore \theta$ lies in II nd or III rd quadrant. Since $\sin\theta$ is positive.

$\therefore \theta$ lies in Ist or IInd quadrant. Hence θ lies in II nd quadrant.

23. (A)

$$\cot A + \tan\left(180^\circ + A\right) + \tan\left(90^\circ + A\right) + \tan\left(360^\circ - A\right)$$

$$= \cot A + \tan A - \cot A - \tan A = 0$$

24. (D)

$$\sin\left[-870^\circ\right] = -\sin\left(870^\circ\right) = -\sin\left(9\times 90^\circ + 60^\circ\right)$$

$$= -\cos 60^\circ = -\frac{1}{2}$$

25. (A)

$\tan\theta = -\frac{1}{\sqrt{5}}$

$\sec^2\theta = 1+\tan^2\theta = 1+\frac{1}{5} = \frac{6}{5}$

$\sec\theta = +\sqrt{\frac{6}{5}}$ [$\because \theta$ lies in the IV th quadrant]

$\therefore \cos\theta = \sqrt{\frac{5}{6}}$

26. (C)

$\sin 15^0 \cos 15^0 = \frac{1}{2}\sin 30^0 = \frac{1}{4}$

27. (A)

$\frac{\cos 17^0 + \sin 17^0}{\cos 17^0 - \sin 17^0} = \frac{1+\tan 17^0}{1-\tan 17^0}$

$= \tan\left(45^0 + 17^0\right) = \tan 62^0$

28. (A)

$\tan 75^0 - \cot 75^0 = -2\cot 150^0$

$= 2\tan 60^0 = 2\sqrt{3}$

29. (B)

$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

$\Rightarrow \cos^2 48^0 - \sin^2 12^0 = \cos 60^0 \cos 36^0 = \frac{\sqrt{5}+1}{8}$

30. (D)

As A & B are the acute angles hence,

$\sin A = \frac{1}{\sqrt{10}} \Rightarrow \cos A = \frac{3}{\sqrt{10}}$ & $\sin B = \frac{1}{\sqrt{5}} \Rightarrow \cos B = \frac{2}{\sqrt{5}}$

Now cos (A + B) = cos A cos B – sin A sin B

$\Rightarrow \cos(A+B) = \frac{3}{\sqrt{10}}\times\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}}\times\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$

$\Rightarrow A+B = \frac{\pi}{4}$

31. (A)

$\cos^2\left(\frac{\pi}{6}+\theta\right) - \sin^2\left(\frac{\pi}{6}-\theta\right)$

$= \cos\left(\frac{\pi}{6}+\theta+\frac{\pi}{6}-\theta\right)\cos\left(\frac{\pi}{6}+\theta-\frac{\pi}{6}+\theta\right)$

$\left[\because \cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B\right]$

$$= \cos\left(\frac{\pi}{3}\right)\cos(2\theta) = \frac{1}{2}\cos 2\theta$$

Hence (A) holds.

32. (C)

$$(1+\tan A)(1+\tan B)$$

$$= (1+\tan A)\left(1+\tan\left(\frac{\pi}{4}-A\right)\right)$$

$$= (1+\tan A)\left(1+\frac{1-\tan A}{1+\tan A}\right)$$

$$= (1+\tan A)\left(\frac{1+\tan A+1-\tan A}{1+\tan A}\right) = 2$$

$\therefore$ (C) holds

33. (B)

$$\sin 163^\circ = \sin\left(180^\circ - 17^\circ\right) = \sin 17^\circ$$

$$\cos 347^\circ = \cos\left(360^\circ - 13^\circ\right) = \cos 13^\circ$$

$$\sin 73^\circ = \sin\left(90^\circ - 17^\circ\right) = \cos 17^\circ$$

$$\sin 167^\circ = \sin\left(180^\circ - 13^\circ\right) = \sin 13^\circ$$

$\therefore$ given expression

$$= \sin 17^\circ \cos 13^\circ + \cos 17^\circ \sin 13^\circ$$

$$= \sin\left(17^\circ + 13^\circ\right) = \sin\left(30^\circ\right) = \frac{1}{2}$$

34. (D)

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\therefore A+B = \frac{\pi}{4}$$

35. (B)

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\left[\cos\theta = \sqrt{1-\frac{144}{165}} = \frac{5}{13} \because 0 < \theta < \frac{\pi^-}{2} \sin\phi = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5} \because \pi < \theta < \frac{3\pi}{2}\right]$$

$$= \frac{12}{13}\left(-\frac{3}{5}\right) + \frac{5}{13}\left(-\frac{4}{5}\right)$$

$$= -\frac{36}{65} - \frac{20}{65} = -\frac{56}{65}$$

36. (D)

$A+B=45^\circ \Rightarrow \frac{\cot A\cot B-1}{\cot B+\cot A}=1$

$(\cot A-1)(\cot B-1)$

$=\cot A\cot B-(\cot A+\cot B)+1$

$=1+1=2$

$\therefore$ (D) holds

37. (C)

$32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)=16(\cos 2A-\cos 3A)$

$=16\left(2\cos^2 A-1-4\cos^3 A+3\cos A\right)$

$=16\left(2\times\frac{9}{16}-1-4\times\frac{27}{64}+3\times\frac{3}{4}\right)=11$

38. (B)

$\sin 50^0-\sin 70^0+\sin 10^0=-2\cos 60^0\sin 10^0+\sin 10^0=0$

39. (D)

The given expression $=\cos 20^\circ+\cos\left(180^\circ-80^\circ\right)+\cos\left(180^\circ-40^\circ\right)$

$=\cos 20^\circ-\left(\cos 80^\circ+\cos 40^\circ\right)$

$=\cos 20^\circ-2\cos 60^\circ\cos 20^\circ=0$

$\therefore$ (D) holds

40. (A)

$\cos 52^\circ+\cos 68^\circ+\cos 172^\circ$

$=\cos 52^\circ+\cos 68^\circ+\cos\left(180^\circ-8^\circ\right)$

$=\cos 52^\circ+\cos 68^\circ-\cos 8^\circ$

$=2\cos\frac{52^\circ+68^\circ}{2}\cos\frac{68^\circ-52^\circ}{2}-\cos 8^\circ$

$=2\cos 60^\circ\cos 8^\circ-\cos 8^\circ$

$=\cos 8^\circ-\cos 8^\circ=0$

41. (C)

$\frac{\sin(2A+B)}{\sin B}=\frac{5}{1}$. By componendo and dividendo we shall get $\frac{\tan(A+B)}{\tan A}=\frac{3}{2}$

42. (C)

Given value $=\left(\cos 12^\circ+\cos 132^\circ\right)+\left(\cos 84^\circ+\cos 156^\circ\right)=-\frac{1}{2}$

43. (B)

$$\frac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ} = \frac{\sin 70^\circ + \sin 50^\circ}{\cos 70^\circ + \cos 50^\circ}$$

$$= \frac{2\sin 60^\circ \cos 10^\circ}{2\cos 60^\circ \cos 10^\circ} = \tan 60^\circ = \sqrt{3}$$

44. (D)

$$\frac{\sin 55^\circ - \cos 55^\circ}{\sin 10^\circ} = \frac{\sin 55^\circ - \sin 35^\circ}{\sin 10^\circ}$$

$$= \frac{2\cos 45^\circ \sin 10^\circ}{\sin 10^\circ} = 2\cos 45^\circ = 2\frac{1}{\sqrt{2}} = \sqrt{2}$$

$\therefore$ (D) holds

45. (D)

$$1 - 2\sin^2\left(\frac{\pi}{4} + \theta\right) = \cos 2\left(\frac{\pi}{4} + \theta\right)$$

$$= \cos\left(\frac{\pi}{2} + 2\theta\right) = -\sin 2\theta$$

46. (B)

$$2\cos^2\theta - 2\sin^2\theta = 1 \quad \Rightarrow \quad \cos 2\theta = \frac{1}{2}$$

$\therefore \ \theta = 30^o$

47. (B)

$$2\sin A\cos^3 A - 2\sin^3 A\cos A = 2\sin A\cos A\left(\cos^2 A - \sin^2 A\right)$$

$$= \sin 2A\cos 2A = \frac{\sin 4A}{2}$$

48. (C)

$\sin A + \cos A = 1$, we get $1 + \sin 2A = 1$

$\Rightarrow \sin 2A = 0$

49. (C)

$$\frac{\tan 80° - \tan 10°}{\tan 70°} = \frac{\cot 10° - \tan 10°}{\tan 70°}$$

$$= \frac{2\cot 20°}{\tan 70°} = 2$$

50. (A)

Use $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \text{tam}^2\theta}$

51. (B)

$$\tan 67\frac{1}{2}^{\circ} + \cot 67\frac{1}{2}^{2}$$

$$= \tan 67\frac{1}{2}^{\circ} + \frac{1}{\tan 67\frac{1}{2}^{\circ}}$$

$$= \frac{1+\tan^2 67\frac{1}{2}^{\circ}}{2\tan 67\frac{1}{2}^{\circ}}.2$$

$$= 2.\frac{1}{\sin 2\left(67\frac{1}{2}^{\circ}\right)}$$

$$\left[\because \sin 2A = \frac{2\tan A}{1+\tan^2 A}\right]$$

$$= \frac{2}{\sin\left(135^{\circ}\right)} = \frac{2}{\sin\left(180^{\circ}-45^{\circ}\right)}$$

$$= \frac{2}{\sin 45^{\circ}} = \frac{2}{\frac{1}{\sqrt{2}}} = 2\sqrt{2}$$

$\therefore$ (B) holds

52. (B)

$$\sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+\sqrt{2\left(1+\cos 4\theta\right)}}$$

$$= \sqrt{2+\sqrt{2.2\cos^2 2\theta}}$$

$$= \sqrt{2+2\cos 2\theta} = \sqrt{2\left(1+\cos 2\theta\right)}$$

$$= \sqrt{2\left(2\cos^2\theta\right)} = 2\cos\theta$$

$\therefore$ (B) holds

53. (B)

$$\cos 15^{\circ} \cos 7\frac{1}{2}^{\circ} \sin 7\frac{1}{2}^{\circ}$$

$$= \frac{1}{2}\cos 15^{\circ} \sin 15^{\circ} = \frac{1}{4}\sin 30^{\circ}$$

$$= \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

$\therefore$ (B) holds

54. (A)

$$\frac{1-t^2}{1+t^2} = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \cos\theta$$

55. (D)

Since $1+\cos x = K \quad \therefore 2\cos^2\frac{x}{2} = K$

$\Rightarrow \cos^2\frac{x}{2} = \frac{K}{2} \quad \Rightarrow 1-\sin^2\frac{x}{2} = \frac{K}{2}$

$\Rightarrow \sin^2\frac{x}{2} = 1-\frac{K}{2} = \frac{2-K}{2}$

$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{2-K}{2}}$

56. (A)

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

57. (A)

$$\frac{\sin 3\theta - \cos 3\theta}{\sin\theta+\cos\theta}+1 = \frac{3\sin\theta - 4\sin^3\theta - 4\cos^3\theta + 3\cos\theta}{\sin\theta+\cos\theta}+1$$

$$= 4\frac{\sin\theta+\cos\theta-\sin^3\theta-\cos^3\theta}{\sin\theta+\cos\theta}$$

$$= 4\frac{\sin\theta+\cos\theta-(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)}{\sin\theta+\cos\theta}$$

$$= 4\sin\theta\cos\theta = 2\sin 2\theta$$

58. (C)

$\cos^2 A(3-4\cos^2 A)^2 + \sin^2 A\ (3-4\sin^2 A)^2 = (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$

$= \cos^2 3A + \sin^2 3A = 1$

59. (C)

$$\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta} = \frac{3\cos\theta+(4\cos^3\theta-3\cos\theta)}{3\sin\theta-(3\sin\theta-4\sin^3\theta)} = \cot^3\theta$$

60. (C)

$\sin 20^0 \sin 40^0\ \sin 60^0 \sin 80^0 = (\sin(60^0-20^0)\sin 20^0 \sin(60^0+20^0))\sin 60^0$

$$= \frac{1}{4}\sin(3\times 20^0)\sin 60^0 = \frac{1}{4}\sin^2 60^0 = \frac{3}{16}$$

61. (D)

$$\cos 20^0\cos 40^0\cos 80^0 = \frac{2\sin 20^0\cos 20^0\cos 40^0\cos 80^0}{2\sin 20^0}$$

$$= \frac{2\sin 40^0\cos 40^0\cos 80^0}{4\sin 20^0} = \frac{2\sin 80^0\cos 80^0}{8\sin 20^0}$$

$= \frac{\sin 160^0}{8\sin 20^0}$, but sin 160^0 = sin (180^0 – 20^0) = sin 20^0, hence

$\cos 20^0 \cos 40^0 \cos 80^0 = \frac{1}{8}$

Alternately

$\cos 20^0 \cos 40^0 \cos 80^0 = \cos(60^0 - 20^0)\cos 20^0 \cos(60^0 + 20^0)$

$\frac{1}{4}\cos\left(3\times 20^0\right) = \frac{1}{8}$

62. (D)

$4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)$

$= \cos 3\theta$

63. (C)

$\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right) + \tan\left(\theta-\frac{\pi}{3}\right)$

$= \tan\theta + \frac{\tan\theta+\sqrt{3}}{1-\tan\theta.\sqrt{3}} + \frac{\tan\theta-\sqrt{3}}{1+\tan\theta.\sqrt{3}}$

$= \tan\theta + \frac{\tan\theta+\sqrt{3}+\sqrt{3}\tan^2\theta+3\tan\theta+\tan\theta-\sqrt{3}-\tan^2\theta\sqrt{3}+3\tan\theta}{1-3\tan^2\theta}$

$= \frac{\tan\theta - 3\tan^3\theta + 8\tan\theta}{1-3\tan^2\theta}$

$= \frac{9\tan\theta - 3\tan^3\theta}{1-3\tan^2\theta}$

$= \frac{3\left[3\tan\theta - \tan^3\theta\right]}{1-3\tan^2\theta} = 3\tan 3\theta$

$\therefore K = 3$

Hence (C) holds.

64. (A)

$x = \tan 15^\circ = 2-\sqrt{3}$

$y = \text{cosec}\, 75^\circ = \sqrt{6}-\sqrt{2}$

$(z) = 4\sin 18^\circ = \sqrt{5}-1$

It follows that $x < y < z$

$\therefore$ (A) holds

65. (D)

$-\sqrt{3^2+4^2}+8 \le 3\cos x + 4\sin x + 8 \le \sqrt{3^2+4^2}+8$

$\Rightarrow 3 \le 3\cos x + 4\sin x + 8 \le 13$

66. (A)

$\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{6}\right)$ acquires maximum at $\alpha+\frac{\pi}{6}=\frac{\pi}{4}$ i.e. $\theta=\frac{\pi}{12}$

67. (D)

$$5\sin^2\theta + 4\cos^2\theta = 4 + \sin^2\theta \geq 4$$

68. (B)

Maximum value of $\sin\theta\cos\theta$

$=$Maximum value of $\left(\frac{\sin 2\theta}{2}\right)$

$= \frac{1}{2}$ Maximum Value of $\sin 2\theta = \frac{1}{2}(1) = \frac{1}{2}$

69. (C)

Minimum value of $\sin\theta\cos\theta$

$= \frac{1}{2}$ Minimum value of $\sin 2\theta = \frac{1}{2}(-1) = -\frac{1}{2}$

70. (C)

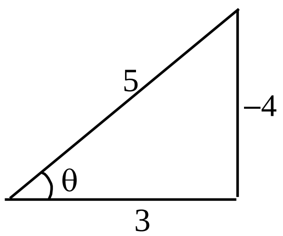
$$\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) \leq \sqrt{2}$$

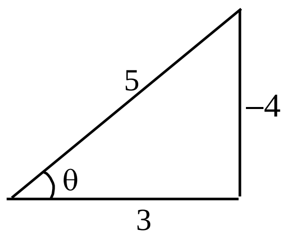
$\therefore$ Maximum value $= \sqrt{2}$

71. (D)

Put $-\left.\begin{matrix} 3 = r\cos\alpha \\ 4 = r\sin\alpha \end{matrix}\right] \begin{matrix} \Rightarrow r^2 = 9 + 16 = 25 \\ \Rightarrow \qquad r = 5 \end{matrix}$

$\therefore 3\cos\theta + 4\sin\theta = r(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$





$$= r\cos(\theta - \alpha) = 5\cos(\theta - \alpha)$$

Its Maximum value $= 5(1) = 5$

And Minimum value $= -5$

72. (A)

Given, $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$

$\frac{1}{2}(1 - \cos^2\theta) + \frac{1}{3}\cos^2\theta = \frac{1}{2} - \frac{1}{6}\cos^2\theta$

Since, $0 \leq \cos^2\theta \leq 1 \Rightarrow -\frac{1}{6} \leq -\frac{1}{6}\cos^2\theta \leq 0$

$\Rightarrow \frac{1}{3} \leq \frac{1}{2} - \frac{1}{6}\cos^2\theta \leq \frac{1}{2} \Rightarrow \frac{1}{3} \leq P \leq \frac{1}{2}$

$\therefore$ (A) holds

73. (C)

$$A+B+C=\pi$$

$$\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C$$

74. (A)

$$\frac{A+B+C}{2}=\frac{\pi}{2}$$

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{c}{2}\right)$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{c}{2}}$$

$$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{A}{2}\tan\frac{C}{2}=1$$

$$\tan\frac{C}{2}=1-\frac{2}{9}=\frac{7}{9}$$

75. (D)

$$A+B+C=\pi$$

$$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$$

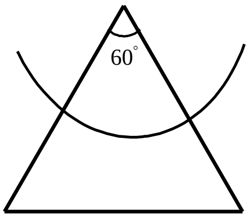
$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{C}{2}}$$

$$\Rightarrow\sum\tan\frac{A}{2}\tan\frac{B}{2}=1$$

EXERCISE - 1 [B]

1. (A)



Area of $\Delta = \frac{\sqrt{3}}{4}.4^2 = 4\sqrt{3}$

2π _____ πr^2

1 ______ $\frac{\pi r^2}{2\pi}$

$\frac{\pi}{3}$ _____ $\frac{\pi r^2}{2\pi}.\frac{\pi}{3}$

Given, $\frac{\pi r^2}{6} = \frac{4\sqrt{3}}{2}$

$\pi r^2 = 12\sqrt{3}$

$r = \sqrt{\frac{12\sqrt{3}}{\pi}}$

2. (C)

Angle covered from 6 A.M. to 3.15 P.M.

$= 277\frac{1}{2}^\circ = \frac{555}{2}\times\frac{\pi}{180} = \frac{111}{2}\times\frac{\pi}{36}$

$\therefore \theta = \frac{37\pi}{24}$ radians

Length of hour hand $= 12$cm

i.e. $r = 12$cm

Since $\theta = \frac{l}{r}$ $\quad\quad \therefore l = r\theta = \frac{12\times 37}{24} = \frac{37\pi}{2}$

Hence reqd. distance $= \frac{37\pi}{2}$ cm.

3. (D)

An hour hand in 1 hour i.e., 60 minute traces $= \frac{360^\circ}{12} = 30^\circ$

$\therefore$ An hour hand in 1 minute traces $= \frac{30^\circ}{60^\circ} = \left(\frac{1}{2}\right)^\circ$

$\therefore$ An hour hand in 40 minutes traces

$= \left(\frac{1}{2}\times 40\right)^\circ = 20^\circ$

$\therefore$ reqd. angle $= 90^\circ - 20^\circ = 70^\circ$

$= \frac{70 \times \pi}{180} = \frac{7\pi}{18}$

$\therefore$ (D) holds

4. (B)

$\sin^2 x + \text{cosec}^2 x + 2 + \cos^2 x + \sec^2 x + 2 - \tan^2 x - \cot^2 x - 2$

$= 3 + \left(\text{cosec}^2 x - \cot^2 x\right) + \left(\sec^2 x - \tan^2 x\right)$

$= 5$

5. (D)

$\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = \text{cosec}^2\theta\sec^2\theta$

6. (A)

Let $\lambda = 3\cos\beta - 5\sin\beta$

Let $S = 3\sin\beta + 5\cos\beta$

$\lambda^2 + S^2 = 9 + 25$

$\therefore \lambda^2 = 9$

7. (A)

$\tan^2\alpha + \cot^2\alpha + 2 = m^2$

$\left(\tan^2\alpha + \cot^2\alpha\right)^2 = \left(m^2 - 2\right)^2$

$\tan^4\alpha + \cot^4\alpha + 2 = m^4 + 4 - 4m^2$

$\tan^4\alpha + \cot^4\alpha = m^4 - 4m^2 + 2$

8. (B)

$\frac{\cot\theta + \text{cosec}\theta - 1}{\cot\theta - \text{cosec}\theta + 1}$

$= \frac{\frac{1}{\text{cosec}\theta - \cot\theta} - 1}{\cot\theta - \text{cosec}\theta + 1} \quad \left(\because (\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta) = 1\right)$

$= \frac{1 - \text{cosec}\theta + \cot\theta}{(\text{cosec}\theta - \cot\theta)(\cot\theta - \text{cosec}\theta + 1)}$

$= \frac{1}{\text{cosec}\theta - \cot\theta} = \text{cosec}\theta + \cot\theta$

$= \frac{1 + \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 - \cos\theta}$

9. (D)

Since $\sin x + \sin^2 x = 1$

$\therefore \sin x = 1 - \sin^2 x = \cos^2 x$

$\therefore \sin^2 x = \cos^4 x$

$\Rightarrow 1 - \cos^2 x = \cos^4 x$

$\Rightarrow 1 = \cos^4 x + \cos^2 x$

$\Rightarrow 1 = \left(\cos^4 x + \cos^2 x\right)^2$

$=\cos^8 x+2\cos^6 x+\cos^4 x$

Thus $\cos^8 x+2\cos^6 x+\cos^4 x=1$

10. (C)

$\sin\left(-566^\circ\right)=-\sin 566^\circ=-\sin\left(6\times 90^\circ+26^\circ\right)$

$=-\left(-\sin 26^\circ\right)=\sin 26^\circ$

11. (C)

$f(x)=3\left[\sin^4\left(\frac{\pi}{2}-x\right)+\sin^4 x\right]-2\left[\cos^6 x+\sin^6 x\right]$

$=3\left[\left(\cos^2 x\right)^2+\left(\sin^2 x\right)^2\right]-2\left[\left(\sin^2 x\right)^3+\left(\cos^2 x\right)^3\right]$

$=3\left[1-2\sin^2 x\cos^2 x\right]-2\left[1-3\sin^2 x\cos^2 x\right]$

$=1$

12. (A)

$2\cos 10+\sin(90+10)+\sin(3\times 360-80)+\sin(27\times 360+280)$

$=2\cos 10+\cos 10-2\sin 80$

$=\cos 10+2\left(\cos 10-\sin(90-10)\right)$

$=\cos 10+2\left(\cos 10-\cos 10\right)=\cos 10$

13. (C)

$\cos^2(90-17)+\cos^2 47-\sin^2(90-47)+\sin^2(90+17)$

$=\sin^2 17+\cos^2 47-\cos^2 47+\cos^2 17$

$=1$

14. (D)

(i) $\sin(2\times 360+45)=\sin 45=\frac{1}{\sqrt{2}}$

(ii) $\frac{1}{-\sin(3\times 360+330)}=\frac{1}{-\sin(360-30)}=\frac{1}{\sin 30}=2$

(iii) $\tan\left(4\pi+\frac{\pi}{3}\right)=\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$

(iv) $\frac{-1}{\tan\left(4\pi-\frac{\pi}{4}\right)}=1$

15. (C)

$\ell^2=\left[\sin\theta-\sin\left(\frac{\pi}{2}-\theta\right)\right]^2+\left[\cos\theta+\cos\left(\frac{\pi}{2}-\theta\right)\right]$

$=(\sin\theta-\cos\theta)^2+(\cos\theta+\sin\theta)^2$

$=1+1=2$

$\ell=\sqrt{2}$

16. (B)

$\sin(180+20)+\cos(180+20)$

$=-\sin 20-\cos 20<0$

17. (B)

$a=\sin 170^{\circ}+\cos 170^{\circ}$

$=\sin(180-10)+\cos(180-10)$

$=\sin 10-\cos 10$

$\Rightarrow a=-ve$

$\{\because \cos 10^{\circ}>\sin 10^{\circ}\}$

18. (C)

$\cos\left(\frac{\pi}{2}-x\right)=\sin x,\ \cos\left(\frac{3\pi}{2}+x\right)=\sin x$

$\cos(\pi-x)=-\cos x, \cos(2\pi-x)=\cos x$

$\therefore$ required $=\sin x\cdot\sin x+\cos x\cdot\cos x$

$=1$

19. (B)

$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}, \cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$

$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}, \cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$

$\Rightarrow x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3} \Rightarrow x=-\frac{y}{2}=-\frac{z}{2} \Rightarrow y=z=-2x$

Now, $xy + yz + zx = x(-2x) + (-2x)(-2x) + (-2x)x = 0$

20. (A)

$\cos^2\alpha+\cos^2(\alpha+120^0)+\cos^2(\alpha-120^0)$

$=\frac{1+\cos 2\alpha}{2}+\frac{1+\cos 2(\alpha+120^0)}{2}+\frac{1+\cos 2(\alpha-120^0)}{2}$

$=\frac{3+\cos 2\alpha+2\cos 2\alpha\cos 120^0}{2}=\frac{3+\cos 2\alpha-\cos 2\alpha}{2}=\frac{3}{2}$

21. (B)

$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}\Rightarrow \tan(A+B)=\frac{-\frac{1}{2}-\frac{1}{3}}{1-\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)}$

$\Rightarrow \tan(A+B)=-1$

$\Rightarrow A+B=\frac{3\pi}{4}$

22. (D)

$\tan 45^0 = \tan\left(180^0 + 45^0\right) \Rightarrow \tan 225^0 = \tan\left(100^0 + 125^0\right)$

$\Rightarrow \dfrac{\tan 100^0 + \tan 125^0}{1 - \tan 100^0 \tan 125^0} = 1$

$\Rightarrow \tan 100^0 + \tan 125^0 + \tan 100^0 \tan 125^0 = 1$

23. (B)

$$\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)} = \frac{\sin(B+A)+\sin\left(\frac{\pi}{2}-(B-A)\right)}{\cos\left(\frac{\pi}{2}-(B-A)\right)+\cos(B+A)}$$

$$= \frac{2\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)}{2\cos\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)} = \tan\left(\frac{\pi}{4}+A\right)$$

$$= \frac{1+\tan A}{1-\tan A} = \frac{\cos A + \sin A}{\cos A - \sin A}$$

24. (C)

Tan 3A=tan(2A+A)

$= \dfrac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$

$\Rightarrow \tan 3A - \tan 3A \tan 2A \tan A$

$= \tan 2A + \tan A$

$\therefore \tan 3A - \tan 2A - \tan A$

$= \tan A \tan 2A \tan 3A$

25. (C)

$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= \sin^2 A\left(1 - \sin^2 B\right) - \sin^2 B\left(1 - \sin^2 A\right)$

$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B$

26. (B)

$\sin^2 A + \sin^2(A-B) + 2\sin A \cos B \sin(B-A)$

$= \sin^2 A + \sin^2(A-B) + \left[\sin(A+B) + \sin(A-B)\right]\sin(B-A)$

$= \sin^2 A + \sin^2(A-B) + \sin(A+B)\sin(B-A) - \sin^2(A-B)$

$= \sin^2 A + \sin^2 B - \sin^2 A$

$= \sin^2 B$

27. (A)

$$\frac{\dfrac{\sin\left(\dfrac{3\theta}{2}\right)}{\sin\left(\dfrac{\theta}{2}\right)}\sin\left(\theta+\dfrac{2\theta}{2}\right)}{\dfrac{\sin\left(\dfrac{3\theta}{2}\right)}{\sin\dfrac{\theta}{2}}\cos\left(\theta+\dfrac{2\theta}{2}\right)}=\frac{\sin\alpha}{\cos\alpha}$$

$$\tan 2\theta=\tan\alpha$$

$$\theta=\frac{\alpha}{2}$$

28. (B)

$$\sin\alpha+\sin\beta=a\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=a \qquad \ldots(i)$$

$$\&\ \cos\alpha-\cos\beta=b\Rightarrow 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}=-b \qquad \ldots(ii)$$

From (i) & (ii) $\tan\dfrac{\alpha-\beta}{2}=-\dfrac{b}{a}$

29. (C)

$$\frac{\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta}{\cos 3\theta+\cos 5\theta+\cos 7\theta+\cos 9\theta}=\frac{(\sin 3\theta+\sin 9\theta)+(\sin 5\theta+\sin 7\theta)}{(\cos 3\theta+\cos 9\theta)+(\cos 5\theta+\cos 7\theta)}$$

$$=\frac{2\sin 6\theta\cos 3\theta+2\sin 6\theta\cos\theta}{2\cos 6\theta\cos 3\theta+2\cos 6\theta\cos\theta}=\tan 6\theta$$

30. (D)

Given value

$$=\left(\sin 47^{\circ}+\sin 61^{\circ}\right)-\left(\sin 11^{\circ}+\sin 25^{\circ}\right)$$

$$=2\sin 54^{\circ}\cos 7^{\circ}-2\sin 18^{\circ}\cos 7^{\circ}$$

$$=2\cos 7^{\circ}\left(\sin 54^{\circ}-\sin 18^{\circ}\right)$$

$$=2\cos 7^{\circ}\,\frac{2\sin 18^{\circ}\cos 18^{\circ}}{\cos 18^{\circ}}\cos 36^{\circ}$$

$$=\cos 7^{\circ}\,\frac{2\sin 36^{\circ}\cos 36^{\circ}}{\cos 18^{\circ}}$$

$$=\cos 7^{\circ}\,\frac{\sin 72^{\circ}}{\cos 18^{\circ}}=\cos 7^{\circ}\left[\because \sin 72^{\circ}=\cos 18^{\circ}\right]$$

31. (D)

$$\cos 40^{\circ}+\cos 80^{\circ}+\cos 160^{\circ}+\cos 240^{\circ}$$

$$=2\cos 60^{\circ}\cos 20^{\circ}+\cos\left(180^{\circ}-20^{\circ}\right)+\cos\left(180^{\circ}+60^{\circ}\right)$$

$$=2\left(\frac{1}{2}\right)\cos 20^{\circ}-\cos 20^{\circ}-\cos 60^{\circ}=-\cos 60^{\circ}=-\frac{1}{2}$$

32. (A)

$\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 228^\circ$

$= \left(\cos 12^\circ + \cos 228^\circ\right) + \left(\cos 84^\circ + \cos 156^\circ\right)$

$= 2\cos\left(120^\circ\right)\cos\left(108^\circ\right) + 2\cos 120^\circ \cos 36^\circ$

$= 2\cos\left(120^\circ\right)\left[\cos\left(108^\circ + \cos 36^\circ\right)\right]$

$= 2\left(-\frac{1}{2}\right).2\cos 72^\circ \cos\left(36^\circ\right)$

$= -2\cos 72^\circ \cos 36^\circ = -2.\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4}$

$= -2\left(\frac{5-1}{16}\right) = -\frac{1}{2}$

33. (B)

$\cot\theta - 2\cot 2\theta$

$\frac{1}{\tan\theta} - \frac{2\times\left(1-\tan^2\theta\right)}{2\tan\theta}$

$= \frac{\tan^2\theta}{\tan\theta} = \tan\theta$

34. (D)

$\frac{\cos 10 - \sqrt{3}\sin 10}{\sin 10.\cos 10}$

$= 2.2\frac{\left(\frac{1}{2}\cos 10 - \frac{\sqrt{3}}{2}\sin 10\right)}{2\sin 10\cos 10}$

$= \frac{4\sin\left(30-10\right)}{\sin 20} = 4$

35. (B)

$\cos\alpha + \cos\beta = 0 \Rightarrow \cos^2\alpha + \cos^2\beta = -2\cos\alpha\cos\beta$

$\&\sin\alpha + \sin\beta = 0 \Rightarrow \sin^2\alpha + \sin^2\beta = -2\sin\alpha\sin\beta$

Now $\cos 2\alpha + \cos 2\beta = 2\left(\cos^2\alpha + \cos^2\beta - 1\right) = 2\left(1 - \sin^2\alpha - \sin^2\beta\right)$

$\Rightarrow \cos 2\alpha + \cos 2\beta = 2\left(-2\cos\alpha\cos\beta - 1\right) \quad ...(i)$

$\& \cos 2\alpha + \cos 2\beta = 2\left(1 + 2\sin\alpha\sin\beta\right) \quad \ldots(ii)$

(i) + (ii)

$\Rightarrow \cos 2\alpha + \cos 2\beta = -2\cos\left(\alpha+\beta\right)$

36. (C)

$$\sqrt{3}\operatorname{cosec}20^0 - \sec 20^0 = \frac{\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0}{\frac{1}{2}\cos 20^0 \sin 20^0} = \frac{4\sin\left(60^0 - 20^0\right)}{2\cos 20^0 \sin 20^0} = \frac{4\sin 40^0}{\sin 40^0} = 4$$

37. (C)

$$\sqrt{3}\left[\cot\theta+\tan\theta\right]=4$$

$$\Rightarrow \sqrt{3}\left[\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right]=4$$

$$\Rightarrow \frac{1}{\sin\theta\cos\theta}=\frac{4}{\sqrt{3}} \Rightarrow \frac{1}{2\sin\theta\cos\theta}=\frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin 2\theta=\frac{\sqrt{3}}{2}=\sin\frac{\pi}{3}$$

$$\Rightarrow 2\theta=\frac{\pi}{3} \qquad \therefore \theta=\frac{\pi}{6}$$

38. (B)

Since, $\cos 20^\circ-\sin 20^\circ=p$

$$\Rightarrow \cos^2 20^\circ+\sin^2 20^\circ-2\sin 20^\circ\cos 20^\circ=p^2$$

$$\Rightarrow 1-p^2=\sin 40^\circ$$

$$\Rightarrow 1-p^2=\sqrt{1-\cos^2 40^\circ}$$

$$\Rightarrow \left(1-p^2\right)^2=1-\cos^2 40^\circ$$

$$\Rightarrow \cos^2 40^\circ=1-\left(1+p^4-2p^2\right)$$

$$\Rightarrow \cos 40^\circ=\sqrt{2p^2-p^4}$$

$$\Rightarrow \cos 40^\circ=p\sqrt{2-p^2}$$

$\therefore$ (B) holds

39. (B)

$$\text{LHS}=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

$$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$=\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}=\frac{1}{4}\left[\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right]^2=\frac{1}{8}$$

40. (C)

$$\sin\theta=\frac{2t}{1+t^2}=\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \text{ if } t=\tan\frac{\theta}{2}$$

$$\therefore \cos\theta=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}=\frac{1-t^2}{1+t^2}$$

41. (D)

$$\frac{1}{4}\left(4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right)=\frac{\cos 3\theta}{4}$$

42. (B)

$$\frac{\cos 20+4(\cos 40-\cos 60)\sin 70}{\cos^2 10}$$

$$=\frac{\cos 20+4\cos 40\cos 20-2\cos 20}{\cos^2 10}$$

$$=\frac{2(\cos 20+2\cos 60+2\cos 20-2\cos 20)}{2\cos^2 10}$$

$$=\frac{2(\cos 20+1)}{(1+\cos 20)}=2$$

43. (C)

$$\sin 12^0 \sin 48^0 \sin 54^0=\sin(60^0-12)\sin 12^0 \sin(60^0+12^0)\frac{\sin 54^0}{\sin 72^0}$$

$$=\frac{\sin 36^0 \sin 54^0}{4\sin 72^0}=\frac{\sin 36^0 \sin 54^0}{8\sin 36^0 \cos 36^0}$$

$$=\frac{\sin 54^0}{8\cos 36^0}\text{, but } \sin 54^0=\sin(90^0-36^0)=\cos 36^0$$

$$\therefore \sin 12^0 \sin 48^0 \sin 54^0=\frac{1}{8}$$

44. (B)

$$x+\frac{1}{x}=2\cos\theta \ \ \& \ \ x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$\Rightarrow x^3+\frac{1}{x^3}=8\cos^3\theta-6\cos\theta=2\cos 3\theta$$

45. (C)

$$\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$$

$$=\sin\frac{\pi}{10}\sin\left(\pi+\frac{3\pi}{10}\right)$$

$$=-\sin\frac{\pi}{10}\sin\frac{3\pi}{10}=-\sin 18^\circ \sin 54^\circ$$

$$=-\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4}=-\frac{5-1}{16}=-\frac{1}{4}$$

46. (C)

$$3\sin 2\theta=2\sin 3\theta$$

$$\Rightarrow 6\sin\theta\cos\theta=2(3\sin\theta-4\sin^3\theta)$$

$$\Rightarrow 4\cos^2\theta-3\cos\theta-1=0$$

$$\Rightarrow \cos\theta=1,\frac{1}{4}$$

Since $\cos\theta\neq 1(\because \theta\neq 0)$

$$\therefore \cos\theta=\frac{1}{4}\Rightarrow \sin\theta=\frac{\sqrt{15}}{4}$$

47. (C)

$f(\theta)=\sin\theta\left(\sin\theta+3\sin\theta-4\sin^3\theta\right)$

$f(\theta)=4\sin^2\theta-4\sin^4\theta$

$f(\theta)=4\sin^2\theta\left(1-\sin^2\theta\right)\geq 0\ \ \forall\theta$

48. (D)

$f(\theta)=5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$

$=5\cos\theta+3\cos\theta.\cos\frac{\pi}{3}-3\sin\theta\sin\frac{\pi}{3}+3$

$=\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta+3$

$-\sqrt{\frac{13^2}{4}+\frac{27}{4}}+3\leq f(\theta)\leq\sqrt{\frac{13^2}{4}+\frac{27}{4}}+3$

$-4\leq f(\theta)\leq 10$

49. (C)

$\sin\theta+\cos\beta$

Maximum Value $=2$

50. (C)

$y=\frac{12}{9+5\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)}$

$=\frac{12}{9+5\sin(\alpha+x)}$ let $\left(\sin\alpha=\frac{3}{5}\right)$

$y_{maximum}=\frac{12}{4}=3$

51. (D)

$y_{minimum}=\frac{1}{9}$

52. (A)

$B=2\sin^2 x-\cos 2x$

$=2\sin^2 x-\left(1-2\sin^2 x\right)$

$=4\sin^2 x-1$

Since $0\leq\sin^2 x\leq 1$ $\quad\therefore 0\leq 4\sin^2 x\leq 4$

$\Rightarrow\quad 0-1\leq 4\sin^2 x-1\leq 3\Rightarrow -1\leq B\leq 3$

53. (B)

$\sin\theta+\cos\theta=\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)$ is Max when $\theta+\frac{\pi}{4}=\frac{\pi}{2}$ i.e. when $\theta=\frac{\pi}{4}=45^\circ$

54. (B)

$2\sin^2\theta + 3\cos^2\theta$

$= 2\left[\sin^2\theta + \cos^2\theta\right] + \cos^2\theta = 2 + \cos^2\theta$

Since least value of $\cos^2\theta = 0$

$\therefore$ least value of $2\sin^2\theta + 3\cos^2\theta = 2 + 0 = 2$

55. (C)

Let $y = \cos\theta - \cos 2\theta$

$\frac{dy}{dx} = \sin\theta + 2\sin 2\theta = 2.2\sin\theta\cos\theta - \sin\theta$

$= \sin\theta(4\cos\theta - 1)$

For Max or Min of $y, \frac{dy}{d\theta} = 0$

$\therefore$ $\sin\theta = 0$ or $\cos\theta = \frac{1}{4}$

i.e. $\theta = 0, \pi, 2\pi ...... \text{or} \cos\theta = \frac{1}{4}$

$\frac{d^2y}{d\theta^2} = -\cos\theta + 4\cos 2\theta > 0$ for $\theta = 0, \pi 2\pi, ...$ and $4\cos 2\theta - \cos\theta = 4\left[2\cos^2\theta - 1\right] - \cos\theta$

$= 8\left(\frac{1}{4}\right)^2 - 4 - \frac{1}{4} = \frac{1}{2} - 4 - \frac{1}{4} < 0$

$\therefore$ y max. when $\cos\theta = \frac{1}{4}$

$\therefore$ $\max.y \quad = \cos\theta - \left(2\cos^2\theta - 1\right)$

$= \frac{1}{4} - \left[\frac{2}{16} - 1\right]$

$= \frac{1}{4} + 1 - \frac{1}{8} = \frac{1}{4} + \frac{7}{8}$

$= \frac{2+7}{8} = \frac{9}{8}$

$\therefore$ (C) holds

56. (D)

$\frac{8\sin x}{8} \cdot \frac{\cos x}{8} \cdot \frac{\cos x}{4} \cdot \frac{\cos x}{2}$

$= \theta = \frac{x}{7}$

$8\sin\theta \cdot \cos\theta \cdot \cos 2\theta \cdot \cos 4\theta$

$= 8\sin\theta \cdot \frac{\sin\left(2^3\theta\right)}{2^3\sin\theta} = \sin 8\theta = \sin x$

57. (C)

We know that

$$\cos A \cos 2A \cos 2^2 A \cos 2^3 A \; \cos 2^{n-1} = \frac{\sin 2^n A}{2^n \sin A}$$

$$\therefore \cos A \cos 2A \cos 2^2 A = \frac{\sin 2^3 A}{2^3 \sin A} \qquad \text{[By putting } n = 3\text{]}$$

$$\therefore \cos A \cos 2A \cos 4A = \frac{\sin 8A}{8 \sin A}$$

$$\therefore \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} = \frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}$$

$$= \frac{\sin\left(\pi + \frac{\pi}{7}\right)}{8\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}} = -\frac{1}{8}$$

Hence (C) holds

58. (D)

The given expression

$$= \frac{1}{2\sin\frac{\pi}{65}}\left(2\sin\frac{\pi}{65}\cos\frac{\pi}{65}\right)\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$$

$$= \frac{1}{4\sin\frac{\pi}{65}}\left(2\sin\frac{2\pi}{65}\cos\frac{2\pi}{65}\right)\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$$

$$= \frac{1}{8\sin\frac{\pi}{65}}\left(2\sin\frac{4\pi}{65}\cos\frac{4\pi}{65}\right)\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$$

$$= \frac{1}{16\sin\frac{\pi}{65}}\left(2\sin\frac{8\pi}{65}\cos\frac{8\pi}{65}\right)\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$$

$$= \frac{1}{32\sin\frac{\pi}{65}}\left(2\sin\frac{16\pi}{65}\cos\frac{16\pi}{65}\right)\cos\frac{32\pi}{65}$$

$$= \frac{1}{64\sin\frac{\pi}{65}}\left(2\sin\frac{32\pi}{65}\cos\frac{32\pi}{65}\right)$$

$$= \frac{1}{64\sin\frac{\pi}{65}}\sin\frac{64\pi}{65}$$

$$= \frac{1}{64\sin\frac{\pi}{65}}\sin\left(\pi - \frac{\pi}{65}\right) = \frac{\sin\frac{\pi}{65}}{64\sin\frac{\pi}{65}} = \frac{1}{64}$$

59. (A)

$\cos A \cos 2A \cos 4A....\cos 2^{n-1} A$

$= \dfrac{\sin 2^n A}{2^n \sin A}$ [Standard Result]

$\therefore$ (A) hold.

60. (B)

$A + B + C = \dfrac{3\pi}{2}$

$\cos 2A + \cos 2B + \cos 2C + 4\sin A \sin B \sin C$

$= 2\cos(A+B)\cos(A-B) + 2\cos^2 C - 1 + 4\sin A \sin B \sin C$

$= 2\cos\left(3\dfrac{\pi}{2} - c\right)\cos(A-B) + 2\cos^2 C - 1 + 4\sin A.\sin B.\sin C$

$= -2\sin C\cos(A-B) + 1 - 2\sin^2 C + 4\sin A \sin B \sin C$

$= -2\sin C\left[\cos(A-B) - \cos(A+B)\right] + 4\sin A \sin B \sin C$

$= -4\sin A \sin B \sin C + 4\sin A \sin B \sin C + 1$

61. (C)

If $A + B + C = n\pi$ then

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

62. (B)

$\left(\dfrac{\cos B}{\cos B} + \dfrac{\cos C}{\sin C}\right)\left(\dfrac{\cos C}{\sin C} + \dfrac{\cos A}{\sin A}\right)\left(\dfrac{\cos A}{\sin A} + \dfrac{\cos B}{\sin B}\right)$

$= \dfrac{\sin(B+C).\sin(A+C).\sin(A+B)}{\sin^2 A.\sin^2 B \sin^2 C}$

$= \text{cosec}A.\text{cosec}B.\text{cosec}C$

$A + B = \pi - C$

$\Rightarrow \sin(A+B) = \sin C$

63. (A)

$A + B = C = 180^\circ \Rightarrow \dfrac{A}{2} + \dfrac{B}{2} = 90^\circ - \dfrac{C}{2}$

$\therefore \tan\left(\dfrac{A}{2} + \dfrac{B}{2}\right) = \tan\left(90^\circ - \dfrac{C}{2}\right) = \cot\dfrac{C}{2}$

$\therefore \dfrac{\tan\dfrac{A}{2} + \tan\dfrac{B}{2}}{1 - \tan\dfrac{A}{2}\tan\dfrac{B}{2}} = \dfrac{1}{\tan\dfrac{C}{2}}$

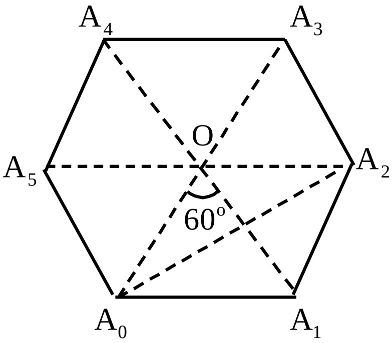
$\Rightarrow \tan\dfrac{A}{2}\tan\dfrac{C}{2} + \tan\dfrac{B}{2}\tan\dfrac{C}{2} = 1 - \tan\dfrac{A}{2}\tan\dfrac{B}{2}$

$\Rightarrow \tan\dfrac{B}{2}\tan\dfrac{C}{2} + \tan\dfrac{C}{2}\tan\dfrac{A}{2} + \tan\dfrac{A}{2}\tan\dfrac{B}{2} = 1$

Divide thro' out by $\tan\dfrac{A}{2}\tan\dfrac{B}{2}\tan\dfrac{C}{2}$, we get

$\cot\dfrac{A}{2} + \cot\dfrac{B}{2} + \cot\dfrac{C}{2} = \cot\dfrac{A}{2}\cot\dfrac{B}{2}\cot\dfrac{C}{2}$

64. (C)



OA_0A_1 is equilateral triangle

$A_0A_1 = 1$

$\cos 120^\circ = \frac{1^2 + 1^2 - A_0A_2{}^2}{2.1.1.}$ **(cosine law)**

$-\frac{2}{2} = 2 - A_0A_2{}^2$

$A_0A_2{}^2 = 3$

$A_0A_2 = \sqrt{3}$

$A_0A_4 = \sqrt{3}$

$A_0A_1 \times A_0A_2 + A_0A_4 = 1.\sqrt{3}.\sqrt{3} = 3$

65. (C)

$\frac{b}{y} = \cot\theta$ & $\frac{a}{x} = \operatorname{cosec}\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = \frac{a^2}{x^2} - \frac{b^2}{y^2}$

$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

66. (A)

Required number of sides $= \frac{360^\circ}{36^\circ} = 10$

$\therefore$ (A) holds

67. (A)

$\cos A = m\cos B \Rightarrow \frac{\cos A}{\cos B} = m$

Apply component to get $\frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$

$\Rightarrow -\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}} = \frac{m+1}{m-1}$

$\Rightarrow \cot\frac{A+B}{2} = \frac{m+1}{m-1}\tan\frac{B-A}{2}$

68. (A)

$$\frac{\sin 2x}{\sin 2y}=n \Rightarrow \frac{\sin 2x+\sin 2y}{\sin 2x-\sin 2y}=\frac{n+1}{n-1} \text{ \{By componendo \& dividendo\}}$$

$$\Rightarrow \frac{2\sin(x+y)\cos(x-y)}{2\cos(x+y)\sin(x-y)}=\frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan(x+y)}{\tan(x-y)}=\frac{n+1}{n-1}$$

69. (D)

$$\frac{\sin\alpha}{\sin\beta}=\frac{5}{3}$$

$$\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\frac{8}{2}=4\ (C\&D)$$

$$\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}=4$$

$$\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}=4$$

70. (B)

$$\operatorname{cosec} A(\sin B\cos C+\cos B\sin C)$$

$$=\operatorname{cosec} A\sin(B+C)$$

$$=\operatorname{cosec} A\sin(180^\circ-A) \qquad [\because A+B+C=180^\circ]$$

$$=\operatorname{cosec} A\sin A=1$$

Hence, (B) holds

EXERCISE - 1 [C]

1. (0.625)

$$\sin 2\theta+\sin 2\phi=\frac{1}{2} \qquad \& \qquad \cos 2\theta+\cos 2\phi=\frac{3}{2}$$

$$\Rightarrow 2\sin(\theta+\phi)\cdot\cos(\theta-\phi)=\frac{1}{2} \qquad ...(1)$$

$$2\cos(\theta+\phi)\cdot\cos(\theta-\phi)=\frac{3}{2} \qquad ...(2)$$

Square & add (1) & (2)

$$\Rightarrow 4\cos^2(\theta-\phi)\cdot\left[\sin^2(\theta+\phi)+\cos^2(\theta+\phi)\right]=\frac{1}{4}+\frac{9}{4}$$

$$\Rightarrow \cos^2(\theta-\phi)=\frac{10}{16}$$

2. (0.0625)

$$\left[\cos 20^{o}\cdot\cos\left(60-20^{o}\right)\cdot\cos\left(60+20^{o}\right)\right]\cdot\cos 60^{o}$$

$$=\frac{1}{4}\cdot\cos 60^{o}\cdot\cos 60^{o}=\frac{1}{16}$$

3. (0.78)

$$\cos(A+B)=\cos A\cdot\cos B-\sin A\cdot\sin B$$

$$\sin A=\frac{1}{\sqrt{5}}\ \Rightarrow\ \cos A=\sqrt{1-\sin^{2}A}=\frac{2}{\sqrt{5}}$$

$$\cos B=\frac{3}{\sqrt{10}}\ \Rightarrow\ \sin B=\sqrt{1-\cos^{2}B}=\frac{1}{\sqrt{10}}$$

$$\therefore\ \cos(A+B)=\frac{2}{\sqrt{5}}\cdot\frac{3}{\sqrt{10}}-\frac{1}{\sqrt{5}}\cdot\frac{1}{\sqrt{10}}=\frac{1}{\sqrt{2}}$$

$$\therefore\ A+B=\frac{\pi}{4}$$

4. (2)

$$A-B=\frac{\pi}{4}\ \Rightarrow\ \tan(A-B)=1$$

$$\Rightarrow\ \tan A-\tan B=1+\tan A\cdot\tan B$$

$$\therefore\ (1+\tan A)\cdot(1-\tan B)=1+(\tan A-\tan B)-\tan A\tan B$$

$$=2$$

5. (0.57)

Divide numerator & denominator by $\cos\theta$

$$\frac{2\tan\theta-5}{4-5\tan\theta}=\frac{3-5}{4-5\cdot\frac{3}{2}}=\frac{-2}{-\frac{7}{2}}=\frac{4}{7}$$

6. (0.26)

$$\frac{2\sin\left(x+\frac{\pi}{6}\right)\cdot\cos\left(x+\frac{\pi}{6}\right)}{2}=\frac{\sin\left(2x+\frac{\pi}{3}\right)}{2}$$

Maximum $=\frac{1}{2}$ if $\sin\left(2x+\frac{\pi}{3}\right)=1$

$$\Rightarrow\ x=\frac{\pi}{12}$$

7. (0.75)

$$\cos^{2}108^{o}+\cos^{2}144^{o}$$

$$=\sin^{2}18^{o}+\cos^{2}36^{o}$$

$$=\left(\frac{\sqrt{5}-1}{4}\right)^{2}+\left(\frac{\sqrt{5}+1}{4}\right)^{2}=\frac{12}{16}=\frac{3}{4}$$

8. (1)

$$x\cdot 1\cdot \frac{1}{2}=\frac{\sqrt{3}}{2}\cdot \frac{1}{\sqrt{3}}$$

$$\Rightarrow\ x=1$$

9. (1.5)

$$\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12}$$

$$=\cos^2 15^\circ+\cos^2 45^\circ+\cos^2 75^\circ$$

$$=\left(\sin^2 75^\circ+\cos^2 75^\circ\right)+\cos^2 45^\circ$$

$$=1+\frac{1}{2}=\frac{3}{2}$$

10. (1)

$$\frac{\sin 85^\circ-\sin 35^\circ}{\cos 65^\circ}$$

$$=\frac{2\sin 25^\circ\cdot\cos 60^\circ}{\cos 65^\circ}\qquad \left\{\sin 25^\circ=\cos 65^\circ\right\}$$

$$=1$$

11. (0.19)

$$\cos\left(\frac{7\pi}{6}\right)=\cos\left(\pi+\frac{\pi}{6}\right)=-\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$$

$$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}=-\frac{1}{2}$$

$$\therefore\ \text{Ans}=\left(1+\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right)\cdot\left(1-\frac{1}{2}\right)\cdot\left(1-\frac{\sqrt{3}}{2}\right)$$

$$=\left(1-\frac{\sqrt{3}^2}{2^2}\right)\cdot\left(1-\frac{1}{2^2}\right)$$

$$=\left(1-\frac{3}{4}\right)\cdot\left(1-\frac{1}{4}\right)=\frac{3}{16}$$

12. (2)

$$\cos 54^\circ=\cos\left(90^\circ-36^\circ\right)=\tan 36^\circ$$

13. (1)

$$\cot\left(102^\circ\right)=\cot\left(90^\circ+12^\circ\right)=-\tan 12^\circ$$

$$\therefore\ \text{Required}=\cot 12^\circ\left(-\tan 12^\circ\right)-\tan 12^\circ\cdot\cot 66^\circ+\cot 66^\circ\cot 12^\circ$$

$$=-1+\cot 66^\circ\cdot\left[\cot 12^\circ-\tan 12^\circ\right]$$

$$=-1+\cot 66^\circ\left(2\cot 24^\circ\right)$$

$$=-1+\left(\tan 24^\circ\right)\cdot\left(2\cot 24^\circ\right)$$

$$=-1+2=1\qquad \left\{\cot 66^\circ=\tan 24^\circ\right\}$$

14. (1.69)

$$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta)$$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)} \quad ...(1)$$

$$\cos(\alpha+\beta) = \frac{4}{5}$$

$$\Rightarrow \tan(\alpha+\beta) = \sqrt{\sec^2(\alpha+\beta)-1}$$

$$= \frac{3}{4} \quad ...(2)$$

$$\sin(\alpha-\beta) = \frac{5}{13}$$

$$\Rightarrow \tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\sqrt{1-\sin^2(\alpha-\beta)}}$$

$$= \frac{5}{12} \quad ...(3)$$

Put (2) & (3) in (1)

$$\tan 2\alpha = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \frac{\frac{14}{12}}{1-\frac{5}{16}} = \frac{56}{33}$$

15. (0.40)

$$\cos^2 A - \sin^2 B = \cos(A+B)\cdot\cos(A-B)$$

$$= \cos(48+12)\cdot\cos(48-12)$$

$$= \cos 60^o \cdot \cos 36^o$$

$$= \frac{1}{2}\times\left(\frac{\sqrt{5}+1}{4}\right)$$

16. (1.73)

$$A = \frac{2\pi}{5},\ B = \frac{\pi}{15}$$

$$A - B = \frac{2\pi}{5} - \frac{\pi}{15} = \frac{\pi}{3}$$

$$\Rightarrow \tan(A-B) = \sqrt{3}$$

$$\Rightarrow \frac{\tan A - \tan B}{1+\tan A\cdot\tan B} = \sqrt{3}$$

$$\Rightarrow \tan A - \tan B = \sqrt{3} + \sqrt{3}\tan A \tan B$$

$$\Rightarrow \text{Ans.} = \sqrt{3}$$

17. (3.87)

$\frac{\sin x}{\sin y}\cdot\frac{\cos y}{\cos x}=\frac{1/2}{3/2}$ & $\frac{\tan x}{\tan y}=\frac{1}{3}$

$\Rightarrow \tan x=\frac{\tan y}{3}$ ...(1)

Square & add

$\sin^2 x+\cos^2 x=1$

$\Rightarrow \frac{\sin^2 y}{4}+\frac{9}{4}\cos^2 y=1$

$\Rightarrow 1-\cos^2 y+9\cos^2 y=4$

$\Rightarrow \cos^2 y=\frac{3}{8}$

$\Rightarrow \tan y=\sqrt{\sec^2 y-1}$

$\tan y=\sqrt{\frac{5}{3}}$ ...(2)

(1) & (2)

$\therefore \tan x=\frac{1}{3}\sqrt{\frac{5}{3}}$

$\therefore \tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\cdot\tan y}$

$=\frac{\frac{4\tan y}{3}}{\frac{1-\tan^2 y}{3}}=\frac{4\tan y}{3-\tan^2 y}$

$=\frac{4\sqrt{5}/3}{3-\frac{5}{3}}=\sqrt{15}$

18. (1.5)

$\sin\frac{7\pi}{8}=\sin\left(\pi-\frac{\pi}{8}\right)=\sin\frac{\pi}{8}$

$\sin\frac{5\pi}{8}=\sin\left(\pi-\frac{3\pi}{8}\right)=\sin\frac{3\pi}{8}$

Further $\sin\frac{3\pi}{8}=\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)=\cos\frac{\pi}{8}$

$\therefore$ Required $=\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}=2$

19. (0.5)

$A+B=90^o$

$\therefore \sin A\sin B$

$=\sin A\cdot\sin(90-A)$

$=\sin A\cos A$

$=\frac{1}{2}\sin 2A$

$\therefore$ Maximum $=\frac{1}{2}$

20. (2)

$$\frac{\cot x-\tan x}{\cot 2x}$$

$$=\frac{\cos^2 x-\sin^2 x}{\left(\frac{2\cos x\cdot\sin x}{2}\right)\cdot\cos 2x}=\frac{2\cos 2x}{\sin 2x\cot 2x}=2$$

JEE Main : PYQ

1. (C)

Take $\cos 72^\circ=\frac{\sqrt{5}-1}{4}$

Let $\sin 36^\circ=\alpha$

$\Rightarrow\ 1-2\sin^2 36^\circ=\frac{\sqrt{5}-1}{4}\Rightarrow 4-8\alpha^2=\sqrt{5}-1$

$\Rightarrow\ 5-8\alpha^2=\sqrt{5}$

Take square both sides,

$\Rightarrow\ \left(5-8\alpha^2\right)^2=5\ \Rightarrow\ 25+64\alpha^4-80\alpha^2=5$

$\Rightarrow\ 64\alpha^4-80\alpha^2+20=0\ \Rightarrow\ 16\alpha^4-20\alpha^2+5=0$

2. (C)

Given, $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$$=2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}=2\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}\cdot\frac{1}{2}$$

$$=\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}=\sin^2\frac{\pi}{8}\sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$$

$$=\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}=\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right)=\frac{1}{8}$$

3. (B)

$$3\left(\sin\theta-\cos\theta\right)^4+6\left(\sin\theta+\cos\theta\right)^2+4\sin^6\theta$$

$$=3\left(1-2\sin\theta\cos\theta\right)^2+6\left(1+2\sin\theta\cos\theta\right)+4\sin^6\theta$$

$$=9+12\sin^2\theta\cos^2\theta+4\sin^6\theta$$

$$=9+12\cos^2\theta\left(1-\cos^2\theta\right)+4\left(1-\cos^2\theta\right)^3$$

$$=9+12\cos^2\theta-12\cos^4\theta+4\left(1-\cos^6\theta-3\cos^2\theta+3\cos^4\theta\right)$$

$$=9+4-4\cos^6\theta$$

$$=13-4\cos^6\theta$$

4. (B)

Given $2\cos\theta+\sin\theta=1$

Squaring both sides, we get $\left(2\cos\theta+\sin\theta\right)^2=1^2$

$\Rightarrow\quad 4\cos^2\theta+\sin^2\theta+4\sin\theta\cos\theta=1$

$\Rightarrow\quad 3\cos^2\theta+\left(\cos^2\theta+\sin^2\theta\right)+4\sin\theta\cos\theta=1$

$\Rightarrow\quad 3\cos^2\theta+1+4\sin\theta\cos\theta=1$

$\Rightarrow\quad 3\cos^2\theta+4\sin\theta\cos\theta=0$

$\Rightarrow\quad \cos\theta\left(3\cos\theta+4\sin\theta\right)=0$

$\Rightarrow\quad 3\cos\theta+4\sin\theta=0$

$\Rightarrow\quad 3\cos\theta=-4\sin\theta$

$\Rightarrow\quad \frac{-3}{4}=\tan\theta=-\sqrt{\sec^2\theta-1}=\frac{-3}{4}\qquad\left(\because\ \tan\theta=\sqrt{\sec^2\theta-1}\right)$

$\Rightarrow\quad \sec^2\theta-1=\left(\frac{-3}{4}\right)^2=\frac{9}{16}$

$\Rightarrow\quad \sec^2\theta=\frac{9}{16}+1=\frac{25}{16}\quad\Rightarrow\quad \sec\theta=\frac{5}{4}$

or $\quad \cos\theta=\frac{4}{5}\qquad\qquad ...(i)$

Now, $\sin^2\theta+\cos^2\theta=1\ \Rightarrow\ \sin^2\theta+\left(\frac{4}{5}\right)^2=1$

$\sin^2\theta+\left(\frac{4}{5}\right)^2=1\Rightarrow\sin^2\theta+\left(\frac{4}{5}\right)=1$

$\sin\theta=\pm\frac{3}{5}\qquad\qquad ...(ii)$

Taking $\left(\sin\theta=-\frac{3}{5}\right)$ because $\left(\sin\theta=\frac{3}{5}\right)$ cannot satisfy the given equation,

Therefore; $7\cos\theta+6\sin\theta=7\times\frac{4}{5}-6\times\frac{3}{5}=\frac{28}{5}-\frac{18}{5}=2$

5. (B)

Given : $\sin\theta=\frac{1}{2}$ and θ is acute angle

$\therefore\quad \theta=\frac{\pi}{6}$

Also given, $\cos\phi=\frac{1}{3}$ and ϕ is acute angle.

$\therefore\quad 0<\frac{1}{3}<\frac{1}{2}$

$\Rightarrow\quad \cos\frac{\pi}{2}<\cos\phi<\cos\frac{\pi}{3}$ or $\frac{\pi}{3}<\phi<\frac{\pi}{2}$

$\therefore\quad \frac{\pi}{3}+\frac{\pi}{6}<\theta<\phi<\frac{\pi}{2}+\frac{\pi}{6}$ or $\frac{\pi}{2}<\theta+\phi<\frac{2\pi}{3}$

$\Rightarrow \quad \theta+\phi \in\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

6. (B)

Since, $\sin\left(\frac{\pi}{22}\right)=\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)=\cos\frac{5\pi}{11}=-\cos\frac{16\pi}{11}$

$\sin\frac{3\pi}{22}=\cos\frac{4\pi}{11}, \sin\frac{5\pi}{22}=\cos\frac{3\pi}{11}=-\cos\frac{8\pi}{11}$

$\sin\frac{7\pi}{22}=\cos\frac{2\pi}{11}, \sin\frac{9\pi}{22}=\cos\frac{\pi}{11}$

Now, $2\sin\frac{\pi}{22}\cdot\sin\frac{3\pi}{22}\cdot\sin\frac{5\pi}{22}\cdot\sin\frac{7\pi}{22}\cdot\sin\frac{9\pi}{22}$

$=2\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{8\pi}{11}\cos\frac{16\pi}{11}$

$=\frac{2\times\sin 2^5\frac{\pi}{11}}{2^5\sin\frac{\pi}{11}}=\frac{1}{16} \quad \left(\because \sin 2^5\frac{\pi}{11}=\sin\frac{\pi}{11}\right)$

7. (A)

We are given that

$\cot\alpha=1, \sec\beta=\frac{-5}{3},$

$\Rightarrow \quad \cos\beta=-\frac{3}{5}, \tan\beta=-\frac{4}{3}, \tan\alpha=1$

$\Rightarrow \quad \tan(\alpha+\beta)=\frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1}=\frac{-1}{7}$

$\because \quad \tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}$

$\Rightarrow \quad \tan(\alpha+\beta)=\frac{-1}{7}$ lies in IVth quadrant

8. (B)

Given trigonometric equation is

$\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$

$=\left(\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)\right).$

$=\cos\left(\frac{4\pi}{7}\right)\left[2\cos\left(\frac{2\pi}{7}\right)+1\right]$

$$= \cos\left(\frac{4\pi}{7}\right)\left[2\left(1-2\sin^2\left(\frac{\pi}{7}\right)+1\right]\right.$$

$$= \cos\left(\frac{4\pi}{7}\right)\left[3-4\sin^2\left(\frac{\pi}{7}\right)\right] = \frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}}\times\cos\left(\frac{4\pi}{7}\right)$$

Multiply both sides by 2

$$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}}\times\cos\left(\frac{4\pi}{7}\right)$$

Apply $2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$$= \frac{\sin\left(\frac{7\pi}{7}\right)+\sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$$, Here, $\sin(\pi)=0$.

$$= \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = -\frac{1}{2}$$

9. (B)

$16\sin 20°\sin 40°\sin 80°$

$= 16\sin 40°\sin 20°\sin 80°$

$= 4\left(4\sin(60-20)\sin(20)\sin(60+20)\right)$

$= 4\times\sin(3\times 20°)$ $\quad\left[\because \sin 3\theta = 4\sin(60-\theta)\times\sin\theta\times\sin(60+\theta)\right]$

$= 4\times\sin 60°$

$= 4\times\frac{\sqrt{3}}{2} = 2\sqrt{3}$

10. (D)

Given trigonometric equation is

$\sin 12° + \sin 12° - \sin 72°$

$= \sin 12° - 2\cos 42°\sin 30°$

$= \sin 12° - \sin(90° - 48°) = \sin 12° - \sin 48°$

$= -2\cos 30°\sin 18° = -2\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{5}-1}{4} = \frac{\sqrt{3}}{4}\left(1-\sqrt{5}\right)$

11. (D)

$$L+M=1-2\sin^2\frac{\pi}{8}=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}} \qquad ...(\text{i})$$

and $L-M=-\cos\frac{\pi}{8}$ ...(ii)

From equations (i) and (ii),

$$L=\frac{1}{2}\left(\frac{1}{\sqrt{2}}-\cos\frac{\pi}{8}\right)=\frac{1}{2\sqrt{2}}-\frac{1}{2}\cos\frac{\pi}{8} \text{ and } M=\frac{1}{2}\left(\frac{1}{\sqrt{2}}+\cos\frac{\pi}{8}\right)=\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$$

12. (B)

$$\cos^2 10°-\cos 10°\cos 50°+\cos^2 50°$$

$$=\left(\frac{1+\cos 20°}{2}\right)+\left(\frac{1+\cos 100°}{2}\right)-\frac{1}{2}(2\cos 10°\cos 50°)$$

$$=1+\frac{1}{2}(\cos 20°+\cos 100°)-\frac{1}{2}[\cos 60°+\cos 40°]$$

$$=\left(1-\frac{1}{4}\right)+\frac{1}{2}[\cos 20°+\cos 100°-\cos 40°]$$

$$=\frac{3}{4}+\frac{1}{2}[2\cos 60°\times\cos 40°-\cos 40°]=\frac{3}{4}$$

13. (A)

$$f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x);\quad f_4(x)=\frac{1}{4}[\sin^4 x+\cos^4 x]$$

$$=\frac{1}{4}\left[(\sin^2 x+\cos^2 x)^2-\frac{(\sin 2x)^2}{2}\right]=\frac{1}{4}\left[1-\frac{(\sin 2x)^2}{2}\right]$$

$$f_6(x)=\frac{1}{6}[\sin^6 x+\cos^6 x]$$

$$=\frac{1}{6}\left[(\sin^2 x+\cos^2 x)^3-\frac{3}{4}(\sin 2x)^2\right]=\frac{1}{6}\left[1-\frac{3}{4}(\sin 2x)^2\right]$$

Now $f_4(x)-f_{(6)}(x)=\frac{1}{4}-\frac{1}{6}-\frac{(\sin 2x)^2}{8}+\frac{1}{8}(\sin 2x)^2=\frac{1}{12}$

14. (A)

We have

$$5\tan^2 x-5\cos^2 x=2(2\cos^2 x-1)+9$$

$$\Rightarrow\quad 5\tan^2 x-9\cos^2 x=-2+9$$

$$\Rightarrow\quad 5\tan^2 x=9\cos^2 x+7$$

$$\Rightarrow\quad 5(\sec^2 x-1)=9\cos^2 x+7$$

Let $\cos^2 x=t\Rightarrow\frac{5}{t}-9t-12=0$

$$\Rightarrow\quad 9t^2+12t-5=0\ \Rightarrow\ 9t^2+15t-3t-5=0$$

$\Rightarrow \quad (3t-1)(3t+5)=0 \Rightarrow t=\frac{1}{3}$ as $t \neq -\frac{5}{3}$.

$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

15. (A)

$4+\frac{1}{2}\sin^2 2x - 2\cos^4 x$

$4+2(1-\cos^2 x)\cos^2 x - 2\cos^4 x = -4\left\{\cos^4 x - \frac{\cos^2 x}{2} - 1 + \frac{1}{16} - \frac{1}{16}\right\}$

$= -4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\}; \ 0 \le \cos^2 x \le 1$

$\Rightarrow \quad -\frac{1}{4} \le \cos^2 x - \frac{1}{4} \le \frac{3}{4} \Rightarrow 0 \le \left(\cos^2 x - \frac{1}{4}\right)^2 \le \frac{9}{16}$

$-\frac{17}{16} \le \left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16} \le \frac{9}{16} - \frac{17}{16}$

$\frac{17}{4} \ge -4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\} \ge \frac{4}{2}$

$M = \frac{17}{4} \Rightarrow m = 2; \ M - m = \frac{17}{4} - 2 = \frac{9}{4}$

16. (B)

Let $\cos\alpha + \cos\beta = \frac{3}{2}$

$\Rightarrow \quad 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{3}{2}$ ...(i)

and $\sin\alpha + \sin\beta = \frac{1}{2}$

$\Rightarrow \quad 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{1}{2}$ ...(ii)

On dividing (ii) by (i), we get $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{3}$

Given: $0 = \frac{\alpha+\beta}{2} \Rightarrow 2\theta = \alpha+\beta$

Consider $\sin 2\theta + \cos 2\theta = \sin(\alpha+\beta) + \cos(\alpha+\beta)$

$= \frac{\frac{2}{3}}{1+\frac{1}{9}} + \frac{1-\frac{1}{9}}{1+\frac{1}{9}} = \frac{6}{10} + \frac{8}{10} = \frac{7}{5}$

17. (9)

Let $f(x)=\frac{4}{\sin x}+\frac{1}{1-\sin x}$

$\Rightarrow \quad f'(x)=0 \Rightarrow \sin x=\frac{2}{3} \qquad \left(\because \; x\in\left(0,\frac{\pi}{2}\right)\cos x\neq 0\right)$

$\therefore \quad f(x)_{\min}=\frac{4}{2/3}+\frac{1}{1-2/3}=9;\;\; f(x)_{\max.}\to\infty$

(x) is continuous function $\therefore\; \alpha_{\min}=9$

18. (80)

$$\sin 10^\circ\left(\frac{1}{2}.2\sin 20^\circ\sin 40^\circ\right).\sin 10^\circ\sin\left(60^\circ-10^\circ\right)\sin\left(60^\circ+10^\circ\right)$$

$$=\sin 10^\circ\frac{1}{2}\left(\cos 20^\circ-\cos 60^\circ\right)\cdot\frac{1}{4}\sin 30^\circ$$

$$=\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\sin 10^\circ\left(\cos 20^\circ-\frac{1}{2}\right)$$

$$=\frac{1}{16}\cdot\frac{1}{2}\left(2\sin 10^\circ\cos 20^\circ-\sin 10^\circ\right)$$

$$=\frac{1}{32}\left(\sin 30^\circ-\sin 10^\circ-\sin 10^\circ\right)$$

$$=\frac{1}{32}\left(\frac{1}{2}-2\sin 10^\circ\right)=\frac{1}{64}\left(1-4\sin 10^\circ\right)$$

$$=\frac{1}{64}-\frac{1}{16}\left(\sin 10^\circ\right)$$

Hence, $\alpha=\frac{1}{64} \Rightarrow \alpha^{-1}=64$

Hence, $16+\alpha^{-1}=80$

19. (1)

$\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$

$\Rightarrow \quad \frac{\sqrt{2}\sin\beta}{\sqrt{2}}=\frac{1}{\sqrt{10}}$

$\therefore \quad \tan\alpha=\frac{1}{7}$ and $\sin\beta=\frac{1}{\sqrt{10}}$; $\tan\beta=\frac{1}{3}$

$$\therefore \quad \tan 2\beta=\frac{2\tan\beta}{1-\tan^2\beta}=\frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}}=\frac{\frac{2}{3}}{\frac{8}{9}}=\frac{3}{4}$$

$$\tan\left(\alpha+2\beta\right)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}}=\frac{\frac{4+21}{28}}{\frac{25}{28}}=1$$