

EXERCISE - 1 [A]

1. (B)

$$\pi \text{ radian} = 180 \text{ degree} \Rightarrow 1 \text{ radian} = \frac{180}{\pi} \text{ degree} \approx 57.3 \text{ degree, hence } \sin 1^c > \sin 1^0.$$

2. (B)

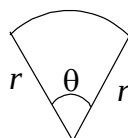
Length of arc of circle of radius r subtending θ at the center $= r\theta$.

$$\text{Hence } 15 = r \times \frac{3}{4} \text{ or } r = 20 \text{ cm}$$

3. (D)

$$\text{Perimeter} = r\theta + 2r = m \cdot r$$

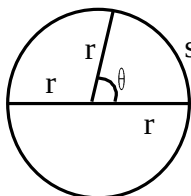
$$\theta = (m-2)^c$$



4. (C)

$$\pi r = S + 2r$$

$$\Rightarrow S = (\pi - 2)r$$



5. (B)

At half past 4, hour hand will be at $4\frac{1}{2}$ and minute hand will be at 6 (for 30 minutes). In 12 hours,

angle made by hour hand

$$\frac{360}{12} \times \frac{9}{2} = 135^\circ$$

In 60 minutes, angle made by minute hand

$$= 360^\circ$$

In 30 minutes, angle made by minute hand

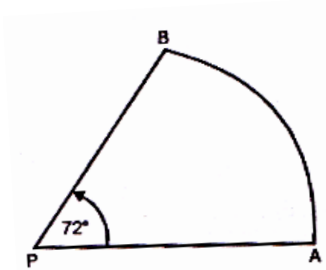
$$= \frac{360^\circ}{60} \times 30 = 180^\circ$$

$$\therefore \text{reqd. angle} = 180^\circ - 135^\circ = 45^\circ = \frac{\pi}{4}$$

Hence (B) holds.

6. (B)

Let the post be at point P and let PA be the length of the rope in tight position. Suppose the horse moves along the arc AB so that $\angle APB = 72^\circ$ and arc AB = 88m. Let r be the length of the rope i.e. PA = r metres.



$$\text{Now, } \theta = 72^\circ = \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$$

$$\text{And } s = 88\text{m} \quad \therefore \theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \frac{2\pi}{5} = \frac{88}{r} \Rightarrow r = 88 \times \frac{5}{2\pi} = 70\text{m}$$

Hence (B) holds.

7. (C)

Since $180^\circ = \pi$ radians

$$\therefore 1^\circ = \frac{\pi}{180} \text{ radians } \therefore p^\circ = \frac{p\pi}{180} \text{ radians}$$

$$\therefore \frac{p\pi}{180} = q$$

$$[\because p \text{ degrees} = q \text{ radians}]$$

$$\Rightarrow \frac{p}{180} = \frac{q}{\pi} \quad \text{i.e. } \frac{p}{90} = \frac{2q}{\pi}$$

Hence (C) holds.

8. (B)

Clearly length of the wire

= circumference of circle of radius 7

$$= 2\pi(7) = 14\pi$$

$$\therefore l = 14\pi, r = 12\text{cm}$$

$$\text{Since } \theta = \frac{l}{r} = \frac{14\pi}{12} = \frac{7\pi}{6} = \frac{7}{6} \times 180^\circ = 210^\circ$$

Hence angle subtended at the centre = 210°

9. (A)

$$\tan(90^\circ - \theta) = \cot \theta \Rightarrow \tan 89^\circ = \cot 1^\circ, \tan 88^\circ = \cot 2^\circ, \tan 87^\circ = \cot 3^\circ, \dots \text{ etc.}$$

$$\text{Also, } \tan \theta \times \cot \theta = 1, \text{ hence } \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ = 1$$

10. (A)

$$\sin \theta - \cos \theta = 1 \Rightarrow \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = 1$$

$$\Rightarrow \sin \theta \cos \theta = 0$$

11. (C)

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\tan\theta - 3}{5\tan\theta + 2}$$

$$5\tan\theta = 4 \Rightarrow \frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{4-3}{4+2} = \frac{1}{6}$$

12. (A)

$$\sin x + \operatorname{cosec} x = 2 \Rightarrow \sin^2 x - 2\sin x + 1 = 0$$

$$\Rightarrow \sin x = 1 \Rightarrow \sin^n x + \operatorname{cosec}^n x = 2$$

13. (C)

$$x \sin 45^\circ \cos^2 60^\circ = \frac{\tan^2 60^\circ \operatorname{cosec} 30^\circ}{\sec 45^\circ \cot^2 30^\circ} \Rightarrow x \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{2} \right)^2 = \frac{(\sqrt{3})^2 (2)}{(\sqrt{2})(\sqrt{3})^2} \Rightarrow x = 8$$

14. (A)

$$\tan^2 30 + 4\sin^2 45 + \frac{1}{3}\cos^2 30 = \frac{1}{3} + \frac{4}{2} + \frac{1}{3} \times \frac{3}{4}$$

$$= \frac{1}{3} + \frac{1}{4} + 2$$

$$= 2\frac{7}{12}$$

15. (D)

$$\frac{\tan^2 60 - 2\tan^2 45 + \sec^2 30}{3\sin^2 45 - \sin 90 + \cos^2 60 \cdot \cos^3 0} = \frac{3 - 2 + \frac{4}{3}}{3 \cdot \frac{1}{2} + \frac{1}{4}}$$

$$= \frac{1 + \frac{4}{3}}{\frac{1}{2} \left(3 + \frac{1}{2} \right)} = \frac{\frac{7}{3}}{\frac{1}{2} \cdot \frac{7}{2}} = \frac{4}{3}$$

16. (C)

$$2P_6 - 3P_4 + 1$$

$$= 2(1 - 3\sin^2 x \cos^2 x) - 3 \left[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x \right] + 1$$

$$= 2 - 6\sin^2 x \cos^2 x - 3(1 - 2\sin^2 x \cos^2 x) + 1 = 0$$

17. (A)

$$90^\circ < 130^\circ < 135^\circ \text{ hence } \sin A > 0 \text{ \& } \cos A < 0 \text{ \& } |\sin A| > |\cos A|$$

$$\Rightarrow \sin A + \cos A > 0$$

18. (A)

$$\cos 24^\circ + \cos 5^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ$$

$$= \cos 24^\circ + \cos 5^\circ + \cos(180^\circ - 5^\circ) + \cos(180^\circ + 24^\circ) + \cos(360^\circ - 60^\circ)$$

$$= \cos 24^\circ + \cos 5^\circ - \cos 5^\circ - \cos 24^\circ + \cos 60^\circ$$

$$= \cos 60^\circ = \frac{1}{2}$$

19. (C)
 $\sin \theta < 0$ & $\tan \theta > 0 \Rightarrow \theta$ lies in 3rd quadrant.

20. (C)
 Given expression

$$= \frac{-\sin(660^\circ) \tan(1050^\circ) \sec(420^\circ)}{\cos(180^\circ + 45^\circ) \operatorname{cosec}(360^\circ - 45^\circ) \cos(360^\circ + 150^\circ)}$$

$$= \frac{-\sin(7 \times 90 + 30^\circ) \tan(3 \cdot 360^\circ - 30^\circ) \sec(360^\circ + 60^\circ)}{(-\cos 45^\circ) - \operatorname{cosec}(45^\circ) \cos 150^\circ}$$

$$= \frac{\cos(30^\circ) (-\tan 30^\circ) \sec 60^\circ}{-(\cos 45^\circ) (-\operatorname{cosec} 45^\circ) (-\cos 30^\circ)}$$

$$= \frac{\cos 30^\circ \tan 30^\circ \sec 60^\circ}{\cos 45^\circ \operatorname{cosec} 45^\circ \cos 30^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}} \cdot 2}{\frac{1}{\sqrt{2}} \cdot 1} = \frac{2}{\sqrt{3}}$$

Hence (C) holds

21. (C)
 Given expression

$$= \frac{(-\sin \theta)(\sec \theta)(-\tan \theta)}{(\sin \theta)(-\sec \theta) \tan \theta} = -1$$

Hence (C) holds.

22. (B)
 Since $\sec \theta$ i.e. $\cos \theta$ is negative.
 $\therefore \theta$ lies in IInd or IIIrd quadrant. Since $\sin \theta$ is positive.
 $\therefore \theta$ lies in Ist or IInd quadrant. Hence θ lies in IInd quadrant.

23. (A)
 $\cot A + \tan(180^\circ + A) + \tan(90^\circ + A) + \tan(360^\circ - A)$
 $= \cot A + \tan A - \cot A - \tan A = 0$

24. (D)
 $\sin[-870^\circ] = -\sin(870^\circ) = -\sin(9 \times 90^\circ + 60^\circ)$
 $= -\cos 60^\circ = -\frac{1}{2}$

25. (A)

$$\tan \theta = -\frac{1}{\sqrt{5}}$$

$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + \frac{1}{5} = \frac{6}{5}$$

$$\sec \theta = +\sqrt{\frac{6}{5}} \quad [\because \theta \text{ lies in the IV th quadrant}]$$

$$\therefore \cos \theta = \sqrt{\frac{5}{6}}$$

26. (C)

$$\sin 15^\circ \cos 15^\circ = \frac{1}{2} \sin 30^\circ = \frac{1}{4}$$

27. (A)

$$\begin{aligned} \frac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ - \sin 17^\circ} &= \frac{1 + \tan 17^\circ}{1 - \tan 17^\circ} \\ &= \tan(45^\circ + 17^\circ) = \tan 62^\circ \end{aligned}$$

28. (A)

$$\begin{aligned} \tan 75^\circ - \cot 75^\circ &= -2 \cot 150^\circ \\ &= 2 \tan 60^\circ = 2\sqrt{3} \end{aligned}$$

29. (B)

$$\begin{aligned} \cos^2 A - \sin^2 B &= \cos(A+B)\cos(A-B) \\ \Rightarrow \cos^2 48^\circ - \sin^2 12^\circ &= \cos 60^\circ \cos 36^\circ = \frac{\sqrt{5}+1}{8} \end{aligned}$$

30. (D)

As A & B are the acute angles hence,

$$\sin A = \frac{1}{\sqrt{10}} \Rightarrow \cos A = \frac{3}{\sqrt{10}} \text{ \& } \sin B = \frac{1}{\sqrt{5}} \Rightarrow \cos B = \frac{2}{\sqrt{5}}$$

$$\text{Now } \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\Rightarrow \cos(A+B) = \frac{3}{\sqrt{10}} \times \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow A+B = \frac{\pi}{4}$$

31. (A)

$$\begin{aligned} &\cos^2\left(\frac{\pi}{6} + \theta\right) - \sin^2\left(\frac{\pi}{6} - \theta\right) \\ &= \cos\left(\frac{\pi}{6} + \theta + \frac{\pi}{6} - \theta\right) \cos\left(\frac{\pi}{6} + \theta - \frac{\pi}{6} + \theta\right) \\ &\left[\because \cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B\right] \\ &= \cos\left(\frac{\pi}{3}\right) \cos(2\theta) = \frac{1}{2} \cos 2\theta \end{aligned}$$

Hence (A) holds.

32. (C)

$$\begin{aligned}& (1 + \tan A)(1 + \tan B) \\&= (1 + \tan A) \left(1 + \tan \left(\frac{\pi}{4} - A \right) \right) \\&= (1 + \tan A) \left(1 + \frac{1 - \tan A}{1 + \tan A} \right) \\&= (1 + \tan A) \left(\frac{1 + \tan A + 1 - \tan A}{1 + \tan A} \right) = 2\end{aligned}$$

$\therefore$ (C) holds

33. (B)

$$\begin{aligned}\sin 163^\circ &= \sin(180^\circ - 17^\circ) = \sin 17^\circ \\ \cos 347^\circ &= \cos(360^\circ - 13^\circ) = \cos 13^\circ \\ \sin 73^\circ &= \sin(90^\circ - 17^\circ) = \cos 17^\circ \\ \sin 167^\circ &= \sin(180^\circ - 13^\circ) = \sin 13^\circ \\ \therefore \text{ given expression} \\&= \sin 17^\circ \cos 13^\circ + \cos 17^\circ \sin 13^\circ \\&= \sin(17^\circ + 13^\circ) = \sin(30^\circ) = \frac{1}{2}\end{aligned}$$

34. (D)

$$\begin{aligned}\tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\&= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1 \\ \therefore A + B &= \frac{\pi}{4}\end{aligned}$$

35. (B)

$$\begin{aligned}\sin(\theta + \phi) &= \sin \theta \cos \phi + \cos \theta \sin \phi \\ \left[\cos \theta = \sqrt{1 - \frac{144}{165}} = \frac{5}{13} \because 0 < \theta < \frac{\pi}{2} \right. & \left. \sin \phi = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5} \because \pi < \theta < \frac{3\pi}{2} \right] \\&= \frac{12}{13} \left(-\frac{3}{5} \right) + \frac{5}{13} \left(-\frac{4}{5} \right) \\&= -\frac{36}{65} - \frac{20}{65} = -\frac{56}{65}\end{aligned}$$

36. (D)

$$A + B = 45^\circ \Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$$

$$(\cot A - 1)(\cot B - 1)$$

$$= \cot A \cot B - (\cot A + \cot B) + 1$$

$$= 1 + 1 = 2$$

$\therefore$ (D) holds

37. (C)

$$32 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right) = 16(\cos 2A - \cos 3A)$$

$$= 16(2 \cos^2 A - 1 - 4 \cos^3 A + 3 \cos A)$$

$$= 16\left(2 \times \frac{9}{16} - 1 - 4 \times \frac{27}{64} + 3 \times \frac{3}{4}\right) = 11$$

38. (B)

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = -2 \cos 60^\circ \sin 10^\circ + \sin 10^\circ = 0$$

39. (D)

$$\text{The given expression} = \cos 20^\circ + \cos(180^\circ - 80^\circ) + \cos(180^\circ - 40^\circ)$$

$$= \cos 20^\circ - (\cos 80^\circ + \cos 40^\circ)$$

$$= \cos 20^\circ - 2 \cos 60^\circ \cos 20^\circ = 0$$

$\therefore$ (D) holds

40. (A)

$$\cos 52^\circ + \cos 68^\circ + \cos 172^\circ$$

$$= \cos 52^\circ + \cos 68^\circ + \cos(180^\circ - 8^\circ)$$

$$= \cos 52^\circ + \cos 68^\circ - \cos 8^\circ$$

$$= 2 \cos \frac{52^\circ + 68^\circ}{2} \cos \frac{68^\circ - 52^\circ}{2} - \cos 8^\circ$$

$$= 2 \cos 60^\circ \cos 8^\circ - \cos 8^\circ$$

$$= \cos 8^\circ - \cos 8^\circ = 0$$

41. (C)

$$\frac{\sin(2A + B)}{\sin B} = \frac{5}{1}. \text{ By componendo and dividendo we shall get } \frac{\tan(A + B)}{\tan A} = \frac{3}{2}$$

42. (C)

$$\text{Given value} = (\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ) = -\frac{1}{2}$$

43. (B)

$$\frac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ} = \frac{\sin 70^\circ + \sin 50^\circ}{\cos 70^\circ + \cos 50^\circ}$$

$$= \frac{2 \sin 60^\circ \cos 10^\circ}{2 \cos 60^\circ \cos 10^\circ} = \tan 60^\circ = \sqrt{3}$$

44. (D)

$$\frac{\sin 55^\circ - \cos 55^\circ}{\sin 10^\circ} = \frac{\sin 55^\circ - \sin 35^\circ}{\sin 10^\circ}$$

$$= \frac{2 \cos 45^\circ \sin 10^\circ}{\sin 10^\circ} = 2 \cos 45^\circ = 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$$

$\therefore$ (D) holds

45. (D)

$$1 - 2 \sin^2 \left(\frac{\pi}{4} + \theta \right) = \cos 2 \left(\frac{\pi}{4} + \theta \right)$$

$$= \cos \left(\frac{\pi}{2} + 2\theta \right) = -\sin 2\theta$$

46. (B)

$$2 \cos^2 \theta - 2 \sin^2 \theta = 1 \Rightarrow \cos 2\theta = \frac{1}{2}$$

$\therefore \theta = 30^\circ$

47. (B)

$$2 \sin A \cos^3 A - 2 \sin^3 A \cos A = 2 \sin A \cos A (\cos^2 A - \sin^2 A)$$

$$= \sin 2A \cos 2A = \frac{\sin 4A}{2}$$

48. (C)

$\sin A + \cos A = 1$, we get $1 + \sin 2A = 1$
 $\Rightarrow \sin 2A = 0$

49. (C)

$$\frac{\tan 80^\circ - \tan 10^\circ}{\tan 70^\circ} = \frac{\cot 10^\circ - \tan 10^\circ}{\tan 70^\circ}$$

$$= \frac{2 \cot 20^\circ}{\tan 70^\circ} = 2$$

50. (A)

$$\text{Use } \cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

51. (B)

$$\begin{aligned}
 & \tan 67 \frac{1^\circ}{2} + \cot 67 \frac{1^\circ}{2} \\
 &= \tan 67 \frac{1^\circ}{2} + \frac{1}{\tan 67 \frac{1^\circ}{2}} \\
 &= \frac{1 + \tan^2 67 \frac{1^\circ}{2}}{2 \tan 67 \frac{1^\circ}{2}} \cdot 2 \\
 &= 2 \cdot \frac{1}{\sin 2 \left(67 \frac{1^\circ}{2} \right)} \\
 & \left[\because \sin 2A = \frac{2 \tan A}{1 + \tan^2 A} \right] \\
 &= \frac{2}{\sin(135^\circ)} = \frac{2}{\sin(180^\circ - 45^\circ)} \\
 &= \frac{2}{\sin 45^\circ} = \frac{2}{\frac{1}{\sqrt{2}}} = 2\sqrt{2}
 \end{aligned}$$

$\therefore$ (B) holds

52. (B)

$$\begin{aligned}
 & \sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}} \\
 &= \sqrt{2 + \sqrt{2 \cdot 2 \cos^2 2\theta}} \\
 &= \sqrt{2 + 2 \cos 2\theta} = \sqrt{2(1 + \cos 2\theta)} \\
 &= \sqrt{2(2 \cos^2 \theta)} = 2 \cos \theta
 \end{aligned}$$

$\therefore$ (B) holds

53. (B)

$$\begin{aligned}
 & \cos 15^\circ \cos 7 \frac{1^\circ}{2} \sin 7 \frac{1^\circ}{2} \\
 &= \frac{1}{2} \cos 15^\circ \sin 15^\circ = \frac{1}{4} \sin 30^\circ \\
 &= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}
 \end{aligned}$$

$\therefore$ (B) holds

54. (A)

$$\frac{1 - t^2}{1 + t^2} = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \cos \theta$$

55. (D)

$$\text{Since } 1 + \cos x = K \quad \therefore 2 \cos^2 \frac{x}{2} = K$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{K}{2} \quad \Rightarrow 1 - \sin^2 \frac{x}{2} = \frac{K}{2}$$

$$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{K}{2} = \frac{2-K}{2}$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{2-K}{2}}$$

56. (A)

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

57. (A)

$$\frac{\sin 3\theta - \cos 3\theta}{\sin \theta + \cos \theta} + 1 = \frac{3\sin \theta - 4\sin^3 \theta - 4\cos^3 \theta + 3\cos \theta}{\sin \theta + \cos \theta} + 1$$

$$= 4 \frac{\sin \theta + \cos \theta - \sin^3 \theta - \cos^3 \theta}{\sin \theta + \cos \theta}$$

$$= 4 \frac{\sin \theta + \cos \theta - (\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta)}{\sin \theta + \cos \theta}$$

$$= 4 \sin \theta \cos \theta = 2 \sin 2\theta$$

58. (C)

$$\cos^2 A (3 - 4\cos^2 A)^2 + \sin^2 A (3 - 4\sin^2 A)^2 = (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$$

$$= \cos^2 3A + \sin^2 3A = 1$$

59. (C)

$$\frac{3\cos \theta + \cos 3\theta}{3\sin \theta - \sin 3\theta} = \frac{3\cos \theta + (4\cos^3 \theta - 3\cos \theta)}{3\sin \theta - (3\sin \theta - 4\sin^3 \theta)} = \cot^3 \theta$$

60. (C)

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = (\sin (60^\circ - 20^\circ) \sin 20^\circ \sin (60^\circ + 20^\circ)) \sin 60^\circ$$

$$= \frac{1}{4} \sin (3 \times 20^\circ) \sin 60^\circ = \frac{1}{4} \sin^2 60^\circ = \frac{3}{16}$$

61. (D)

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{2\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

$$= \frac{2\sin 40^\circ \cos 40^\circ \cos 80^\circ}{4\sin 20^\circ} = \frac{2\sin 80^\circ \cos 80^\circ}{8\sin 20^\circ}$$

$$= \frac{\sin 160^\circ}{8 \sin 20^\circ}, \text{ but } \sin 160^\circ = \sin (180^\circ - 20^\circ) = \sin 20^\circ, \text{ hence}$$

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$$

Alternately

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \cos (60^\circ - 20^\circ) \cos 20^\circ \cos (60^\circ + 20^\circ)$$

$$\frac{1}{4} \cos(3 \times 20^\circ) = \frac{1}{8}$$

62. (D)

$$4 \cos \theta \cos \left(\frac{\pi}{3} + \theta \right) \cos \left(\frac{\pi}{3} - \theta \right) \\ = \cos 3\theta$$

63. (C)

$$\tan \theta + \tan \left(\theta + \frac{\pi}{3} \right) + \tan \left(\theta - \frac{\pi}{3} \right) \\ = \tan \theta + \frac{\tan \theta + \sqrt{3}}{1 - \tan \theta \cdot \sqrt{3}} + \frac{\tan \theta - \sqrt{3}}{1 + \tan \theta \cdot \sqrt{3}} \\ = \tan \theta + \frac{\tan \theta + \sqrt{3} + \sqrt{3} \tan^2 \theta + 3 \tan \theta + \tan \theta - \sqrt{3} - \tan^2 \theta \sqrt{3} + 3 \tan \theta}{1 - 3 \tan^2 \theta} \\ = \frac{\tan \theta - 3 \tan^3 \theta + 8 \tan \theta}{1 - 3 \tan^2 \theta} \\ = \frac{9 \tan \theta - 3 \tan^3 \theta}{1 - 3 \tan^2 \theta} \\ = \frac{3[3 \tan \theta - \tan^3 \theta]}{1 - 3 \tan^2 \theta} = 3 \tan 3\theta$$

$$\therefore K = 3$$

Hence (C) holds.

64. (A)

$$x = \tan 15^\circ = 2 - \sqrt{3}$$

$$y = \operatorname{cosec} 75^\circ = \sqrt{6} - \sqrt{2}$$

$$(z) = 4 \sin 18^\circ = \sqrt{5} - 1$$

It follows that $x < y < z$

$\therefore$ (A) holds

65. (D)

$$-\sqrt{3^2 + 4^2} + 8 \leq 3 \cos x + 4 \sin x + 8 \leq \sqrt{3^2 + 4^2} + 8 \\ \Rightarrow 3 \leq 3 \cos x + 4 \sin x + 8 \leq 13$$

66. (A)

$$\sin \left(\theta + \frac{\pi}{6} \right) + \cos \left(\theta + \frac{\pi}{6} \right) \text{ acquires maximum at } \alpha + \frac{\pi}{6} = \frac{\pi}{4} \text{ i.e. } \theta = \frac{\pi}{12}$$

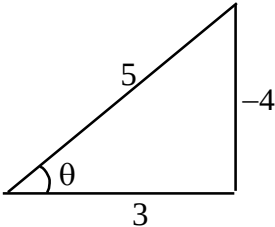
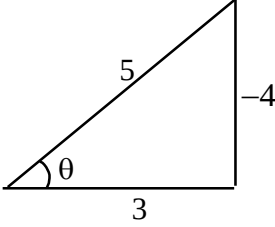
67. (D)
 $5\sin^2 \theta + 4\cos^2 \theta = 4 + \sin^2 \theta \geq 4$

68. (B)
 Maximum value of $\sin \theta \cos \theta$
 $= \text{Maximum value of } \left(\frac{\sin 2\theta}{2} \right)$
 $= \frac{1}{2} \text{ Maximum Value of } \sin 2\theta = \frac{1}{2}(1) = \frac{1}{2}$

69. (C)
 Minimum value of $\sin \theta \cos \theta$
 $= \frac{1}{2} \text{ Minimum value of } \sin 2\theta = \frac{1}{2}(-1) = -\frac{1}{2}$

70. (C)
 $\sin x + \cos x = \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) \leq \sqrt{2}$
 $\therefore \text{Maximum value} = \sqrt{2}$

71. (D)
 Put $\begin{cases} 3 = r \cos \alpha \\ 4 = r \sin \alpha \end{cases} \Rightarrow r^2 = 9 + 16 = 25$
 $\Rightarrow r = 5$
 $\therefore 3\cos \theta + 4\sin \theta = r(\cos \theta \cos \alpha + \sin \theta \sin \alpha)$

$= r \cos(\theta - \alpha) = 5 \cos(\theta - \alpha)$

Its Maximum value $= 5(1) = 5$

And Minimum value $= -5$

72. (A)
 Given, $P = \frac{1}{2} \sin^2 \theta + \frac{1}{3} \cos^2 \theta$
 $\frac{1}{2}(1 - \cos^2 \theta) + \frac{1}{3} \cos^2 \theta = \frac{1}{2} - \frac{1}{6} \cos^2 \theta$
 Since, $0 \leq \cos^2 \theta \leq 1 \Rightarrow -\frac{1}{6} \leq -\frac{1}{6} \cos^2 \theta \leq 0$
 $\Rightarrow \frac{1}{3} \leq \frac{1}{2} - \frac{1}{6} \cos^2 \theta \leq \frac{1}{2} \Rightarrow \frac{1}{3} \leq P \leq \frac{1}{2}$
 $\therefore (A) \text{ holds}$

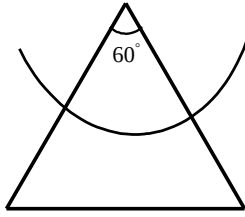
73. (C)
 $A + B + C = \pi$
 $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

74. (A)
 $\frac{A + B + C}{2} = \frac{\pi}{2}$
 $\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$
 $\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$
 $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{A}{2} \tan \frac{C}{2} = 1$
 $\tan \frac{C}{2} = 1 - \frac{2}{9} = \frac{7}{9}$

75. (D)
 $A + B + C = \pi$
 $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$
 $\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$
 $\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$
 $\Rightarrow \sum \tan \frac{A}{2} \tan \frac{B}{2} = 1$

EXERCISE - 1 [B]

1. (A)



$$\text{Area of } \Delta = \frac{\sqrt{3}}{4} \cdot 4^2 = 4\sqrt{3}$$

$$2\pi \text{ } \pi r^2$$

$$1 \text{ } \frac{\pi r^2}{2\pi}$$

$$\frac{\pi}{3} \text{ } \frac{\pi r^2}{2\pi} \cdot \frac{\pi}{3}$$

$$\text{Given, } \frac{\pi r^2}{6} = \frac{4\sqrt{3}}{2}$$

$$\pi r^2 = 12\sqrt{3}$$

$$r = \sqrt{\frac{12\sqrt{3}}{\pi}}$$

2. (C)

Angle covered from 6 A.M. to 3.15 P.M.

$$= 277 \frac{1^\circ}{2} = \frac{555}{2} \times \frac{\pi}{180} = \frac{111}{2} \times \frac{\pi}{36}$$

$$\therefore \theta = \frac{37\pi}{24} \text{ radians}$$

Length of hour hand = 12cm

i.e. $r = 12\text{cm}$

$$\text{Since } \theta = \frac{l}{r} \quad \therefore l = r\theta = \frac{12 \times 37}{24} = \frac{37\pi}{2}$$

$$\text{Hence reqd. distance} = \frac{37\pi}{2} \text{ cm.}$$

3. (D)

$$\text{An hour hand in 1 hour i.e., 60 minute traces} = \frac{360^\circ}{12} = 30^\circ$$

$$\therefore \text{An hour hand in 1 minute traces} = \frac{30^\circ}{60^\circ} = \left(\frac{1}{2}\right)^\circ$$

$\therefore$ An hour hand in 40 minutes traces

$$= \left(\frac{1}{2} \times 40\right)^\circ = 20^\circ$$

$$\therefore \text{reqd. angle} = 90^\circ - 20^\circ = 70^\circ$$

$$= \frac{70 \times \pi}{180} = \frac{7\pi}{18}$$

$\therefore$ (D) holds

4. (B)

$$\begin{aligned} & \sin^2 x + \operatorname{cosec}^2 x + 2 + \cos^2 x + \sec^2 x + 2 - \tan^2 x - \cot^2 x - 2 \\ &= 3 + (\operatorname{cosec}^2 x - \cot^2 x) + (\sec^2 x - \tan^2 x) \\ &= 5 \end{aligned}$$

5. (D)

$$\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} = \operatorname{cosec}^2 \theta \sec^2 \theta$$

6. (A)

$$\text{Let } \lambda = 3\cos\beta - 5\sin\beta$$

$$\text{Let } S = 3\sin\beta + 5\cos\beta$$

$$\lambda^2 + S^2 = 9 + 25$$

$$\therefore \lambda^2 = 9$$

7. (A)

$$\tan^2 \alpha + \cot^2 \alpha + 2 = m^2$$

$$(\tan^2 \alpha + \cot^2 \alpha)^2 = (m^2 - 2)^2$$

$$\tan^4 \alpha + \cot^4 \alpha + 2 = m^4 + 4 - 4m^2$$

$$\tan^4 \alpha + \cot^4 \alpha = m^4 - 4m^2 + 2$$

8. (B)

$$\frac{\cot \theta + \operatorname{cosec} \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1}$$

$$= \frac{\frac{1}{\operatorname{cosec} \theta - \cot \theta} - 1}{\cot \theta - \operatorname{cosec} \theta + 1} \quad (\because (\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta) = 1)$$

$$= \frac{1 - \operatorname{cosec} \theta + \cot \theta}{(\operatorname{cosec} \theta - \cot \theta)(\cot \theta - \operatorname{cosec} \theta + 1)}$$

$$= \frac{1}{\operatorname{cosec} \theta - \cot \theta} = \operatorname{cosec} \theta + \cot \theta$$

$$= \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 - \cos \theta}$$

9. (D)

$$\text{Since } \sin x + \sin^2 x = 1$$

$$\therefore \sin x = 1 - \sin^2 x = \cos^2 x$$

$$\therefore \sin^2 x = \cos^4 x$$

$$\Rightarrow 1 - \cos^2 x = \cos^4 x$$

$$\Rightarrow 1 = \cos^4 x + \cos^2 x$$

$$\Rightarrow 1 = (\cos^4 x + \cos^2 x)^2$$

$$= \cos^8 x + 2\cos^6 x + \cos^4 x$$

Thus $\cos^8 x + 2\cos^6 x + \cos^4 x = 1$

10. (C)

$$\begin{aligned}\sin(-566^\circ) &= -\sin 566^\circ = -\sin(6 \times 90^\circ + 26^\circ) \\ &= -(-\sin 26^\circ) = \sin 26^\circ\end{aligned}$$

11. (C)

$$\begin{aligned}f(x) &= 3\left[\sin^4\left(\frac{\pi}{2} - x\right) + \sin^4 x\right] - 2[\cos^6 x + \sin^6 x] \\ &= 3[(\cos^2 x)^2 + (\sin^2 x)^2] - 2[(\sin^2 x)^3 + (\cos^2 x)^3] \\ &= 3[1 - 2\sin^2 x \cos^2 x] - 2[1 - 3\sin^2 x \cos^2 x] \\ &= 1\end{aligned}$$

12. (A)

$$\begin{aligned}2\cos 10 + \sin(90 + 10) + \sin(3 \times 360 - 80) + \sin(27 \times 360 + 280) \\ = 2\cos 10 + \cos 10 - 2\sin 80 \\ = \cos 10 + 2(\cos 10 - \sin(90 - 10)) \\ = \cos 10 + 2(\cos 10 - \cos 10) = \cos 10\end{aligned}$$

13. (C)

$$\begin{aligned}\cos^2(90 - 17) + \cos^2 47 - \sin^2(90 - 47) + \sin^2(90 + 17) \\ = \sin^2 17 + \cos^2 47 - \cos^2 47 + \cos^2 17 \\ = 1\end{aligned}$$

14. (D)

$$\begin{aligned}\text{(i) } \sin(2 \times 360 + 45) &= \sin 45 = \frac{1}{\sqrt{2}} \\ \text{(ii) } \frac{1}{-\sin(3 \times 360 + 330)} &= \frac{1}{-\sin(360 - 30)} = \frac{1}{\sin 30} = 2 \\ \text{(iii) } \tan\left(4\pi + \frac{\pi}{3}\right) &= \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \\ \text{(iv) } \frac{-1}{\tan\left(4\pi - \frac{\pi}{4}\right)} &= 1\end{aligned}$$

15. (C)

$$\begin{aligned}\ell^2 &= \left[\sin \theta - \sin\left(\frac{\pi}{2} - \theta\right)\right]^2 + \left[\cos \theta + \cos\left(\frac{\pi}{2} - \theta\right)\right]^2 \\ &= (\sin \theta - \cos \theta)^2 + (\cos \theta + \sin \theta)^2 \\ &= 1 + 1 = 2 \\ \ell &= \sqrt{2}\end{aligned}$$

16. (B)
 $\sin(180+20) + \cos(180+20)$
 $= -\sin 20 - \cos 20 < 0$
17. (B)
 $a = \sin 170^\circ + \cos 170^\circ$
 $= \sin(180-10) + \cos(180-10)$
 $= \sin 10 - \cos 10$
 $\Rightarrow a = -ve$
 $\{\because \cos 10^\circ > \sin 10^\circ\}$
18. (C)
 $\cos\left(\frac{\pi}{2} - x\right) = \sin x, \cos\left(\frac{3\pi}{2} + x\right) = -\sin x$
 $\cos(\pi - x) = -\cos x, \cos(2\pi - x) = \cos x$
 $\therefore \text{required} = \sin x \cdot \sin x + \cos x \cdot \cos x$
 $= 1$
19. (B)
 $\cos \frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}, \cos \frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}$
 $\cos \frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}, \cos \frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}$
 $\Rightarrow x = y \cos \frac{2\pi}{3} = z \cos \frac{4\pi}{3} \Rightarrow x = -\frac{y}{2} = -\frac{z}{2} \Rightarrow y = z = -2x$
 Now, $xy + yz + zx = x(-2x) + (-2x)(-2x) + (-2x)x = 0$
20. (A)
 $\cos^2 \alpha + \cos^2(\alpha + 120^\circ) + \cos^2(\alpha - 120^\circ)$
 $= \frac{1 + \cos 2\alpha}{2} + \frac{1 + \cos 2(\alpha + 120^\circ)}{2} + \frac{1 + \cos 2(\alpha - 120^\circ)}{2}$
 $= \frac{3 + \cos 2\alpha + 2 \cos 2\alpha \cos 120^\circ}{2} = \frac{3 + \cos 2\alpha - \cos 2\alpha}{2} = \frac{3}{2}$
21. (B)
 $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \Rightarrow \tan(A+B) = \frac{-\frac{1}{2} - \frac{1}{3}}{1 - \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)}$
 $\Rightarrow \tan(A+B) = -1$
 $\Rightarrow A+B = \frac{3\pi}{4}$

22. (D)

$$\tan 45^\circ = \tan(180^\circ + 45^\circ) \Rightarrow \tan 225^\circ = \tan(100^\circ + 125^\circ)$$

$$\Rightarrow \frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ} = 1$$

$$\Rightarrow \tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ = 1$$

23. (B)

$$\frac{\sin(B+A) + \cos(B-A)}{\sin(B-A) + \cos(B+A)} = \frac{\sin(B+A) + \sin\left(\frac{\pi}{2} - (B-A)\right)}{\cos\left(\frac{\pi}{2} - (B-A)\right) + \cos(B+A)}$$

$$= \frac{2 \sin\left(\frac{\pi}{4} + A\right) \cos\left(\frac{\pi}{4} - B\right)}{2 \cos\left(\frac{\pi}{4} + A\right) \cos\left(\frac{\pi}{4} - B\right)} = \tan\left(\frac{\pi}{4} + A\right)$$

$$= \frac{1 + \tan A}{1 - \tan A} = \frac{\cos A + \sin A}{\cos A - \sin A}$$

24. (C)

$$\tan 3A = \tan(2A + A)$$

$$= \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

$$\Rightarrow \tan 3A - \tan 2A \tan A = \tan A$$

$$\therefore \tan 3A - \tan 2A - \tan A = 0$$

$$= \tan A \tan 2A \tan 3A$$

25. (C)

$$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B$$

$$= \sin^2 A (1 - \sin^2 B) - \sin^2 B (1 - \sin^2 A)$$

$$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$$

$$= \sin^2 A - \sin^2 B$$

26. (B)

$$\sin^2 A + \sin^2(A-B) + 2 \sin A \cos B \sin(B-A)$$

$$= \sin^2 A + \sin^2(A-B) + [\sin(A+B) + \sin(A-B)] \sin(B-A)$$

$$= \sin^2 A + \sin^2(A-B) + \sin(A+B) \sin(B-A) - \sin^2(A-B)$$

$$= \sin^2 A + \sin^2 B - \sin^2 A$$

$$= \sin^2 B$$

27. (A)

$$\frac{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \sin\left(\theta + \frac{2\theta}{2}\right)}{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\frac{\theta}{2}} \cos\left(\theta + \frac{2\theta}{2}\right)} = \frac{\sin \alpha}{\cos \alpha}$$

$$\tan 2\theta = \tan \alpha$$

$$\theta = \frac{\alpha}{2}$$

28. (B)

$$\sin \alpha + \sin \beta = a \Rightarrow 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = a \quad \dots (i)$$

$$\& \cos \alpha - \cos \beta = b \Rightarrow 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} = -b \quad \dots (ii)$$

$$\text{From (i) \& (ii) } \tan \frac{\alpha - \beta}{2} = -\frac{b}{a}$$

29. (C)

$$\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} = \frac{(\sin 3\theta + \sin 9\theta) + (\sin 5\theta + \sin 7\theta)}{(\cos 3\theta + \cos 9\theta) + (\cos 5\theta + \cos 7\theta)}$$

$$= \frac{2 \sin 6\theta \cos 3\theta + 2 \sin 6\theta \cos \theta}{2 \cos 6\theta \cos 3\theta + 2 \cos 6\theta \cos \theta} = \tan 6\theta$$

30. (D)

Given value

$$= (\sin 47^\circ + \sin 61^\circ) - (\sin 11^\circ + \sin 25^\circ)$$

$$= 2 \sin 54^\circ \cos 7^\circ - 2 \sin 18^\circ \cos 7^\circ$$

$$= 2 \cos 7^\circ (\sin 54^\circ - \sin 18^\circ)$$

$$= 2 \cos 7^\circ \frac{2 \sin 18^\circ \cos 18^\circ}{\cos 18^\circ} \cos 36^\circ$$

$$= \cos 7^\circ \frac{2 \sin 36^\circ \cos 36^\circ}{\cos 18^\circ}$$

$$= \cos 7^\circ \frac{\sin 72^\circ}{\cos 18^\circ} = \cos 7^\circ \left[\because \sin 72^\circ = \cos 18^\circ \right]$$

31. (D)

$$\cos 40^\circ + \cos 80^\circ + \cos 160^\circ + \cos 240^\circ$$

$$= 2 \cos 60^\circ \cos 20^\circ + \cos(180^\circ - 20^\circ) + \cos(180^\circ + 60^\circ)$$

$$= 2 \left(\frac{1}{2} \right) \cos 20^\circ - \cos 20^\circ - \cos 60^\circ$$

$$= -\cos 60^\circ = -\frac{1}{2}$$

32. (A)

$$\begin{aligned} & \cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 228^\circ \\ &= (\cos 12^\circ + \cos 228^\circ) + (\cos 84^\circ + \cos 156^\circ) \\ &= 2 \cos(120^\circ) \cos(108^\circ) + 2 \cos 120^\circ \cos 36^\circ \\ &= 2 \cos(120^\circ) [\cos(108^\circ) + \cos 36^\circ] \\ &= 2 \left(-\frac{1}{2}\right) \cdot 2 \cos 72^\circ \cos(36^\circ) \\ &= -2 \cos 72^\circ \cos 36^\circ = -2 \cdot \frac{\sqrt{5}-1}{4} \cdot \frac{\sqrt{5}+1}{4} \\ &= -2 \left(\frac{5-1}{16}\right) = -\frac{1}{2} \end{aligned}$$

33. (B)

$$\begin{aligned} & \cot \theta - 2 \cot 2\theta \\ &= \frac{1}{\tan \theta} - \frac{2 \times (1 - \tan^2 \theta)}{2 \tan \theta} \\ &= \frac{\tan^2 \theta}{\tan \theta} = \tan \theta \end{aligned}$$

34. (D)

$$\begin{aligned} & \frac{\cos 10^\circ - \sqrt{3} \sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ} \\ &= 2 \cdot \frac{\left(\frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ\right)}{2 \sin 10^\circ \cos 10^\circ} \\ &= \frac{4 \sin(30^\circ - 10^\circ)}{\sin 20^\circ} = 4 \end{aligned}$$

35. (B)

$$\begin{aligned} & \cos \alpha + \cos \beta = 0 \Rightarrow \cos^2 \alpha + \cos^2 \beta = -2 \cos \alpha \cos \beta \\ & \& \sin \alpha + \sin \beta = 0 \Rightarrow \sin^2 \alpha + \sin^2 \beta = -2 \sin \alpha \sin \beta \\ & \text{Now } \cos 2\alpha + \cos 2\beta = 2(\cos^2 \alpha + \cos^2 \beta - 1) = 2(1 - \sin^2 \alpha - \sin^2 \beta) \\ & \Rightarrow \cos 2\alpha + \cos 2\beta = 2(-2 \cos \alpha \cos \beta - 1) \quad \dots(i) \\ & \& \cos 2\alpha + \cos 2\beta = 2(1 + 2 \sin \alpha \sin \beta) \quad \dots(ii) \\ & (i) + (ii) \\ & \Rightarrow \cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta) \end{aligned}$$

36. (C)

$$\begin{aligned}\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ &= \frac{\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ}{\frac{1}{2} \cos 20^\circ \sin 20^\circ} \\ &= \frac{4 \sin (60^\circ - 20^\circ)}{2 \cos 20^\circ \sin 20^\circ} = \frac{4 \sin 40^\circ}{\sin 40^\circ} = 4\end{aligned}$$

37. (C)

$$\begin{aligned}\sqrt{3} [\cot \theta + \tan \theta] &= 4 \\ \Rightarrow \sqrt{3} \left[\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right] &= 4 \\ \Rightarrow \frac{1}{\sin \theta \cos \theta} &= \frac{4}{\sqrt{3}} \Rightarrow \frac{1}{2 \sin \theta \cos \theta} = \frac{2}{\sqrt{3}} \\ \Rightarrow \sin 2\theta &= \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \\ \Rightarrow 2\theta &= \frac{\pi}{3} \quad \therefore \theta = \frac{\pi}{6}\end{aligned}$$

38. (B)

$$\begin{aligned}\text{Since, } \cos 20^\circ - \sin 20^\circ &= p \\ \Rightarrow \cos^2 20^\circ + \sin^2 20^\circ - 2 \sin 20^\circ \cos 20^\circ &= p^2 \\ \Rightarrow 1 - p^2 &= \sin 40^\circ \\ \Rightarrow 1 - p^2 &= \sqrt{1 - \cos^2 40^\circ} \\ \Rightarrow (1 - p^2)^2 &= 1 - \cos^2 40^\circ \\ \Rightarrow \cos^2 40^\circ &= 1 - (1 + p^4 - 2p^2) \\ \Rightarrow \cos 40^\circ &= \sqrt{2p^2 - p^4} \\ \Rightarrow \cos 40^\circ &= p\sqrt{2 - p^2} \\ \therefore \text{(B) holds}\end{aligned}$$

39. (B)

$$\begin{aligned}\text{LHS} &= \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right) \\ &= \left(1 - \cos^2 \frac{\pi}{8}\right) \left(1 - \cos^2 \frac{3\pi}{8}\right) \\ &= \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} = \frac{1}{4} \left[\cos \frac{\pi}{4} - \cos \frac{\pi}{2} \right]^2 = \frac{1}{8}\end{aligned}$$

40. (C)

$$\sin \theta = \frac{2t}{1+t^2} = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \text{ if } t = \tan \frac{\theta}{2}$$

$$\therefore \cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \frac{1 - t^2}{1 + t^2}$$

41. (D)

$$\frac{1}{4} \left(4 \cos \theta \cos \left(\frac{\pi}{3} + \theta \right) \cos \left(\frac{\pi}{3} - \theta \right) \right) \\ = \frac{\cos 3\theta}{4}$$

42. (B)

$$\frac{\cos 20 + 4(\cos 40 - \cos 60) \sin 70}{\cos^2 10} \\ = \frac{\cos 20 + 4 \cos 40 \cos 20 - 2 \cos 20}{\cos^2 10} \\ = \frac{2(\cos 20 + 2 \cos 60 + 2 \cos 20 - 2 \cos 20)}{2 \cos^2 10} \\ = \frac{2(\cos 20 + 1)}{(1 + \cos 20)} = 2$$

43. (C)

$$\sin 12^\circ \sin 48^\circ \sin 54^\circ = \sin(60^\circ - 12^\circ) \sin 12^\circ \sin(60^\circ + 12^\circ) \frac{\sin 54^\circ}{\sin 72^\circ} \\ = \frac{\sin 36^\circ \sin 54^\circ}{4 \sin 72^\circ} = \frac{\sin 36^\circ \sin 54^\circ}{8 \sin 36^\circ \cos 36^\circ} \\ = \frac{\sin 54^\circ}{8 \cos 36^\circ}, \text{ but } \sin 54^\circ = \sin(90^\circ - 36^\circ) = \cos 36^\circ \\ \therefore \sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{8}$$

44. (B)

$$x + \frac{1}{x} = 2 \cos \theta \quad \& \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x} \right)^3 - 3 \left(x \right) \left(\frac{1}{x} \right) \left(x + \frac{1}{x} \right) \\ \Rightarrow x^3 + \frac{1}{x^3} = 8 \cos^3 \theta - 6 \cos \theta = 2 \cos 3\theta$$

45. (C)

$$\sin \frac{\pi}{10} \sin \frac{13\pi}{10}$$

$$\begin{aligned}
&= \sin \frac{\pi}{10} \sin \left(\pi + \frac{3\pi}{10} \right) \\
&= -\sin \frac{\pi}{10} \sin \frac{3\pi}{10} = -\sin 18^\circ \sin 54^\circ \\
&= -\frac{\sqrt{5}-1}{4} \cdot \frac{\sqrt{5}+1}{4} = -\frac{5-1}{16} = -\frac{1}{4}
\end{aligned}$$

46. (C)

$$3 \sin 2\theta = 2 \sin 3\theta$$

$$\Rightarrow 6 \sin \theta \cos \theta = 2(3 \sin \theta - 4 \sin^3 \theta)$$

$$\Rightarrow 4 \cos^2 \theta - 3 \cos \theta - 1 = 0$$

$$\Rightarrow \cos \theta = 1, \frac{1}{4}$$

Since $\cos \theta \neq 1 (\because \theta \neq 0)$

$$\therefore \cos \theta = \frac{1}{4} \Rightarrow \sin \theta = \frac{\sqrt{15}}{4}$$

47. (C)

$$f(\theta) = \sin \theta (\sin \theta + 3 \sin \theta - 4 \sin^3 \theta)$$

$$f(\theta) = 4 \sin^2 \theta - 4 \sin^4 \theta$$

$$f(\theta) = 4 \sin^2 \theta (1 - \sin^2 \theta) \geq 0 \quad \forall \theta$$

48. (D)

$$f(\theta) = 5 \cos \theta + 3 \cos \left(\theta + \frac{\pi}{3} \right) + 3$$

$$= 5 \cos \theta + 3 \cos \theta \cdot \cos \frac{\pi}{3} - 3 \sin \theta \sin \frac{\pi}{3} + 3$$

$$= \frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 3$$

$$-\sqrt{\frac{13^2}{4} + \frac{27}{4}} + 3 \leq f(\theta) \leq \sqrt{\frac{13^2}{4} + \frac{27}{4}} + 3$$

$$-4 \leq f(\theta) \leq 10$$

49. (C)

$$\sin \theta + \cos \beta$$

Maximum Value = 2

50. (C)

$$y = \frac{12}{9 + 5 \left(\frac{3}{5} \cos x + \frac{4}{5} \sin x \right)}$$

$$= \frac{12}{9 + 5 \sin(\alpha + x)} \text{ let } \left(\sin \alpha = \frac{3}{5} \right)$$

$$y_{\text{maximum}} = \frac{12}{4} = 3$$

51. (D)

$$y_{\text{minimum}} = \frac{1}{9}$$

52. (A)

$$\begin{aligned} B &= 2\sin^2 x - \cos 2x \\ &= 2\sin^2 x - (1 - 2\sin^2 x) \\ &= 4\sin^2 x - 1 \end{aligned}$$

$$\begin{aligned} \text{Since } 0 \leq \sin^2 x \leq 1 \quad \therefore 0 \leq 4\sin^2 x \leq 4 \\ \Rightarrow 0 - 1 \leq 4\sin^2 x - 1 \leq 3 \Rightarrow -1 \leq B \leq 3 \end{aligned}$$

53. (B)

$$\sin \theta + \cos \theta = \sqrt{2} \sin \left(\theta + \frac{\pi}{4} \right) \text{ is Max when } \theta + \frac{\pi}{4} = \frac{\pi}{2} \text{ i.e. when } \theta = \frac{\pi}{4} = 45^\circ$$

54. (B)

$$\begin{aligned} 2\sin^2 \theta + 3\cos^2 \theta \\ = 2[\sin^2 \theta + \cos^2 \theta] + \cos^2 \theta = 2 + \cos^2 \theta \end{aligned}$$

$$\begin{aligned} \text{Since least value of } \cos^2 \theta = 0 \\ \therefore \text{least value of } 2\sin^2 \theta + 3\cos^2 \theta = 2 + 0 = 2 \end{aligned}$$

55. (C)

$$\text{Let } y = \cos \theta - \cos 2\theta$$

$$\begin{aligned} \frac{dy}{dx} &= \sin \theta + 2\sin 2\theta = 2.2\sin \theta \cos \theta - \sin \theta \\ &= \sin \theta (4\cos \theta - 1) \end{aligned}$$

$$\text{For Max or Min of } y, \frac{dy}{d\theta} = 0$$

$$\therefore \sin \theta = 0 \text{ or } \cos \theta = \frac{1}{4}$$

$$\text{i.e. } \theta = 0, \pi, 2\pi, \dots \text{ or } \cos \theta = \frac{1}{4}$$

$$\frac{d^2y}{d\theta^2} = -\cos \theta + 4\cos 2\theta > 0 \text{ for } \theta = 0, \pi, 2\pi, \dots \text{ and } 4\cos 2\theta - \cos \theta = 4[2\cos^2 \theta - 1] - \cos \theta$$

$$= 8\left(\frac{1}{4}\right)^2 - 4 - \frac{1}{4} = \frac{1}{2} - 4 - \frac{1}{4} < 0$$

$$\therefore y \text{ max. when } \cos \theta = \frac{1}{4}$$

$$\therefore \text{max. } y = \cos \theta - (2\cos^2 \theta - 1)$$

$$= \frac{1}{4} - \left[\frac{2}{16} - 1 \right]$$

$$= \frac{1}{4} + 1 - \frac{1}{8} = \frac{1}{4} + \frac{7}{8}$$

$$= \frac{2+7}{8} = \frac{9}{8}$$

∴ (C) holds

56. (D)

$$\frac{8 \sin x}{8} \cdot \frac{\cos x}{8} \cdot \frac{\cos x}{4} \cdot \frac{\cos x}{2}$$

$$= \theta = \frac{x}{7}$$

$$8 \sin \theta \cdot \cos \theta \cdot \cos 2\theta \cdot \cos 4\theta$$

$$= 8 \sin \theta \cdot \frac{\sin(2^3 \theta)}{2^3 \sin \theta} = \sin 8\theta = \sin x$$

57. (C)

We know that

$$\cos A \cos 2A \cos 2^2 A \cos 2^3 A \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$

$$\therefore \cos A \cos 2A \cos 2^2 A = \frac{\sin 2^3 A}{2^3 \sin A} \quad [\text{By putting } n = 3]$$

$$\therefore \cos A \cos 2A \cos 4A = \frac{\sin 8A}{8 \sin A}$$

$$\therefore \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = \frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}}$$

$$= \frac{\sin \left(\pi + \frac{\pi}{7} \right)}{8 \sin \frac{\pi}{7}} = \frac{-\sin \frac{\pi}{7}}{8 \sin \frac{\pi}{7}} = -\frac{1}{8}$$

Hence (C) holds

58. (D)

The given expression

$$= \frac{1}{2 \sin \frac{\pi}{65}} \left(2 \sin \frac{\pi}{65} \cos \frac{\pi}{65} \right) \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$= \frac{1}{4 \sin \frac{\pi}{65}} \left(2 \sin \frac{2\pi}{65} \cos \frac{2\pi}{65} \right) \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$= \frac{1}{8 \sin \frac{\pi}{65}} \left(2 \sin \frac{4\pi}{65} \cos \frac{4\pi}{65} \right) \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$= \frac{1}{16 \sin \frac{\pi}{65}} \left(2 \sin \frac{8\pi}{65} \cos \frac{8\pi}{65} \right) \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$\begin{aligned}
&= \frac{1}{32 \sin \frac{\pi}{65}} \left(2 \sin \frac{16\pi}{65} \cos \frac{16\pi}{65} \right) \cos \frac{32\pi}{65} \\
&= \frac{1}{64 \sin \frac{\pi}{65}} \left(2 \sin \frac{32\pi}{65} \cos \frac{32\pi}{65} \right) \\
&= \frac{1}{64 \sin \frac{\pi}{65}} \sin \frac{64\pi}{65} \\
&= \frac{1}{64 \sin \frac{\pi}{65}} \sin \left(\pi - \frac{\pi}{65} \right) = \frac{\sin \frac{\pi}{65}}{64 \sin \frac{\pi}{65}} = \frac{1}{64}
\end{aligned}$$

59. (A)

$$\cos A \cos 2A \cos 4A \dots \cos 2^{n-1} A$$

$$= \frac{\sin 2^n A}{2^n \sin A} \quad [\text{Standard Result}]$$

$\therefore$ (A) hold.

60. (B)

$$A + B + C = \frac{3\pi}{2}$$

$$\cos 2A + \cos 2B + \cos 2C + 4 \sin A \sin B \sin C$$

$$= 2 \cos(A+B) \cos(A-B) + 2 \cos^2 C - 1 + 4 \sin A \sin B \sin C$$

$$= 2 \cos \left(3\frac{\pi}{2} - C \right) \cos(A-B) + 2 \cos^2 C - 1 + 4 \sin A \sin B \sin C$$

$$= -2 \sin C \cos(A-B) + 1 - 2 \sin^2 C + 4 \sin A \sin B \sin C$$

$$= -2 \sin C [\cos(A-B) - \cos(A+B)] + 4 \sin A \sin B \sin C$$

$$= -4 \sin A \sin B \sin C + 4 \sin A \sin B \sin C + 1$$

61. (C)

If $A + B + C = n\pi$ then

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

62. (B)

$$\begin{aligned}
&\left(\frac{\cos B}{\cos B} + \frac{\cos C}{\sin C} \right) \left(\frac{\cos C}{\sin C} + \frac{\cos A}{\sin A} \right) \left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} \right) \quad \left| \begin{array}{l} A + B = \pi - C \\ \Rightarrow \sin(A+B) = \sin C \end{array} \right. \\
&= \frac{\sin(B+C) \cdot \sin(A+C) \cdot \sin(A+B)}{\sin^2 A \cdot \sin^2 B \sin^2 C} \\
&= \operatorname{cosec} A \cdot \operatorname{cosec} B \cdot \operatorname{cosec} C
\end{aligned}$$

63. (A)

$$A + B + C = 180^\circ \Rightarrow \frac{A}{2} + \frac{B}{2} = 90^\circ - \frac{C}{2}$$

$$\therefore \tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right) = \cot \frac{C}{2}$$

$$\therefore \frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$$

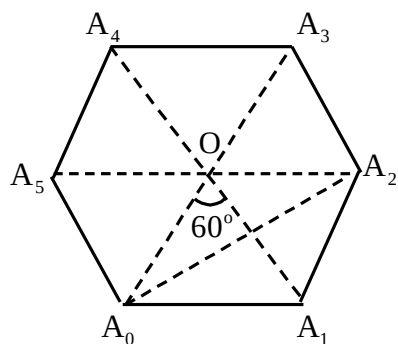
$$\Rightarrow \tan \frac{A}{2} \tan \frac{C}{2} + \tan \frac{B}{2} \tan \frac{C}{2} = 1 - \tan \frac{A}{2} \tan \frac{B}{2}$$

$$\Rightarrow \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} + \tan \frac{A}{2} \tan \frac{B}{2} = 1$$

Divide thro' out by $\tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2}$, we get

$$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$$

64. (C)



OA_0A_1 is equilateral triangle

$$A_0A_1 = 1$$

$$\cos 120^\circ = \frac{1^2 + 1^2 - A_0A_2^2}{2 \cdot 1 \cdot 1} \quad (\text{cosine law})$$

$$-\frac{1}{2} = 2 - A_0A_2^2$$

$$A_0A_2^2 = 3$$

$$A_0A_2 = \sqrt{3}$$

$$A_0A_4 = \sqrt{3}$$

$$A_0A_1 \times A_0A_2 + A_0A_4 = 1 \cdot \sqrt{3} \cdot \sqrt{3} = 3$$

65. (C)

$$\frac{b}{y} = \cot \theta \quad \& \quad \frac{a}{x} = \operatorname{cosec} \theta$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = \frac{a^2}{x^2} - \frac{b^2}{y^2}$$

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

66. (A)

$$\text{Required number of sides} = \frac{360^\circ}{36^\circ} = 10$$

$\therefore$ (A) holds

67. (A)

$$\cos A = m \cos B \Rightarrow \frac{\cos A}{\cos B} = m$$

$$\text{Apply componendo to get } \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$$

$$\Rightarrow -\frac{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot \frac{A+B}{2} = \frac{m+1}{m-1} \tan \frac{B-A}{2}$$

68. (A)

$$\frac{\sin 2x}{\sin 2y} = n \Rightarrow \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{n+1}{n-1} \quad \{\text{By componendo \& dividendo}\}$$

$$\Rightarrow \frac{2 \sin(x+y) \cos(x-y)}{2 \cos(x+y) \sin(x-y)} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan(x+y)}{\tan(x-y)} = \frac{n+1}{n-1}$$

69. (D)

$$\frac{\sin \alpha}{\sin \beta} = \frac{5}{3}$$

$$\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{8}{2} = 4 \quad (\text{C \& D})$$

$$\frac{2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)}{2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)} = 4$$

$$\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$$

70. (B)

$$\operatorname{cosec} A (\sin B \cos C + \cos B \sin C)$$

$$= \operatorname{cosec} A \sin(B+C)$$

$$= \operatorname{cosec} A \sin(180^\circ - A) \quad \left[\because A+B+C=180^\circ \right]$$

$$= \operatorname{cosec} A \sin A = 1$$

Hence, (B) holds

EXERCISE - 1 [C]

1. (0.625)

$$\sin 2\theta + \sin 2\phi = \frac{1}{2} \quad \& \quad \cos 2\theta + \cos 2\phi = \frac{3}{2}$$

$$\Rightarrow 2 \sin(\theta + \phi) \cdot \cos(\theta - \phi) = \frac{1}{2} \quad \dots(1)$$

$$2 \cos(\theta + \phi) \cdot \cos(\theta - \phi) = \frac{3}{2} \quad \dots(2)$$

Square & add (1) & (2)

$$\Rightarrow 4 \cos^2(\theta - \phi) \cdot [\sin^2(\theta + \phi) + \cos^2(\theta + \phi)] = \frac{1}{4} + \frac{9}{4}$$

$$\Rightarrow \cos^2(\theta - \phi) = \frac{10}{16}$$

2. (0.0625)

$$[\cos 20^\circ \cdot \cos(60 - 20^\circ) \cdot \cos(60 + 20^\circ)] \cdot \cos 60^\circ$$

$$= \frac{1}{4} \cdot \cos 60^\circ \cdot \cos 60^\circ = \frac{1}{16}$$

3. (0.78)

$$\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\sin A = \frac{1}{\sqrt{5}} \Rightarrow \cos A = \sqrt{1 - \sin^2 A} = \frac{2}{\sqrt{5}}$$

$$\cos B = \frac{3}{\sqrt{10}} \Rightarrow \sin B = \sqrt{1 - \cos^2 B} = \frac{1}{\sqrt{10}}$$

$$\therefore \cos(A + B) = \frac{2}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} - \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$$\therefore A + B = \frac{\pi}{4}$$

4. (2)

$$A - B = \frac{\pi}{4} \Rightarrow \tan(A - B) = 1$$

$$\Rightarrow \tan A - \tan B = 1 + \tan A \cdot \tan B$$

$$\therefore (1 + \tan A) \cdot (1 - \tan B) = 1 + (\tan A - \tan B) - \tan A \tan B$$

$$= 2$$

5. (0.57)

Divide numerator & denominator by $\cos \theta$

$$\frac{2 \tan \theta - 5}{4 - 5 \tan \theta} = \frac{3 - 5}{4 - 5 \cdot \frac{3}{2}} = \frac{-2}{-\frac{7}{2}} = \frac{4}{7}$$

6. (0.26)

$$\frac{2 \sin\left(x + \frac{\pi}{6}\right) \cdot \cos\left(x + \frac{\pi}{6}\right)}{2}$$

$$= \frac{\sin\left(2x + \frac{\pi}{3}\right)}{2}$$

$$\text{Maximum} = \frac{1}{2} \text{ if } \sin\left(2x + \frac{\pi}{3}\right) = 1$$

$$\Rightarrow x = \frac{\pi}{12}$$

7. (0.75)

$$\cos^2 108^\circ + \cos^2 144^\circ$$

$$= \sin^2 18^\circ + \cos^2 36^\circ$$

$$= \left(\frac{\sqrt{5}-1}{4}\right)^2 + \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{12}{16} = \frac{3}{4}$$

8. (1)

$$x \cdot 1 \cdot \frac{1}{2} = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 1$$

9. (1.5)

$$\cos^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{4} + \cos^2 \frac{5\pi}{12}$$

$$= \cos^2 15^\circ + \cos^2 45^\circ + \cos^2 75^\circ$$

$$= (\sin^2 75^\circ + \cos^2 75^\circ) + \cos^2 45^\circ$$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

10. (1)

$$\frac{\sin 85^\circ - \sin 35^\circ}{\cos 65^\circ}$$

$$= \frac{2 \sin 25^\circ \cdot \cos 60^\circ}{\cos 65^\circ} \quad \{\sin 25^\circ = \cos 65^\circ\}$$

$$= 1$$

11. (0.19)

$$\cos\left(\frac{7\pi}{6}\right) = \cos\left(\pi + \frac{\pi}{6}\right) = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\cos \frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$\therefore \text{Ans} = \left(1 + \frac{\sqrt{3}}{2}\right) \left(1 + \frac{1}{2}\right) \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\begin{aligned}
&= \left(1 - \frac{\sqrt{3}^2}{2^2}\right) \cdot \left(1 - \frac{1}{2^2}\right) \\
&= \left(1 - \frac{3}{4}\right) \cdot \left(1 - \frac{1}{4}\right) = \frac{3}{16}
\end{aligned}$$

12. (2)

$$\cos 54^\circ = \cos(90^\circ - 36^\circ) = \tan 36^\circ$$

13. (1)

$$\cot(102^\circ) = \cot(90^\circ + 12^\circ) = -\tan 12^\circ$$

$$\begin{aligned}
\therefore \text{Required} &= \cot 12^\circ (-\tan 12^\circ) - \tan 12^\circ \cdot \cot 66^\circ + \cot 66^\circ \cot 12^\circ \\
&= -1 + \cot 66^\circ \cdot [\cot 12^\circ - \tan 12^\circ] \\
&= -1 + \cot 66^\circ (2 \cot 24^\circ) \\
&= -1 + (\tan 24^\circ) \cdot (2 \cot 24^\circ) \\
&= -1 + 2 = 1 \qquad \qquad \qquad \{\cot 66^\circ = \tan 24^\circ\}
\end{aligned}$$

14. (1.69)

$$\begin{aligned}
\tan 2\alpha &= \tan(\alpha + \beta + \alpha - \beta) \\
&= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} \quad \dots(1)
\end{aligned}$$

$$\cos(\alpha + \beta) = \frac{4}{5}$$

$$\begin{aligned}
\Rightarrow \tan(\alpha + \beta) &= \sqrt{\sec^2(\alpha + \beta) - 1} \\
&= \frac{3}{4} \quad \dots(2)
\end{aligned}$$

$$\sin(\alpha - \beta) = \frac{5}{13}$$

$$\begin{aligned}
\Rightarrow \tan(\alpha - \beta) &= \frac{\sin(\alpha - \beta)}{\sqrt{1 - \sin^2(\alpha - \beta)}} \\
&= \frac{5}{12} \quad \dots(3)
\end{aligned}$$

Put (2) & (3) in (1)

$$\tan 2\alpha = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{14}{12}}{1 - \frac{5}{16}} = \frac{56}{33}$$

15. (0.40)

$$\begin{aligned}
\cos^2 A - \sin^2 B &= \cos(A + B) \cdot \cos(A - B) \\
&= \cos(48 + 12) \cdot \cos(48 - 12) \\
&= \cos 60^\circ \cdot \cos 36^\circ
\end{aligned}$$

$$= \frac{1}{2} \times \left(\frac{\sqrt{5}+1}{4} \right)$$

16. (1.73)

$$A = \frac{2\pi}{5}, B = \frac{\pi}{15}$$

$$A - B = \frac{2\pi}{5} - \frac{\pi}{15} = \frac{\pi}{3}$$

$$\Rightarrow \tan(A - B) = \sqrt{3}$$

$$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = \sqrt{3}$$

$$\Rightarrow \tan A - \tan B = \sqrt{3} + \sqrt{3} \tan A \tan B$$

$$\Rightarrow \text{Ans.} = \sqrt{3}$$

17. (3.87)

$$\frac{\sin x}{\sin y} \cdot \frac{\cos y}{\cos x} = \frac{1/2}{3/2} \quad \& \quad \frac{\tan x}{\tan y} = \frac{1}{3}$$

$$\Rightarrow \tan x = \frac{\tan y}{3} \quad \dots(1)$$

Square & add

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \frac{\sin^2 y}{4} + \frac{9}{4} \cos^2 y = 1$$

$$\Rightarrow 1 - \cos^2 y + 9 \cos^2 y = 4$$

$$\Rightarrow \cos^2 y = \frac{3}{8}$$

$$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$$

$$\tan y = \sqrt{\frac{5}{3}} \quad \dots(2)$$

(1) & (2)

$$\therefore \tan x = \frac{1}{3} \sqrt{\frac{5}{3}}$$

$$\therefore \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$= \frac{\frac{4 \tan y}{3}}{1 - \tan^2 y} = \frac{4 \tan y}{3 - \tan^2 y}$$

$$= \frac{\frac{4 \sqrt{5}/3}{3}}{3 - \frac{5}{3}} = \sqrt{15}$$

18. (1.5)

$$\sin \frac{7\pi}{8} = \sin \left(\pi - \frac{\pi}{8} \right) = \sin \frac{\pi}{8}$$

$$\sin \frac{5\pi}{8} = \sin \left(\pi - \frac{3\pi}{8} \right) = \sin \frac{3\pi}{8}$$

$$\text{Further } \sin \frac{3\pi}{8} = \sin \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \\ = \cos \frac{\pi}{8}$$

$$\therefore \text{ Required } = \sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \\ = 2$$

19. (0.5)

$$A + B = 90^\circ$$

$$\therefore \sin A \sin B$$

$$= \sin A \cdot \sin(90 - A)$$

$$= \sin A \cos A$$

$$= \frac{1}{2} \sin 2A$$

$$\therefore \text{ Maximum } = \frac{1}{2}$$

20. (2)

$$\frac{\cot x - \tan x}{\cot 2x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\left(\frac{2 \cos x \cdot \sin x}{2} \right) \cdot \cos 2x} = \frac{2 \cos 2x}{\sin 2x \cot 2x} = 2$$

JEE Main : PYQ

1. (C)

$$\text{Take } \cos 72^\circ = \frac{\sqrt{5}-1}{4}$$

$$\text{Let } \sin 36^\circ = \alpha$$

$$\Rightarrow 1 - 2 \sin^2 36^\circ = \frac{\sqrt{5}-1}{4} \Rightarrow 4 - 8\alpha^2 = \sqrt{5} - 1$$

$$\Rightarrow 5 - 8\alpha^2 = \sqrt{5}$$

Take square both sides,

$$\Rightarrow (5 - 8\alpha^2)^2 = 5 \Rightarrow 25 + 64\alpha^4 - 80\alpha^2 = 5$$

$$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0 \Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$$

2. (C)

$$\text{Given, } 2 \sin \left(\frac{\pi}{8} \right) \sin \left(\frac{2\pi}{8} \right) \sin \left(\frac{3\pi}{8} \right) \sin \left(\frac{5\pi}{8} \right) \sin \left(\frac{6\pi}{8} \right) \sin \left(\frac{7\pi}{8} \right)$$

$$= 2 \sin^2 \frac{\pi}{8} \sin^2 \frac{2\pi}{8} \sin^2 \frac{3\pi}{8} = 2 \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} \cdot \frac{1}{2}$$

$$\begin{aligned}
&= \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} = \sin^2 \frac{\pi}{8} \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \\
&= \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} = \frac{1}{4} \sin^2 \left(\frac{\pi}{4} \right) = \frac{1}{8}
\end{aligned}$$

3. (B)

$$\begin{aligned}
&3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4\sin^6 \theta \\
&= 3(1 - 2\sin \theta \cos \theta)^2 + 6(1 + 2\sin \theta \cos \theta) + 4\sin^6 \theta \\
&= 9 + 12\sin^2 \theta \cos^2 \theta + 4\sin^6 \theta \\
&= 9 + 12\cos^2 \theta (1 - \cos^2 \theta) + 4(1 - \cos^2 \theta)^3 \\
&= 9 + 12\cos^2 \theta - 12\cos^4 \theta + 4(1 - \cos^6 \theta - 3\cos^2 \theta + 3\cos^4 \theta) \\
&= 9 + 4 - 4\cos^6 \theta \\
&= 13 - 4\cos^6 \theta
\end{aligned}$$

4. (B)

$$\text{Given } 2\cos \theta + \sin \theta = 1$$

$$\text{Squaring both sides, we get } (2\cos \theta + \sin \theta)^2 = 1^2$$

$$\begin{aligned}
&\Rightarrow 4\cos^2 \theta + \sin^2 \theta + 4\sin \theta \cos \theta = 1 \\
&\Rightarrow 3\cos^2 \theta + (\cos^2 \theta + \sin^2 \theta) + 4\sin \theta \cos \theta = 1 \\
&\Rightarrow 3\cos^2 \theta + 1 + 4\sin \theta \cos \theta = 1 \\
&\Rightarrow 3\cos^2 \theta + 4\sin \theta \cos \theta = 0 \\
&\Rightarrow \cos \theta (3\cos \theta + 4\sin \theta) = 0 \\
&\Rightarrow 3\cos \theta + 4\sin \theta = 0 \\
&\Rightarrow 3\cos \theta = -4\sin \theta \\
&\Rightarrow \frac{-3}{4} = \tan \theta = -\sqrt{\sec^2 \theta - 1} = \frac{-3}{4} \quad \left(\because \tan \theta = \sqrt{\sec^2 \theta - 1} \right) \\
&\Rightarrow \sec^2 \theta - 1 = \left(\frac{-3}{4} \right)^2 = \frac{9}{16} \\
&\Rightarrow \sec^2 \theta = \frac{9}{16} + 1 = \frac{25}{16} \Rightarrow \sec \theta = \frac{5}{4} \\
&\text{or } \cos \theta = \frac{4}{5} \quad \dots \text{(i)}
\end{aligned}$$

$$\text{Now, } \sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta + \left(\frac{4}{5} \right)^2 = 1$$

$$\sin^2 \theta + \left(\frac{4}{5} \right)^2 = 1 \Rightarrow \sin^2 \theta + \left(\frac{4}{5} \right) = 1$$

$$\sin \theta = \pm \frac{3}{5} \quad \dots \text{(ii)}$$

Taking $\left(\sin \theta = -\frac{3}{5}\right)$ because $\left(\sin \theta = \frac{3}{5}\right)$ cannot satisfy the given equation,

$$\text{Therefore; } 7 \cos \theta + 6 \sin \theta = 7 \times \frac{4}{5} - 6 \times \frac{3}{5} = \frac{28}{5} - \frac{18}{5} = 2$$

5. (B)

Given : $\sin \theta = \frac{1}{2}$ and θ is acute angle

$$\therefore \theta = \frac{\pi}{6}$$

Also given, $\cos \phi = \frac{1}{3}$ and ϕ is acute angle.

$$\therefore 0 < \frac{1}{3} < \frac{1}{2}$$

$$\Rightarrow \cos \frac{\pi}{2} < \cos \phi < \cos \frac{\pi}{3} \text{ or } \frac{\pi}{3} < \phi < \frac{\pi}{2}$$

$$\therefore \frac{\pi}{3} + \frac{\pi}{6} < \theta < \phi < \frac{\pi}{2} + \frac{\pi}{6} \text{ or } \frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$$

$$\Rightarrow \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$$

6. (B)

$$\text{Since, } \sin\left(\frac{\pi}{22}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{22}\right) = \cos \frac{5\pi}{11} = -\cos \frac{16\pi}{11}$$

$$\sin \frac{3\pi}{22} = \cos \frac{4\pi}{11}, \sin \frac{5\pi}{22} = \cos \frac{3\pi}{11} = -\cos \frac{8\pi}{11}$$

$$\sin \frac{7\pi}{22} = \cos \frac{2\pi}{11}, \sin \frac{9\pi}{22} = \cos \frac{\pi}{11}$$

$$\text{Now, } 2 \sin \frac{\pi}{22} \cdot \sin \frac{3\pi}{22} \cdot \sin \frac{5\pi}{22} \cdot \sin \frac{7\pi}{22} \cdot \sin \frac{9\pi}{22}$$

$$= 2 \cos \frac{\pi}{11} \cdot \cos \frac{2\pi}{11} \cdot \cos \frac{4\pi}{11} \cdot \cos \frac{8\pi}{11} \cos \frac{16\pi}{11}$$

$$= \frac{2 \times \sin 2^5 \frac{\pi}{11}}{2^5 \sin \frac{\pi}{11}} = \frac{1}{16} \quad \left(\because \sin 2^5 \frac{\pi}{11} = \sin \frac{\pi}{11} \right)$$

7. (A)

We are given that

$$\cot \alpha = 1, \sec \beta = \frac{-5}{3},$$

$$\Rightarrow \cos \beta = -\frac{3}{5}, \tan \beta = -\frac{4}{3}, \tan \alpha = 1$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{1 - \frac{4}{3}}{\frac{4}{3} \times 1} = \frac{-1}{7}$$

$$\therefore \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{-1}{7} \text{ lies in IVth quadrant}$$

8. (B)

Given trigonometric equation is

$$\begin{aligned} & \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right) \\ &= \left(\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right)\right) \\ &= \cos\left(\frac{4\pi}{7}\right) \left[2\cos\left(\frac{2\pi}{7}\right) + 1\right] \\ &= \cos\left(\frac{4\pi}{7}\right) \left[2\left(1 - 2\sin^2\left(\frac{\pi}{7}\right)\right) + 1\right] \\ &= \cos\left(\frac{4\pi}{7}\right) \left[3 - 4\sin^2\left(\frac{\pi}{7}\right)\right] = \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin \frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right) \end{aligned}$$

Multiply both sides by 2

$$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin \frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

Apply $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin \frac{\pi}{7}}, \text{ Here, } \sin(\pi) = 0.$$

$$= \frac{-\sin \frac{\pi}{7}}{2\sin \frac{\pi}{7}} = -\frac{1}{2}$$

9. (B)

$$16 \sin 20^\circ \sin 40^\circ \sin 80^\circ$$

$$= 16 \sin 40^\circ \sin 20^\circ \sin 80^\circ$$

$$= 4(4 \sin(60 - 20) \sin(20) \sin(60 + 20))$$

$$= 4 \times \sin(3 \times 20^\circ) \quad \left[\because \sin 3\theta = 4 \sin(60 - \theta) \times \sin \theta \times \sin(60 + \theta) \right]$$

$$\begin{aligned}
&= 4 \times \sin 60^\circ \\
&= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}
\end{aligned}$$

10. (D)

Given trigonometric equation is

$$\begin{aligned}
&\sin 12^\circ + \sin 12^\circ - \sin 72^\circ \\
&= \sin 12^\circ - 2 \cos 42^\circ \sin 30^\circ \\
&= \sin 12^\circ - \sin(90^\circ - 48^\circ) = \sin 12^\circ - \sin 48^\circ \\
&= -2 \cos 30^\circ \sin 18^\circ = -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4} = \frac{\sqrt{3}}{4}(1-\sqrt{5})
\end{aligned}$$

11. (D)

$$L + M = 1 - 2 \sin^2 \frac{\pi}{8} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \dots(i)$$

$$\text{and } L - M = -\cos \frac{\pi}{8} \quad \dots(ii)$$

From equations (i) and (ii),

$$L = \frac{1}{2} \left(\frac{1}{\sqrt{2}} - \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} - \frac{1}{2} \cos \frac{\pi}{8} \text{ and } M = \frac{1}{2} \left(\frac{1}{\sqrt{2}} + \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

12. (B)

$$\begin{aligned}
&\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ \\
&= \left(\frac{1 + \cos 20^\circ}{2} \right) + \left(\frac{1 + \cos 100^\circ}{2} \right) - \frac{1}{2} (2 \cos 10^\circ \cos 50^\circ) \\
&= 1 + \frac{1}{2} (\cos 20^\circ + \cos 100^\circ) - \frac{1}{2} [\cos 60^\circ + \cos 40^\circ] \\
&= \left(1 - \frac{1}{4} \right) + \frac{1}{2} [\cos 20^\circ + \cos 100^\circ - \cos 40^\circ] \\
&= \frac{3}{4} + \frac{1}{2} [2 \cos 60^\circ \times \cos 40^\circ - \cos 40^\circ] = \frac{3}{4}
\end{aligned}$$

13. (A)

$$\begin{aligned}
f_k(x) &= \frac{1}{k} (\sin^k x + \cos^k x); \quad f_4(x) = \frac{1}{4} [\sin^4 x + \cos^4 x] \\
&= \frac{1}{4} \left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2} \right] = \frac{1}{4} \left[1 - \frac{(\sin 2x)^2}{2} \right] \\
f_6(x) &= \frac{1}{6} [\sin^6 x + \cos^6 x] \\
&= \frac{1}{6} \left[(\sin^2 x + \cos^2 x)^3 - \frac{3}{4} (\sin 2x)^2 \right] = \frac{1}{6} \left[1 - \frac{3}{4} (\sin 2x)^2 \right]
\end{aligned}$$

$$\text{Now } f_4(x) - f_{(6)}(x) = \frac{1}{4} - \frac{1}{6} - \frac{(\sin 2x)^2}{8} + \frac{1}{8}(\sin 2x)^2 = \frac{1}{12}$$

14. (A)

We have

$$5 \tan^2 x - 5 \cos^2 x = 2(2 \cos^2 x - 1) + 9$$

$$\Rightarrow 5 \tan^2 x - 9 \cos^2 x = -2 + 9$$

$$\Rightarrow 5 \tan^2 x = 9 \cos^2 x + 7$$

$$\Rightarrow 5(\sec^2 x - 1) = 9 \cos^2 x + 7$$

$$\text{Let } \cos^2 x = t \Rightarrow \frac{5}{t} - 9t - 12 = 0$$

$$\Rightarrow 9t^2 + 12t - 5 = 0 \Rightarrow 9t^2 + 15t - 3t - 5 = 0$$

$$\Rightarrow (3t - 1)(3t + 5) = 0 \Rightarrow t = \frac{1}{3} \text{ as } t \neq -\frac{5}{3}.$$

$$\cos 2x = 2 \cos^2 x - 1 = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$$

$$\cos 4x = 2 \cos^2 2x - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$$

15. (A)

$$4 + \frac{1}{2} \sin^2 2x - 2 \cos^4 x$$

$$\begin{aligned} 4 + 2(1 - \cos^2 x) \cos^2 x - 2 \cos^4 x &= -4 \left\{ \cos^4 x - \frac{\cos^2 x}{2} - 1 + \frac{1}{16} - \frac{1}{16} \right\} \\ &= -4 \left\{ \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \right\}; \quad 0 \leq \cos^2 x \leq 1 \end{aligned}$$

$$\Rightarrow -\frac{1}{4} \leq \cos^2 x - \frac{1}{4} \leq \frac{3}{4} \Rightarrow 0 \leq \left(\cos^2 x - \frac{1}{4} \right)^2 \leq \frac{9}{16}$$

$$-\frac{17}{16} \leq \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \leq \frac{9}{16} - \frac{17}{16}$$

$$\frac{17}{4} \geq -4 \left\{ \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \right\} \geq \frac{4}{2}$$

$$M = \frac{17}{4} \Rightarrow m = 2; \quad M - m = \frac{17}{4} - 2 = \frac{9}{4}$$

16. (B)

$$\text{Let } \cos \alpha + \cos \beta = \frac{3}{2}$$

$$\Rightarrow 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = \frac{3}{2} \quad \dots(i)$$

$$\text{and } \sin \alpha + \sin \beta = \frac{1}{2}$$

$$\Rightarrow 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = \frac{1}{2} \quad \dots (ii)$$

$$\text{On dividing (ii) by (i), we get } \tan \left(\frac{\alpha + \beta}{2} \right) = \frac{1}{3}$$

$$\text{Given: } 0 = \frac{\alpha + \beta}{2} \Rightarrow 2\theta = \alpha + \beta$$

$$\text{Consider } \sin 2\theta + \cos 2\theta = \sin(\alpha + \beta) + \cos(\alpha + \beta)$$

$$= \frac{\frac{2}{3}}{1 + \frac{1}{9}} + \frac{1 - \frac{1}{9}}{1 + \frac{1}{9}} = \frac{6}{10} + \frac{8}{10} = \frac{7}{5}$$

17. (9)

$$\text{Let } f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$$

$$\Rightarrow f'(x) = 0 \Rightarrow \sin x = \frac{2}{3} \quad \left(\because x \in \left(0, \frac{\pi}{2} \right) \cos x \neq 0 \right)$$

$$\therefore f(x)_{\min} = \frac{4}{2/3} + \frac{1}{1 - 2/3} = 9; f(x)_{\max} \rightarrow \infty$$

(x) is continuous function $\therefore \alpha_{\min} = 9$

18. (80)

$$\sin 10^\circ \left(\frac{1}{2} \cdot 2 \sin 20^\circ \sin 40^\circ \right) \cdot \sin 10^\circ \sin (60^\circ - 10^\circ) \sin (60^\circ + 10^\circ)$$

$$= \sin 10^\circ \cdot \frac{1}{2} (\cos 20^\circ - \cos 60^\circ) \cdot \frac{1}{4} \sin 30^\circ$$

$$= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \sin 10^\circ \left(\cos 20^\circ - \frac{1}{2} \right)$$

$$= \frac{1}{16} \cdot \frac{1}{2} (2 \sin 10^\circ \cos 20^\circ - \sin 10^\circ)$$

$$= \frac{1}{32} (\sin 30^\circ - \sin 10^\circ - \sin 10^\circ)$$

$$= \frac{1}{32} \left(\frac{1}{2} - 2 \sin 10^\circ \right) = \frac{1}{64} (1 - 4 \sin 10^\circ)$$

$$= \frac{1}{64} - \frac{1}{16} (\sin 10^\circ)$$

$$\text{Hence, } \alpha = \frac{1}{64} \Rightarrow \alpha^{-1} = 64$$

$$\text{Hence, } 16 + \alpha^{-1} = 80$$

19. (1)

$$\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7} \text{ and } \sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$$

$$\Rightarrow \frac{\sqrt{2} \sin \beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$$

$$\therefore \tan \alpha = \frac{1}{7} \text{ and } \sin \beta = \frac{1}{\sqrt{10}}; \tan \beta = \frac{1}{3}$$

$$\therefore \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \cdot \frac{1}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$

EXERCISE - 2 [A]

1. (B)

$$\tan A = \sqrt{2} \quad \tan^2 A = 2$$

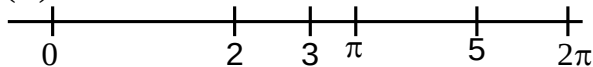
$$\frac{\sin^4 A - 3\sin^2 A \cos^2 A + 7\cos^4 A}{1 + \sin^2 A \cos^2 A + 5\cos^4 A}$$

$$= \frac{\sin^4 A - 3\sin^2 A \cos^2 A + 7\cos^4 A}{(\sin^2 A + \cos^2 A)^2 + \sin^2 A \cos^2 A + 5\cos^4 A}$$

$$= \frac{\tan^4 A - 3\tan^2 A + 7}{(\tan^2 A + 1)^2 + \tan^2 A + 5} \quad (\text{divide N \& D by } \cos^4 x)$$

$$= \frac{4 - 6 + 7}{9 + 2 + 5} = \frac{5}{16}$$

2. (A)



$$\sin 2 > 0$$

$$\sin 3 > 0$$

$$\sin 5 < 0$$

$$\sin 2 \sin 3 \sin 5 < 0$$

3. (A)

$$f(x) = \sec x - \tan x$$

$$g(x) = \sec x + \tan x$$

$$f(x) \cdot g(x) = 1$$

$$f(A) \cdot f(B) \cdot f(C) = g(A) \cdot g(B) \cdot g(C)$$

$$f^2(A) \cdot f^2(B) \cdot f^2(C) = f(A)g(A)f(B)g(B)f(C)g(C)$$

$$f(A)f(B)f(C) = \pm 1$$

4. (B)

$$\cos(A) = \cos B \cos C$$

$$\cos(\pi - (B + C)) = \cos B \cos C$$

$$-[\cos B \cos C - \sin B \sin C] = \cos B \cos C$$

$$\sin B \sin C = 2 \cos B \cos C$$

$$\tan B \tan C = 2$$

5. (D)

$$\sum_{r=0}^{10} \cos^3\left(\frac{\pi r}{3}\right)$$

$$1 + \left(\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^3 + (-1)^3 + \left(-\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 + (1)^3 + \left(\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^3 + (-1)^3 + \left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^3$$

$$= -\frac{1}{8}$$

6. (B)

$$U_n = 2 \cos n\theta$$

$$U_1 U_n - U_{n-1}$$

$$= 2(2 \cos n\theta \cdot \cos \theta) - 2 \cos(n-1)\theta$$

$$= 2(\cos(n+1)\theta + \cos(n-1)\theta) - 2 \cos(n-1)\theta$$

$$= 2 \cos(n+1)\theta$$

$$= U_{n+1}$$

7. (C)

$$\cos 5\theta = a \cos^5 \theta + b \cos^3 \theta + c \cos \theta$$

$$\theta = 0$$

$$1 = a + b + c$$

$$\theta = \frac{\pi}{3}$$

$$\frac{1}{2} = \frac{a}{32} + \frac{b}{8} + \frac{c}{2}$$

$$\theta = \frac{\pi}{4}$$

$$\frac{-1}{\sqrt{2}} = \frac{a}{4\sqrt{2}} + \frac{b}{2\sqrt{2}} + \frac{c}{\sqrt{2}}$$

Solve: $a = 16, b = -20, c = 5$

8. (D)

$$f(x) = \frac{\sin 3x}{\sin x} \quad \sin x \neq 0$$

$$3 - 4 \sin^2 x$$

$$\text{Range in } [-1, 3)$$

9. (C)

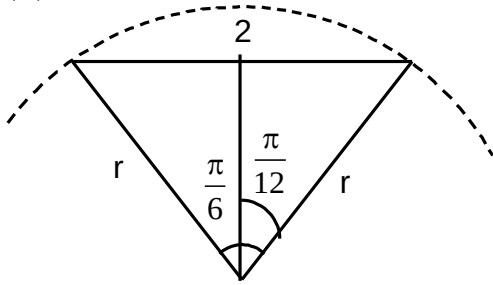
$$\cos(\theta + \phi) = m(\cos(\theta - \phi))$$

$$\frac{\cos \theta \cos \phi - \sin \theta \sin \phi}{\cos \theta \cos \phi + \sin \theta \sin \phi} = \frac{m}{1}$$

$$\frac{\sin \theta \sin \phi}{\cos \theta \cos \phi} = \frac{1 - m}{1 + m}$$

$$\tan \theta = \left(\frac{1-m}{1+m} \right) \cot \phi$$

10. (B)



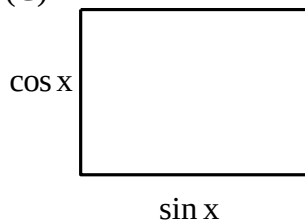
$$\sin \frac{\pi}{12} = \frac{1}{r}$$

$$r = \frac{1}{\sin \frac{\pi}{12}} = \sqrt{6} + \sqrt{2}$$

11. (B)

$$\begin{aligned} \frac{1 - 2\sin^2 \frac{\alpha}{2}}{1 + \sin \alpha} &= \frac{\cos \alpha}{1 + \sin \alpha} \\ &= \frac{\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}}{\left(\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} \right)^2} \\ &= \frac{\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}} \\ &= \frac{1-m}{1+m} \end{aligned}$$

12. (C)



$$A = \sin x \cos x$$

$$A = \frac{1}{2} \sin 2x$$

$$A \leq \frac{1}{2}$$

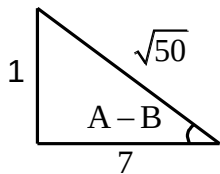
13. (D)

$$\begin{aligned} &\sin^8 75^\circ - \cos^8 75^\circ \\ &= (\sin^4 75^\circ + \cos^4 75^\circ)(\sin^2 75^\circ - \cos^2 75^\circ)(1) \end{aligned}$$

$$\begin{aligned}
 &= \left(1 - \frac{1}{2} \sin^2 150^\circ\right) (-\cos 150^\circ) \\
 &= \left(1 - \frac{1}{8}\right) \left(+\frac{\sqrt{3}}{2}\right) \\
 &= \frac{7\sqrt{3}}{16}
 \end{aligned}$$

14. (D)
 $\tan A = 3$
 $\tan B = 2$

$$\tan(A - B) = \frac{3-2}{1+6} = \frac{1}{7}$$

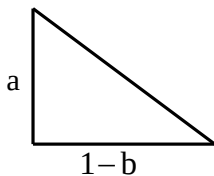


$$\sin(A - B) = \frac{1}{\sqrt{50}}$$

$$\begin{aligned}
 \sin 2(A - B) &= 2 \times 1 \times \frac{1}{\sqrt{50}} \times \frac{7}{\sqrt{50}} \\
 &= \frac{7}{25}
 \end{aligned}$$

15. (A)
 $\tan A + \tan B = a$
 $\tan A \cdot \tan B = b$

$$\tan(A + B) = \frac{a}{1-b}$$



$$\sin(A + B) = \frac{a}{\sqrt{a^2 + (1-b)^2}}$$

$$\sin^2(A + B) = \frac{a^2}{a^2 + (1-b)^2}$$

16. (A)
 $\tan B = \frac{n \sin A \cos A}{1 - n \cos^2 A}$
 $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\begin{aligned}
 &= \frac{\frac{\sin A}{\cos A} + \frac{n \sin A \cos A}{1 - n \cos^2 A}}{1 - \frac{\sin A}{\cos A} \cdot \frac{n \sin A \cos A}{1 - n \cos^2 A}}
 \end{aligned}$$

$$\frac{\sin A - n \cos^2 A \sin A + n \cos^2 A \sin A}{\cos A (1 - n \cos^2 A)} = \frac{\sin A}{\cos A (1 - n)}$$

$$= \frac{1 - n \cos^2 A - n \sin^2 A}{1 - n \cos^2 A}$$

17. (B)

$$\tan \beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$$

$$\tan \alpha = \frac{1}{7}$$

$$\tan(\alpha + 2\beta) = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{3}{28}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$

$$\alpha + 2\beta = \frac{\pi}{4}$$

18. (C)

$$\sin A = \sin B$$

$$\cos A = \cos B$$

$$\Rightarrow A = B + 2n\pi$$

$$\sin\left(\frac{A-B}{2}\right) = 0$$

19. (C)

$$1 - \frac{1}{\sin \theta + \cos \theta} (\sin^3 \theta + \cos^3 \theta)$$

$$= 1 - \frac{1}{\sin \theta + \cos \theta} (\sin \theta + \cos \theta) (1 - \sin \theta \cos \theta)$$

$$= \sin \theta \cos \theta$$

20. (B)

$$\frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{a}{1}$$

$$\cot x \cot y = \frac{1+a}{a-1}$$

21. (D)

$$2 \sin A = \sqrt{3} \sin B \quad \text{_____ (1)}$$

$$2 \cos A = \sqrt{5} \cos B \quad \text{_____ (2)}$$

Square and add (1) & (2)

$$3 \sin^2 B + 5 \cos^2 B = 4$$

$$3 \tan^2 B + 5 = 4 \sec^2 B \text{ or } \tan B = 1$$

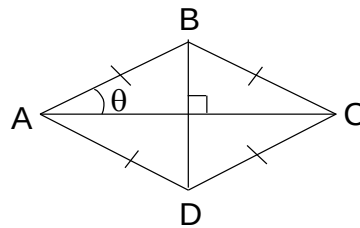
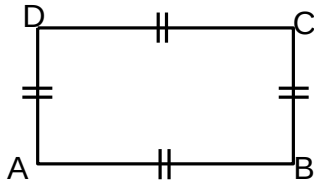
$$\tan A = \frac{\sqrt{3}}{\sqrt{5}}$$

$$\tan A + \tan B = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5}}$$

22. (B)

$$\begin{aligned} P_n - P_{n-2} &= \cos^n \theta + \sin^n \theta - \cos^{n-2} \theta - \sin^{n-2} \theta \\ &= \cos^{n-2} \theta (-\sin^2 \theta) + \sin^{n-2} \theta (-\cos^2 \theta) \\ &= -\sin^2 \theta \cos^2 \theta (\cos^{n-4} \theta + \sin^{n-4} \theta) \\ &= k P_{n-4} \\ K &= -\sin^2 \theta \cos^2 \theta \end{aligned}$$

23. (C)
Given



$$AB^2 = AC \times BD$$

$$AC^2 + BD^2 = 4AB^2 \text{ \& } \tan \theta = \frac{BD}{AC}$$

$$\Rightarrow AC^2 + BD^2 = 4AC BD$$

$$\Rightarrow \left(\frac{BD}{AC}\right)^2 + 1 = 4\left(\frac{BD}{AC}\right)$$

$$\tan^2 \theta + 1 = 4 \tan \theta$$

$$\sin 2\theta = \frac{1}{2} \Rightarrow \theta = 15^\circ$$

$$\text{Ans. } 2\theta = 30^\circ$$

24. (A)

$$\sin\left(\frac{A}{2} + \left(\frac{A}{2} + \frac{C}{2}\right)\right) = K \sin\left(\frac{\pi - (A+B)}{2}\right)$$

$$\sin\left(\frac{A}{2} + \frac{\pi - B}{2}\right) = K \cos\left(\frac{A+B}{2}\right)$$

$$\cos\left(\frac{A-B}{2}\right) = K \cos\left(\frac{A+B}{2}\right)$$

$$\frac{\cos \frac{A}{2} \cos \frac{B}{2} + \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} \cos \frac{B}{2} - \sin \frac{A}{2} \sin \frac{B}{2}} = \frac{K}{1}$$

$$\frac{K-1}{K+1} = \tan \frac{A}{2} \tan \frac{B}{2}$$

25. (A)

$$\frac{3 + \cot 76^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ}$$

$$\cot 76^\circ + \cot 16^\circ = \frac{\sin 92^\circ}{\sin 76^\circ \sin 16^\circ} = \frac{\sin 88^\circ}{\sin 76^\circ \sin 16^\circ}$$

$$1 + \cot 76^\circ \cot 16^\circ = \frac{1}{2 \sin 76^\circ \sin 16^\circ}$$

$$\text{LHS} = \frac{3 + \cot 76^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ} = \frac{2 + \frac{1}{2 \sin 76^\circ \sin 16^\circ}}{\frac{\sin 88^\circ}{\sin 76^\circ \sin 16^\circ}} = \frac{2(\cos 60^\circ - \cos 92^\circ) + 1}{2 \sin 88^\circ}$$

$$= \frac{2 \sin^2 46^\circ}{2 \sin 44^\circ \cos 44^\circ} = \cot 44^\circ = \tan 46^\circ$$

26. (A)

$$\frac{2(\sin 1^\circ + \sin 2^\circ - \dots - \sin 89^\circ)}{2(\cos 1^\circ + \cos 2^\circ - \dots - \cos 44^\circ) + 1}$$

$$N^r = 2(\sin 1^\circ + \sin 2^\circ - \dots - \sin 44^\circ) = \frac{2 \sin\left(\frac{89}{2}\right) \sin 45^\circ}{\sin\left(\frac{1}{2}\right)}$$

$$D^r = 2(\cos 1^\circ + \cos 2^\circ - \dots - \cos 44^\circ) = \frac{2 \sin 22^\circ \cos \frac{45}{2}}{\sin \frac{1}{2}} + 1$$

$$= \frac{-\sin \frac{1}{2} + \sin \frac{89}{2}}{\sin \frac{1}{2}} + 1$$

$$\frac{N^r}{D^r} = \frac{\frac{2 \sin \frac{89}{2} \sin 45^\circ}{\sin \frac{1}{2}}}{\frac{-\sin \frac{1}{2} + \sin \frac{89}{2}}{\sin \frac{1}{2}} + 1} = \sqrt{2}$$

27. (A)

$$x \in \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right)$$

$$\sin x < -\frac{1}{2}$$

$$\cos x > -\frac{1}{2}$$

$$\sin x - \cos x < 0$$

$$\sin x - \cos x = t$$

$$1 - \sin 2x = t^2$$

$$t^2 = 1 - \frac{2024}{2025}$$

$$t^2 = \frac{1}{2025}$$

$$t = \frac{-1}{45}$$

28. (D)

$$\begin{aligned} & (\tan 4\theta + \tan 2\theta)(1 - \tan^2 3\theta \tan^2 \theta) \\ &= \frac{\sin 6\theta}{\cos 4\theta \cos 2\theta} (1 - \tan 3\theta \tan \theta)(1 + \tan 3\theta \tan \theta) \\ &= \frac{\sin 6\theta}{\cos^2 3\theta \cos^2 \theta} \\ &= 2 \tan 3\theta \sec^2 \theta \end{aligned}$$

29. (D)

$$\begin{aligned} & \frac{1}{\tan x} + \frac{1 - \sqrt{3} \tan x}{\sqrt{3} + \tan x} + \frac{1 + \sqrt{3} \tan x}{\tan x - \sqrt{3}} \\ &= \frac{3}{\tan 3x} = \frac{3(1 - 3 \tan^2 x)}{3 \tan x - \tan^3 x} \end{aligned}$$

30. (B)

$$\begin{aligned} & \frac{a \cos \alpha - b \sin \alpha}{\sin \alpha \cos \alpha} \\ &= 2\sqrt{a^2 + b^2} \frac{(\sin \theta \cos \alpha - \cos \theta \sin \alpha)}{\sin 2\alpha} \\ &= 2\sqrt{a^2 + b^2} \frac{\sin(\theta - \alpha)}{\sin 2\alpha} \quad (\theta = 3\alpha) \\ &= 2\sqrt{a^2 + b^2} \end{aligned}$$

31. (D)

$$\begin{aligned} 2 \sin \frac{A}{2} &= \sin \frac{A}{2} + \cos \frac{A}{2} - \left(\cos \frac{A}{2} - \sin \frac{A}{2} \right) \\ &= -\sqrt{1 + \sin A} + \sqrt{1 - \sin A} \end{aligned}$$

32. (A)

$$\begin{aligned} p^2 &= 1 - \sin 40^\circ \\ \sin 40^\circ &= 1 - p^2 \quad (p < 0) \\ \cos 40^\circ &= \sqrt{1 - (1 - p^2)^2} \\ &= -p\sqrt{2 - p^2} \end{aligned}$$

33. (C)

$$\begin{aligned} & \tan \frac{\alpha}{2} + \cot \frac{\alpha}{2} \\ &= \frac{2}{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} \\ &= \frac{2}{p} \\ & x^2 - \frac{2}{p}x + 1 = 0 \\ & px^2 - 2x + p = 0 \end{aligned}$$

34. (B)

$$\begin{aligned} & \frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3} \sin 250^\circ} \\ & \frac{1}{\sin 20^\circ} - \frac{1}{\sqrt{3} \cos 20^\circ} \\ & \frac{2(\sqrt{3} \cos 20^\circ - \sin 20^\circ)}{\sqrt{3} 2 \sin 20^\circ \cos 20^\circ} \\ &= \frac{4}{\sqrt{3}} \frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} = \frac{4}{\sqrt{3}} \end{aligned}$$

35. (B)

$$\begin{aligned} & \sin x + \sin y = a \\ & \cos x + \cos y = b \\ &= 2 \\ & a = 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) \\ & b = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) \\ & \frac{a}{b} + \frac{b}{a} = \frac{1}{\sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x+y}{2} \right)} \\ & \frac{a^2 + b^2}{ab} = \frac{2}{\sin(x+y)} \\ & \sin(x+y) = \frac{2ab}{a^2 + b^2} \end{aligned}$$

36. (C)

$$\begin{aligned} & \sqrt{2} \sin \frac{3\pi}{20} + \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin \frac{\pi}{10} - \frac{1}{\sqrt{2}} \cos \frac{\pi}{10} \right) \\ &= \sqrt{2} \sin \frac{3\pi}{20} - \sqrt{2} \left(\sin \left(\frac{\pi}{4} - \frac{\pi}{10} \right) \right) \\ &= 0 \end{aligned}$$

37. (A)

$$\text{If } A + B = \frac{\pi}{4}$$

$$\tan(A + B) = 1$$

$$\tan A + \tan B + \tan A \tan B + 1 = 2$$

$$(1 + \tan A)(1 + \tan B) = 2$$

$$A = 11^\circ \quad B = 34^\circ$$

$$(1 + \tan A)(1 + \tan 34^\circ) = 2$$

$$A = 17^\circ \quad B = 28^\circ$$

$$(1 + \tan 17^\circ)(1 + \tan 28^\circ) = 2$$

$$\text{Hence } \frac{(1 + \tan 11^\circ)(1 + \tan 34^\circ)}{(1 + \tan 17^\circ)(1 + \tan 28^\circ)} = 1$$

38. (C)

$$\begin{aligned} & \frac{4 \sin 9^\circ \sin 51^\circ \sin 39^\circ \sin 51^\circ \sin 69^\circ \sin 81^\circ}{\sin 54^\circ} \\ &= \frac{4(\sin 9^\circ \sin 51^\circ \sin 69^\circ)(\sin 21^\circ \sin 39^\circ \sin 81^\circ)}{\sin 54^\circ} \\ &= \frac{4\left(\frac{\sin 27^\circ}{4}\right)\left(\frac{\sin 63^\circ}{4}\right)}{\sin 54^\circ} \\ &= \frac{1}{4} \frac{\sin 27^\circ \cos 27^\circ}{\sin 54^\circ} = \frac{1}{8} \end{aligned}$$

39. (C)

$$\begin{aligned} & \left(\sin \frac{\pi}{10} + \sin \frac{13\pi}{10} \right) \cdot \left(\cos^2 \frac{\pi}{6} - \cos^2 \frac{\pi}{10} \right) \\ &= \sin \frac{13\pi}{10} = \sin \left(\pi + \frac{3\pi}{10} \right) = -\sin \frac{3\pi}{10} \\ &\therefore (\sin 18^\circ - \sin 54^\circ) \cdot \left(\frac{3}{4} - \cos^2 \frac{\pi}{10} \right) \\ &= \left(\frac{\sqrt{5}-1}{4} \right) - \left(\frac{\sqrt{5}+1}{4} \right) \cdot \frac{3\cos \frac{\pi}{10} - 4\cos^3 \frac{\pi}{10}}{4\cos \frac{\pi}{10}} \\ &= -\frac{1}{2} \cdot \frac{-\cos \frac{3\pi}{10}}{4\cos \frac{\pi}{10}} = \frac{\cos 54^\circ}{8\cos 18^\circ} = \frac{\sin 36^\circ}{8\cos 18^\circ} \\ &= \frac{1}{4} \sin 18^\circ = \frac{\sqrt{5}-1}{16} \end{aligned}$$

40. (A)

$$\sin^2 \theta = \frac{1}{2} \left(\frac{x}{y} + \frac{y}{x} \right)$$

Possible only when $x = y \left(\frac{x}{y} + \frac{y}{x} \in (-\infty, -2] \cup [2, \infty) \right)$

41. (B)

$$\cos \frac{6\pi}{7} = -\cos \frac{\pi}{7}$$

$$\cos \frac{5\pi}{7} = -\cos \frac{2\pi}{7}$$

$$\cos \frac{3\pi}{7} = -\cos \frac{4\pi}{7}$$

$$\left(\cos \frac{\pi}{7} + \cos \frac{6\pi}{7} \right) + \left(\cos \frac{2\pi}{7} + \cos \frac{5\pi}{7} \right) + \left(\cos \frac{3\pi}{7} + \cos \frac{4\pi}{7} \right) + \cos \pi = -1$$

42. (D)

$$\cot(\alpha + \beta) = 0$$

$$\alpha + \beta = \frac{\pi}{2}$$

$$\sin(\alpha + 2\beta) = \sin\left(\frac{\pi}{2} + \beta\right) = \cos \beta$$

43. (C)

$$\sin x = 1 - \sin^2 x$$

$$\sin x = \cos^2 x$$

$$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x - 2$$

$$= (\sin x + \sin^2 x)^3 - 2$$

$$1 - 2 = -1$$

44. (D)

$$\frac{1}{4} [\sqrt{3} \cos 23^\circ - \sin 23^\circ]$$

$$\frac{1}{2} [\sin 60^\circ \cos 23^\circ - \cos 60^\circ \sin 23^\circ]$$

$$\frac{1}{2} \sin 37^\circ$$

45. (B)

$$\tan 60^\circ = \sqrt{3}$$

$$\tan(20^\circ + 40^\circ) = \sqrt{3}$$

$$\tan 20^\circ + \tan 40^\circ = \sqrt{3} - \sqrt{3} \tan 20^\circ \tan 40^\circ$$

$$\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ = \sqrt{3}$$

46. (A)
 $\tan(\alpha + \beta) = 1$
 $\tan \alpha + \tan \beta = 1 - \tan \alpha \tan \beta$
 $(1 + \tan \alpha)(1 + \tan \beta) = 2$
 $f(\theta) = \frac{1}{1 + \tan \theta}$
 $f(\alpha)f(\beta) = \frac{1}{(1 + \tan \alpha)(1 + \tan \beta)} = \frac{1}{2}$
47. (A)
 If $A + B + C = \pi$ $\tan A + \tan B + \tan C = \tan A + \tan B + \tan C$
 $\tan A \tan B \tan C = 6$ $\tan A + \tan B = 3$
 $\tan A \tan B = 2$ $\tan A = 1$
 $\Rightarrow \tan C = 3$ $\tan B = 2$

48. (D)

$$\frac{(\sin \alpha + \cos \alpha)^2}{\cos 2\alpha \left(\frac{\tan \alpha + 1}{\tan \alpha - 1} \right)} - \frac{1}{4} \sin 2\alpha \left[\cot \frac{\alpha}{2} - \tan \frac{\alpha}{2} \right]$$

$$\frac{(\sin \alpha + \cos \alpha)^2 (\sin \alpha - \cos \alpha)}{(\cos \alpha - \sin \alpha)(\cos \alpha + \sin \alpha)(\sin \alpha + \cos \alpha)} - \frac{2}{4} \sin \alpha \cos \alpha \frac{\cos \alpha}{\left(\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \right)}$$

$$1 - \cos^2 \alpha = \sin^2 \alpha$$

49. (C)
 $x = \sqrt{2 + \sqrt{2 - \sqrt{2 + x}}}$
 $x = 2 \cos \theta$

$$2 \cos \theta = \sqrt{2 + \sqrt{2 - 2 \cos \frac{\theta}{2}}}$$

$$= \sqrt{2 + \sqrt{2 \left(1 - \cos \left(\frac{\theta}{2} \right) \right)}}$$

$$= \sqrt{2 + 2 \sin \frac{\theta}{4}}$$

$$= \sqrt{2 \left(1 + \cos \left(\frac{\pi}{2} - \frac{\theta}{4} \right) \right)}$$

$$2 \cos \theta = 2 \cos \left(\frac{\pi}{4} - \frac{\theta}{8} \right)$$

$$\theta = \frac{\pi}{4} - \frac{\theta}{8}$$

$$\frac{9\theta}{8} = 45^\circ$$

$$\theta = 40^\circ$$
 Hence $x = 2 \cos 40^\circ$

50. (B)

$$\frac{1}{\sin 1^\circ} \left(\frac{\sin(46^\circ - 45^\circ)}{\sin 46 \sin 45} + \frac{\sin(48 - 47)}{\sin 48 \sin 47} - \dots - \right)$$

$$\frac{1}{\sin 1^\circ} (\cot 45 - \cot 46 + \cot 47 - \cot 48 - \dots -)$$

$$= \frac{1}{\sin 1^\circ} = \operatorname{cosec} 1^\circ$$

51. (A)

$$3 \sin x + 4 \cos x + 12 \sin y + 5 \cos y = 18$$

$$5 \cos \left(x - \tan^{-1} \frac{3}{4} \right) + 13 \cos \left(y - \tan^{-1} \frac{12}{5} \right) = 18$$

Possible $x = \tan^{-1} \frac{3}{4}$

$$y = \tan^{-1} \frac{12}{5}$$

$$\tan(x + y) = \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4} \times \frac{12}{5}} = -\frac{63}{16}$$

52. (D)

$$\sum \tan B \tan C = x$$

$$(1 - \tan B \tan C) = \frac{\cos(B + C)}{\cos B \cos C} = \frac{-\cos A}{\cos B \cos C}$$

$$\tan B \tan C = 1 + \frac{\cos A}{\cos B \cos C}$$

$$\sum \tan B \tan C = 3 + \sum \frac{\cos A}{\cos B \cos C}$$

$$= 3 + \sum \frac{\cos^2 A}{\cos A \cos B \cos C}$$

$$\sum \cos^2 A = \frac{3 + \cos 2A + \cos 2B + \cos 2C}{2}$$

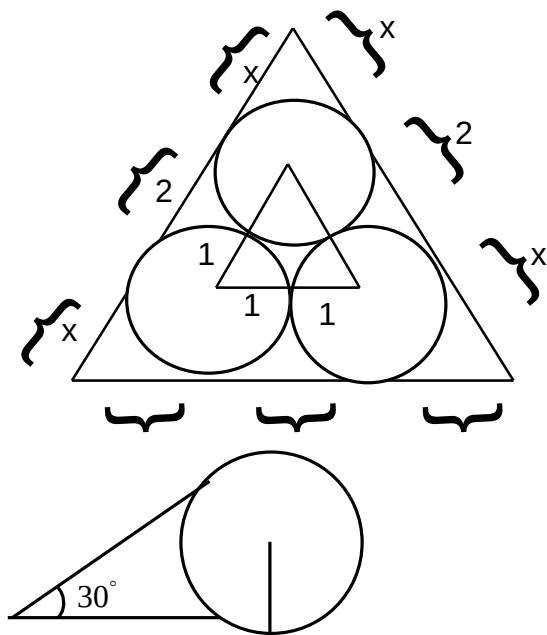
$$\frac{2 - 4\pi \cos A}{2} = 1 - 2\pi(\cos A)$$

$$\sum \tan B \tan C = 3 + \frac{1 - 2\pi \cos A}{\pi \cos A}$$

$$= 1 + \frac{1}{\pi \cos A} = \frac{q + 1}{q}$$

53. (B)

$$\tan 30^\circ = \frac{1}{x} = \frac{1}{\sqrt{3}}$$



$$x = \sqrt{3}$$

Side of equilateral triangle

$$= 2 + 2\sqrt{3}$$

$$\text{Area} = \frac{\sqrt{3}}{4} (2 + 2\sqrt{3})^2$$

$$= 4\sqrt{3} + 6$$

54. (C)

$$\sum_{m=1}^6 \operatorname{cosec} \left(\theta + \frac{(m-1)\pi}{4} \right) \operatorname{cosec} \left(\theta + \frac{m\pi}{4} \right) = 4\sqrt{2}$$

$$\frac{1}{\sin \frac{\pi}{4}} \sum_{m=1}^6 \frac{\sin \frac{\pi}{4}}{\sin \left(\theta + (m-1) \frac{\pi}{4} \right) \sin \left(\theta + \frac{m\pi}{4} \right)} = 4\sqrt{2}$$

$$= \sqrt{2} \sum_{m=1}^6 \frac{\sin \left(\theta + m \frac{\pi}{4} - \left(\theta + (m-1) \frac{\pi}{4} \right) \right)}{\sin \left(\theta + (m-1) \frac{\pi}{4} \right) \sin \left(\theta + \frac{m\pi}{4} \right)}$$

$$\frac{\sin(A-B)}{\sin A \sin B} = \cot B - \cot A$$

$$\sqrt{2} \sum_{m=1}^6 \left(\cot \left(\theta + (m-1) \frac{\pi}{4} \right) - \cot \left(\theta + \frac{m\pi}{4} \right) \right)$$

$$\sqrt{2} \left(\cot \theta - \cot \left(\theta + \frac{3\pi}{2} \right) \right) = 4\sqrt{2}$$

$$\cot \theta + \tan \theta = 4$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12} \quad \text{or} \quad \frac{5\pi}{12}$$

55. (B)

$$3\sin^2 A + 2\sin^2 B = 1$$

$$\Rightarrow 3\sin^2 A = \cos 2B \quad \dots\dots (1)$$

$$\text{Also } 3\sin 2A = 2\sin 2B \quad \dots\dots (2)$$

From (1) & (2)

$$\frac{3\sin^2 A}{3(2\sin A \cos A)} = \frac{\cos 2B}{2\sin 2B}$$

$$\Rightarrow \tan A = \cot 2B$$

$$\tan A = \tan\left(\frac{\pi}{2} - 2B\right)$$

$$A + 2B = \frac{\pi}{2}$$

56. (A)

$$\tan 20^\circ \tan 80^\circ \cot 50^\circ$$

$$= \tan 20 \tan (60 + 20) \tan (60 - 20)$$

$$= \tan (3 \times 20^\circ)$$

$$= \sqrt{3}$$

57. (B)

$$a \cos x + b \sin x = c$$

$$\text{Put } \cos x = \frac{1-t^2}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}$$

$$\text{Where } t = \tan \frac{x}{2}$$

$$a\left(\frac{1-t^2}{1+t^2}\right) + b\left(\frac{2t}{1+t^2}\right) = c$$

$$(a+c)t^2 - 2bt + c - a = 0$$

$$\text{Roots of this equation are } \tan \frac{x_1}{2} \text{ and } \tan \frac{x_2}{2}$$

$$\tan \frac{x_1}{2} + \tan \frac{x_2}{2} = \frac{2b}{a+c}$$

$$\tan \frac{x_1}{2}, \tan \frac{x_2}{2} = \frac{c-a}{a+c}$$

$$\tan\left(\frac{x_1+x_2}{2}\right) = \frac{\frac{2b}{a+c}}{1 - \frac{c-a}{c+a}} = \frac{2b}{2a} = \frac{b}{a}$$

58. (B)

$$\sin x + \cos x = \frac{\sqrt{7}}{2}$$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \frac{\sqrt{7}}{2} \quad \text{Where } t = \tan \frac{x}{2}$$

$$(\sqrt{7}+2)t^2 - 4t + (\sqrt{7}-2) = 0$$

$$t = \frac{4 \pm \sqrt{16 - 4(\sqrt{7}+2)(\sqrt{7}-2)}}{2(\sqrt{7}+2)} = \frac{1}{3}(\sqrt{7}-2) \text{ or } (\sqrt{7}-2)$$

$$\tan \frac{x}{2} < \tan \frac{\pi}{8}$$

$$x \in \left[0, \frac{\pi}{4}\right] \quad \frac{x}{2} \in \left[0, \frac{\pi}{8}\right]$$

So $\tan \frac{x}{2}$ will have lower value

$$\tan \frac{x}{2} = \frac{\sqrt{7}-2}{3}$$

59. (C)

$$\cos x + \cos y - \cos(x+y) = \frac{3}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right) + 1 = \frac{3}{2}$$

$$4\cos^2\left(\frac{x+y}{2}\right) - 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + 1 = 0$$

$$D \geq 0$$

$$16\cos^2\left(\frac{x-y}{2}\right) - 16 \geq 0$$

$$-16\sin^2\left(\frac{x-y}{2}\right) \geq 0$$

Possible only at $x = y$

60. (D)

$$\tan x = n \tan y \quad (\tan y = t)$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{nt - t}{1 + nt^2}$$

$$\tan(x-y) = \frac{t(n-1)}{1 + nt^2} = \alpha$$

$\sec^2(x-y)$ will be max if $\tan(x-y)$ is max

$$\alpha + n\alpha t^2 = (n-1)t$$

$$D \geq 0$$

$$(n-1)^2 - 4n\alpha^2 \geq 0$$

$$\alpha^2 \leq \frac{(n-1)^2}{4n}$$

$$\tan^2(x-y) \leq \frac{(n-1)^2}{4n}$$

$$\sec^2(x-y) \leq \frac{(n-1)^2}{4n} + 1$$

$$\leq \frac{(n+1)^2}{4n}$$

EXERCISE - 2 [B]

1. **(ABC)**

$$a = \cos x + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x + \frac{4\pi}{3}\right)$$

$$= \cos x + 2\cos x \cdot \cos\left(\frac{2\pi}{3}\right)$$

$$= \cos x - \cos x = 0$$

$$b = \sin x + \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x + \frac{4\pi}{3}\right)$$

$$= \sin x + 2\sin x \cdot \cos\left(\frac{2\pi}{3}\right)$$

$$= \sin x - \sin x = 0$$

2. **(BD)**

$$\left. \begin{array}{l} \sin 1 \approx \sin 57^\circ \\ \sin 2 \approx \sin 114^\circ \approx \sin 66^\circ \\ \sin 3 \approx \sin 171^\circ \approx \sin 9^\circ \end{array} \right\} \sin 2 > \sin 1 > \sin 3$$

$$\left. \begin{array}{l} \cos 1 \approx \cos 57^\circ \\ \cos 2 \approx \cos 114^\circ \approx -\cos 66^\circ \\ \cos 3 \approx \cos 171^\circ \approx -\cos 9^\circ \end{array} \right\} \cos 1 > \cos 2 > \cos 3$$

3. **(AB)**

$$x = r \sin A \cos B$$

$$y = r \sin A \sin B$$

$$z = r \cos A$$

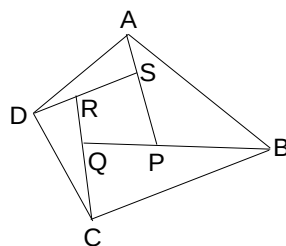
$$x^2 + y^2 + z^2 = r^2 \sin^2 A + r^2 \cos^2 A = r^2$$

4. **(BCD)**

$$\left. \begin{array}{l} \text{Quadrilateral is cyclic hence} \\ P + R = \pi \\ Q + S = \pi \end{array} \right\}$$

$$P = \frac{A}{2} + \frac{B}{2}$$

$$R = \frac{C}{2} + \frac{D}{2}$$



5. **(ABD)**

$$\tan \frac{x}{2} \in \theta$$

$$(i) \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \in \theta$$

$$(ii) \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \in \theta$$

$$(iii) \sec \frac{x}{2} = \sqrt{1 + \tan^2 \frac{x}{2}} \quad (\text{not necessarily})$$

$$(iv) \tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} \in \theta$$

6. **(ABCD)**

$$2 \sin \frac{A}{2} = \pm \sqrt{1 + \sin A} \pm \sqrt{1 - \sin A}$$

7. **(AD)**

$$\begin{aligned} & (4 \cos^2 9^\circ - 3)(4 \cos^2 27^\circ - 3) \\ &= \frac{(4 \cos^3 9^\circ - 3 \cos 9^\circ)(4 \cos^3 27^\circ - 3 \cos 27^\circ)}{\cos 9^\circ \cdot \cos 27^\circ} \\ &= \frac{\cancel{\cos 27^\circ} \cdot \cos 81^\circ}{\cos 9^\circ \cdot \cancel{\cos 27^\circ}} = \frac{\sin 9^\circ}{\cos 9^\circ} = \tan 9^\circ = \cot 81^\circ \end{aligned}$$

8. **(AD)**

$$\begin{aligned} \cos \beta &= \sqrt{\sin \alpha \cdot \cos \alpha} \\ \Rightarrow \cos^2 \beta &= \sin \alpha \cdot \cos \alpha \\ \Rightarrow \frac{1 + \cos 2\beta}{2} &= \frac{\sin 2\alpha}{2} \Rightarrow \cos 2\beta = -(1 - \sin 2\alpha) \\ &= -\left(1 - \cos\left(\frac{\pi}{2} - 2\alpha\right)\right) \\ &= -2 \sin^2\left(\frac{\pi}{4} - \alpha\right) \\ &= -2 \cos^2\left(\frac{\pi}{4} + \alpha\right) \end{aligned}$$

9. **(BC)**

$$\begin{aligned} & \frac{\sqrt{1 + \cos \alpha} + \sqrt{1 - \cos \alpha}}{\sqrt{1 + \cos \alpha} - \sqrt{1 - \cos \alpha}} ; \quad \alpha \in (\pi, 2\pi) \\ &= \frac{\sqrt{2} \left(\left| \cos \frac{\alpha}{2} \right| + \left| \sin \frac{\alpha}{2} \right| \right)}{\sqrt{2} \left(\left| \cos \frac{\alpha}{2} \right| - \left| \sin \frac{\alpha}{2} \right| \right)} = \frac{-\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}}{-\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}} = \frac{1 - \tan \frac{\alpha}{2}}{1 + \tan \frac{\alpha}{2}} = \tan \left(\frac{\pi}{4} - \frac{\alpha}{2} \right) \\ &= \cot \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \end{aligned}$$

10. **(C)**

$$\cos \theta \csc \theta - \cot \theta = q$$

$$\cos \theta \csc \theta + \cot \theta = \frac{1}{q}$$

$$\text{So, } \cos \theta \csc \theta = \frac{\left(q + \frac{1}{q}\right)}{2}$$

$$\cot \theta = \frac{\left(\frac{1}{q} - q\right)}{2}$$

11. **(AD)**

$$\cos A \cdot \cos B + \sin A \sin B = \frac{3}{5} \quad \dots(1)$$

$$\sin A \sin B = 2 \cos A \cos B \quad \dots(2)$$

$$\text{So, } \cos A \cos B = \frac{1}{5}$$

$$\sin A \sin B = \frac{2}{5}$$

12. **(BCD)**

$1^\circ \approx 57^\circ$

$\sin 1, \sin 2, \sin 3 \rightarrow \text{positive}$

$\sin 4, \sin 5 \rightarrow \text{negative}$

$\cos 1, \cos 5 \rightarrow \text{positive}$

$\cos 2, \cos 3, \cos 4 \rightarrow \text{negative}$

$\tan 1, \tan 4 \rightarrow \text{positive}$

$\tan 2, \tan 3, \tan 5 \rightarrow \text{negative}$

Ans. B, C, D

13. **(ABD)**

$$\frac{\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ + \cos 157^\circ \cos 97^\circ}$$

$$= \frac{\frac{1}{2}(\sin 30^\circ + \sin 14^\circ + \cos 256^\circ + \cos 60^\circ)}{\frac{1}{2}(\sin 30^\circ + \sin 16^\circ + \cos 254^\circ + \cos 60^\circ)}$$

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$

$$\cos 254^\circ = \cos(270^\circ - 16^\circ) = -\sin 16^\circ$$

$$\cos 256^\circ = \cos(270^\circ - 14^\circ) = -\sin 14^\circ$$

$$\therefore = \frac{\frac{1}{2}\left(\frac{1}{2} + \sin 14^\circ - \sin 14^\circ + \frac{1}{2}\right)}{\frac{1}{2}\left(\frac{1}{2} + \sin 16^\circ - \sin 16^\circ + \frac{1}{2}\right)}$$

$$\sec(-100\pi) = 1$$

$$\cos \sec\left(\frac{-3\pi}{2}\right) = \cos \sec\left(\frac{\pi}{2}\right) = 1 \quad \dots(B)$$

$$\sin\left(\frac{7\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1 \neq 1$$

$$\cot\left(\frac{5\pi}{4}\right) = 1 \quad \dots(D)$$

14. **(AD)**

$$\frac{\sqrt{1-\sin A}}{\sqrt{1+\sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$$

$$\frac{\sqrt{1-\sin A}}{\sqrt{1+\sin A}}$$

$$\Rightarrow \sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)\left(\frac{1+\sin A}{1+\sin A}\right)}$$

$$\Rightarrow \sqrt{\frac{1-\sin^2 A}{(1+\sin A)^2}} = \sqrt{\frac{\cos^2 A}{(1+\sin A)^2}}$$

$$\Rightarrow \left| \frac{\cos A}{1+\sin A} \right| \text{ now } 1+\sin A \text{ is always positive}$$

$$\Rightarrow \left| \frac{\cos A}{(1+\sin A)} \cdot \frac{(1-\sin A)}{(1-\sin A)} \right|$$

$$\Rightarrow \left| \frac{\cos A(1-\sin A)}{(1-\sin^2 A)} \right| \Rightarrow \left| \frac{1-\sin A}{\cos A} \right|$$

L.H.S

$$\Rightarrow \frac{1-\sin A}{|\cos A|} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$$

$$\frac{1-\sin A}{|\cos A|} = \frac{1-\sin A}{\cos A}$$

$$\Rightarrow \sin A = 1 \text{ or } |\cos A| = \cos A$$

But $\sin A \neq 1$

$\therefore$ then $\cos A = 0$

$|\cos A| = \cos A$ means $\cos A$ is positive i.e. 1st and 4th Quadrant

15. **(ABCD)**

$$\sec A = \frac{17}{8} \quad \csc B = \frac{5}{4}$$

$$\cos A = \frac{8}{17} \quad \sin B = \frac{4}{5}$$

$$\sin A = \pm \frac{15}{17} \quad \cos B = \pm \frac{3}{5}$$

$$\therefore \sec(A+B) = \frac{1}{\cos(A+B)} = \frac{1}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{1}{\left(\frac{8}{17}\right)\left(\pm \frac{3}{5}\right) - \left(\pm \frac{15}{17}\right)\left(\frac{4}{5}\right)}$$

$$= \frac{85}{\pm 24 \pm 60}$$

Possible answer are

$$\frac{85}{84}, \frac{85}{-84}, \frac{85}{36}, \frac{85}{-36}$$

16. (ABD)

$$\begin{aligned} \text{(A)} \quad & \frac{1 - 2 \sin^2 \alpha}{2 \cot\left(\frac{\pi}{4} + \alpha\right) \cos^2\left(\frac{\pi}{4} - \alpha\right)} \\ &= \frac{\cos 2\alpha}{2 \tan\left(\frac{\pi}{4} - \alpha\right) \cos^2\left(\frac{\pi}{4} - \alpha\right)} \\ &= \frac{\cos 2\alpha}{2 \sin\left(\frac{\pi}{4} - \alpha\right) \cos\left(\frac{\pi}{4} - \alpha\right)} \\ &= \frac{\cos 2\alpha}{\sin\left(2\left(\frac{\pi}{4} - \alpha\right)\right)} \\ &= \frac{\cos 2\alpha}{\sin\left(\frac{\pi}{2} - 2\alpha\right)} = \frac{\cos 2\alpha}{\cos 2\alpha} = 1 \end{aligned}$$

$$\begin{aligned} \text{(B)} \quad & \frac{\sin(\pi - \alpha)}{\sin \alpha - \cos \alpha \tan \frac{\alpha}{2}} + \cos(\pi - \alpha) \\ &= \frac{\sin \alpha}{\left(\sin \alpha \cos \frac{\alpha}{2} - \cos \alpha \sin \frac{\alpha}{2}\right)} - \cos \alpha \\ &= \frac{\sin \alpha}{\sin\left(\alpha - \frac{\alpha}{2}\right)} - \cos \alpha \\ &= \frac{\left(2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}\right) \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} - \cos \alpha \\ &= 2 \cos^2 \frac{\alpha}{2} - \cos \alpha \\ &= 1 + \cos \alpha - \cos \alpha = 1 \end{aligned}$$

$$\begin{aligned} \text{(C)} \quad & \frac{(1 - \tan^2 \alpha)}{4 \tan^2 \alpha} \\ &= \left(\frac{1 - \tan^2 \alpha}{2 \tan \alpha}\right)^2 = \cot^2 2\alpha \\ &= \frac{1}{4 \sin^2 \alpha \cos^2 \alpha} = \operatorname{cosec}^2 2\alpha \\ \therefore \quad & \operatorname{cosec}^2 2\alpha - \cot^2 2\alpha = 1 \end{aligned}$$

$$(D) \quad 1 + \sin 2\alpha = (\sin \alpha + \cos \alpha)^2$$

$$\therefore \frac{1 + \sin 2\alpha}{(\cos \alpha + \sin \alpha)^2} = 1$$

17. (AD)

$$\begin{aligned} & \frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} \\ &= \frac{\sin A \cos A + \sin B \cos B + \sin C \cos C}{\sin A \sin B \sin C} \end{aligned}$$

$$A + B + C = \pi$$

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$$

$$\therefore \text{Numerator} = 2 \sin A \sin B \sin C$$

$$\text{L.H.S} = 2.$$

18. (ABC)

$$\frac{\sqrt{3} \sin(\alpha + \beta) - \frac{2}{\cos\left(\frac{\pi}{6}\right)} \cos(\alpha + \beta)}{\sin \alpha}$$

$$= \frac{\sqrt{3} \sin(\alpha + \beta) - \frac{4}{\sqrt{3}} \cos(\alpha + \beta)}{\sin \alpha}$$

$$\sin \beta = \frac{4}{5}$$

$$\text{If } \beta \in \left(0, \frac{\pi}{2}\right) \text{ and } \tan \beta > 0$$

$$\text{Then } \cos \beta = \frac{3}{5}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \frac{3}{5} \sin \alpha + \frac{4}{5} \cos \alpha$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \frac{3}{5} \cos \alpha - \frac{4}{5} \sin \alpha$$

$$= \frac{1}{\sqrt{3} \sin \alpha} (3 \sin(\alpha + \beta) - 4 \cos(\alpha + \beta))$$

$$= \frac{1}{\sqrt{3} \sin \alpha} \left(\frac{9}{5} \sin \alpha + \frac{12}{5} \cos \alpha - \frac{12}{5} \cos \alpha + \frac{16}{5} \sin \alpha \right)$$

$$= \frac{1}{\sqrt{3} \sin \alpha} \left(\frac{25}{5} \sin \alpha \right) = \frac{5}{\sqrt{3}}$$

(A) & (B) true

$$\text{For } \tan \beta < 0 \quad \cos \beta = \frac{-3}{5}$$

$$\sin(\alpha + \beta) = \frac{-3}{5} \sin \alpha + \frac{4}{5} \cos \alpha$$

$$\cos(\alpha + \beta) = \frac{-3}{5} \cos \alpha - \frac{4}{5} \sin \alpha$$

$$\begin{aligned}
\text{L.H.S.} &= \frac{1}{\sqrt{3} \sin \alpha} (3 \sin(\alpha + \beta) - 4 \cos(\alpha + \beta)) \\
&= \frac{1}{\sqrt{3} \sin \alpha} \left(\frac{-9}{5} \sin \alpha + \frac{12}{5} \cos \alpha + \frac{12}{5} \cos \alpha + \frac{16}{5} \sin \alpha \right) \\
&= \frac{1}{\sqrt{3} \sin \alpha} \left(\frac{24}{5} \cos \alpha + \frac{7}{5} \sin \alpha \right) \\
&= \left(\frac{24 \cos \alpha + 7 \sin \alpha}{5 \sqrt{3} \sin \alpha} \right) \\
&= \frac{\sqrt{3}}{15} (24 \cot \alpha + 7) \\
\therefore (A) (B) (C)
\end{aligned}$$

19. **(ACD)**

$$\begin{aligned}
&\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \dots + \cos 140^\circ \\
&= (\cos 40^\circ + \cos 140^\circ) + (\cos 60^\circ + \cos 120^\circ) + \dots + (\cos 80^\circ + \cos 100^\circ) + \cos 20^\circ
\end{aligned}$$

$$\text{As } \cos(\pi - \theta) = -\cos \theta$$

$$\therefore \cos(\pi - \alpha) + \cos \theta = 0$$

L.H.S

$$= 0 + \cos 20^\circ \quad (A)$$

$$= \cos 20^\circ$$

$$= \sin 70^\circ \quad (D)$$

$$= \cos 20^\circ$$

$$= \cos(30^\circ - 10^\circ)$$

$$= \frac{\sqrt{3}}{2} \cos 10^\circ + \frac{1}{2} \sin 10^\circ \quad (C)$$

20. **(ABC)**

$$x \cos \alpha + y \sin \alpha = K \quad \text{where}$$

$\alpha = A, B$ are roots of this equation

$$x \cos \theta = k - y \sin \theta$$

Square

$$x^2 \cos^2 \theta = (k - y \sin \theta)^2$$

$$x^2 (1 - \sin^2 \theta) = k^2 + y^2 \sin^2 \theta - 2ky \sin \theta$$

$$x^2 - x^2 \sin^2 \theta = k^2 + y^2 \sin^2 \theta - 2ky \sin \theta$$

$$0 = (x^2 + y^2) \sin \theta - 2ky \sin \theta + (k^2 - x^2)$$

Quadratic in $\sin \theta$ roots are $\sin A, \sin B$

$$\sin A \sin B = \frac{k^2 - x^2}{x^2 + y^2}$$

$$\sin A + \sin B = \frac{2ky}{x^2 + y^2}$$

$$y \sin \theta = k - x \cos \theta$$

Square

$$y^2 \sin^2 \theta = (k - x \cos \theta)^2$$

$$y^2 (1 - \cos^2 \theta) = k^2 + x^2 \cos^2 \theta - 2kx \cos \theta$$

$$(x^2 + y^2) \cos^2 \theta - 2kx \cos \theta + k^2 - y^2 = 0$$

Quadratic in $\cos \theta$ having roots $\cos A, \cos B$

$$\cos A + \cos B = \frac{2kx}{x^2 + y^2}$$

$$\cos A \cos B = \frac{k^2 - y^2}{x^2 + y^2}$$

21. (AC)

$$\frac{2 \sin 2^\circ + 4 \sin 4^\circ + \dots + 180 \sin 180^\circ}{90} = A$$

$$A = \frac{(2 \sin 2^\circ + 4 \sin 4^\circ + \dots + 176 \sin 176^\circ + 178 \sin 178^\circ)}{90}$$

Supplementary angles have the same sign value

$$\sin 2^\circ = \sin 178^\circ$$

$$\sin 4^\circ = \sin 176^\circ \text{ etc}$$

$$90A = (2 \sin 2^\circ + 178 \sin 178^\circ) + (4 \sin 4^\circ + 176 \sin 176^\circ) + \dots + (88 \sin 88^\circ + 92 \sin 92^\circ) + 90 \sin 90^\circ$$

$$= (2 \sin 2^\circ + 178 \sin 2^\circ) + (4 \sin 4^\circ + 176 \sin 176^\circ) + \dots + (88 \sin 88^\circ + 92 \sin 88^\circ) + 90$$

$$= (180 \sin 2^\circ + 180 \sin 4^\circ + \dots + 180 \sin 88^\circ) + 90$$

$$90A = 180(\sin 2^\circ + \sin 4^\circ + \dots + \sin 88^\circ) + 90$$

$$\sin 2^\circ + \sin 4^\circ + \dots + \sin 88^\circ$$

$$= \frac{\sin(44^\circ) \sin(45^\circ)}{\sin 1^\circ}$$

$$90A = 180 \frac{\sin 44^\circ \sin 45^\circ}{\sin 1^\circ} + 90^\circ$$

$$= 90 \left(\frac{2 \sin 44^\circ \sin 45^\circ}{\sin 1^\circ} + 1 \right)$$

$$= 90 \left(\frac{\cos 1^\circ - \cos 89^\circ}{\sin 1^\circ} + 1 \right)$$

$$= 90 \left(\cot 1^\circ - \frac{\cos 89^\circ}{\sin 1^\circ} + 1 \right)$$

$$\cos 89^\circ = \sin 1^\circ$$

$$\therefore 90A = 90(\cot 1^\circ - 1 + 1)$$

$$= 90 \cot 1^\circ$$

$$A = \cot 1^\circ \quad (A)$$

$$(B) \text{ is } \tan 1^\circ$$

$$(C) = \frac{\cos(39^\circ) \cos 1^\circ}{\sin 51^\circ \sin 1^\circ}$$

$$(D) \text{ is } \tan 1^\circ$$

22. (ABCD)

$$\sin \theta + \cos \theta = -\frac{1}{5}$$

$$\frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} + \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \frac{-1}{5}$$

$$\text{Let } \tan \frac{\theta}{2} = x$$

$$\frac{2x + 1 - x^2}{1 + x^2} = \frac{-1}{5}$$

$$5(2x + 1 - x^2) + 1(1 + x^2) = 0$$

$$10x + 5 - 5x^2 + 1 + x^2 = 0$$

$$6 + 10x - 4x^2 = 0$$

$$2x^2 - 5x - 3 = 0 \quad (C)$$

$$x = \frac{5 \pm \sqrt{25 + 4(2)(3)}}{2}$$

$$= \frac{5 \pm 7}{2} = 6, -1$$

Now question is value of $\tan \frac{\theta}{2}$ is a root of which equation. If we see the other equations putting $x = -1$, or 6 will satisfy them. So All (A) (B) (C) (D) Have roots either -1 or 6

23. (AD)

$$\cos(A - B) = \frac{3}{5}, \quad \cos(A + B) = \frac{2}{5}$$

$$\text{Add} \quad 2 \cos A \cos B = 1$$

$$\text{Subtract} \quad 2 \sin A \sin B = \frac{1}{5}$$

24. (ABCD)

$$\sin a + \sin b = \frac{1}{\sqrt{2}} \quad \dots(1)$$

$$\cos a + \cos b = \frac{\sqrt{3}}{\sqrt{2}} \quad \dots(2)$$

Square and add the equations we get

$$2 + 2 \cos a \cos b + 2 \sin a \sin b = \frac{1}{2} + \frac{3}{2}$$

$$\therefore 2 \cos(a - b) = 0$$

$$\cos(a - b) = 0$$

$$2 \cos^2\left(\frac{a - b}{2}\right) - 1 = 0$$

$$\cos^2\left(\frac{a - b}{2}\right) = \frac{1}{2}$$

$$\sec^2\left(\frac{a - b}{2}\right) = 2$$

$$\tan^2\left(\frac{a - b}{2}\right) = 1$$

$$\therefore \cot^2\left(\frac{a-b}{2}\right)=1 \quad (C)$$

$$\sin a + \sin b = \frac{1}{\sqrt{2}}$$

$$2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{1}{\sqrt{2}}$$

$$\cos a + \cos b = \frac{\sqrt{3}}{\sqrt{2}}$$

$$2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{\sqrt{3}}{\sqrt{2}}$$

$$\text{Hence } \tan\left(\frac{a+b}{2}\right)=\frac{1}{\sqrt{3}} \quad (D)$$

$$\sin(a+b)=\frac{2\tan\left(\frac{a+b}{2}\right)}{1+\tan^2\left(\frac{a+b}{2}\right)}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\sqrt{3}}{2}$$

$$\sin(a+b)=\frac{\sqrt{3}}{2} \quad (A)$$

25. **(AC)**

$$\frac{\sin A}{\sin B}=p, \frac{\cos A}{\cos B}=q$$

$$\text{Dividing we get } \frac{\tan A}{\tan B}=\frac{p}{q}$$

$$\text{Now } \sin A = p \sin B \quad \dots(1)$$

$$\cos A = q \cos B \quad \dots(2)$$

$$\sin^2 A + \cos^2 A = 1$$

$$p^2 \sin^2 B + q^2 \cos^2 B = 1$$

$$p^2 \sin^2 B + q^2 (1 - \sin^2 B) = 1$$

$$(p^2 - q^2) \sin^2 B = 1 - q^2$$

$$\sin^2 B = \left(\frac{1 - q^2}{p^2 - q^2} \right)$$

$$\cos^2 B = 1 - \sin^2 B$$

$$= \left(\frac{p^2 - 1}{p^2 - q^2} \right)$$

$$\tan^2 B = \left(\frac{1 - q^2}{p^2 - 1} \right) = \frac{q^2 - 1}{1 - p^2} \quad (C)$$

From (1) & (2)

$$\sin^2 A = p^2 \sin^2 B$$

$$= p^2 \left(\frac{1 - q^2}{p^2 - q^2} \right)$$

$$\cos^2 A = q^2 \left(\frac{p^2 - 1}{p^2 - q^2} \right)$$

$$\tan^2 A = \frac{p^2(1-q^2)}{q^2(p^2-1)} \quad (A)$$

26. **(ABCD)**

$$0 \leq \theta \leq \pi$$

$$81^{\sin^2 \theta} + 81^{\cos^2 \theta} = 30$$

$$81^{\sin^2 \theta} + 81^{1-\sin^2 \theta} = 30$$

$$\text{Let } 81^{\sin^2 \theta} = x$$

$$x + \frac{81}{x} = 30$$

$$x^2 + 81 = 30x$$

$$x^2 - 30x + 81 = 0$$

$$x = 27 \text{ or } 3$$

$$\therefore 81^{\sin^2 \theta} = 27, 3$$

$$3^{4\sin^2 \theta} = 3^3, 3^1$$

$$\therefore 4\sin^2 \theta = 3, 1$$

$$\sin^2 \theta = \frac{3}{4}, \frac{1}{4}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}, \pm \frac{1}{2}$$

$$\text{For } 0 \leq \theta \leq \pi \quad \theta = 30^\circ, 60^\circ, 120^\circ, 150^\circ$$

27. **(BCD)**

For a cyclic Quadrilateral $B + D = \pi$ or $B = \pi - D$

$$\sin B = \sin D \Rightarrow \operatorname{cosec} B = \operatorname{cosec} D$$

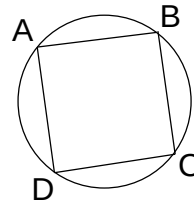
$$\cot B = -\cot D \text{ or } \tan B = -\tan D$$

$$\text{similarly } A = \pi - C$$

$$\cot A = -\cot C$$

$$\cot A + \cot C = 0$$

$$\sec B = -\sec D$$



28. **(ABCD)**

$$\sin\left(\frac{11\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)$$

$$= \frac{1}{2}\left(\cos\left(\frac{6\pi}{12}\right) - \cos\left(\frac{16\pi}{12}\right)\right)$$

$$= \frac{1}{2}\left(0 - \cos\left(\frac{4\pi}{3}\right)\right)$$

$$= \frac{1}{2}\left(+\frac{1}{2}\right) = \frac{1}{4} \quad \dots (A)$$

$$\operatorname{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)$$

$$= \frac{1}{-\sin \frac{\pi}{10} \cos \left(\frac{\pi}{5} \right)} = \frac{1}{(-1) \left(\frac{\sqrt{5}-1}{4} \right) \left(\frac{\sqrt{5}+1}{4} \right)}$$

$$= (-1)(4) \quad \dots (B)$$

$$\sec^4 \theta + \cos^4 \theta$$

$$= 1 - 2 \sin^2 \theta \cos^2 \theta$$

$$= 1 - \frac{\sin^2 2\theta}{2} \quad \theta = \frac{\pi}{8}$$

$$= 1 - \frac{\sin^2 \frac{\pi}{4}}{2}$$

$$= 1 - \frac{1}{2}$$

$$= \frac{3}{4} \quad \dots (C)$$

$$\left(1 + \cos \frac{2\pi}{9} \right) \left(1 + \cos \frac{4\pi}{9} \right) \left(1 + \cos \frac{8\pi}{9} \right)$$

$$= 2 \cos^2 \frac{\pi}{9} \times 2 \cos^2 \frac{2\pi}{9} \times 2 \cos^2 \frac{4\pi}{9}$$

$$= 8 \left(\cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9} \right)$$

$$= 8 \left(\frac{1}{8} \frac{\sin \left(\frac{8\pi}{9} \right)}{\sin \frac{\pi}{9}} \right)^2$$

$$= \frac{8}{64} = \frac{1}{8} \quad \dots (D)$$

29. (ABC)

(A) $\tan \alpha \tan 2\alpha \tan 3\alpha$

$$\tan 3\alpha - \tan 2\alpha - \tan \alpha$$

Is true for all angles

Hint: $2\alpha + \alpha = 3\alpha$

$\therefore$ take tan on both sides to get the answer

(B) $\operatorname{cosec} 2\alpha + \operatorname{cosec} 4\alpha - \operatorname{cosec} \alpha$

$$= \frac{1}{\sin \frac{2\pi}{7}} + \frac{1}{\sin \frac{4\pi}{7}} - \frac{1}{\sin \frac{\pi}{7}}$$

$$= \frac{2 \sin \left(\frac{3\pi}{7} \right) \cos \left(\frac{\pi}{7} \right)}{\left[2 \sin \left(\frac{\pi}{7} \right) \cos \left(\frac{\pi}{7} \right) \right] \sin \frac{4\pi}{7}} - \frac{1}{\sin \frac{\pi}{7}}$$

$$= \frac{1}{\sin \frac{\pi}{7}} = \frac{1}{\sin \frac{\pi}{7}} = 0$$

(D) $8 \cos \alpha \cos 2\alpha \cos 4\alpha$

$$= \frac{\sin 8\alpha}{\sin \alpha} = \frac{\sin \frac{8\pi}{7}}{\sin \frac{\pi}{7}} = -1$$

(C) $\cos \alpha + \cos 3\alpha - \cos 2\alpha$
 $= 2 \cos 2\alpha \cos \alpha - \cos 2\alpha$
 $= \cos 2\alpha (2 \cos \alpha - 1)$

$$= \cos \left(\frac{2\pi}{7} \right) \left(2 \cos \frac{\pi}{7} - 1 \right)$$

$$= \cos \left(\frac{2\pi}{7} \right) \left(2 \cos \frac{\pi}{7} - 1 \right) \frac{\sin \frac{\pi}{7}}{\sin \frac{\pi}{7}}$$

$$= \cos \left(\frac{2\pi}{7} \right) \left(\frac{2 \sin \frac{\pi}{7} \cos \frac{\pi}{7} - \sin \frac{\pi}{7}}{\sin \frac{\pi}{7}} \right)$$

$$= \frac{\cos \left(\frac{2\pi}{7} \right)}{\sin \frac{\pi}{7}} \left(\sin \frac{2\pi}{7} - \sin \frac{\pi}{7} \right)$$

$$= \frac{\cos \left(\frac{2\pi}{7} \right) \left(2 \sin \frac{\pi}{14} \right) \left(\cos \frac{3\pi}{14} \right)}{2 \sin \frac{\pi}{14} \cos \frac{\pi}{14}}$$

$$= \frac{\cos \frac{3\pi}{14} \cos \frac{2\pi}{7}}{\left(\cos \frac{\pi}{14} \right)}$$

$$= \frac{1}{2} \frac{\left(2 \cos \left(\frac{3\pi}{14} \right) \sin \frac{3\pi}{14} \right)}{\cos \frac{\pi}{14}} = \frac{1}{2} \frac{\sin \left(\frac{6\pi}{14} \right)}{\cos \frac{\pi}{14}}$$

$$\frac{6\pi}{14}, \frac{\pi}{14} \text{ are complementary}$$

$$\therefore \text{L.H.S.} = \frac{1}{2} \times 1 = \frac{1}{2}$$

30. **(AC)**
 $\cot \alpha - \tan \alpha = 2 \cot 2\alpha$... (A)
 (A) is true
 $\tan \alpha + \cot \alpha = 2 \operatorname{cosec} 2\alpha$... (D)
 (D) is wrong
 $\tan(45^\circ + \alpha) - \tan(45^\circ - \alpha)$
 $= \cot(45^\circ - \alpha) - \tan(45^\circ - \alpha)$
 $= 2 \cot(2(45^\circ - \alpha))$
 $2 \cot(90^\circ - 2\alpha)$
 $= 2 \tan 2\alpha$
 (B) (C) wrong

31. **(A)**
 $45^\circ = \frac{45}{180} \times 200 = 50^\circ$

32. **(D)**
 $\frac{23\pi^C}{4} = \frac{23 \times 180^\circ}{4} = 1035^\circ$

33. **(B)**
 $200^\circ = \frac{200}{200} \times 180^\circ = 180^\circ$
 $= \frac{200}{200} \times \pi^C = \pi^C$

34. **(B)**
 Sum of angles of a hexagon $= 180(n-2)$
 $= 180 \times 4 = 720^\circ = 800^\circ = 4\pi^C$

35. **(A)**
 $\sin \alpha + \sin \beta = \frac{1}{3}$; $\cos \alpha + \cos \beta = \frac{1}{4}$

36. **(B)**
 $2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$ _____ (1)

37. **(D)**
 $2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4}$ _____ (2)
 $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{4}{3}$

$$\text{Hence, } \sin(\alpha + \beta) = \frac{2 \tan\left(\frac{\alpha + \beta}{2}\right)}{1 + \tan^2\left(\frac{\alpha + \beta}{2}\right)} = \frac{2 \times \frac{4}{3}}{1 + \frac{16}{9}} = \frac{8 \times 3}{25} = \frac{24}{25}$$

$$\cos(\alpha + \beta) = \frac{1 - \tan^2\left(\frac{\alpha + \beta}{2}\right)}{1 + \tan^2\left(\frac{\alpha + \beta}{2}\right)} = \frac{1 - \frac{16}{9}}{1 + \frac{16}{9}} = \frac{-7}{25}$$

$$\tan(\alpha + \beta) = \frac{24}{25} \times \frac{25}{7} = -\frac{24}{7}$$

38. (B)

$$\begin{aligned} P_n - P_{n-2} &= \sin^n \theta + \cos^n \theta - \sin^{n-2} \theta - \cos^{n-2} \theta \\ &= \sin^{n-2} \theta (\sin^2 \theta - 1) + \cos^{n-2} \theta (\cos^2 \theta - 1) \\ &= -\sin^2 \theta \cos^2 \theta (\sin^{n-4} \theta + \cos^{n-4} \theta) \\ &= -\sin^2 \theta \cdot \cos^2 \theta \cdot P_{n-4} \\ \Rightarrow k &= -\sin^2 \theta \cdot \cos^2 \theta \end{aligned}$$

39. (B)

$$\begin{aligned} \sin \theta + \cos \theta = m &\Rightarrow \sin \theta \cdot \cos \theta = (m^2 - 1)/2 \\ 4(1 - P_6) &= 4(1 - (\sin^6 \theta + \cos^6 \theta)) \\ &= 4(1 - (1 - 3\sin^2 \theta \cos^2 \theta)) \\ &= 12\sin^2 \theta \cos^2 \theta = 12 \left(\frac{m^2 - 1}{2} \right)^2 \\ &= 3(m^2 - 1)^2 \end{aligned}$$

40. (D)

$$\begin{aligned} P_{n-2} - P_n &= \sin^2 \theta \cdot \cos^2 \theta P_\lambda \\ \lambda &= n - 4 \end{aligned}$$

41. (C)

$$\begin{aligned} 2P_6 - 3P_4 + 10 &= 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 10 \\ &= 2(1 - 3\sin^2 \theta \cos^2 \theta) - 3(1 - 2\sin^2 \theta \cos^2 \theta) + 10 \\ &= 2 - 3 + 10 = 9 \end{aligned}$$

42. (B)

$$\begin{aligned} \sin \frac{\pi}{7} + \sin \frac{2\pi}{7} + \sin \frac{3\pi}{7} \\ = \frac{2 \sin \frac{2\pi}{7} \cdot \sin \frac{3\pi}{14}}{2 \cdot \sin \frac{\pi}{14}} = \frac{\cos\left(\frac{\pi}{14}\right) - \cos\left(\frac{\pi}{2}\right)}{2 \sin \frac{\pi}{14}} = \frac{1}{2} \cot\left(\frac{\pi}{14}\right) \end{aligned}$$

43. (B)

$$\sin \frac{\pi}{n} + \sin \frac{3\pi}{n} + \sin \frac{5\pi}{n} + \dots \text{---n terms}$$

$$= \frac{\sin \left(\frac{\pi}{n} + \frac{(n-1) 2\pi}{2n} \right) \cdot \sin \left(\frac{2\pi \cdot n}{2n} \right)}{\sin \left(\frac{2\pi}{2n} \right)}$$

$$= \frac{\sin \pi \cdot \sin \pi}{\sin \frac{\pi}{4}} = 0$$

44. (C)

$$\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$$

$$= \frac{\cos \left(\frac{\pi}{11} + \frac{(5-1) 2\pi}{2 \cdot 11} \right) \cdot \sin \left(\frac{5 \cdot 2\pi}{2 \cdot 11} \right)}{\sin \left(\frac{2\pi}{11} / 2 \right)}$$

$$= \frac{2 \cos \frac{5\pi}{11} \cdot \sin \frac{5\pi}{11}}{2 \sin \frac{\pi}{11}} = \frac{\sin \frac{10\pi}{11}}{2 \sin \frac{\pi}{11}}$$

$$= \frac{\sin \frac{\pi}{11}}{2 \sin \frac{\pi}{11}} = \frac{1}{2}$$

45. (C)

$$\sum_{r=0}^n \sin^2 \frac{r\pi}{n} = \frac{1}{2} \sum_{r=0}^n \left(1 - \cos \frac{2r\pi}{n} \right)$$

$$= \frac{n+1}{2} - \frac{1}{2} \left(\cos 0 + \cos \frac{2\pi}{n} + \cos \frac{4\pi}{n} + \dots \cos 2\pi \right)$$

$$= \frac{n+1}{2} - \frac{\cos \left(0 + \frac{2\pi}{n} \right) \cdot \sin \left(\frac{2\pi}{n} \cdot \frac{(n+1)}{2} \right)}{2 \sin \left(\frac{2\pi}{n \cdot 2} \right)}$$

$$= \frac{n+1}{2} - \frac{1}{2} \frac{\sin \left(\pi + \frac{\pi}{n} \right) (-1)}{\sin \frac{\pi}{n}} = \frac{n+1}{2} - \frac{1}{2} = \frac{n}{2}$$

46. (B)

$$\cos 7 \approx \cos (6.28 + 0.72)$$

$$\approx \cos (0.72)$$

So, $\cos 7 > \cos 1$

(but not a correct explanation)

47. (A)

$$\left(27^{\cos 2x} \cdot 81^{\sin 2x}\right)_{\max}$$

$$= \left(3^{3\cos 2x + 4\sin 2x}\right)_{\min}$$

$$= 3^{-5} = \frac{1}{3^5}$$

48. (B)

$$\sin 3 \approx \sin 9^\circ$$

$$\sin 1 \approx \sin 57$$

$$\sin 2 \approx \sin 66^\circ$$

Sin x is possible but and in clearly as decreasing
(Not a correct explanation)

49. (A)

$$\sin^2 A + \sin^2 B + \sin^2 C = 2$$

$$\Rightarrow 2\sin^2 A + 2\sin^2 B + 2\sin^2 C = 4$$

$$\Rightarrow 1 - \cos 2A + 1 - \cos 2B + 1 - \cos 2C = 4$$

$$\Rightarrow \cos 2A + \cos 2B + \cos 2C = -1$$

$$\Rightarrow -1 - 4\cos A \cos B \cos C = -1$$

$$\Rightarrow \cos A \cos B \cos C = 0$$

Hence are of them is 90°

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$$

50. (B)

$$xy + yz + zx = 1$$

Let $x = \tan \frac{A}{2}, y = \tan \frac{B}{2}, z = \tan \frac{C}{2}$

$$xy + yz + zx = \tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} + \tan \frac{C}{2} \cdot \tan \frac{A}{2} = 1$$

Hence, $\left(\frac{A}{2} + \frac{B}{2} + \frac{C}{2}\right) = n\pi + \frac{\pi}{2} \Rightarrow A + B + C = 2n\pi + \pi$

We know $\frac{2x}{1+x^2} = \sin A, \frac{2y}{1+y^2} = \sin B, \frac{2z}{1+z^2} = \sin C$

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cos \frac{C}{2}$$

$$\Rightarrow \frac{2x}{1+x^2} + \frac{2y}{1+y^2} + \frac{2z}{1+z^2} = 4 \frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+y^2}} \cdot \frac{1}{\sqrt{1+z^2}}$$

The identify $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$ is true but not used here

51. (A) - Q, (B) - S, (C) - P, (D) - R

(A) $A = \sin^2 \theta + \cos^4 \theta$

$$= \sin^2 \theta + (1 - \sin^2 \theta)^2$$

$$= \sin^4 \theta - \sin^2 \theta + 1$$

$A \in \left[\frac{3}{4}, 1\right]$ (minimum at $\sin^2 \theta = \frac{1}{2}$, maximum of $\sin^2 \theta = 1$)

Ans: (Q)

$$(B) A = 3\cos^2 \theta + \sin^4 \theta$$

$$= \sin^4 \theta - 3\sin^2 \theta + 3$$

$$A \in [1, 3]$$

$$(\text{min of } \sin^2 \theta = 0, \text{ max of } \sin^2 \theta = 1)$$

Ans: (S)

$$(C) A = \sin^2 \theta - \cos^4 \theta$$

$$= (1 - \cos^2 \theta) - \cos^4 \theta$$

$$= -(\cos^4 \theta + \cos^2 \theta) + 1$$

$$A \in [-1, 1]$$

Ans: (P)

$$(D) A = \tan^2 \theta + 2\cot^2 \theta$$

$$\text{By A.M} \geq \text{G.M.}, A \geq 2\sqrt{2}$$

$$A \in [2\sqrt{2}, \infty)$$

$$52. (A) - R, (B) - P, (C) - Q, (D) - S$$

$$\cos \alpha + \cos \beta = \frac{1}{2}; \sin \alpha + \sin \beta = \frac{1}{3}$$

$$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2} \quad \dots\dots(i)$$

$$2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3} \quad \dots\dots(ii)$$

$$(C) \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1/3}{1/2} = \frac{2}{3}$$

$$(A) \cos\left(\frac{\alpha+\beta}{2}\right) = \pm \frac{3}{\sqrt{13}}$$

$$(i)^2 + (ii)^2$$

$$4\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4} + \frac{1}{9} = \frac{13}{36}$$

$$\cos\left(\frac{\alpha-\beta}{2}\right) = \pm \frac{\sqrt{13}}{12}$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \pm \frac{\sqrt{131}}{\sqrt{13}}$$

$$53. (A) - S, (B) - R, (C) - P, (D) - Q$$

$$(A) \cos 20^\circ + \cos 80^\circ - \sqrt{3} \cos 50^\circ$$

$$= 2\cos 50^\circ \cos 30^\circ - \sqrt{3} \cos 50^\circ$$

$$= 0$$

$$(B) 1 + \cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \dots\dots + \cos \frac{6\pi}{7}$$

$$= 1 + \frac{\cos\left(\frac{\pi}{7} + \frac{5}{2} \cdot \frac{\pi}{7}\right) \cdot \sin\left(\frac{6}{2} \cdot \frac{\pi}{7}\right)}{\sin\left(\frac{\pi}{14}\right)}$$

$$= 1 + \frac{\cos\left(\frac{\pi}{2}\right) \cdot \sin\left(\frac{3\pi}{7}\right)}{\sin\frac{\pi}{14}} = 1 + 0 = 1$$

$$\begin{aligned} \text{(C)} \quad & \cos 20^\circ + \cos 40^\circ + \cos 60^\circ - 4\cos 10^\circ \cdot \cos 20^\circ \cdot \cos 30^\circ \\ &= 2\cos 30^\circ \cdot \cos 10^\circ + 2\cos^2 30^\circ - 1 - 4\cos 10^\circ \cdot \cos 20^\circ \cdot \cos 30^\circ \\ &= 2\cos 30^\circ (\cos 10^\circ + \cos 30^\circ) - 1 - 4\cos 10^\circ \cdot \cos 20^\circ \cdot \cos 30^\circ \\ &= 4\cos 10^\circ \cdot \cos 20^\circ \cdot \cos 30^\circ - 1 - 4\cos 10^\circ \cdot \cos 20^\circ \cdot \cos 30^\circ \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad & \frac{1}{2} [2\cos 20^\circ \cdot \cos 100^\circ + 2\cos 100^\circ \cdot \cos 140^\circ - 2\cos 140^\circ \cdot \cos 200^\circ] \\ &= \frac{1}{2} [\cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ] \\ &= \frac{1}{2} \left[-\frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \cos 80^\circ + \cos 40^\circ - \cos 20^\circ \right] \\ &= -\frac{3}{4} + \frac{1}{2} (2\cos 60^\circ \cdot \cos 20^\circ - \cos 20^\circ) \\ &= -\frac{3}{4} \end{aligned}$$

54. (A) - R, (B) - P, (C) - S, (D) - Q

$$\begin{aligned} \text{(A)} \quad & \tan \theta (\cot \theta \cdot \cos \theta + \sin \theta) \\ &= \tan \theta \cdot \cot \theta \cdot \cos \theta + \frac{\sin \theta}{\cos \theta} \sin \theta \\ &= \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} = \sec \theta \end{aligned}$$

Ans: (R)

$$\begin{aligned} \text{(B)} \quad & \frac{\tan \theta \cdot \operatorname{cosec}^2 \theta}{1 + \tan^2 \theta} \\ &= \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin^2 \theta} \cdot \cos^2 \theta = \cot \theta \end{aligned}$$

Ans: (P)

$$\begin{aligned} \text{(C)} \quad & \frac{\operatorname{cosec}^2 \theta - 1}{\operatorname{cosec} \theta \cdot \cos^2 \theta} = \frac{\cot^2 \theta}{\frac{1}{\sin \theta} \cdot \cos^2 \theta} \\ &= \frac{\cos^2 \theta}{\sin^2 \theta \cdot \cos^2 \theta} \cdot \sin \theta \\ &= \operatorname{cosec} \theta \end{aligned}$$

Ans: (S)

$$\begin{aligned} \text{(D)} \quad & \frac{\cot \theta \cdot \sec^2 \theta}{1 + \cot^2 \theta} = \frac{\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos^2 \theta}}{\frac{1}{\sin^2 \theta}} \\ &= \frac{\sin \theta}{\cos \theta} = \tan \theta \end{aligned}$$

Ans: (Q)

55. (A) - RS, (B) - RT, (C) - PQ
- (A) $3\sin 2\theta + 4\cos 2\theta + 3$
 $\in [-5+3, 5+3] \quad \lambda + \mu = 6 \quad \text{(R)}$
 $\in [-2, 8] \quad \lambda - \mu = 10 \quad \text{(S)}$
- (B) $5\cos \theta + 3\cos \theta \cdot \cos \frac{\pi}{3} - 2\sin \theta \cdot \sin \frac{\pi}{3} + 3$
 $= \left(\frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta \right) + 3$
 $\in \left[\sqrt{\frac{169}{4} + \frac{27}{4}} + 3, \sqrt{\frac{169}{4} + \frac{27}{4}} + 3 \right]$
 $\in [-4, 10] \quad \lambda + \mu = 6 \quad \text{(R)}$
 $\lambda - \mu = 14 \quad \text{(T)}$
- (C) $1 + \sin \theta \cdot \frac{1}{\sqrt{2}} + \cos \theta \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{\cos \theta}{\sqrt{2}} + \frac{2\sin \theta}{\sqrt{2}}$
 $= 1 + \frac{3}{\sqrt{2}} \cos \theta + \frac{3}{\sqrt{2}} \sin \theta$
 $= \in \left[1 - \frac{3\sqrt{2}}{\sqrt{2}}, 1 + \frac{3\sqrt{2}}{\sqrt{2}} \right]$
 $\in [-2, 4] \quad \lambda + k = 2 \quad \text{(P)}$
 $\lambda - k = 6 \quad \text{(Q)}$

56. (A) - R, (B) - S, (C) - Q, (D) - P

$$\cos A = \frac{1}{3}$$

$$A \in (1350^\circ, 1440^\circ)$$

$$\Rightarrow \in (270^\circ, 360^\circ) \quad \text{IV quad}$$

$$\sin A = \frac{-2\sqrt{2}}{3}$$

$$\frac{A}{2} \in (675^\circ, 720^\circ) \quad \text{IV quad}$$

$$\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}} = \sqrt{\frac{1 - \frac{1}{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\cos \frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}} = \sqrt{\frac{2}{3}} \quad \tan \frac{A}{2} = -\frac{1}{\sqrt{2}}$$

57. (A) - R, (B) - P, (C) - S, (D) - Q

(A) $\sqrt{3} \sin x - \cos x \in [-2, 2] \quad \text{(R)}$

(B) $4\cos^2 x - 4\cos x + 3$

$$= (2\cos x - 1)^2 + 2$$

$$\in [0, 9] + 2$$

$$\in [2, 11] \quad \text{(P)}$$

$$(C) \frac{2 \tan x}{\tan^2 x + 1} = \sin 2x \in [-1, 1] \quad (S)$$

$$(D) \sin^4 x + \cos^4 x = 1 - 2 \sin^2 x \cos^2 x \\ = 1 - \frac{1}{2} \sin^2 2x \in [0, 1]$$

$$\text{So, } \in \left[\frac{1}{2}, 1 \right] \quad (Q)$$

58. (A) - Q, (B) - P, (C) - R, (D) - S

Standard solutions

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$$

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$$

$$\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos 2A \cos B \cos C$$

$$\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$$

59. (A) - S, (B) - P, (C) - Q, (D) - R

$$\sin 2\theta = k$$

$$(A) \operatorname{cosec} 2\theta + \cot 2\theta - \cos 2\theta$$

$$= \frac{1}{\sin 2\theta} + \frac{1 - \sin^2 2\theta}{\sin 2\theta} = \frac{1}{k} + \frac{1 - k^2}{k} = \frac{2 - k^2}{k} \quad (S)$$

$$(B) \left(\frac{1 + \tan \theta}{1 - \tan \theta} \right)^2 = \frac{1 + \sin 2\theta}{1 - \sin 2\theta} = \frac{1 + k}{1 - k} \quad (P)$$

$$(C) \sin 2\theta - \frac{1}{2} (2 \cos^2 2\theta) \\ = k - (1 - k^2) = k^2 + k - 1 \quad (Q)$$

$$(D) \sin 6\theta = 3 \sin 2\theta - 4 \sin^3 2\theta \\ = 3k - 4k^3 \quad (R)$$

60. (A) - R, (B) - S, (C) - P, (D) - Q

$$(A) \sin B = \frac{4}{5}, \tan \left(\frac{A}{2} \right) = 1 \Rightarrow A = 90^\circ$$

$$B + C = 90^\circ$$

$$\cos C = \frac{4}{5}$$

$$\frac{a}{b} = \frac{\sin A}{\sin B} = \frac{1}{\frac{4}{5}} = \frac{5}{4} \quad (R)$$

$$(B) \frac{b}{c} = \frac{\sin B}{\sin C} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3} \quad (S)$$

$$(C) \frac{c + b}{a} = \frac{\sin C + \sin B}{\sin A} = \frac{3}{5} + \frac{45}{5} = \frac{7}{5} \quad (P)$$

$$(D) \sqrt{\frac{a^2 + c^2 - b^2}{2ac}} = \sqrt{\frac{\sin^2 A + \sin^2 C - \sin^2 B}{2 \sin A \sin C}} = \sqrt{\frac{3}{5}} \quad (Q)$$

1. (3)

$$\frac{\sin 2\alpha + \sin 4\alpha - \sin 3\alpha}{\cos 2\alpha + \cos 4\alpha - \cos 3\alpha}$$

$$= \frac{2\sin 3\alpha \cos \alpha - \sin 3\alpha}{2\cos 3\alpha \cos \alpha - \cos 3\alpha} = \frac{\sin 3\alpha}{\cos 3\alpha} = \tan 3\alpha$$

$$k = 3$$

2. (1)

$$\tan 45^\circ = 1 \Rightarrow \tan(27^\circ + 18^\circ) = 1$$

$$\Rightarrow \frac{\tan 27^\circ + \tan 18^\circ}{1 - \tan 27^\circ \tan 18^\circ} = 1$$

$$\Rightarrow \tan 27^\circ + \tan 18^\circ = 1 - \tan 27^\circ \tan 18^\circ$$

$$\Rightarrow \tan 27^\circ + \tan 18^\circ + \tan 27^\circ \tan 18^\circ = 1$$

3. (3)

$$\text{LHS } \frac{\cos 96^\circ + \sin 96^\circ}{\cos 96^\circ - \sin 96^\circ} = \frac{1 + \tan 96^\circ}{1 - \tan 96^\circ}$$

$$\Rightarrow \frac{\tan 45^\circ + \tan 96^\circ}{1 - \tan 45^\circ \tan 96^\circ} = \tan(141)$$

$$\text{Or } \tan(180 + 141) = \tan(321)$$

$$\therefore n = 3$$

4. (3)

$$\text{In } \triangle ABC \quad \sum \tan A = \pi \tan A$$

$$6 = 2 \times \tan C$$

$$\Rightarrow \tan C = 3$$

5. (0)

$$a \sin \theta - \frac{a}{\sin \theta} = \frac{b}{\cos \theta} - b \cos \theta$$

$$a \frac{(\sin^2 \theta - 1)}{\sin \theta} = b \frac{(1 - \cos^2 \theta)}{\cos \theta}$$

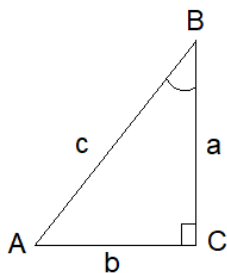
$$\Rightarrow -a \cos^3 \theta = b \sin^3 \theta \Rightarrow a \cos^3 \theta + b \sin^3 \theta = 0$$

Square both side

$$a^2 \cos^6 \theta + b^2 \sin^6 \theta + 2ab \cos^3 \theta \sin^3 \theta = 0$$

6. (8)

$$\left(\frac{c}{a} + \frac{c}{b}\right)^2$$



$$= (\sec \theta + \operatorname{cosec} \theta)^2$$

$$= 4 \frac{(\sin \theta + \cos \theta)^2}{(\sin 2\theta)^2} \Rightarrow 4 \left(\frac{1}{\sin^2 2\theta} + \frac{1}{\sin 2\theta} \right)$$

$\therefore$ For $\min(\sin 2\theta)$ $\max = 1$

Hence $\min = 8$

7. (1)

$$\sqrt{2 \cot \alpha + \operatorname{cosec}^2 \alpha} + \cot \alpha$$

$$\sqrt{(1 + \cot \alpha)^2} + \cot \alpha$$

$$|1 + \cot \alpha| + \cot \alpha$$

$$\text{Since } \frac{3\pi}{4} < \alpha < \pi \Rightarrow 1 + \cot \alpha < 0$$

$$\Rightarrow 1 - \cot \alpha + \cot \alpha$$

Ans: 1

8. (3)

Let α & β be the roots

$$\alpha + \beta = -\cos \theta + 1, \alpha\beta = \frac{1}{2} \cos^2 \theta$$

$$\therefore \alpha^2 + \beta^2 = (1 - \cos \theta)^2 - 2 \times \frac{1}{2} \cos^2 \theta$$

$$= 1 - 2 \cos \theta$$

Maximum as 3

9. (1)

$$\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$$

$$= 1 - \frac{\sin^2 2\theta}{2}$$

For maximum $\sin 2\theta = 0$

i.e. 1

10. (0)

$$\text{If } \sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$$

That is only possible when all

$$\sin \theta = 1 \Rightarrow \cos \theta_1 = 0$$

$$\therefore \cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 0$$

11. (1)

$$\tan \frac{\pi}{4n} \tan \frac{2\pi}{4n} \dots \tan \frac{n\pi}{4n} \dots \tan \left(\frac{\pi}{2} - \frac{2\pi}{4n} \right) \tan \left(\frac{\pi}{2} - \frac{\pi}{4n} \right)$$

(use $\left[\text{Use } \tan \left(\frac{\pi}{2} - \alpha \right) = \cot \alpha \right]$)

OR

$$\tan \frac{4\pi}{4n} \cdot \tan \frac{2\pi}{4n} \dots 1 \dots \cot \frac{2\pi}{4n} \cdot \cot \frac{\pi}{4n}$$

(use $\tan \theta \cot \theta = 1$)

Ans: 1

12. (5)

$\sin x = 1 - \sin^2 x$ or $\sin x = \cos^2 x$ of $\cos^2 x + \cos^4 x = 1$

Expression

$$\cos^6 x [\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1] + \cos^4 x + \cos^2 x + 3$$

$$= \cos^6 x [\cos^2 x + 1]^3 + 4$$

$$= (\cos^4 x + \cos^2 x)^3 + 4$$

$$= 1 + 4 = 5$$

13. (1)

$$\tan(100^\circ + 125^\circ) = \frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \cdot \tan 125^\circ}$$

$$\tan 225^\circ = 1 - \tan 100^\circ \tan 125^\circ = \tan 100^\circ + \tan 125^\circ$$

14. (1)

On simplification the expression reduces to

$$\tan^2 \frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2}$$

When is minimum when all angles equal = 60

$\therefore$ Min value is 1

15. (0)

$$\frac{x}{\cos \theta} = \frac{y}{\cos \left(\theta - \frac{2\pi}{3} \right)} = \frac{z}{\cos \left(\theta + \frac{2\pi}{3} \right)} = k$$

$$\Rightarrow x + y + z = k \left(\cos \theta + \cos \left(\theta - \frac{2\pi}{3} \right) + \cos \left(\theta + \frac{2\pi}{3} \right) \right)$$

$$= k \left(\cos \theta + 2 \cos \theta \cos \frac{2\pi}{3} \right)$$

$$= k \left(\cos \theta - 2 \cos \theta \times \frac{1}{2} \right)$$

$$= 0$$

16. (1)

$$\frac{\tan 6^\circ \tan 54^\circ \tan 66^\circ \tan 42^\circ \tan 78^\circ}{\tan 54^\circ}$$

$$\frac{\tan 18^\circ \tan 42^\circ \tan 78^\circ}{\tan 54^\circ}$$

Use $\tan \theta \tan (60 - \theta) \tan (160^\circ + \theta) = \tan 3\theta$

17. (2)

Angle moved in 60 min = 360°
 Angle moved in 20 min = 120°

$$\therefore d = 10 \times 120^\circ \times \frac{\pi}{180}$$

$$d = \frac{20\pi}{3} \quad \text{Ans : 2}$$

18. (7)

During every motion particle rotates by 1 radian angle (Approx. 57.16°)
 $\therefore$ To rotate 360° it must be in its 7th round

19. (1)

$$\sec^4 A - \tan^4 A - 2 \tan^2 A$$

$$= (\sec^2 A - \tan^2 A)(\sec^2 A + \tan^2 A) - 2 \tan^2 A$$

$$= \sec^2 A + \tan^2 A - 2 \tan^2 A$$

$$= \sec^2 A - \tan^2 A = 1$$

20. (1)

We know $\sin^2 \theta \geq \sin^8 \theta$ (1)
 $\cos^2 \theta \geq \cos^{14} \theta$ (2)
 Add (1) & (2)
 $\Rightarrow \sin^8 \theta + \cos^{14} \theta \leq 1$
 $0 < A \leq 1$
 $a = 0, b = 1$

21. (3)

We know $x + \frac{1}{x}$ lies in interval
 $(-\infty, -2] \text{ or } [2, \infty)$
 $\therefore$ For is to be equal to $2 \cos \theta$
 $\cos \theta$ must be ± 1
 $\therefore$ in $[0, 2\pi]$ only 3 possible angle $0, \pi, 2\pi$

22. (2)

Use $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$
 $\therefore \sin^2 24 - \sin^2 6 = \sin 30 \sin 18$

$$= \frac{1}{2} \times \frac{\sqrt{5}-1}{4}$$

$$= \frac{\sqrt{5}-1}{8}$$

$$\therefore a=5, b=1, c=8$$

Ans: 2

23. (2)

$$A+B=225^\circ \quad (\text{Similar to question 2})$$

$$\Rightarrow \tan A \tan B + \tan A + \tan B = 1$$

$$\text{Now } \frac{(1+\cot A)(1+\cot B)}{\cot A \cot B}$$

$$\text{Simplifying to } 1 + \tan A + \tan B + \tan A \tan B = 2$$

24. (7)

$$\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$$

$$\text{Or } \cos^3 \theta = \frac{\cos 3\theta + 3\cos \theta}{4}$$

$\therefore$ Expression is

$$\frac{\cos 3\theta + 3\cos \theta}{4} + \frac{\cos(360+3\theta) + 3\cos(120+\theta)}{4} + \frac{\cos(720+3\theta) + 3\cos(240+\theta)}{4}$$

$$= \frac{3}{4}\cos 3\theta + \left(\frac{\cos \theta + \cos(120+\theta) + \cos(240+\theta)}{4} \right) \cdot 3$$

$$= \frac{3}{4}\cos 3\theta + 0$$

$$a+b=7$$

25. (2)

$$\frac{\sin(A+B)\cos A}{\cos(A+B)\sin A} = 3$$

Apply Componendo & dividendo

$$\frac{\sin(2A+B)}{\sin(B)} = \frac{4}{2} = 2$$

26. (2)

$$\cos A = \cos(\pi - (B+C))$$

$$= -\cos(B+C)$$

$$\therefore \frac{\cos A}{\sin B \sin C} = -\frac{\cos B \cos C + \sin B \sin C}{\sin B \sin C}$$

$$\frac{\cos A}{\sin B \sin C} = (1 - \cot B \cot C)$$

$\therefore$ Expression becomes

$$= 3 - (\cot A \cot B + \cot B \cot C + \cot C \cot A)$$

$$\text{In } \triangle ABC \sum \cot A \cot B = 1$$

$$= 2$$

27. (4)

$$\begin{aligned} & \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \left(-\frac{\cos \pi}{15} \right) \\ &= - \left[\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \right] \\ &= - \left[\frac{\sin \left(\frac{16\pi}{15} \right)}{16 \sin \frac{\pi}{15}} \right] \end{aligned}$$

Use formula

$$\begin{aligned} \prod_{r=0}^{n-1} \cos 2^r \theta &= \frac{\sin 2^n \theta}{2^n \sin \theta} \\ &= \frac{1}{16} \\ \frac{1}{\sqrt{x}} &= 4 \end{aligned}$$

28. (3)

$$\begin{aligned} \sin A \cos B &= \frac{1}{4} & 3 \sin A \cos B &= \cos A \sin B \\ \therefore \cos A \sin B &= \frac{3}{4} \\ &= \sin(A+B) = 1, \sin(A-B) = -\frac{1}{2} \\ &= A+B = 90^\circ & \& A-B = -30^\circ \\ A &= 30^\circ, B = 60^\circ \\ &= \cot^2 A = 3 \end{aligned}$$

29. (4)

$$\begin{aligned} a^2 + b^2 &= 2 - 2 \sin A \sin B + 2 \cos A \cos B \\ &= 2 + 2 \cos(A+B) \\ &= 4 \cos^2 \left(\frac{A+B}{2} \right) \\ \text{Maximum} &= 4 \end{aligned}$$

30. (8)

$$\begin{aligned} & \tan \frac{\pi}{3} + 2 \tan \frac{2\pi}{3} + 4 \tan \frac{4\pi}{3} + 8 \tan \frac{8\pi}{3} \\ &= \sqrt{3} + 2 \left(\tan \left(\pi - \frac{\pi}{3} \right) \right) + 4 \tan \left(\pi + \frac{\pi}{3} \right) + 8 \tan \left(3\pi - \frac{\pi}{3} \right) \\ &= \sqrt{3} - 2\sqrt{3} + 4\sqrt{3} - 8\sqrt{3} \\ &= 5\sqrt{3} - 10\sqrt{3} = -5\sqrt{3} \end{aligned}$$

31. (8)

$$\begin{aligned} & \text{For minimum in } \triangle ABC, A = B = C = 60^\circ \\ \therefore |\sec A \sec B \sec C| &= 8 \end{aligned}$$

32. (1)

$$\cot 6^\circ \cot 42^\circ \cot 66^\circ \cot 78^\circ$$

$$= \left(\frac{\cot 6^\circ \cdot \cot 54^\circ \cot 66^\circ}{\cot 54^\circ} \right) \cdot \left(\frac{\cot 18^\circ \cot 42^\circ \cot 78^\circ}{\cot 18^\circ} \right)$$

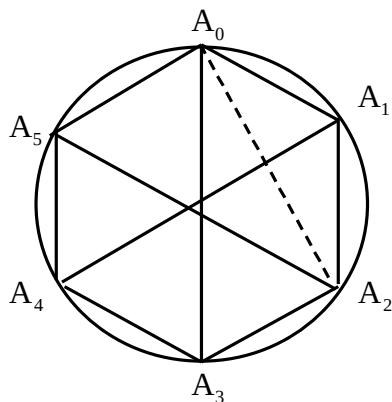
$$= \frac{\cot 18^\circ}{\cot 54^\circ} \cdot \frac{\cot 54^\circ}{\cot 18^\circ} = 1$$

33. (3)

Use cosine Rule

$$A_0 A_4 = A_0 A_2 = \sqrt{1^2 + 1^2 - 2 \cos 120^\circ}$$

$$= \sqrt{3}$$



$$\therefore A_0 A_1 = 1$$

$$A_0 A_2 = \sqrt{3}$$

$$A_0 A_4 = \sqrt{3}$$

$$\therefore \text{Ans. } 3$$

34. (8)

Expression is

$$3 + 2 \sin^2 \theta - 3 \sin 2\theta$$

Or $3 + 1 - \cos 2\theta - 3 \sin 2\theta$

$$4 - [\cos 2\theta + 3 \sin 2\theta]$$

Minimum $4 \pm \sqrt{10}$

Ans:-8

35. (0)

$$|9 \tan^2 A + 4 \cot^2 B|$$

For min put both $\tan^2 A = 0$ & $\cot^2 B = 0$

36. (2)

Use $AM \geq GM$

$$\therefore |\tan A + \cot A| \geq 2$$

37. (2)

$$\frac{A}{2} + \frac{B}{2} = 45^\circ$$

$$\Rightarrow \left(1 + \tan \frac{A}{2} \right) \left(1 + \tan \frac{B}{2} \right) = 2$$

38. (1)

$$\begin{aligned} & \cos 20^\circ + 1 - \cos 110^\circ - \sqrt{2} \sin 65^\circ \\ & 1 + \cos 20^\circ - \cos 110^\circ - \sqrt{2} \sin 65^\circ \\ & 1 + 2 \sin 65^\circ \sin 45^\circ - \sqrt{2} \sin 65^\circ \\ & 1 + \sqrt{2} \sin 65^\circ - \sqrt{2} \sin 65^\circ \\ & \text{Ans: } 1 \end{aligned}$$

39. (3)

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\text{OR } \frac{3 \tan \frac{\pi}{9} - \tan^3 \frac{\pi}{9}}{1 - 3 \tan^2 \frac{\pi}{9}} = \sqrt{3}$$

Square

$$9 \tan^2 \frac{\pi}{9} + \tan^6 \frac{\pi}{9} - 6 \tan^4 \frac{\pi}{9} = 3 + 27 \tan^4 \frac{\pi}{9} - 18 \tan^2 \frac{\pi}{9}$$

Solving to get 3

40. (3)

$$\begin{aligned} \sum \cos A &= 0 \\ \sum \sin A &= 0 \\ \Rightarrow B &= 120^\circ + A, C = 240^\circ + A \\ \text{Expression become } \frac{3 \cos 3A}{\cos 3A} &= 3 \end{aligned}$$

JEE Advanced : PYQ

1. (b)

$$\begin{aligned} A &= \sin^2 \theta + \cos^4 \theta = \sin^2 \theta + (1 - \sin^2 \theta)^2 \\ &= \sin^4 \theta - \sin^2 \theta + 1 \Rightarrow A = \left(\sin^2 \theta - \frac{1}{2} \right)^2 + \frac{3}{4} \end{aligned}$$

Now, $0 \leq \left(\sin^2 \theta - \frac{1}{2} \right)^2 \leq \frac{1}{4}$

$$\Rightarrow \frac{3}{4} \leq \left(\sin^2 \theta - \frac{1}{2} \right)^2 + \frac{3}{4} \leq 1$$

$$\Rightarrow \frac{3}{4} \leq A \leq 1$$

2. (a)

$$\alpha + \beta + \gamma = 2\pi \Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$$

$$\therefore \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(\pi - \frac{\gamma}{2}\right) = -\tan\frac{\gamma}{2}$$

$$\Rightarrow \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}$$

3. (b)

Given expression can be written as

$$\frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A} \quad \left(\because \tan A = \frac{\sin A}{\cos A} \text{ and } \cot A = \frac{\cos A}{\sin A} \right)$$

$$= \frac{1}{\sin A - \cos A} \left\{ \frac{\sin^3 A - \cos^3 A}{\cos A \sin A} \right\}$$

$$\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$= \frac{\sin^2 A + \sin A \cos A + \cos^2 A}{\sin A \cos A} = 1 + \sec A \operatorname{cosec} A$$

4. (c)

$$\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$$

$$= \sum_{k=1}^{13} \frac{1}{\sin\frac{\pi}{6}} \left[\frac{\sin\left\{\frac{\pi}{6} + \frac{k\pi}{6} - \left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right\}}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)} \right]$$

$$\sum_{k=1}^{13} 2 \left[\cot\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{k\pi}{6}\right) \right]$$

$$= 2 \left[\left\{ \cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right) \right\} + \left\{ \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{2\pi}{6}\right) \right\} + \dots + \left\{ \cot\left(\frac{\pi}{4} + \frac{12\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right) \right\} \right]$$

$$= 2 \left[\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right) \right] = 2 \left[1 - \cot\frac{5\pi}{12} \right]$$

$$= 2 \left[1 - \frac{\sqrt{3}-1}{\sqrt{3}+1} \right] = 2 \left[1 - (2 - \sqrt{3}) \right] = 2(\sqrt{3} - 1)$$

5. (b)

$$\text{Given : } \theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan \theta < 1 \text{ and } \cot \theta > 1$$

Let $\tan \theta = 1 - x$ and $\cot \theta = 1 + y$, where $x, y > 0$ and are very small, then

$$\therefore t_1 = (1-x)^{1-x}, t_2 = (1-x)^{1+y}, t_3 = (1+y)^{1-x}, t_4 = (1+y)^{1+y}$$

Clearly, $t_4 > t_3$ and $t_1 > t_2$ also, $t_3 > t_1$

$$\therefore t_4 > t_3 > t_1 > t_2.$$

6. (c)

$$\text{Given : } \alpha + \beta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2} - \beta$$

$$\Rightarrow \tan \alpha = \tan \left(\frac{\pi}{2} - \beta \right) = \cot \beta = \frac{1}{\tan \beta}$$

$$\Rightarrow \tan \alpha \tan \beta = 1 \Rightarrow 1 + \tan \alpha \tan \beta = 2.$$

$$\text{Now, } \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\Rightarrow \tan \gamma = \frac{\tan \alpha - \tan \beta}{2}$$

$$\Rightarrow 2 \tan \gamma = \tan \alpha - \tan \beta \Rightarrow \tan \alpha = 2 \tan \gamma + \tan \beta$$

7. (a)

$$\text{Given : } (\cot \alpha_1) \cdot (\cot \alpha_2) \cdot \dots \cdot (\cot \alpha_n) = 1$$

$$\Rightarrow (\cos \alpha_1)(\cos \alpha_2) \cdot \dots \cdot (\cos \alpha_n)$$

$$= (\sin \alpha_1)(\sin \alpha_2) \cdot \dots \cdot (\sin \alpha_n) \quad \dots (i)$$

$$\text{Let } y = (\cos \alpha_1)(\cos \alpha_2) \cdot \dots \cdot (\cos \alpha_n) \text{ (to be max.)}$$

$$\Rightarrow y^2 = (\cos^2 \alpha_1)(\cos^2 \alpha_2) \cdot \dots \cdot (\cos^2 \alpha_n)$$

$$= \cos \alpha_1 \sin \alpha_1 \cos \alpha_2 \sin \alpha_2 \cdot \dots \cdot \cos \alpha_n \sin \alpha_n \quad [\text{From (i)}]$$

$$= \frac{1}{2^n} [\sin 2\alpha_1 \sin 2\alpha_2 \cdot \dots \cdot \sin 2\alpha_n]$$

$$\text{Now, } 0 \leq \alpha_1, \alpha_2, \dots, \alpha_n \leq \frac{\pi}{2}$$

$$\therefore 0 \leq 2\alpha_1, 2\alpha_2, \dots, 2\alpha_n \leq \pi$$

$$\Rightarrow 0 \leq \sin 2\alpha_1, \sin 2\alpha_2, \dots, \sin 2\alpha_n \leq 1$$

$$\therefore y^2 \leq \frac{1}{2^n} \cdot 1 \Rightarrow y \leq \frac{1}{2^{n/2}}$$

$$\therefore \text{Max. value of } y \text{ i.e. } (\cos \alpha_1) \cdot (\cos \alpha_2) \cdot \dots \cdot (\cos \alpha_n) = \frac{1}{2^{n/2}}$$

8. (c)

$$f(\theta) = \sin \theta (\sin \theta + \sin 3\theta)$$

$$= \sin \theta (\sin \theta + 3\sin \theta - 4\sin^3 \theta)$$

$$= \sin \theta (4\sin \theta - 4\sin^3 \theta) = \sin^2 \theta (4 - 4\sin^2 \theta)$$

$$= 4\sin^2 \theta (1 - \sin^2 \theta)$$

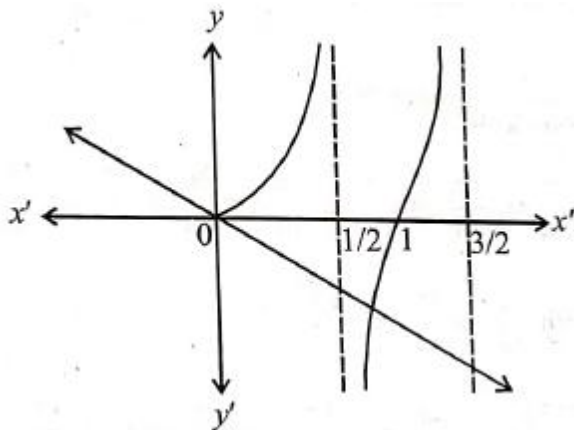
$$= 4\sin^2 \theta \cos^2 \theta = (2\sin \theta \cos \theta)^2 = (\sin 2\theta)^2 \geq 0, \text{ which is true for all } \theta.$$

9. (b, c)

Given : $f(x) = x \sin \pi x, x > 0$

$$\Rightarrow f'(x) = \sin \pi x + x\pi \cos \pi x$$

Now, $f'(x) = 0 \Rightarrow \tan \pi x = -\pi x$



From graph of $y = \tan \pi x$ and $y = -\pi x$, it is clear that they intersect each other at unique point in the intervals

$$(n, n+1) \text{ and } \left(n + \frac{1}{2}, n+1\right)$$

10. (a, c, d)

As $\tan(2\pi - \theta) > 0$ and $-1 < \sin \theta < -\frac{\sqrt{3}}{2}, \theta \in [0, 2\pi]$

Hence $\frac{3\pi}{2} < \theta < \frac{5\pi}{3}$

Now $2 \cos \theta (1 - \sin \varphi) = \sin^2 \theta \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) \cos \varphi - 1$

$$\Rightarrow 2 \cos \theta (1 - \sin \varphi) = 2 \sin \theta \cos \varphi - 1$$

$$\Rightarrow 2 \cos \theta + 1 = 2 \sin(\theta + \varphi)$$

As $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right), 1 < 2 \sin(\theta + \varphi) < 2$

As $\theta + \varphi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ or $(\theta + \varphi) \in \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

We have $\varphi \in \cup \left(\frac{2\pi}{3}, \frac{7\pi}{6}\right)$

11. (a, b)

Given : $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5} \Rightarrow 3 \sin^4 x + 2 \cos^4 x = \frac{6}{5}$

$$\Rightarrow \sin^4 x + 2[\sin^4 x + \cos^4 x] = \frac{6}{5}$$

$$\Rightarrow \sin^4 x + 2[1 - 2\sin^2 x \cos^2 x] = \frac{6}{5}$$

$$\Rightarrow \sin^4 x + 2 - 4\sin^2 x(1 - \sin^2 x) = \frac{6}{5}$$

$$\Rightarrow 5\sin^4 x - 4\sin^2 x + 2 - \frac{6}{5} = 0$$

$$\Rightarrow 25\sin^4 x - 20\sin^2 x + 4 = 0$$

$$\Rightarrow (5\sin^2 x - 2)^2 = 0 \Rightarrow \sin^2 x = \frac{2}{5}$$

$$\Rightarrow \cos^2 x = \frac{3}{5} \text{ and } \tan^2 x = \frac{2}{3}$$

$$\text{Also } \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{625} + \frac{3}{625} = \frac{5}{625} = \frac{1}{125}$$

12. (a, b, c, d)

Note that multiplicative loop is very important approach in IIT Mathematics

$$\left(\tan \frac{\theta}{2}\right)(1 + \sec \theta) = \frac{\sin \theta/2}{\cos \theta/2} \cdot \left[1 + \frac{1}{\cos \theta}\right]$$

$$= \frac{(\sin \theta/2) 2 \cos^2 \theta/2}{(\cos \theta/2) \cos \theta}$$

$$= \frac{(2 \sin \theta/2) \cos \theta/2}{\cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\therefore f_n(\theta) = (\tan \theta/2)(1 + \sec \theta)$$

$$(1 + \sec 2\theta)(1 + \sec 2^2 \theta) \dots (1 + \sec 2^n \theta)$$

$$= (\tan \theta)(1 + \sec 2\theta)(1 + \sec 2^2 \theta) \dots (1 + \sec 2^n \theta)$$

$$\text{Now, } f_2\left(\frac{\pi}{16}\right) = \tan\left(2^2 \cdot \frac{\pi}{16}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

Therefore, (a) is the correct option.

$$f_3\left(\frac{\pi}{32}\right) = \tan\left(2^3 \cdot \frac{\pi}{32}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

Therefore, (b) is the correct option.

$$f_4\left(\frac{\pi}{64}\right) = \tan\left(2^4 \cdot \frac{\pi}{64}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

Therefore, (c) is the correct option.

$$f_5\left(\frac{\pi}{128}\right) = \tan\left(2^5 \cdot \frac{\pi}{128}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

Therefore, (d) is the correct option.

13. (A) $\rightarrow r; B \rightarrow p$

(p) If $\frac{13\pi}{48} < \alpha < \frac{14\pi}{48}$ then $\frac{13\pi}{16} < 3\alpha < \frac{14\pi}{16}$ and $\frac{13\pi}{24} < 2\alpha < \frac{14\pi}{24}$

$\Rightarrow 3\alpha \in \text{II quad and } 2\alpha \in \text{II quad} \Rightarrow \sin 3\alpha = +ve$

$\cos 2\alpha = -ve \quad \therefore \frac{\sin 3\alpha}{\cos 2\alpha} = -ve$

$\therefore$ (B) corresponds to (p).

(q) If $\alpha \in \left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$ then $\frac{14\pi}{16} < 3\alpha < \frac{18\pi}{16}$ and $\frac{14\pi}{24} < 2\alpha < \frac{18\pi}{24}$

$\Rightarrow 3\alpha \in \text{II or III quad and } 2\alpha \in \text{II quad}$

$\Rightarrow$ Nothing can be said about the sing of $\frac{\sin 3\alpha}{\cos 2\alpha}$ over this interval.

(r) If $\alpha \in \left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$ then $\frac{18\pi}{16} < 3\alpha < \frac{23\pi}{16}$ and $\frac{18\pi}{24} < 2\alpha < \frac{23\pi}{24}$

$\Rightarrow 3\alpha \in \text{III quad and } 2\alpha \in \text{II quad}$

$\Rightarrow \sin 3\alpha = -ve, \cos 2\alpha = -ve, \therefore \frac{\sin 3\alpha}{\cos 2\alpha} = +ve$

$\therefore$ (A) corresponds to (r)

(s) If $\alpha \in \left(0, \frac{\pi}{2}\right)$, then $0 < 3\alpha < \frac{3\pi}{2}$ and $0 < 2\alpha < \pi$

$\Rightarrow$ Nothing can be said about the sing of $\frac{\sin 3\alpha}{\cos 2\alpha}$ over the given interval.

14. (True)

$$\tan A = \frac{1 - \cos B}{\sin B} = \frac{2 \sin^2 B / 2}{2 \sin B / 2 \cos B / 2} = \tan B / 2$$

$$\text{Now, } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \tan B / 2}{1 - \tan^2 B / 2} = \tan B$$

Hence, statement is true.

15. (1)

Rearrange the given expression

$$\begin{aligned} & \left(\frac{\sin \alpha}{\cos \beta} + \frac{\cos \beta}{\sin \alpha} + \frac{\cos \alpha}{\sin \beta} + \frac{\sin \beta}{\cos \alpha} \right)^2 \\ &= \left(\frac{\cos(\alpha - \beta)}{\sin \beta \cos \beta} + \frac{\cos(\alpha - \beta)}{\sin \alpha \cdot \cos \alpha} \right)^2 \\ &= \left(\frac{4}{3} \left\{ \frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha} \right\} \right)^2 \quad \left[\because \cos(\alpha - \beta) = \frac{2}{3} \right] \\ &= \frac{16}{9} \left[\frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha \cdot \sin 2\beta} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{64}{9} \left(\frac{2 \sin(\alpha + \beta) \cdot \cos(\alpha - \beta)}{2 \sin 2\alpha \cdot \sin 2\beta} \right)^2 \\
&= \frac{64}{9} \left(\frac{2 \cdot \frac{1}{3} \cdot \frac{2}{3}}{\cos(2\alpha - 2\beta) - \cos(2\alpha + 2\beta)} \right)^2 \\
&= \frac{64}{9} \left(\frac{\frac{4}{9}}{2 \cos^2(\alpha - \beta) - 1 - 1 + 2 \sin^2(\alpha + \beta)} \right)^2 \\
&= \frac{64}{9} \left(\frac{\frac{4}{9}}{\frac{8}{9} - 2 + \frac{2}{9}} \right)^2 = \frac{64}{9} \left(\frac{1}{-2} \right)^2 = \left[\frac{16}{9} \right] = [1.7] = 1
\end{aligned}$$

16. (2)

Let $f(\theta) = \frac{1}{g(\theta)}$, where $g(\theta) = \sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta$

Clearly, f is maximum when g is minimum

$$\text{Now, } g(\theta) = \frac{1 - \cos 2\theta}{2} + \frac{3}{2} \sin 2\theta + \frac{5}{2} (1 + \cos 2\theta)$$

$$= 3 + 2 \cos 2\theta + \frac{3}{2} \sin 2\theta \geq 3 + \left(-\sqrt{4 + \frac{9}{4}} \right)$$

$$\therefore g_{\min} = 3 - \frac{5}{2} = \frac{1}{2} \Rightarrow f_{\max} = 2.$$

17. (1)

$$\text{Let } \pi x - \frac{\pi}{4} = \theta \in \left[\frac{-\pi}{4}, \frac{7\pi}{4} \right]$$

$$\therefore f(x) \geq 0$$

$$\text{So, } \left(3 - \sin \left(\frac{\pi}{2} + 2\theta \right) \right) \sin \theta \geq \sin(\pi + 3\theta)$$

$$\Rightarrow (3 - \cos 2\theta) \sin \theta \geq -\sin 3\theta$$

$$\Rightarrow \sin \theta [3 - 4 \sin^2 \theta + 3 - \cos 2\theta] \geq 0$$

$$\Rightarrow \sin \theta (6 - 2(1 - \cos 2\theta) - \cos 2\theta) \geq 0$$

$$\Rightarrow \sin \theta (4 + \cos 2\theta) \geq 0 \Rightarrow \sin \theta \geq 0$$

$$\Rightarrow \theta \in [0, \pi] \Rightarrow 0 \leq \pi x - \frac{\pi}{4} \leq \pi \Rightarrow x \in \left[\frac{1}{4}, \frac{5}{4} \right]$$

$$\Rightarrow [\alpha, \beta] = \left[\frac{1}{4}, \frac{5}{4} \right]; \therefore \beta - \alpha = \frac{5}{4} - \frac{1}{4} = 1$$

18. (1/3)

$$\text{Given : } A+B=\frac{\pi}{3} \Rightarrow \tan(A+B)=\sqrt{3}$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = \sqrt{3} \Rightarrow \frac{\tan A + \frac{y}{\tan A}}{1 - y} = \sqrt{3} \quad [\text{Let } y = \tan A \tan B]$$

$$\Rightarrow \tan^2 A + \sqrt{3}(y-1)\tan A + y = 0$$

$$\text{For real value of } \tan A, 3(y-1)^2 - 4y \geq 0$$

$$\Rightarrow 3y^2 - 10y + 3 \geq 0 \Rightarrow (y-3)\left(y - \frac{1}{3}\right) \geq 0$$

$$\Rightarrow y \leq \frac{1}{3} \text{ or } \geq 3$$

$$\text{But } A, B > 0 \text{ and } A+B=\frac{\pi}{3} \Rightarrow A, B < \frac{\pi}{3}$$

$$\Rightarrow \tan A \tan B < 3$$

$$\Rightarrow y \leq \frac{1}{3} \Rightarrow \text{Max. value of } y \text{ is } \frac{1}{3}.$$

19. $[-\pi/2, -\pi/10] \cup [3\pi/10, \pi/2]$

$$\text{Given : } 2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}, t \in [-\pi/2, \pi/2]$$

$$\Rightarrow (6\sin t - 5)x^2 + 2(1-2\sin t)x - (1+2\sin t) = 0$$

The given equation will hold, if x be some real number, and hence, $D \geq 0$

$$\Rightarrow 4(1-2\sin t)^2 + 4(6\sin t - 5)(1+2\sin t) \geq 0$$

$$\Rightarrow 16\sin^2 t - 8\sin t - 4 \geq 0$$

$$\Rightarrow (4\sin^2 t - 2\sin t - 1) \geq 0$$

$$\Rightarrow \sin t \leq -\left(\frac{\sqrt{5}-1}{4}\right) \text{ or } \sin t \geq \frac{\sqrt{5}+1}{4}$$

$$\Rightarrow \sin t \leq \sin\left(-\frac{\pi}{10}\right) \text{ or } \sin t \geq \sin\left(\frac{3\pi}{10}\right)$$

$$\Rightarrow t \leq -\frac{\pi}{10} \text{ or } t \geq \frac{3\pi}{10} \quad [\text{Note that } \sin x \text{ is an increasing function from } -\frac{\pi}{2} \text{ to } \frac{\pi}{2}]$$

$$\therefore \text{ range of } t \text{ is } \left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right].$$

20. (1/8)

$$\begin{aligned} K &= \sin \frac{\pi}{18} \sin \frac{5\pi}{18} \sin \frac{7\pi}{18} \\ &= \cos\left(\frac{\pi}{2} - \frac{\pi}{18}\right) \cos\left(\frac{\pi}{2} - \frac{5\pi}{18}\right) \cos\left(\frac{\pi}{2} - \frac{7\pi}{18}\right) \end{aligned}$$

$$= \cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9} = \frac{1}{2^3 \sin \frac{\pi}{9}} \cdot \sin \frac{8\pi}{9}$$

$$[\because \cos \alpha \cos 2\alpha \cos 2^2 \alpha \dots \cos 2^{n-1} \alpha = \frac{1}{2^n \sin \alpha} \cdot \sin(2^n \alpha)]$$

$$= \frac{1}{8 \sin \frac{\pi}{9}} \cdot \sin \frac{\pi}{9} = \frac{1}{8}$$

21. (1/64)

$$\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14} \sin \frac{9\pi}{14} \sin \frac{11\pi}{14} \sin \frac{13\pi}{14}$$

$$= \left(\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \right)^2 \times 1 \quad [\because \sin \frac{13\pi}{14} = \sin \frac{\pi}{14}, \sin \frac{11\pi}{14} = \sin \frac{3\pi}{14} \text{ and } \sin \frac{9\pi}{14} = \sin \frac{5\pi}{14}]$$

$$= \left[\cos \left(\frac{\pi}{2} - \frac{\pi}{14} \right) \cos \left(\frac{\pi}{2} - \frac{3\pi}{14} \right) \cos \left(\frac{\pi}{2} - \frac{5\pi}{14} \right) \right]^2$$

$$= \left[\cos \frac{3\pi}{7} \cos \frac{2\pi}{7} \cos \frac{\pi}{7} \right]^2 = \left[\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} \right]^2 \quad [\because \cos \alpha \cos 2\alpha \cos 2^2 \alpha \dots \cos 2^{n-1} \alpha]$$

$$= \left[\frac{1}{2^n \sin \alpha} \cdot \sin(2^n \alpha) \right]^2$$

$$= \left(\frac{1}{8 \sin \frac{\pi}{7}} \sin \frac{8\pi}{7} \right)^2 = \left(\frac{\sin(\pi + \pi/7)}{8 \sin \pi/7} \right)^2$$

$$= \left(\frac{-\sin \pi/7}{8 \sin \pi/7} \right)^2 = \left(\frac{1}{8} \right)^2 = \frac{1}{64}$$