Sequence & Series

EXERCISE - 1 [A]

1. **(a)**

$\Rightarrow 5t_5 8t$

$\Rightarrow 5(a+4d)=8(a+7d)$

$\Rightarrow 3a=-36d$

$\Rightarrow d=-\frac{1}{12}a$

Now, $T_{13}=a+12d$

$\Rightarrow a+12\left(-\frac{1}{12}a\right)=0$

2. **(c)**

$\Rightarrow T_7=a+6d$

$\Rightarrow a+6d+34$ ....(1)

$\Rightarrow T_{13}=a+12d$

$\Rightarrow a+12d=64$ ....(2)

Solving (1) and (2)

$\Rightarrow a=4$ and $d=5$

$\Rightarrow \therefore T_{18}=a+17d$

$\Rightarrow 4+17\times 5=89$

3. **(b)**

$\Rightarrow T_7=a+6d$

$\Rightarrow a+6d=40$

$\Rightarrow a=40-6d$

$\Rightarrow S_{13}=\frac{12}{2}[2a+12d]$

$\Rightarrow \frac{13}{2}[2(40-6d)+12d]$

$\Rightarrow \frac{13}{2}[80]=13\times 40=520$

4. **(a)**

$\Rightarrow S_{40} = \frac{40}{2}[2a + 39d]$

$\Rightarrow 20[2(2) + 39(4)] = 3200$

5. **(c)**

Let the terms of A. P are $(a-d), a, (a+d)$

Now, $(a-d)+(a+d)=12$

$\Rightarrow 2a = 12$

$\Rightarrow a = 6$

$\Rightarrow$ and$(a-d)a = 24$

$\Rightarrow (6-d) = 6 = 24$

$\Rightarrow d = 2$

$\Rightarrow \therefore$ first tern $a - d = 6 - 2 = 4$

6. **(c)**

$\Rightarrow S_{2m} = \frac{2n}{2}[2(2)+(2n-1)(3)] = n[a+6n]$ ……….(1)

$\Rightarrow S_n = \frac{n}{2}[2(57)+(n-1)(2)] = n[56+n]$ ……….(2)

Solving (1) and (2)

$\Rightarrow 1 + 6n = 56 + n$

$\Rightarrow n = 11$

7. **(a)**

$\Rightarrow S_{10} = 4S_5$

$\Rightarrow \frac{10}{2} = [2a+9d] = 4 \times \frac{5}{2}[2a+4d]$

$\Rightarrow 2a = d$

$\Rightarrow \frac{a}{d} = \frac{1}{2}$

8. **(a)**

$\Rightarrow Sp = \frac{p}{2}[2a+(p-1)d] = x$ ………(1)

$\Rightarrow Sq = \frac{q}{2}[2a+(q-1)d] = y$ …..….(2)

$\Rightarrow Sr = \frac{r}{2}[2a+(q-1)d] = z$ ………(3)

Now substituting value from (1), (2), (3) in

$\Rightarrow \frac{x}{p} = (q-r) + \frac{y}{q}(r-p) + \frac{z}{r}(p-q)$

$\Rightarrow 2[2a-(p-1)d](q-r) + 2[2a-(q-1)d](r-p) + 2[2a-(r-1)d](p-q) = 0$

9. **(a)**

Odd two digit number will be 11, 13, 15, ………99 – total 45 numbers

$$\Rightarrow S=\frac{45}{2}\left[2(11)+(45-1)2\right]$$

$$\Rightarrow \frac{45}{2}[22+88]=2475$$

10. **(d)**

$$\Rightarrow S=\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+.......+\frac{1}{\sqrt{2n}+\sqrt{2n+1}}$$

$$\Rightarrow \frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}=\sqrt{3}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+.......+\frac{\sqrt{2n+1}-\sqrt{2n}}{2}$$

$$\Rightarrow \frac{1}{2}\left(\sqrt{2n+1}-1\right)$$

11. **(d)**

$\Rightarrow a_1, a_2.........a_{n+1}$ are in A. P.

Let $a_1=a$ and common difference be d

Then, $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+........+\frac{1}{a_na_{n+1}}$

$$\Rightarrow \frac{1}{a(a+d)}+\frac{1}{(a+d)(a+2d)}+........+\frac{1}{(a+nd)(a+(n-1)d)}$$

$$\Rightarrow \frac{1}{d}\left[\frac{d}{a(a+d)}+\frac{d}{(a+d)(a+2d)}+........+\frac{d}{(a+nd)(a+(n-1)d)}\right]$$

$$\Rightarrow \frac{1}{d}\left[\left(\frac{1}{d}-\frac{1}{a+d}\right)+\left(\frac{1}{a+d}-\frac{1}{a+2d}\right)+........+\left(\frac{2}{a+(n-1)d}-\frac{1}{a+nd}\right)\right]$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{a}-\frac{1}{a+nd}\right]$$

$$\Rightarrow \frac{1}{d}\left[\frac{a+nd-a}{a(a+nd)}\right]$$

$$\Rightarrow \frac{n}{a_1a_{n+1}}$$

12. **(d)**

Let first be a and common difference be d.

$a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$

$\Rightarrow 6a+23d=75$

Now, $S_{24}=\frac{24}{2}[2a+23d]=12[75]=900$

13. **(a)**

a, b, c are in A. P.

$\Rightarrow \frac{a+c}{2} = b$

$\Rightarrow \frac{a+c}{2abc} = \frac{1}{ac}$

$\Rightarrow \frac{ab+cb}{2abc} = \frac{1}{ac}$

$\Rightarrow \frac{\frac{1}{ab}+\frac{1}{bc}}{2} = \frac{1}{ac}$

$\Rightarrow \therefore \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A. P.

14. **(b)**

$\Rightarrow \log 2 \log\left(2^n - 1\right), \log\left(2^n + 3\right)$ are in A.P.

$\Rightarrow \therefore \log\left(2^n - 1\right) = \frac{\log 2 + \log 2^n + 3}{2}$

$\Rightarrow 2\log\left(2^n - 1\right) = \log\left(2 \times \left(2^n + 3\right)\right)$

$\Rightarrow \log\left(2^n - 1\right)^2 = \log\left(2^{n+1} + 6\right)$

$\Rightarrow \left(2^n - 1\right)^2 = 2^{n+1} + 6$

$\Rightarrow 2^{2n} + 1 - 2^{n+1} = 2^{n+1} + 6$

$\Rightarrow 2^{2n} - 4.2^n - 5 = 0$

Let $^n = t$

$\Rightarrow t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$\Rightarrow t = 5$ or $t = -1$

$\Rightarrow 2^n = 5$ or $2^n = -1$ (not possible)

$\Rightarrow \log_2^5 = n$

15. **(c)**

$x, |x+1|, |x-1| = A.P$

For x < - 1

$\Rightarrow x, -x-1, -x+1 = A.P$

$\Rightarrow \therefore -x-1-x = -2x-1$

$\Rightarrow -x+1+x+1 = 2$

From (1) and (2)

$\Rightarrow -2x-1 = 2$

$\Rightarrow -2x = 3x = \frac{-3}{2}$

$\Rightarrow \therefore S_{20} = \frac{20}{2}\left[2\left(\frac{-3}{2}\right) + (19)2\right] = 350$

16. **(b)**

$\Rightarrow a = 2n - 1$

$\Rightarrow n = \frac{a+1}{2}$

$\Rightarrow (1+3+5+.......p)+(1+3+5+......+q)=(1+3+5+.....+r)$

$\Rightarrow \left(\frac{p+1}{2}\right)^2 + \left(\frac{q+1}{2}\right)^2 = \left(\frac{r+1}{2}\right)^2$

$\Rightarrow (p+1)^2 + (q+1)^2 = (r+1)^2$

P > hence smallest pythagaion to put will be 6, 8, 10.

Therefore p = 7, q = 5, r = 9

Least value $p+q+r=21$

17. **(b)**

Let first term of G. P be A and common ratio be R.

$\Rightarrow T_p = AR^{p-1} = a$

$\Rightarrow T_q = AR^{-q-1} = b$

$\Rightarrow T_r = AR^{r-1} = c$

Now, $a^{q-r}.b^{r-p}.c^{p-q} = \left(AR^{(p-1)^{(q-r)}}\right).\left(AR^{(q-1)^{(r-1)}}\right).\left(AR^{(r-1)}\right)^{(p-q)}$

$\Rightarrow = A^0R^0 = 1$

18. **(c)**

Let the first term of G.P. be $\frac{a}{r^2}, \frac{1}{r}, a, ar, ar^2$

If third term is 4

$\Rightarrow a = 4$

$\therefore$ their product $= (a)^5 = (4)^5$

19. **(d)**

$\Rightarrow x, 2x+2, 3x+3$ are in G. P.

then $(2x+2)^2 = x(3x+3)$

$\Rightarrow 4x^2 + 4 + 8x = 3x^2 + 3x$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+1)(a+4) = 0$

$\Rightarrow x = -4$ or $x = -1$

if $x = -1$ then term will be – 1, 0, 0 Not possible

if $x = -4$

Then term will be -4, -6, -8

$\Rightarrow a = -4$

$\Rightarrow r = \frac{-6}{-4} = \frac{3}{2}$

$\Rightarrow T_4 = ar^3 = -4 \times \left(\frac{3^3}{2}\right) = -4 \times \frac{27}{8} = -13.5$

20. **(b)**

$a = x$ let common ratio be r.

$\Rightarrow S_{\infty} = 5$

$\Rightarrow \frac{x}{1-r} = 5$

$\Rightarrow r = \frac{5-x}{5}$

Or $r \in (-1,1)$ for an infinite G. P.

$\Rightarrow -1 < \frac{5-x}{5} < 1$

$\Rightarrow 10 > x > 0$

21. **(a)**

A, b, c are in A.P.

$\Rightarrow \therefore \frac{a+c}{2} = b$ ....(1)

and $c-b = b-a$ ....(2)

and $b-a, c-b a$ ar in G. P.

then $(c-b)^2 = a(b-a)$

form (2)

$\Rightarrow (b-a)^2 = a(b-a)$

$\Rightarrow b-a = a$ ....(3)

From (3)

$\Rightarrow b = 2a$

$\Rightarrow \frac{b}{a} = 2$

From (3)

$\Rightarrow b = 2a$

$\Rightarrow \frac{a+c}{2} = 2a$

$\Rightarrow \frac{c}{a} = 3$

$\Rightarrow \therefore a : b : c = 1 : 2 : 3$

22. **(b)**

Let $S = 3 + 33 + 333 + ...33.....33$

$\Rightarrow S = 3(1 + 11 + 111 + ... + 111.....111)$

$\Rightarrow 3\left(1 + (10+1) + (10^2 + 10 + 1) + ... + \left(10^n + 10^{a-1} + 10^{a-2} + ... + 10 + 1\right)\right)$

$\Rightarrow 3\left(n + 10(n-1) + 10^2(n-2) + ...10^n\right)$ .....(1)

Let $S^1 = n + 10(n-1) + 10^2(n-2) + ... + 10^n$ ......(2)

$\Rightarrow 10S^1 = 10n + 10^2(n-1) + ...210^n + 10^{n+1}$ ......(3)

(2) – (3)

$\Rightarrow -9S^1 = n - 10 - 10^2 - 10^3......10^{a+1}$

$\Rightarrow n-\left(10+10^{2}+10^{3}+.....10^{n}+10^{n+1}\right)$

$\Rightarrow n-\dfrac{10\left(10^{n}-1\right)}{10-1}=n-\dfrac{10^{n+1}-10}{9}$

$\Rightarrow \dfrac{9n-10^{n+1}+10}{9}$

$\Rightarrow S^{1}=\dfrac{10^{n+1}-10-9n}{81}$

$\Rightarrow \therefore$ From (1)

$\Rightarrow S=\dfrac{10^{n+1}-10-9n}{27}$

23. (b)

1234, 2345, 3456 ……….

d = 1111

$T_{n}=1234+\left(n-1\right)1111$

$=123+1111n$

24. (a)

$a=2+d$

$b=2+2d$

$c=\left(2+2d\right)d$

$2\left(a+d\right)d.d=160$

$\Rightarrow d.d\left(1+d\right)=4.4.5$

$\Rightarrow d=4$

$a=6, b=10$

$c=40$

$a+b+c=56$

25. (a)

$T_{6}=8T_{3}$

$\Rightarrow ar^{5}=8ar^{2}$

$\Rightarrow \ell^{3}=8$

$\Rightarrow r=2$

$T_{7}+t_{8}=192$

$\Rightarrow ar^{6}+ar^{7}=192$

$\Rightarrow a\left(64+128\right)=192$

$\Rightarrow a=1$ ……..(1)

$\Rightarrow a=1$ …….(2)

$T_{5}+T_{6}+.......T_{11}=\dfrac{2^{4}\left(2^{7}-1\right)}{2-1}=2032$

$T_{6}+T_{9}=2^{5}+2^{8}=288$

26. **(c)**

$$\Rightarrow \sum_{n=1}^{\infty} \sin^{2n}\theta = \frac{1}{1-\sin^2\theta} \Rightarrow x = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \sum_{n=1}^{\infty} \cos^{2n}\phi = \frac{1}{1-\cos^2\phi} \Rightarrow y = \frac{1}{\sin^2\phi}$$

$$\Rightarrow \sum_{n=1}^{\infty} \cos^{n}(\theta+\phi)\cos^{n}(\theta-\phi) = \frac{1}{1-\cos(\theta+\phi)\cos(\theta-\phi)}$$

$$\Rightarrow 2 = \frac{1}{1-\cos^2\theta+\sin^2\phi}$$

Now, $z = \dfrac{1}{1-\frac{1}{x}+\frac{1}{y}}$ or $z(xy-y+x) = xy$

$$\Rightarrow xyz - xy = yz - zx$$

27. **(b)**

Let $x = \sqrt{2}+1$

$\Rightarrow y = 1$

$\Rightarrow z = \sqrt{2}-1$

$$\Rightarrow \frac{x}{y} = \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{1} = \frac{y}{z}$$

$\Rightarrow \therefore x, y, z$ are in G. P.

28. **(d)**

$$\Rightarrow (a)\frac{1}{b-a}+\frac{1}{b-c} = \frac{1}{b}$$

$$\Rightarrow \frac{b(b-c)+b(b-a)}{b(b-a)(b-c)} = (b-a)(b-c)$$

$$\Rightarrow b^2+bc+b^2-ab$$

$$\Rightarrow b^2+ac-ab-bc$$

$$\Rightarrow b^2 = ac$$

a, b, c are in G.P.

but a, b, c are in H. P. so not correct

(b) as a, b, c, are in H. P.

$$\Rightarrow b = \frac{2ac}{a+c}$$

But $b = \dfrac{2ac}{a+c}$ is given so not correct

$$\Rightarrow (c)\frac{b+a}{b-a}+\frac{b+c}{b-c} = 1$$

$$\Rightarrow (b+a)(b-c)+(b+c)(b-a) = (b-a)(b-c)$$

$$\Rightarrow b^2+bc+ab+ac+b^2-ab+bc-ac$$

$$\Rightarrow b^2-bc-ab+ac$$

$$\Rightarrow b^2+bc+ab = 3ac$$

No result

$\therefore$ Answer is none.

29. **(c)**

$\Rightarrow b = \frac{2ac}{a+c}$

Now, $\frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{\frac{b}{a}+1}{\frac{b}{a}-1} + \frac{\frac{b}{c}+1}{\frac{b}{a}-1}$

$\Rightarrow \frac{\frac{2c}{a+c}+1}{\frac{2c}{a+c}-1} + \frac{\frac{2a}{a+c}+1}{\frac{2a}{a+c}-1} = \frac{3c+a}{c-a} + \frac{3a+c}{c-a} = 2$

30. **(a)**

$\Rightarrow y = \frac{2ab}{a+b}, x = \frac{2ay}{a+y}, z = \frac{2by}{b+y}$

or $y = \frac{2ab}{a+b}, x = \frac{4ab}{a+3b}, z = \frac{4ab}{3a+b}$

Now, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{a+b}{2ab} + \frac{a+3b}{4ab} + \frac{3a+b}{4ab}$

$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{10}{9}$

31. **(c)**

$\Rightarrow a, b, c$ are in H. P.

$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$ and $\frac{1}{c} = \frac{2}{b} - \frac{1}{a}$

Now, $\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right) = \left(\frac{1}{b} + \frac{2}{b} - \frac{1}{a} - \frac{1}{a}\right)\left(\frac{2}{b} - \frac{1}{b}\right)$

$\Rightarrow \left(\frac{3}{2} - \frac{2}{a}\right)\left(\frac{1}{b}\right) = \frac{3}{b^2} - \frac{2}{ab}$

32. **(c)**

$\Rightarrow \frac{a}{b}, \frac{b}{c}, \frac{c}{a} = \text{H.P.}$

$\Rightarrow \frac{b}{c} = \frac{2\frac{a}{b}\frac{c}{a}}{\frac{a}{b} + \frac{c}{a}}$

$\Rightarrow \frac{b}{c} = \frac{2\frac{c}{b}}{\frac{a^2+c^2}{ab}}$

$\Rightarrow a^2b + b^2c = 2ac^2$

33. **(c)**

$\Rightarrow a, b, c = G.P.$

$\Rightarrow b^2 = ac$

Now, $\frac{1}{\log_a^x} + \frac{1}{\log_b^x} = \log_x^a + \log_x^b = \log_x^{ab} = \log_x^{b^2}$

$\Rightarrow 2\log_x^b = 2\frac{1}{\log_b^x}$

$\Rightarrow \therefore \log_a^x, \log_b^x, \log_c^x = H.P$

34. **(b)**

Let $a = \frac{1}{\frac{1}{b} - d}$ and $c\frac{1}{\frac{1}{b} + d}$

$\Rightarrow a = \frac{b}{1-bd}$ and $c = \frac{b}{1+bd}$

Now, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{4}$

$\Rightarrow \frac{1-bd}{b} + \frac{1}{b} + \frac{1+bd}{b} = \frac{1}{4}$ hence b = 12

Now, a + b + c = 37

$\Rightarrow \frac{12}{1-12d} + 12 + \frac{12}{1+12d} = 37$

$\Rightarrow \frac{24}{1-144d^2} = 25$

$\Rightarrow d = \frac{1}{60}$. Hence numbers are 15, 12, 10

35. **(a)**

$\Rightarrow d = \frac{19-3}{3+1} = \frac{16}{4} = 4$

$\Rightarrow A_1 = a + d = 3 + 4 = 7$

$\Rightarrow A_2 = a + 2d = 11$

$\Rightarrow A_3 = a + 3d = 15$

36. **(b)**

A, b, c, d, e, f i.e. A. M. ‘s between 2 and 12

$\Rightarrow d = \frac{b-a}{n+1} = \frac{12-2}{6+1} = \frac{10}{7}$

$\Rightarrow S = \frac{8}{2}[2a + 7d] = 4[4 + 10] = 56$

$\Rightarrow \therefore a + b + c + d + e + f = 200 - a - b$

$\Rightarrow 56 - 2 - 12 = 42$

37. **(b)**

$\Rightarrow r=\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow n=2$

$\Rightarrow r=\left(\frac{64}{1}\right)^{\frac{1}{n+1}}=4$

$\Rightarrow G_1=ar=4$

$\Rightarrow G_2=ar^2=16$

38. **(b)**

$\Rightarrow G.M=3^{\frac{n+1}{2}}$

39. **(a)**

$\Rightarrow \frac{a+b}{2}=\frac{2ab}{a+b}$

$\Rightarrow a=b$

40. **(a)**

$\Rightarrow A_2+A_2=a+b, G_1G_2=ab$

$\Rightarrow \frac{1}{H_1}+\frac{1}{H_2}=\frac{a+b}{ab}$

$\Rightarrow \therefore \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2}$

$\Rightarrow \frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$

41. **(d)**

Let the two number be a and b, then

$\Rightarrow \frac{\frac{2ab}{a+b}}{\sqrt{ab}}=\frac{12}{13}$

$\Rightarrow \frac{2\sqrt{ab}}{a+b}=\frac{12}{13}$

$\Rightarrow \frac{\left(a+b\right)^2-\left(2\sqrt{ab}\right)^2}{a+b}=\frac{5}{13}$

$\Rightarrow \frac{a-b}{a+b}=\frac{5}{13}$

$\Rightarrow 13a-13b-5a+5b$

$\Rightarrow \frac{a}{b}=\frac{9}{4}$

42. **(c)**

$\Rightarrow \frac{a+b}{2} - \sqrt{ab} = 2$

$\Rightarrow \frac{a}{b} = \frac{4}{1} = a = 4b.$

$\Rightarrow \frac{4b+b}{2} - \sqrt{4b^2} = 2$

$\Rightarrow \frac{5}{2}b - 2b = 2$

$\Rightarrow b = 4$ and a = 16

43. **(c)**

$\frac{\frac{a+b}{2}}{\frac{2ab}{a+b}} = \frac{m}{n}$

$\Rightarrow \frac{(a+b)^2}{4ab} = \frac{m}{n}$

$\Rightarrow \frac{(a+b)^2}{(a+b)^2 - 4ab} = \frac{m}{m-n}$

$\Rightarrow \frac{a+b}{a-b} = \frac{\sqrt{m}}{\sqrt{m-n}}$

$\Rightarrow \frac{a}{b} = \frac{\sqrt{m}+\sqrt{m-n}}{\sqrt{m}-\sqrt{m-n}}$

44. **(c)**

$x = \frac{\log 3}{\log 5} + \frac{\log 5}{\log 7} + \frac{\log 7}{\log 9}$

$\frac{x}{3} \geq \left(\frac{\log 3}{\log 5} \cdot \frac{\log 5}{\log 7} \cdot \frac{\log 7}{2og3}\right)$

(By Am $\geq$ am)

$\Rightarrow \frac{x}{3} \geq \left(\frac{1}{2}\right)^{1/3}$

$\Rightarrow x \geq \frac{3}{3\sqrt{2}}$

45. **(a)**

$A_1 = G_1 = H_1 = A$

$A_{3n-1} = G_{2n-1} = H_{2n-1} = 8$

So, $An = \frac{A+B}{2}, Gn = \sqrt{AB}, Hn = \frac{2AB}{A+B}$

$\Rightarrow \boxed{b^2 = ac}$

46. **(d)**

If A. M. are inserted between two given number then product of rth A.M. from being and rth H.M. form and is equal to the product 0 there numbers.

Hence, $a_4 \times h_7 = 2\times 3$ i.e. 6

47. **(b)**

$\Rightarrow a_1 + a_{2n} = a_2 + a_{2n-1} = a_3 + a_{2n-2} = a + b$

$\Rightarrow g_1 g_{2n} = g_2 g_{2n-1} = g_3 g_{2n-2} = ... = ab$

Hence, $\frac{a_1 + b_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \frac{a_3 + a_{2n-2}}{g_3 g_{2n-3}} + ... = n\frac{a+b}{ab}$

But $\frac{2ab}{a+b} = h$, therefore $\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \frac{a_3 + a_{2n-2}}{g_3 g_{2n-3}} + ... = \frac{2n}{h}$

48. **(b)**

$\Rightarrow \frac{a+b}{2} = \frac{3}{2}$

$\Rightarrow a + b = 3$

$\Rightarrow \frac{2ab}{a+b} = \frac{4}{3}$

$\Rightarrow 2ab = 4$

$\Rightarrow ab = 2$

$\Rightarrow \therefore x^2 - 3x + 2 = 0$

49. **(b)**

$$\Rightarrow \frac{\pm\sqrt{\frac{c}{a}}}{\pm\sqrt{\frac{n}{1}}} = \pm\sqrt{\frac{cl}{an}}$$

50. **(b)**

$$\Rightarrow \frac{1}{xy - x^2} + \frac{1}{xy - y^2} = \frac{1}{x(y-x)} - \frac{1}{y(y-x)}$$

$$\frac{y-x}{(y-x)xy} = \frac{1}{xy} = \frac{1}{G^2}$$

51. **(b)**

$$\Rightarrow d = \frac{b-a}{n+1} = \frac{38-2}{n+1} = \frac{36}{n+1}$$

If n A. M. are inserted between 2 and 38 then total numbers of terms A. P. is n + 2

$$\Rightarrow S_{n+2} = \frac{n+2}{2}\left[2a + (n+2-1)d\right]$$

$$\Rightarrow \frac{n+2}{2}\left[2(2) + (n+1)\frac{36}{n+1}\right] = 200$$

$$\Rightarrow \frac{n+2}{2}[4+36]=200$$
$$\Rightarrow n+2=10$$
$$\Rightarrow n=8$$

52. **(b)**

Let $S=1+2.2+3.2^2+4.2^3+.....+100.2^{99}$ ....(1)

$\Rightarrow 2S=2+2.2+3.2^2+.......+99.2^{99}+100.2^{100}$ ....(2)

(1) - (2)

$$\Rightarrow -1S=1+2+2^2+2^3+.......2^{99}-100.2^{100}$$
$$\Rightarrow 1\frac{\left(2^{100}-1\right)}{2-1}-100.2^{100}$$
$$\Rightarrow 2^{1000}-1-100.2^{100}$$
$$\Rightarrow S=99.2^{100}+1$$

53. **(c)**

$$\Rightarrow a^3+2^3+3^3+4^3+......15^3\text{is}$$
$$\Rightarrow S=\left(\frac{n(n+1)}{2}\right)^2=\left(\frac{15(15+1)}{2}\right)^2=(120)^2=14400$$

54. **(d)**

$$\Rightarrow \left(1^2-t_1\right)+\left(2^2-t_2\right)+......+\left(n^2-t_n\right)=\frac{1}{3}n\left(n^2-1\right)$$
$$\Rightarrow 1^2+2^2+.........n^2-\left(t_1+t_2+........t_n\right)=\frac{1}{3}n\left(n^2-1\right)$$
$$\Rightarrow \frac{n(n+1)}{2}$$
$$\Rightarrow t_n=n$$

55. **(d)**

$$\Rightarrow \frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+.....$$
$$\Rightarrow \frac{1}{4}\left[\frac{1}{3}-\frac{1}{7}\right]+\frac{1}{4}\left[\frac{1}{7}-\frac{1}{11}\right]+\frac{1}{4}\left[\frac{1}{11}-\frac{1}{15}\right]$$
$$\Rightarrow=\frac{1}{4}\left[\frac{1}{3}-\frac{1}{\infty}\right]=\frac{1}{12}$$

56. **(b)**

$$\Rightarrow \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+......$$
$$\Rightarrow 1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}........=1$$

57. **(a)**

$$\Rightarrow S=\frac{\left(1+2+3+.......+n\right)^2-\left(1^2+2^2+3^2+.......+n^2\right)}{2}$$

$$\Rightarrow \frac{n^2\left(n+1\right)^2}{8}-\frac{n\left(n+1\right)\left(2n+1\right)}{12}$$

$$\Rightarrow \frac{n\left(n+1\right)\left(n-1\right)\left(3n+1\right)}{24}$$

58. **(b)**

$$\Rightarrow \frac{1}{3}+\frac{1}{3^n}+\frac{1}{3^3}+.....\infty=\frac{1}{2}$$

Hence $y=\left(0.64\right)^{\log_{0.25}^{0.5}}$

$$\Rightarrow y=\left(0.64\right)^{\frac{1}{2}}=0.8$$

59. **(a)**

$S=1.3^2+2.5^2 3.7^2+......$

$\Rightarrow T_n=n.\left(2n+1\right)^2$

$\Rightarrow T_n=4n^3+4n^2+n$

$\Rightarrow S_n\sum 4n^3+4n^2+n$

$$\Rightarrow 4\left(n\frac{n+1}{2}\right)^2+4\left(\frac{n\left(n+1\right)\left(2n+1\right)}{6}\right)+\frac{n\left(n+1\right)}{2}\Rightarrow 188090$$

60. **(b)**

Let $S=1+\frac{3}{2}+\frac{5}{2^2}+\frac{7}{2^3}+.......$ …….(1)

$\Rightarrow \frac{1}{2}s=\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^2}+......$ ……..(2)

(1) – (2)

$$\Rightarrow \frac{1}{2}s=1+\frac{2}{2}+\frac{2}{2^2}+\frac{2}{2^3}+.......$$

$$\Rightarrow 1+2\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....\right)$$

$$\Rightarrow 1+2\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)=1+2=3$$

$$\Rightarrow S=6$$

EXERCISE - 1 [B]

1. **(b)**
-4, -1, +2, +5 + ….
Is an A.P. with
First term a = -4
And common difference d = 3
Therefore

$$T_n = a + (n-1)d$$

$$\Rightarrow T_{10} = -4 + (10-1)*3$$

$$\Rightarrow T_{10} = 23$$

2. **(a)**
First term a = 2
Common difference d = 4
n = 40

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[4 + (40-1)4\right]$$

=1600

3. **(d)**
4, 9, 14, ….., 104
First term a = 4
Common difference d = 5
n^{th} tem is $T_n = 104$

$$T_n = a + (n-1)d$$

$$\Rightarrow 104 = 4 + (n-1)5$$

$$\Rightarrow n = 21$$

Therefore, middle term will be 11^{th} term

$$T_{10} = 4 + (11-1)5$$

= 54

4. **(b)**

$$T_9 = 0$$

$$\Rightarrow a + (9-1)d = 0$$

$$\Rightarrow a = -8d$$

Now,

$$T_{29/}T_{19} = \frac{a + (29-1)d}{a + (19-1)d} = \frac{a + 28d}{a + 18d} \qquad = \frac{8d + 28d}{-8d + 18d} = \frac{2}{1}$$

$$T_{29} : T_{19} = 2:1$$

5. **(b)**

Number lying between 10 and 200 are the numbers which are multiple of 7

14, 21, 28, ……, 196

a = 14

d = 7

$T_n = 196$

$T_n = a + (n-1)d$

$\Rightarrow 196 = 14 + (n-1)7$

$\Rightarrow n = 27$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{27}{2}[2*14 + (27-1)7]$

= 2835

6. **(b)**

Let first term = a

Common difference = d

Then, A.P. be

$a, (a+d), (a+2d), (a+3d), ...$

$T_4 = a + 3d$

$\Rightarrow a + 3d + 3a$

$\Rightarrow a = \frac{3}{2}d$ ……(1)

$T_7 - 2(T_3) = 1$

$\Rightarrow a + 6d - 2(a + 2d) = 1$

$\Rightarrow 2d - a = 1$

Substituting value of a from (1)

$2d - \frac{3}{2}d = 1$

$\Rightarrow d = 2$

7. **(a)**

Let the term of A.P. is a

And common difference is d

So,

$T_p = a + (p-1)d = A$

$T_Q = a + (Q-1)d = B$

$T_r = a + (r-1)d = C$

Therefore,

$A(Q-r) + B(r-p) + C(p-Q)$

$= a(a + (p-1)d)(Q-r) + (a + (Q-1)d)(a + (r-1)d)(p-Q)$

= 0

8. **(b)**

$$\frac{Sn}{Sn}=\left(\frac{n}{2}\right)\left(2a+(n-1)d\right)/\left(\frac{n}{2}\right)\left(2a'+(n-1)d'\right)$$

$$\frac{Sn}{Sn'}=\frac{2a+(n-1)d}{2a+(n-1)d'}$$

$$\frac{2a+(n-1)d}{2a+(n-1)d'}=\frac{3n+8}{7n+15}$$

Let, substituting n = 23

$$\frac{2a+(23-1)d}{2a+(23-1)d'}=\frac{3*23+8}{7*23+15}$$

$$\frac{a+11d}{a+11d'}=\frac{77}{176}$$

$$T_{12/}T_{12'}=7/16$$

9. **(b)**

$$S_n : n/2\left(2a+(n-1)d\right)=2n^2+5n$$

$$S_1 : ½(2a)=2+5=7$$

$$\Rightarrow a=7$$

$$S_2 : (14+d)=18$$

$$\Rightarrow d=4$$

$$T_n=a+(n-1)d=7+(n-1)4=4n+3$$

10. **(b)**

Let the three terms of A.P. are a – d, a, a + d

Sum of first terns

$$a-d+a+a+d=3a=51$$

$$\Rightarrow a=17$$

Product of first and third term

$$(a-d)(a+d)=a^a-d^a$$

$$\Rightarrow 17^2-d^2=273$$

$$\Rightarrow d^2=16$$

$$\Rightarrow d=4$$

So, third term

$$a+d=17+4=21$$

11. **(b)**

Let the four terms of A.P. are

$$a-3d, a-d, a+d, a+3d$$

Then,

$$a-3d+a-d+a+d+a+3d=20$$

$$\Rightarrow 4a=20$$

$$\Rightarrow 4=5$$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{2}{3}$$

$$\Rightarrow \frac{3(a^2-9d^2)=2(a^a-d^a)}{(a-d)(a+d)}=\frac{2}{3}$$

$$\Rightarrow 3(a^2-9d^2)=2(a^a-d^a)$$

$$\Rightarrow a^2 25d^2$$

$$\Rightarrow a=5d$$

$$\Rightarrow d=1$$

Smallest term

$$a-3d=5-3=2$$

12. **(a)**

Let the three numbers are

$$a-d, a, a+d$$

$$(a-d)(a+d)=5a$$

$$\Rightarrow (a^2-d^2)=5a \qquad \ldots\ldots (1)$$

$$a+a+d=8(a-d)$$

$$\Rightarrow 6a=9d$$

$$\Rightarrow 2a=3d \qquad (2)$$

Solving (1) and (2), we get

$$a=9, d=6$$

So, the numbers are 3, 9, 15.

13. **(c)**

Sum of interior angles of an $n=\text{gon}=(n-2)\times 180^o$

Sum of n terms of A.P. $(a=120^0, d=5^o)=\frac{n}{2}\{2\times 120^o+(n-1)\times 5^o\}$.

Hence $\frac{n}{2}\{2\times 120^o+(n-1)\times 5^o\}=(n-2)\times 180^o$

$\Rightarrow n^2-25+144=0 \Rightarrow n=9$ or 16.

But for $n=16,$ greatest angle exceeds 180^o hence only 9 is correct.

14. **(b)**

Common difference of the two A.P.s are 4 & 5, hence common difference of A.P. formed by common terms will be 20. Also the first common term is 21. Now

$S=100(2\times 21\times 20)=402200$.

15. **(c)**

m^{th} term of first series $=2m+61, m^{th}$ term of second series $=7m-4$. $7m-4=2m+61 \Rightarrow m=13$.

16. **(c)**

$d_1=3 \,\&\, d_2=2 \Rightarrow d(\text{common terms})=6$

First common term = 5

Hence common term are 5, 11, 17,…
Now general term = 6n – 1.
60^{th} term of first A.P. = 179
50^{th} term of second A.P. = 101
Comparing $6n-1$ with 101 gives n = 17

17. **(a)**
$a+e=b+d=2c \Rightarrow a-4b+6c-4d+2=0$.

18. **(b)**
Given $11+11+d+11+2d+11+3d=56$ &
$11+(n-4)d+11+(n-3)d+11+(n-2)d+11+(n-1)d=12$
$\Rightarrow d=2$ & $(2n-5)d=34$ or $n=11$.

19. **(c)**
$\frac{2n}{2}\{2\times 2+(2n-1)\times 3\}=\frac{n}{2}\{2\times 57+(n-1)\times 2\} \Rightarrow n=11.$

20. **(c)**
$(a+6d)-(a+d)=20 \Rightarrow d=4$ & $a+2d=9 \Rightarrow a=1$.
Now n^{th} term $=4n-3=2001 \Rightarrow n=501.$

21. **(a)**
$(1+3+5+...p\text{ terms})+(1+3+5+...q\text{ terms})=(1+3+5+...r\text{ terms}) \Rightarrow p^2+q^2=r^2$
Now smallest pythagorian triplet will be 3, 4 ,5, hence least value of $p+q+r=12.$

22. **(b)**
As a, x, y, z, b are in A.P. therefore $x+z=a+b$ & $y=\frac{a+b}{2}$
$\Rightarrow x+y+z=\frac{3}{2}(a+b)$. Hence $a+b=10$

23. **(a)**
Let the number be $a-d, a, a+d.$
Now $a-d+a+a+d=15 \Rightarrow a=5$
As given $a-d+1, a+4, a+d+19$ are in G.P. hence
$(a+4)^2=(a-d+1)(a+d+19) \Rightarrow 81=16=(6-d)(24+d) \Rightarrow d=3$.
Numbers are 2, 5, 8.

24. **(b)**
Let the first term is a
Common difference is d
Then,
$T_2=a$
$T_3=a+d$
$T_6=a+4d$

T_2, T_3 and T_6 are in G.P., Then

$(a+d)^2 = a(a+4d)$

$\Rightarrow a^2 + 2ad + d^2 = a^2 + 4ad$

$\Rightarrow d^2 = 2nd$

$\Rightarrow d = 2a$

Common ratio

$T_{3/}T_2 = (3a/a) = 3$

25. **(a)**

18, -12, 8, …….. - is in G.P.

Common ratio

$r = -\frac{12}{18} = -\frac{2}{3}$

$T_r = ar^a$

$\Rightarrow \frac{512}{729} = 18\left(-\frac{2}{3}\right)^n$

$\Rightarrow n - 1 = 8$

$\Rightarrow n = 9$

26. **(c)**

Let the first term of G.P. is a

And the common ratio is r

Then, the five consecutive terms of G.P. are

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

$\Rightarrow a = 4$

Then,

$\frac{a}{r^2} * \frac{a}{r} * a * ar * ar^2 = a^5 = 4^5$

27. **(c)**

Let the first term of G.P. is a

And the common ratio is r

Then,

$T_3 = ar^2 = 15$ (1)

$T_7 = ar^6 = 135$ (2)

Solving (1) and (2), we get

$r^4 = 9$

$a = 5$

Therefore,

$T_5 = ar^4 = 5*9 = 45$

28. **(b)**

$1, x^2, 6 - x^2$ are in G.P. then

$$\frac{x^2}{1}=\frac{6-x^2}{x^2}$$
$$\Rightarrow x^4=6-x^2$$
$$\Rightarrow x^4+x^2=6=0$$

29. (a)

$1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+....$ are in G.P.

With common ratio $-\frac{1}{3}$

Then, the sum infinite G.P. is

$$S_n=\frac{a}{a-r}=\frac{1}{\left(1+\frac{1}{3}\right)}=\frac{3}{4}$$

30. (b)

$$1+\frac{2}{x}+\frac{4}{x^3}+\frac{8}{x^3}+....$$

Sum of infinite term is finite when common ratio is less than 1

i.e. $\left|\frac{2}{x}\right|<1$

$$\Rightarrow |x|$$

31. (b)

$$96+48+24+12+....+\frac{3}{16}$$

Then, the common ratio $\frac{48}{96}=\frac{1}{2}$

$$T_n=ar^n$$
$$\Rightarrow \frac{3}{16}=96\left(\frac{1}{2}\right)^{n-1}$$
$$\Rightarrow \frac{3}{2^{n-6}}=\frac{3}{16}$$
$$\Rightarrow n=10$$

32. (c)

$3+3a+3a^2+...=\frac{45}{8}$ is a G.P.

$$S_n=\frac{a}{1-r}$$
$$\Rightarrow \frac{3}{1-a}=\frac{45}{8}$$
$$\Rightarrow 24=45(1-a)$$

$\Rightarrow 45a = 45 - 24 = \frac{21}{45} = \frac{7}{15}$

33. **(c)**

Let the number be a, ar, ar^2

Then,

$a + ar + ar^2 = 155$

$\Rightarrow a\left(1 + r + r^2\right) = 155$ (1)

And,

$ar^2 - a = 120$

$\Rightarrow a\left(r^2 - 1\right) = 120$ (2)

Solving (1) and (2), we get

$r = 5$ and $a = 5$

34. **(d)**

Let the numbers be a, ar, ar^2

Then there sum

$a + ar + ar^2 = 14$ (1)

And sum of their square

$a^2 + a^2r^2 + a^2r^4 = 84$ (2)

Squaring (1) and subtracting (2), we get

$\left(a + ar + ar^2\right)^2 - a^2 + a^2r^2 + a^2r^4 = 196 - 84$

$\Rightarrow 2ar\left(a + ar + ar^2\right) = 12$

$\Rightarrow ar = 4$

Subtracting this in (1) and solving, we get

$r = 2$ and $a = 2$

Therefore three numbers are 2, 4, 8

35. **(b)**

Let the four tern be a, ar, ar^2, ar^3

Then,

$a + ar^2 = 40$

$\Rightarrow a\left(1 + r^2\right) = 40$

$\Rightarrow \left(1 + r^2\right) = \frac{40}{a}$ (1)

And

$ar + Ar^2 = 80$

$\Rightarrow ar\left(1 + r^2\right) = 80$

From (1)

$ar\left(\frac{40}{a}\right) = 80$

$\Rightarrow r = 2$ and $a = 8$

36. **(b)**

a, b, c are in G.P.

Let the common ration is r

i.e. $\frac{b}{a} = \frac{c}{a} = r$

Then, for a^{-1}, b^{-1}, c^{-1}

$\frac{b^{-1}}{a^{-1}} = \frac{a}{b} = \frac{1}{r}$ and $\frac{c^{-1}}{b^{-1}} = \frac{b}{c} = \frac{1}{r}$

Therefore, a^{-1}, b^{-1}, c^{-1} are also in G.P.

37. **(a)**

Given $a \times ar \times ar^2 = 216$ & $a \times ar + ar \times ar^2 + ar^2 \times a = 126.$

Or $(ar)^3 = 216$ & $a^2 r(1 + r + r^2) = 126.$

$\Rightarrow 2r^2 - 5r + 2 = 0.$ hence $r = \frac{1}{2}$ & $a = 12.$

Now $a = 12, b = 6, c = 3.$

38. **(a)**

$x = \log_{0.4}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...\infty \text{ terms}\right) \Rightarrow c = \log_{0.4}\left(\frac{1}{2}\right)$

Hence $(0.16)^x = (0.16)^{-\log_{0.4}{}^{2}} = 2^{-\log_{0.4}{}^{0.16}}$

Therefore $(0.16)^x = 2^{-2} = \frac{1}{4}.$

39. **(c)**

$t_n = 3 \times 2^{n-1}$. Now $12288 = 3 \times 2^{12}$.

Hence $m = 13.$

40. **(a)**

$S_{10} = \frac{a(r^{10} - 1)}{r - 1}$ & $S_5 = \frac{a(r^5 - 1)}{r - 1}$. Now $\frac{S_{10}}{S_5} = 244 \Rightarrow \frac{r^{10} - 1}{r^5 - 1} = 244$ or $r = 3$.

41. **(a)**

Let the first term a and common ratio be b, then

$x = ab^{p-1}, y = ab^{q-1}, z = ab^{r-1} \Rightarrow \frac{y}{x} = b^{q-p}, \frac{z}{y} = b^{r-q}, \frac{x}{z} = b^{p-r}$

Now $x^{q-r} y^{r-p} z^{p-q} = \left(\frac{y}{z}\right)^r \left(\frac{z}{y}\right)^p \left(\frac{x}{z}\right)^q = b^{r(q-p)+p(r-p)+q(p-r)}$

Or $x^{q-r} y^{r-p} z^{p-q} = b^0 = 1.$

42. **(b)**

$9^{1/3}.9^{1/9}.9^{1/27}...\infty \text{ terms} = 9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...\infty \text{ terms}}$

$= 9^{\frac{1/3}{9^{1-1/3}}} = 9^{1/2} = 3$.

43. **(d)**

$x = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\infty \text{ terms} = \frac{1}{2}$

Now $x^{\log_b a} = \left(\frac{1}{2}\right)^{\log_{\sqrt{5}} 0.2} = 4.$

44. **(c)**

Given $a + ar + ar^2 + ... + ar^9 = S_1 \;\&\; ar^{10} + ar^{11} + ar^{12} + ...ar^{19} = S_2$

$\Rightarrow \frac{a\left(1-r^{10}\right)}{1-r} = S_1 \;\&\; \frac{ar^{10}\left(1-r^{10}\right)}{1-r} = S_2$

Or $\frac{S_2}{S_1} = r^{10}$.

45. **(b)**

P, , r are in A.P.

$\Rightarrow Q., -p = r - p$ (1)

$T_p = ar^{(p-1)}$

$T_Q = ar^{(Q-1)}$

$Tr = ar^{(r-1)}$

$\frac{ar^{Q-1}}{ar^{p-1}} = r^{Q.-p}$

And

$\frac{ar^{r-1}}{ar^{Q.-1}} = r^{r-Q.}$

From (1) we get

Common ratio is same

Then T_p, T_Q, T_r are in G.P.

46. **(c)**

$T_m = \frac{1}{a + (m-1)d} = n$

$\Rightarrow n\left(a + (m-1)d\right) = 1$ (1)

$T_n = \frac{1}{a + (n-1)d} = m$

$\Rightarrow m\left(a + (n-1)d\right) = 1$ (2)

From (1) and (2)

$n\left(a + (m-1)d\right) = m\left(a + (n-1)d\right)$

$na + (m-1)nd = ma + m(n-1)d$

$(n-m)a = (n-m)d$

$a = d$

$$T_m = \frac{1}{a+(m-1)a} = n$$

$$\Rightarrow a = \frac{1}{mn}$$

$$T_r = \frac{1}{a+(r-1)d} = \frac{1}{a+(r-1)a} = \frac{mn}{r}$$

47. **(d)**

First term is 1

N A.M's are inserted between the 1 and 51 then it become a A.P. of n+2 terms

Let the common difference is d

Then,

4^{th} A.M. will be the 5^{th} term of the A.P.

And 7^{th} A.M. will be the 8^{th} term of the A.P.

$$T_5 = 1+(5-1)d = 1+4d$$

$$T_8 = 1+(8-1)d = 1+7d$$

$$\frac{1+4d}{1+7d} = \frac{3}{5}$$

$$\Rightarrow d = 2$$

So, $T_{(n+2)} = 1+(n+2-1)d = 51$

$$\Rightarrow (n+1)2 = 50$$

$$\Rightarrow n = 24$$

48. **(b)**

x, y, z are in A.P.

a is the A.M. of x and y

$$\Rightarrow a = \frac{x+y}{2} \qquad (1)$$

b is the A.M. of y and z

$$\Rightarrow b = \frac{y+z}{2} \qquad (2)$$

Adding (1) and (2)

$$\frac{a+b}{2} = y$$

49. **(b)**

Let the common difference is d

Then,

$\frac{1}{3}, \frac{1}{3}+d, \frac{1}{4}+2d, \frac{1}{24}$ are in A.P.

$$d = \frac{1}{24} = \frac{1}{3} - 2d$$

$$\Rightarrow d = \frac{-7}{72}$$

$$A_1 = \frac{1}{3} + \left(\frac{-7}{24}\right) = \frac{17}{72}$$

$$A_2 = \frac{1}{3} + 2\left(\frac{-7}{24}\right) = \frac{5}{36}$$

50. **(c)**

H.M. between $\frac{a}{b}, \frac{b}{a}$ is

$$H = \frac{2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)}{\left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)} = \frac{2ab}{a^2b^2}$$

51. **(b)**

$\frac{2}{3}, a, b, c, d, \frac{2}{13}$ are in H.P

Then,

$\frac{3}{2}, \frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{13}{2}$ are in A.P.

So, then Second H.M is the second A.M and it will be the 3rd term of the A.P.

$$T_6 = \frac{3}{2} + (6-1)d = \frac{13}{2}$$

$\Rightarrow d = 1$

Therefore,

$$\frac{1}{b} = \frac{3}{2} + (3-1)d$$

$$\Rightarrow b = \frac{2}{7}$$

52. **(b)**

Let the one number be a the other number will be 4a

Then,

$AM + 2 = GM$

$$\Rightarrow \frac{a+4a}{2} + 2 = \sqrt{a4a}$$

$$\Rightarrow \frac{5a}{2} + 2 = 2a$$

$\Rightarrow a = 4$

53. **(c)**

Let the two numbers is a, b

Then,

$$\frac{a+b}{2} = 34 \qquad (1)$$

And,

$$16^2 = ab \qquad (2)$$

Solving (1) and (2)
$a = 4, b = 64$

54. **(c)**
Let the two numbers is a, b
$\frac{a+b}{2} = A$
$ab = G^2$
$\frac{2ab}{a+b} = 4$
$\Rightarrow 8\left(\frac{a+b}{2}\right) = 2ab$
$\Rightarrow 4(A) = G^2$
$2A + G^2 = 27$
$\Rightarrow A = 4.5$ (1)
$\Rightarrow ab = 18$ (2)
Solving (1) and (2) we get
$a = 6$
$b = 3$

55. **(c)**
Let the two numbers is a, b
Then,
$\frac{a+b}{2} = x \quad ab = y$
$\frac{2ab}{a+b} = Z$
So, $z < y < x$

56. **(a)**
Let the two numbers is a, b
$AM = GM + 5$
$\frac{a+b}{2} = \sqrt{ab} + 5$ (1)
$GM = HM + 4$
$\sqrt{ab} = \frac{2ab}{a+b} + 4$ (2)
From (1), subtracting the value of $\sqrt{ab}$
$\frac{a+b}{2} = \frac{2ab}{a+b} + 4$ (3)
From (1)
$ab = \left(\frac{a+b}{2} - 5\right)^2$ (4)
Subtracting value of ab from (4) in (3) we get

$$\frac{a+b}{2}-5=\frac{2}{a+b}\left(\frac{a+b}{2}-5\right)^2+4$$

Solving this we get

$a=10$

$b=40$

57. **(c)**

Let the sum is S

$S=1+3x+5x^2+7x^3+....$ (1)

$xS= \quad x+3x^2+5x^2+....$ (2)

(1) – (2)

$(1-x)S=1+2x+2x^2+2x^3$

$(1-x)S=1+2\left(x+x^2+x^3+......\right)$

$(1-x)S=1+2\left(\frac{x}{1-x}\right)$

$$S=\frac{1+x}{(1-x)^2}$$

58. **(d)**

$S=1+2\left(1+\frac{1}{n}\right)+3\left(1-\frac{1}{n}\right)^2+.....$ (1)

$\left(1-\frac{1}{n}\right)S=\left(1-\frac{1}{n}\right)+2\left(1-\frac{1}{n}\right)^2+3\left(1-\frac{1}{n}\right)^2+.....$ (2)

(1) – (2)

$\frac{1}{n}S=1+\left(1-\frac{1}{n}\right)+\left(1-\frac{1}{n}\right)^2+\left(1-\frac{1}{n}\right)^3+......$

$\frac{1}{n}S=\frac{1}{1-\frac{1}{n}}=\frac{n}{n-1}$

$$S=\frac{n^2}{n-1}$$

59. **(b)**

$$\sum_{k=1}^{n}k^3=\left(\frac{n(n+1)}{2}\right)^2=\left(\sum_{k=1}^{n}k\right)^2$$

60. **(b)**

$S=\left(1+3+5+...20\text{ terms}\right)+\left(2+4+8+...20\text{ terms}\right)$

$\Rightarrow S=20^2+\frac{2\left(2^{20}-1\right)}{2-1}$ or $398+2^{21}$

EXERCISE - 1 [C]

1. (6534)

All number divisible by 6 are 6, 12, 18, …., 198

Sum $= \frac{33(6+198)}{2} = 3366$

Now sum of all the even numbers less than $200 = \frac{99(2+198)}{2} = 9900$

Hence required Sum $= 9900 - 3366 = 6534.$

2. (10)

$S = \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots n$ terms = $1 - \frac{1}{2} + 1 - \frac{1}{4} + 1 - \frac{1}{8} + 1 - \frac{1}{16} + \ldots n$ terms

$\Rightarrow S = n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots . n \text{ terms}\right)$ Or $S = n - \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = n - 1 + 2^{-n}$.

As given $S = 9 + 2^{-10}$ hence n = 10.

3. (64)

$S = 1 + n + n^2 + \ldots + n^{127} = \frac{n^{128} - 1}{n - 1}$

$\Rightarrow S = (n^2+1)(n^4+1)(n^8+1)(n^{1+}+1)(n^{32}+1)(n^{64}+1)$

Hence $n^m + 1$ will divides s for $n = 2, 4, 8, 16, 32, 64.$

4. (11)

$S = \frac{1}{2} + \frac{1}{4} + \ldots \infty$ terms = 2 & $S_n = 1 + \frac{1}{2} + \frac{1}{4} + \ldots n$ terms = $2\left(1 - \frac{1}{2^n}\right)$

Now $2 - 2\left(1 - \frac{1}{2^n}\right) < \frac{1}{1000} \Rightarrow \frac{1}{2^n} < \frac{1}{2000}$ or $2^n > 2000.$

Hence ≥ 11.

5. (900)

In first rebound ball will travel $2 \times 100 \times \frac{4}{5}$, in second rebound ball will travel $2 \times 100 \times \left(\frac{4}{5}\right)^2$, in second rebound will travel $2 \times 100 \times \left(\frac{4}{5}\right)^3$, and so on infinitely.

Hence total distance travelled $= 100 + 200 \times \left(\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \ldots \infty \text{ terms}\right) = 900$ mts.

6. **(2)**

As given $a+ar+ar^2+...+ar^{2n-1}=3\left(a+ar^2+ar^4+...+ar^{2n-2}\right)$

$$\Rightarrow \frac{a\left(1-r^{2n}\right)}{1-r}=3\frac{a\left(1-r^{2n}\right)}{1-r^2}\Rightarrow r=2.$$

7. **(16)**

$$\left(1+r\right)\left(1+r^2\right)\left(1+r^4\right)\left(1+r^8\right)=\frac{1-r^{16}}{1-r}\Rightarrow n=16.$$

8. **(6)**

Sum of n terms after first n terms $=S_{2n}-s_n=2S_n\Rightarrow S_{2n}\Rightarrow S_n=3S_n$

$$\Rightarrow \frac{2n}{2}\left\{2a+\left(2n-1\right)d\right\}=3\times\frac{n}{3}\left\{2a+\left(n-1\right)d\right\}$$

$$\Rightarrow a=\left(n+1\right)\frac{d}{2}.$$

Now $\frac{S_{3n}}{S_n}=\frac{\frac{3n}{2}\left\{2a+\left(3n-1\right)d\right\}}{\frac{n}{2}\left\{2a+\left(n-1\right)d\right\}}\Rightarrow\frac{S_n}{S_n}=\frac{3\left\{\left(n+1\right)d+\left(3n-1\right)d\right\}}{\left\{\left(n+1\right)d+\left(n-1\right)d\right\}}$

$$\Rightarrow \frac{S_{3n}}{S_n}=6.$$

9. **(1600)**

For integral roots discrmin ant must be a perfect square, hence $9+4a_i=k^2$.

The values of a_i for which it's a perfect square are $4,10,18,28,40.....,270$.

Now Let

$S_n=4+10+18+28+...+t_n$

$S_n=\quad 4+10+18+...+t_{n-1}+t_n$

$0=4+6+8+10+...n\text{ terms - }t_n$

$\Rightarrow t_n=\frac{n\left(n+3\right)}{2}$. Also 270 is 15th term.

Now $S_{15}=\frac{1}{2}\sum_{r=1}^{15}r^2+\frac{3}{2}\sum_{r=1}^{15}r$ or $S_{15}=\frac{15\times16\times31}{12}+\frac{3\times15\times16}{4}=1600$

10. **(8)**

$$\sum_{k=1}^{\infty}\frac{2^{k+2}}{3^k}=2^2\times\sum_{r=1}^{\infty}\left(\frac{2}{3}\right)^k\Rightarrow s=4\times\left(\frac{\frac{2}{3}}{1-\frac{2}{3}}\right)=8.$$

11. **(14)**

$\frac{a+b}{2}+\frac{2ab}{a+b}=25\ \&\ ab=144\Rightarrow\left(a+b\right)^2-50\left(a+b\right)+576=0$. Hence $a+b=18$ or 32.

12. **(2)**

Let $b = ar \,\&\, c = ar^2, 2p = a + ar, 2q = ar + ar^2 \Rightarrow 2p = a(1+r), 2q = ar(1+r)$

$$\frac{a}{p} + \frac{c}{q} = \frac{2}{1+r} + \frac{2ar^2}{ar(1+r)} \Rightarrow \frac{a}{p} + \frac{c}{q} = 2.$$

13. **(188090)**

$S = 13^2 + 2.5^2 + 3.7^2 + ... \Rightarrow t_n = n(2n+1)^2$

$$S_{20} \sum_{r=1}^{20} \left(4r^3 + 4r^2 + r\right) \Rightarrow S_{20} = 4 \times \frac{20^2 \times 21^2}{4} + 4 \times \frac{20 \times 21 \times 41}{6} + \frac{20 \times 21}{2}$$

Hence $S_{20} = 188090.$

14. **(1)**

Given $t_n = \frac{1}{n(n+1)} \Rightarrow t_n = \frac{1}{n} - \frac{1}{n+1}$

Hence $S = \sum_{r=1}^{\infty} \left(\frac{1}{r} - \frac{1}{r+1}\right) = 1$

15. **(100)**

$S_n = 1 + 3 + 6 + 10 + 15 + 21 + ... + t_n$

$S_n = \quad 1 + 3 + 6 + 10 + 15 + ... + t_{n-1} + t_n$

$0 = 1 + 2 + 3 + 4 + 5 + 6 + ...$n terms $-t_n$

$\Rightarrow t_n = \frac{n(n+1)}{2}$ Now $t_n = 5050$ gives $n = 100$.

Section-I

1. (d)

Let $t_n = \frac{1}{a_n . a_{n+1}} = \frac{1}{d}\frac{(a_{n+1} - a_n)}{a_{n+1}a_n}$

$\Rightarrow t_n = \frac{1}{d}\left(\frac{1}{a_n} - \frac{1}{a_{n+1}}\right)$

$\Rightarrow \sum_{n=1}^{20} t_n = \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \frac{1}{a_{20}} - \frac{1}{a_{21}}\right)$

$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{21}}\right) = \frac{(a_{21} - a_1)}{da_1a_{21}}$

$= \frac{20d}{da_{21}.a_1} = \frac{20}{a_1a_{21}} = \frac{4}{9}$

$\therefore\ a_1a_{21} = 45$ ....(i)

$\Rightarrow 189 = \frac{21}{2}(a_1 + a_{21})$

$\Rightarrow a_1 + a_{21} = 18$ (Using (i))

$\Rightarrow a_1 + \frac{45}{a_1} = 18 \Rightarrow a_1^2 - 18a_1 + 45 = 0$

$(a_1 - 15)(a_1 - 3) = 0 \Rightarrow a_1 = 3$ or 15

Case I : When $a_1 = 3$

$\Rightarrow a_{21} = 15 = 3 + 20d \Rightarrow d = \frac{3}{5}$

$\therefore\ a_6 = 3 + 5 \times \frac{3}{5} = 6$

And $a_{16} = 3 + 15 \times \frac{3}{5} = 12$

$\therefore a_6a_{16} = 72$

Case II : When $a_1 = 15$

$\Rightarrow a_{21} = 3 = 15 + 20d \Rightarrow d = \frac{-3}{5}$

$\therefore\ a_6 = 15 + 5 \times \left(\frac{-3}{5}\right) = 12$

And $a_{16} = 15 + 15 \times \left(\frac{-3}{5}\right) = 6$

$\therefore a_6a_{16} = 72$

2. (c)

3. (a)

We have, $S_{10}=530$ and $S_5=140$

$\Rightarrow S_{10}=\frac{10}{2}[2a+9d]=530$

$\Rightarrow 5[2a+9d]=530$

$\Rightarrow 2a+9d=106$ ....(i)

And $S_5=\frac{5}{2}[2a+4d]=140$

$2a+4d=56$ ....(ii)

$\therefore$ By (i) and (ii), we get a = 8 and d = 10

$\therefore\ S_{20}-S_6=\frac{20}{2}[2a+19d]-\frac{6}{2}[2a+5d]$

$=10[2a+19d]-3(2a+5d)$

$=20a+19d-6a-15d=14a+175d$

Putting a = 8 and d = 10 , we get

$S_{20}-S_6=14\times 8+175\times 10=1862$

4. (b)

5. (d)

We have,

$\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)\in$ A.P.

$\Rightarrow 2x=\tan\left(\frac{\pi}{9}\right)+\tan\left(\frac{7\pi}{18}\right)$

$\Rightarrow 2x=\tan\left(\frac{\pi}{9}\right)+\tan\left[\frac{\pi}{2}-\frac{\pi}{9}\right]$

$\Rightarrow x=\frac{1}{2}\left[\tan\frac{\pi}{9}+\cot\frac{\pi}{9}\right]$ .....(i)

Similarly, $y=\frac{1}{2}\left[\tan\frac{\pi}{9}+\tan\frac{5\pi}{18}\right]$

$\Rightarrow 2y=\left[\tan\frac{\pi}{9}+\tan\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)\right]$

$\Rightarrow 2y=\tan\frac{\pi}{9}+\cot\frac{2\pi}{9}$

$\therefore\ |x-2y|$

$=\left|\frac{1}{2}\tan\frac{\pi}{9}+\frac{1}{2}\cot\frac{\pi}{9}-\tan\frac{\pi}{9}-\cot\frac{2\pi}{9}\right|$

$=\left|\frac{1}{2}\left(\cot\frac{\pi}{9}-\tan\frac{\pi}{9}\right)-\cot\frac{2\pi}{9}\right|$

$=\left|\frac{1}{2}\left[\frac{1-\tan^2\pi/9}{\tan\pi/9}\right]-\cot\frac{2\pi}{9}\right|$

$$=\left|\cot\frac{2\pi}{9}-\cot\frac{2\pi}{9}\right|=0$$

6. (b)

We have,

$$S_1=\frac{2n}{2}\left(2a+\left(2n-1\right)d\right) \quad .....(i)$$

And $S_2=\frac{4n}{2}\left(2a+\left(4n-1\right)d\right) \quad .....(ii)$

Subtracting (i) from (ii), we get

$$S_2-S_1=2na+nd\left(6n-1\right)=n\left(2a+\left(6n-1\right)d\right)$$

$$\Rightarrow \frac{1000}{n}=2a+\left(6n-1\right)d$$

$$\left(\because\ S_2-S_1=1000\text{ (given)}\right)$$

Now, $S_{6n}=\frac{6n}{2}\left(2a+\left(6n-1\right)d\right)$

$$\Rightarrow S_{6n}=\frac{6n}{2}\times\frac{1000}{n}=3000$$

7. (c)

8. (b)

Let, $S=\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+....$

$$=\left(2-\frac{1}{2}\right)x^2+\left(2-\frac{1}{3}\right)x^3+\left(2-\frac{1}{4}\right)x^4+.....$$

$$=2\left(x^2+x^3+x^4+....\right)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+....\right)$$

$$=2\frac{x^2}{1-x}+\log_e\left(1-x\right)+x$$

$$=\frac{2x^2+x-x^2}{1-x}+\log_e\left(1-x\right)$$

$$=x\frac{\left(1+x\right)}{1-x}+\log_e\left(1-x\right)$$

9. (b)

We have, $y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}+......\text{upto}\infty\text{ terms}$

$$\Rightarrow y=\log_{10}x+\frac{1}{3}\log_{10}x+\frac{1}{9}\log_{10}x+.....\text{upto}\infty\text{ terms}$$

$$\Rightarrow y=\log_{10}x\left\{1+\frac{1}{3}+\frac{1}{3^2}+....\right\}$$

$$\Rightarrow y=\frac{3}{2}\log_{10}x \quad .....(i)$$

Also, $\dfrac{2+4+6+......+2y}{3+6+9+.....+3y}=\dfrac{4}{\log_{10}x}$

$\Rightarrow \dfrac{2}{3}\left(\dfrac{1+2+3+.....+y}{1+2+3+.....+y}\right)=\dfrac{4}{\log_{10}x}$

$\Rightarrow \log_{10}x=6 \Rightarrow x=10^6$

$\therefore$ From (i), we have, y = 9

So, ordered pair $(x,y)=(10^6,9)$

10. (a)

We have,

$y=\left(1-\dfrac{1}{2}\right)x^2+\left(1-\dfrac{1}{3}\right)x^3+\left(1-\dfrac{1}{4}\right)x^4+....$

$=\left(x^2+x^3+x^4+....\right)-\left(\dfrac{x^2}{2}+\dfrac{x^3}{3}+\dfrac{x^4}{4}+....\right)$

$=\dfrac{x^2}{1-x}+x-\left(x+\dfrac{x^2}{2}+\dfrac{x^3}{3}+\dfrac{x^4}{4}+....\right)$

$\dfrac{x}{1-x}+\log(1-x)$

At $x=\dfrac{1}{2}, y=1-\log 2$

$\therefore\ e^{1+y}=e^{1+1-\log 2}=\dfrac{1}{2}e^2$

11. (d)

Let a be the first term and it is given that r is the common ratio of G.P.

Also, we have $T_4=3r^2 \Rightarrow ar^3=3r^2$

$\Rightarrow ar=3 \Rightarrow a=\dfrac{3}{r}$

So, the G.P. becomes, $\dfrac{3}{r},3,3r,3r^2,.....$

According to question, we have

$\dfrac{3}{r},6,3r\in$ A.P.

$\Rightarrow 6-\dfrac{3}{r}=3r-6 \Rightarrow r^2-4r+1=0$

$\Rightarrow r=2\pm\sqrt{3}$

Since, the G.P. is increasing.

So, $r=2+\sqrt{3}$

$\Rightarrow d=6-\dfrac{3}{r}=6-\dfrac{3}{2+\sqrt{3}}$

$=6-3\left(2-\sqrt{3}\right)=3\sqrt{3}$

$\therefore\ r^2-d=\left(2+\sqrt{3}\right)^2-3\sqrt{3}$

$=7+4\sqrt{3}-3\sqrt{3}=7+\sqrt{3}$

12. (c)

$100^{\alpha}-199\beta=(100)^2+(100-1)(100+1)+(100-2)(100+2)+....+(100-99)(100+99)$

$=(100)^2+(100^2-1^2)+(100^2-2^2)+......+(100^2-99^2)$

$=(100)^2+99(100)^2-(1^2+2^2....+99^2)$

$=(100).(100)^2-\frac{(99\times100\times199)}{6}$

$=(100^3)-199(1650)$

$\Rightarrow \alpha=3$ and $\beta=1650$

So, required slope, $m=\frac{1650-0}{3-0}=550$

13. (b)

$e^{(\cos^2 x+\cos^5 x+\cos^6 x+.....\infty)}\log_e 2$

$=2^{\cos^2 x+\cos^4 x+....\infty}$

$=2^{\cos^2 x\left(\frac{1}{1-\cos^2 x}\right)}=2^{\cot^2 x}$

Now, $t^2-9t+8=0\Rightarrow t=1,8$

$\Rightarrow 2^{\cot^2 x}=1,8\Rightarrow \cot^2 x=0,3$

Since, $0<x<\frac{\pi}{2}\Rightarrow \cot x=\sqrt{3}$

$\therefore \frac{2\sin x}{\sin x+\sqrt{3}\cos x}=\frac{2}{1+\sqrt{3}\cot x}=\frac{2}{4}=\frac{1}{2}$

14. (b)

15. (d)

Given, $T_2+T_6=\frac{25}{2}$

$\Rightarrow ar+ar^5=\frac{25}{2}$ ....(i)

Also, $T_3.T_5=25$

$\Rightarrow ar^2ar^4=25\Rightarrow a^2r^6=25$

$\Rightarrow ar^3=5$ ....(ii)

Dividing (i) by (ii), we get

$\frac{r+r^5}{r^3}=\frac{5}{2}\Rightarrow 2+2r^4=5r^2$

$\Rightarrow 2r^4-5r^2+2=0\Rightarrow r^2=2,\frac{1}{2}$

$\therefore\ r^2=2$ [Rejecting $r^2=\frac{1}{2}$]

Now, $T^4+T^6+T^8=ar^3+ar^5+ar^7$

$b_1, b_2, b_3, \ldots$

16. (b)

Let $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots..\infty$ ....(i)

And $\frac{1}{3}S = + \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} \ldots..\infty$ ....(ii)

Subtracting (ii) from (i), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \ldots..\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \left(\frac{5}{3^2} + \frac{5}{3^3} + \ldots..\infty\right)$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \left(\frac{\frac{5}{3^2}}{1 - \frac{1}{3}}\right) = \frac{4}{3} + \frac{5}{6} = \frac{13}{6}$$

$$\therefore \quad S = \frac{13}{6} \times \frac{3}{2} = \frac{13}{4}$$

17. (b)

Given, $f(x) = a^{a^x} + a^{1-a^x}$

Since A.M. $\geq$ G.M.

$$\therefore \quad \frac{a^{a^x} + \frac{a}{a^{a^a}}}{2} \geq \left(a^{a^x} . \frac{a}{a^{a^x}}\right)^{1/2}$$

$$\Rightarrow a^{a^x} + \frac{a}{a^{a^x}} \geq 2\sqrt{a}$$

$\therefore$ Minimum value is $2\sqrt{a}$

18. (b)

19. (d)

Let $t_n = \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4}$

$$\lim_{x \to 2} t_n = \frac{2}{4n(n+1) + 4(2n+1) + 4}$$

$$= \frac{1}{2(n^2 + 3n + 2)} = \frac{1}{2}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

Now, $t_1 = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{3}\right)$

$t_2 = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{4}\right); t_3 = \frac{1}{2}\left(\frac{1}{4} - \frac{1}{5}\right)$

$$\therefore \sum_{n=1}^{9} t_n = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \ldots.. + \frac{1}{9} - \frac{1}{10} + \frac{1}{10} - \frac{1}{11}\right)$$

$$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{1}{2}\left(\frac{9}{22}\right)=\frac{9}{44}$$

20. (d)

The given series is

$$\frac{3}{1^2\times 2^2}+\frac{5}{2^2\times 3^2}+\frac{7}{3^2\times 4^2}+.....$$

$$\Rightarrow \frac{2^2-1^2}{1^2\times 2^2}+\frac{3^2-2^2}{2^2\times 3^2}+\frac{4^2-3^2}{3^2\times 4^2}+.....$$

$$\Rightarrow \left(\frac{1}{1^2}-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{3^2}-\frac{1}{4^2}\right)+....$$

$\therefore$ Required sum

$$=1-\frac{1}{2^2}+\frac{1}{2^2}-\frac{1}{3^2}+....+\frac{1}{10^2}-\frac{1}{11^2}$$

$$=1-\frac{1}{121}=\frac{120}{121}$$

21. (d)

We have,

$$\log_{9^{1/2}} x+\log_{9^{1/3}} x+\log_{9^{1/4}} x+....+\log_{9^{1/22}} x=504$$

$$\Rightarrow 2\log_9 x+3\log_9 x+4\log_9 x+....+22\log_9 x=504$$

$$\Rightarrow (2+3+4+.....+22)\log_9 x=504$$

$$\Rightarrow \left(\frac{22\times 23}{2}-1\right)\log_9 x=504$$

$$\Rightarrow 252\log_9 x=504 \Rightarrow \log_9 x=2 \Rightarrow x=81$$

22. (b)

$$T_r=\frac{1}{(2r+1)^2-1}=\frac{1}{4r(r+1)}$$

$$=\frac{1}{4}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

Now, $\sum_{r=1}^{100} T_r=\frac{1}{4}\left(1-\frac{1}{101}\right)$

$$=\frac{1}{4}\times\frac{100}{101}=\frac{25}{101}$$

23. (d)

24. (b)

Given : $\frac{S_5}{S_9}=\frac{5}{17}$

$$\Rightarrow \frac{\frac{5}{2}[2a+4d]}{\frac{9}{2}[2a+8d]} = \frac{5}{17}$$

$$\Rightarrow 17(2a+4d) = 9(2a+8d)$$

$$\Rightarrow 17(a+2d) = 9(a+4d)$$

$$\Rightarrow 8a = 2d \Rightarrow d = 4a$$

It is given that, $110 < a_{15} < 120$

$$\Rightarrow 110 < a + 14d < 120$$

$$\Rightarrow 110 < 57a < 120 \Rightarrow \frac{110}{57} < a < \frac{120}{57}$$

As a is integer so, $a = 2 \Rightarrow d = 4a = 8$

Then, $S_{10} = \frac{10}{2}[4 + 9 \times 8] = 380$

25. (b)

26. (d)

We have,

$a_1 = 2, a_{10} = 3, a_1 b_1 = 1 = a_{10} b_{10}$

Let d_1 be the common difference of an A.P. $a_1, a_2, a_3, ...$

and d_2 be the common difference of an A.P. $b_1, b_2, b_3, ...$

$$a_{10} = a_1 + 9d_1 \Rightarrow 3 = 2 + 9d_1 \Rightarrow d_1 = \frac{1}{9}$$

$$b_{10} = b_1 + 9d_2 \Rightarrow \frac{1}{3} = \frac{1}{2} + 9d_2 \Rightarrow d_2 = -\frac{1}{54}$$

$$a_4 = a_1 + 3d_1 = 2 + 3\left(\frac{1}{9}\right) = \frac{7}{3}$$

$$b_4 = b_1 + 3d_2 = \frac{1}{2} + 3\left(\frac{-1}{54}\right) = \frac{8}{18}$$

$$\therefore \quad a_4 b_4 = \frac{28}{27}$$

27. (c)

Let $A_1, A_2, A_3....A_n$ be n arthimetic means between a and 100

$a, A_1, A_2, A_3.......100.$

Common difference is $d = \frac{100-a}{n+1}$

Ratio of first mean to last mean is 1 : 7,

$$\frac{A_1}{A_n} = \frac{1}{7} \Rightarrow \frac{1+d}{1+nd} = \frac{1}{7}$$

$$\Rightarrow \frac{a+\frac{100-a}{n+1}}{a+\frac{(100-a)n}{n+1}}=\frac{1}{7} \Rightarrow \frac{an+100}{a+100n}=\frac{1}{7}$$

$\Rightarrow 7an+700=a+100n$

It is given that a + n = 33,

$\Rightarrow 7(33-n)n+700=33-n+100n$

$\Rightarrow 7n^2-132n-667=0$

$\Rightarrow n=23, \frac{-29}{7}$ (Igonre negative value)

So, value of n is 23.

28. (a)

Sum of first 21 terms of A.P.

$S_{21}=\frac{21}{2}\left[20ar+(21-1)10ar^2\right]$

$=\frac{21}{2}\left[20ar+(20)10ar^2\right]$

$=21\left[10ar+(10)10ar^2\right]$

$=21\left[10ar+(11-1)10ar^2\right]=21a_{11}$

29. (c)

$f(x+y)=2f(x)f(y)$

$f(1+1)=2f(1)f(1); f(2)=2\times2\times2=8$

$f(3)=2f(1).f(2)=2\times2\times8=32$

$f(4)=128; f(5)=512$

So, $f(1),f(2),f(3),f(4),.....$ forms a G.P. of common ratio 4.

We know that sum of G.P. for 10 terms is

$S_{10}=\frac{a(4^{10}-1)}{4-1}=\frac{a(2^{20}-1)}{3}$

Comparing with $\sum_{k=1}^{10} f(\alpha+k)$

$=\frac{512}{3}(2^{20}-1)$

$\Rightarrow a=512$

For $f(5)=512$, So, $\alpha=4$

30. (d)

31. (b)

We have, $S=1+2.3+3.3^2+.......+10.3^9$ .....(i)

$3S=1.3+2.3^2+.....+9.3^9+10.3^{10}$ .....(ii)

Subtracting (ii) from (i), we get

$$-2S=\left(1+3+3^2+.....+3^9\right)-10.3^{10}$$

$$\Rightarrow -2S=\frac{1\left(3^{10}-1\right)}{2}-10.3^{10}$$

$$\Rightarrow 2S=\frac{1-3^{10}}{2}+10.3^{10}$$

$$=\frac{1-3^{10}+20.3^{10}}{2}\Rightarrow S=\frac{1}{4}\left[19.3^{10}+1\right]$$

32. (c)

33. (c)

$$x=\sum_{n=0}^{\infty}a^n \qquad |a|<1$$

$$x=\frac{1}{1-a}\Rightarrow 1-\frac{1}{x}=a$$

$$y=\sum_{n=0}^{\infty}b^n \qquad |b|<1$$

$$=\frac{1}{1-b}\Rightarrow 1-\frac{1}{y}=b$$

$$z=\sum_{n=0}^{\infty}c^n \qquad |c|<1$$

$$=\frac{1}{1-c}\Rightarrow 1-\frac{1}{z}=c$$

For A.P., 2b = a + c

$$2\left(1-\frac{1}{y}\right)=1-\frac{1}{x}+1-\frac{1}{z}$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}=\frac{2}{z}$$

Clearly, $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P.

34. (c)

$$S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+.... \qquad ....(i)$$

$$\frac{1}{7}S=\frac{2}{7}+\frac{6}{7^2}+\frac{12}{7^3}+\frac{20}{7^4}+.... \qquad ....(ii)$$

Subtract (ii) from (i), we get

$$\frac{6S}{7}=2+\frac{4}{7}+\frac{6}{7^2}+\frac{8}{7^3}+.... \qquad ....(iii)$$

$$\frac{6S}{7}=\frac{2}{7}+\frac{4}{7^2}+\frac{6}{7^3}+\frac{8}{7^4}.... \qquad ....(iv)$$

Subtract (iv) from (iii), we get

$$\left(\frac{6}{7}\right)^2 S=2+\frac{2}{7}+\frac{2}{7^2}+\frac{2}{7^3}+....$$

$$=2\left(1+\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+....\right)=2\left[\frac{1}{1-\frac{1}{7}}\right]=\frac{7}{3}$$

$$\therefore \quad 4S=\frac{7}{3}\times\frac{7^2}{6^2}\times4=\left(\frac{7}{3}\right)^3$$

35. (c)

Let the G.P. be $a, ar, ar^2.....$

Now, $A_1A_3A_5A_7=\frac{1}{1296}$

$$\Rightarrow (a)(ar^2)(ar^4)(ar^6)=\frac{1}{1296}$$

$$\Rightarrow a^4r^{12}=\frac{1}{1296}\Rightarrow ar^3=\frac{1}{6} \qquad .....(i)$$

Also, $A_2+A_4=\frac{7}{36}$

$$\Rightarrow ar+ar^3=\frac{7}{36}\Rightarrow ar=\frac{7}{36}-\frac{1}{6}=\frac{1}{36} \qquad \text{(Using (i))}$$

Now, $\frac{ar^3}{ar}=\frac{1/6}{1/36}\Rightarrow r^2=6$

$$\therefore\ A_6+A_8+A_{10}=ar^5+ar^7+ar^9=ar^5\left(1+r^2+r^4\right)$$

$$=ar.r\left(1+r^2+r^4\right)=\frac{1}{36}.6.6\left(1+6+36\right)=43$$

36. (b)

$\{a_n\}_{n=0}^{\infty}$ is a sequence such that

$a_0=a_1=0$ and $a_{n+2}=2a_{n+1}-a_n+1$

$\therefore\quad a_2=2a_1-a_0+1=1$; $a_3=2a_2-a_1+1=3$

$a_4=2a_3-a_2+1=6; a_5=2a_4-a_3+1=10$

$$S=\sum_{n=2}^{\infty}\frac{a_n}{7^n}=\frac{1}{7^2}+\frac{3}{7^3}+\frac{6}{7^4}+\frac{10}{7^5}+....$$

$$\frac{1}{7}S=\frac{1}{7^3}+\frac{3}{7^4}+\frac{6}{7^5}+....$$

$$\therefore\ S-\frac{1}{7}S=\frac{1}{7^2}+\frac{2}{7^3}+\frac{3}{7^4}+\frac{4}{7^5}....$$

$$\Rightarrow\frac{6}{7}S=\frac{1}{7^2}+\frac{2}{7^3}+\frac{3}{7^4}+\frac{4}{7^5}....$$

$$\Rightarrow\frac{6}{49}S=\frac{1}{7^3}+\frac{2}{7^4}+\frac{3}{7^5}+....$$

$$\therefore\quad \frac{6}{7}S-\frac{6}{49}S=\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\frac{1}{7^5}+.....$$

$$\Rightarrow\frac{36}{49}S=\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\frac{1}{7^5}+.....$$

$$\Rightarrow \frac{36}{49}S=\frac{\frac{1}{7^2}}{1-\frac{1}{7}} \Rightarrow \frac{36}{49}S=\frac{1}{42}$$

$$S=\frac{1}{42}\times\frac{49}{36}=\frac{7}{216}$$

37. (c)

Let $S=1+\frac{5}{6}+\frac{12}{6^2}+...$

$$\Rightarrow \frac{S}{6}=\frac{1}{6}+\frac{5}{6^2}+\frac{12}{6^3}+...$$

$$S-\frac{S}{6}=1+\frac{4}{6}+\frac{7}{6^2}+....$$

$$\Rightarrow \frac{5S}{6}=1+\frac{4}{6}+\frac{7}{6^2}+\frac{10}{6^3}+.....$$

$$\Rightarrow \frac{5S}{6^2}=\frac{1}{6}+\frac{4}{6^2}+\frac{7}{6^3}+.....$$

$$\therefore \quad \frac{5S}{6}-\frac{5S}{6^2}=1+\frac{3}{6}+\frac{3}{6^2}+.....$$

$$\Rightarrow \frac{25S}{6}=1+\frac{3}{6}+\frac{3}{6^2}+....$$

$$\Rightarrow \frac{25S}{6}-1=\frac{3.\frac{3}{6}}{1-\frac{1}{6}}$$

$$\Rightarrow \frac{25S}{6}=\frac{3}{5}+1 \Rightarrow S=\frac{8\times36}{125}=\frac{288}{125}$$

38. (a)

Since, we know that

$$\frac{4^x+4^{-x}}{2}\geq\sqrt{4^x\times4^{-x}}=1$$

$\therefore \quad \cos\left(\frac{x^2+x}{6}\right)=\frac{4^x+x^{-x}}{2}=1$, Which is possible only when x = 0

Possible only when x = 0

$\therefore$ Number of elements in the set S = 1

39. (d)

A.M. $\geq$ G.M.

$$\Rightarrow \frac{x+x+x+y+y}{5}\geq\left(x^3y^2\right)^{1/5}$$

$$\Rightarrow 3x+2y\geq5\left(2^{15}\right)^{1/5}$$

$$\left(\because x^3y^2=2^{15} \text{ (given)}\right)$$

$$\Rightarrow 3x+2y\geq40$$

40. (b)

We have, $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$

$= \frac{3}{4}\sum_{n=1}^{21}\left[\frac{1}{4n-1} - \frac{1}{4n+3}\right]$

$= \frac{3}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{15}\right)+...\right]$

$= \frac{3}{4}\left[\frac{1}{3}-\frac{1}{87}\right] = \frac{7}{29}$

41. (c)

G.M. of (60 + n) terms

$= \left(2.2^2....2^{60}.4^1.4^2....4^n\right)\frac{1}{60+n}$

$\therefore$ [Given]

$= \left(2^{1+2+....60}.4^{1+2+....+n}\right)\frac{1}{60+n} = 2^{225/8}$

$\Rightarrow \left[2^{30\times 61}.2^{\frac{2\times n(n+1)}{2}}\right]^{\frac{1}{60+n}} = 2^{225/8}$

$\Rightarrow \left[2^{30\times 61+n\times(n-1)}\right]^{\frac{1}{60+n}} = 2^{225/8}$

$\Rightarrow \frac{30\times 61+n\times(n+1)}{60+n} = \frac{225}{8}$

$\Rightarrow 8n^2 - 217n + 1140 = 0 \Rightarrow n = 20$

Now, $\sum_{k=1}^{20} k(20-k) = \sum_{k=1}^{20} 20k - \sum_{k=1}^{20} k^2$

$= \frac{20\times 20\times 21}{2} - \frac{20\times 21\times 41}{6}$

$= 4200 - 2870 = 1330$

42. (c)

$\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + ... + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$

$\Rightarrow \frac{1}{20}\left[\frac{(40-a)-(20-a)}{(20-a)-(40-a)} + \frac{(60-a)-(40-a)}{(40-a)-(60-a)} + ... + \frac{(200-a)-(180-a)}{(180-a)-(200-a)}\right] = \frac{1}{256}$

$\Rightarrow \frac{1}{20}\left[\frac{1}{(20-a)} - \frac{1}{(40-a)} + \frac{1}{(40-a)} - \frac{1}{(60-a)} + ... + \frac{1}{(180-a)} - \frac{1}{(200-a)}\right] = \frac{1}{256}$

$\Rightarrow \frac{1}{20}\left[\frac{1}{20-a} - \frac{1}{200-a}\right] = \frac{1}{256}$

$\Rightarrow \frac{1}{20}\left[\frac{(200-a)-(20-a)}{(20-a)(200-a)}\right] = \frac{1}{256}$

$$\Rightarrow \frac{180}{20(20-a)(200-a)}=\frac{1}{256}$$
$$\Rightarrow (20-a)(200-a)=9\times256$$
$$\Rightarrow 4000-220a+a^2=2304$$
$$\Rightarrow a^2-220a+1696=0$$
$$\Rightarrow a^2-212a-8a+1696=0$$
$$\Rightarrow (a-212)(a-8)=0 \Rightarrow a=8,212$$

Hence, Maximum value of a is 212

43. (b)

Consider $\sum_{r=1}^{20}(r^2+1)(r)!$

$$=\sum_{r=1}^{20}\left[(r+1)^2-2r\right](r)!$$
$$=\sum_{r=1}^{20}(r+1)(r+1)(r)!-2\sum_{i=1}^{20}(r)(r)!$$
$$=\sum_{r=1}^{20}(r+1)(r+1)!-\sum_{i=1}^{20}(r)(r)!-\sum_{i=1}^{20}(r)(r)!$$
$$=\sum_{r=1}^{20}\left[(r+1)(r+1)!-r(r)!\right]-\sum_{i=1}^{20}(r+1-1)(r)!$$
$$=\sum_{r=1}^{20}\left[(r+1)(r+1)!-r(r)!\right]-\sum_{i=1}^{20}(r+1)!-r!$$
$$=(21\times21!-1)-(-1+21!)=(22-1)21!-21!$$
$$=22\times21!-21!-21!=22!-2(21!)$$

44. (d)

Given that $s_1, s_2, s_3, ..., s_{10}$

respectively be the sun of 12 term of 10 Aps, whose first terms are 1,2,3,…….,10 and the common difference are 1,3,5,…19, respectively.

The first terms are $a_i = i$ and the common difference are $d_i = 2i-1$.

Thus, $S_i=\frac{12}{2}\left[2i+11\times(2i-1)\right]$

$$=6[24i=11]$$
$$\sum_{i=1}^{10}S_i=6\sum_{i=1}^{10}(24i-11)$$
$$=6\left[24\sum_{i=1}^{10}i-110\right]$$
$$=6\left[24\frac{(10)(10+1)}{2}-110\right]$$
$$=6[12\times10\times11-110]$$
$$=6\times1210=7260$$

45. (c)

Given,

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}\right)$$

Now, $\frac{1}{\sqrt{a_1+\sqrt{a_2}}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+....+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$

$$=\frac{\sqrt{a_2-\sqrt{a_1}}}{a_2-a_1}+\frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2}+...+\frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}$$

$$=\frac{\sqrt{a_2}-\sqrt{a_1}+\sqrt{a_3}-\sqrt{a_2}+...+\sqrt{a_n}-\sqrt{a_{n-1}}}{d}$$

$(\because a_2-a_1=a_3-a_2=...a_n-a_{n-1}=d)$

$$=\frac{\sqrt{a_n}-\sqrt{a_1}}{d}$$

$$=\frac{\sqrt{a_1+(n-1)d}-\sqrt{a_1}}{d}$$

$$\therefore \lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_1+(n-1)d}-\sqrt{a_1}}{d}\right)$$

$$=\lim_{n\to\infty}\left[\frac{1}{\sqrt{d}}\left(\frac{\sqrt{a_1+(n-1)d}-\sqrt{a_1}}{\sqrt{n}}\right)\right]$$

$$=\lim_{n\to\infty}\left[\frac{1}{\sqrt{d}}\left(\sqrt{\frac{a_1}{n}+\left(d-\frac{d}{n}\right)}-\sqrt{\frac{a_1}{n}}\right)\right]$$

$$=\frac{1}{\sqrt{d}}\left(\sqrt{0+d-0}-\sqrt{0}\right)=\frac{\sqrt{d}}{\sqrt{d}}=1$$

46. (d)

We have, $a_7=3$

$\Rightarrow$ $a+6d=3$

$\Rightarrow$ $a=3=6d$

Let $P=a_1a_4=a(a+3d)$

$=(3-6d)(3-6d+3d)$

$=9-27d+18d^2$

Now, $\frac{dP}{dd}=-27+36d$

For maxima or minima,

$$\frac{dP}{dd}=0$$

$\Rightarrow$ $-27+36d=0\Rightarrow d=\frac{27}{36}=\frac{3}{4}$

Again, $\frac{d^2P}{dd^2} = 36 > 0$

So, P is minimum at $d = \frac{3}{4}$

Now, $a = 3 - 6d$

$\Rightarrow \quad a = 3 - 6 \times \frac{3}{4} = \frac{(-3)}{2}$

Again, $S_n = 0$

$\frac{n}{2}\left[2a + (n-1)d\right] = 0$

$\Rightarrow \quad 2a + (n-1)d = 0$

$\Rightarrow \quad 2 \times \left(-\frac{3}{2}\right) + (n-1)\left(\frac{3}{4}\right) = 0$

$\frac{n}{2}\left[2a + (n-1)d\right] = 0$

$\Rightarrow \quad n = 5$

Now, $n! - 4a_{n(n+1)} = 5! - 4a_{35}$

$= 120 - 4(a + 34d)$

$= 120 - 4\left[\frac{-3}{2} + 34 \times \frac{3}{4}\right]$

$= 120 + 6 - 102 = 24$

47. (b)

Given, $x^{pq^2} = y^{qr} = z^{p^2r} = \lambda$ (say)

Since, $x^{pq^2} = y^{qr} = z^{p^2r} = \lambda$ (given)

$pq^2 = \log_x \lambda$

$\Rightarrow \quad \log_\lambda x = \frac{1}{pq^2}$ … (i)

$qr = \log_y \lambda$

$\Rightarrow \quad \log_\lambda y = \frac{1}{qr}$ … (ii)

$\Rightarrow \quad \log_\lambda z = \frac{1}{p^2r}$ … (iii)

Now, $\log_x x = \frac{\log_\lambda x}{\log_\lambda y} = \frac{qr}{pq^2} = \frac{r}{pq}$ … (iv) [Using Eqs.(i) and (iii)]

$\log_z y = \frac{\log_{\lambda y}}{\log_\lambda z} = \frac{\frac{1}{qr}}{\frac{1}{p^2r}} = \frac{p^2r}{qr} = \frac{p^2}{q}$ … (v) [Using Eqs.(ii) and (iii)]

And $\log_x z = \frac{\log_\lambda z}{\log_\lambda x} = \frac{\frac{1}{p^2 r}}{\frac{1}{pq^2}} = \frac{q^2}{pr}$ … (vi) [Using Eqs.(i) and (ii)]

Given, $3, \frac{3r}{pq}, \frac{3p^2}{q}, \frac{7q^2}{pr}$ are in AP.

Then, $\frac{3r}{pq} - 3 = \frac{1}{2} \Rightarrow r = \frac{7}{6} pq$ … (vii)

Also given, $r = pq + 1$

$\Rightarrow \quad pq = 6$ … (viii)

From Eq. (vii) $r = 7$ … (ix)

Now, $\frac{3p^2}{q} - \frac{3r}{pq} = \frac{1}{2}$

$\Rightarrow \quad \frac{3p^2}{q} = \frac{1}{2} + \frac{3r}{pq} = \frac{8}{2} = 4$ … (x)

From Eq. (viii), $q = \frac{6}{q} \Rightarrow \frac{1}{q} = \frac{p}{6}$

From Eq.(x), $3p^2 \times \frac{p}{6} = 4 \Rightarrow p = 2$

$\therefore \quad pq = 6$ gives us $q = 3$

Hence, $r - p - q = 7 - 2 - 3 = 2$

48. (a)

a, A_1, A_2, b in AP

$\frac{a+b}{2} = \frac{A_1 + A_2}{2}$

$\Rightarrow \quad a + b = A_1 + A_2$ …. (i)

a, G_1, G_3, b in GP

$b = ar^4 \Rightarrow \sqrt[4]{\frac{b}{a}} = r$

$G_1 = ar = a\sqrt[4]{\frac{b}{a}},$

$G_2 = ar^2 = \left(4\sqrt{\frac{b}{a}}\right)^2 = a\sqrt{\frac{b}{a}}$

$G_3 = ar^3 = a\left(\sqrt[4]{\frac{b}{a}}\right)^3 = a\left(\frac{b}{a}\right)^{3/4}$

$G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$

$= \left(a\left(\frac{b}{a}\right)^{1/4}\right)^4 + \left(a\left(\frac{b}{a}\right)^{1/2}\right)^4 + \left[a\left(\frac{b}{a}\right)^{3/4}\right]^4 + \left[a\left(\frac{b}{a}\right)^{1/4}\right]^2 \left[\left(\frac{b}{a}\right)^{3/4}\right]^2$

$$= a^4 \times \frac{b}{a} + a^4 \times \frac{b^3}{a^3} + a^2 \frac{b^{1/2}}{a^{1/2}} \times \frac{a^2 \times b \times b^{1/2}}{a \times a^{1/2}}$$

49. (b)

Let $a_1, a_2, a_3, ...$ be a GP of increasing positive numbers and given that sum of its 6th and 8th terms be 2.

So, $ar^5 + ar^7 = 2$ ... (i)

And also given that, product if its 3rd and 5th terms be $\frac{1}{9}$

So, $\left(ar^2\right)\left(ar^4\right) = \frac{1}{9} \Rightarrow a^2r^6 = \frac{1}{9}$ ... (ii)

Since, $r > 0$

So, from Eq.(i), we get

$ar^5\left(1+r^2\right) = 2$ ... (iii)

And from Eq.(ii), we get $\left(ar^3\right)^2 = \frac{1}{9}$

$\Rightarrow \quad ar^3 = \pm 1/3$

Since, $r > 0$ and $a > 0 \Rightarrow ar > 0$, so

$ar^3 = \frac{1}{3}$

From Eq. (iii), we get

$ar^3 \cdot r^2\left(1+r^2\right) = 2$

$\Rightarrow \quad \frac{1}{3}\left(r^2 + r^4\right) = 2$

$\Rightarrow \quad r^4 + r^2 - 6 = 0$

$\Rightarrow \quad r^2 = 2,\ r^2 = -3$ (not possible)

$\Rightarrow \quad r = \sqrt{2};\quad \therefore a = \frac{1}{6\sqrt{2}}$

Now, $6\left(a_2 + a_4\right)\left(a_4 + a_6\right)$

$= 6\left(ar + ar^3\right)\left(ar^3 + ar^5\right)$

$= 6a^2r^4\left(1+r^2\right)^2$

$= 6 \times \frac{1}{36 \times 2} \times 4 \times (1+2)^2$

$= \frac{1}{12} \times 4 \times 9 = 3$

50. (c)

Given that, the first term a and common ratio r of a geometric progression be positive integer. So their first three terms are a, ar, ar^2

$a^2 + a^2r^2 + a^2r^4 = 33033$

$\Rightarrow \quad a^2\left(a+r^2+r^4\right)$

$= 3\times 7\times 11\times 11\times 13\times$

$= 3\times 7\times 13\times 11^2$

$\therefore \quad a^2 = 11^2$

$\Rightarrow \quad a = 11$

And $1+r^2+r^4 = 273$

$\Rightarrow \quad r^2+r^4 = 272$

$\Rightarrow \quad r^4+r^2-272 = 0$

$\Rightarrow \quad \left(r^2+17\right)\left(r^2-16\right) = 0 \quad [\because r^2 = -17 \text{ not possible }]$

$\Rightarrow \quad r = \pm 4$

$\Rightarrow \quad r = 4\left(r > 0\right)$

So, sum of these first three terms are

$a + ar + ar^2$

$\Rightarrow \quad 11+44+176 = 231$

51. (b)

Given, $a, b, c, d > 0$ and a,b,c,

$d \in R$

And $a+b+c+d = 11$

Also, $\left(a^5b^3c^2d\right)_{max} = 3750\beta$ [Given]

Clearly, we know that for all the real numbers if min/max is given or asked then use $AM \geq GM$.

So, $\left(\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}\right)+\left(\frac{b}{3}+\frac{b}{3}+\frac{b}{3}\right)+\left(\frac{c}{2}+\frac{c}{2}\right)+d = 11$

$\Rightarrow \quad AM \geq GM$

$$\Rightarrow \quad \left(\frac{\frac{a}{5}\times 5+\frac{b}{3}\times 3+\frac{c}{2}\times 2+d}{5+3+2+1}\right) \geq \left[\left(\frac{a}{5}\right)^5\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2 d\right]^{\frac{1}{11}}$$

$$\Rightarrow \quad \left(\frac{11}{11}\right)^{11} \geq \frac{a^5b^3c^2d}{5^5\times 3^3\times 2^2}$$

$\Rightarrow \quad \left(a^5b^3c^2d\right)_{max} = 1^{11}\times 5^5\times 3^3\times 2^2$

$\Rightarrow \quad 3750\,\beta = 5^5\times 3^3\times 2^2$

$\Rightarrow \quad \beta = 5\times 3^2\times 2 \Rightarrow \beta = 90$

Hence, value of β is 90.

52. (d)

Given that $ax^2+2bx+cy = 0$ and $dx^2+2ex+fy = 0$ intersect on the line y = 1

$ax^2+2bx+c = 0 \qquad \ldots \text{(ii)}$

and $dx^2+2ex+f = 0 \qquad \ldots \text{(iii)}$

a, b, c are in GP.

So, $b^2 = ac \Rightarrow b = \sqrt{a}\sqrt{c}$

Put $b = \sqrt{ac}$ in Eq.(i), we get

$$ax^2 + 2\sqrt{a}\sqrt{c}x + c = 0$$

$$\left(\sqrt{ax} + \sqrt{c}\right)^2 = 0 \Rightarrow x = -\frac{\sqrt{c}}{\sqrt{a}}$$

53. (a)

Given that a^3, b^3 and c^3 be in Ap. Then

$$2b^3 = a^3 + c^3 \quad \ldots\ldots \text{(i)}$$

And $\log_a b, \log_c a$ and $\log_b c$ in GP

$$\left(\log_c a\right)^2 = \left(\log_a b\right)\left(\log_b c\right)$$

$$= \log_a b \times \frac{\log_a c}{\log_a b} \quad \left[\because \log_b x = \frac{\log_a x}{\log_a b}\right]$$

$$\Rightarrow \quad \left(\log_c a\right)^3 = 1 \Rightarrow a = c$$

Put in eq. (i), we get

$$2b^3 = a^3 + a^3 \Rightarrow b = a$$

$$\therefore \quad a = b = c$$

Also given,

$$a_1 = \frac{a + 4b + c}{3} \text{ and } d = \frac{a - 8b + c}{10}$$

And $S_{20} = -444$ (Given)

$$\Rightarrow \quad -444 = \frac{20}{2}\left[2 \times a_1 + 19d\right]$$

$$= 10\left[\frac{2\left(a + 4b + c\right)}{3} + 19\frac{\left[a - 8b + c\right]}{10}\right]$$

$$= 10\left[4a + 19\left(-\frac{3a}{5}\right)\right]$$

$$= 10\left(\frac{20a - 57a}{5}\right) = 2\left(-37a\right)$$

$$\Rightarrow \quad a = \frac{222}{37} = 6$$

Now, abc $= a^3$ $\quad \left[\because a = b = c\right]$

$$= 6^3 = 216$$

54. (a)

We have,

Where, $x_1, x_2, ..., x_{100}$ are in AP with first term as 2.

$$\text{Mean} = \frac{\sum_{i=1}^{100} x_i}{100} = 200$$

$\Rightarrow \quad \frac{100}{2}\times[2\times 2+99d]=20000$

$4+99d=400 \Rightarrow d=4$

Also, $y_1 = i(x_i - i) = i[2+(i-1)4-i]$

$= i[3i-2] = 3i^2 - 2i$

Required mean $\frac{\sum_{i=1}^{100} y_i}{100}$

$= \frac{1}{100}\left[\sum_{i=1}^{100}(3i^2-2i)\right]$

$= \frac{1}{100}\left[\frac{3\times 100\times 101\times 201}{6} = 2\times\frac{100\times 101}{2}\right]$

$= \frac{20301}{2} - 101 = \frac{20099}{2} = 10049.5$

55. (b)

We have,

$S_n = 4+11+21+34+50+...+T_n \quad 0 = 4+7+10+13+...+(T_n - T_{n-1}) - T_n$

$\therefore \quad T_n = 4+7+10+13+...n$ terms This from an AP with $a=4, d=3$

$T_n = \frac{n}{2}[2\times 4+(n-1)3] = \frac{3}{2}n^2 + \frac{5}{2}n$

$S_n = \Sigma T_n = \frac{3}{2}\Sigma n^2 + \frac{5}{2}\Sigma n$

$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2}\frac{n(n+1)}{2}$

$= \frac{2n(n+1)(n+3)}{4}$

$= \frac{n(n+1)(n+3)}{2}$

$\frac{1}{60}(S_{29} - S_9)$

$= \frac{1}{60}\left[\frac{29\times 30\times 32}{2} - \frac{9\times 10\times 12}{2}\right]$

$= 223$

56. (b)

$S_n = 5+8+14+23+35+50+...+a_{n-1}+a_n$

Or $S_n = 5+8+14+23+35+...+a_{n-1}+a_n$

$0 = 5+[3+6+9+12+15+...\text{ to }(n-1)\text{ terms}] - a_n$

Or $a_n = 5+\frac{(n-1)}{2}[2\times 3+(n-1-1)3]$

Or $a_n = 5 + \frac{(n-1)\cdot 3n}{2} = \frac{10 + 3n^2 - 3n}{2}$

$\Rightarrow \quad a_n = \frac{3n^2 - 3n + 10}{2}$

Now, $S_{30} = \frac{3\sum_{n=1}^{30} n^2 - 3\sum_{n=1}^{30} n + 10\sum_{n=1}^{30} 1}{2} = \frac{3\times\frac{30\times 31\times 61}{6} - 3\times\frac{30\times 31 + 10\times 30}{2}}{2}$

$S_{30} = 13635$

$a_{40} = \frac{3(40)^2 - 3\times 40 + 10}{2} = 2345$

$\therefore \quad S_{30} - a_{40} = 11290$

57. (a)

$S_K = \frac{1 + 2 + ... + K}{K}$

$= \frac{K(K+1)}{2K} = \frac{K+1}{2}$

$\sum_{K=1}^{n} S_K^2 = \sum_{K=1}^{n}\left(\frac{K+1}{2}\right)^2$

$= \frac{1}{4}\sum_{K=1}^{n}(K+1)^2$

$= \frac{1}{4}\left[2^2 + 3^2 + ... + (n+1)^2 + 1^2 - 1^2\right]$

$= \frac{1}{4}\left[\frac{(n+1)(n+2)(2(n+1)+1)}{6} - 1\right]$

$= \frac{(n+1)(n+2)(2n+3) - 6}{24}$

$= \frac{n(2n^2 + 9n + 13)}{24}$

$\frac{n}{A}(Bn^2 + Cn + D)$ [given]

58. (a)

We have,

$1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$

$\Rightarrow \quad (1-2)(1+2) + (3-4)(3+4) + ... + (2021-2022)(2021+2022) + (2023)^2 = 1012m^2n$

$\Rightarrow \quad (-1)(1+2) + (-1)(3+4) + ...(-1)(2021+2022) + (2023)^3 = 1012m^2n$

$\Rightarrow \quad (-1)(1+2+3+4... + 2021 + 2022) + (2023)^2 = 1012m^2n$

$\Rightarrow \quad (-1).\frac{(2022)(2023)}{2}+(2023)^2$

$=1012m^2n$

$\Rightarrow \quad (2023)(2023-1011)=1012m^2n$

$\Rightarrow \quad 2023\times 1012=1012m^2n$

$\Rightarrow \quad m^2n=2023 \Rightarrow m^2n=289\times 7$

$\Rightarrow \quad m^2=289$ and $n=7$

$\Rightarrow \quad m=17$ and $n=7$ such that $\gcd(m,n)=1)$

$\therefore \quad m^2-n^2=289-49=240$

59. (c)

Let

$S=5+11+19+29+41+...+T_n \qquad \ldots \quad$ (i)

$S=5+11+19+29+...+T_{n-1}+T_n \qquad \ldots \quad$ (ii)

$0=5+6+8+10+12+...-T_n$

[shifting one terms and subtracting Eqs. (i) and (ii) we get]

$=5+\frac{n-1}{2}\left[2\times 6+(n-2)(2)\right]-T_n$

$\Rightarrow \quad T_n=5+(n-1)(n+4)$

$\Rightarrow \quad T_n=5+n^2+3n-4$

$\Rightarrow \quad T_n=n^2+3n+1$

$\Sigma T_n=\Sigma n^2+3\Sigma n+\Sigma 1$

$\Rightarrow \quad S_n=\frac{n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}+n$

When $n=20$

Then, $S_{20}=\frac{20\times 21\times 41}{6}+\frac{3\times 20\times 21}{2}+20$

$=2870+630+20=3520$

60. (b)

We have, $T_n=\frac{n}{1+n^2+n^4}$

$=\frac{n}{(n^2+n+1)(n^2-n+1)}$

$=\frac{1}{2}\left[\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right]$

Now, $S_{10}=\sum_{n=1}^{10}T_n$

$=\frac{1}{2}\sum_{n=1}^{10}\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)$

$$=\frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)+...+\left(\frac{1}{91}-\frac{1}{111}\right)\right]$$
$$=\frac{1}{2}\left[-\frac{1}{111}\right]=\frac{1}{2}\left[\frac{110}{111}\right]=\frac{55}{111}$$

61. (a)

Given, $a_n=\frac{-2}{4n^2-16n+15}$,

$$=\frac{-2}{4n^2-6n-10n+15}$$
$$\frac{-2}{2n(2n-3)-5(2n-3)}$$
$$\Rightarrow\quad a_n=\frac{-2}{(2n-3)(2n-5)}$$
$$=\frac{1}{2n-3}-\frac{1}{2n-5}$$
$$\therefore\quad a_1=\frac{1}{(-1)}-\frac{1}{(-3)}$$
$$a_2=\frac{1}{1}-\frac{1}{(-1)}$$
$$a_3=\frac{1}{3}-\frac{1}{1}$$

62. (c)

$$a_1+a_2+...a_n=\frac{n^2+3n}{(n+1)(n+2)}$$
$$\Rightarrow\quad S_n=\frac{n^2+3n+2-2}{n^2+3n+2}$$
$$=1-\frac{2}{n^2+3n+2}$$
$$=1-\frac{2}{(n+1)(n+2)}$$
$$S_{n-1}=1-\frac{2}{n(n+1)}$$
$$\therefore\quad a_n=\frac{2}{n(n+1)}-\frac{2}{(n+1)(n+2)}$$
$$\Rightarrow\quad \frac{2[n+2-n]}{n(n+1)(n+2)}=\frac{4}{n(n+1)(n+2)}$$
$$\Rightarrow\quad \frac{1}{a_n}=\frac{n(n+1)(n+2)}{4}=\frac{n^3+3n^2+2n}{4}$$

Now, $28\sum_{k=1}^{10}\frac{1}{a_k}$

$$=7\left(\sum_{k=1}^{10}k^3+3\sum_{k=1}^{10}k^2+2\sum_{k=1}^{10}k\right)$$

$$=7\left(\frac{10^2\times11^2}{4}+\frac{3\times10\times11\times21}{6}+\frac{2\times10\times11}{2}\right)$$

$$=7\left(5^2\times11^2+5\times11\times21+10\times11\right)$$

$$=7\times5\times11\left(5\times11+21+2\right)$$

$$=7\times2\times11\times78=2\times3\times5\times7\times11\times13$$

Which is the products of first 6 prime numbers.

63. (d)

$$17m=m+(m-4)+(m-4\times2)+...+(m-4\times24)$$

$$\Rightarrow\quad 17m=25m-4(1+2+...+24)$$

$$\Rightarrow\quad 8m=\frac{4\cdot24\cdot25}{2}=150$$

64. (a)

$$4^{1+x}+4^{1-x},\frac{K}{2},16^x+16^{-x}\to AP$$

$$K=4^{1+x}+4^{1-x}+16^x+16^{-x}$$

$$=4\left(4^x+\frac{1}{4^x}\right)+\left(4^{2x}+\frac{1}{4^{2x}}\right)$$

$$\because\quad P+\frac{1}{P}\geq2\ \text{ for all }\ P>0$$

$$\therefore\quad K_{min}=4\times2+2=10$$

65. (b)

Given that

$$S_{10}=390\ \text{ and }\ \frac{a_{10}}{a_5}=\frac{15}{7}$$

$$S_{10}=\frac{10}{2}[2a+9d]=390$$

$\Rightarrow\quad 2a+9d=78$ …. (i)

$$\frac{a_{10}}{a_5}=\frac{a+9d}{a+4d}=\frac{15}{7}$$

$$\Rightarrow\quad 7a+63d=15a+60d$$

$\Rightarrow\quad 8a=3d$ …. (ii)

On solving Eqs.(i) and (ii), we get

$a=3, d=8$

Now,

$S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$

$= \frac{5}{2}[4a + 38d] = 5(2a + 19d)$

$= 5(6 + 19 \times 8) = 790$

66. (d)

Given that

$S_{20} = 790$ and $S_{10} = 145$

$S_{20} = \frac{20}{2}[2a + 19d] = 790$

$\Rightarrow \quad 2a + 19d = 79 \quad \ldots. \quad (i)$

$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

And $S_{10} = 145$

$\Rightarrow \quad \frac{10}{2}[2a + 9d] = 145$

$2a + 9d = 29 \quad \ldots \quad (ii)$

On solving Eqs. (i) and (ii),

$a = -8, d = 5$

Now, $S_{15} - S_5$

$= \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$

$= \frac{15}{2}[-16 + 70] - \frac{5}{2}[-16 + 20]$

$= 405 - 10 = 395$

67. (b)

$\log_e q, \log_e c$ are in AP

$\Rightarrow \quad 2\log b = \log a + \log c \Rightarrow b^2 = ac$

$\because \log_e a - \log_e 2b,$

$\log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are in AP

$\Rightarrow \quad \log_e \frac{a}{2b}, \log_e \frac{2b}{3c}, \log_e \frac{3c}{a}$ are in AP

$\Rightarrow \quad 2\log_e \frac{2b}{3c} = \log_e \frac{a}{2b} + \log_e \frac{3c}{a}$

$\Rightarrow \quad \left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$

$\frac{4b^2}{9c^2} = \frac{3c}{2b} \Rightarrow 8b^3 = 27c^3$

$2b = 3c$

$\because \quad b^2 = ac$

$$\left(\frac{3c}{2}\right)^2 = ac \Rightarrow 9c = 4a$$

$\therefore$ a : b : c

$$\frac{9c}{4} : \frac{3c}{2} : c \Rightarrow 9 : 6 : 4$$

68. (b)

Given progression is

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ... \left(-129\frac{1}{4}\right)$$

i.e $20, \frac{77}{4}, \frac{37}{2}, \frac{71}{3}, ..., \left(-\frac{517}{4}\right)$

i.e. last term (t_n) or $l = -\frac{517}{4}$

As, we know that $a + (n-1)d = 1$

$\Rightarrow \quad -\frac{517}{4} = 20 + (n-1)\left(-\frac{3}{4}\right)$

$\Rightarrow \quad -517 = 80 + (-3n + 3)$

$\Rightarrow \quad -600 = -3n \Rightarrow n = 200$

Since, r th term from end

$= \quad (n - r + 1)$ th the from beginning

$\Rightarrow$ 20^{th} term from end

$= \quad (200 - 20 + 1)$ th term from beginning

$\Rightarrow$ 20^{th} term from end = 181 th term from beginning

Hence, $t_{181} = 20 + (181-1)\left(-\frac{3}{4}\right)$

$= 20 - 135 = -115$

Alternative Solution

If we consider last of given progression as first term.

i.e. $a = -129\frac{1}{4} = \frac{517}{4}$, then

$d = \text{common difference} = \frac{3}{4}$

Hence, $T_{20} = -\frac{517}{4} + 19\left(\frac{3}{4}\right) = \frac{460}{4} = -115$

69. (d)

First progression is 4,9,14,19,….

Up to 25^{th} term.

Second progression is 3,6,9,12,……

Up to 37^{th} term.

Now, T_{25} (for first) $= 4 + (25-1) \times 5$

$= 4 + 120 = 124$

And T_{37} (for second)

$= 3 + (37-1)\times 3 = 3 + 108 = 111$

Let d_1 = common difference of

1^{st} sequence = 5

$d_2 =$ common difference of

2^{nd} sequence = 3

And first common difference = 15 (which is LCM of d_1 and d_2)

Hence, common terms are 9,24,39,54,69,84,99.

Thus, total number of common terms = 7

70. (b)

$f(x) = \dfrac{1}{2+\sin 3x + \cos 3x} \quad -\sqrt{2} \le \sin 3x + \cos 3x \le \sqrt{2}$

Or $2-\sqrt{2} \le 2 + \text{Sin} 3x + \text{Cos } 3x \le 2+\sqrt{2}$

Or $\dfrac{1}{2+\sqrt{2}} \le \dfrac{1}{2+\sin 3x + \cos 3x} \le \dfrac{1}{2-\sqrt{2}}$

$\therefore \quad a = \dfrac{1}{2+\sqrt{2}}$ and $b = \dfrac{1}{2-\sqrt{2}}$

Now, $\alpha = \left(\dfrac{\dfrac{1}{2+\sqrt{2}} + \dfrac{1}{2-\sqrt{2}}}{2} \right)$

$= \dfrac{2-\sqrt{2}+2+\sqrt{2}}{(4-2)\times 2} = \dfrac{4}{2\times 2} = 1$

And $\beta = \sqrt{\dfrac{1}{(2+\sqrt{2})} \times \dfrac{1}{(2-\sqrt{2})}}$

$= \sqrt{\dfrac{1}{(4-2)}} = \dfrac{1}{\sqrt{2}}$

$\therefore \quad \dfrac{\alpha}{\beta} = \dfrac{1}{\left(\dfrac{1}{\sqrt{2}}\right)} = \sqrt{2}$

71. (a)

$\sum_{n=0}^{\infty} ar^n = 57 \Rightarrow \dfrac{a}{1-r} = 57$

$a = 57(1-r)$ …. (i)

$\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$

$\dfrac{a^3}{1-r^3} = 9747$ …. (ii)

By Eqs. (i) and (ii), we get

$$\frac{(57)^3(1-r)^3}{(1-r)\left(1+r+r^2\right)}=9747$$

By Eqs.(i) and (ii), we get

$$\frac{(57)^3(1-r)^3}{(1-r)\left(1+r+r^2\right)}=9747$$

$$\Rightarrow \quad \frac{1+r^2-2r}{1+r^2+r}=\frac{9747}{57\times57\times57}=\frac{1}{19}$$

$$\Rightarrow \quad 19+19r^2-38r=1+r^2+r$$

$$\Rightarrow \quad 18r^2-39r+18=0$$

$$\Rightarrow \quad 6r^2-9r-4r+6=0$$

$$\Rightarrow \quad 3r(2r-3)-2(2r-3)=0$$

$$r=\frac{3}{2},\frac{2}{3} \qquad \left(\because r=\frac{3}{2}\text{ reject}\right)$$

$$\therefore \quad r=2/3\Rightarrow a=19$$

$$\therefore \quad a+18r=19+18\times\frac{2}{3}=31$$

72. (d)

$$T_2+T_6=\frac{70}{3}$$

$$\Rightarrow \quad ar+ar^5=\frac{70}{3}$$

$$\Rightarrow \quad ar\left(1+r^4\right)=\frac{70}{3} \qquad \ldots \quad \text{(i)}$$

And $T_3\cdot T_5=49\Rightarrow ar^2\cdot ar^4=49$

$$\Rightarrow \quad a^2r^6=49\Rightarrow ar^2\cdot ar^4=49$$

$$\Rightarrow \quad a^2r^6=49\Rightarrow ar^3=7$$

$$\Rightarrow \quad a=\frac{7}{r^3}$$

Put the values of a in Eq. (i),

$$\frac{7}{r^3}\times r\left(1+r^4\right)=\frac{70}{3}$$

$$\Rightarrow \quad \frac{1+r^4}{r^2}=\frac{10}{3}\Rightarrow 3+3r^4=10r^2$$

$$\Rightarrow \quad 3\left(r^2\right)^2-10r^2+3=0$$

Let $r^2=t$

$$3t^2-10t+3=0$$

$$\Rightarrow \quad 3t^2-9t-t+3=0$$

$$\Rightarrow \quad 3t(t-3)-1(t-3)=0$$

$\Rightarrow \quad (t-3)(3t-1)=0 \Rightarrow t=3,1/3$

So, $r^2=3$ or $r^2=1/3$

$\Rightarrow \quad r \neq \frac{1}{\sqrt{3}}$ (GP is increasing)

$\Rightarrow \quad r=\sqrt{3}$

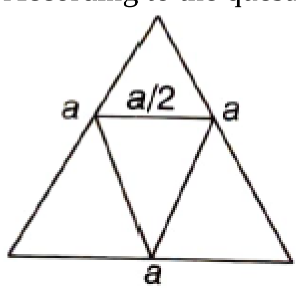
$T_4+T_6+T_8=ar^3+ar^5+ar^7$

$=ar^3\left(1+r^2+r^4\right)$

$=7(1+3+9)=91$

73. (a)

According to the question,



Area of first $\Delta=\frac{\sqrt{3}a^2}{4}$

Area of second $\Delta=\frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2=\frac{\sqrt{3}a^2}{16}$

Area of third $\Delta=\frac{\sqrt{3}a^2}{64}$

Sum of area $=\frac{\sqrt{3}a^2}{4}\left(1+\frac{1}{4}+\frac{1}{16}+\ldots\right)$

$Q=\frac{\sqrt{3}a^2}{4}\left(\frac{4}{3}\right)=\frac{a^2}{\sqrt{3}}$ … (i)

Perimeter of first triangle $=3a$

Perimeter of second triangle $=\frac{3a}{2}$

Perimeter of third triangle $=\frac{3a}{4}$

$P=3a\left(1+\frac{1}{2}+\frac{1}{4}+\ldots\right)$

$P=3a\cdot 2=6a$

$a=P/6$ … (ii)

By Eqs. (i) and (ii), we get

$Q=\frac{1}{\sqrt{3}}\frac{P^2}{36} \Rightarrow P^2=36\sqrt{3}Q$

74. (b)

Given, a,b,c are in AP

$\Rightarrow \quad 2b = a + c$

And $(a+1), b, c+3$ are in GP

$\Rightarrow \quad b^2 = (a+1)(c+3)$

AM of $a, b, c = 8$

$\frac{a+b+c}{3} = 8 \Rightarrow a+b+c = 24$

$\Rightarrow \quad 3b = 24 \Rightarrow b = 8$

$a + c = 16$ and $(a+1)(c+3) = 64$

$c = 16 - a$

Then, $(a+1)(16-a+3) = 64$

$\Rightarrow \quad (a+1)(19-a) = 64$

$\Rightarrow \quad 19a - a^2 + 19 - a = 64$

$\Rightarrow \quad a^2 - 18a + 45 = 0$

$\Rightarrow \quad (a-15)(a-3) = 0$

$a = 15, 3; \because a > 10$

$\Rightarrow \quad a = 15$ and $c = 1$

$\therefore \quad a = 15, b = 8, c = 1$

$\therefore \quad$ GM of $a, b, c = (abc)^{1/3}$

Cube of GM $= 120$

75. (d)

Since, 2,p,q are in GP.

So, $\quad p^2 = 2q \quad \ldots$ (i)

According to question,

$a + 6d = 2 \quad \ldots$ (ii)

$a + 7d = p \quad \ldots$ (iii)

$a + 12d = q \quad \ldots$ (iv)

Where d = common difference subtracting Eq. (iii) from Eq.(ii),

$p - 2 = d$

Subtracting Eq. (iv) from Eq.(iii),

$q - p = 5d = 5(p-2)$(From Eq.(v))

$\Rightarrow \quad q = 6p - 10$

From Eq. (i), $p^2 = 2(6p-10)$

$\Rightarrow \quad p^2 = 12p + 20 = 0 \Rightarrow p = 10, 2$

When $p = 10$, then $q = 50$ and $d = 8$ and $a = -46$

So, GP is 2,10,50,250,1250

Now, $ar^4 = a + (n-1)d$

$\Rightarrow \quad 1250 = -46 + (n-1)8$

$\therefore \quad n = 163$

76. (d)

We have, $px^2 + qx - r = 0$

Given that p,q and r be the consecutive terms of a non-constant.
GP, then let

$p = \frac{a}{r_1}, q = a, r = ar_1$

And given, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$

$\Rightarrow \quad \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{4} \Rightarrow \frac{-\frac{q}{p}}{-\frac{r}{p}} = \frac{3}{4}$

$\Rightarrow \quad \frac{q}{r} = \frac{3}{4} \Rightarrow \frac{1}{r_1} = \frac{3}{4}$

$\Rightarrow \quad r_1 = \frac{4}{3}$ ... (i)

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= \left(\frac{-q}{p}\right)^2 - 4\left(\frac{-r}{p}\right)$

$= (r_1)^2 + 4\left(r_1^2\right) = 5r_1^2$

$= 5\left(\frac{4}{3}\right)^2$ [from Eq.(i)]

$= \frac{80}{9}$

77. (b)

Let AP be $3, 3+d, 3+2d, 3+3d$

So, $3+d = a, 3+2d = b, 3+3d = c$

Also, ;let GP be $3r = a-1, 3r^2 = b+1, 3r^3 = c+9$

On comparing, we get

$3r = 3 + d - 1$

$\Rightarrow \quad d = 3r - 2$ ... (i)

And $3r^2 = 3 + 2d - 1$

$\Rightarrow \quad 3r^2 = 2d + 4$ ... (ii)

On solving Eqs. (i) and (ii), we get

$r = 2, a = 7, b = 11, c = 15$

Hence, AM of a,b and c

$= \frac{7+11+15}{3} = 11$

78. (b)

Let first term of AP be a and common difference be d.

Given, $a = 1$

2^{nd} term of AP, $T_2 = a + d = 1 + d$

8^{th} term of AP, $T_8 = a + 7d = 1 + 7d$

44^{th} term of AP,

$T_{44} = a + 43d = 1 + 43d$

Given, $1+d, 1+7d, 1+43d$ are in GP

$$(1+7d)^2 = (1+d)(1+43d)$$

$\Rightarrow \quad 1 + 49d^2 + 14d = 1 + 44d + 43d^2$

$\Rightarrow \quad 6d^2 - 30d = 0$

$\Rightarrow \quad d = 5$ [given, non-constant AP]

So, $S_{20} = \frac{20}{2}\left[2 \times 1 + (20-1) \times 5\right]$

$= 970$

79. (b)

For 1^{st} GP,

$$T_1 = a$$

$$T_3 = b = ar_1^2 \qquad \ldots \text{(i)}$$

For 2^{nd} GP,

$$T_1' = a,$$

$$T_5' = b = ar_2^4 \qquad \ldots \text{(ii)}$$

According to the question,

$T_{11} = T_P'$

$\left[\because \text{general term of GP} = ar^{n-1}\right]$

i) $\Rightarrow \quad ar_1^{10} = a \cdot r_2^{p-1}$

$\Rightarrow \quad a\left(\frac{b}{a}\right)^5 = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

[by Eqs. (i) and (ii)]

$\therefore \quad 5 = \frac{p-1}{4} \Rightarrow p = 21$

80. (b)

$a_1, a_2, a_3, \ldots$ are in GP

$a_1 = \frac{1}{8}$

$a_2 = \frac{a_3 + a_4}{2} \qquad \ldots \text{(i)}$

$\therefore \quad a_1 = a$ and r the common ratio

$\therefore \quad$ Eq.(i) becomes,

$ar = \frac{ar^2 + ar^3}{2}$

$2 = r + r^2$

$r^2+r-2=0; r^2+2r-r-2=0$

$(r+2)(r-1)=0$

$r=-2,1$

$a_2 \neq a_1 \Rightarrow r \neq 1$

$\therefore \quad r=-2$

Now, $S_{20}-S_{18}=a_{19}+a_{20}$

$=ar^{18}+ar^{19}=ar^{18}(1+r)$

$=\frac{1}{8}(-2)^{18}(1-2)$

$=\frac{-2^{18}}{2^3}=-2^{15}$

81. (b)

Let a be the first term and r be the common ratio of the GP

$\therefore \quad (a+ar+ar^2+...+ar^{63}) = 7(a+ar^2+...+ar^{62})$

$\Rightarrow \quad \frac{a(1-r^{64})}{1-r}=\frac{7a(1-r^{64})}{1-r^2}$

$\Rightarrow \quad 1=\frac{7}{1+r} \Rightarrow 1+r=7$

$\Rightarrow \quad r=6$

82. (a)

Given, $2\tan^2\theta-5\sec\theta=1$

$\Rightarrow \quad 2(\sec^2\theta-1)-5\sec\theta-1=0$

$\Rightarrow \quad 2\sec^2\theta-5\sec\theta-3=0$

$\Rightarrow \quad 2\sec^2\theta-6\sec\theta+\sec\theta-3=0$

$\Rightarrow \quad (2\sec\theta+1)(\sec\theta-3)=0$

$\because \quad \sec\theta=-\frac{1}{2}$ (not possible)

$\therefore \quad \sec\theta=3 \Rightarrow \cos\theta=\frac{1}{3}$

From graph,

2 solution in $[0,2\pi]$,

2 solution in $[2\pi,4\pi]$,

2 solution in $[4\pi,6\pi]$,

And 1 solution in $\left[6\pi,\frac{13\pi}{2}\right]$.

Hence, $n=13$

$\sum_{k=1}^{13}\frac{k}{2^k}=S$ [say]

So, $S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+...+\frac{13}{2^{13}}$

$\frac{1}{2}S=\frac{1}{2^2}+\frac{2}{2^3}+...+\frac{13}{2^{14}}$

$\frac{1}{2}S=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{13}}-\frac{13}{2^{14}}$

$\Rightarrow \quad \frac{1}{2}S=\frac{1}{2}\left[\frac{\left(1-2^{13}\right)}{\left(1-\frac{1}{2}\right)}-\frac{13}{2^{14}}\right]$

$\Rightarrow \quad S=2\left(\frac{2^{13}-1}{2^{13}}\right)-\frac{13}{2^{13}}=\frac{2^{14}-15}{2^{13}}$

83. (d)

Given, $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$

$\because$ 1 satisfies the equation.

α and 1 are roots of equation.

Product of roots

$\alpha\cdot 1=\frac{c+a-2b}{a+b-2c}$

$\alpha=\frac{c+a-2b}{a+b-2c}$

Statement I

$\alpha\in(-1,0)$

$-1<\alpha<0$

$-1<\frac{c+a-2b}{a+b-2c}<0;$

$\because \quad a>b>c>0$

$0<\frac{c+a-2b+a+b-2c}{a+b-2c}<1$

$\because \quad a+b>c+c$

$a+b>2c$

$a+b-2c>0$

$0<\frac{2a-b-c}{a+b-2c}<1$

$\therefore \quad 2a-b-c>0$

$2a>b+c$

$a>\frac{b+c}{2}$

$\therefore$ b can't be GM between a and c

Statement II $\alpha\in(0,1)$

$0<\alpha<1$

$0<\frac{c+a-2b}{a+b-2c}<1$

Here, $b > c$ and $b < \frac{a+c}{2}$

$\therefore$ b may be GM between a and c.

$\therefore$ Statement I and statement II both are true.

84. (c)

Given,

$$\frac{1}{1\cdot(1+d)}+\frac{1}{(1+d)(1+2d)}+...+\frac{1}{(1+9d)(1+10d)}=5$$

$$\Rightarrow \frac{1}{d}\left[\frac{(1+d)-1}{1\cdot(1+d)}+\frac{(1+2d)-(1+d)}{(1+d)(1+2d)}+...+\frac{(1+10d)-(1+9d)}{(1+9d)(1+10d)}\right]=5$$

$$\Rightarrow \frac{1}{d}\left[\left(1-\frac{1}{1+d}\right)+\left(\frac{1}{1+d}-\frac{1}{1+2d}\right)+...+\left(\frac{1}{1+9d}-\frac{1}{1+10d}\right)\right]=5$$

$$\Rightarrow \frac{1}{d}\left[1-\frac{1}{10d}\right]=5$$

$$\Rightarrow \frac{10d}{d(1+10d)}=5 \Rightarrow 50d=5$$

85. (a)

We have, $m=\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{99}+\sqrt{100}}$

$$\Rightarrow m=\frac{\sqrt{2}-\sqrt{1}}{(\sqrt{1}+\sqrt{2})(\sqrt{2}-\sqrt{1})}+\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{2}+\sqrt{3})(\sqrt{3}-\sqrt{2})}+...+\frac{\sqrt{100}-\sqrt{99}}{(\sqrt{99}+\sqrt{100})(\sqrt{100}-\sqrt{99})}$$

$$\Rightarrow m=(\sqrt{2}-\sqrt{1})+(\sqrt{3}-\sqrt{2})+...+(\sqrt{100}-\sqrt{99})$$

$$=\sqrt{100}-\sqrt{1}=10-1=9$$

Also, $n=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+...+\frac{1}{99\cdot 100}$

$$n=\frac{2-1}{1\cdot 2}+\frac{3-2}{2\cdot 3}+...+\frac{100-99}{99\cdot 100}$$

$$\Rightarrow n=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+...+\frac{1}{99}-\frac{1}{100}$$

$$\Rightarrow n=1-\frac{1}{100}=\frac{99}{100}$$

$$\therefore (m,n)=\left(9,\frac{99}{100}\right)$$

Which lies on the line $11x-100y=0$

86. (c)

$$\frac{1\times 2^2+2\times 3^2+...100\times(101)^2}{1^2\times 2+2^2\times 3+...100^2\times 101}$$

$$\Rightarrow \frac{\sum_{n=1}^{100} r(r+1)^2}{\sum_{n=1}^{100} r^2(r+1)} = \frac{\Sigma\left(r^3+2r^2+r\right)}{\Sigma r^3+r^2}$$

$$= \frac{\left[\frac{n}{2}(n+1)\right]^2 + \frac{2n}{6}(n+1)(2n+1) + \frac{n}{2}(n+1)}{\left[\frac{n}{2}(n+1)\right]^2 + \frac{n}{6}(n+1)(2n+1)}$$

$$= \frac{\frac{n}{2}(n+1) + \frac{2(2n+1)}{3} + 1}{\frac{n}{2}(n+1) + \frac{2n+1}{3}}$$

$$= \frac{3n(n+1)+4(2n+1)+6}{3(n)(n+1)+4n+2}$$

$$\Rightarrow \frac{3n^2+3n+8n+10}{3n^2+7n+2}$$

$$\Rightarrow \frac{3n^2+11n+10}{3n^2+7n+2} = 1 + \frac{4n+8}{3n^2+7n+2}$$

Put $n = 100$

$$\Rightarrow 1 + \frac{408}{30702} = 1 + \frac{4}{301} = \frac{305}{301}$$

87. (b)

$$S = \frac{1}{1-3\cdot1^2+1^4} + \frac{2}{1-3\cdot2^2+2^4} + \frac{3}{1-3\cdot3^2+3^4} + \ldots 10 \text{ term}$$

$$T_n = \frac{n}{1-3n^2+n^4}$$

$$= \frac{n}{\left(n^2-n-1\right)\left(n^2+n-1\right)}$$

$$= \frac{1}{2}\left[\frac{1}{n^2-n-1} - \frac{1}{n^2+n-1}\right]$$

$$S = \sum_{n=1}^{10} T_n$$

$$= \frac{1}{2}\left[\sum_{n=1}^{10} \frac{1}{n^2-n-1} - \frac{1}{n^2+n-1}\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{-1}-\frac{1}{1}\right) + \left(\frac{1}{1}-\frac{1}{5}\right) + \left(\frac{1}{5}-\left(-\frac{1}{11}\right)\right) + \ldots \left(\frac{1}{89}-\frac{1}{109}\right)\right]$$

$$= \frac{1}{2}\left(-1-\frac{1}{109}\right) = \frac{-55}{109}$$

Section-II

1. (7744)

$$S=\left(209+220+231+....+495\right)-\left(231+319+418+341+451\right)$$

$$=\frac{27}{2}\times 704-1760=9504-1760=7744$$

2. (5143)

Total 4-digit numbers

$=9\times 10\times 10\times 10=9000$

Numbers divisible by 7 are, 1001, 1008, ….., 9996

$\Rightarrow 9996=1001+(n-1)7\Rightarrow n=1286$

Again, Numbers divisible by 3 are, 1002, 1005, ……, 9999

$\Rightarrow 9999=1002+\left(m-1\right)3\Rightarrow m=3000$

Numbers which are divisible by both 7 and 3 are $1008,\ 1029,\ \ldots..,9996\Rightarrow 9996$

$=1008+\left(p-1\right)21\Rightarrow p=429$

$\therefore$ Required number of 4-digit numbers which are neither multiple of 7 nor multiple of 3

$=9000-\left(1286+3000-429\right)=9000-3857$

$=5143$

3. (3)

$\because\ \log_3 2,\log_3\left(2^x-5\right)$

$\log_3\left(2^x-\frac{7}{2}\right)\in$ A.P.

$\Rightarrow 2\log_3\left(2^x-5\right)=\log_3 2+\log_3\left(2^x-\frac{7}{2}\right)$

$\Rightarrow \left(2^x-5\right)^2=2\left(2^x-\frac{7}{2}\right)$

$\Rightarrow \left(2^x-5\right)^2=\left(2^{x+1}-7\right)$

$\Rightarrow 2^{2x}+25-5.2^{x+1}=2^{x+1}-7$

$\Rightarrow 2^{2x}+12.2^x+32=0$

Let $2^x=t\Rightarrow t^2-12t+32=0$

$\Rightarrow \left(t-8\right)\left(t-4\right)=0$

$2^x=2^3$ or $2^x=2^2\Rightarrow x=3$ or 2

But for $x=2,\ \left(2^x-5\right)<0$ which is not possible. $\therefore\ \ x=3$

4. (832)

We have,

$A=\{1,2,3,.....,100\}\quad\left(\because 101^2=10201>10101\right)$

$B=\{4,7,10,....,100,103,106,....\}$

$C=\{2,4,6,8,.......,100,102,104,106,....\}$

Now, $B-C=\{7,13,19,25,......,97,\quad 103,......\}$

$\Rightarrow A\cap(B-C)=\{7,13,19,25,.....,97\}$

We can see that the elements of the set $A\cap(B-C)$ form an A.P. with a = 7, d = 6.

$\therefore\ 97=7+(n-1)6\Rightarrow n=16.$

So, $S_{16}=\frac{16}{2}[7+97]=832$

5. (2021)

Clearly, $c_2=a_2+b_2$

$=a_1-3+2b_1=12$ (Given)

$\Rightarrow a_1+2b_1=15$ .....(i)

And $c_3=a_3+b_3=a_1-6+4b_1=13$ (Given)

$\Rightarrow a_1+4b_1=19$ .....(ii)

On solving (i) and (ii), we get $b_1=2$ and $a_1=11$

Now, $\sum_{k=1}^{10}c_k=\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}b_k$, where

$\sum_{k=1}^{10}a_k=\frac{10}{2}(2a_1+(10-1)d)$

$=\frac{10}{2}\times(22+9\times(-3))$

$=5\times(22-27)=-25$

And $\sum_{k=1}^{10}b_k=\frac{b_1(r^{10}-1)}{(2-1)}=\frac{2\times(2^{10}-1)}{2-1}$

$=2\times(2^{10}-1)=2096$

$\Rightarrow\sum_{k=1}^{10}c_k=2021$

6. (305)

We have,

$S=\frac{7}{5}+\frac{9}{5^2}+\frac{13}{5^3}+\frac{19}{5^4}+....$ (i)

$\Rightarrow\frac{S}{5}=\frac{7}{5^2}+\frac{9}{5^3}+\frac{13}{5^4}+....$ (ii)

Subtracting (ii) from (i), we get

$\left(S-\frac{S}{5}\right)=\frac{7}{5}+\frac{2}{5^2}+\frac{4}{5^3}+\frac{6}{5^4}+....$

$\frac{4S}{5}=\frac{7}{5}+\frac{2}{5^2}+\frac{4}{5^3}+\frac{6}{5^4}+....$

$\frac{4S}{5}=\frac{7}{5}+\frac{2}{5^2}+\frac{4}{5^3}+\frac{6}{5^4}+....$

$\frac{4S}{5}=\frac{7}{5}+S_1\text{(say)}$ ....(iii)

$$\text{Now, } S_1 = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + ...$$

$$\Rightarrow \frac{S_1}{5} = \frac{2}{5^3} + \frac{4}{5^4} + \frac{6}{5^5} + ...$$

$$\Rightarrow \left(S_1 - \frac{S_1}{5}\right) = \frac{2}{5^2} + 5\left(\frac{1}{5^3} + \frac{1}{5^4} + \frac{1}{5^5} + ...\infty\right)$$

$$\Rightarrow \frac{4S_1}{5} = \frac{2}{5^2} + \frac{2 \times \frac{1}{5^3}}{1 - \frac{1}{5}}$$

$$\Rightarrow S_1 = \frac{5}{4}\left[\frac{2}{25} + \frac{1}{50}\right] = \frac{1}{8}$$

From (iii), we have, $\frac{4S}{5} = \frac{7}{5} + \frac{1}{8} = \frac{61}{40}$

$$\Rightarrow S = \frac{61}{40} \times \frac{5}{4} = \frac{61}{32} \Rightarrow 160S = 305$$

7. (7)

We have, $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$ ....(i)

$$\text{Now, } \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = \sum_{n=1}^{\infty} \frac{a_n}{8^n} = S\,(\text{say})$$

Dividing both sides of (i) by 8^n, we get

$$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$$

$$\Rightarrow 64\frac{a_{n+2}}{8^{n+2}} = \frac{16a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$$

$$\Rightarrow 64\sum_{n=1}^{\infty}\frac{a_{n+2}}{8^{n+2}} = 16\sum_{n=1}^{\infty}\frac{a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty}\frac{a_n}{8^n}$$

$$\Rightarrow 64\left(S - \frac{a_1}{8} - \frac{a_2}{8^2}\right) = 16\left(S - \frac{a_1}{8}\right) + S$$

$$\Rightarrow 64\left(S - \frac{1}{8} - \frac{1}{64}\right) = 16\left(S - \frac{1}{8}\right) + S$$

$$\Rightarrow 64S - 9 = 16S - 2 + S \Rightarrow 47S = 7$$

$$47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}} = 7$$

8. (3)

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + ...\text{upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...\text{upto } \infty\right)} = l$$

$$= \left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + ...\right)^{\log_{\frac{1}{4}}\left(\frac{1}{3}\left(\frac{1}{1 - \frac{1}{3}}\right)\right)} = l$$

Let $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+....=x$

$\Rightarrow (x-1)=\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+....$ ....(i)

$\Rightarrow \frac{1}{3}(x-1)=\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}....$ .....(ii)

Subtracting (ii) from (i), we get

$$\Rightarrow \frac{2}{3}(x-1)=\frac{2}{3}+\frac{4}{3^2}\times\left(\frac{1}{1-\frac{1}{3}}\right)$$

$$\Rightarrow \frac{2}{3}(x-1)=\frac{2}{3}+\frac{4}{3^2}.\frac{3}{2}$$

$$\Rightarrow \frac{2}{3}(x-1)=\frac{2}{3}+\frac{2}{3}$$

$\Rightarrow x-1=2$ or $x=3$

$\therefore\ 3^{\log_{\frac{1}{4}}\frac{1}{2}}=l \qquad 3^{\frac{1}{2}}=l \Rightarrow l^2=3$

9. (3)

Possible A.P. is 11, 16, 21, 26…….with possible last term = 9996

Also, possible G.P. is 4, 8, 16, 32……with possible last term = 8192

Now, for common terms, we have

General term of A.P. = General term of G.P.

$$\Rightarrow 11+(n-1)5=4\left(2^{n-1}\right)$$

$$\Rightarrow 5n+6=2^{n+1}$$

$$\Rightarrow n=\frac{2^{n+1}-6}{5}$$

This is only possible when, unit digit of 2^{n+1} is 6.

i.e., for n = 3, 7, 11. So, only 3 common terms exist

10. (14)

Given, $\frac{1}{16}$, a, b are in G.P.

$$\Rightarrow 16a^2=b$$

Also, $\frac{1}{a},\frac{1}{b},6$ are in A.P. $\Rightarrow \frac{2}{b}=\frac{1}{a}+6$

$$\Rightarrow \frac{2}{16a^2}=\frac{1}{a}+6 \Rightarrow \frac{1}{8a^2}-\frac{1}{a}-6=0$$

$$\Rightarrow \frac{1}{a^2}-\frac{8}{a}-48=0 \Rightarrow \frac{1}{a}-\frac{1}{a}-6=0$$

$$\Rightarrow a=\frac{1}{12},\frac{-1}{4}$$

But $a>0 \Rightarrow a=\frac{1}{12}$

$\therefore\ b = 16\left(\frac{1}{12}\right)^2 = \frac{1}{9}$

$\therefore\ 72(a+b) = 72\left(\frac{1}{12}+\frac{1}{9}\right) = 14$

11. (3)

Let the terms are a, ar, ar^2, ar^3

$\therefore\ a\left(\frac{r^4-1}{r-1}\right) = \frac{65}{12}$ .....(i)

Also, $\frac{1}{a}\frac{\left(\frac{1}{r^4}-1\right)}{\frac{1}{r}-1} = \frac{65}{18} \Rightarrow \frac{1}{a}\left(\frac{\frac{1-r^4}{r^4}}{\frac{1-r}{r}}\right) = \frac{65}{18}$

$\Rightarrow \frac{1}{ar^3}\left(\frac{1-r^4}{1-r}\right) = \frac{65}{18}$ .....(ii)

Dividing (i) by (ii) , we get $a^2r^3 = \frac{3}{2}$

Also, we have $a^2r^3 = 1$, i.e., ar = 1

Now, from (iii), we get

$(a^2r^2)r = \frac{3}{2} \Rightarrow r = \frac{3}{2}$

$\therefore\ a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$

Now, third term,

$ar^2 = \frac{2}{3}\left(\frac{3}{2}\right)^2 = \frac{3}{2} = \alpha$ (Given)

$\Rightarrow 2\alpha = 3$

12. (9)

Given, length of side of $A_1 = 12$

Length of diagonal of $A_2 = 12$

$\therefore$ Length of side of $A_2 = \frac{12}{\sqrt{2}}$

Length of side of $A_3 = \frac{12}{(\sqrt{2})^2}$

Length of side of $A_n = \frac{12}{(\sqrt{2})^{n-1}}$

$\therefore$ Area of $A_n = \left(\frac{12}{(\sqrt{2})^{n-1}}\right)^2 < 1$

$$\Rightarrow \frac{144}{2^{(n-1)}} < 1 \Rightarrow 2^{(n-1)} > 144$$

$$\Rightarrow n-1 > 7 \Rightarrow n > 8$$

Since, n is a natural number

$\therefore$ Least value of n is 9.

13. (10)

14. (9)

We have,

$$\frac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)}$$

$$= \sum_{k=0}^{20} \frac{A_k}{\alpha+k}, k \in N$$

$$\Rightarrow \frac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)}$$

$$= \frac{A_0}{\alpha} + \frac{A_1}{\alpha+1} + \frac{A_2}{\alpha+2} + ... + \frac{A_{20}}{\alpha+20}$$

$$\Rightarrow 1 = A_0(\alpha+1)(\alpha+2).....(\alpha+20) + A_1(\alpha)(\alpha+2).....(\alpha+20) + ... + A_{20}(\alpha)(\alpha+1)...(\alpha+19)$$

Now, Putting $\alpha = -13$ in above relation, we get

$$A_{13} = \frac{1}{(-13)(-12)(-11)...(-13+12)(-13+14)...(-13+20)}$$

$$= -\frac{1}{\lfloor 13 . \lfloor 7}$$

Similarly,

$$A_{14} = \frac{1}{(-14)(-13)...(-14+13)(-14+15)...(-14+20)} = \frac{1}{\lfloor 14 . \lfloor 6}$$

And

$$A_{15} = \frac{1}{(-15)(-14)....(-15+20)} = \frac{1}{\lfloor 15 . \lfloor 5}$$

Now, $\frac{A_{14}}{A_{13}} = \frac{-\lfloor 13 \lfloor 7}{\lfloor 14 \lfloor 6} = -\frac{7}{14} = -\frac{1}{2}$

$$\frac{A_{15}}{A_{13}} = \frac{-\lfloor 13 . \lfloor 7}{\lfloor 15 . \lfloor 5} = \frac{1}{5}$$

Value of 100 $\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2 = 100\left(\frac{A_{14}}{A_{13}} + \frac{A_{15}}{A_{13}}\right)^2$

$$= 100\left(-\frac{1}{2} + \frac{1}{5}\right)^2 = 100 \times \left(\frac{-3}{10}\right)^2 = 9$$

15. (16)

$$S_n(x) = (2+3+6+11+18+27+.......+T_n)\log_a x$$

Let $S = 2+3+6+11+18+27+......+T_{n-1}+T_n$ .....(i)

Also, $S = 2+3+6+11+18+........+T_n$ .....(ii)

Subtracting (i) from (ii), we get

$T_n = 2+\left[1+3+5+......+(n-1)\text{terms}\right]$

$\therefore T_n = 2+(n-1)^2$

$S = \sum T_n = 2n + \frac{(n-1)n(2n-1)}{6}$

Thus, $S_n(x) = \left(2n + \frac{n(n-1)(2n-1)}{6}\right)\log_a x$

Also, $S_{24}(x) = 1093$

$\Rightarrow \log_a(x)\left(48 + \frac{23\times24\times47}{6}\right) = 1093$

$\Rightarrow \log_a(x) = \frac{1}{4}$ ....(iii)

Also, $S_{12}(2x) = 265$

$\Rightarrow \log_a(2x)\left(24 + \frac{11\times12\times23}{6}\right) = 265$

$\Rightarrow \log_a 2x = \frac{1}{2}$ .....(iv)

Subtracting (iii) from (iv), we get

$\log_a 2x - \log_a x = \frac{1}{4}$

$\Rightarrow \log_a 2 = \frac{1}{4} \Rightarrow a = 16$

16. (38)

P and r are the roots of the equation,

$x^2 - 8ax + 2a = 0$

$\therefore \quad p + r = 8a, pr = 2a \Rightarrow \frac{1}{p} + \frac{1}{r} = 4$ .....(i)

Also, q and s are the roots of

$x^2 + 12ax + 6b = 0$

$\therefore \quad q + s = -12b, qs = 6b \Rightarrow \frac{1}{q} + \frac{1}{s} = -2$ ....(ii)

Given that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P.

$\therefore \quad \frac{1}{q} - \frac{1}{p} = \frac{1}{r} - \frac{1}{q} \Rightarrow \frac{2}{q} = \frac{1}{r} + \frac{1}{p} = 4$ [Using (i)]

$\Rightarrow q = \frac{1}{2}$ and $s = \frac{-1}{4}$

Clearly, $\frac{1}{r} - \frac{1}{q} = \frac{1}{s} - \frac{1}{r} \Rightarrow \frac{2}{r} = \frac{1}{q} + \frac{1}{s}$

$\Rightarrow \frac{2}{r} = -2 \Rightarrow r = -1$ and $p = \frac{1}{5}$ [Using (ii)]

$\therefore\ \frac{1}{a}-\frac{1}{b}=\frac{2}{pr}-\frac{6}{qs}=38$

17. (6993)
Let a_i denotes the i^{th} set.
$\therefore$ Number of elements in $a_1 = 1$
Number of elements in $a_2 = 3$
Number of elements in $a_3 = 5$

$\therefore$ Number of elements in a_{11}
$=1+(11-1).2=21$
Total number of elements in set 1 + set 2 +…….+ set 10
$=1+3+5+....+1+(9\times 2)=100$
$\therefore$ First term of $a_{11}=101\times 3=303$
And last term of $a_{11}=121\times 3=363$
$\therefore$ Sum of elements of a_{11}
$=\frac{21}{2}(303+363)=6993$

18. (53)
Let the common difference of the A.P'.s having 3, 4, …..,33 terms be $d_1, d_2, ..., d_{31}$
So, $d_1=\frac{199-100}{2}=49.5\notin I,$
$d_2=\frac{199-100}{3}=33\in I,$
$d_3=\frac{199-100}{4}=24.75\notin I,$
$d_4=\frac{199-100}{5}=19.8\notin I,$
Continuting in this way, we get only 3 possibilities of d i.e., 9, 11, 33.
So, required sum = 9 + 11 + 33 = 53

19. (50)
We have,
$f(x)=(x-p)^2-q,\ x\in R, q>0$
$f(x)=0$
$\Rightarrow (x-p)^2-q=0\Rightarrow x=p+\sqrt{q}, p-\sqrt{q}$
Let $a_1=a, a_2=a+d, a_3=a+2d$ and $a_4=a+3d$
$\therefore\quad P=\frac{4a+6d}{4}\Rightarrow a=\left(p-\frac{3}{2}d\right)$
Now, $|f(a_i)|=500$
$\Rightarrow |f(a_1)|=500\Rightarrow \left|(a_1-p)^2-q\right|=500$

$$\Rightarrow\left[\left\{\left(p-\frac{3}{2}d\right)-p\right\}^2-q\right]=500$$

$$\Rightarrow\frac{9}{4}d^2-q=500$$

Now, $|f(a_1)|^2=|f(a_2)|^2$ .....(i)

$$\Rightarrow\left[(a_1-p)^2-q\right]^2=\left[(a_2-p)^2-q\right]^2$$

$$\Rightarrow(a_1-p)^4-q^2-2q(a_1-p)^2=(a_1-p)^4+q^2-2q(a_2-p)^2$$

$$\Rightarrow(a_1-p)^4-2q(a_1-p)^2=(a_2-p)^4-2q(a_2-p)^2$$

$$\Rightarrow\left(\frac{-3}{2}d\right)^4-2q\left(\frac{-3}{2}d\right)^2$$

$$\Rightarrow\left(\frac{-3}{2}d+d\right)^4-2q\left(\frac{-3}{2}d+d\right)^2$$

$$\Rightarrow d^2=\frac{4q}{5}$$

From (i), we have $\frac{9}{4}\times\frac{4q}{5}-q=500$

$$\Rightarrow\frac{4q}{5}=500\Rightarrow q=625$$

$\therefore$ Absolute difference between the roots is $=2\sqrt{q}=2\times25=50$

20. (16)

$a_1, a_2, ...a_n, ...$ be an A.P. and $\sum_{r=1}^{\infty}\frac{a_r}{2^r}=4$

Let $S=\sum_{r=1}^{\infty}\frac{a_r}{2^r}=\frac{a_1}{2}+\frac{a_2}{2^2}+\frac{a_3}{2^3}+....$

$$\frac{S}{2}=\frac{a_1}{2^2}+\frac{a_2}{2^3}+\frac{a_3}{2^4}+....$$

$$S-\frac{S}{2}=\frac{a_1}{2}+\frac{a_2-a_1}{2^2}+\frac{a_3-a_2}{2^3}+...a$$

$$\Rightarrow\frac{S}{2}=\frac{a_1}{2}+d\left(\frac{1}{2^2}+\frac{1}{2^3}+...\right)$$

$$=\frac{a_1}{2}+d\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)=\frac{a_1}{2}+\frac{d}{2}$$

$\Rightarrow S=a_1+d=a_2=4$ (Given)

$$\Rightarrow 4a_2=4\times4=16$$

21. (702)

22. (2223)

S : 3, 6, 9, 12………upto 78 terms and

T : 5, 9, 13, 17…….upto 59 terms are given

Two series,

For S : 3, 6, 9, 12,…..upto 78 term

It is an A.P. where a = 3, d = 3

$S_{78} = 3 + (77 \times 3) = 234$

Now, T : 5, 9, 13, …….upto 59 terms is also an A.P. where a = 5 and d = 4;

$T_{59} = 5 + (58)4 = 237$

Now, common difference of common terms A.P. = LCM (3, 4) = 12

$\therefore$ Common terms = {9, 21, 33,….n terms}, where n^{th} terms ≤ 234 i.e., $9 + (n-1)12 \le 234$

$\Rightarrow (n-1) \le \frac{225}{12} \Rightarrow n \le 19.75 \Rightarrow n = 19$

$\therefore$ A.P. of common terms having 19 terms is given by, C : {9, 21, 23,…..225}

Sum of terms common to both the series is given by

$C_n = \frac{19}{2}\{18 + (18 \times 12)\} = 2223$

23. (142)

$x_1, x_2, x_3, ..., x_{20}$ is G.P.

$x_1 = 3, r = \frac{1}{2}, x_i = 3.\left(\frac{1}{2}\right)^{i-1}$

$\therefore \sum_{i=1}^{20} (x_i - i)^2 = \sum_{i=1}^{20} (x_i^2 + i^2 - 2x_i i)$

$= S_1 + S_2 - 2S_3$ ....(i)

Let $S_1 = \sum_{i=1}^{20} x_i^2 = \sum_{i=1}^{20} 9\left(\frac{1}{4}\right)^{i-1}$

$= 9\left(1 + \frac{1}{4} + \frac{1}{4^2} + ... + \frac{1}{4^{19}}\right)$

$= 9\frac{\left(1 - \left(\frac{1}{4}\right)^{20}\right)}{1 - \frac{1}{4}} = 12\left(1 - \frac{1}{2^{40}}\right)$

$S_2 = \sum_{i=1}^{20} i^2 = \frac{20 \times 21 \times 41}{6} = 2870$

$S_3 = \sum_{i=1}^{20} x_i.i = \sum_{i=1}^{20} 3\left(\frac{1}{2}\right)^{i-1}.i$

$= 3\sum_{i=1}^{20}\left(\frac{1}{2}\right)^{i-1}.i = 3\left[1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + ... + \frac{20}{2^{19}}\right]$ ....(ii)

$\frac{S_3}{2} = 3\left[\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + ... + \frac{19}{2^{19}} + \frac{20}{2^{20}}\right]$ ....(iii)

Subtract (iii) from (ii) , we get

$$\frac{S_3}{2}=3\left[1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....+\frac{1}{2^{19}}-\frac{20}{2^{20}}\right]$$

$$\therefore \quad S_3=6\left(2-\frac{22}{2^{20}}\right)$$

From (i) , we have

$$\sum_{i=1}^{20}(x_i-i)^2=\frac{12\left(1-\frac{1}{2^{40}}\right)+2870-12\left(2-\frac{22}{2^{20}}\right)}{20}$$

$$\therefore \ \bar{x}=\frac{12\left(1-\frac{1}{2^{40}}\right)+2870-12\left(2-\frac{22}{2^{20}}\right)}{20}$$

$$\therefore \ |\bar{x}|=142$$

24. (12)

We have,

$$\frac{6}{3^{12}}+\frac{10}{3^{11}}+\frac{20}{3^{10}}+\frac{40}{3^9}+.......+\frac{10240}{3}$$

$$=\frac{6}{3^{12}}+10\left(\frac{1}{3^{11}}+\frac{2}{3^{10}}+\frac{2}{3^9}+.......+\frac{2^{10}}{3}\right)$$

$$=\frac{6}{3^{12}}+10.\frac{1}{3^{11}}\left(\frac{6^{11}-1}{6-1}\right)$$

$$\left[\because \ 2^{nd} \text{ expression is a G.P. with } a=\frac{1}{3^{11}}, r=6\right]$$

$$=\frac{6}{3^{12}}+\frac{2}{3^{11}}\left(6^{11}-1\right)$$

$$=\frac{2\times 3}{3^{12}}+\frac{2\times 2^{11}\times 3^{11}}{3^{11}}-\frac{2}{3^{11}}$$

$$=\frac{2}{3^{11}}+2\times 2^{11}-\frac{2}{3^{11}}$$

$$=2^{12}.1=2^n.m \quad \therefore m=1, n=12$$

$$\Rightarrow m.n=12$$

25. (98)

We have, $\frac{1}{3},\frac{5}{9},\frac{19}{27},\frac{65}{81},......$

Let, $S_{100}=\left(1-\frac{2}{3}\right)+\left(1-\frac{4}{9}\right)+\left(1-\frac{8}{27}\right)+......100 \text{ terms}$

$$=100-\left[\frac{2}{3}+\left(\frac{2}{3}\right)^2+....\right]$$

$$=100-\frac{\frac{2}{3}\left(1-\left(\frac{2}{3}\right)^{100}\right)}{1-\frac{2}{3}}$$

$$=100-2\left(1-\left(\frac{2}{3}\right)^{100}\right)=98+2\left(\frac{2}{3}\right)^{100}$$

$$\left|S_{100}\right|=98$$

26. (40)

Given , a_1,a_2,a_3,a_4,a_5 are in G.P.

Let common ratio is r.

$a_2+a_4=2a_3+1$

$\Rightarrow a_1r+a_1r^3=2a_1r^2+1$ ....(i)

And $\Rightarrow 3a_2+a_3=2a_4 \Rightarrow 3a_1r+a_1r^2=2a_1r^3$ ....(i)

$\Rightarrow 2r^2-r-3=0$

$\Rightarrow (2r-3)(r+1)=0 \Rightarrow r=-1,3/2$

Since $a_1>0$ So, r = - 1, rejected $r=\frac{3}{2}$

Put $r=\frac{3}{2}$ in equation (i)

$\Rightarrow a_1\left(\frac{3}{2}+\frac{27}{8}-\frac{9}{2}\right)=1 \Rightarrow a_1=\frac{8}{3}$

So, $a_2+a_4+2a_5=a_1r+a_1r^3+2a_1r^4$

$=\frac{3}{2}\left(\frac{8}{3}\right)+\frac{8}{3}\left(\frac{27}{8}\right)+2\times\left(\frac{8}{3}\right)\left(\frac{81}{16}\right)$

$=4+9+27=40$

27. (41651)

$$S_n=\frac{n^2}{1-\frac{1}{(n+1)^2}}=\left(n^2+1\right)-\frac{2}{n+2}$$

It is given that,

$$\frac{1}{26}+\sum_{n=1}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)$$

$$=\frac{1}{26}+\sum_{n=1}^{50}\left(\left(n^2-n\right)+2\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right)$$

$$=\frac{1}{26}+\frac{50\times 51\times 101}{6}-\frac{50\times 50}{2}+2\left(\frac{1}{2}-\frac{1}{52}\right)$$

$=41651$

28. (27560)

We have given $a_1 = b_1 = 1$,

$a_n = a_{n-1} + 2, b_n = a_n + b_{n-1}$

$$\left.\begin{array}{l} a_2 = 3 \\ a_3 = 5 \\ a_4 = 7 \\ a_5 = 9 \\ a_6 = 11 \\ a_8 = 13 \end{array}\right| \begin{array}{l} \\ b_2 = 4 \\ b_3 = 9 \\ b_4 = 16 \\ \therefore\ b_n = n^2 \\ \end{array}$$

$$\therefore \sum_{n=1}^{15} a_n.b_n = \sum_{n=1}^{15}(2n-1)n^2 = \sum_{n=1}^{15}(2n^3 - n^2)$$

$$= 2\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{15^2(16)^2}{2} - \frac{15\times16\times31}{6} = 27560$$

29. (166)

$\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}, m$

And n are co-prime

$$\Rightarrow \sum_{k=1}^{10} \frac{1}{2}\left(\frac{2k}{(k^2+1)^2 - k^2}\right) = \frac{m}{n}$$

$$\Rightarrow \frac{1}{2}\sum_{k=1}^{10}\left(\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right) = \frac{m}{n}$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{1-1+1} - \frac{1}{10^2+10+1}\right] = \frac{m}{n}$$

$$\Rightarrow \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{m}{n}$$

$$\Rightarrow \frac{55}{111} = \frac{m}{n} \Rightarrow m = 55, n = 111$$

$\therefore \quad m + n = 166$

30. (10620)

Given , $|f(n)| \le 800$ and $f(n) = 2n^2 - n - 1$

$\Rightarrow -800 \le f(n) \le 800$

$\Rightarrow f(n) + 800 \ge 0$ and $f(n) - 800 \le 0$

When $f(n) + 800 \ge 0$

$\Rightarrow 2n^2 - n - 1 + 800 \ge 0$

$\Rightarrow 2n^2 - n + 799 \ge 0$

It is true for all $n \in Z$ because $D < 0$. When $f(n) - 800 \le 0$

$$\Rightarrow 2n^2 - n - 1 - 800 \le 0$$
$$\Rightarrow 2n^2 - n - 801 \le 0$$
$$\Rightarrow n(2n-1) \le 801$$
$$\Rightarrow n = -19, -18, ..., 18, 19, 20$$
$$\therefore \sum_{n\in S} f(n) = \sum_{n=-19}^{20} 2n^2 - n - 1$$
$$= 4\left[1^2 + 2^2 + ... + 19^2\right] + 2\times 20^2 - 20 - 40$$
$$= \frac{4\times 19\times 20\times 39}{6} + 800 - 60 = 10620$$

31. (120)

Then n^{th} term of the sequence can be represented as

$$T_n = \frac{-1 + 2^3 - 3^3 + 4^3 + ... + (2n)^3}{n(4n+3)}$$
$$= \frac{2\left(2^3 + 4^3 + 6^3 + ... + (2n)^3\right) - \left(1^3 + 2^3 + 3^3 + ... + (2n)^3\right)}{n(4n+3)}$$
$$= \frac{2^4\left(\frac{n(n+1)}{2}\right)^2}{n(4n+3)} - n^2(2n+1)^2$$
$$= \frac{4n^2(n+1)^2 - n^2(2n+1)^2}{n(4n+3)}$$
$$= \frac{n^2\left(4n^2 + 8n + 4 - 4n^2 - 4n - 1\right)}{n(4n+3)}$$
$$= \frac{n^2(4n+3)}{n(4n+3)} = n$$
$$\Rightarrow \sum_{n=1}^{15} T_n = \sum_{n=1}^{15} n = \frac{15\times 16}{2} = 120$$

32. (286)

$$\frac{1}{2.3.4} + \frac{1}{3.4.5} + ... + \frac{1}{100.101.102} = \frac{k}{101}$$
$$\frac{4-2}{2.3.4} + \frac{5-3}{3.4.5} + ... + \frac{102-100}{100.101.102} = \frac{2k}{101}$$
$$\Rightarrow \frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + ... + \frac{1}{100.101} - \frac{1}{101.102} = \frac{2k}{101}$$
$$\Rightarrow \frac{1}{6} - \frac{1}{101.102} = \frac{2k}{101}$$
$$\Rightarrow 2k = \frac{101}{6} - \frac{1}{102}$$
$$\Rightarrow 17\times 2k = \frac{17\times 101}{6} - \frac{17}{102}$$

$$=\frac{1717}{6}-\frac{1}{6}=\frac{1716}{6}$$
$$\Rightarrow 34k=286$$

33. (1100)

$$A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min(i,j),$$
$$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max(i,j),$$
$$\therefore\ A+B=\sum_{i=1}^{10}\sum_{j=1}^{10}(i+j)=\sum_{i=1}^{10}(10i+55)$$
$$=10\times55+55\times10=1100$$

34. (276)

$$\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+....$$
$$\Rightarrow t_r=\frac{r}{4r^4+1}$$
$$\Rightarrow t_r=\frac{r}{(2r^2+1)^2-(2r)^2}$$
$$\Rightarrow t_r=\frac{1}{4}\times\frac{4r}{(2r^2+1-2r)^2-(2r^2+1+2r)}$$
$$=\frac{1}{4}\left[\frac{1}{2r^2+1-2r}-\frac{1}{2r^2+1+2r}\right]$$
$$\Rightarrow \sum_{r=1}^{10}t_r=\frac{1}{4}\sum_{r=1}^{10}\left(\frac{1}{2r^2+1-2r}-\frac{1}{2r^2+1+2r}\right)$$
$$=\frac{1}{4}\left[\left(\frac{1}{1}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{25}\right)+....\left(\frac{1}{181}-\frac{1}{221}\right)\right]$$
$$=\frac{1}{4}\times\frac{220}{221}=\frac{55}{221}=\frac{m}{n}$$
$$\therefore\ m+n=55+221=276$$

35. (9525)

The sum of all those term, of the arithmetic progression 3,8,13,…373, which one not divisible by 3 is

$$=(3+8+13+18+...+373)\ -(3+18+33...+363)$$
$$=\frac{75}{2}(3+373)-\frac{25}{2}(3+363)$$
$$=\frac{75}{2}\times376-\frac{25}{2}\times366$$
$$=75\times188-25\times183$$
$$=14100-4575=9525$$

36. (321)

We have

$S_1 : 3, 7, 11, 15, ..., 399$

$S_2 : 2, 5, 8, 11, ..., 359$

$S_3 : 2, 7, 12, 17, ..., 197$

The common terms in the three AP's are

47,107,167.

$\therefore$ Required sum $= 47 + 107 + 167 = 321$

37. (514)

Numbers which are divisible by:

Are 100,102,104,….,998

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 998 = 100 + (n-1)2$

$\Rightarrow \quad n = 450$

Numbers which are divisible by 3 are

102, 105, 108, …, 999

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 999 = 102 + (n-1)\times 3 \Rightarrow n = 300$

Numbers which are divisible by 7 are

105,112,……,994

$\therefore \quad 994 = 105 + (n-1)7 = 889$

$\Rightarrow \quad n = 127 + 1 = 128$

Numbers which are divisible by 2 and 3

Are 102,108,114,…996

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 996 = 102 + (n-1)\times 6$

$\Rightarrow \quad n = 150$

Numbers which are divisible by 2 and 7

Are 112, 126, 140,….,994

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 994 = 112 + (n-1)14 \Rightarrow n = 64$

Numbers which ar e divisible by 3 and 7 are 105,126,147,………,987

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 187 = 105 + (n-1)21$

$\Rightarrow \quad n = 43$

Numbers which are divisible by 2,3,7

Are 126,168,….,966

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 966 = 126 + (n-1)42$

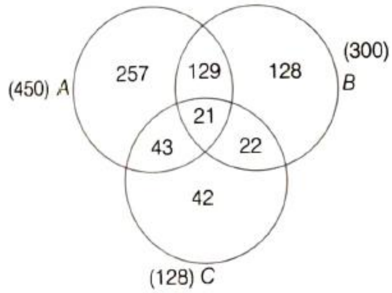
$\Rightarrow \quad n = 21$

Now, let

A : Numbers which are divisible by 2

B : Numbers which are divisible by 3

C : Numbers which are divisible by 7



$\therefore$ Total required number
$= 257 + 129 + 128 = 514$

38. (754)
We have, $a_1 = 8, a_2, a_3, ..., a_n$ are in AP.
Now, $S_4 = 50$

$\Rightarrow \frac{4}{2}[2 \times 8 + (4-1)d] = 50$

$\Rightarrow d = 3$
Again, sum of last four term $= 170$
$\Rightarrow n = 14$
$\therefore$ Middle terms are a_7 and a_8.
Required product $= a_7 \times a_8$
$= (a + 6d)(a + 7d)$
$= 26 \times 29 = 754$

39. (8)
Given,
$a_5 = 2a_7$ and $a_{11} = 18$
Now, $a_5 = 2a_7$
$\Rightarrow a + 4d = 2(a + 6d)$
$\Rightarrow a + 4d = 2a + 12d$
$\Rightarrow a = -8d$ ... (i)
And $a_{11} = 18 \Rightarrow a + 10d = 18$ ... (ii)
From Eqs. (i) and (ii), we get
$a = -72, d = 9$

$$\text{Now, } 12\left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + ... + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}}\right)$$

$$= 12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{a_{11} - a_{10}} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{a_{12} - a_{11}} + ... + \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{a_{18} - a_{17}}\right)$$

$$= 12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + ... + \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d}\right)$$

$$= \frac{12}{d}\left(\sqrt{a_{18}} - \sqrt{a_{10}}\right)$$

$$= \frac{12}{9}\left(\sqrt{a+17d}-\sqrt{a+9d}\right)$$

$$= \frac{12}{9}\left[\sqrt{-72+135}-\sqrt{-72+81}\right]$$

$$= \frac{12}{9}[9-3]=8$$

40. (495)

A_1, A_2, A_3 are in AP

$A_1 = A, A_2 = A+1, A_3 = A+2,$

Common difference = d

$a = 7$th term of $A_1 = A+6d$

$b = 9$th term of $A_2 = A+1+8d$

$c = 17$th term of $A_3 = A+2+16d$

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ 2(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix} + 70 = 0$$

41. (150)

$\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP

$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$ … (i)

$x, \sqrt{2}y, z$ are in GP $\Rightarrow \left(2\sqrt{2}y\right)^2 = xz$

$\Rightarrow 2y^2 = xz$ … (ii)

$xy + yz + zx = \frac{3}{\sqrt{2}} xyz$

$\Rightarrow \frac{1}{z} + \frac{1}{x} + \frac{1}{y} = \frac{3}{\sqrt{2}}$ … (iii)

From Eqs. (i) (iii),

$\frac{2}{y} + \frac{1}{y} = \frac{3}{\sqrt{2}} \Rightarrow y = \sqrt{2}$

From Eq.(ii),

$2\left(\sqrt{2}\right)^2 = xz \Rightarrow xz = 4$ … (iv)

From Eq.(iii),

$\frac{1}{z} + \frac{1}{x} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}} \Rightarrow \frac{x+2}{xz} = \sqrt{2}$

$\Rightarrow x + z = 4\sqrt{2}$ $[\because xz = 4]$

$\therefore \quad 3(x+y+z)^2 = 3\left(4\sqrt{2}+\sqrt{2}\right)^2$

$= 3\times 50 = 150$

42. (400)

$(20)^{19} + 2(21)(20)^{18} \;\; +3(21)^2(20)^{17} + ... + 20(21)^{19}$

$= k(20)^{19}$

$\Rightarrow (20)^{19}\left[1+2\left(\frac{21}{20}\right)+3\left(\frac{21}{20}\right)^2 + ... + 20\left(\frac{21}{20}\right)^{19}\right] = k(20)^{19}$

$\Rightarrow \quad k = 1+2\left(\frac{21}{20}\right)+3\left(\frac{21}{20}\right)+...+20\left(\frac{21}{20}\right)^{19} \qquad \ldots \quad (i)$

Now, $k\left(\frac{21}{20}\right) = \left(\frac{21}{20}\right)+2\left(\frac{21}{20}\right)^2 + 3\left(\frac{21}{20}\right)^3 + ... + 20\left(\frac{21}{20}\right)^{20} \qquad \ldots \quad (ii)$

From Eq.(i). Eq.(ii), we get

$k\left(\frac{-1}{20}\right) = 1+\frac{21}{20}+\left(\frac{21}{20}\right)^2 + ... + \left(\frac{21}{20}\right)^{19} - 20\left(\frac{21}{20}\right)^{20}$

$\Rightarrow \quad k\left(\frac{-1}{20}\right) = \frac{1\left(\left(\frac{21}{20}\right)^{20}-1\right)}{\frac{21}{20}-1} - 20\left(\frac{21}{20}\right)^{20}$

$\Rightarrow \quad k\left(\frac{-1}{20}\right) = 20\left(\left(\frac{21}{20}\right)^{20}-1\right) - 20\left(\frac{21}{20}\right)^{20}$

$\Rightarrow \quad 20\left(\frac{21}{20}\right)^{20} - 20 - 20\left(\frac{21}{20}\right)^{20}$

$\Rightarrow \quad K\left(\frac{-1}{20}\right) = -20 \Rightarrow K = 400$

43. (9)

We know that kth term of GP of

$a_k = a_1 r_1^{k-1}$

$b_k = b_1 r_2^{k-1} \qquad [\because \{a_k\} \text{ and } \{b_k\} \text{ in GP}]$

Given that $c_k = a_k + b_k$

$\Rightarrow \quad c_2 = a_2 + b_2$

$\Rightarrow \quad 5 = a_1 r_1^1 + b_1 r_2^1$

$\Rightarrow \quad r_1 + r_2 = \frac{5}{4} \qquad \ldots \quad (i)$

And $c_3 = a_3 + b_3$

$\Rightarrow \quad \frac{13}{4} = a_1 r_1^2 + b_1 r_2^2$

$\Rightarrow \quad r_1^2 + r_2^2 = \frac{13}{16}$ … (ii)

From Eq. (i) squaring both sides,
We get

$r_1^2 + r_2^2 + 2r_1r_2 = \frac{25}{16} \Rightarrow r_1r_2 = \frac{3}{8}$

Now, $r_1 - r_2 = \sqrt{(r_1 + r_2)^2 - 4r_1r_2}$

$= \sqrt{\frac{25}{16} - \frac{3}{2}} = \frac{1}{4}$ … (iii)

From Eqs. (i) and (iii), we get

$r_1 = \frac{3}{4}$

And $r_2 = \frac{2}{4}$ but, here $r_1 > r_2$

But, we need as per question, $r_1 < r_2$, so lets assume

$r_1 = \frac{2}{4} = \frac{1}{2}$ and $r_1 = \frac{3}{4}$

Now, $a_6 = 4\left(\frac{1}{2}\right)^5 \Rightarrow 12a_6 = \frac{3}{2}$

And $b_4 = 4\left(\frac{3}{4}\right)^3 \Rightarrow 8b_4 = \frac{27}{2}$

So, $12a_6 + 8b_4 = 15$ and $\sum_{k=1}^{\infty} C_k = \sum_{k=1}^{\infty}(a_k + b_k)$

$= \frac{a_1}{1 - r_1} + \frac{b_1}{1 - r_2}$

$= \frac{4}{1 - \frac{1}{2}} + \frac{4}{1 - \frac{3}{4}} = 24$

So, $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4) = 24 - 15 = 9$

44. (60)

We have, $a_1, a_2, a_3, ...$ an increasing GP positive numbers.

Let common ration 'r'.

Given that $a_4 \cdot a_6 = 9$

$\Rightarrow \quad a_1r^3 \cdot a_1r^5 = 9 \Rightarrow (a_1r^4)^2 = 9$

$\Rightarrow \quad a_1r^4 = 3$ … (i)

Now, $a_1a_9 + a_2a_4a_9 + (a_5 + a_7)$

$= a_1 \cdot a_1r^5 = 9 \Rightarrow (a_1r^4)^2 = 9$

$= a_1r^4 = 3$

Now, $a_1a_9 + a_2a_4a_9 + (a_5 + a_7)$

$= a_1 \cdot a_1r^8 + (a_1r)(a_1r^3)(a_1r^8) + (24)$

$= (a_1r^4)^2 + (a_1r^4)^3 + 24$

$= 3^2 + 3^3 + 24$ [From Eq.(i)]

$= 60$

45. (3)

If a and b are positive numbers and $a, b, \frac{1}{18}$ are in GP.

So, $b^2 = a \cdot \frac{1}{18}$ or $a = 18b^2$ … (i)

$\because \quad \frac{1}{a}, 10, \frac{1}{b}$ are in AP.

So, $2(10) = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \quad 20 = \frac{a+b}{ab}$ or $a + b = 20ab$

From Eq.(i), we get

$18b^2 + b = 20(18b^2)b$

Or $360b^3 - 18b^2 - b = 0$

$b(12b-1)(30b+1) = 0$

$b = 0$ or $b = \frac{1}{12}$ or $b = -\frac{1}{30}$

$b = \frac{1}{12}$, as $b > 0 \quad \Rightarrow a = \frac{1}{8}$

Now, $16a + 12b = 16\left(\frac{1}{8}\right) + 12\left(\frac{1}{12}\right)$

Or $16a + 12b = 3$

46. (12)

Let a = first of GP Common ratio of $GP = \frac{1}{m}, m \in N$

[Given] … (i)

Also, $T_4 = 500$ [Given]

$\Rightarrow \frac{a}{m^3} = 500$ [Using Eq. (i)]

Since, $S_n - S_{n-1} = ar^{n-1}$ … (ii)

Given, $S_6 > S_5 + 1$ and $S_7 - S_6 < \frac{1}{2}$

$$\Rightarrow \quad S_6 - S_5 > 1 \text{ and } \frac{a}{m^6} < \frac{1}{2} \quad \text{[Using Eq. (ii)]}$$

$$\Rightarrow ar^5 > 1 \text{ and } \frac{500}{m^3} < \frac{1}{2}$$

$$\Rightarrow \frac{500}{m^2} > 1 \text{ and } m^3 > 1000$$

$$\Rightarrow m^2 < 500 \qquad \dots \text{ (iii)}$$

And $m^3 > 10^3$ … (iv)

From Eqs. (iii) and (iv), we get

$m = 11, 12, 13, \dots, 22$ because $m \in N$

Hence, number of possible values of m is 12.

47. (7)

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2\cdot 3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2\cdot 3}+\frac{1}{2\cdot 3^2}-\frac{1}{3^3}\right)+\dots$$

$$=\left(-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\dots\right)$$

$$+\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{3^2}+\dots\right)+\frac{1}{2^2}\left(1-\frac{1}{3}+\frac{1}{3^2}+\dots\right)+\dots$$

$$=\left(-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\dots\right)+\left(\frac{1}{2}+\frac{1}{2^2}+\dots\right).\left(1-\frac{1}{3}+\frac{1}{3^2}+\dots\right)+\dots$$

48. (825)

Now, $\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+\dots+\left[\sqrt{120}\right]$

$$=\sum_{n=1}^{10}(2n+1)\cdot n$$

$$=\sum_{n=1}^{10}2n^2+\sum_{n=1}^{10}n$$

$$=2\times\frac{10\times 11\times 21}{6}+\frac{10\times 11}{2}$$

$$=\frac{10\times 11}{2}\left\{\frac{42}{3}+1\right\}$$

$5\times 11\times 15$

$= 75\times 11 = 825$

49. (1310)

$2\cdot 2^2 - 3^3 + 2\cdot 4^2 - 5^2 + 2\cdot 6^2 - 7^2 + \dots$

upto 20 terms

$$= 2\cdot 2^2 - 3^2 + 2\cdot 4^2 - 5^2 + 2\cdot 6^2 - 7^2 + \dots + 2\cdot(20)^2 - (21)^2$$

$$= 3\left(2^2 + 4^2 + 6^2 + \dots + (20)^2\right) - \left(2^2 + 3^2 + \dots + (21)^2\right)$$

$$= 3\cdot 4\cdot\left(1^2 + 2^2 + \dots + (10)^2\right) - \left(2^2 + 3^2 + \dots + (21)^2\right)$$

$$=12\left\{\frac{10\cdot(10+1)\cdot(20+1)}{6}\right\}-\left\{\frac{21\cdot(21+1)\cdot(42+1)}{6}-1\right\}$$
$$=2(10\cdot 11\cdot 21)-(7\times 11\times 43-1)$$
$$=2(2310)-(3310)$$
$$=4620-3310=1310$$

50. (2)

Given, sum of series,

$1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots$ is 10

$\forall k\in N$

Now, $10=1+\frac{4}{k+\frac{8}{k^2}}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots$ … (i)

Multiply the above equation by $\frac{1}{k}$ and shift it to right.

$\Rightarrow \frac{10}{k}=\frac{1}{k}+\frac{4}{k^2}+\frac{8}{k^3}+\frac{13}{k^4}+\ldots$ … (ii)

Now, Eq.(i) –Eq.(ii),

$10-\frac{10}{k}$

$=1+\left(\frac{3}{k}+\frac{4}{k^2}+\frac{5}{k^3}+\frac{6}{k^4}+\ldots\right)$ … (iii)

$S=\frac{3}{k}+\frac{4}{k^2}+\frac{5}{k^3}+\frac{6}{k^4}+$ … (A)

Then, $\frac{S}{k}=\frac{3}{k^2}+\frac{4}{k^3}+\frac{5}{k^4}$ … (B)

From Eq. (A) – Eq. (B),

$$S-\frac{S}{k}=\frac{3}{k}+\left(\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}+\ldots\right)$$

$$\Rightarrow S=\frac{S}{k}=\frac{3}{k}+\frac{\frac{1}{k^2}}{1-\frac{1}{k}}$$

$$\Rightarrow \frac{(k-1)S}{k}=\frac{3}{k}+\frac{1}{k(k-1)}$$

$$\Rightarrow S=\frac{3k-2}{(k-1)^2}$$

Now, from Eq. (iii),

$$10-\frac{10}{k}=1+\frac{3k-2}{(k-1)^2}$$

$\Rightarrow \quad \frac{(k-1)\cdot 10}{k}=1+\frac{3k-2}{(k-1)^2}$

$\Rightarrow \quad 10(k-1)=k+\frac{(3k-2)k}{(k-1)^2}$

$\Rightarrow \quad (9k-10)(k-1)^2=(3k-2)k$

$\Rightarrow \quad (9k^3-28k^2+29k-10=3k^2-2k$

$\Rightarrow \quad 9k^3-31k^2+31k=10=0$

Use hit and trial method,

$k=1; 9-31+31-10=-1$

$k=2; 72-214+62-10=0$

Thus, $k=2$

51. (2175)

We have,

$S=109+\frac{108}{5}+\frac{107}{5^2}+...+\frac{2}{5^{107}}+\frac{1}{5^{108}}$ ... (i)

$\frac{S}{5}=\frac{109}{5}+\frac{108}{5^2}+\frac{107}{5^3}+...+\frac{2}{5^{108}}+\frac{1}{5^{109}}$ ... (ii)

Subtract Eq. (ii) from Eq. (i),

$\frac{4S}{5}=109-\left[\frac{1}{5}-\frac{1}{5^2}+...+\frac{1}{5^{108}}+\frac{1}{5^{109}}\right]$

This from a GP with $r=\frac{1}{5}$

$\frac{4\,S}{5}=109-\frac{1}{5}\left[\frac{1-\frac{1}{5^{109}}}{1-\frac{1}{5}}\right]$

$=109-\frac{1}{4}\left[1-\frac{1}{5^{109}}\right]$

$=109-\frac{1}{4}+\frac{1}{4}\times\frac{1}{5^{109}}$

$=109-\frac{1}{4}+\frac{1}{4}\times\frac{1}{5^{109}}$

$4S=\frac{5}{4}\left[435+\frac{1}{5^{109}}\right]$

$\Rightarrow \quad 16S=2175+\frac{1}{5^{108}}$

$16S-(25)^{-54}=2175$

52. (16)

We haven $a_1, a_2, 2, a_3, a_4$ be an arithmetic-geometric progression.

Let d be the common difference of the corresponding arithmetic progression

$$\frac{2-2d}{4},\frac{2-d}{2},2,2(2+d),4(2+2d)$$

Ne the arithmetic –geometric progression

$$\frac{2-2d}{4}+\frac{2-d}{2}+2+2(2+d)+4(2+2d)=\frac{49}{2}$$

$\Rightarrow\quad 1-d+2-d+4+8+4d+16+16d=49$

$d=1$

$\therefore\quad a_4=(2+2d)=4(2+2)=16$

53. (6952)

Let

$$S=1^2-2\cdot3^2+3\cdot5^2-4\cdot7^2\ 5\cdot9^2-...+15\cdot29^2$$

$$=1+\sum_{r=1}^{7}\left[(2r+1)(4r+1)^2-2r(4r-1)^2\right]$$

$$=1+\sum_{r=1}^{7}\left(48r^2+8r+1\right)$$

$$=1+48\sum_{r=1}^{7}r^2+8\sum_{r=1}^{7}r+\sum_{r=1}^{7}1$$

$$=1+48\left(\frac{7(7+1)(14+1)}{6}\right)+8\left(\frac{7(7+1)}{2}\right)+7$$

$$=1+8\times7\times8\times15+4\times7\times8+7$$

$$=6952$$

Alternate solution

Let $=\left(1.1^2+3.5^2+...15.(29)^2\right)-\left(2.3^2+4.7^2+...14.(27)^2\right)$

$$=\sum_{n=1}^{8}(2n-1)(4n-3)^2-\sum_{n=1}^{7}(2n)(4n-1)^2$$

$$=\sum_{n=1}^{8}\left(32n^3-64n^2+42n-9\right)-2\sum_{n=1}^{7}\left(16n^3-8n^2+n\right)$$

$$=\left\{32\left(\frac{8\times9}{2}\right)^2-64\left(\frac{8\times9\times17}{6}\right)+42\left(\frac{8\times9}{2}\right)-72\right\}$$

$$-2\left\{16\left(\frac{7\times8}{2}\right)^2-8\left(\frac{7\times8\times15}{6}\right)+\left(\frac{7\times8}{2}\right)\right\}$$

$$=\left\{32\times(36)^2-64\times204+42\times36-72\right\}-2\left\{16\times(28)^2-8(140)+28\right\}=6952$$

54. (26)

$$\sum_{n=0}^{\infty}\frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)}$$

$$=\sum_{n=3}^{\infty}\frac{1}{(n-3)!}+\sum_{n=2}^{\infty}\frac{3}{(n-2)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}-\sum_{n=0}^{\infty}\frac{1}{(2n)!}$$

$$=e+3e+e+\frac{1}{2}\left(e-\frac{1}{e}\right)-\frac{1}{2}\left(e+\frac{1}{e}\right)$$

$$=5e-\frac{1}{e}ae+\frac{b}{e}+c,\ \text{where } a,b,c\in Z$$

On comparing; $a=5, b=-1$

$\therefore \quad a^2-b+c=25-(-1)+0=26$

55. (461)

Let $a_1=b_1=1$ and

$a_n=a_{n-1}+(n-1)$,

$b_n=b_{n-1}+a_{n-1}, \forall n\geq 2$

$a_2=a_1+1\Rightarrow a_2-a_1=1$

$a_3-a_2=2$

$\therefore \quad b_2-b_1=a_1$

$a_4-a_3=3$

$b_3-b_2=a_2$

$\vdots$

$a_n-a_{n-1}=n-1$,

$b_n-b_{n-1}=a_{n-1}$

Adding,

$a_n-a_1=1+2+...+(n-1)$

Adding,

$b_n-b_1=a_1+a_2+...+a_{n-1}$

$$\Rightarrow \quad a_n=\frac{n(n-1)}{2}+1=\frac{n^2-n+2}{2}$$

$$\Rightarrow \quad b_n-b_1=\sum_{K=1}^{n-1}\frac{K^2-K+2}{2}$$

$\therefore \quad b_n-b_1$

$$=\frac{1}{2}\left(\sum_{K=1}^{n-1}K^2-\sum_{K=1}^{n-1}K+2\sum_{K=1}^{n-1}1\right)$$

$$=\frac{1}{2}\left\{\frac{(n-1)n(2n-1)}{6}-\frac{n(n-1)}{2}+2(n-1)\right\}$$

$$\Rightarrow \quad b_n=\frac{1}{12}\left(2n^3-6n^2+16n-12\right)+1$$

Now, $S=\sum_{n=1}^{10}\frac{b_n}{2^n}$

$$= \sum_{n=1}^{10} \frac{1/12\left(2n^3 - 6n^2 + 16n - 12\right) + 1}{2^n}$$

$= 3.761$

And $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}} = 3.921$

$\therefore \quad 2^7 (2S - T) = 2^7 (2 \times 3.761 - 3.921)$

$= 461$

56. (5)

We have,

$$\frac{1^3 + 2^3 + 3^3 + ...\text{n terrms}}{1.3 + 2.5 + 3.7 + ...\text{n terms}} = \frac{9}{5}$$

$$\Rightarrow \quad \frac{\left(\frac{n(n+1)}{2}\right)^2}{\sum_{n=1}^{n}\left(n(2n+1)\right)} = \frac{9}{5} \qquad \ldots \quad \text{(i)}$$

$$\because \quad \sum_{n=1}^{n}\left(n(2n+1)\right) = \sum_{n=1}^{n}\left(2n^2 + n\right)$$

$$= 2\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left(\frac{4n+2+3}{3}\right)$$

$\therefore$ From Eq.(i),

$$\frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2}\left(\frac{4n+5}{3}\right)} = \frac{9}{5} \Rightarrow \frac{3(n+1)}{2(4n+5)} = \frac{9}{5}$$

$\Rightarrow \quad (5n+6)(3n-15) = 0$

$\therefore \quad (5n+6)(3n-15) = 0$

$\therefore \quad n = \frac{-6}{5}, n = 5$

$\therefore \quad n = 5$

57. (1010)

Given, $f(m+n) = f(m) + f(n)$

So, $f(x) = mx$

$\because \quad f(1)1 \Rightarrow m \times 1 \Rightarrow m = 1$

$\therefore \quad f(x) = x$

Now, $\sum_{k=1}^{2022} f(\lambda+k) \le (2022)^2$

$\Rightarrow \quad \sum_{k=1}^{2022} (\lambda+k) \le (2022)$

$\Rightarrow \quad (\lambda+\lambda+\lambda+\ldots+\lambda)$

$+(1+2+3+\ldots+2022) \le (2022)^2$ (2022 terms)

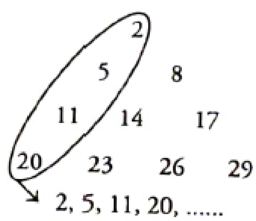
$\Rightarrow \quad 2022\lambda + \frac{2022 \times 2023}{2} \le (2022)^2$

$\Rightarrow \quad \lambda \le 2022 - \frac{2023}{2}$

$\Rightarrow \quad \lambda \le \frac{2021}{2} = 1010.5$

$\therefore$ Largest natural number of λ is 1010.

58. (1505)



General term $= \frac{3n^2 - 3n + 4}{2}$

$T_{10} = \frac{3 \times 100 - 3 \times 10 + 4}{2}$

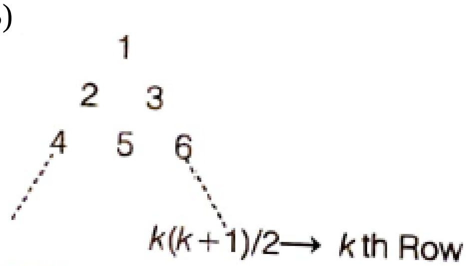
$= \frac{300 - 26}{2} = 137$

$d = 3$

So, sum $= \frac{10}{2}[2 \times 137 + 9 \times 3]$

$= 5(274 + 27) = 1505$

59. (103)



For 5310

$\frac{k(k+1)}{2} < 5310 \Rightarrow k(k+1) < 10620$

For $k = 102$ (maximum value of k

$$k(k+1) < 10620$$

That means, 5310 will contain in next row.

So, required row $= 102 + 1 = 103$

60. (910)

$A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 a_{2k-1}^2 - a_{2k}^2$

$= (-d)(a_1 + a_2 + ... + a_{2k})$

$= (-d)\left(\frac{2k}{2}\right)(2a_1 + (2k-1)d)$

$\Rightarrow \quad A_3 = (-3d)(2a_1 + 5d) = -153$

$\Rightarrow \quad (d)(2a_1 + 5d) = 51 \qquad \ldots \text{(i)}$

Also, $A_5 = (-5d)(2a_1 + 9d) = -435 = (d)(2a_1 + 9d) = 87 \qquad \ldots \text{(ii)}$

From Eqs. (i) and (ii), we get

$4d^2 = 36 \Rightarrow d \pm 3$

$\therefore \quad 3(2a_1 + 9d) = 87 \Rightarrow 2a_1 + 9d = 29$

$\Rightarrow \quad 2a_1 = 29 - 27 = 2 \Rightarrow a_1 = 1$

$\therefore \quad a_{17} - A_7 = (a = 16d) - (-7d)(2 + 13d)$

$= (1 + 16 \times 3) - (-7 \times 3)(2 + 13 \times 3)$

$= 49 + 7 \times 3(2 + 39)$

$= 49 + 21 \times 41 = 910$

61. (6699)

Ist $AP \Rightarrow 3, 7, 11, ..., 403$

$\Rightarrow \quad d_1 = 4$

IInd $AP \Rightarrow 2, 5, 8, ..., 404 \Rightarrow d_2 = 3$

Common difference of common AP

$\Rightarrow \quad D = LCM(d_1, d_2) \Rightarrow 12$

First common term $\Rightarrow a = 11$

Common $AP \Rightarrow 11, 23, ..., T_n \; T_n \le 403$

$\Rightarrow \quad 11 + (n-1)12 \le 403$

$\Rightarrow \quad (n-1) \le \frac{392}{12} \Rightarrow n - 1 \le 3266$

$\Rightarrow \quad n \le 33.66 \Rightarrow n = 33$

Sum of common terms

$S = \frac{33}{2}[2.11 + (33-1)12]$

$S = 6699$

62. (96)

$f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$

$$=\frac{\sin^4\theta+\cos^2\theta+2\cos^2\theta}{\sin^4\theta+\cos^2\theta}$$

$$=1+\frac{2\cos^2\theta}{\sin^4\theta+\cos^2\theta}$$

$$=1+\frac{2\cos^2\theta}{1+\cos^4\theta-2\cos^2\theta+\cos^2\theta}$$

$$=1+\frac{2\cos^2\theta}{1+\cos^4\theta-\cos^2\theta}$$

$$=1+\frac{2}{\frac{1}{\cos^2\theta}+\cos^2\theta-1}$$

[divide and multiply by $\cos^2\theta$]

Let $g(\theta)=\frac{2}{\frac{1}{\cos^2\theta}+\cos^2\theta-1}$

Maximum of $g(\theta)\Rightarrow$ when denominator is minimum and

Minimum of $\left\{\frac{1}{\cos^2\theta}+\cos^2\theta-1\right\}=1$

So, maximum of $g(\theta)=2$

Similarly, maximum of $g(\theta)=\frac{2}{\infty}=0$

So, range of

$$f(\theta)=1+g(\theta)=[0+1,2+1]=\left[\underset{\alpha}{\underset{\downarrow}{1}},\underset{\beta}{\underset{\downarrow}{3}}\right]$$

Required sum of GP,

$$S_\infty=\frac{a}{1-r}=\frac{64}{\left(1-\frac{1}{3}\right)}=\frac{64\times 3}{2}=96$$

63. (1011)

$$\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+...+\frac{1}{\alpha+1012}\right)-\left(\frac{1}{2\cdot 1}+\frac{1}{4\cdot 3}+\frac{1}{6\cdot 5}+...-\frac{1}{2024\cdot 2023}\right)=\frac{1}{2024}$$

$$\Rightarrow\quad\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+...+\frac{1}{\alpha+1012}\right)-\left\{\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...\left(\frac{1}{2023}-\frac{1}{2024}\right)\right\}=\frac{1}{2024}$$

$$=\left(\frac{1}{\alpha+1}-\frac{1}{\alpha+2}+...\frac{1}{\alpha+1012}\right)-\left\{\left(1+\frac{1}{2}+\frac{1}{3}+...\frac{1}{2023}\right)-\frac{1}{2024}-2\left(\frac{1}{2}+\frac{1}{4}+...\frac{1}{2022}\right)\right\}=\frac{1}{2024}$$

$$\Rightarrow\quad\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+...+\frac{1}{1012}-\left(\frac{1}{1}+\frac{1}{2}+...+\frac{1}{2023}\right)+\frac{1}{2024}+2\left(\frac{1}{2}+\frac{1}{4}+...+\frac{1}{2022}\right)=\frac{1}{2024}$$

$$\Rightarrow\quad\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+...+...\frac{1}{\alpha+1012}=\frac{1}{1012}+\frac{1}{1013}+...+\frac{1}{2023}$$

$\therefore\quad\alpha=1011$

64. (5)

$$\alpha=\sum_{r=0}^{n}\left(4r^2+2r+1\right)\cdot{}^{n}C_r$$

$$=\sum_{r=0}^{n}4r^2\cdot{}^{n}C_r+\sum_{r=0}^{n}2r\cdot{}^{n}C_2+\sum_{r=0}^{n}{}^{n}C_r$$

$$=4n(n+1)\times 2^{n-2}+2n\cdot 2^{n-1}+2^n$$

$$=2^n\left[n(n+1)+n+1\right]$$

$$2^n(n+1)^2$$

And $\beta=\left(\sum_{r=0}^{n}\frac{{}^{n}C_r}{n+1}\right)+\frac{1}{n+1}\quad=\frac{2^{n+1}-1}{n+1}+\frac{1}{n+1}=\frac{2^{n+1}}{n+1}$

So, $\frac{2\alpha}{\beta}$

$$=\frac{2^{n+1}(n+1)^2}{2^{n+1}}\times(n+1)=(n+1)^3$$

According to the question,

$$140<(n+1)^3<281$$

So, $n+1=6\Rightarrow n=5$

65. (3660

$$S(x)=(1+x)+2(1+x)^2+3(1+x)^3+\ldots+60(1+x)^{60}$$

$$(1+x)S(x)=(1+x)^2+2(1+x)^3+\ldots+59(1+x)^{60}+60(1+x)^{61}$$

Put $x=60$

$$(60)^2S(60)=(60)^2\times(61)^{61}+61-(61)^{61}=(61\times 59)(61)^2+61$$

$\therefore\quad a=61\times 59$ and $b=61$

So, $a+b=3660$

66. (6)

$$T_r=3T_{r-1}+6^r;\ r=2,3,4\ldots,n$$

$$T_2=3T_1+6^2=3\cdot 6+6^2 \qquad \ldots\ (i)$$

$$T_3=3T_2+6^3=3\left(3\cdot 6+6^2+6^3\right)$$

$$=3^2\cdot 6+3\cdot 6^2+6^3 \qquad \ldots\ (ii)$$

$$\Rightarrow\quad T_r=3^{r-1}\cdot 6+3^{r-2}\cdot 6^2+\ldots+6^r$$

$$\Rightarrow\quad T_r=3^{r-1}.6\left[1+2+2^2+\ldots+2^{r-1}\right]$$

$$T_r=6\cdot 3^{r-1}\cdot 1\cdot\frac{\left(1-2^r\right)}{(-1)}$$

$$T_r=\frac{6.3^r}{3}\left(2^r-1\right)$$

$T_r = 2\left(6^r - 3^r\right) \Rightarrow S_n = 2\Sigma\left(6^r - 3^r\right)$

$\Rightarrow \quad S_n = \left[\frac{6}{5}\left(6^n - 1\right) - \frac{3}{2}\left(3^n - 1\right)\right]$

$S_n = 2\left[\frac{12\left(6^n - 1\right) - 15\left(3^n - 1\right)}{10}\right]$

$= \frac{3}{5}\left[4 \cdot 6^4 - 5 \cdot 3^n + 1\right]$

$\therefore \quad n^2 - 12n + 39 = 3$

$\Rightarrow \quad n = 6$

67. (76)

$S = 1 + \frac{x}{2\sqrt{3}} + \frac{x^2}{18} + \frac{x^3}{36\sqrt{3}} + \frac{x^4}{180} + \ldots\infty$

Put $\frac{x}{\sqrt{3}} = t$, where $x = \sqrt{3} - \sqrt{2}$

$S = 1 + \frac{t}{2} + \frac{t^2}{6} + \frac{t^3}{12} + \frac{t^4}{20} + \ldots \; = 1\left(1 - \frac{1}{2}\right)t + \left(\frac{1}{2} - \frac{1}{3}\right)t^2 + \left(\frac{1}{3} - \frac{1}{4}\right)t^3 + \ldots$

$S = \left(1 + t + \frac{t^2}{2} + \frac{t^3}{3} + \ldots\right) - \left(\frac{t}{2} + \frac{t^2}{3} + \frac{t^3}{4} + \ldots\right).$

$S = \left(t + \frac{t^2}{2} + \frac{t^3}{3} + \ldots\right) - \frac{1}{2}\left(t + \frac{t^2}{2} + \frac{t^3}{3} + \ldots\right) + 2$

$S = 2 - \log(1 - t)\left(1 - \frac{1}{t}\right)$

$= 2 + \left(\frac{1}{t} - 1\right)\log(1 - t)$

$= 2 + \left(\frac{\sqrt{3}}{\sqrt{3} - \sqrt{2}} - 1\right)\log\left(1 - \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}}\right)$

$= 2 + \left(\frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}}\right)\log\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$

$= 2 + \sqrt{2}\left(\sqrt{3} + \sqrt{2}\right)\log\left(\frac{2}{3}\right)^{1/2}$

$= 2 + \left(\frac{\sqrt{6} + 2}{2}\right)\log\frac{2}{3}$

$= 2 + \left(\sqrt{\frac{3}{2}} + 1\right)\text{in}\,\frac{2}{3}$

$\therefore \; a = 2, b = 3 \Rightarrow 11a + 18b = 76$

68. (9)

$AP = 3, 7, 11, \ldots$

$\therefore \quad S_k = \frac{k}{2}\left[6+(k-1)4\right]$

$= k(3+2k-2)$

$= k(2k+1) = 2k^2 + k$

$\Rightarrow \quad \sum_{k=1}^{n} S_k = \Sigma 2k^2 + \Sigma k$

$= 2\left[\frac{n}{6}(n+1)(2n+1)\right] + \frac{n}{2}(n+1)$

$= n(n+1)\left[\frac{2n+1}{3} + \frac{1}{2}\right]$

$= \frac{n(n+1)}{6}(4n+5)$

$\Rightarrow \quad \frac{6\Sigma S_k}{n(n+1)} = 4n+5$

$\because \quad 40 < 4n+5 < 42$

$35 < 4n < 37$

$\Rightarrow \quad \frac{35}{4} < n < \frac{37}{4}$

$\therefore \quad n = 9$

69. (353)

We have ,

$\alpha = 1^2 + 4^2 + 8^2 + 13^2 + \ldots$ up to 10

$\beta = \sum_{n=1}^{10} n^4$

Let $S = 1+4+8+13+\ldots+t_n$

$S = 1+4+8+13 \ldots t_{n-1} + t_n$

$\Rightarrow \quad t_n = 1 + |3+4+5+\ldots(n-1)\,\text{term}|$

$\Rightarrow \quad t_n = 1 + \frac{n-1}{2}(6+(n-2)1)$

$\Rightarrow \quad t_n = 1 + \frac{(n-1)}{2}(n+4)$

$= \frac{(n-1)(n+4)+2}{2}$

$= \frac{n^2+3n-4+2}{2} = \frac{n^2+3n-2}{2}$

$\alpha = \sum_{n=1}^{10}\left(\frac{n^2+3n-2}{2}\right)^2$

$$4\alpha=\sum_{n=1}^{10}\left(n^2+3n-2\right)^2=\sum_{n=1}^{10}\left(n^4+9n^2+4+6n^3-12n-4n^2\right)$$

$$4\alpha-\beta=\sum_{n=1}^{10}\left(6n^3+n^2-12n+4\right)$$

$$=6\sum_{n=1}^{10}n^3+\sum_{n=1}^{10}n^2-12\sum_{n=1}^{10}n+4\sum_{n=1}^{10}1$$

$$=6\left[\frac{10(11)}{2}\right]^2+\frac{10\times11\times21\times5}{6}-12\times\frac{10\times11}{2}+40$$

$$\Rightarrow\quad 55(330+35-12)+40=55\times353+40$$

$$k=353$$

70. (9)

Given,

$$8=3+\frac{1}{4}(3+p)+\frac{1}{4^2}(3+2p)+\frac{1}{4^3}(3+3p)+\ldots\infty$$

$$\Rightarrow\quad 8=\frac{3}{1-\frac{1}{4}}+\frac{p\times\left(\frac{1}{4}\right)}{\left(1-\frac{1}{4}\right)^2}$$ [as we know, sum of infinite terms of AGP $=\frac{1}{1-r}+\frac{r\cdot d}{(1-r)^2}$]

$$\Rightarrow\quad 8=\frac{3}{1}\times\frac{4}{3}+\frac{p}{4}\times\frac{16}{9}\Rightarrow 4=\frac{16p}{36}\Rightarrow p=9$$

EXERCISE - 2 [A]

1. **(c)**

$S=\log a+\log\frac{a^3}{b^2}+...n\text{ terms}$

$\Rightarrow S=(1+2+3+..n\text{ terms})\log a-(1+2+3+...n-1\text{ terms})\log b$

$\Rightarrow S=\frac{n(n+1)}{2}\log a-\frac{n(n-1)}{2}\log b$

$\Rightarrow S=\frac{n^2}{2}\log\frac{a}{b}+\frac{n}{2}\log ab$.

2. **(c)**

First term = a, second term = b, last term = c.

Common difference, $d=b-a\Rightarrow c=a+(n-1)(b-a)$

Hence $n=\frac{b+c-2a}{b-a}$. Now sum of n terms will be

$S_n=\frac{a+c}{2}\left(\frac{b+c-2a}{b-a}\right)$.

3. **(b)**

$\ell=a+(n-1)d\Rightarrow d=\frac{\ell-a}{n-1}$. Also $S=\frac{n}{2}(a+\ell)\Rightarrow n=\frac{2S}{a+\ell}$.

Hence $d=\frac{\ell-a}{\frac{2S}{a+\ell}}$ i.e. $d=\frac{\ell^2-a^2}{2S-\ell-a}$

4. **(b)**

$2d=z-x,$ also $2y=z+x\Rightarrow 4y^2=(z-x)^2+4zx$

Hence $4y^2=4d^2+4zx$ i.e. $d^2=y^2-zx$

5. **(b)**

$S_p=\frac{p}{2}\{2a+(p-1)d\}=0\Rightarrow d=-\frac{2a}{p-1}$

Now sum of next q term $=S_{p+q}-S_p$

$=\frac{p+q}{2}\{2a+(p+q-1)d\}$

$\Rightarrow$ Now Sum of next q term $=\frac{p+q}{2}\left\{2a-(p+1-1)\frac{2a}{p-1}\right\}=\frac{-(p+q)q}{p-1}$.

6. **(a)**

Total number of terms by the end of n^{th} group $=1+2+3+...+n=\frac{n(n+1)}{2}$

Sum of all the terms till n^{th} group $=\dfrac{\dfrac{n(n+1)}{2}\left(\dfrac{n(n+1)}{2}+1\right)}{2}$

Sum of all the terms till $(n-1)^{th}$ group $=\dfrac{\dfrac{n(n-1)}{2}\left(\dfrac{n(n-1)}{2}+1\right)}{2}$

Sum of n^{th} group $=\dfrac{\dfrac{n(n+1)}{2}\left(\dfrac{n(n+1)}{2}+1\right)}{2}-\dfrac{\dfrac{n(n-1)}{2}\left(\dfrac{n(n-1)}{2}+1\right)}{2}$

$\Rightarrow$ Sum of n^{th} group $=\dfrac{n(n^2+1)}{2}$

7. **(c)**

Let the numbers be $a-3d, a-d, a+d, a+3d$.

As given $a-3d+a-d+a+d+a+3d=48 \Rightarrow a=12$

Also $\dfrac{a^2-9d^2}{a^2-d^2}=\dfrac{27}{35} \Rightarrow \dfrac{144-9d^2}{144-d^2}=\dfrac{27}{35} \Rightarrow d=\pm 2$

Hence the numbers are 6, 10, 14, 18.

8. **(a)**

$$S=\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\ldots+10 \text{ terms}$$

$$\Rightarrow S=\left(x^2+x^4+x^6+\ldots+10 \text{ terms}\right)+\left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}+\ldots+10 \text{ terms}\right)+20$$

Or $S=\dfrac{x^2\left(x^{20}-1\right)}{x^2-1}+\dfrac{\dfrac{1}{x^2}\left(1-\dfrac{1}{x^{20}}\right)}{1-\dfrac{1}{x^2}}+20$ i.e. $S=\left(\dfrac{x^{20}-1}{x^2-1}\right)\left(\dfrac{x^{22}+1}{x^{20}}\right)+20.$

9. **(d)**

$$S=\sqrt{ax}\left(1+\sqrt{b}+b+b\sqrt{b}+\ldots\infty \text{ terms}\right)+x\left(1+\sqrt{y}+y+y\sqrt{y}+\ldots\infty \text{ terms}\right)$$

$$\Rightarrow S=\frac{\sqrt{ax}}{1-\sqrt{b}}+\frac{x}{1-\sqrt{y}}.$$

10. **(b)**

$a+ar+ar^2=S, a\times ar\times ar^2=P \;\&\; \dfrac{1}{a}+\dfrac{1}{ar}+\dfrac{1}{ar^2}=R$.

$\Rightarrow a\left(1+r+r^2\right)=S, a^3a^3=P \;\&\; \dfrac{a+r+r^2}{ar^2}=R$

$\Rightarrow P^2R^3\times\dfrac{\left(1+r+r^2\right)^3}{a^3r^6}$ or $P^2R^3=a^3\left(1+r+r^2\right)^3=S^3$

11. **(c)**

$S = 1 + r + r^2 + ...\infty$ terms $= \dfrac{1}{1-r} \Rightarrow r = \dfrac{S-1}{S}$.

Also $1 + r^2 + r^4 + ...\infty$ terms $= \dfrac{1}{1-r^2}$

Hence $1 + r^2 + r^4 + ...\infty$ terms $= \dfrac{S^2}{S^2 - (S-1)^2}$ i.e. $\dfrac{S^2}{2S-1}$.

12. **(b)**

As given $x = a + ar + ar^2 + ...\infty$ terms $= \dfrac{a}{1-r}$

and $y = a^2 + a^2r^2 + a^2r^4 + ...\infty$ terms $\dfrac{a^2}{1-r^2}$

$\Rightarrow r = \dfrac{x^2 - y}{x^2 + y}$.

13. **(b)**

$0.7 + 0.77 + 0.777 + ... = \dfrac{7}{9}(0.9 + 0.99. + 0.999 + ...)$

$= \dfrac{7}{9}\left(10 - \dfrac{1}{10} - \dfrac{1}{10^2} - \dfrac{1}{10^3} - ...\right) = \dfrac{7}{81}\left(89 + \dfrac{1}{10^{10}}\right)$

14. **(a)**

$f(2n) = \sum_{r=1}^{2n} r^4 \Rightarrow f(2n) = \sum_{r=1}^{n} (2r-1)^4 + \sum_{r=1}^{n} (2r)^4$

$\Rightarrow f(2n) = \sum_{r=1}^{n} (2r-1)^4 + 16 \times \sum_{r=1}^{n} r^4$

$\Rightarrow f(2n) = \sum_{r=1}^{n} (2r-1)^4 + 16f(n)$

$\Rightarrow f(2n) - 16f(n) = \sum_{r=1}^{n} (2r-1)^4$

15. **(a)**

For roots to be real $q^2 \geq 4pr$. But as p, q, r are in A.P. hence $q = \dfrac{p+r}{2}$.

Thus $(p+r)^2 \geq 16pr$ or $p^2 - 14pr + r^2 \geq 0 \Rightarrow \left(\dfrac{p}{r} - 7\right)^2 \geq 48$

$\Rightarrow \left|\dfrac{p}{r} - 7\right| \geq 4\sqrt{3}$. Similarly $\left|\dfrac{r}{p} - 7\right| \geq 4\sqrt{3}$.

16. **(b)**

$\log_{5.2^x+1} 2, \log_{2^{1-x}+1} 4, 1$ are in H.P. hence $\log_2(5.2^x + 1), \log_2(2^{1-x} + 1)^{1/2}, \log_2 2$ are in A.P.

$\Rightarrow (5.2^x + 1), (2^{1-x} + 1)^{1/2}, 2$ are in G.P.

$\Rightarrow 2^{1-x}+1=\left(5.2^x+1\right)\times 2$ or $\frac{2}{2^x}+1=10.2^x+2$

$\Rightarrow 10\left(2^x\right)^2+2^x-2=0$

$\Rightarrow 2^x=\frac{4}{5}$

Hence x is a negative real number.

17. **(a)**

$$\frac{S_{kx}}{S_x}=\frac{\frac{kx}{2}\left\{2a+(kx-1)d\right\}}{\frac{x}{2}\left\{2a+(x-1)d\right\}}\Rightarrow\frac{S_{kx}}{S_x}=\frac{k\left\{2a-d+kxd\right\}}{\left\{2a-d+xd\right\}}$$

Clearly if $2a=d$, then $\frac{S_{kx}}{S_x}=k^2$ i.e. independent of x.

18. **(d)**

p, q, r are in H.P. hence q is H.M. of p & r. Also $\sqrt{pr}$ is G.M. of p & r.

Now H.M. < G.M. $\Rightarrow q<\sqrt{pr}$ or $q^2<pr$.

As discriminant is $4\left(q^2-pr\right)$ hence roots must be imaginary.

19. **(b)**

$$S_n=\frac{a\left(r^n-1\right)}{r-1},S_{2n}=\frac{a\left(r^{2n}-1\right)}{r-1}\&S_{3n}=\frac{a\left(r^{3n}-1\right)}{r-1}$$

$$S_{2n}=\left(1+r^n\right)S_n\ \&S_{3n}=\left(1-r^n+r^{2n}\right)S_n$$

$$S_{2n}-S_n=\left(r^n\right)S_n,S_{3n}-S_{2n}=r^n\left(r^n-2\right)S_n,S_{3n}-S_n=r^n\left(r^n-1\right)S_n$$

20. **(a)**

$$S=\frac{1}{2.4}+\frac{1.3}{2.4.6}+\frac{1.3.5}{2.4.6.8}+\frac{1.3.5.7}{2.4.6.8.10}+...\infty \text{ terms}$$

$$S=\frac{1.(4-3)}{2.4}+\frac{1.3.(6-5)}{2.4.6}+\frac{1.3.5.(8-7)}{2.4.6.8}+\frac{1.3.5.7.(10-9)}{2.4.6.8.10}+...\infty \text{ terms}$$

$$S=\frac{1}{2}-\frac{1.3}{2.4}+\frac{1.3}{2.4}-\frac{1.3.5}{2.4.6}+\frac{1.3.5}{2.4.6}-\frac{1.3.5.7}{2.4.6.8}+\frac{1.3.5.7}{2.4.6.8}-\frac{1.3.5.7.9}{2.4.6.8.10}+...\infty \text{ terms}$$

$$S=\frac{1}{2}.$$

21. **(c)**

$$S = 1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} - \frac{5^2}{5^4} + ...\infty \text{ terms}$$

$$\frac{1}{5}S = \quad \frac{1^2}{5} - \frac{2^2}{5^2} + \frac{3^2}{5^3} - \frac{4^2}{5^4} - ...\infty \text{ terms}$$

$$\frac{6}{5}S = 1 - \frac{3}{5} + \frac{3}{5^2} - \frac{7}{5^3} + \frac{9}{5^4} - ...\infty \text{ terms}$$

$$\frac{6}{5}S = 1 - \frac{3}{5} + \frac{3}{5^2} - \frac{7}{5^3} + \frac{9}{5^4} - ...\infty \text{ terms}$$

$$\frac{6}{25}S = \quad \frac{3}{5} - \frac{3}{5^2} + \frac{7}{5^3} - \frac{9}{5^4} + ...\infty \text{ terms}$$

$$\frac{36}{25}S = 1 - 2\left(\frac{1}{5} - \frac{1}{5^2} + \frac{1}{5^3} - \frac{1}{5^4} + ...\infty \text{ terms}\right)$$

Hence $S = \frac{25}{54}$.

22. **(b)**

According to Cauchy – Schwarz inequality

$$\left(x_1{}^2 + x_2{}^2 + x_3{}^2 + ... + x_{n-1}{}^2\right)\left(x_2{}^2 + x_3{}^2 + x_4{}^2 + ... + x_3{}^2\right) \ge \left(x_1x_2 + x_2x_3 + x_3x_4 + ... + x_{n-1}x_n\right)$$

Hence as given in question

$$\left(x_1{}^2 + x_2{}^2 + x_3{}^2 + ... + x_{n-1}{}^2\right)\left(x_2{}^2 + x_3{}^2 + x_4{}^2 + ... + x_3{}^2\right) = \left(x_1x_2 + x_2x_3 + x_3x_4 + ... + x_{n-1}x_n\right)$$

Now in Cauchy – Schwarz inequality equality occurs when

$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = ...\frac{x_{n-1}}{x_n}$. Hence numbers are in G.P.

23. **(b)**

$$S_n = 1 + \frac{3}{2} + \frac{5}{3} + ...\frac{2n-1}{n} \Rightarrow S_n = 2 - 1 + 2 - \frac{1}{2} + 2 - \frac{1}{3} + ... + 2 - \frac{1}{n}$$

$$\Rightarrow S_n = 2n - \left(1 + \frac{1}{2} + \frac{1}{3} + ...\frac{1}{n}\right) \text{ or } S_n = 2n - H_n.$$

24. **(c)**

$$4x^2 + 9y^2 + 16z^2 = 6xy - 12yz - 8zx = 0$$

$$\Rightarrow 8x^2 + 18y^2 + 32x^2 - 12xy - 34yz - 16zx = 0$$

$$\Rightarrow (2x - 3y)^2 + (3y - 4z)^2 (3y - 4z)^2 + (4z - 2x)^2 = 0$$

$$\Rightarrow 2x = 3y = 4z \text{ or } \frac{x}{6} = \frac{y}{4} = \frac{z}{3}.$$

Hence x, y, z are in H.P.

25. **(c)**

$$a_n = 1 + 10 + 10^2 + ... + 10^{n-1} \Rightarrow a_n = \frac{10^n - 1}{9}$$

Now $a_{124} = \frac{10^{224} - 1}{9}$. Observe that $271 \times 369 = 10^5 - 1$

Rewrite a_{124} as $\frac{\left(\left(10^5\right)^{124} - 1\right)10^4 + 10^4 - 1}{9}$ or $\frac{27\ln}{9} + 1111$

Now remainder when 1111 is divided by 27 is 27.

26. **(a)**

$a=\frac{a}{1-r}, y=\frac{b}{1+r} \,\&\, z=\frac{c}{1-r^2} \Rightarrow \frac{xy}{z}=\frac{ab}{c}$.

27. **(c)**

$1+|\cos x|+|\cos x|^2+...\infty$ terms $=\frac{1}{1-|\cos x|} \Rightarrow 8^{\frac{1}{1-|\cos x|}}=4^3$

Or $2^{\frac{3}{1-|\cos x|}}=2^6$ hence $\cos x=\pm\frac{1}{2}$

Now in $(-\pi,\pi)$, $x=-\frac{2\pi}{3},-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$.

28. **(a)**

Let $b=ar, c=ar^2+d=ar^3$, then

$(a-c)^2+(b+c)^2+(b-d)^2-(a-d)^2=(a-ar^2)^2+(ar-ar^2)^2+(ar-ar^3)^2-(a-ar^3)^2$

$=a^2(1-r)^2(1+r)^2+a^2r^2(1-r)^2+a^2r^2(1-r)^2(1+r)^2-a^2(1+r+r^2)^2$

$=a^2(1-r)^2(1+r)^2(1+r^2)-a^2(1-r)^2(1+r^2)(1+r)^2=0$.

29. **(c)**

$S=1+(1+x)+(1+x+x^2)+(1+x+x^2x^3)+...n$ terms

$\Rightarrow (1-x)S=(1-x)+(1-x^2)+(1-x^3)+(1-x^4)+...n$ terms

$\Rightarrow (1-x)S=n-(x+x^2+x^3+...n \text{ terms})$

$\Rightarrow S=\frac{n}{1-x}-\frac{(x^n-1)}{(1-x)^2}$.

30. **(a)**

$2,\frac{7}{4},\frac{14}{9},...$ is an H.P. hence $\frac{1}{2},\frac{4}{7},\frac{9}{14},....$ will be an A.P. with $d=\frac{1}{14}$

6^{th} term $=\frac{1}{2}+5\times\frac{1}{14}$ i.e. $\frac{6}{7}$.

Hence 6^{th} term of the given series will be $\frac{7}{6}$.

31. **(d)**

A.M. of n A.M.s between a & b is single A.M. between a & b, hence
Sum of n A.M.s between a & b, S = nA.

$\Rightarrow \frac{S}{A}=n$.

32. **(b)**

$2b=a+c, x^2=ab, y^2=bc \Rightarrow \frac{x^2+y^2}{2}=b^2$.

33. **(b)**

Let the roots be $x_1 \& x_2$, then $x_1+x_2=2A \& x_1+x_2=G^2$.

Required equation is $x^2-2Ax+G^2=0$.

34. **(c)**

$\frac{p+q}{2}=2\sqrt{pq} \Rightarrow (p+q)^2=16pq$

$\Rightarrow \left(\frac{p}{q}\right)^2-14\left(\frac{p}{q}\right)+1=0$. Hence $\frac{p}{q}=7+2\sqrt{12}$. Now $7+2\sqrt{12}=\left(2+\sqrt{3}\right)^2=\frac{2+\sqrt{3}}{2-\sqrt{3}}$.

35. **(a)**

G.M. of roots of $x^2-2ax+b^2=0$, $\sqrt{b^2}=|b|$

Now for $x^2-2bx+a^2=0, b=\frac{\text{sum of roots}}{2}$ i.e. AM.

36. **(b)**

$a+b+c=3A, abc=G^3 \& \frac{3abc}{ab+bc+ca}=H \Rightarrow ab+bc+ca=\frac{3G^3}{H}$

Now required equation is $x^3-3Ax^2+\left(\frac{3G^3}{H}\right)x-G^3=0$.

37. **(c)**

Given $t_n=(2n-1)(2n+1)(2n+3)$. Let $T_n=(2n-1)(2n+1)(2n+3)(2n+5)$

$T_{n-1}=(2n=3)(2n-1)(2n+1)(2n+3)$.

Now $T_n-T_{n-1}=8t_n$

$S_n\sum_{r=1}^{n}t_r=\frac{1}{8}\times\sum_{r=1}^{n}(T_r-T_{r=1})$

Hence $S_n=\frac{T_n-T_0}{8}$ i.e. $S_n=\frac{(2n-1)(2n+1)(2n+3)(2n+5)+15}{8}$

38. **(a)**

$S=1^2+2^2+3^3x^2+4^2x^2+\ldots\infty$ terms

$xS=\quad 1^2x+2^2x^2+3^2c^2+\ldots\infty$ terms

$(1-x)S=1+3x+5x^2+7x^3+\ldots\infty$ terms

$x(1-x)S=\quad x+3x^2+5x^2+\ldots\infty$ terms

$(1-x)^2S=1+2\left(x+x^2+x^3+\ldots\infty \text{ terms}\right)$

$\Rightarrow S=\frac{1}{(1-x)}+\frac{2x}{(1-x)^3}$.

39. **(a)**

$S=1-\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\ldots\infty$ terms

$$\frac{1}{2}S=\frac{1}{2}-\frac{3}{4}+\frac{5}{8}...\infty \text{ terms}$$

$$\frac{3}{2}S=1-2\left(\frac{1}{2}-\frac{1}{2^2}+\frac{1}{2^3}-\frac{1}{2^4}+...\infty \text{ terms}\right)$$

$$\Rightarrow S=\frac{2}{9}.$$

40. (c)

$S=1+3+7+15+...n \text{ terms}$

$\Rightarrow S=2-1+4-1+8-1+16-1+...$

$\Rightarrow S=(2+4+8+16+...n \text{ terms})-n$

$\Rightarrow S=2^{n+1}-2-n$.

41. (d)

a,b,c,d are in A.P. $\Rightarrow \frac{a}{abcd},\frac{b}{abcd},\frac{c}{abcd},\frac{d}{abcd}$ will also be in A.P.

i.e. $\frac{1}{bcd},\frac{1}{acd},\frac{1}{abd},\frac{1}{abc}$ are in A.P.

$\Rightarrow bcd,acd,abd,abc$ are in H.P.

42. (a)

Roots are equal therefore Discrminant = 0.

$(c-a)^2=4(b-c)(a-b)\Rightarrow a^2+4b^2+c^2-4ab-4bc+2ax=0$.

$\Rightarrow (a+c+2b)^2=0$

$\Rightarrow a+c=2b$.

43. (b)

Clearly the given expression is in form of sum of a G.P.

44. (c)

$x=\frac{1}{1-a},y=\frac{1}{a-b},z=\frac{1}{1-c}$

Now a, b, c are in A.P. hence $1-a,1-b,1-c$ will also be in A.P.

Hence $x=\frac{1}{1-a},y=\frac{1}{1-b},z\frac{1}{1-c}$ will be in H.P.

45. (b)

$(a^2+b^2)x^2-2ab(a+c)x+(b^2+x^2)=0\Rightarrow(ax-b)^2+(bx-c)^2=0$

Hence $\frac{b}{a}=\frac{c}{-}a=x$. Therefore a, b, c are in G.P.

EXERCISE - 2 [B]

1. **(a, c)**

$b=\frac{2ac}{a+c}$

Now, discriminate $=4\left(b^2-ac\right)<0$ $\left(\begin{array}{l}\because b=1+m \text{ of } a,b,c \\ a+m\le\left(4m=\sqrt{ac}\right)\end{array}\right)$

Thus non-real roots

$\text{sum}=\frac{-2b}{a}=\text{real}$

$\boxed{\text{Ans}:a,c}$

2. **(b, d)**

$\left(a^2+b^2+c^2\right)p^2-2\left(ab+bc+cd\right)p+\left(b^2+c^2+a^2\right)\le 0$

For real p, discriminate ≤ 0 as expression is everywhere non-positive

But $a^2+b^2+c^2>0\Rightarrow$ expression ≥ 0

Thus, $4\left(ab+bc+cd\right)^2=\left(a^2+b^2+c^2\right)$

Solving, we get ad = bc

Also, $\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

$\boxed{\text{Ans}:b,d}$

3. **(c, d)**

$S_{even}=S_{odd}+50d$

$S_{even}+S_{odd}=S_n=-1$

$1+\left(1+50d\right)=-1$

Also, $\frac{100}{2}\left[2a+99\times\frac{3}{50}a\right]=-1$

$\Rightarrow 100a+\frac{99\times 3\times 100}{100}=-1$

$\Rightarrow 100a=-148$

Solving $t_{100}=\frac{74}{25}$

$\boxed{\text{Ans}:c,d}$

4. **(a, b, c)**

$a=AR^{p-1}$

$a=AR^{q-1}$

$c=AR^{r-1}$

$\log a=\log A+\left(p-1\right)\log R$

$\log b=\log A+\left(q-1\right)\log R$

$$\log c=\log A+(r-1)\log R$$

$$(q-r)\log a+(r-p)\log b+(p-q)\log c$$

$$=(\log A)(q-r+r-p+p-q)$$

$$+\left[(q-r)(p-1)+(r-p)(q-1)+(p-q)(r-1)\right]\log R$$

$$=0$$

5. **(a, b, c)**

$$\alpha+\beta=-\frac{b}{a},\alpha\beta=\frac{c}{a}$$

$$(\alpha-\beta)^2=\left(\frac{-b}{a}\right)^2-\frac{4c}{a}=\frac{+b^2-4ac}{a^2}$$

$$y+\delta=\frac{-q}{p},y\delta=\frac{r}{p}$$

(a) $\rightarrow \dfrac{b^2-4ac}{q^2-4pr}=\dfrac{a^2}{p^2}\times\dfrac{\left(\frac{b}{a}\right)^2-4\frac{c}{a}}{\left(\frac{q}{p}\right)^2-4\frac{q}{b}}$

$$=\frac{a^2}{p^2}\times\frac{(\alpha+\beta)^2-4\alpha\beta}{(y+\delta)^2-y\delta}$$

$$=\left(\frac{a^2}{p^2}\right)\left(\frac{\alpha-\beta}{y-\delta}\right)^2=\frac{a^2}{p^2}\times\frac{\frac{b^2-4ac}{a^2}}{\frac{q^2-4pr}{p^2}}$$

$$=\frac{c^2}{r^2}$$

(b) $\dfrac{\alpha-\beta}{\alpha\beta}=\left(\dfrac{\frac{b^2-4ac}{a^2}}{\frac{a}{c}}\right)^{\frac{1}{2}}=\left(\dfrac{b^2 4ac}{ac}\right)^{\frac{1}{2}}$

Also, $\dfrac{1}{\beta}+\dfrac{1}{y}=\dfrac{1}{\alpha}+\dfrac{1}{\delta}$

$$\Rightarrow\frac{y-\delta}{ys}=\frac{\alpha-\beta}{\alpha\beta}$$

6. **(a, b, c)**

$$f(0)=1$$

$$f'(x)=3ax^2+26x-1$$

f ' exist everywhere

f '(0) has two distinct roots.

So $(2b)^2+4(3a)>0$

$$4b^2 12a>0$$

$b^2 + 3a > 0$

Let the roots be $p, \frac{2pq}{p+q}$ and q.

Substituting them and eliminating p and q we get a, b, c

7. **(b, c, d)**

$\log \frac{a}{2b}, \log \frac{b}{1.5c}, \log \frac{3c}{a}$ is ab

So, $\left(\frac{b}{1.5c}\right)^2 = \frac{3c}{a} \times \frac{a}{2b}$

$\frac{b^2}{2.25c^2} = \frac{3c}{2b}$

$2b^3 = 6.7c^3$

$\left(\frac{b}{c}\right)^3 = 3.375$

$\Rightarrow \frac{b}{c} = \frac{3}{2}$

$a = 4.5k$

$a = 3k$

$c = 2k$

$\text{Thus}, 7 \rightarrow a, b, c, d$

8. **(a, b, c)**

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$E = \left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right)$

$= \left(\frac{1}{c} + D\right)\left(\frac{1}{c} - D\right)$ (D = common diff)

$= \frac{1}{c^2 - D^2}$

$= \frac{1}{c^2} - \left(\frac{1}{b} - \frac{1}{a}\right)^2$

$= \frac{1}{c^2} - \frac{1}{a^2} - \frac{1}{b^2} + \frac{2}{ab}$

Substituting $b = \frac{2ac}{ab}$

This simplifies to $\boxed{\text{Ans}: b, c, d}$

9. **(a, b, c)**

$A_1 = a + \frac{b-a}{3}$

$A_2 = \frac{a+2b}{3}$

$G_1 = a\sqrt[3]{\frac{ab}{a}} = \left(a^2 b\right)^{\frac{1}{3}}$

$G_2 = \left(b^2 a\right)^{\frac{1}{3}}$

$\mu_1 = \frac{9ab}{3a+6b}, \mu_2 = \frac{9ab}{3b+6a}$

Solving the choices we find $\boxed{\text{Ans}: a, b, c}$

10. **(a, b)**

$$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{3}{q} \times \frac{1}{3} = \frac{1}{q}$$

Option $a \to (0.25)^{-1} = 4$

Option $b \to (0.008)^{\cos(\frac{1}{1})} = 8$

11. **(a, b, c, d)**

a, c

$1, 5, 25, ...$

12. **(a, b, c, d)**

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ is A.P.

So $\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ is A.P

So $\frac{a+b+c}{a} - 2, \frac{a+b+c}{b} - 2, \frac{a+b+c}{c} - 2$ is A.P.

$\Rightarrow \frac{b+c-a}{a}, \frac{a+c-b}{b}, \frac{a+b-c}{c}$ is A.P.

Thus $\frac{a}{b+c-a}, \frac{b}{a+c-b}, \frac{c}{a+b-c}$ is A.P.

Similarly rest of options.

$12 \to a, b, c$.

13. **(a, b)**

$S = 1^2 + 2(2)^2 + 3^2 + 2(4)^2 ...$

$= (1^2 + 2^2 + 3^2 + 4^2 + ...)$

$4\left(1^2 + 2^2 + 3^2 + ...\left(\frac{n}{2}\right)^2\right)$

N is even, we have

$$S = \frac{n(n+1)(2n+1)}{6} + \frac{4\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)(n+1)}{6}$$

$$= \frac{n(n+1)^2}{2}$$

If n is odd, we have

$$S=\left(1^2+2^2+...+n^2\right)+4\left(1^2+2^2+...+\left(\frac{n-1}{2}\right)^2\right)$$

$$=\frac{n(n+1)(2n+1)}{6}+\left(\frac{\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)(n-1)}{6}\right)$$

$$=\frac{1}{2}n^2$$

14. (a, b, d)

$t_a=2, t_b=31$

$d=\frac{31-2}{b-a}=\frac{29}{b-a}=\text{rational}$

So all terms rational

$\boxed{\text{Ans}:14\to a,b,d}$

15. (a, b)

$\underset{p}{a}\ \underset{q}{ar}\ \underset{r}{ar^2}$

$p^2+q^2+2pq\cos R=r^2$

$\Rightarrow a^2r^4=a^2+a^2r^2+2a^2r\cos R$

$\Rightarrow r^4=1+r^2+2r\cos R$

$\Rightarrow \cos R=\frac{r^4-r^2-1}{2r}$

$0<\frac{r^4-r^2-1}{2r}<1$

16. (c, d)

$b^2=\frac{2a^2c^2}{a^2+c^2}$ & $b=\frac{a+c}{2}$

$\Rightarrow\left(a^2+c^2+2ac\right)\left(a^2+c^2\right)=8a^2c^2$

or $\left(a^2+c^2\right)+2ac\left(a^2+c^2\right)-8(ac)^2=0$

$\Rightarrow\left(a^2+c^2-2ac\right)\left(a^2+c^2+4ac\right)=0$

As $a\neq b\neq c$ hence $a^2+c^2-2ac\neq 0$

Now $a^2+4ac+c^2=0$ gives $c=\left(-2\pm\sqrt{3}\right)a$ & $2b=\left(-1\pm\sqrt{3}\right)a$

17. (a, b, c, d)

$a_n=(\underbrace{111......11}_{n\text{ times}})$

If n is divisible by 3.

Then $a_n 3$

Thus not a prime

$\boxed{Ans:17\rightarrow a,b,c}$

18. (a, b, c, d)

$ar^2 > 4(ar)-3a \qquad (b_1 = a, b_2 = ar, b_3 = ar^3)$

$\Rightarrow r^2 < 4r-3$

$r^2 - 4r + 3 > 0$

$r < 1 \& r > 3$

$\boxed{Ans:19\rightarrow b,c}$

19. (c, d)

$a_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2^n-1}$

$<1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+(\text{missing})$

$\Rightarrow a_n < n$

By induction, for option $b, a(k) < \frac{k}{2}$

$a(k+1) = \frac{k}{2}+\frac{1}{2^k}+...+\frac{1}{2^k-1} < \frac{k}{2}+1 < \frac{(k+1)}{2}$

Thus by induction, $a(n) < \frac{n}{2}$

Similarly $a(2n) > n$

20. (a, b, c)

Solving purely by principle of mathematical inductions.

(a) $\rightarrow \alpha(2\times 1) = -\frac{1}{2} = \frac{1}{2}$

$\frac{1}{1+1}+\frac{1}{a+2}+...+\frac{1}{2} \neq \frac{1}{2}$ wrong

(b) $\rightarrow$ for $n = 1$, $\alpha(2n) < 1$

for $n = k, \alpha(2k) < 1$ (suppose)

$\alpha(2(k+1)) = \alpha(2k+2)$

21. (c, d)

22. (a, b, c, d)

23. (b, c, d)

24. (a, b, c, d)

25. (a, b)

26. (a, d)

27. **(b, d)**

28. **(a, b)**

29. **(b, c, d)**

30. **(a, b, c, d)**

31. **(b, d)**

32. **(a, c, d)**

33. **(b, c)**

34. **(a, c, d)**

35. **(a, d)**

36. **(b, c)**

37. **(a, b, c, d)**

38. **(a)**

39. **(a, b, c)**

40. **(b, c)**

41. **(a, b, c, d)**

42. **(a, b, c)**

43. **(a, b, c, d)**

44. **(a, b, c, d)**

45. **(a, b, c, d)**

Paragraph Type

Passage-I

1. **(a)**

$a = 2r, d = 2r - 2$

$$Ar = \frac{r}{2}\left(4r + (r-1)(2r-2)\right)$$

$$= \frac{r}{2}\left(4r + 2r^2 - 2r - 2r + 2\right)$$

$$=\frac{r}{2}\left(2r^2+2\right)$$
$$=r^3+r$$
$$\sum_{r=1}^{n} Ar=\sum r^3+\sum r$$
$$=\left|\left[\frac{r\left(r+1\right)^2}{2}\right]+\frac{r\left(r+1\right)}{2}\right| r=n$$
$$=\left|\frac{r\left(r+1\right)}{2}\left(\frac{r^2+r+12}{2}\right)\right| r=n$$
$$=\frac{n}{4}\left(n+1\right)\left(n^2+n+2\right)$$
$\boxed{\text{Ans : a}}$

2. (c)

$$B_{10}=A_{12}-A_{11}$$
$$=12^3+12+11^3-11$$
$$=1728+12-1331-11$$
$$=1740-1342$$
$$=398$$

3. (b)

$$C_r=B_{r+1}-B_r$$
$$=A_{r+3}-A_{r+2}-A_{r+2}+A_{r+1}$$
$$=A_{r+3}+A_r-2A_{r+2}$$
$$=\left(r+3\right)^3+r^3-2\left(r+2\right)^3+r+3+r-2r-4$$
$$=r^3+27+27r+9r^2+r^3-2r^3-16-24r-12r^2-1$$
$$=-3r^2+3r+8$$
$$\frac{d}{r}=-6r+3.\left|\text{diff}\right|=6$$

Passage-II

4. (d)

$3, B, C, D$

Now, $\frac{2\times 3\times c}{c+3}=b$ or $\frac{c}{c+3}=\frac{b}{6}$

Also, $2c=b+d$

$$c=\frac{b+d}{2}=\frac{b+\left(b+4\right)}{2}=b+2$$

So $\frac{b+2}{b+5}=\frac{b}{6}$

Or $6b+12=b^2+5b$

$$b^2-b-12=0$$

$c=3,\quad b=4\ ,\ c=6\ ,\ d=8$

$\boxed{\text{Ans}:(d)}$

5. **(c)**

$4, A_1, A_2, A_3, 6$

$\Rightarrow 4, 4.5, 5, 5.5, 6$

$5.5=\frac{11}{2}=\frac{11}{4}=\frac{14+8}{4}=\frac{k+8}{4}$

$\boxed{\text{Ans}:(c)}$

6. **(b)**

$k=14, b=4, d=8$

$kb^2=14.4^7.kd^7=14.8^7$

$r=\left(\frac{14.8^7}{14.4^7}\right)=2$

Passage-III

7. **(a)**

$$\frac{\frac{a}{3}+\frac{a}{3}+\frac{a}{3}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{4}+\frac{c}{4}+\frac{c}{4}+\frac{c}{4}}{4}\geq 10\sqrt{\left(\frac{a}{3}\right)^3\left(\frac{b}{3}\right)^3\left(\frac{c}{4}\right)^4}$$

$\boxed{\text{Ans}:(a)}$

8. **(d)**

$$\frac{a+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}}{10}$$

$$\geq \frac{10}{\frac{1}{a}+\frac{3}{b}+\frac{3}{b}+\frac{3}{b}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}}$$

or $\frac{1}{a}+\frac{9}{b}+\frac{36}{c}\geq\frac{100}{20}$ i.e. 5

9. **(b)**

$$\frac{3(a+x)+4(y+2)+5(z+4)}{12}\geq\sqrt[12]{(1+x)^3(y+2)^4(z+4)^5}$$

$$\Rightarrow\frac{5+3+8+20}{12}\geq\sqrt[12]{(1+x)^3(y+2)^4(z+4)^5}$$

3^{12} is max value.

$\boxed{\text{Ans}:(b)}$

Passage-IV

10. **(c)**

11. (d)

12. (b)

Passage-V

13. (c)

14. (d)

15. (a)

Passage-VI

16. (b)

17. (a)

18. (d)

Matrix-Match Type

1. (A) → (P), (B) → (R), (C) → (Q), (D) → (P)

$(A) \to (P)$

So $\frac{p}{64} = 2^{9-6} = 2^3 = 8$

(B) $\to (r)$

$$\frac{4^{x+\frac{1}{2}} + 4^{\frac{3}{2}} - x}{4} = \frac{1}{2}\left(\frac{x^{x+\frac{1}{2}} + 4^{\frac{3}{2}-x}}{x + \frac{1}{2} + \frac{3}{2} - x}\right)$$

This is greater than $\sqrt{4^{x+\frac{1}{2}} \times 4^{\frac{3}{2}-x}} = 4$

$\frac{1}{2}(4) = 2$

(C) $\to (q)$

$(x+2y-2)(2y+z-x)(z+x-y)$

$= (a+2a=2d-a-2d)(2a+2d+a+2d-a)(a+2d+a-a-d)$

$= (2a)(2a+4d)(a+d)$

$= 2x \times 2z \times y$

$= 4xyz$

(D) $\to (p)$

$t_1 = 7(t_2 + ... + ...)$

$a = 7\left(\frac{a}{1-r} - a\right)$

$$\Rightarrow 8a = \frac{7a}{1-r}$$

$$1-r = \frac{7}{8}$$

2. **(A) → (R), (B) → (P), (C) → (Q), (D) → (P)**

3. **(A) → (Q), (B) → (R), (C) → (Q), (D) → (R)**

4. **(A) → (T), (B) → (T), (C) → (R), (D) → (Q)**

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ is A.P.

So $\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ is A.P.

So $\frac{a+b+c}{a}-2, \frac{a+b+c}{b}-2, \frac{a+b+c}{c}-2$ is A.P.

$\Rightarrow \frac{b+c-a}{a}, \frac{a+c-b}{b}, \frac{a+b-c}{b}, \frac{a+b+c}{c}$ is A.P.

Thus $\frac{a}{b+c-a}, \frac{b}{a+c-b}, \frac{c}{a+b-c}$ is A.P.

EXERCISE - 2 [C]

1. **(6)**

$$\frac{\frac{n}{2}\left[10+(n-1)4\right]}{\frac{n}{2}\left[14+(n-1)2\right]}=\frac{5}{4}$$

$$\frac{4n+6}{2n+12}=\frac{5}{4}$$

$$\frac{2n+3}{n+6}=\frac{5}{4}$$

$$\Rightarrow \boxed{n=6}$$

2. **(5)**

$$\frac{n}{2}\left[2\times 2+(n-1)3\right]=950$$

$$\frac{n}{2}\left[4+3n-3\right]=950$$

$$3n^2+n=1900$$

$$\boxed{n=25}$$

3. **(8)**

Let n be number of terms, then

$$S_{even}+S_{odd}=S_n$$

$$S_{even}=S_{odd}+\left(\frac{n}{2}\right)d$$

$$\Rightarrow 30=24+\frac{nd}{2}$$

$$\Rightarrow nd=12$$

also $a+(n-1)d-q=\frac{21}{2}$

4. **(7)**

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a'+(n-1)d'\right]}=\frac{7n+1}{4n+27}$$

For $n=21$

$$\frac{2a+20d}{2a'+20d'}=\frac{147+1}{111}$$

$$\Rightarrow \frac{a+10d}{a'+20d'}=\frac{148}{111}$$

$$\Rightarrow \frac{t_{11}}{t_{11}}=\frac{148}{111}=\frac{4}{3}$$

$$4+3=\boxed{7}$$

5. **(6)**

$LCM = 60$

So (div. by 15) + (div. by 20) – (div. by 60)

$= 20 + 15 - 5$

$= \boxed{30}$

6. **(3)**

$$\frac{\frac{n(n+1)}{2} - r}{n-1} = 5 \Rightarrow n^2 - 9n + 10 = 2r$$

Now, $2 \le n^2 - 9n + 10 \le 2n$

(i) $n^2 - 9n + 10 \ge 2$ $\quad n^2 - 9n + 12 \ge 0$

$\Rightarrow n \le \frac{9-\sqrt{51}}{2}$ or $n \le \frac{9+\sqrt{51}}{2}$

(ii) $n^2 - 9n + 10 \le 2n$ $\quad n^2 - 11n + 10 \le 0$

$\Rightarrow 1 \le n \le 5 + 10$

From (i) & (ii), $\frac{9\sqrt{51}}{2} \le n \le 10$

Hence n may be 8, 9 or 10

Hence sum of values of n = 27

7. **(4)**

D = common diff

$-2D + kD^2 + 8D^3 = -6D + 4D^2 - D^3$

$9D^3 + (k-4)D^2 + 4D = 0$

$9D^2 + (k-4)D + 4 = 0$

$(k-4)^2 - 4 \times 4 \times 9 \ge 0$

$(k-4)^2 - (12)^3 \ge 0$

$(k-16)(k+8) \ge 0$

$\boxed{k \ge 16}$

8. **(6)**

Numbers be a – d, a, a + d

$(a^2)^2 = (a-d)^2 (a+d)^2$

$\Rightarrow a^4 = (a^2 - d^2)^2$

$= a^4 + d^4 + 2a^2d^2$

$\Rightarrow d^4 = 2a^2a^2$

$\Rightarrow d^2 = 2a^2$

$$r = \frac{a^2}{(a-d)^2} = \left(\frac{a}{a-d}\right)^2 = \left(\frac{1}{1-\frac{d}{a}}\right)^2$$

$\frac{d}{a} = \pm\sqrt{2}$

So $r=\left(\frac{1}{a+\sqrt{2}}\right)^2$ or $\left(\frac{1}{1-\sqrt{2}}\right)^2$

$=\left(\sqrt{2}-1\right)^2$ or $\left(\sqrt{2}+1\right)^2$

$r_1+r_2=2(2+1)=6$

9. **(3)**

Let the number be $100ar^2+10at+a$

Now, $100ar^2+10at+a-400=100\left(ar^2-4\right)+10ar+a$

Given that $a+ar^2-4=2ar$

Also as each of a, ar, ar^2 is an integer lying between 1 & 9, hence $r=2$ or 3

For $r=2, a+ar^2-4=2ar \Rightarrow a=4,$ but $ar^2>9$

For $r=3, a+ar^2-4=2ar \Rightarrow a=1$

Hence the number is 931 & sum of digits = 13

10. **(7)**

$\frac{a\left(r^n-1\right)}{r-1}>1000$

$\frac{\left(3^n-1\right)}{2}>1000$

$3^n>2001$

$n=7$

11. **(7)**

n includes all number div by 2, 3 or 5

We have

$$\sum \perp^n=\left(1+\frac{1}{2}+\frac{1}{4}+....\right)+\left(1+\frac{1}{3}+\frac{1}{9}+.....\right)$$
$$+\left(1+\frac{1}{5}+\frac{1}{25}....\right)-\left(1+\frac{1}{6}+\frac{1}{x}....\right)-\left(1+\frac{1}{15}+\frac{1}{225}\right)$$
$$-\left(1=\frac{1}{10}+\frac{1}{100}.....\right)+2\left(1+\frac{1}{30}+\frac{1}{900}+....\right)$$

12. **(4)**

$\frac{a}{r}, a, ar=GP$

$a^3=216 \qquad \Rightarrow a=6$

$\left(\frac{6}{r}\times 6\right)+\left(6\times 6r\right)+\left(6\right)^2=156$

$\Rightarrow \frac{36}{r}+36+36r=156$

$r=3$

$\text{sum}=18+6+2$

$=\boxed{26}$

13. **(6)**

$\frac{a}{r}, a, ar, ar^2.....$

$\frac{a\left((r)^{24}-1\right)}{(r)-1}=1 \qquad \Rightarrow \frac{\left(\frac{1}{b}\right)\left(1-r^{24}\right)}{\left(r^{23}\right)(1-r)}$

$\Rightarrow ar^{23}=2$

$t_{24}=2$

We have $\left(t_1 \times t_{24}\right)=\left(\sqrt{2}\right)^2$

14. **(1)**

We have

$\frac{p}{q}=1+\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)^2+3\left(\frac{1}{6}\right)^3.....$

Now, $S=1+x+2x^2+3x^3....$

$Sx=x+x^2+2x^3+3x^4.....$

$S(1-x)=\left(x^2+x^3+3x^4\right)$

$=\frac{x^2}{1-x}+\frac{(1-x)^2}{(1-x)}$

$S=\frac{x^2}{(1-x)^2}+\frac{1}{1-x}$

$\frac{p}{q}=\frac{\frac{1}{36}}{\frac{25}{36}}+\frac{\frac{1}{5}}{\frac{5}{6}}=\frac{1}{25}+\frac{6}{5}=\frac{31}{25}$

$\left|p^2-q^2\right|=4$

15. **(1)**

$=0.9\left(100-1+1000-1+....10^{101}-1\right)$

$=0.9\left(10^2 \_....+10^{101}-100\times1\right)$

$=0.9\left(10^3+....10^{101}\right)$

$=0.9\frac{\left(10^3\right)\left(10^{99}-1\right)}{10-1}=100\left(10^{99}-1\right)$

$=10^{99}-100$

16. **(3)**

$t_1=b-2$

$t_3=b+6$

$r=\sqrt{\frac{b+6}{b-2}}$

$\frac{t_1+t_3}{2}=b+2$

$$\frac{b+2}{b-2\sqrt{\frac{b+6}{b-2}}}=\frac{5}{3}\Rightarrow\frac{b+2}{\sqrt{(b-2)(b+6)}}=\left(\frac{5}{3}\right)^2$$

$$\Rightarrow\frac{(b+2)^2}{(b-2)(b+6)}=\left(\frac{5}{3}\right)^2\Rightarrow\boxed{b=3}$$

17. **(9)**

After withdrawal of Acid = 729 – 3x

After II withdrawal of Acid $=729-3x-\left(\frac{729-3x}{729}\right)\times 3x$

i.e. $\frac{(729-3x)^2}{729}$

After III withdrawal Acid $=\frac{(729-3x)^2}{729}-\left(\frac{729-3x}{729}\right)^2\times 3x$

i.e. $\frac{(729-3x)^3}{729^2}$

After VI withdrawal Acid $=\frac{(729-3x)^6}{729^5}=64$

Hence, $(729-3x)^6=2^6 3^{30}$ or $x=81$

18. **(5)**

$$y=\frac{a+b}{2}$$

$$x=\frac{a+y}{2}=\frac{a}{2}+\frac{a}{4}+\frac{b}{4}=\frac{3a}{4}+\frac{b}{4}=\frac{3a+b}{4}$$

$$z=\frac{a+3b}{4}$$

$$xyz=55\Rightarrow 55=\frac{(a+b)(3a+b)(a+3b)}{a+b}$$

Again, It $1+p, y=\frac{2ab}{a+b}$

$$x=\frac{2ay}{a+y}=\frac{2a\times\frac{2ab}{a+b}}{a+\frac{2ab}{a+b}}=\frac{4a^2b}{a^2+3ab}$$

$$z=\frac{4a^2b}{a^2+3ab}$$

$$xyz=\frac{4a^2b\times 4ab^2\times 2ab}{ab(a+b)(b+3a)(a+3b)}$$

$$\frac{343}{55}=\frac{32a^3b^3}{ab(a+b)(3a+b)(a+3b)}$$

$$\therefore 55\times\frac{343}{55}=a^3b^3$$

$$ab=7$$

19. **(6)**

$$\frac{1}{t_1} = 2.5 = \frac{5}{2}$$

$$\frac{1}{t_2} = \frac{23}{12}$$

Smaller the t r_1, greater the term.

Common diff of $AP = \frac{23}{11} - \frac{30}{12} = \frac{-7}{12}$

So we have $\frac{30}{12}, \frac{23}{12}, \frac{16}{12}, \frac{2}{12}, \frac{-5}{12}$

so $\frac{2}{12}$

Reciprocal = $\frac{12}{2} = 6$

20. **(1)**

$$\sum_{r=0}^{88} \frac{1}{\cos r^0 \cos(r+1)^0}$$

$$\frac{1}{\sin 1^0} \sum_{\ell=0}^{88} \frac{\sin 1^0}{\cos r^0 . \cos(r+1)^0}$$

$$= \frac{1}{\sin 1^0} \sum_{\ell=0}^{88} \frac{\sin\left[(r+1) - r\right]}{\cos r^0 . \cos(r+1)^0}$$

$$= \frac{1}{\sin 1^o} \left(\sum_{r=0}^{88} \frac{\sin(r+1)\cos r^0 - \cos(r+1)\sin r}{\cos r . \cos(r+1)} \right)$$

$$= \frac{1}{\sin^0} \left(\sum_{\ell=0}^{88} \tan(r+1)^0 - \tan r^0 \right)$$

$$= \frac{1}{\sin^0} \left(\tan 89^0 - \tan 0^0 \right)$$

$$= \frac{\cot^0}{\sin 1^0} \qquad \Rightarrow \theta = 1^0$$

21. **(6)**

roots are

$-3x, -x, x, 3x$

So, $10x^2 = 3m + 2$

$9x^4 = m^2$

$$\Rightarrow \frac{100}{9} = \frac{9m^2 + 4 + 12m}{m^2}$$

$\Rightarrow 100m^2 = 81m^2 + 108m + 36$

$\Rightarrow 19m^2 - 108m - 36 = 0$

$\Rightarrow 19m^2 - 114m + 6m - 36 = 0$

$\Rightarrow (19m + 6)(m - 6) = 0$

$\Rightarrow m = 6$

22. **(5)**

17, 21, 25 ……….. 417

16, 21, 26 ……….. 466

Common terms $\rightarrow$

21, 41, 61 ……………….

$T_n = 20n + 1 \le 417$

$\Rightarrow 20n \le 416$

$n \le 20.8$

$\boxed{n = 20} \Rightarrow K = 5$

23. **(8)**

$P2^x + \frac{4}{2^x} = 5$

$\Rightarrow pt^2 - 5t + 4 = 0$

Where $\left(t = 2^x > 0\right)$

So, $D \ge 0$

$\Rightarrow 25 - 16p \ge 0$

$p < \frac{25}{16}$

But, $p > 0$

So, $p = 0, p = 1$

at $p = 1$, 2 solution

at $p = 0$, 1 solution

No. of value of p is $Q = 1$

AP : $1, \frac{1}{x_1}, \frac{1}{x_2} \ldots\ldots\ldots \frac{1}{x_{20}}, \frac{1}{6}$

AP : $6, \frac{1}{x_1}, \frac{6}{x_2} \ldots\ldots \frac{6}{x_{20}}$

AP :

24. **(1)**

$a_1 = \frac{1}{2}$

$(n-1)a_{n-1} = (n+1)a_n$

$\Rightarrow n(n-1)a_{n-1} - n(n+1)a_n = 0$

$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_{n-1} - n(n+1)a_n = \sum 0$

$\Rightarrow 2.1.a, -n(n+1)a_n = 0$

$\Rightarrow a_n = \frac{1}{n(n+1)}$

$S_n = \sum_{r=1}^{n} \frac{1}{n(n+1)}$

$= \sum_{r=1}^{\infty} \frac{1}{r} - \frac{1}{r+1}$

$= 1 - 0 = 1$

25. **(4)**

(1), (2, 3, 4), (5, 6, 7, 8, 9) ……..

No. of terms in n^{th} jump is N_n

$N_1 = 1, N_2 = 3, N_3 = 5$

$N_n = (2n-1)$

So, total no. of elements till $(n-1)$ groups

is $1+3+5+......(2n-3) = (n-1)^2$

total no. of elements till n groups

is $+3+5......2n-1 = n^2$

sum of elements till $(n-1)$ groups is

$1+2+3+4..........(2-1)^2$

=

$$\frac{(n-1)^2\left((n-1)^2+1\right)}{2}$$

Sum of elements till n groups is

$$1+2+3+.........n^2 = \frac{n^2(n^2+1)}{2}$$

Sum of elements in 12 groups is

Ans. -4 $\frac{12^2(11^2+1)}{2} - \frac{11^2(11^2+1)}{2} = 6238$

26. **(1)**

t^{th}**M**ean $= (r+1)^{th}$ term

$$T_{r+1} = x + r.\frac{(2y-x)}{n+1}$$

$$= \frac{(n+1-r)x + 2xy}{n+1}$$

$$T_{r+1} = 2x + r\frac{(y-23)}{n+1}$$

$$\Rightarrow \frac{(n+1-\ell)2x+2y}{n+1}$$

$\Rightarrow (n+1-r)x + 2xy = (n+1-r)2x + xy$

$\Rightarrow (n+1-\ell)x = xy$

$\Rightarrow \frac{y}{x} = \frac{x+1}{r} - 1$

$\Rightarrow \frac{n+1}{r} - \frac{y}{z} = 1$

Ans – 1

27. **(4)**

$$\left(\frac{1}{5}\right)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+........\infty\right)}$$

$$=\left(\sqrt{5}\right)^{-2\log_{\sqrt{5}}\left(\frac{1/4}{1-1/2}\right)}$$

$$=\left(\sqrt{5}\right)^{\log_{\sqrt{5}}\left(1/2\right)^{-2}}$$

$$=4$$

28. **(5)**

$$\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)........\infty$$

$$=\frac{\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)}{\left(1-\frac{1}{3}\right)}.........\infty$$

$$=\frac{\left(1-\frac{1}{3^2}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)}{2/3}..........\infty$$

29. **(9)**

$$x=\frac{a+b}{2}$$

$$y=\sqrt{ab}$$

$$z=\frac{2ab}{a+b}$$

$$y^2=xz$$

$$z=\frac{x}{9}\left(\text{given}\right)$$

$$\Rightarrow y^2=\frac{x^2}{9}$$

$$\Rightarrow \frac{x^2}{y^2}=9$$

30. **(0)**

$$\frac{a^n+b^n}{a^{n-1}+b^{n-1}}=\frac{2ab}{a+b}$$

$$\Rightarrow a^{n+1}+b^{n+1}+ab\left(b^{n-1}+a^{n-1}\right)$$

$$=2ab\left(a^{n-1}+b^{n-1}\right)$$

$$\Rightarrow a^{n+1}+b^{n+1}=ab\left(a^{n-1}+b^{n-1}\right)$$

$$\Rightarrow a^n\left(a-b\right)=b^n\left(a-b\right)$$

$$\Rightarrow a^n=b^n$$

$$\Rightarrow n=0$$

31. **(8)**

$$S=\frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+a^8+a^{10}$$

By $AM \ge GM$

$$\frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3+8^{10}}$$

$$\Rightarrow \boxed{S \ge 1}$$

32. **(1)**

(32)

$$\frac{a+b}{1-ab},\ b,\ \frac{b+c}{1-bc} \longrightarrow A.P.$$

$$\therefore \frac{a+b}{1-ab}+\frac{b+c}{1-bc}=2b$$

$$\therefore (a+b)(1-bc)+(b+c)(1-ab)=ab(1-ab)(1-bc)$$

after solving

$$(1+b^2)(a+c-2abc)=0$$

$$\therefore a+c-2abc=0 \Rightarrow \frac{a+c}{2ac}=b$$

α, β are roots of given eqn.

$$\therefore \alpha+\beta=-\frac{2abc}{2ac}=-b$$

$$\alpha\beta=\frac{a+c}{2ac}=b$$

$$(1+\alpha)(1+\beta) \Rightarrow \alpha+\beta+\alpha\beta+1 \Rightarrow -b+b+1 \Rightarrow 1$$

33. **(5)**

(33)

$$a+b=-\frac{(-4)}{11}=+\frac{4}{11} \qquad ab=\frac{-2}{11}$$

$$a,b=\frac{4\pm\sqrt{16+88}}{2\cdot 11} \Rightarrow |a|,|b|<1$$

$$\therefore 1+a+a^2+\cdots=\frac{1}{1-a} \ \& \ 1+b+b^2+\cdots=\frac{1}{1-b}$$

$$x=\frac{1}{1-a}\cdot\frac{1}{1-b}=\frac{1}{1+ab-(a+b)}=\frac{1}{1-\frac{2}{11}-\frac{4}{11}}=\frac{11}{5}$$

$$\therefore \frac{6}{x}+\frac{6}{x^2}+\cdots=\frac{6}{x}\cdot\frac{1}{1-\frac{1}{x}}=\frac{6}{x}\cdot\frac{x}{x-1}=\frac{6}{\frac{11}{5}-1}=5$$

34. **(3)**

$$a_k=2a_{k-1}-a_{k-2} \Rightarrow a_k+a_{k-2}=2a_{k-1}$$

$$\therefore a_1\ a_2\ a_3 \cdots a_{11} \longrightarrow A.P.$$

$$a_1^2+a_2^2+\cdots+a_{11}^2=90\cdot 11$$

$$\Rightarrow 15^2+(15+d)^2+\cdots+(15+10d)^2=90\cdot 11$$

$$11\cdot 15^2+30d(1+2+\cdots+10)+d^2(1^2+2^2+\cdots+10^2)=90\cdot 11$$

$$15^2+30\cdot d\cdot\frac{10}{2}+d^2\cdot 10\cdot\frac{21}{6}=90$$

$$5\cdot 7d^2+5\cdot 30\cdot d+15^2-90=0$$

$$7d^2+30d+27=0$$

$$7d^2+21d+9d+27=0$$

$$\therefore (7d+9)(d+3)=0$$

$$d=\frac{-9}{7},-3$$

$$\therefore a_2<\frac{27}{2} \qquad \therefore d=-3$$

$$\therefore \frac{a_1+a_2+\cdots+a_{11}}{110} \Rightarrow \frac{\frac{11}{2}[2\cdot 15+10\cdot(-3)]}{110}=0$$

35. **(5)**

$$\frac{n(n+1)}{2} - k - (k+1) = 1224$$

$$\therefore n^2 + n - 2(2k+1) = 2448$$

$$\therefore n^2 + n - 2450 = 4k$$

$$(n+50)(n-49) = 4k$$

$$\therefore n = 50 \ \& \ k = 25$$

$$\therefore k - 20 \Rightarrow 25 - 20 \Rightarrow 5$$

36. **(6)**

(36)

$$A_n = \frac{3}{4}\left[\frac{1-\left(-\frac{3}{4}\right)^n}{1-\left(-\frac{3}{4}\right)}\right] = \frac{3}{7}\left[1-\left(-\frac{3}{4}\right)^n\right]$$

$$B_n > A_n \Rightarrow 1 - A_n > A_n \Rightarrow 1 > 2A_n \Rightarrow A_n < \frac{1}{2}$$

$$\therefore \frac{3}{7}\left[1-\left(-\frac{3}{4}\right)^n\right] < \frac{1}{2} \Rightarrow \left(-\frac{3}{4}\right)^n > -\frac{1}{6}$$

Given condition is true for all even values of n.
Now lets consider n as odd natural no.

$\therefore \quad -\left(\frac{3}{4}\right)^n > -\frac{1}{6} \Rightarrow \left(\frac{3}{4}\right)^n < 1/6$

$\therefore \quad n > \log_{(3/4)}(1/6) \approx 6.22$

$\therefore \quad n = 7, 9, 11, \ldots$

$\therefore$ If we consider even & odd both

$n = 2, 4, 6, 7, 8, 9, 10, 11, \ldots$

$\therefore \quad n_0 = 6$

37. **(6)**

(37)

$T_1 = a \qquad T_2 = a + d$

$S_n = (1 + 2T_n)(1 - T_n)$

$n = 1 \qquad S_1 = (1 + 2T_1)(1 - T_1)$

$\therefore \quad T_1 = (1 + 2T_1)(1 - T_1)$ [for $n = 1$, $S_1 = T_1$]

$\therefore \quad 2T_1^2 = 1$

$\therefore \quad T_1 = \frac{1}{\sqrt{2}}$

$S_2 = (1 + 2T_2)(1 - T_2)$

$T_1 + T_2 = 1 - 2T_2^2 + T_2$

$\therefore T_2^2 = \frac{1 - T_1}{2} = \frac{\sqrt{2} - 1}{2\sqrt{2}} = \frac{2 - \sqrt{2}}{4} = \frac{\sqrt{4} - \sqrt{2}}{4}$

$\therefore \quad a + b = 4 + 2 = 6$

38. **(9)**

(38)

$T_{20} = \log 20 = a + 19d$

$T_{32} = \log 32 = a + 31d$

$a + 19d = 1 + \log 2 \quad - (1)$

$a + 31d = 5\log 2 \quad - (2)$

(1) 5 − (2)

$4a + 64d = 5$

$a + 16d = \frac{5}{4}$

$\therefore T_{17} = \frac{5}{4}$

$\therefore p = 5$ & $q = 4$

$p + q = 9$

39. (3)

(39)

$$\frac{a^2+1}{2} \ge a \Rightarrow a^2+1 \ge 2a \Rightarrow \frac{a^2+1}{b+c} \ge \frac{2a}{b+c}$$

Similarly $\frac{b^2+1}{c+a} \ge \frac{2b}{c+a}$ & $\frac{c^2+1}{a+b} \ge \frac{2c}{a+b}$

$$\therefore \frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b} \ge \frac{2a}{b+c} + \frac{2b}{c+a} + \frac{2c}{a+b}$$

$$\frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b} \ge \frac{2a}{b+c} + 2 + \frac{2b}{c+a} + 2 + \frac{2c}{a+b} + 2$$

$$\frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b} \ge 2(a+b+c)\left[\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right] - 6 \quad \text{— (1)}$$

Now
$$\frac{\frac{b+c}{a+b+c} + \frac{c+a}{a+b+c} + \frac{a+b}{a+b+c}}{3} \ge \frac{3}{\frac{a+b+c}{b+c} + \frac{a+b+c}{c+a} + \frac{a+b+c}{a+b}}$$

$$\therefore \quad \frac{2}{3} \ge \frac{3}{(a+b+c)\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right)}$$

$$\therefore (a+b+c)\left[\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right] \ge \frac{9}{2}$$

$\therefore$ from eqn (1)

$$\frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b} \ge 2 \cdot \frac{9}{2} - 6$$

$$\frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b} \ge 3$$

40. (3)

(40)

$$\sum_{r=1}^{n} T_r = 3n^2 + 5n + 2$$

$r=1 \quad T_1 = 3+5+2$

$r=2 \quad T_1+T_2 = 3\cdot 2^2 + 5\cdot 2 + 2 \Rightarrow T_2 = 3(2^2-1) + 5(2-1)$

$r=3 \quad T_1+T_2+T_3 = 3\cdot 3^2 + 5\cdot 3 + 2 \Rightarrow T_3 = 3(3^2-2^2) + 5(3-2)$

$r=4 \quad T_1+T_2+T_3+T_4 = 3\cdot 4^2 + 5\cdot 4 + 2 \Rightarrow T_4 = 3(4^2-3^2) + 5(4-3)$

$\therefore T_m = 3(m^2-(m-1)^2) + 5(m-(m-1)) \quad [m>1]$

$= 3\cdot(2m-1) + 5 \Rightarrow 6m+2 \quad [m>1]$

$$\sum_{r=1}^{n} T_r^2 = T_1^2 + T_2^2 + \cdots + T_n^2$$

$$= 10^2 + \sum_{m=2}^{n}(6m+2)^2$$

$$= 10^2 + \sum_{m=2}^{n}(36m^2 + 2^2 + 24m)$$

$$= 100 + 36\left[\frac{n(n+1)(2n+1)}{6} - 1^2\right] + 2^2\cdot(n-1) + 24\left(\frac{n(n+1)}{2} - 1\right)$$

$\therefore d = 100 - 36\cdot 1 - 2^2 - 24 = 36$

$b = 36\cdot\frac{3}{6} + \frac{24}{2} = 30$

$c = \frac{36}{6} + 2^2 + \frac{24}{2} = 22$

$\therefore \frac{b+d}{c} = \frac{30+36}{22} = 3$

1. (a, d)

$S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + ...$

$= (3^2 + 7^2 + 11^2 + ...) + (4^2 + 8^2 + 12^2 + ...) - (1^2 + 5^2 + 9^2 + ...) - (2^2 + 6^2 + 10^2 + ...)$

$= \sum_{r=1}^{n}(4r-1)^2 + \sum_{r=1}^{n}(4r)^2 - \sum_{r=1}^{n}(4r-3)^2 - \sum_{r=1}^{n}(4r-2)^2$

$= \left[\sum_{r=1}^{n}(4r-1)^2 - (4r-3)^2\right] + 4\left[\sum_{r=1}^{n}(2r)^2 - (2r-1)^2\right]$

$= 8\sum_{r=1}^{n}(2r-1) + 4\sum_{r=1}^{n}(4r-1)$

$= 8\left[2\frac{n(n+1)}{2} - n\right] + 4\left[4\frac{n(n+1)}{2} - n\right]$

$= 8n^2 + 8n^2 + 4n = 16n^2 + 4n$

For $n = 8, 16n^2 + 4n = 1056$

And for $n = 9, 16n^2 + 4n = 1332$

2. (b)

$\log_e b_1, \log_e b_2, ..., \log_e b_{101}$ are in A.P.

$\Rightarrow b_1, b_2, ..., b_{101}$ are in G.P.

Also, $a_1, a_2, ...a_{101}$ are in A.P., where $a_1 = b_1$ are $a_{51} = b_{51}$

$\therefore$ $b_2, b_3, ..., b_{50}$ are GM's and $a_2, a_{3,.....,} a_{50}$ are AM's between b_1 and b_{51}

$\because$ $GM < AM \Rightarrow b_2 < a_2, b_3, ..., b_{50} < a_{50}$

$\because$ $b_1 + b_2 + ... + b_{51} < a_1 + a_2 + ... + a_{51}$

$\Rightarrow t < s$

Also a_1, a_{51}, a_{101} are in A.P and b_1, b_{51}, b_{101} are in G.P.

$\because$ $a_1 = b_1$ and $a_{51} = b_{51}$, $\therefore b^{101} > a_{101}$

3. (b, c)

Given $a_1 = 7, d = 8$

Hence, $a_n = 7 + (n-1)8$ and $T_1 = 3$

Also, $T_{n+1} = T_n + a_n$

$T_n = T_{n-1} + a_{n-1}$

$\vdots$

$T_2 = T_1 + a_1$

$\therefore$ $T_{n+1} = (T_{n-1} + a_{n-1}) + a_n$

$= T_{n-2} + a_{n-2} + a_{n-1} + a_n$

$\vdots$

$\Rightarrow T_{n+1} = T_1 + a_1 + a_2 + ... + a_n$

$$\Rightarrow T_{n+1} = T_1 + \frac{n}{2}\left[2(7)+(n-1)8\right]$$

$$\Rightarrow T_{n+1} = 3 + n(4n+3)$$

Hence, for n = 19; $T_{20} = 3+(19)(79) = 1504$ ....(i)

For n = 29; $T_{30} = 3+(29)(119) = 3454 \rightarrow (C)$

$$\sum_{k=1}^{20} T_k = 3 + \sum_{k=1}^{20} T_k = 3 + \sum_{k=1}^{19} T_k \left(3+4n^2+3n\right)$$

$$= 3 + 3(19) + \frac{3(19)(20)}{2} + \frac{4(19)(20)(39)}{6}$$

$$= 3 + 10507 = 10510 \rightarrow (b)$$

And similarly $\sum_{k=1}^{30} T_k = 3 + \sum_{k=1}^{30} T_k \left(4n^2+3n+3\right) = 35615$

4. (3)

We know that $S_\infty = \frac{a}{1-r}$

$$\therefore\ S_k = \begin{cases} \dfrac{\frac{k-1}{k!}}{1-\frac{1}{k}}, k \neq 1 \\ 0, k = 1 \\ \dfrac{1}{(k-1)!}, k \geq 2 \end{cases}$$

Now, $\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right| = \sum_{k=1}^{100}\left|\left(k^2-3k+1\right)\right|\frac{1}{(k-1)!}$

$$= |-1| + \sum_{k=1}^{100}\frac{\left(k^2-1\right)+1-3(k-1)-2}{(k-1)!},$$

Since $k^2-3k+1>0\ \forall k \geq 3$

$$= 1 + \sum_{k=1}^{100}\left(\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right)$$

$$= 1 + \left(1-\frac{1}{2!}\right) + \left(\frac{1}{1!}-\frac{1}{3!}\right) + \left(\frac{1}{2!}-\frac{1}{4!}\right) + \ldots.. + \left(\frac{1}{96!}-\frac{1}{98!}\right) + \left(\frac{1}{97!}-\frac{1}{99!}\right)$$

$$= 3 - \frac{1}{98!} - \frac{1}{99!} = 3 - \frac{9900}{100!} - \frac{100}{100!} = 3 - \frac{10000}{100!} = 3 - \frac{(100)^2}{100!}$$

$$\Rightarrow \frac{100^2}{100!} + \sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right| = 3$$

5. (0)

Given : $a_k = 2a_{k-1} - a_{k-2}$

$\Rightarrow \frac{a_{k-2} + a_k}{2} = a_{k-1}, 3 \le k \le 11$

If a is the first term and D the common difference then $a_1^2 + a_2^2 + ... + a_{11}^2 = 990$

$\Rightarrow 11a^2 + d^2\left(1^2 + 2^2 + ... + 10^2\right) + 2ad\left(1 + 2 + ... + 10\right)$

$= 990$

$\Rightarrow 11a^2 + \frac{10 \times 11 \times 21}{6} d^2 + 2ad \times \frac{10 \times 11}{2} = 990$

$\Rightarrow a^2 + 35d^2 + 10d = 90$

Since $a = a_1 = 15$

$\therefore 35d^2 + 150d + 135 = 0 \Rightarrow 7d^2 + 30d + 27 = 0$

$\Rightarrow (d+3)(7d+9) = 0 \Rightarrow d = -3 \text{ or } -9/7$

Then $a_2 = 15 - 3 = 12$ or $15 - \frac{9}{7} = \frac{96}{7} > \underline{2}$

$\therefore \ d \ne -9/7$

Hence $\frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{\frac{11}{2}\left[2 \times 15 + 10(-3)\right]}{11} = 0$

6. (3 or 9)

$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}\left[2 \times 3 + (5n-1)d\right]}{\frac{n}{2}\left[6 + (n-1)d\right]}$ $\quad [\because m = 5n]$

$= \frac{5\left[(6-d) + 5nd\right]}{(6-d) + nd}$, which will be independent of n if d = 6 or d = 0

For a proper A.P., we take d = 6

$\therefore a_2 = a_1 + d = 3 + 6 = 9$

7. (8)

Since, $AM \ge GM$

$\Rightarrow \frac{\frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3} + 1 + a^8 + a^{10}}{8} \ge \left(\frac{1}{a^5} \times \frac{1}{a^4} \times \frac{1}{a^3} \times \frac{1}{a^3} \times \frac{1}{a^3} \times 1 \times a^8 \times a^{10}\right)^{\frac{1}{8}}$

$\Rightarrow \frac{\frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3} + 1 + a^8 + a^{10}}{8} \ge (1)^{\frac{1}{8}}$

$\Rightarrow \frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3} + 1 + a^8 + a^{10} \ge 8(1)^{\frac{1}{8}}$

$\Rightarrow \frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3} + 1 + a^8 + a^{10} \ge 8$

So, the minimum value is 8.

8. (5)

Let k, k+1 be removed from pack.

$\therefore (1+2+3+...+n)-(k+k+1)=1224$

$\frac{n(n+1)}{2}-2k=1225 \Rightarrow k=\frac{n(n+1)-2450}{4}$

For n = 50, k = 25, $\therefore$ k – 20 = 5

9. (4)

Since a, b, c are in G.P., $\therefore$ $b = ar$ and $c = ar^2$

Also, $\frac{b}{a}$ is an integer $\Rightarrow$ r is an integer

$\because$ A.M. of a, b, c is b + 2

$\Rightarrow \frac{a+b+c}{3} = b+2 \Rightarrow a+ar+ar^2 = 3ar+6$

$\Rightarrow a(r^2-2r+1)=6, \quad \therefore a(r-1)^2=6$

Since, a and r are integers

Hence, the only possible values of a and r can be 6 and 2 respectively.

$\therefore \frac{a^2+a-14}{a+1}=\frac{36+6-14}{6+1}=\frac{28}{7}=4$

10. (9)

$\frac{S_7}{S_{11}}=\frac{6}{11} \Rightarrow \frac{\frac{7}{2}(2a+6d)}{\frac{11}{2}(2a+10d)}=\frac{6}{11} \Rightarrow a=9d$

$a_7 = a+6d=15d$

Given $130 < a_7 < 140$

$\Rightarrow 130 < 15d < 140 \Rightarrow d = 9$

[Since d is a natural number because all trms are natural numbers.]

11. (6)

Let the sides be a – d, a, a + d where d is positive.

Using Pythagoras theorem,

$(a+d)^2=(a-d)^2+a^2 \Rightarrow a=4d$

$\therefore$ Sides are 3d, 4d, 5d

Area = 24 $\Rightarrow \frac{1}{2}\times 3d \times 4 = 24 \Rightarrow d = 4$

$\therefore$ Smallest side = 3d = 12

12. (3748)

The given sequences upto 2018 terms are

1, 6, 11, 16,……,10086

And 9, 16, 23,…..,14128

The common terms are

16, 15, 86,…..,upto n terms, where $T_n \le 10086$

$\Rightarrow 16+(n-1)35 \le 10086$

$\Rightarrow 35n-19 \le 10086$

$\Rightarrow n \le \dfrac{10105}{35} = 288.7$

$\therefore\ n = 288$

$\therefore\ n(X\cup Y) = n(X) + n(Y) - n(X\cap Y)$

$= 2018 + 2018 - 288 = 3748$

13. (157)

AP(1, 3) : 1, 4, 7, 10, 13,….

AP(2, 3) : 2, 7, 12, 17, 22,….

AP(3, 7) : 3, 10, 17, 24, 31,…..

For $AP(1,3)\cap AP(2,5)\cap AP(3,7)$

First term will be the minimum common

$\therefore$ We need to find that minimum common which.

When divided by 7 leaves remainder $3 \rightarrow (7m+3)$

And when divided by 5 leaves remainder $2 \rightarrow (5p+2)$

And when divided by 3 leaves remainder $1 \rightarrow (3q+1)$

By hit and trial 52 is such number $(7\times 7+3)$

$\therefore$ First term 'a' of intersection AP = 52

Also common difference 'd' of intersection AP

= LCM (7, 5, 3) = 105

$\therefore$ a + d = 52 + 105 = 157

14. (1)

It is given that

$2(a_1 + a_2 + ... + a_n) = b_1 + b_2 + ... + b_n$

$\Rightarrow 2\times \dfrac{n}{2}\left(2c + (n-2)x_2\right) = c\left(\dfrac{2^n - 1}{2-1}\right)$ $[\because a_1 = c, b_1 = c]$

$\Rightarrow c(2^n - 1 - 2n) = 2n^2 - 2n$

$\Rightarrow c = \dfrac{2n^2 - 2n}{2^n - 1 - 2n} \in \mathbb{N}$

So, $2n^2 - 2n \ge 2^n - 1 - 2n$

$\Rightarrow 2n^2 + 1 \ge 2^n \Rightarrow n < 7$

$\because c \in \mathbb{N} \Rightarrow c > 0 \Rightarrow n > 2$

$\Rightarrow$ n can be 3, 4, 5, or 6

Checking c against these values of n

When n = 3, c = 14

When n = 4, c = $\dfrac{24}{7}$ which is not possible

When n = 5 , c = $\dfrac{40}{21}$ which is not possible

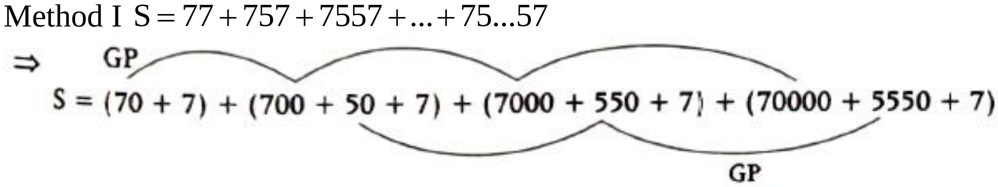
When n = 6, c = $\dfrac{60}{51}$ which is not possible

$\therefore$ We get c = 12 when n = 3
Hence, there exists only one value of c which holds the inequality

15. (8)
By AM-GM inequality
$AM \geq GM$
$$\therefore \frac{3^{y1}+3^{y2}+3^{y3}}{3} \geq \left[3^{(y_1+y_2+y_3)}\right]^{\frac{1}{3}}$$
$$\Rightarrow 3^{y1}+3^{y2}+3^{y3} \geq 3^{y4}$$
$$\Rightarrow \log_3\left(3^{y1}+3^{y2}+3^{y3}\right) \geq 4 \Rightarrow m=4$$
$$\because \log_3 x_1+\log_3 x_2+\log_3 x_3=\log_3\left(x_1x_2x_3\right)$$
Again by AM-GM inequality
$AM \geq GM$
$$\Rightarrow \frac{x_1+x_2+x_3}{3} \geq \sqrt[3]{x_1x_2x_3}$$
$$\Rightarrow x_1x_2x_3 \leq 27$$
$$\Rightarrow \log_3\left(x_1x_2x_3\right) \leq \log_3\left(3^3\right)$$
$$\Rightarrow \log_3 x_1+\log_3 x_2+\log_3 x_3 \leq 3 \Rightarrow M=3$$
Now, $\log_2\left(m^3\right)+\log\left(M^2\right)=6+2=8$

16. (18900)
For T_1 = a and common difference = d_1
And similarly now for A.P. $w_1, w_2, \ldots .. w_{100}$
T_1 = b and common difference = d_2
$$A_{51}-A_{50}=I_{51}w_{51}-I_{50}w_{50}$$
$$=\left(a+50d_1\right)\left(b+50d_2\right)-\left(a+49d_1\right)\left(b+49d_2\right)$$
$$=50bd_1+50ad_2+2500d_1d_2-49ad_2-49bd_1-2401d_1d_2$$
$$=bd_1+ad_2+99d_1d_2=1000$$
$$\Rightarrow bd_1+ad_2=1000-990=10 \quad ..(i)\ (\text{As } d_1d_2=10)$$
$$\therefore \quad A_{100}-A_{90}=l_{100w100}-l_{90w90}$$
$$=\left(a+99d_1\right)\left(b+99d_2\right)-\left(a+89d_1\right)\left(b+89d_2\right)$$
$$=99bd_1+99ad_2+99d_1d_2-89bd_1-89ad_2-89d_1d_2$$
$$=10\left(bd_1+ad_2\right)+1880d_1d_2$$
$$\Rightarrow 10(10)+1880(10)=18900$$

17. (1219)
Method I $S=77+757+7557+\ldots+\overbrace{75\ldots57}^{98}$



$$....+\underbrace{70000....0}_{99}+\underbrace{5555....550}_{98}+7$$

$$\Rightarrow \quad S=\left(10+10^2+10^3+...+10^{99}\right)+50\left(1+11+111+...+\overbrace{111...1}^{98}\right)+\underbrace{(7+7+7...+7)}_{99}$$

$$\Rightarrow \quad S=70\left(\frac{10^{99}-1}{9}\right)+\frac{50}{9}\left[(10-1)+\left(10^2-2\right)+\left(10^3-1\right)+...+\left(10^{98}-1\right)\right]+7\times 99$$

$$=70\times\left(\frac{10^{99}-1}{9}\right)+\frac{50}{9}\left[10\left(\frac{10^{98}-1}{9}\right)-98\right]+7\times 99$$

$$=\frac{7\times 10^{100}}{9}-\frac{70}{9}+\frac{50}{9}\left[\frac{10^{99}-1-9}{9}-98\right]+7\times 99$$

$$=\frac{7\times 10^{100}}{9}-\frac{70}{9}+\frac{50}{9}\left[\overbrace{1111...1}^{99}-99\right]+7\times 99$$

$$=\frac{\overbrace{75555...5}^{99}-70+143\times 9}{9}$$

$$=\frac{\overbrace{75555...57}^{99}+1210}{9}=\frac{\overbrace{7555...57}^{99}+m}{n}\text{ (given)}$$

$\therefore \quad m=1210$ and $n=9$

Hence, $m+n=1210+9=1219$

Method II st difference $\longrightarrow$

$$S=\underset{680}{77+757}+\underset{6800}{757+7557}+\underset{68000}{7557+}......+7\overbrace{555.........}^{98}57)$$

We observe that Ist difference of given series is 680,6800,68000,… which are in GP.
As we know that (by technique),if 1st difference is in GP, then
the general term is $T_n=\alpha(r)^n+\beta$, where r is the common ratio.

From above GP i.e. 680,6800,68000,… its common ratio $=10$

$\therefore \quad T_n=\alpha(10)^n+\beta \qquad \ldots \quad$ (i)

If $n=1$ then $T_1=10\alpha+\beta=77$ (given) i.e. first term of S.

If $n=2$ then $T_2=100\alpha+\beta=757$ (given) i.e. second term of S.

On solving T_1 and T_2, we get

$\alpha=68/9$ and $\beta=13/9$

Now, according to given data in question, we have

$$S=\frac{\overbrace{7555...57}^{99}+m}{n}$$ which takes the from (in general term) is

$$S=\frac{T_{100}+m}{n}=\frac{\left(\alpha\cdot 10^{100}+\beta\right)+m}{n} \qquad \ldots \quad \text{(ii)}$$

Now, $S=\sum_{n=1}^{99} T_n$ [from Eq. (i)]

$$=\sum_{n=1}^{99}\left(\alpha(10)^n+\beta\right)=\alpha\sum_{n=1}^{99}10^n+\sum_{n=1}^{99}\beta$$

$$=\alpha\left[\frac{10\left(10^{99}-1\right)}{9}\right]+99\beta$$

$$=\alpha\left(\frac{10^{100}}{9}-\frac{10}{9}\right)+99\beta$$

$$=\frac{\alpha\cdot10^{100}}{9}-\frac{10\alpha}{9}+99\beta$$

$$=\frac{\alpha\left(10^{100}\right)+\beta}{9}-\frac{\beta}{9}-\frac{10\alpha}{9}+99\beta$$

$$=\frac{T_{100}}{9}-\frac{19}{91}-\frac{10}{9}\times\frac{68}{9}+99\times\frac{13}{9}$$

$$\frac{T_{100}+1210}{9} \quad \ldots \text{(iii)}$$

From Eqs. (ii) and (iii), we get

$m=1210$ and $n=9$

$\therefore \quad m+n=1219$