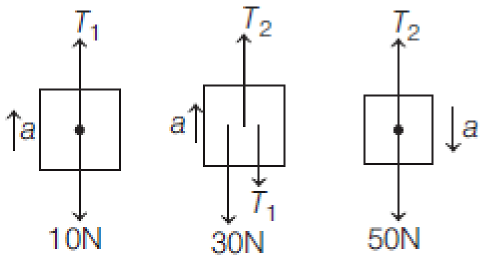
Advanced Booklet Solution

Laws of Motion & Friction

In chapter Exercise – 1 (Problems on Constraints & Simple Pulley System)

1.	$v = 2u$	**2.**	$u = 2v$	**3.**	$u = 2v$	**4.**	$u = 4v$
5.	$v_1 + v_2 = 2v_3$	**6.**	$v_2 + v_3 = 2v_1$	**7.**	$2v_1 = v_2 + 2v_3$	**8.**	$v_2 = v_3 = 2v_1$
9.	$a_2 = 3a_1$	**10.**	$3a_1 = 2a_2$	**11.**	$5a_1 = 4a_2$		

12. $g/9$



$$50 - T_2 = 5a \qquad ...(i)$$

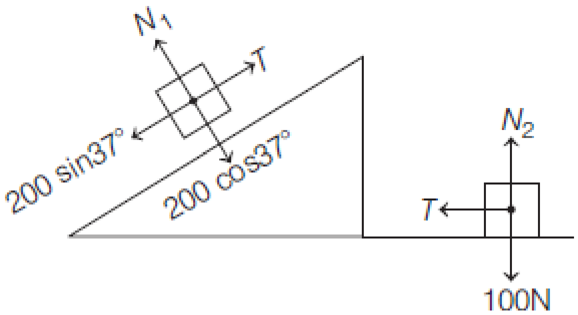
$$T_2 - 30 - T_1 = 3a \qquad ...(ii)$$

$$T_1 - 10 = 1a \qquad ...(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$a = \frac{g}{9}$$

13. $2g/5$



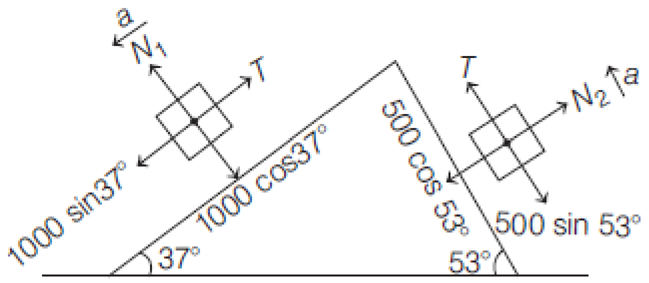
$$200\sin 37^\circ - T = 20a \qquad ...(i)$$

$$T = 10a \qquad ...(ii)$$

Adding Eqs. (i) and (ii), we get

$\Rightarrow \quad a = 4\ \text{m/s}^2$

14. $2g / 15$



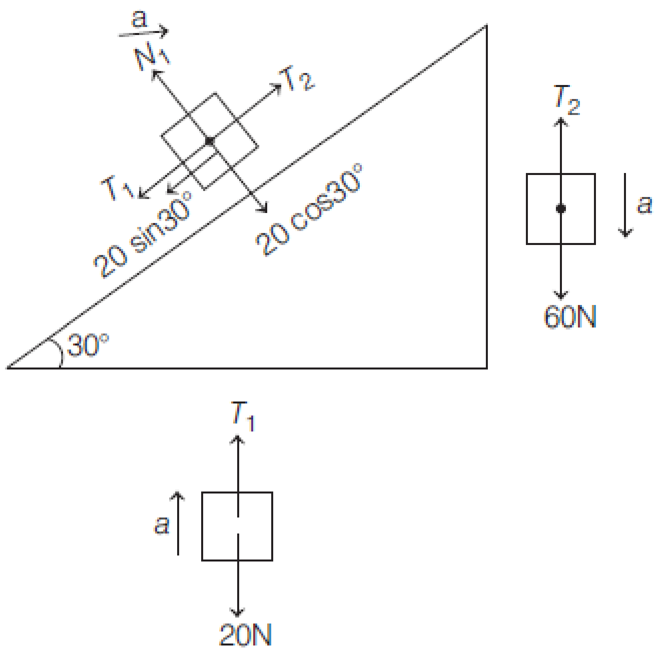
$$1000\sin 37^\circ - T = 100a \qquad ...(i)$$

$$T - 500\sin 53^\circ = 50a \qquad ...(ii)$$

Adding Eqs. (i) and (ii), we get

$$\Rightarrow \quad a = \frac{4}{3}\,\text{m/s}^2$$

15. $3g / 10$



$$60 - T_2 = 6a \qquad ...(i)$$

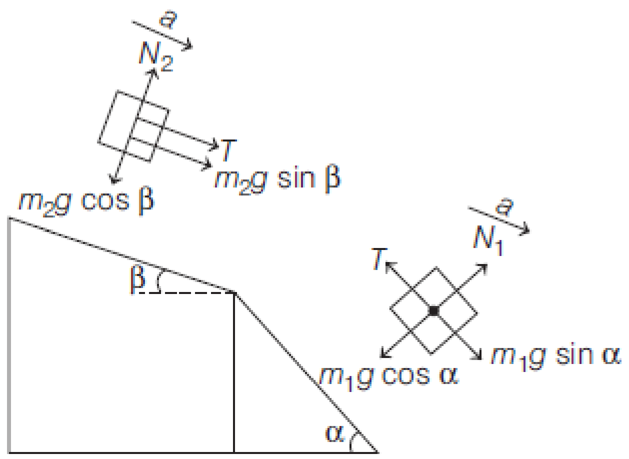
$$T_2 - T_1 - 10 = 2a \qquad ...(ii)$$

$$T_1 - 20 = 2a \qquad ...(iii)$$

Adding Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \quad a = 3\,\text{m/s}^2$$

16. $\dfrac{(m_1 \sin\alpha + m_2 \sin\beta) g}{m_1 + m_2}$



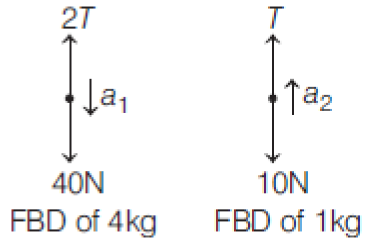
$m_1 g \sin\alpha - T = m_1 a$ ...(i)

$T + m_2 g \sin\beta = m_2 a$ ...(ii)

Adding Eqs. (i) and (ii), we get

$\Rightarrow \quad a = \dfrac{(m_1 \sin\alpha + m_2 \sin\beta) g}{m_1 + m_2}$

17. acceleration of 4 kg = 2.5 m/s^2 down, acceleration of 1 kg = 5 m/s^2 up



$40 - 2T = 4a_1$ ...(i)

$T - 10 = 1a_2$ ...(ii)

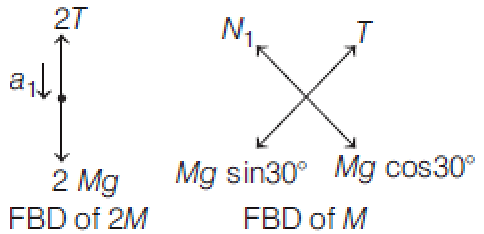
$2a_1 = a_2$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

$a_1 = 2.5 \text{ m/s}^2$, down

and $a_2 = 5 \text{ m/s}^2$, up

18. acceleration of $2M = \dfrac{g}{6}$ down, acceleration of $M = \dfrac{g}{3}$ up the plane



$2Mg - 2T = 2Ma_1$ ...(i)

$T - Mg \sin 30^\circ = Ma_2$ ...(ii)

$2a_1 = a_2$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

$a_1 = \frac{g}{6}$, down

and $a_2 = \frac{g}{3}$, up the plane

19. acceleration of $2M = \frac{2g}{3}$ down, acceleration of $M = \frac{g}{3}$ up

$2Mg - T = 2Ma_1$ ...(i)

$2T - Mg = Ma_2$ ...(ii)

$2a_2 = a_1$ ...(iii)

$a_2 = \frac{g}{2}$, up and $a_1 = \frac{2g}{3}$, down

20. acceleration of 4 kg $= \frac{2g}{7}$ down, acceleration of 5 kg $= \frac{g}{7}$ up

$50 - 2T = 5a_1$ ...(i)

$T - 40 = 4a_2$ ...(ii)

$a_2 = 2a_1$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

$a_1 = -\frac{10}{7}\,\text{m/s}^2 = \frac{10}{7}\,\text{m/s}^2$, up

$a_2 = -\frac{20}{7}\,\text{m/s}^2 = \frac{20}{7}\,\text{m/s}^2$, down

21. acceleration of 3 kg = 20 m/s^2 and acceleration of 5 kg = 10 m/s^2

$170 - 2T = 5a_1$ ...(i)

$T = 3a_2$ ...(ii)

$a_2 = 2a_1$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

$a_1 = 10\,\text{m/s}^2$ and $a_2 = 20\,\text{m/s}^2$

22. acceleration of $A = \frac{9F}{34m}$, acceleration of $B = \frac{3F}{17m}$

$F - 2T = 2ma_1$ ...(i)

$3T = 4ma_2$ ...(ii)

$2a_1 = 3a_2$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

$a_2 = \frac{3F}{17m}$ and $a_1 = \frac{9F}{34m}$

23. acceleration of $3m$ and m are $\frac{g}{13}$ and $\frac{7g}{13}$, respectively.

$$7T - 3mg = 3ma_1 \quad ...(i)$$

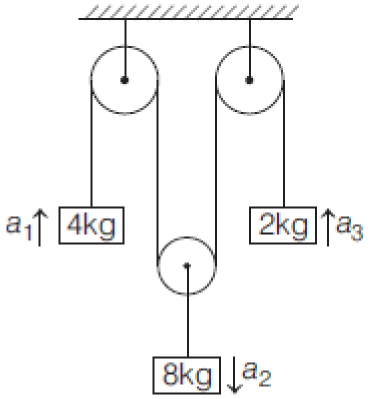
$$mg - T = ma_2 \quad ...(ii)$$

$$7a_1 = a_2 \quad ...(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$a_1 = \frac{7g}{13} \text{ and } a_2 = \frac{g}{13}$$

24. acceleration of 8 kg, 4 kg and 2 kg are 2 m/s^2 up, 2 m/s^2 down, 6 m/s^2, respectively.



$$T - 40 = 4a_1 \quad ...(i)$$

$$80 - 2T = 8a_2 \quad ...(ii)$$

$$T - 20 = 2a_3 \quad ...(iii)$$

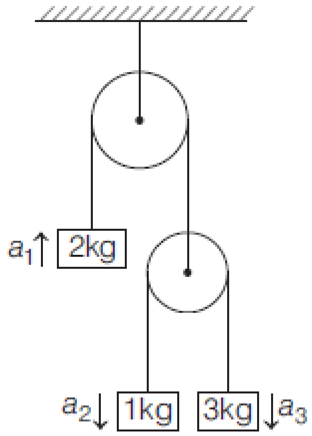
$$2a_2 = a_1 + a_3 \quad ...(iv)$$

$$a_1 = -2\,\text{m/s}^2$$

$$a_2 = +2\,\text{m/s}^2$$

$$a_3 = +6\,\text{m/s}^2$$

25. acceleration of 1 kg, 2 kg and 3 kg are 2 m/s^2 down, 2 m/s^2 and 6 m/s^2 up, respectively.



$$2T - 20 = 2a_1 \quad ...(i)$$

$$10 - T = a_2 \quad ...(ii)$$

$$30 - T = 3a_3 \quad ...(iii)$$

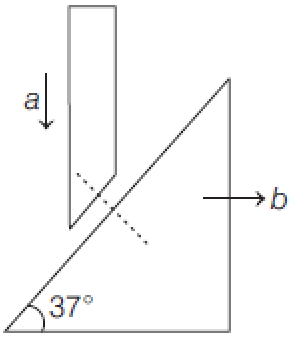
$$2a_1 = a_2 + a_3 \quad ...(iv)$$

Solving Eqs. (i), (ii), (iii) and (iv), we get

$a_1 = 2\,\text{m/s}^2$, $a_2 = -2\,\text{m/s}^2$, $a_3 = 6\,\text{m/s}^2$

In chapter Exercise – 2 (Wedge constraints & Pseudo force (acc. frames))

26. $\frac{4a}{3}\hat{\mathbf{i}}$



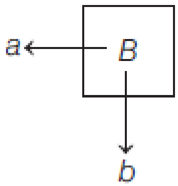
From wedge constraint their acceleration will be same along the common normal at contact.

$\Rightarrow \quad a\cos 37^\circ = b\sin 37^\circ \Rightarrow b = \frac{4a}{3}$

So, $\mathbf{a}_B = \left(\frac{4a}{3}\right)\hat{\mathbf{i}}$

27. $-a\hat{\mathbf{i}} - 4a\hat{\mathbf{j}}$

From string constraint,



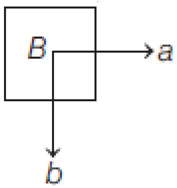
$-4a + b = 0$

$\Rightarrow \quad b = 4a$

$\mathbf{a}_B = -a\hat{\mathbf{i}} - 4a\hat{\mathbf{j}}$

28. $a\hat{\mathbf{i}} - 2(a+c)\hat{\mathbf{j}}$

From string constraint,



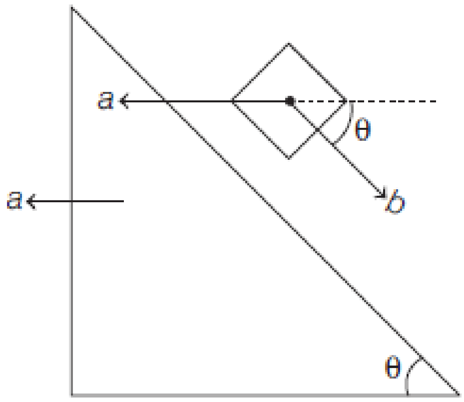
$-2a - 2c + b = 0$

$\Rightarrow \quad b = 2(a+c)$

$\mathbf{a}_B = a\hat{\mathbf{i}} - 2(a+c)\hat{\mathbf{j}}$

29. $a(\cos\theta - 1)\hat{\mathbf{i}} - a\sin\theta\,\hat{\mathbf{j}}$

From string constraint,



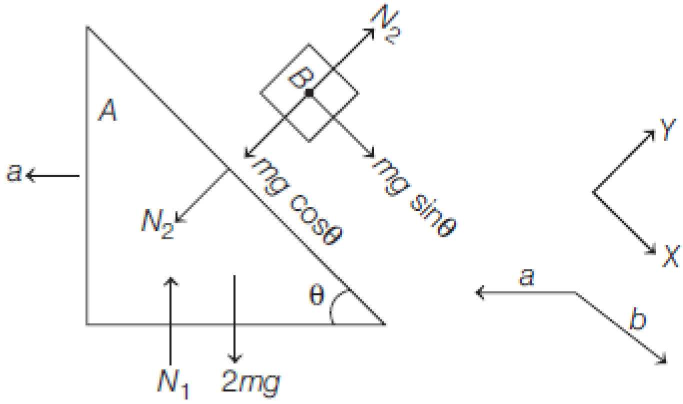
$$-a + b = 0$$

$$\Rightarrow \quad b = a$$

$$\mathbf{a}_B = (b\cos\theta - a)\hat{\mathbf{i}} - b\sin\theta\,\hat{\mathbf{j}}$$

$$= a(\cos\theta - 1)\hat{\mathbf{i}} - a\sin\theta\,\hat{\mathbf{j}}$$

30. $a = \dfrac{b\cos\theta}{3}; b = \dfrac{3g\sin\theta}{3-\cos^2\theta}$



For A, $\sum F_x = ma_x$

$\Rightarrow \quad N_2\sin\theta = 2ma \qquad \ldots(\text{i})$

For B, $\sum F_x = ma_x$

$\Rightarrow \quad mg\sin\theta = m(b - a\cos\theta) \qquad \ldots(\text{ii})$

$\sum F_y = ma_y$

$\Rightarrow \quad mg\cos\theta - N_2 = ma\sin\theta \qquad \ldots(\text{iii})$

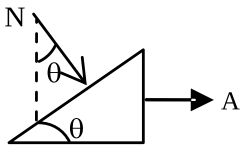
Solving Eqs. (i), (ii) and (iii), we get

$$a = \frac{b\cos\theta}{2}; b = \frac{3g\sin\theta}{3-\cos^2\theta}$$

31. $a_1\sin 45^0 = a_2\sin 15^0$

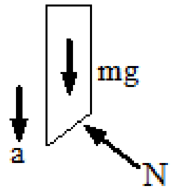
32. $A = \dfrac{mg}{M\cot\theta + m\tan\theta}$ and $a = \dfrac{mg\tan\theta}{M\cot\theta + m\tan\theta}$

Let the acceleration of wedge be 'A'



$N\sin\theta = MA$ ...(1)

Let acceleration of rod be 'a'



$mg - N\cos\theta = ma$ ...(2)

$\tan\theta = y/x$

$y = x\tan\theta$

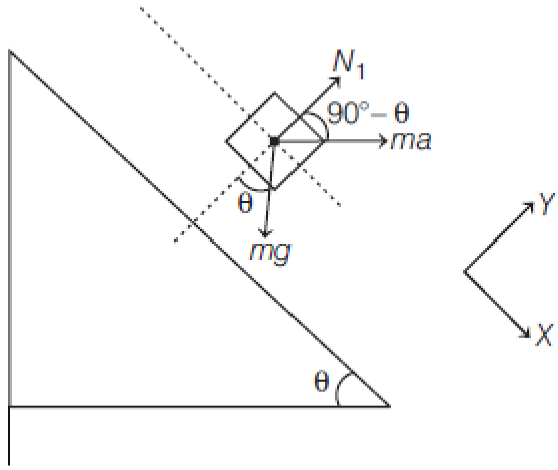
$\dfrac{d^2y}{dt^2} = \dfrac{d^2x}{dt^2}\tan\theta$

$a = A\tan\theta$ ...(3)

Solving (1) (2) & (3), we get

$A = \dfrac{mg}{M\cot\theta + m\tan\theta}$ and $a = \dfrac{mg\tan\theta}{M\cot\theta + m\tan\theta}$

33. $g\cot\theta$



$\sum F_y = 0$

$\Rightarrow N_1 + ma\sin\theta = mg\cos\theta$

$\Rightarrow N_1 = mg\cos\theta - ma\sin\theta$

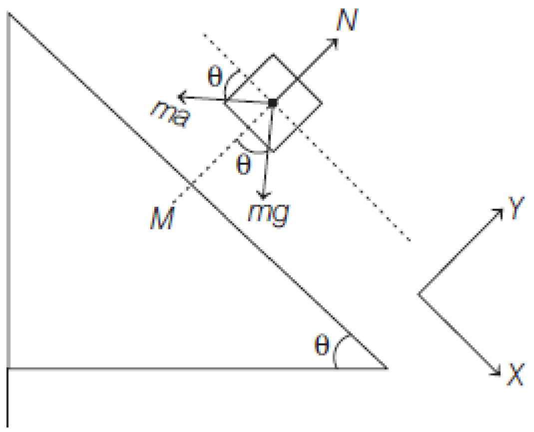
When block loses contact with the wedge

$N_1 = mg\cos\theta - ma\sin\theta = 0$

$\Rightarrow a = g\cot\theta$

34. $(M+m)g\tan\theta$

Lets take acceleration of wedge to be a.



$$\sum F_x = 0$$

$\Rightarrow \quad mg\sin\theta - ma\cos\theta = 0$

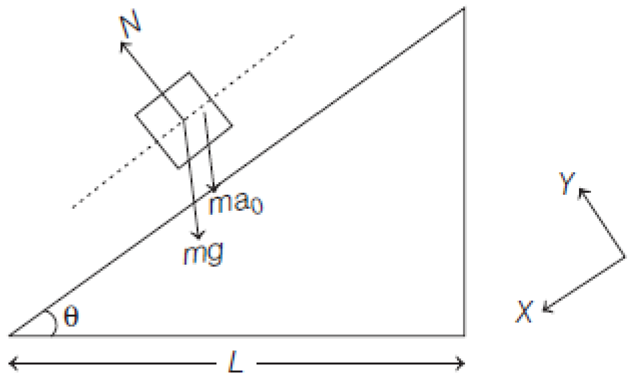
$\Rightarrow \quad a = g\tan\theta$

For $(M+m)$ system,

$$\sum F_x = ma$$

$\Rightarrow \quad F = (M+m)g\tan\theta$

35. $\sqrt{\left(\dfrac{2L}{(g+a_0)\sin\theta\cos\theta}\right)}$



$$\sum F_x = ma_x$$

$\Rightarrow \quad (mg + ma_0)\sin\theta = ma_x$

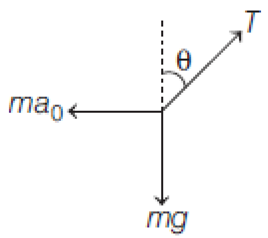
$\Rightarrow \quad a_x = (g+a_0)\sin\theta$

$$s = ut + \frac{1}{2}at^2$$

$\Rightarrow \quad \dfrac{L}{\cos\theta} = 0 + \dfrac{1}{2}(g+a_0)\sin\theta T^2$

$$T = \sqrt{\frac{2L}{(g+a_0)\sin\theta\cos\theta}}$$

36. $\tan^{-1}\left(\frac{a_0}{g}\right)$



FBD of *m*

$$\sum F_x = 0$$

$$\Rightarrow \quad T\sin\theta = ma_0 \qquad \text{...(i)}$$

$$\sum F_y = 0$$

$$\Rightarrow \quad T\cos\theta = mg \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\Rightarrow \quad \tan\theta = \frac{a_0}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a_0}{g}\right)$$

In chapter Exercise – 3 (Spring force)

37. 10 N
System is in equilibrium. Tension in the string will be 10 N, so spring balance reading will be 10 N.

38. 16 N

$$40 - T = 4a \qquad \text{...(i)}$$

$$T - 10 = 1a \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get
$a = 6$ m/s^2 and $T = 16$ N
Spring balance reading = T = 16 N

39. Each will read 10 kg.
Both of them will read 100 N.

40. 0.2 m

$$30 - T_1 = 3a \qquad \text{...(i)}$$

$$10 + T_1 - T_2 = a \qquad \text{...(ii)}$$

$$T_2 - 20 = 2a \qquad \text{...(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$a = \frac{10}{3}\,\text{m/s}^2 \text{ and } T_1 = 20\,\text{N}$$

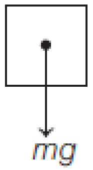
Spring force $= kx = T_1$

$$\Rightarrow \quad 100x = 20$$

$$\Rightarrow \quad x = 0.2\,\text{m}$$

41. $g/2$ upwards, g downwards

Just after cutting the string

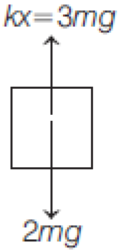


FBD of m

$F = ma$

$\Rightarrow \quad mg = ma$

$\Rightarrow \quad a = g \downarrow$



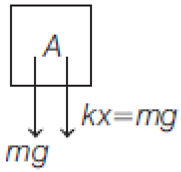
FBD of $2m$

$F = ma$

$\Rightarrow \quad 3mg - 2mg = 2ma$

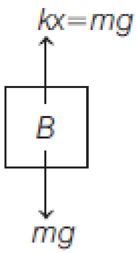
$\Rightarrow \quad a = \dfrac{g}{2} \uparrow$

42. $2g$ downwards, 0



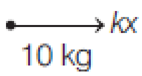
For A, $mg + mg = ma_A$

$a_a = 2g \downarrow$



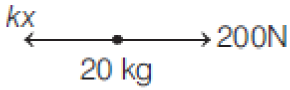
For B, $a_B = 0$

43. 4 ms^{-2}



$\Rightarrow \quad kx = 10(12)$

$\Rightarrow \quad kx = 120\,N$



$200 - kx = 20a$

$\Rightarrow$ $200 - 120 = 20a$

$\Rightarrow$ $a = 4\ \text{m/s}^2$

JEE Main Exercise

1. (D)

Horizontal velocity of ball and person are same so both will cover equal horizontal distance in a given interval of time and after following the parabolic path the ball falls exactly in the hand which threw it up.

2. (B)

$$\vec{a} = \frac{\overrightarrow{v_2} - \overrightarrow{v_1}}{t} = \frac{(-2)-(+10)}{4} = \frac{-12}{4} = -3\ \text{m/s}^2$$

3. (B)

$$F = ma = 10 \times (-3) = -30\ \text{N}$$

4. (B)

Impulse = Force × Time = $-30 \times 4 = -120\ N\text{-}s$

5. (D)

$$R = m(g+a) = m(g+g) = 2mg$$

6. (A)

$$T_1 = m(g+a) = 1 \times \left(g + \frac{g}{2}\right) = \frac{3g}{2}$$

$$T_2 = m(g-a) = 1 \times \left(g - \frac{g}{2}\right) = \frac{g}{2} \quad \therefore \quad \frac{T_1}{T_2} = \frac{3}{1}$$

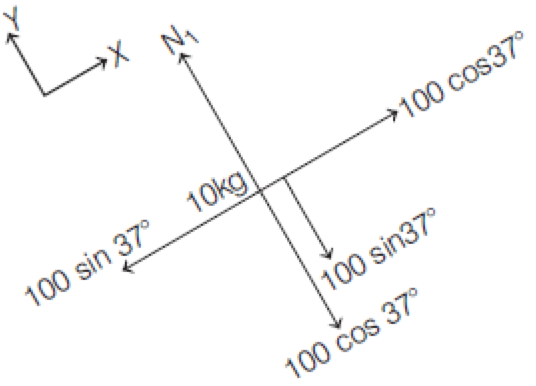
7. (B)

Apparent weight $= m(g-a) = 50(9.8 - 9.8) = 0$

8. (A)

$$m = \frac{F}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = \sqrt{200} = 10\sqrt{2}\ \text{kg}$$

9. (A)



$$\sum F_x = ma_x$$

$$\Rightarrow \quad 100\cos 37^\circ - 100\sin 37^\circ = 10a$$

$$\Rightarrow \quad a = 2\ \text{m/s}^2$$

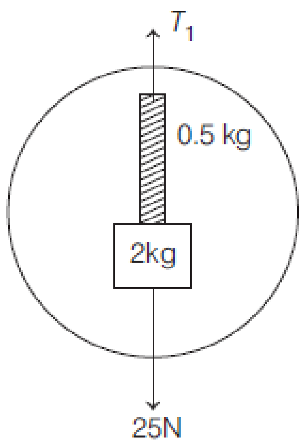
10. (C)

Acceleration of system, $a = \dfrac{F}{m_1 + m_2 + m_3}$

Net force on $m_2 = m_2 a$

$$= m_2\left(\frac{F}{m_1 + m_2 + m_3}\right)$$

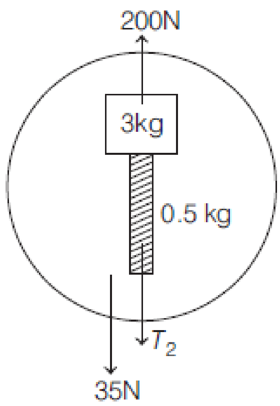
11. (C)



$$\sum F_y = ma_y$$

$$\Rightarrow \quad T_1 - 25 = 2.5(10)$$

$$T_1 = 50\ \text{N}$$



$$\sum F_y = ma_y$$

$\Rightarrow$ $200 - 35 - T_2 = 3.5(10)$

$\Rightarrow$ $T_2 = 130\text{ N}$

12. (C)

$150 - T_1 - T_2 = 15a$ ...(i)

$T_1 = 5a$ ...(ii)

$T_2 = 10a$ ...(iii)

From Eqs. (i), (ii)and (iii), we get

$a = 5\text{ m/s}^2$ and $T_2 = 50\text{ N}$

13. (C)

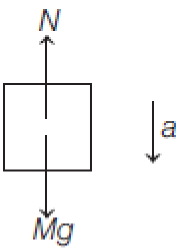
$8g\sin 30^\circ - T = 8a$ ...(i)

$T - 4g = 4a$ ...(ii)

From Eqs. (i) and (ii), we get

$a = 0$

14. (A)



FBD of block

$Mg - N = ma$

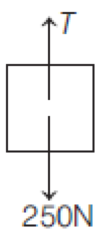
$\Rightarrow$ $Mg - \dfrac{Mg}{4} = Ma \Rightarrow a = \dfrac{3g}{4}$

15. (D)

$T - 6000g = 6000\left(\dfrac{g}{2}\right)$

$\Rightarrow$ $T = 9000\,g$

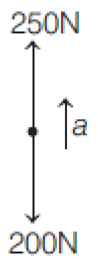
16. (D)



FBD of cage

$\sum F_y = 0$

$\Rightarrow$ $T = 250\text{ N}$

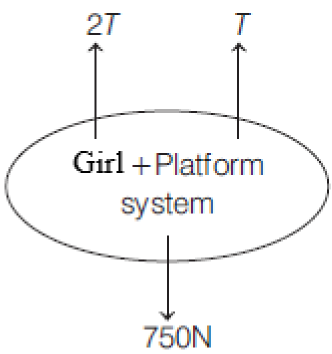


FBD of monkey

$250 - 200 = 20a$

$\Rightarrow \quad a = 2.5 \text{ m/s}^2$

17. (B)



$\sum F_y = ma_y$

$\Rightarrow \quad 3T - 750 = 75(0)$

$\Rightarrow \quad T = 250 \text{ N}$

18. (A)

$m_2 g - 2T = m_2 a_2 \qquad ...(i)$

$T = m_1 a_1 \qquad ...(ii)$

$a_1 = 2a_2 \qquad ...(iii)$

From Eqs. (i), (ii) and (iii), we get

$$a_2 = \frac{m_2 g}{4m_1 + m_2}$$

19. (C)

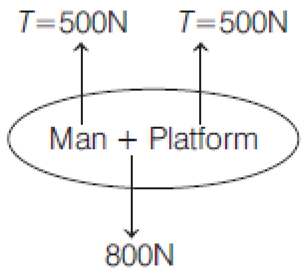
$\Delta p = \sum F \Delta t$

$m\Delta v = 7(1.5) + 5(1.7) + 10(3)$

$\Delta = 4.9 \text{ m/s}$

20. (B)

For (man + platform) system,



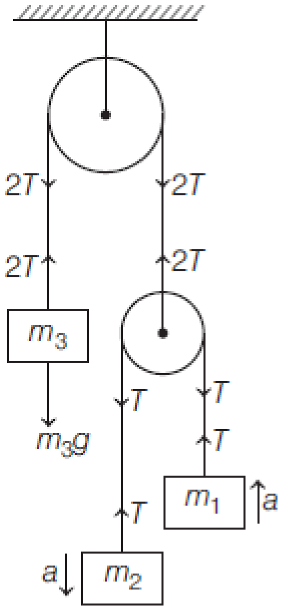
$$\sum F_y = ma_y$$

$\Rightarrow \quad 2T - 800 = 80a$

$\Rightarrow \quad a = 2.5 \text{ m/s}^2$

21. (A)

Since, m_3 is at rest.



$\Rightarrow \quad 2T = m_3 g$

For m_1 and m_2,

$$m_2 g - T = m_2 a \qquad \ldots(\text{i})$$

$$T - m_1 g = m_1 a \qquad \ldots(\text{ii})$$

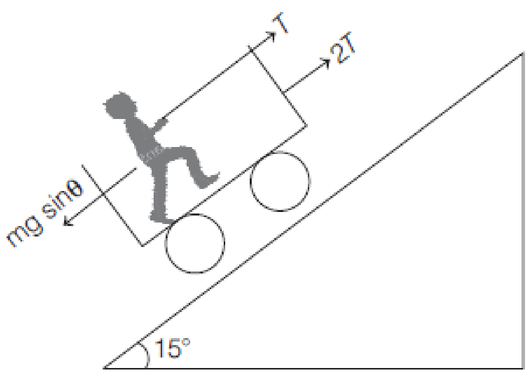
From Eqs. (i) and (ii), we get

$$\frac{m_2 g - T}{m_2} = \frac{T - m_1 g}{m_1}$$

$$T - = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{m_3 g}{2}$$

$$\Rightarrow \quad m_3 = \frac{4m_1\, m_2}{m_1 + m_2}$$

22. (C)



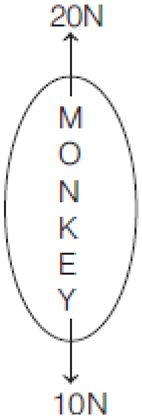
$$3T - mg\sin\theta = ma$$

$$750 - 1000(0.26) = 100a$$

$$a = 4.9 \text{ m/s}^2$$

23. (B)

To just lift the block, tension in the string will be 20 N.



$F = ma$

$\Rightarrow \quad 20 - 10 = 1 \cdot a_1$

$a_1 = 10\ \text{m/s}^2$

When monkey stops moving w.r.t. the string

$20 - T = 2a_2 \qquad \text{...(i)}$

$T - 10 = 1a_2 \qquad \text{...(ii)}$

From Eqs. (i) and (ii), we get

$\Rightarrow \quad a_2 = \dfrac{10}{3}\ \text{m/s}^2$

Change in acceleration $= a_1 - a_2 = 10 - \dfrac{10}{3} = \dfrac{20}{3}\ \text{m/s}^2$

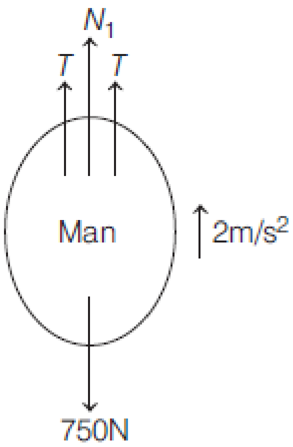
24. (B)

For (man + platform) system

$\Rightarrow \quad \sum F = ma$

$4T - 750 - 250 = 100 \times 2$

$\Rightarrow \quad T = 300\ \text{N}$



For only man, $\sum F = ma$

$\Rightarrow \quad 2T + N_1 - 750 = 75(2)$

$\Rightarrow \quad N_1 = 300\ \text{N}$

Reading $= \dfrac{N_1}{g} = 30\ \text{kg}$

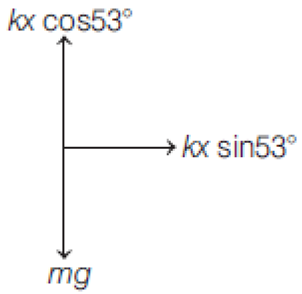
25. (C)

Before burning the string,

$$\sum F_x = 0 \Rightarrow T = kx\sin 53^\circ$$

$$\sum F_y = 0 \Rightarrow kx\cos 53^\circ = mg$$

$$\Rightarrow \quad kx = \frac{5mg}{3}$$



After burning the string,

$$\sum F_x = ma_x$$

$$\Rightarrow \quad kx\sin 53^\circ = ma_x$$

$$\Rightarrow \quad a_x = \frac{4g}{3}$$

$$\sum F_y = ma_y$$

$$\Rightarrow \quad mg - kx\cos 53^\circ = ma_y$$

$$\Rightarrow \quad a_y = 0$$

26. (C)

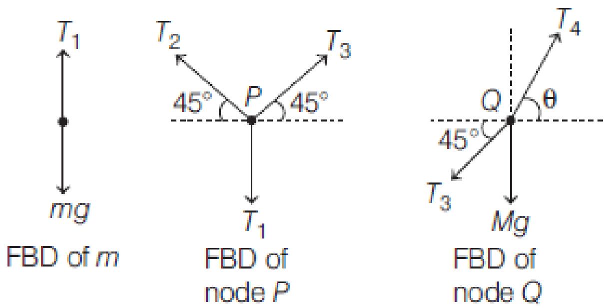
$$T = \frac{2m_1 m_2 g_{\text{eff}}}{m_1 + m_2} = \frac{2(3)(1.5)}{4.5}\left(g + \frac{g}{10}\right)$$

$$= \frac{11g}{5} = 22\,\text{N}$$

$$kx = 2T = 2(22) = 44\,\text{N}$$

Reading of spring balance $= \frac{44}{g} = 4.4\,\text{kg}$

27. (A)



Fro m, $\sum F_y = 0 \Rightarrow T_1 - mg = 0$

$$\Rightarrow \quad T_1 = mg$$

For node P, $\sum F_x = 0$

$$\Rightarrow \quad T_3\cos 45^\circ - T_2\cos 45^\circ = 0$$

$\Rightarrow \quad T_3 = T_2$

$\sum F_y = 0$

$\Rightarrow \quad T_2 \sin 45° + T_3 \sin 45° = T_1$

$\Rightarrow \quad 2T_3 \sin 45° = T_1$

$\Rightarrow \quad T_3 \sin 45° = \frac{T_1}{2} = \frac{mg}{2}$

For node Q,

$\sum F_y = 0$

$\Rightarrow \quad T_4 \sin\theta = Mg + T_3 \sin 45° \qquad ...(i)$

$\sum F_x = 0$

$\Rightarrow \quad T_4 \cos\theta = T_3 \cos 45° \qquad ...(ii)$

Dividing Eq. (i) by Eq. (ii), we get

$$\Rightarrow \quad \tan\theta = \frac{M + \frac{m}{2}}{\frac{m}{2}} = 1 + \frac{2M}{m}$$

28. (10)

Force on particle at 20 cm away $F = kx$

$F = 15 \times 0.2 = 3\ N \qquad [\text{As } k = 15\ N/m]$

$\therefore$ Acceleration = $\frac{\text{Force}}{\text{Mass}} = \frac{3}{0.3} = 10\ m/s^2$

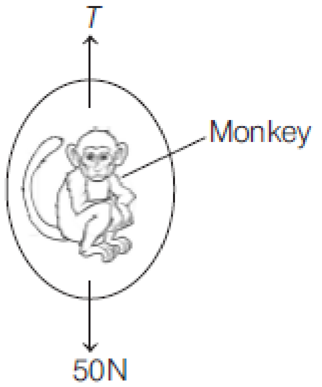
29. (2.5)

Tension the string $= m(g+a) =$ Breaking force

$\Rightarrow 20(g+a) = 25 \times g \Rightarrow a = g/4 = 2.5\ m/s^2$

30. (6)

For clamp, $T \sin 30° = 40 \Rightarrow T = 80\,\text{N}$



For monkey, $T - 50 = 5a$

$a = 6\ \text{m/s}^2$

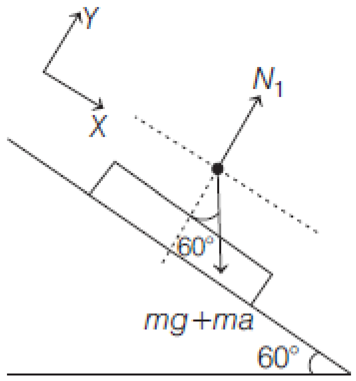
31. (322)

For (A + B + C) system, $\sum F = \sum ma$

$T_D - 100 - 150 - 80 = [10(-2) + 15(0) + 8(1.5)]$

$T_D = 322\,\text{N}$

32. (40)



$\sum F_y = 0$

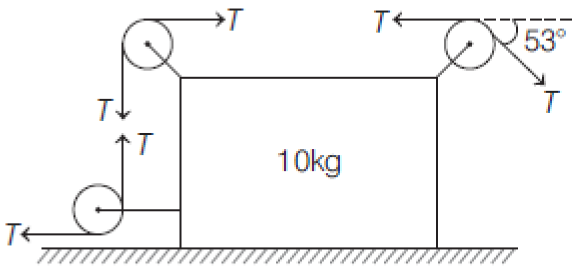
$\Rightarrow \quad N_1 = (mg + ma)\cos 60°$

$\Rightarrow \quad N_1 = 50 \times 16 \times \frac{1}{2}$

$N_1 = 400\,\text{N}$

Reading of weighing machine $= \frac{N_1}{g} = 40$ kg

33. (4)



$T = 100$ N

$\sum F_x = ma_x$

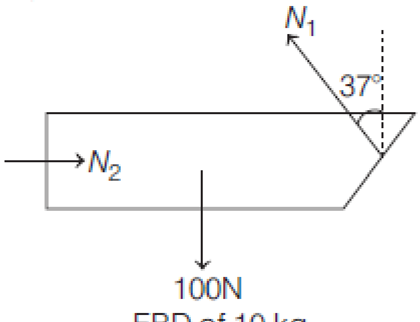
$\Rightarrow \quad T - T\cos 53° = 10a$

$100 - 100\left(\frac{3}{5}\right) = 10a$

$\Rightarrow \quad a = 4\ \text{m/s}^2$ left

34. (75)

$\sum F_y = 0$

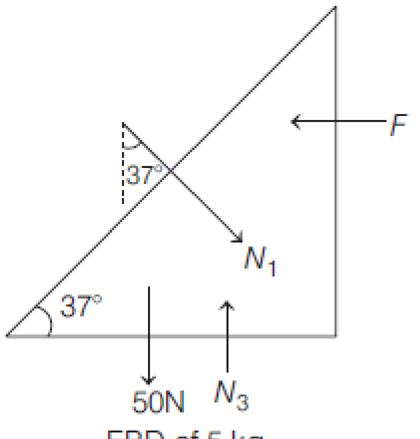


FBD of 10 kg

$\Rightarrow \quad N_1 \cos 37° = 100$

$\Rightarrow \quad N_1 = \frac{500}{4} = 125\,\text{N}$

$\sum F_x = 0$



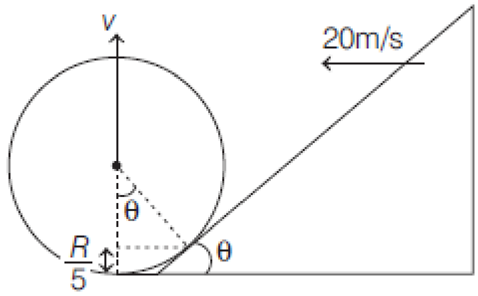
$F = N_1 \sin 37°$

$\Rightarrow \quad F = 125\left(\frac{3}{5}\right)$

$\Rightarrow \quad F = 75\,\text{N}$

35. (15)

$\cos\theta = \frac{4R/5}{R} = \frac{4}{5} \Rightarrow \theta = 37°$

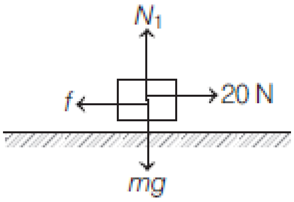


From wedge constraint, $v\cos\theta = 20\sin\theta$

$\Rightarrow \quad v = 20\tan\theta = 20\left(\frac{3}{4}\right) = 15\,\text{m/s}$

Friction

1. (A)



$$\sum F_y = 0$$

$\Rightarrow$ $N_1 - 40 = 0$

$\Rightarrow$ $N_1 = 40\,\text{N}$

Since, the block is not moving.

$$\sum F_x = 0$$

$\Rightarrow$ $20 - f = 0$

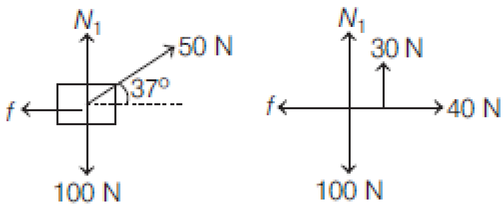
$\Rightarrow$ $f = 20\,\text{N}$ (static)

$$f \le f_L$$

$\Rightarrow$ $20 \le \mu_s (N_1)$

$\Rightarrow$ $\mu_S \ge 0.5$

2. (A)



$$\sum F_y = 0$$

$\Rightarrow$ $N_1 + 50\sin 30^\circ - 100 = 0$

$\Rightarrow$ $N_1 = 70\,\text{N}$

$$f_L = \mu_S N$$

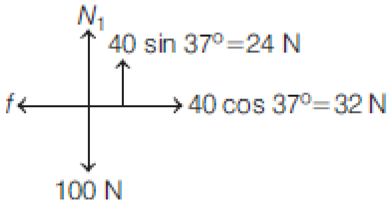
$$= 0.5(70) = 35\,\text{N}$$

$$\sum F_x = ma_x$$

$\Rightarrow$ $40 - 35 = 10a_x$

$\Rightarrow$ $a_x = 0.5\,\text{m/s}^2$

3. (B)



$\sum F_y = 0$

$\Rightarrow \quad +N_1 + 24 - 100 = 0$

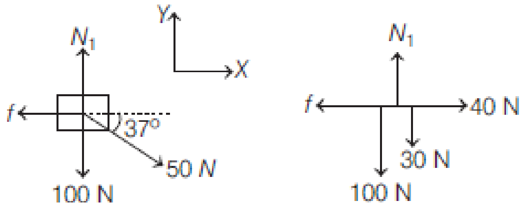
$\Rightarrow \quad N_1 = 76\,\text{N}$

$f_L = \mu_S N = 0.5(76) = 38\,\text{N}$

Since, driving force is less than the limiting friction, the block will not move.
So, $a = 0$.

4. (C)

$\sum F_y = 0$



$\Rightarrow \quad N_1 - 100 - 30 = 0$

$\Rightarrow \quad N_1 = 130\,\text{N}$

$f_L = \mu_S N_1 3$

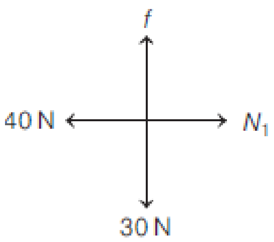
$\Rightarrow \quad f_L = 0.3(130) = 39\,\text{N}$

$\sum F_x = ma_x$

$\Rightarrow \quad 40 - 39 = 10a_x$

$\Rightarrow \quad a_x = 0.1\ \text{m/s}^2$

5. (C)



$\sum F_x = 0$

$\Rightarrow \quad N_1 - 40 = 0$

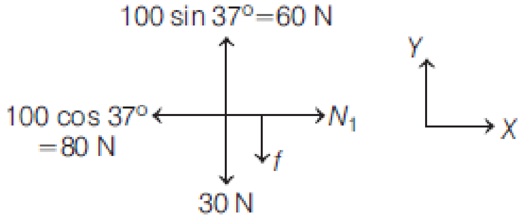
$\Rightarrow \quad N_1 = 40\text{N}$

$f_L = \mu_S N$

$\Rightarrow \quad f_L = 0.8(40) = 32\,\text{N}$

Since, $f_L \geq 30\,\text{N}$, the block won't move.

6. (A)



$$\sum F_x = 0$$

$\Rightarrow$ $+N_1 - 80 = 0$

$\Rightarrow$ $N_1 = 80N$

$$f_L = \mu_S N$$

$$f_L = 0.25(80) = 20\text{ N}$$

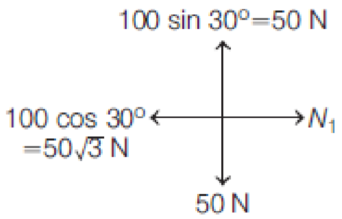
In absence of friction, driving force on the block is 60 – 30 = 30 N up the wall which is more than the limiting friction. So, block will accelerate up the wall.

$$\sum F_y = ma_y$$

$\Rightarrow$ $60 - 30 - 20 = 3a_y$

$\Rightarrow$ $a_y = \dfrac{10}{3}\text{ m/s}^2$

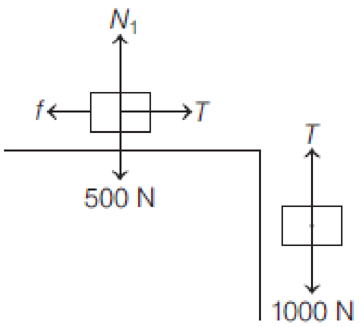
7. (A)



Since, driving force is zero in absence of friction, no friction will act

$\Rightarrow$ $f = 0$ and $a = 0$

8. (A)



For 100 kg block,

$$\sum F_y = 0$$

$\Rightarrow$ $T - 1000 = 0$

$\Rightarrow$ $T = 1000\text{ N}$

For 50 kg block,

$$\sum F_y = 0$$

$$N_1 - 500 = 0$$

$\Rightarrow \quad N_1 = 500\text{N}$

$\sum F_x = 0$

$\Rightarrow \quad T - f = 0$

$\Rightarrow \quad f = T = 1000\text{ N}$

$f_L = \mu N_1 = \mu(500)$

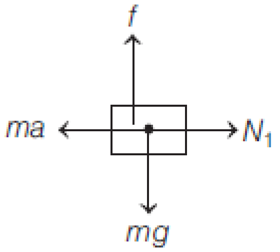
$f \le f_L$

$1000 \le 500(\mu)$

$\Rightarrow \quad \mu \ge 2$

9. (B)

Lets take acceleration of cart as a



$\sum F_x = 0 \Rightarrow N_1 = ma$

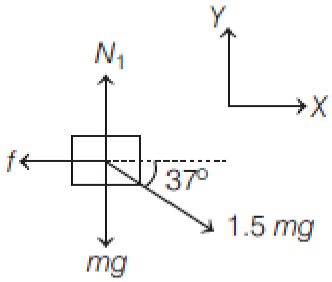
$\sum F_y = 0 \Rightarrow f = mg$

and $f \le f_L$

$\Rightarrow \quad mg \le \mu N_1 \Rightarrow a \ge \dfrac{g}{\mu}$

$\Rightarrow \quad a_{\min} = \dfrac{g}{\mu}$

10. (A)



$\sum F_y = 0$

$\Rightarrow \quad +N_1 - mg - 1.5mg\sin 37^\circ = 0$

$\Rightarrow \quad N_1 = 1.9\,mg$

$\sum F_x = ma_x$

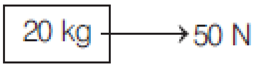
$\Rightarrow \quad 1.5mg\cos 37^\circ - \mu N_1 = m(0)$

$\Rightarrow \quad \mu = \dfrac{1.5mg\cos 37^\circ}{1.9\,mg}$

$\Rightarrow \quad \mu = \dfrac{12}{19}$

11. (A)

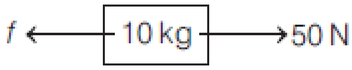
Taking both the block as a system,



$$\sum F = ma$$

$\Rightarrow$ $50 = 20a$

$\Rightarrow$ $a = 2.5\ \text{m/s}^2$



For upper 10 kg block,

$$50 - f = 10(2.5)$$

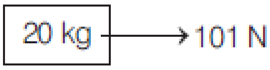
$\Rightarrow$ $f = 25\ \text{N}$

& $f_L = 0.5(100) = 50\ \text{N}$

Since, $f < f_L$

Friction between both the blocks is static & common acceleration is 2.5 m/s^2.

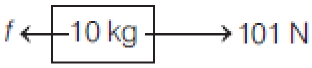
12. (B)



$$\sum F = ma$$

$\Rightarrow$ $101 = 20a$

$\Rightarrow$ $a = 5.05\ \text{m/s}^2$



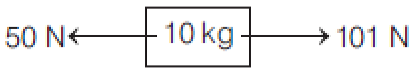
$$\sum F = ma$$

$\Rightarrow$ $101 - f = 10(5.05)$

$\Rightarrow$ $f = 50.5\ \text{N}$

Since $f > f_L$; out assumption is wrong.

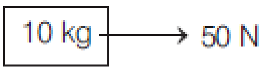
Friction between the blocks will be kinetic.



$$\sum F = ma$$

$\Rightarrow$ $101 - 50 = 10a$

$\Rightarrow$ $a = 5.1\ \text{m/s}^2$



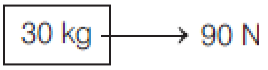
$$\sum F = ma$$

$\Rightarrow$ $50 = 10a$

$\Rightarrow$ $a = 5\ \text{m/s}^2$

13. (A)

For system,

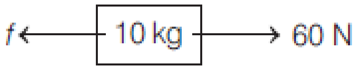


$\sum F = ma$

$\Rightarrow \quad 90 = 30a$

$\Rightarrow \quad a = 3\,\text{m/s}^2$

For 10 kg block,



$\sum F = ma$

$\Rightarrow \quad 60 - f = 10(3)$

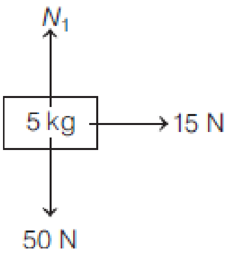
$f = 30\,\text{N}$

& $\quad f_L = 00.5(100) = 50\,\text{N}$

Since, $f < f_L$; out assumption is correct.

Both the blocks will move with common acceleraiton of 3 m/s^2.

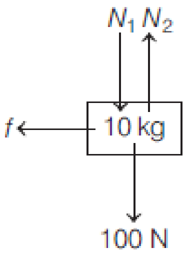
14. (B)



$\sum F = ma$

$\Rightarrow \quad 15 = 5a$

$\Rightarrow \quad a = 3\,\text{m/s}^2$

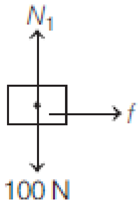


$a = 0$

$f = 0$

15. (C)

FBD of 10 kg block,



$\sum F_y = 0 \Rightarrow N_1 = 100\,\text{N}$

$\sum F_x = ma$

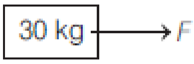
$\Rightarrow \quad f = 10a$

For maximum acceleration,

$10a = f_L = \mu(100)$

$\Rightarrow \quad a_{max} = 5\,\text{m/s}^2$

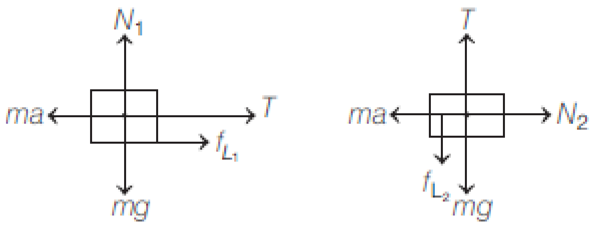
For (20 kg + 10 kg) system,



$\sum F_x = ma$

$\Rightarrow \quad F = 30(5) = 150\,\text{N}$

16. (C)



For keeping smaller blocks at rest w.r.t. the bigger block

$ma = mg + f_{L_2} + f_{L_1}$

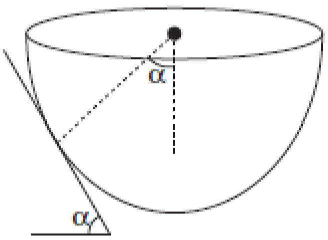
$\Rightarrow \quad ma = mg + \mu N_2 + \mu N_1$

$\Rightarrow \quad ma = mg + \mu(ma) + \mu(mg)$

$\Rightarrow \quad a = \left(\frac{1+\mu}{1-\mu}\right)g$

So, $F = (M + 2m)\left(\frac{1+\mu}{1-\mu}\right)g$

17. (A)



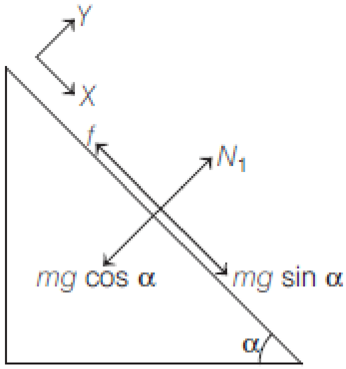
$\sum F_y = 0$

$\Rightarrow \quad N_1 - mg\cos\alpha = 0$

$\Rightarrow \quad N_1 = mg\cos\alpha$

$\sum F_x = 0$

$\Rightarrow \quad mg\sin\alpha = f$



& $\quad f \le f_L$

$\Rightarrow \quad mg\sin\alpha \le \mu mg\cos\alpha$

$\Rightarrow \quad \tan\alpha \le \dfrac{1}{3}$

$\Rightarrow \quad \tan\alpha_{max} = \dfrac{1}{3}$

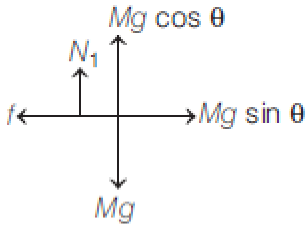
$\Rightarrow \quad \cot\alpha_{max} = 3$

18. (C)

$\sum F_y = 0$

$\Rightarrow \quad N_1 + Mg\cos\theta - Mg = 0$

$\Rightarrow \quad N_1 = Mg(1-\cos\theta)$



For pulling the block,

$Mg\sin\theta \ge f_L$

$\Rightarrow \quad Mg\sin\theta \ge \mu N_1$

$\Rightarrow \quad Mg\sin\theta \ge \mu Mg(1-\cos\theta)$

$\Rightarrow \quad \dfrac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right)} \ge \mu$

$\Rightarrow \quad \cot\left(\dfrac{\theta}{2}\right) \ge \mu$

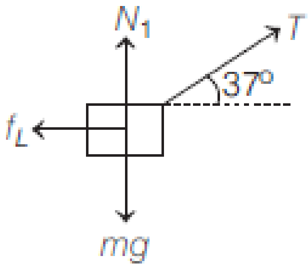
19. (B)

For block *B*,

$\sum F_y = 0$

$\Rightarrow \quad +N_1 + T\sin 37^o = mg$

$\Rightarrow \quad N_1 = mg - T\sin 37^o$



$$\sum F_x = 0$$

$$\Rightarrow \quad T\cos 37^o = f_L$$

$$\Rightarrow \quad \frac{4T}{5} = \mu\left(mg - \frac{3T}{5}\right)$$

$$\Rightarrow \quad \frac{12T}{5} = 100g - \frac{3T}{5}$$

$$\Rightarrow \quad T = \frac{100g}{3}$$

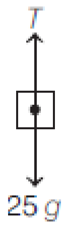
For A,

$$\sum F = ma$$

$$\Rightarrow \quad T - 25g = 25a$$

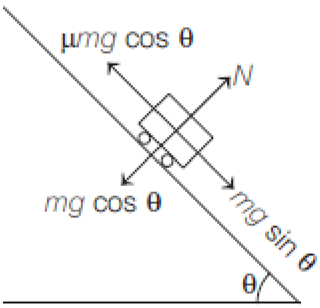
$$\Rightarrow \quad \frac{100g}{3} - 25g = 25a$$

$$\Rightarrow \quad a = \frac{g}{3}$$



20. (C)

$$a = g\sin\theta - \mu g\cos\theta$$



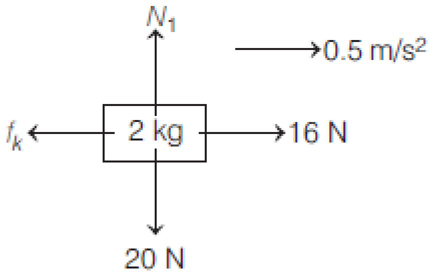
$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad v^2 = (20)^2 + 2\times 10\left[\left(\frac{3}{5}\right) - \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\right](21)$$

$$v = \sqrt{400+84} = \sqrt{484}\ \text{m/s} = 22\ \text{m/s}$$

21. (A)

For 2 kg block,



$$\sum F_y = 0 \Rightarrow N_1 = 20\,\text{N}$$

$$f_k = \mu N_1 = 20\mu_1$$

$$\sum F = ma$$

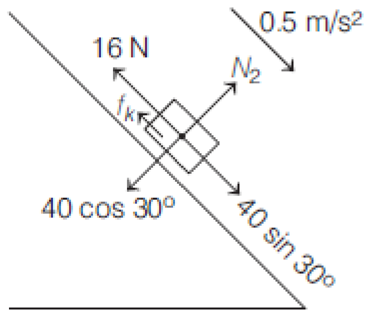
$$\Rightarrow 16 - 20\mu_1 = 2(0.5)$$

$$\Rightarrow \mu_1 = \frac{3}{4}$$

For 4 kg block,

$$\sum F = ma$$

$$\Rightarrow 40\sin 30^\circ - 16 - f_k = 4(0.5)$$



$$\Rightarrow f_k = 2\,\text{N}$$

$$f_k = \mu_k N$$

$$\Rightarrow 2 = \mu_k\left(40\cos 30^\circ\right)$$

$$\Rightarrow \mu_k = \frac{1}{10\sqrt{3}} = 0.0577 = 0.06$$

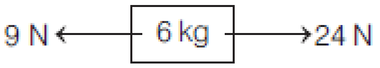
22. (B)

For 6 kg block,

$$\sum F = ma$$

$$\Rightarrow 24 - 9 = 6a$$

$$\Rightarrow a = 2.5\,\text{m/s}^2$$

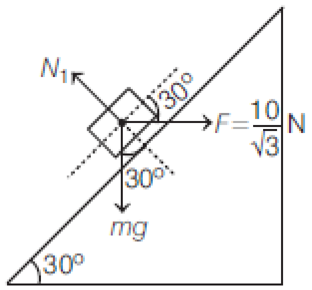


For 3 kg block,

$$\Rightarrow F - 24 = 3(2.5)$$

$$\Rightarrow F = 31.5\,\text{N}$$

23. (A)

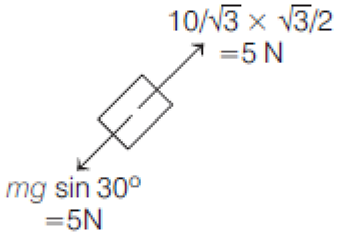


$\Rightarrow \quad +N_1 - mg\cos 30^\circ - F\sin 30^\circ = 0$

$\Rightarrow \quad N_1 = mg\cos 30^\circ + F\sin 30^\circ$

$\Rightarrow \quad N_1 = 5\sqrt{3} + \frac{5}{\sqrt{3}} = \frac{20}{\sqrt{3}}\text{ N}$

$f_L = \mu N_1 = 0.5\left(\frac{20}{\sqrt{3}}\right) = \frac{10}{\sqrt{3}}\text{ N}$



Along the inclined plane, in absence of friction, net force is zero.

So, $f = 0$

24. (0.75)

$F_{\min} = \frac{\mu mg}{\sqrt{\mu^2+1}} = \frac{3mg}{5}$

$\Rightarrow \quad 25\mu^2 = 9\mu^2 + 9$

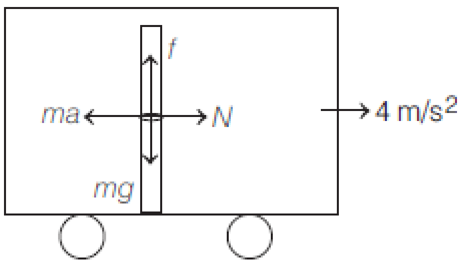
$\Rightarrow \quad 16\mu^2 = 9$

$\Rightarrow \quad \mu = \frac{3}{4} = 0.75$

25. (0.5)

w.r.t. train $\sum F_x = 0$

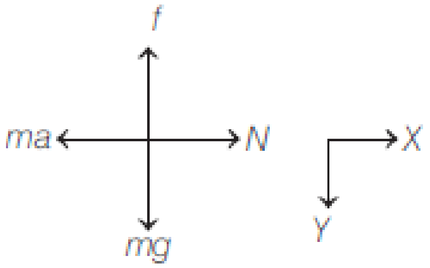
$N = ma$



$\sum F_y = ma_y$

$\Rightarrow \quad mg - \mu N = ma_y$

$\Rightarrow \quad mg - \mu ma = ma_y$



$\Rightarrow \quad a_y = g - \mu a$

$\Rightarrow \quad a_y = 10 - \frac{1}{2}(4) = 8 \text{ m/s}^2$

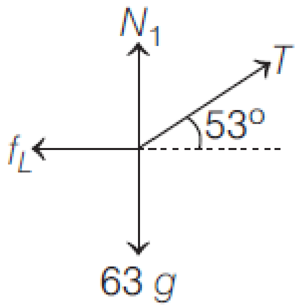
$s_y = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow \quad 1 = 0 + \frac{1}{2}(8) t^2$

$\Rightarrow \quad t = 0.5 \text{ s}$

26. (35)

For man,



$\sum F_y = 0$

$\Rightarrow \quad N_1 + T \sin 53^\circ = 63g$

$\Rightarrow \quad N_1 = 63g - \frac{4T}{5}$

$\sum F_x = 0$

$\Rightarrow \quad T \cos 53^\circ = f_L$

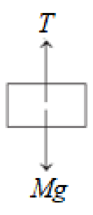
$\Rightarrow \quad \frac{3T}{5} = \mu\left(63g - \frac{4T}{5}\right)$

$\Rightarrow \quad \frac{3T}{5} = \frac{3}{5}\left(63g - \frac{4T}{5}\right)$

$\Rightarrow \quad \frac{9T}{5} = 63g$

$\Rightarrow \quad T = 35g$

For block,



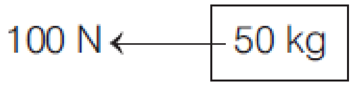
$\sum F_y = Ma_y$

$\Rightarrow \quad T - Mg = 0$

$\Rightarrow \quad M = 35 \text{ kg}$

27. (0.98)

Taking ($A + B$) system,

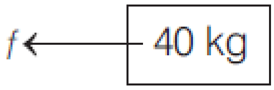


$$\sum F = ma$$

$\Rightarrow \quad 100 = 50a$

$\Rightarrow \quad a = 2 \text{ m/s}^2$

For B,



$$\sum F = ma$$

$\Rightarrow \quad f = 40 \times 2 = 80 \text{ N}$

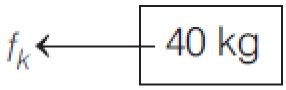
$$f_L = \mu_s N$$

$$= 0.75 \times 98 = 73.5 \text{ N}$$

Since, $f > f_L$, out assumption is wrong.

Friction between the blocks will be kinetic.

For B,



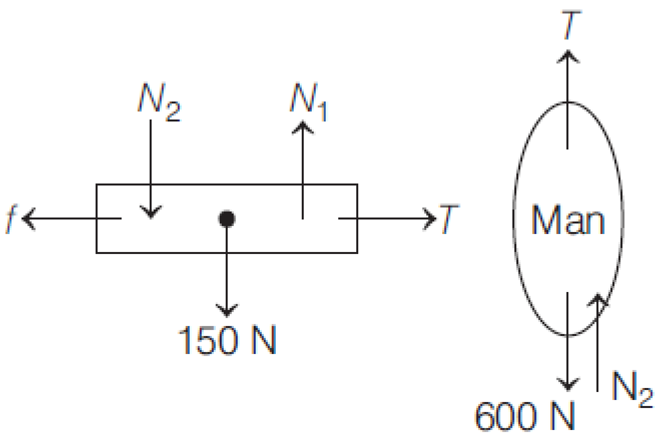
$$\sum F = ma$$

$\Rightarrow \quad f_k = 40a$

$$0.4 \times 10 \times 9.8 = 40a$$

$\Rightarrow \quad a = 0.98 \text{ m/s}^2$

28. (250)



For board,

$$\sum F_x = ma_x$$

$\Rightarrow \quad T - f_L = 0$

$\Rightarrow \quad T = f_L \Rightarrow T = \mu N_1$

$\Rightarrow \quad T = 0.5(N_2 + 150) \qquad \text{...(i)}$

For man,

$$\sum F_y = 0$$

$\Rightarrow \quad T + N_2 - 600 = 0$

$\Rightarrow \quad N_2 = 600 - T \qquad \ldots(\text{ii})$

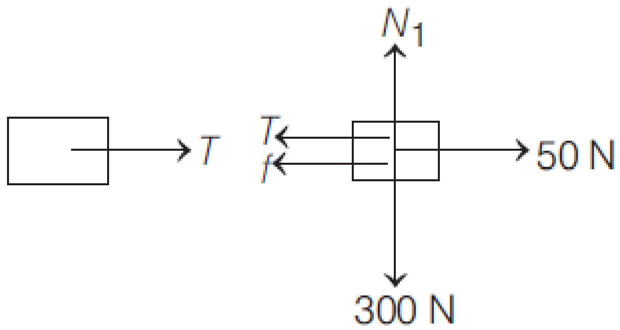
From Eqs. (i) & (ii), we get

$$T = 0.5(600 - T + 150)$$

$\Rightarrow \quad 3T = 750$

$\Rightarrow \quad T = 250\,\text{N}$

29. (0)

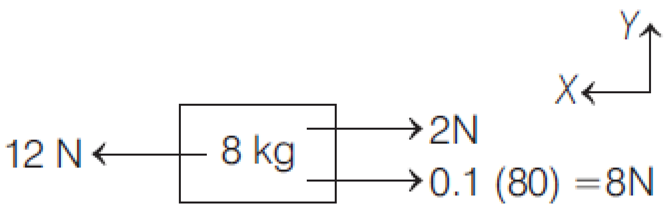


For block *A*,

$$f_L = \mu N = 0.2(300) = 60\,\text{N}$$

Since applied force is less than $f_L, T = 0$ and hence friction on block $B = 0$.

30. (3)



For (4 kg + 4 kg) system,

$$\sum F = ma$$

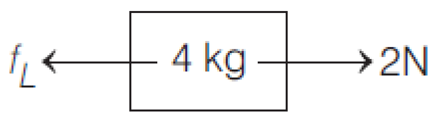
$\Rightarrow \quad 12 - 2 = 8 = 8a$

$\Rightarrow \quad a = 0.25 \text{ m/s}^2$

For upper 4kg block,

$$\sum F = ma$$

$\Rightarrow \quad f_L - 2 = 4(0.25)$



$\Rightarrow \quad f_L = 3N$

and $\quad f_L = 3\,\text{N} = \mu(40) \Rightarrow \mu = \dfrac{3}{40}$

31. **(a)** 0 m/s^2, 0 N **(b)** 0 m/s^2, $f_s = 25\,\text{N}$ **(c)** 0 m/s^2, $f_L = 30$ N **(d)** 1.2 m/s^2, $f_k = 25$ N

$f_L = \mu_s N$

$\Rightarrow \; f_L = 0.6(50) = 30\,\text{N}$

$f_k = \mu_k N$

$\Rightarrow \; f_k = 0.5(50) = 25\,\text{N}$

(a) Since, no force is applied, frictional force will be zero.

(b) Since, applied force is less than limiting friction, block won't move and static friction will be equal to the force applied, i.e. 25 N.

(c) Since, $F = f_L$, the block is on verge of motion. So, a = 0 and f_L = 30 N.

(d) Since, $F > f_L$, the block will be set into motion and nature of friction will be kinetic.

$F - f_k = ma$

$\Rightarrow$ 31 – 25 = 5a

$\Rightarrow \; a = 1.2$ m/s^2

SECTION – I

1. (b)

Impulse, $I =$ change in momentum, ΔP

$$F_{\text{avg}} = \frac{\Delta P}{\Delta t} \qquad \because \ \Delta P_1 = \Delta P_2 \quad \therefore \ I_1 = I_2$$

Given $\Delta t_1 = 3\text{s}$ and $\Delta t_2 = 5\text{s}$

Hence, F_{avg} in case (i), when $\Delta t_2 = 3\text{s}$ is more than (ii) when $\Delta t_2 = 5\text{s}$

2. (b)

Force, $F = \frac{dm}{dt}v = \frac{10}{5} \times 4.5 = 9$ dyne

3. (b)

Initial momentum $\vec{P}_i = 0.15 \times 12\left(\hat{i}\right)$

Final momentum $\vec{P}_f = 0.15 \times 12\left(-\hat{i}\right)$

$\left|\Delta\vec{P}\right| = 3.6$ kg m/s or $3.6 = F\Delta t$

$3.6 = 100\,\Delta t \qquad \therefore \ \Delta t = 0.036\,\text{sec}$

4. (c)

$$F = \frac{dp}{dt} = v\frac{dm}{dt} = 10 \times 1 = 10\,\text{N}$$

$$a = \frac{F}{m} = \frac{10}{2} = 5\ \text{m/s}^2$$

5. (c)

$$a = \frac{vdv}{dx};\ \ adx = vdx;\ \ \int_{0.5}^{1.5} -\frac{kx}{m}dx = \int_{4}^{v} v\,dv$$

$$\Rightarrow -\frac{k}{2m}\left[1.5^2 - 0.5^2\right] = \frac{v^2 - 4^2}{2} \ \Rightarrow -\frac{12}{2\times 2}\left[2\right] = \frac{v^2 - 16}{2}$$

$$\Rightarrow -3 \times 4 = v^2 - 16$$

$$\Rightarrow v^2 = 4 \ \Rightarrow \ v = 2\ \text{m/s}$$

6. (d)

Thrust force on rocket is given by

$$F_{\text{thrust}} = \left(V_{\text{rel}} \cdot \frac{dm}{dt}\right)$$

$$\Rightarrow \left(V_{\text{rel}} \cdot \frac{dm}{dt} - mg\right) = ma$$

$$\Rightarrow 500\left(\frac{dm}{dt}\right) - 10^3 \times 10 = 10^3 \times 20$$

$$\Rightarrow \frac{dm}{dt} = 60 \text{ kg/s}$$

7. (c)

From the Newton's second law of motion,

$$F = ma$$

$$\Rightarrow a = \frac{F}{M} \Rightarrow a = \frac{F_0}{M}\left[1 - \left(\frac{T-t}{T}\right)^2\right]$$

$$\Rightarrow \int_0^v dv = \frac{F_0}{M}\int_0^{2T}\left[1 - \left(\frac{T-t}{T}\right)^2\right]dt$$

$$\Rightarrow V = \frac{F_0}{M}\left[t + \frac{1}{3T^2}(T-t)\right]_0^{2T}$$

$$\Rightarrow V = \frac{F_0}{M}\left\{\left[2T + \frac{1}{3T^2}(T-2T)^3\right] - \left[0 + \frac{T^3}{3T^2}\right]\right\}$$

$$\Rightarrow V = \frac{F_0}{M}\left[\frac{4T}{3}\right]$$

8. (b)

From the Newton's second law,

$$F = \frac{dp}{dt}\frac{d(mv)}{dt} = v\left(\frac{dm}{dt}\right) + \frac{mdv}{dt} \qquad \ldots \text{(i)}$$

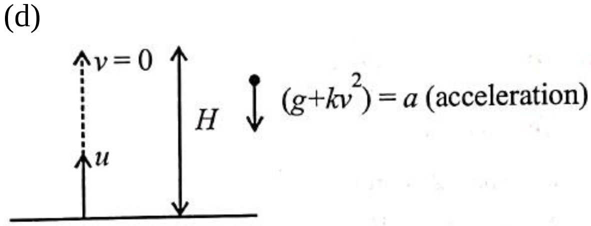
We have given, $\frac{dM(t)}{dt} = bv^2(t)$ … (ii) and $F_{\ell xt} = 0$

Thrust on the satellite,

$$= -v\left(\frac{dm}{dt}\right) = -v\left(bv^2\right) = -bv^3 \text{ [Using (i) and (ii)]}$$

$$= M(t)a = -bv^3 \Rightarrow a = \frac{-bv^3}{M(t)}$$

9. (d)



$\vec{F} = mkv^2 + mg$ ($\because mg$ and mkv^2 act opposite to motion)

$$\vec{a} = \frac{\vec{F}}{-m} = -\left[kv^2 + g\right]$$

$$\Rightarrow v \cdot \frac{dv}{dh} = -\left[kv^2 + g\right] \qquad \left(\because a = v\frac{dv}{dh}\right)$$

$$\Rightarrow \int_u^0 \frac{v \cdot dv}{kv^2 + g} = \int_0^h dh \Rightarrow \frac{1}{2k} \ln\left[kv^2 + g\right]_u^0 = -h$$

$$\Rightarrow \frac{1}{2k} \ln\left[\frac{ku^2 + g}{g}\right] = h$$

10. (a)

Net acceleration

$$\frac{dsv}{dt} = a = -\left(g + \gamma v^2\right)$$

Let time t required to rise to its zenith ($v = 0$) so,

$$\int_{v_0}^0 \frac{-dv}{g + rv^2} = \int_0^t dt \qquad [\text{for } H_{\max}, v = 0]$$

$$\therefore \ t = \frac{1}{\sqrt{\gamma g}} \tan^{-1}\left(\frac{\sqrt{\gamma}\, v_0}{\sqrt{g}}\right)$$

11. (d)

$v^2 = u^2 - 2gh$ or $v = \sqrt{u^2 - 2gh}$

Momentum, $P = mv = m\sqrt{u^2 - 2gh}$

At $h = 0, P = mu$ and at $h = \frac{u^2}{g}$, $P = 0$

Upward direction is positive and downward direction is negative.

12. (b)

From Newton's second law

$$\frac{dp}{dt} = F = kt$$

Integrating both sides we get,

$$\int_p^{3p} dp = \int_0^T kt\, dt \Rightarrow \left[p\right]_p^{3p} = k\left[\frac{t^2}{2}\right]_0^T$$

$$\Rightarrow 2p = \frac{kT^2}{2} \Rightarrow T = 2\sqrt{\frac{p}{k}}$$

13. (a)

From $F = \frac{R}{t^2} v(t) \Rightarrow m\frac{dv}{dt} = \frac{R}{t^2} v(t)$

Integrating both sides $\int \frac{dv}{v} = \int \frac{\text{P}dt}{mt^2}$

In $v \propto \frac{1}{t}$

14. (a)

From questiuon, mass of body, $m = 5\,\text{kg}$

Velocity at $t = 0$,

$u = \left(6i - 2\hat{j}\right)\text{m/s}$

Velocity at $t = 10\,\text{s}$,

$v = +6\ \hat{j}\ \text{m/s}$

Force, $F = ?$

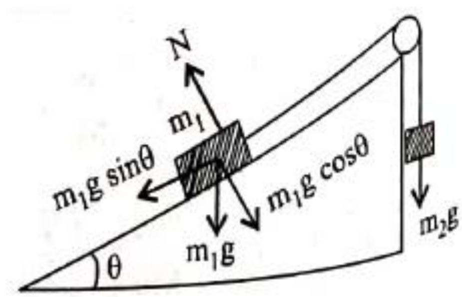
Acceleraiton, $a = \frac{v-u}{t} = \frac{6\hat{j} - \left(6\hat{i} - 2\hat{j}\right)}{10} = \frac{3\hat{i} + 4\hat{j}}{5}\ \text{m/s}^2$

Force, $F = ma$

$= 5 \times \frac{\left(-3\hat{i} + 4\hat{j}\right)}{5} = \left(-3\hat{i} + 4\hat{j}\right)\text{N}$

15. (c)

For equilibrium conduction, $m_2 g = m_1 g \sin\theta$



$\sin\theta = \frac{m_2}{m_1} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

Normal force (N) on $m_1 = 5g\cos\theta$

$= 5 \times 10 \times \frac{4}{5} = 40\,\text{N}$,

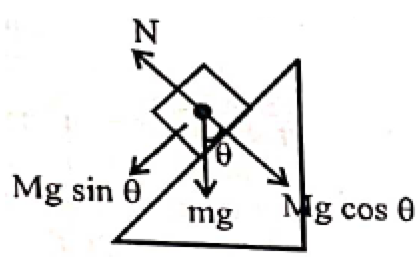
Friction $(f) = m_2 g = 30\,\text{N}$, $F = \sqrt{N^2 + f^2} = 50\,\text{N}$

16. (c)

Acceleration of block on smooth inclined plane,

$a = g\sin\theta$

Using, $s = ut + \frac{1}{2}at^2$



$s = \frac{1}{2} g \sin 30° (2)^2$

When the incline is changed to 45°

$s = \frac{1}{2} g \sin 45° t^2$

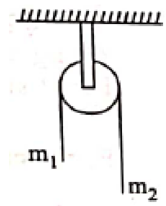
As distance travelled is same

$\therefore\ \left(\frac{1}{2}\right)(4) = \frac{1}{\sqrt{2}} t^2$

$\Rightarrow\ t = 2\sqrt{2} \simeq 1.68$

17. (d)

Acceleration in such system is given as



$$a = \frac{(m_2 - m_1)}{(m_2 + m_1)} g$$

$$\Rightarrow \frac{g}{2} = \frac{(\lambda(L-\ell) - \lambda\ell)}{\lambda L} \quad \Rightarrow \ell = \frac{L}{4} = \frac{L}{x}$$

So, $x = 4$

18. (c)

Given that mass of monkey, m = 50 kg

Acceleration due to gravity, g = 10 m/s^2

Tension (T) = 350 N

Given monkey climbs downward, acceleration of monkey,

a = 4 m/s^2

When monkey climbs upward, acceleration of monkey,

a = 5 m/s^2

(For upward)

$$T = mg = ma$$

$$\Rightarrow T = mg + ma = 50(10+5) = 750\,\text{N}$$

Rope will break while climbing upward

(For downward)

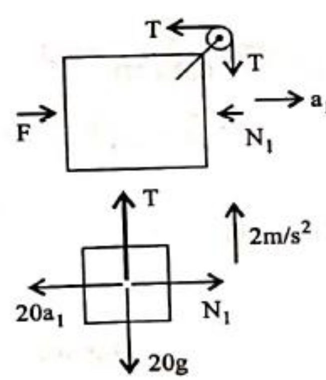
$$T = ma(g-a) = 50(10-4) = 300\,\text{N}$$

Rope will not break while climbing downward

19. (a)

Let a_1 be the acceleration of 100 kg block

FBD of 100 kg block w.r.t. ground



$$F - T - N_1 = 100a_1 \quad ...(i)$$

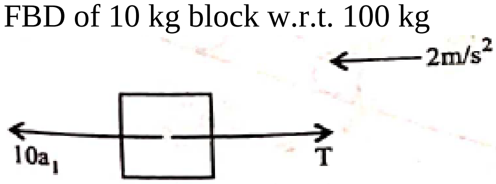
FBD of 20 block w.r.t. 100 kg

$$T - 20g = 20(2) \Rightarrow T = 40 + 200$$

$$\Rightarrow T = 240 \quad ...(ii)$$

$$N_1 = 20a_1 \quad ...(iii)$$

FBD of 10 kg block w.r.t. 100 kg



$$10a_1 - 240 = 10(2)$$

$$\Rightarrow a_1 = 26\ \text{m/s}^2$$

$$F - 240 - 20(26) = 100 \times 26$$

$$\Rightarrow F = 3360\,\text{N}$$

20. (a)

Let addition force required be $=\vec{F}$ -

$$\vec{F}+5i-6\hat{i}+7\hat{j}-8\hat{j}=0$$

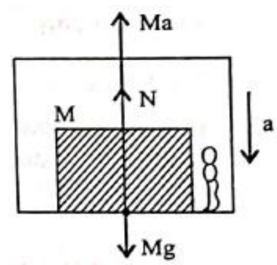
$$\Rightarrow \quad \vec{F}=\hat{i}+\hat{j}, \left|\vec{F}\right|=\sqrt{1^2+1^2}=\sqrt{2}$$

Angle with x-axis: $\tan\theta=\frac{y \text{ component}}{x \text{ component}}=\frac{1}{1}$

So, $\theta=\tan^{-1}(1)=45°$.

21. (c)

For observer in box



$$N+Ma=Mg$$

$$\Rightarrow \quad N=M(g-a)$$

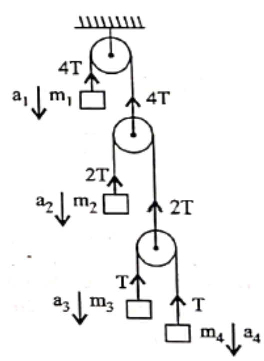
$$\Rightarrow \quad \frac{Mg}{4}=M(g-a)$$

$$\Rightarrow \quad a=\frac{3g}{4}$$

22. (b)

When elevator does downward, then a pseudo force acts on object in upward direction, due to which effective weight of object decreases.

23. (a)



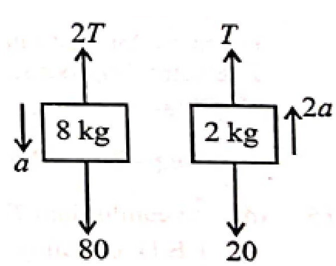
$$\sum \vec{T}\bullet\vec{a}=0$$

$$\Rightarrow -4Ta_1-2Ta_2-Ta_3-Ta_4=0$$

$$\Rightarrow 4a_1+2a_2+a_3+a_4=0$$

24. (d)

From free body diagram,



$$80-2T=8a \qquad \text{...(i)}$$

$$T-20=4a \qquad \text{...(ii)}$$

Multiple equation (ii) by 2 and adding with equation (i) we get

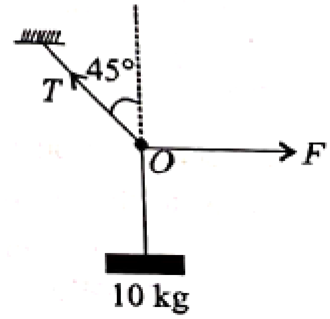
$$(8+8)a=40 \Rightarrow a=\frac{40}{16}=\frac{10}{4} \text{ m/s}^2$$

Using $S=\frac{1}{2}at^2 \Rightarrow t^2=\frac{2S}{a}$

$$\Rightarrow \quad \frac{0.2\times 2\times 4}{10}=t^2 \Rightarrow t=0.4 \sec$$

25. (a)

The free body diagram is by Lami's theorem



$\Rightarrow \quad \dfrac{mg}{135°} = \dfrac{F}{135}$

$\Rightarrow \quad F = mg = 100\,\text{N}$

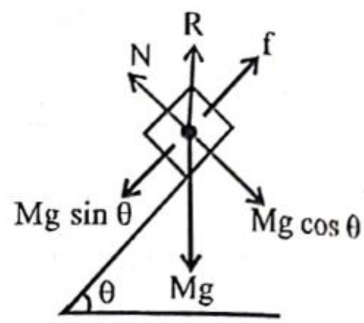
26. (b)

Retardation due to friction $= \dfrac{\mu Mg}{M} = \mu g$

Now, $s = \dfrac{v^2 - u^2}{2a} = \dfrac{0^2 - 2^2}{2g\mu} = \dfrac{2}{0.4 \times 10} = 0.5\,\text{m}$

27. (a)

From the free diagram shown



$N = Mg\cos\theta$

$f = Mg\sin\theta$

Contact force, $R = \sqrt{N^2 - f^2}$

$\Rightarrow \quad R = \sqrt{(Mg\cos\theta)^2 + (Mg\sin\theta)^2}$

$= \sqrt{(Mg)^2(\cos^2\theta + \sin^2\theta)} \Rightarrow R = Mg$

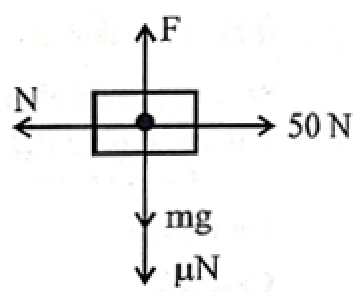
28. (d)

By FBD of block, we have

N = 50 N and $F \le mg + \mu N$

i.e. $F \le 2g + 0.5 \times 50 \le 20 + 25 \le 45\,\text{N}$

So, Maximum force that can be applied is 45 N.



29. (d)

For 4 kg block

$4g - T = 4a \quad \ldots(\text{i})$

For 40 kg block

$T - 40g + 0.02 = 40a \quad [\because f_k = \mu mg]$

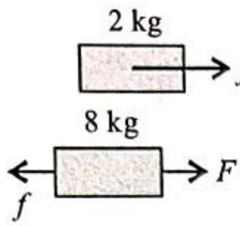
$T - 8 = 40a \quad \ldots(\text{ii})$

Adding (i) & (ii), we get

$40 - 8 = 44a$

$a = \dfrac{32}{44} = \dfrac{8}{11}\,\text{m/s}^2$

30. (c)



$f = 2a$ ...(i)

$F - f = 8a$ ...(ii)

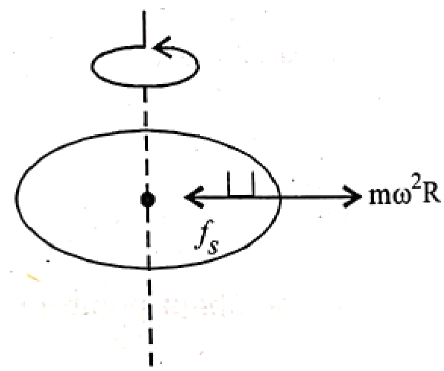
Clearly, from (i) 'a' will be maximum when $f = f_{lim}$

So, $a_{max} = \frac{f_{lim}}{2} = \frac{\mu \times 2g}{2} = \mu g$

From (ii), $F_{max} = 8a_{max} + f_{lim} = 8\mu g + 2\mu g = 10\mu g = 49\,\text{N}$

31. (b)

For beaker to move with disc



$f_s = m\omega^2 R$

So, R will be maximum, when $f_s = f_{lim}$

Therefore, $f_{lim} = m\omega^2 R_{max}$

$\mu_{mg} = m\omega^2 R_{max}$

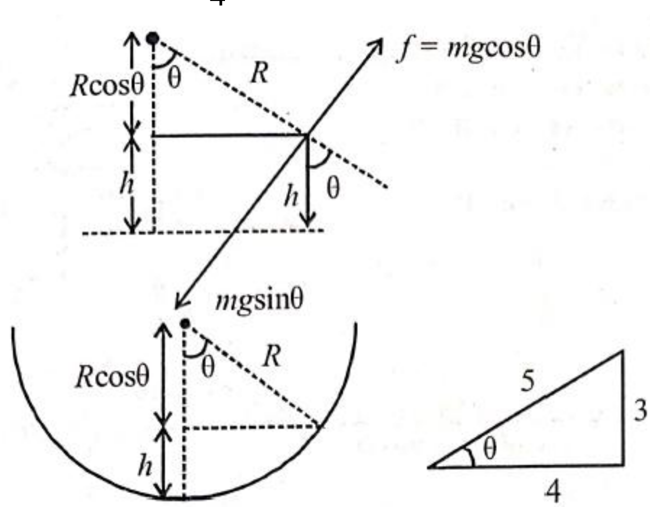
$R_{max} = \frac{\mu g}{\omega^2}$

So, $R \le \frac{\mu g}{\omega^2}$

32. (a)

For balancing, $mg\sin\theta = f = \mu mg\cos\theta$

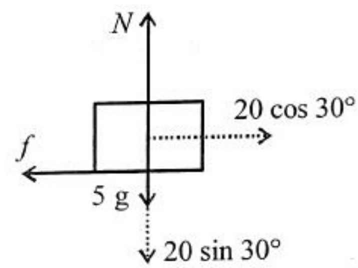
$\Rightarrow \quad \tan\theta = \mu = \frac{3}{4} = 0.75$



$h = R - R\cos\theta = R - R\left(\frac{4}{5}\right) = \frac{R}{5}$

$\therefore \quad h = \frac{R}{5} = 0.2\text{ m} \qquad [\because$ radius, $R = 1$ m$]$

33. (c)



$A:\ N = 5g + 20\sin 30°$

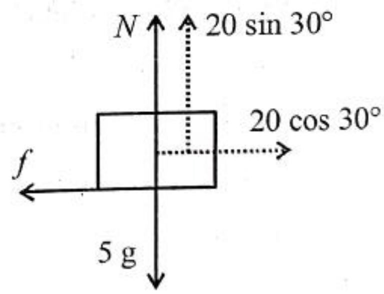
$$= 50 + 20 \times \frac{1}{2} = 60\ \text{N}$$

Acceleration, $a_1 = \frac{F - f}{m}$

$$= \frac{20\cos 30° - \mu N}{5}$$

$$= \left[\frac{20 \times \frac{\sqrt{3}}{2} - 0.2 \times 60}{5}\right]$$

$$= 1.06\ \text{m/s}^2$$



$B:\ N = 5g - 20\sin 30°$

$$= 50 - 20 \times \frac{1}{2} = 40\ \text{N}$$

$$a_2 = \frac{F - f}{m} = \left[\frac{20\cos 30° - 0.2 \times 40}{5}\right] = 1.86\ \text{m/s}^2$$

Now, $a_2 - a_1 = 1.86 - 1.06 = 0.8\ \text{m/s}^2$

34. (b)

Taking (*A* + *B*) as system

$$F - \mu(M + m)g = (M + m)a$$

$$\Rightarrow\ a = \frac{F - \mu(M + m)g}{(M + m)}\ \ \Rightarrow\ \ a = \frac{F - (0.2)4 \times 10}{4} = \left(\frac{F - 8}{4}\right)$$

But, $a_{max} = \mu g = 0.2 \times 10 = 2$

$$\therefore\ \frac{F - 8}{4} = 2\ \Rightarrow\ F = 16\ \text{N}$$

35. (a)

From figure, $2 + mg\sin 30° = \mu mg\cos 30°$ and $10 = mg\sin 30° + \mu mg\cos 30° = 2\mu mg\cos 30° - 2$

$\Rightarrow\ 6 = \mu mg\cos 30°$ and $4 = mg\sin 30°$

By dividing above two $\Rightarrow\ \frac{3}{2} = \mu \times \sqrt{3}$

$\therefore$ Coefficient of friction, $\mu = \frac{\sqrt{3}}{2}$

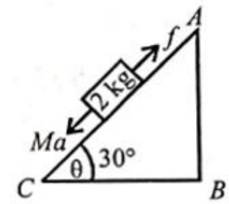
36. (d)

Equation of motion when the mass slides down

$Mg\sin\theta - f = Ma$

$\Rightarrow$ $10 - f = 6$ ($M = 2$ kg, $a = 3$ m/s^2, $\theta = 30°$ given)

$\therefore$ $f = 4\,\text{N}$



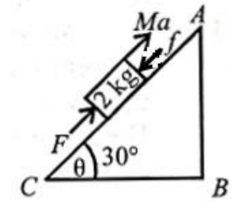
Equation of motion when the block is pushed up

Let the external force required to take the block up the plane with same acceleration be F

$F - Mg\sin\theta - f = Ma$

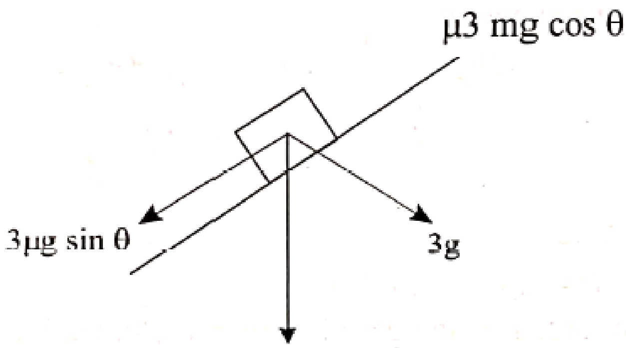
$\Rightarrow$ $F - 10 - 4 = 6$

$F = 20\,\text{N}$



37. (b)

Let μ be the minimum coefficient of friction



At equilibrium, mass does not move so, $3mg\sin\theta = \mu 3mg\cos\theta$

$\therefore$ $\mu_{min} = \tan\theta$

38. (b)

Initial speed at point A, $u = v_0$

Speed at point B, v = ?

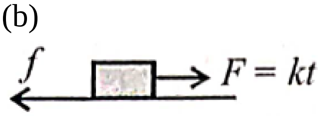
$v^2 - u^2 = 2gh$

$v^2 = v_0^2 + 2gh$

Let ball travels distance 'S' before coming to rest

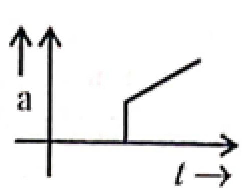
$$S = \frac{v^2}{2\mu g} = \frac{v_0^2 + 2gh}{2\mu g} = \frac{v_0^2}{2\mu g} + \frac{2gh}{2\mu g} = \frac{h}{\mu} + \frac{v_0^2}{2\mu g}$$

39. (b)



$$a = \frac{kt - \mu mg}{m}$$

$$a = \frac{kt}{m} - \mu g$$



So, a-t graph will be as shown

40. (d)

$a = g\sin\theta - \mu g\cos\theta$

$a = g\sin\theta - 0.3\,xg\,\cos\theta$

$= \frac{g}{\sqrt{2}} - \frac{0.3\,gx}{\sqrt{2}} = 5\sqrt{2} - 0.3\left(5\sqrt{2}\right)x = 5\sqrt{2} - 1.5\sqrt{2x}$

Velocity will increase until $a = 0$ and when $v = v_{max}$, then $a = 0$

$0 = 5\sqrt{2} - 1.5\sqrt{2}x$

$x = \frac{5\sqrt{2}}{1.5\sqrt{2}}$

$x = 3.33\,\text{m}$

41. (c)

Given, position vector of particle

$r = \left(10t\hat{i} + 15t^2\hat{j} + 7k\right)\text{m}$

$\therefore$ velocity of particle,

$v = \frac{dr}{dt} = 10\hat{i} + 30t\hat{j}$

Acceleration of particle,

$a = \frac{dv}{dt} = 30\hat{j}$

$\therefore$ Net force applied on the particle,

$F = ma = 30m\hat{j}$

Clearly, net force is applied along Y-axis

42. (c)

Given, $F_1 = 10\text{N}, F_2 = 8\text{N},$

$F_3 = 6\ \text{N}$, mass, $m = 5\ \text{kg}$

According to question, resultant of F_2

And F_3 should be opposite to F_1.

$\Rightarrow\ a = \frac{F}{m} = \frac{10}{5} = 2\text{m}/\text{s}^2$

43. (c)

Given,

$v = 10\sqrt{x}$

$m = 500\text{g} = \frac{500}{1000}\text{kg}$

$\Rightarrow\ a = v\frac{dv}{dx} \Rightarrow v = 10\sqrt{x}$

$\frac{dv}{dx} = 10 \times \frac{1}{2}x^{-1/2} \Rightarrow \frac{dv}{dx} = \frac{5}{\sqrt{x}}$

$\Rightarrow\ a = v\frac{dv}{dx} = 10\sqrt{x}.\frac{5}{\sqrt{x}}$

$\Rightarrow \quad a = 50 m/s^2 \Rightarrow f = ma$

$\Rightarrow \quad f = \frac{500}{1000} \times 50 \Rightarrow f = 25 \text{ N}$

44. (a)

Mass of particle $= 500 \text{ g}$

$= 500 \times 10^{-3} \text{kg} = 0.5 \text{ kg}$

Velocity of the particle is;

$v = 2t\hat{i} + 3t^2\hat{j}(m/s)$

Acceleration of particle is

$a = \frac{dv}{dt} = 2\hat{i} + 6t\hat{j} \; m/s^2$

So, acceleration of the particle at $t = 1s$ is,

$a(t = 1s) = 2\hat{i} + 6\hat{j} \; m/s^2$

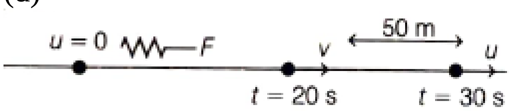
$\therefore$ Force the particle is

$F = ma(\text{at } t = 1s)$

$\Rightarrow \quad F = 0.5(2\hat{i} + 6\hat{j}) = \hat{i} + 3\hat{j} \text{ N}$

Hence, $x = 3$

45. (d)



As particle covers 50m in 10s while moving with a constant velocity $(F = 0, a = 0)$

Velocity of particle is,

$v = \frac{d}{T} = \frac{50}{10} = 5 m/s$

Now, for first 20 s of journey,

$u = 0, v = m/s, t = 20 \text{ s}$

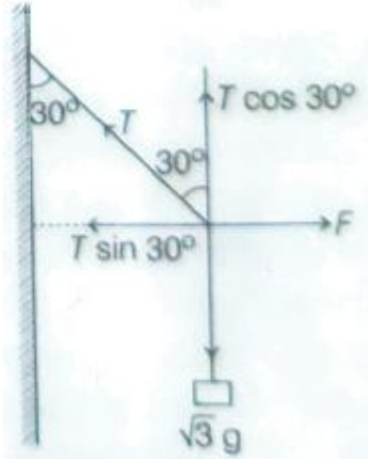
Acceleration, $a = \frac{v - u}{t} = \frac{5}{20} m/s^2$

Magnitude force will be

$F = m \times a = 20 \times \frac{5}{20} = 5N$

46. (d)

From the given free body diagram.



$$T\cos 30^\circ = \sqrt{3}g$$

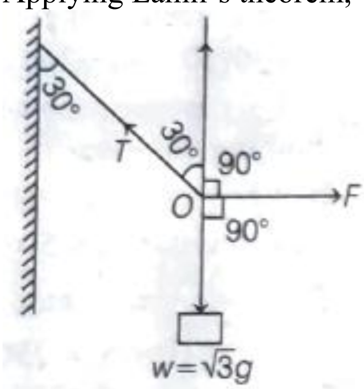
$$\Rightarrow \quad T = \frac{\sqrt{3}}{2} = \sqrt{3}g$$

$$\Rightarrow \quad T = 2g = 2\times 10\left(\because g = 10\ ms^{-2}\right)$$

$$= 20\ N$$

Alternative Solution

Since, three forces T,F and W are acting on a point 0 and the system is in equilibrium, hence Applying Lamil's theorem,



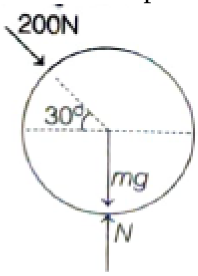
$$\frac{T}{\sin 90^\circ} = \frac{F}{\sin\left(180^\circ - 30^\circ\right)} = \frac{w}{\sin\left(90^\circ + 30^\circ\right)}$$

$$\Rightarrow \quad \frac{T}{\sin 90^\circ} = \frac{w}{\sin\left(90^\circ + 30^\circ\right)}$$

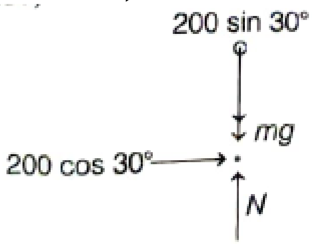
$$\Rightarrow \quad T = \frac{w}{\cos 30^\circ} = \frac{\sqrt{3}g}{\left(\sqrt{3}/2\right)} = 2g = 2\times 10 = 20N$$

47. (b)

FBD of the sphere



Therefore,



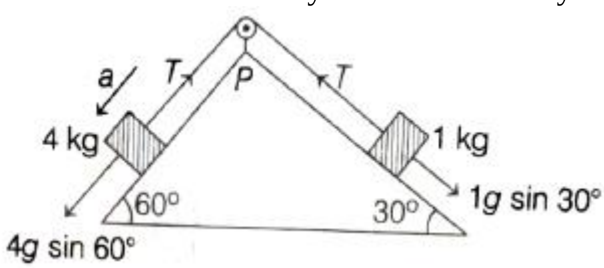
Since, $a_y = 0$

Thus, $N = mg + 200\sin 30^\circ$

$N = 70 \times 10 + 200 \times \sin 30^\circ$
$= 800\text{ N}$

48. (a)

Let a be acceleration of system FBD of the system



On applying Newton's second law, we get

$4 \times a = 4 \times g\sin 60^\circ - T$

$1 \times a = T - 1 \times g\sin 30^\circ$

or, $4a = 20\sqrt{3} - T$ … (i)

And $a = 4\sqrt{3} - 1$ … (ii)

From Eqs. (i) and (ii), we get

$5a = 20\sqrt{3} - 5$

or, $a = 4\sqrt{3} - 1$

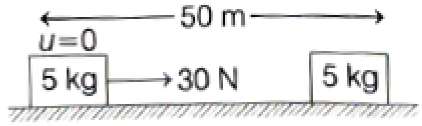
From Eq. (ii),

$4\sqrt{3} - 1 = T - 5$

or, $T = 4\left(\sqrt{3} + 1\right)\text{N}$

49. (a)

The given situation in question is as follows.



Let, the coefficient of kinetic friction be μ.

Acceleration,

$$a = \frac{F - f}{m} = \frac{F - \mu mg}{m}$$

$$a = \frac{30 - \mu \times 5 \times 10}{5}$$

$$= 6 - 10\mu$$

Using second equation of motion,

$$s = ut + \frac{1}{2}at^2$$

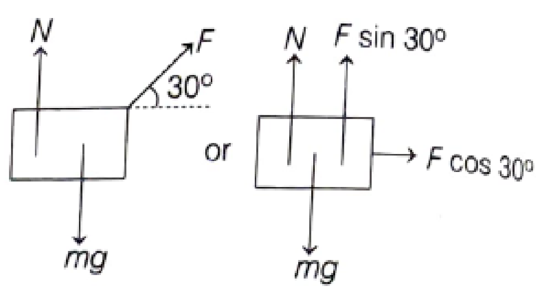
$$\Rightarrow \quad 50 = 0 + \frac{1}{2}(6 - 10\mu) \times 10^2$$

$$\Rightarrow \quad 6 - 10\mu = 1.0$$

$$\Rightarrow \quad \mu = 0.5$$

50. (a)

Drawing FBD of block,



In y-direction,

$$\Sigma F_y = ma_y \qquad (\because a_y = 0)$$

$$N + F\sin 30^\circ = mg$$

Or $\quad N = mg - F\sin 30^\circ$ ….. (i)

in x-direction,

$$\Sigma F_x = ma_x$$

$$\Rightarrow \quad F\cos 30^\circ - \mu N = ma_x$$

As block is on the verge of N from Eq. (i) to Eq.(i) to Eq. (ii), we get

$$F\cos 30^\circ = \mu\left(mg - F\sin 30^\circ\right)$$

$$F\cos 30^\circ = \mu mg - \mu F\sin 30^\circ$$

$$F\left(\cos 30^\circ + \mu\sin 30^\circ\right) = \mu mg$$

$$F = \frac{\mu mg}{\cos 30^\circ + \mu\sin 30^\circ}$$

Given that

$\mu_s = 0.25, m = 10kg, g = 10m/s$

$$F = \frac{0.25 \times 10 \times 10}{\frac{\sqrt{3}}{2} + 0.25 \times \frac{1}{2}}$$

$\Rightarrow \quad F = 25.2N$

51. (c)

Given, $u = 20m/s, t = 5s$

Let, μ be the coefficient of friction, then R = retardation,

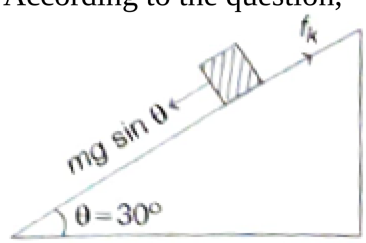
$a = \mu g$

Using, $v = u - at$

$$\Rightarrow 0 = 20 - \mu g \times 5 \Rightarrow \mu = \frac{20}{10 \times 5} = 0.4$$

52. (d)

According to the question,



Acceleration of block

$$= \frac{mg\sin\theta - \mu_k mg\cos\theta}{m}$$

$$= g(\sin\theta - \mu_k \cos\theta) = \frac{g}{4} \text{ [Given]}$$

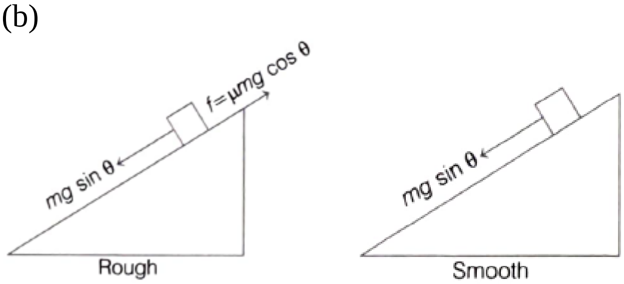
So, $\sin\theta - \mu_k \cos\theta = \frac{1}{4}$

As $\theta = 30^\circ$; we have

$$\frac{1}{2} - \frac{\sqrt{3}}{2}\mu_k = \frac{1}{4}$$

Or $\mu_k = \frac{2 \times 1}{4\sqrt{3}} = \frac{1}{2\sqrt{3}}$

53. (b)



Acceleration in case of smooth inclined plane,

$$a_1 = g\sin\theta = \frac{8}{\sqrt{2}}, \theta = 45^\circ$$

Acceleration over a rough inclined
Plane, $a_2 = g\sin\theta = \mu g\cos\theta$

$= g/\sqrt{2} - \mu g/\sqrt{2} = \frac{g}{\sqrt{2}}(1-\mu)$

Using $s = ut + \frac{1}{2}at^2$

Time taken, $t = \sqrt{\frac{2s}{a}}$ $\quad (\because u = 0)$

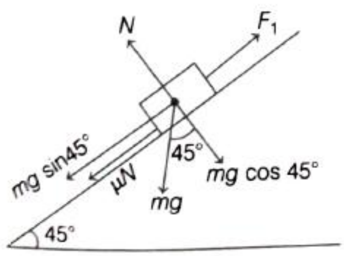
Given, $t_{rough} = nt_{smooth}$

$\sqrt{\frac{2s}{a_2}} = n\sqrt{\frac{2s}{a_1}}$ or $a_1 = n^2 a_2$

$\Rightarrow \quad \frac{g}{\sqrt{2}} = n^2\left(\frac{g}{\sqrt{2}}(1-\mu)\right)$

$\Rightarrow \quad \mu = 1 - \frac{1}{n^2}$

54. (c)
When pushing upwards, friction force will be in downward direction



$F_1 = mg\sin 45^\circ + \mu N$

$= mg\sin 45^\circ + \mu mg\cos 45^\circ$

$F_1 = \frac{mg}{\sqrt{2}} + \frac{\mu mg}{\sqrt{2}}$

When preventing it from sliding,
Friction force will be in upward direction.

$F_2 + \mu N = mg\sin 45^\circ$

$F_2 = mg\sin 45^\circ - \mu mg\cos 45^\circ$

$= \frac{mg}{\sqrt{2}} - \frac{\mu mg}{\sqrt{2}}$

According to given situation,

$F_1 = 2F_2$

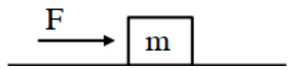
$\frac{mg}{\sqrt{2}} + \frac{\mu mg}{\sqrt{2}} = 2\left(\frac{mg}{\sqrt{2}} - \frac{\mu mg}{\sqrt{2}}\right)$

$3\mu mg = mg$

$\mu = \frac{1}{3} = 0.33$

55. (b)

Given,



From second law of motion,

$$F = ma$$

$$a = \frac{F}{m}$$

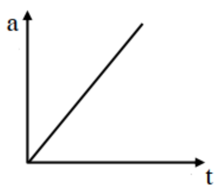
Force applied is linearly depended on time

$F \propto t \Rightarrow F = kt$ (k = constant)

$$a = \frac{kt}{m}$$

Or $a \propto t$

We plot a versus t it will be linear from origin.



SECTION – II

1. (6)

By law of conservation of linear momentum $\vec{P}_i = \vec{P}_f$

$\Rightarrow 60 \times V = (120 + 60) \times 2 \Rightarrow 60V = 360 \Rightarrow V = 6 \text{ m/s}$

2. (12)

Impulse $= \Delta\vec{p}$

$= \vec{P}_f - \vec{P}_i = mv - (-mv) = 2mv = 2 \times 0.4 \times 15 = 12 \text{ Ns}$

3. (500)

$\vec{a} = \frac{\vec{F}}{m} = 10\hat{i} + 5\hat{j}$

Displacement of the box along x-axis,

$x = \frac{1}{2} a_x t^2 = \frac{1}{2} \times 10 \times 100 = 500 \text{ m}$

4. (3)

Along horizontal

$F_1 + 1\cos 45° = 2\sin 45°$

$F_1 = \frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

Along vertical

$F_2 = 1\sin 45 + 2\sin 45$

$F_2 = 3\sin 45 = \frac{3}{\sqrt{2}}$

So, $\frac{F_1}{F_2}=\frac{1}{3}$. So, $x=3$

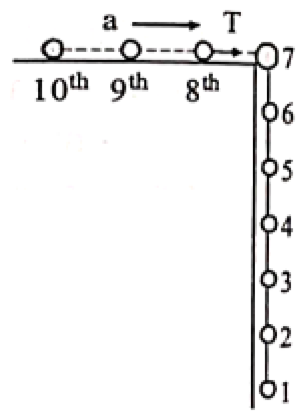
5. (36)

We have, acceleration of system as

$a=\frac{6mg}{10m}=\frac{3g}{5}$

taking 8, 9, 10 together

$T=3ma=3m\times\frac{3g}{5}=\frac{3\times2\times3\times10}{5}=36\text{ N}$



6. (12)

Let draw FBD of block clearly for equilibrium

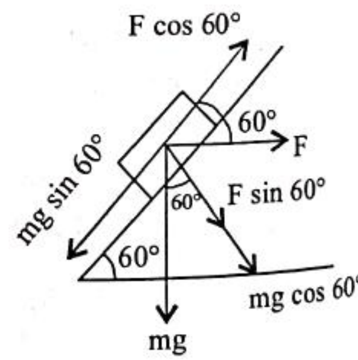
$F\cos60^\circ=mg\sin60^\circ$

$\Rightarrow \frac{F}{mg}=\tan60^\circ$

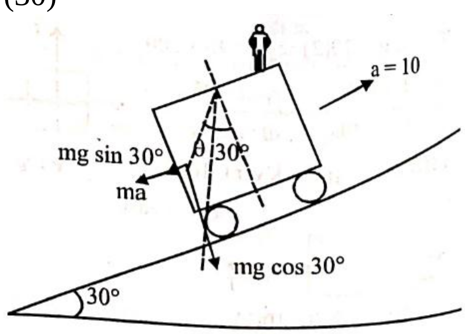
$\Rightarrow \frac{\sqrt{x}}{0.2\times10}=\sqrt{3}$

$\Rightarrow \sqrt{x}=2\sqrt{3}$

$\Rightarrow x=12$



7. (30)



From figure, $\tan\left(30^\circ+\theta\right)=\frac{mg\sin30^\circ+ma}{mg\cos30^\circ}$

$\Rightarrow \tan\left(30^\circ+\theta\right)=\frac{5+10}{5\sqrt{3}}=\frac{1+2}{\sqrt{3}}$

$\Rightarrow \frac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}\tan\theta}$

$\Rightarrow \sqrt{3}\tan\theta+1=3-\sqrt{3}\tan\theta$

$\Rightarrow 2\sqrt{3}\tan\theta=2$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \qquad \therefore \quad \theta = 30°$

8. (82)

$\vec{P}$ makes angle of 35° with AC

So, component along $AC = 100\cos 35 = 81.9\text{ N} \simeq 82\text{ N}$

9. (3)

Acceleration on smooth inclined plane

$a = g\sin 30° = \frac{g}{2}$

Using $S = ut + \frac{1}{2}at^2$

$\Rightarrow \quad S = \frac{1}{2}\frac{g}{2}T^2 = \frac{g}{2}T^2 \qquad ...(\text{i}) \quad (\because u = 0)$

Acceleration on rough inclined plane

$a = g\sin 30° - \mu g\cos 30° = \frac{g}{2} - \frac{\mu g\sqrt{3}}{2}$

$\Rightarrow \quad a = \frac{g}{2}\left(1 - \mu\sqrt{3}\right)$

Using again $S = ut + \frac{1}{2}at^2$

$\Rightarrow \quad S = \frac{1}{4}g\left(1 - \sqrt{3}\mu\right)(\alpha T)^2 \qquad ...(\text{ii})$

By (i) and (ii)

$= \frac{1}{4}gT^2 = \frac{1}{4}g\left(1 - \sqrt{3}\mu\right)\alpha^2T^2 \quad \Rightarrow \quad 1 - \sqrt{3}\mu = \frac{1}{\alpha^2}$

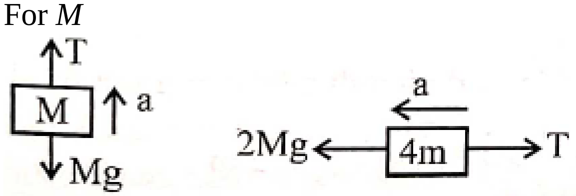
$\Rightarrow \quad \mu = \left(\frac{\alpha^2 - 1}{\alpha^2}\right)\frac{1}{\sqrt{3}} \quad \Rightarrow \quad x = 3.00$

10. (6)

For 4m

$2Mg - T = 4Ma \qquad \ldots (\text{i})$

For M



$T - mg = Ma \qquad \ldots (\text{ii})$

Adding (i) & (ii), we get

$Mg = 5Ma \quad \Rightarrow \quad a = \frac{g}{5}$

So, $T = Ma + Mg = \frac{Mg}{5} + Mg = \frac{6}{5}Mg$

11. (3)

From question,

$$t_a = \frac{1}{2}t_d$$

$$\sqrt{\frac{2s}{a_a}} = \frac{1}{2}\sqrt{\frac{2s}{a_d}}$$

Or, $a_a = 4a_d$ ...(i)

$$g\sin\theta + \mu g\cos\theta = 4(g\sin\theta - \mu g\cos\theta)$$

$$\Rightarrow 5\mu g\cos\theta = 3g\sin\theta \Rightarrow \mu = \frac{3\tan\theta}{5}$$

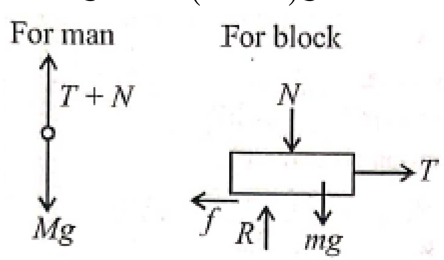
$$\Rightarrow \mu = \frac{\sqrt{3}}{5} \qquad [\because \theta = 30°]$$

So, $x = 3$

12. (30)

From FBD shown, $T = Mg - N$

$$R = mg + N = (M+m)g - T$$



For no movement of block,

$$T \le \mu R \Rightarrow T \le \mu[(M+m)g - T]$$

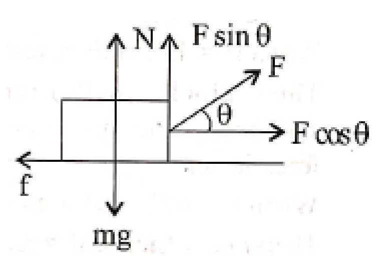
$$\Rightarrow T \le \frac{\mu(M+m)g}{1+\mu} \Rightarrow T = \frac{(0.5)(5+4)\times 10}{1+0.5} = \frac{45}{1.5}$$

$$\therefore T_{max} = 30\,N$$

13. (5)

As block is at rest

So, $F\cos\theta = f = \mu N$



$$F\cos\theta = \mu(mg - F\sin\theta)$$

$$F(\cos\theta + \mu\sin\theta) = \mu mg$$

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

For F_{min}

$(\cos\theta + \mu\sin\theta)$ should be maximum i.e. $\frac{d}{d\theta}(\cos\theta+\mu\sin\theta)=0 \Rightarrow \tan\theta=\mu$

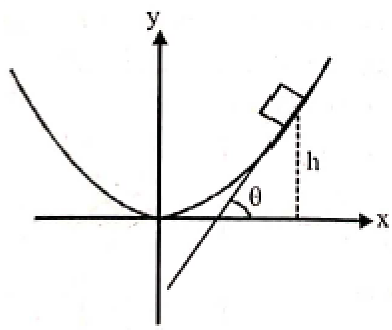
So, $\sin\theta=\frac{\mu}{\sqrt{1+\mu^2}}$ and $\cos\theta=\frac{1}{\sqrt{1+\mu^2}}$

Thus, $F_{\min}=\frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}}+\frac{\mu.\mu}{\sqrt{1+\mu^2}}}=\frac{\mu mg}{\sqrt{1+\mu^2}}$

Putting the value of μ, m and g

$F_{\min}=5\,\text{N}$

14. (25)



Block will fall down when $\theta=$ angle of repose i.e.,

$\tan\theta=\mu$

$\therefore\quad \tan\theta=\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2}{4}\right)=\frac{x}{2}$ and at time of maximum height $\tan\theta=\mu=0.5$

$\Rightarrow\quad x=1$ and therefore $y=0.25\,\text{m}$ = 25 cm

(Assuming that *x* and *y* in the equation are given in meter)

15. (25)

F.B.D. of the block is shown in the diagram.

Since, block is at rest,

$\therefore\quad f_r-mg=0$ ...(i)

$F-N=0$ ...(ii)

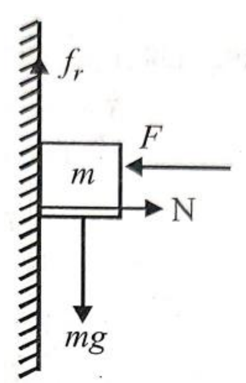
$f_r\le\mu N$

In limiting case,

$f_r=\mu N=\mu F$ ...(iii)

Using equation (i) and (iii),

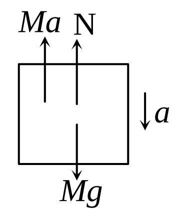
$F=\frac{mg}{\mu}\Rightarrow F=\frac{0.5\times10}{0.2}=25\,\text{N}$



Exercise – I (Only One Option Correct)

1. (C)

F.B.D. of block is as shown



$$Mg = N + Ma$$

$$Mg = \frac{Mg}{4} + Ma$$

$$a = \frac{3g}{4}$$

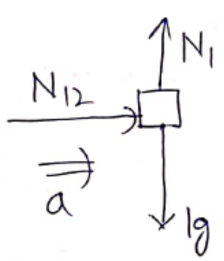
$\therefore$ **(C)**

2. (A)

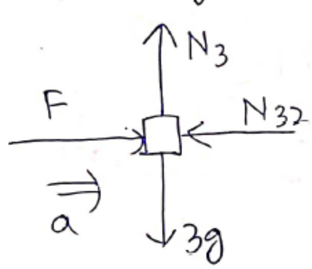
A. External force is in horizontal direction, so al blocks move with same acceleration.

$$a = \frac{F}{3+2+1} = \frac{F}{6}\ \text{m/s}^2$$

B.



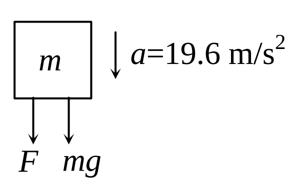
$$N_{12} = ma = 1 \times \frac{F}{6} = \frac{F}{6}\ \text{N}$$



$$F - N_{32} = 3a = \frac{3F}{6} = \frac{F}{2}$$

$$\Rightarrow\ N_{32} = F - \frac{F}{2} = \frac{F}{2} \neq N_{12}$$

3. (A)



$F + mg = ma$

$F = m(a - g) = 2\ (19.6 - 9.8) = 19.6$ N

$\therefore$ **(A)**

4. (B)

$f = \mu (m_1 + m_2 + m_3)\, g = 0.4\ (3 + 2 + 1) \times 10 = 24$ N

To move the blocks $F \geq f$, $3t \geq 24$, $t \geq 8$s

$\therefore$ **(B)**

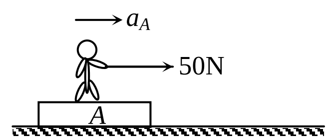
5. (C)

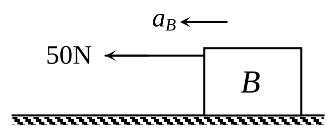
$a = \dfrac{Mg\sin\theta}{2M}$ and $T = Ma$

$\therefore$ **(C)**

6. (A)

Maximum friction force between block A and boy
$= 0.5\times80\times10 = 400\,\text{N} > 50\,\text{N}$





So, $a_A = \dfrac{50}{200} = \dfrac{1}{4}\,\text{m/s}^2$

$a_B = \dfrac{50}{100} = \dfrac{1}{2}\,\text{m/s}^2$

So relative a = 0.75 m/s^2

Relative velocity

$v = 0 + 0.75\times 4 = 3$ m/s

Friction between boy and block A

$f = 120\times\dfrac{1}{4} = 30$ N

$\therefore$ **(A)**

7. (A)

The inclined plane exerts a force of mg cos θ perpendicular to inclination and mg sin θ along inclination.

$\therefore$ **(A)**

8. (C)

For equilibrium of $\sqrt{2}$ M block

$2T\cos\theta = \sqrt{2}Mg$, $T = Mg$, $\cos\theta = \dfrac{1}{\sqrt{2}}$, $\theta = 45°$

$\therefore$ **(C)**

9. (B)

Thrust on the block $F = v\dfrac{dm}{dt} = 5$ N

Acceleration of the block $= \dfrac{F}{M} = \dfrac{5}{2}\,ms^{-2}$

$\therefore$ **(B)**

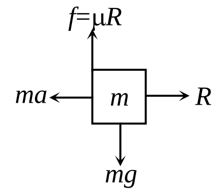
10. (C)

$\Sigma F_y = 0, R = ma$

$Mg = \mu R = \mu ma$

$\mu = \frac{g}{a} = 0.5$

$\therefore$ **(C)**



11. (C)

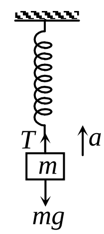
$T - mg = ma$

$T = mg + ma$

$Kx = m(g + a)$

$x = \frac{m(g+a)}{K}$

$\therefore$ **(C)**



12. (B)

$N = m_A(g - a) = 0.5(10 - 2) = 4$ N

$\therefore$ **(B)**

13. (A)

$mg - \eta mg = ma$, $a = g(1 - \eta)$

$\therefore$ **(A)**

14. (D)

Let retardation of body is a and air resistance is f

$v = u + at$

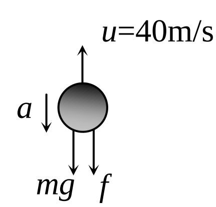
$0 = 40 - 3a$

$a = \frac{40}{3}$ m/s^2

$ma = mg + f$

$f = ma - mg = 1.5\left(\frac{40}{3} - 10\right) = 5$ N

$\therefore$ **(D)**

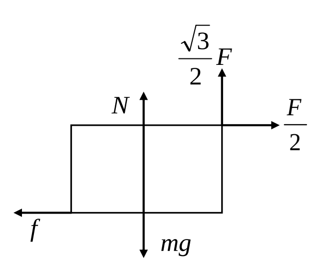


15. (B)

$f = \frac{F}{2} = \mu N$

Also, $N = mg - \frac{\sqrt{3}}{2}F$

$\Rightarrow$ $f = \mu\left(mg - \frac{\sqrt{3}}{2}F\right)$



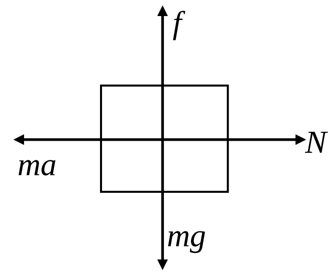
16. (A)

$f = \mu N = mg$

Also, $N = ma$

$\Rightarrow \quad \mu ma = mg$

$\Rightarrow \quad a = \dfrac{g}{\mu}$



17. (C)

$m_1 = 2kg, m_2 = 4kg, m_3 = 6kg$,

$$a = \frac{F-(m_1+m_2+m_3)g\sin 53°}{m_1+m_2+m_3}$$

$$a = \frac{120-12\times 10\times 4/5}{12}, \quad = \frac{24}{12} = 2\,\text{ms}^{-2}$$

$T_1 - m_1 g\sin 53° = m_1 a$,

$T_1 = 4 + 20\times\frac{4}{5} = 20N$

$T_2 - (m_1+m_2)g\sin 53° = (m_1+m_2)a$,

$T_2 = 12 + 60\times\frac{4}{5} = 60N$, $\frac{T_1}{T_2} = \frac{1}{3}$

$\therefore$ **(C)**

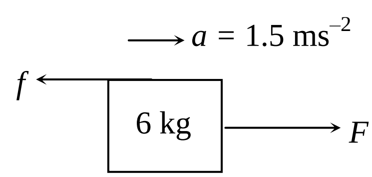
18. (A)

$f = 0.4\times 2\times 10 = 8N$

$F - 8 = 6\times 1.5$

$F = 17N$

$\therefore$ **(A)**



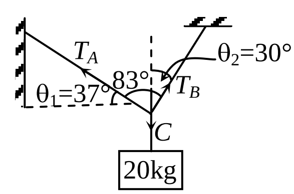
19 (A)

$T_A\cos\theta_1 = T_B\sin\theta_2$

$T_A\cos 37° = T_B\sin 30°$

$T_A\times\frac{4}{5} = T_B\times\frac{1}{2}$; $\quad \frac{T_A}{T_B} = \frac{5}{8}$

$\therefore$ **(A)**



20. (B)

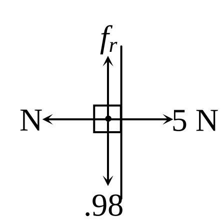
F.B.D. of block is as shown

Maximum frictional force = μN = 0.5 × 5 = 2.5 N

As, maximum friction force > frictional force required to avoid motion

$\therefore \quad f_r = mg = 0.98$ N

$\therefore$ **(B)**



21. (D)

As weight = 0.3 × 10 = 3 N trying to slide the two block system,

but f_{max} = 0.5 × 1 × 10 + 0.5 × 1 × 10 = 10 N,

Hence the system is in equilibrium, and friction of block *B* is sufficient to balance the weight hence tension between *A* and *B* is zero.

∴ **(D)**

22. (C)

$T \sin \theta = R$

$T \cos \theta = W$

Solving

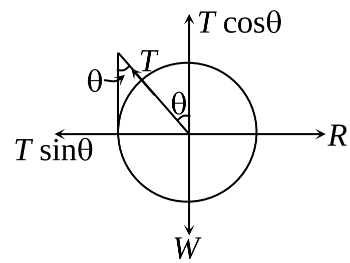
$T^2 = R^2 + W^2$

$R = W \tan \theta$

Vectorially

$\vec{R} + \vec{T} + \vec{W} = 0$

∴ **(c)**



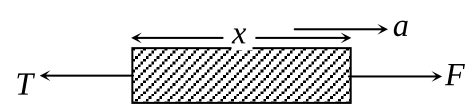
23. (C)

Acceleration $a = \frac{F}{M}$

Drawing F.B.D.

$F - T = \frac{M}{L}(x)a \quad \Rightarrow \quad T = F\left(1 - \frac{x}{L}\right)$

∴ **(C)**



24. (A)

During downward motion: $F = mg\sin\theta - \mu mg\cos\theta$

During upward motion: $2F = mg\sin\theta + \mu mg\cos\theta$

Solving above two equations: we get $\mu = \frac{1}{3}\tan\theta$

∴ **(A)**

25. (A)

Frictional force = $\mu R = \mu(mg + Q\cos\theta)$ and horizontal push = $P + Q\sin\theta$

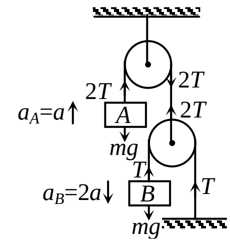
For equilibrium, we have $\mu(mg + Q\cos\theta) = P + Q\sin\theta$

∴ $\mu = \frac{P + Q\sin\theta}{mg + Q\cos\theta}$

∴ **(A)**

26. (D)

$$2T - mg = ma \qquad \ldots\text{(i)}$$
$$mg - T = 2ma \qquad \ldots\text{(ii)}$$



(i) and (ii) $\Rightarrow \quad a = \dfrac{g}{5}$

$\therefore \qquad a_B = \dfrac{2g}{5}$

$\therefore \qquad$ **(D)**

27. (B)

Initially, the weight of load L is the force on the system of mass 8 kg.

Acceleration = $\dfrac{2 \times 10}{8} = \dfrac{20}{8}$ units

Toward the end, force = (2 + 1) × 10 N = 30 N

So, acceleration now is $\dfrac{30}{8}$ units.

$\therefore \qquad$ **(B)**

28. (A)

$m_2g - 2T = m_2a$

or $\qquad 2T = m_2(g - a) \qquad$ ..... (i)

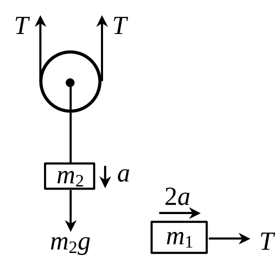
Again, $T = m_1(2a)$

or $\qquad 2T = 4m_1a \qquad$ ..... (ii)

Equating (i) and (ii), $m_2g - m_2a = 4m_1a$

or $\qquad (4m_1 + m_2)a = m_2g$

$$a = \frac{m_2 g}{4m_1 + m_2}.$$



$\therefore \qquad$ **(A)**

29. (B)

$S = \dfrac{1}{2}\mu g t^2$ or $t \propto \dfrac{1}{\sqrt{\mu}}$.

$\therefore \qquad$ **(B)**

30. (A)

On cutting of string QR, the resultant force m_1 remains zero because its weight m_1g is balance by the tension in the spring but on block m_2 a resultant upward force $(m_1 - m_2)g$ is developed.

Thus block m_1 will have no resultant acceleration whereas m_2 does have an upward acceleration given by $\dfrac{(m_1 - m_2)g}{m_2}$.

$\therefore \qquad$ **(A)**

31. (D)

$R = m(g - a)$ for downward motion of lift

If $a = g$, then $R = 0$ $\quad\therefore\quad$ $F = \mu R = 0$

$\therefore$ **(D)**

32. (A)

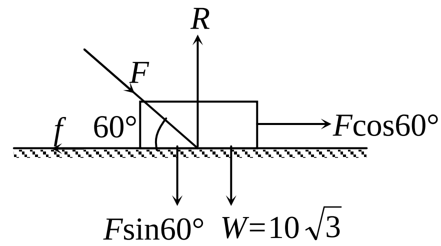
$f = \mu R = \mu(W + F\sin 60°)$

$F\cos 60° = \mu(W + F\sin 60°)$

Substituting $\mu = \dfrac{1}{2\sqrt{3}}$ and $W = 10\sqrt{3}$,

we get $F = 20$ N

$\therefore$ **(A)**



33. (A)

Its velocity becomes $\dfrac{v_0}{2}$ under a retardation of μg in time t_0.

$\therefore \quad \dfrac{v_0}{2} = v_0 - \mu g\, t_0$ or $\mu g\, t_0 = \dfrac{v_0}{2}$ or $\mu = \dfrac{v_0}{2g\, t_0}$

$\therefore$ **(A)**

34. (A)

At equilibrium, let tension in each spring be T. Then

$2T\cos 60° = Mg$

$T = Mg$

When right spring breaks, the net force on the block is T.

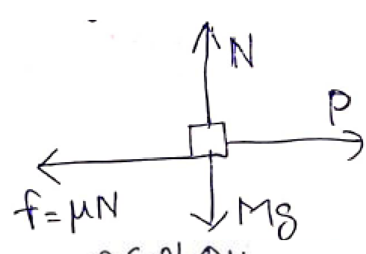
$\therefore \quad a = \dfrac{T}{M} = 10\ \text{m/s}^2$

$\therefore$ **(A)**

35. (C)

A. From FBD,

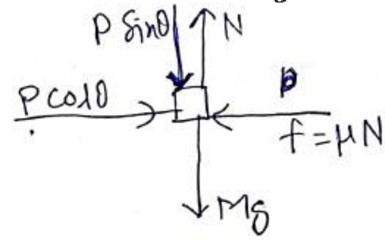
$P = \mu N = \mu Mg$



B. $N = P\sin\theta + Mg$

$P\cos\theta = \mu N = \mu\left(P\sin\theta + Mg\right)$

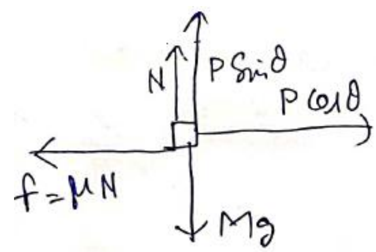
$\Rightarrow P = \dfrac{\mu Mg}{\cos\theta - \mu\sin\theta}$



C. $N = Mg - P\sin\theta$

$P\cos\theta = \mu Mg - \mu P\sin\theta$

$\Rightarrow P = \dfrac{\mu Mg}{\cos\theta + \mu\sin\theta}$



D. No external force in horizontal direction, so block cannot move in this direction.
P is least in *C*.

36. (D)

Consider FBD of m (boy) in triangular block frame.

For vertical direction N = mg – mg sin^2 60^0 = $\dfrac{mg}{4}$

For horizontal direction f = mg sin 60^0 cos 60^0 = $\sqrt{3}\dfrac{mg}{4}$

As $f \le \mu N$

$\Rightarrow \sqrt{3}\dfrac{mg}{4} \le \mu\dfrac{mg}{4}$

$\Rightarrow \mu \ge \sqrt{3}$

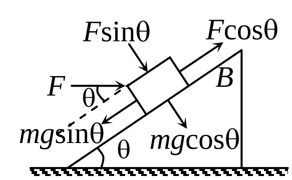
$\therefore$ **(D)**

37. (B)

Resultant force on block along the incline plane is

$= F\cos\theta - mg\sin\theta$

$\therefore$ **(B)**



38. (C)

$\mu mg = m\left(\dfrac{mg}{4m}\right) \Rightarrow \mu = \dfrac{1}{4}$

$\therefore$ **(C)**

39. (A)

$a_{\max} = \mu g$

$\therefore$ **(A)**

40. (D)

For constant velocity $F = mg$,

So, acceleration of man $a = \dfrac{F}{2m} = \dfrac{g}{2}$

$\therefore$ **(D)**

41. (A)

Let tension be T then. $T = ma$. For block M, $F - T = MA \Rightarrow A = \frac{F - ma}{M}$

$\therefore$ **(A)**

42. (C)

From constraint relation $v_B = \frac{v}{3}$

$\therefore$ **(C)**

43. (B)

$T_1 = \frac{mg}{\cos\theta}$, $T_2 = mg\cos\theta$

$\frac{T_1}{T_2} = \sec^2\theta = 2$

$\therefore$ **(B)**

44. (A)

$N_A = N_B$

$\therefore$ **(A)**

45. (A)

Maximum friction force is 50 N which is greater than 40 N. Block does not move.

$\therefore$ **(A)**

46. (C)

From constraint relation, $a_B = 8a_A$

$\therefore$ **(C)**

47. (A)

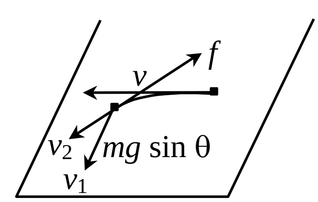
$m_3 g = 2T \quad \Rightarrow \quad m_3 = 1$ kg

$\therefore$ **(A)**

48. (B)

Here friction force will be

$f = \mu N = \tan\theta\, mg\cos\theta = mg\sin\theta$



At any instant acceleration opposite to motion is equal in magnitude to the acceleration down the incline.

$\because$ $mg\sin\theta$ is acting down the plane.

So for small interval of time speed the block looses along its direction of motion exactly equals the speed it gain down the incline. Let v_2 be the speed of the block and v_1 is the component of velocity down the incline then

$v_2 + v_1 = \text{Constant} = C$

Initially $v_1 = 0$, so $C = v$

Finally $v_2 = v_1 = v_f$ after long time

$$2v_2 = v$$

$$v_2 = \frac{v}{2} = v_1$$

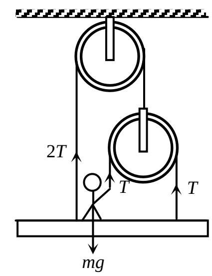
$\therefore$ **(B)**

49. (A)

$$4T = mg$$

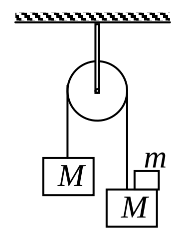
$\therefore$ $T = \frac{60 \times 10}{4}$ = 150 N

$\therefore$ **(a)**



50. (B)

Acceleration of blocks = $\frac{(M+m)g - Mg}{2M+m} = \frac{mg}{2M+m}$



Considering free body diagram of block m only

$$mg - N = ma$$

$$N = m(g-a) = m\left[g - \frac{mg}{2M+m}\right]$$

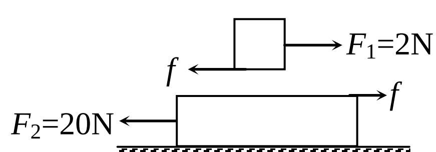
$$N = \frac{2mMg}{2M+m}$$

$\therefore$ **(B)**

51. (A)

Free body diagram of the two bodies are as follows

Let acceleration of both the blocks towards left is a.



Then $a = \frac{f-2}{2} = \frac{20-f}{4}$

or $2f - 4 = 20 - f$ or f = 8 N

Maximum friction between the two blocks can be $f_{\max} = \mu mg = (0.5)(2)(10) = 10$ N

Now since $f < f_{\max}$

Therefore, friction force between the two blocks is 8 N.

$\therefore$ **(A)**

52. (B)

Force at the surface of BC

$N = 2m(a)$, a = acceleration of system

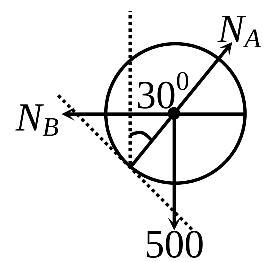
$\therefore$ $N = 2m\frac{F}{5m} = \frac{2F}{5}$

To prevent slipping of block B, $\mu N = mg$

$\Rightarrow \quad \frac{\mu 2F}{5} = mg$, $\quad F = \frac{5}{2\mu} mg$

$\therefore$ **(b)**

53. (C)



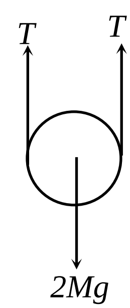
$N_A \cos 30° = 500$

$N_A \sin 30° = N_B$

$\sqrt{3} = \frac{500}{N_B}$

$N_B = \frac{500}{\sqrt{3}}$ N $\quad \therefore$ **(C)**

54. (D)



For pulley, $2T - 2Mg = 0 \Rightarrow T = Mg$

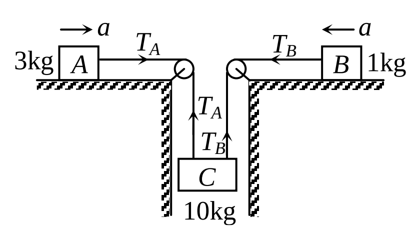
For man,

$T = Ma$

$Mg = Ma$

$a = g \quad \therefore$ **(D)**

55. (A)



By constraint relation,

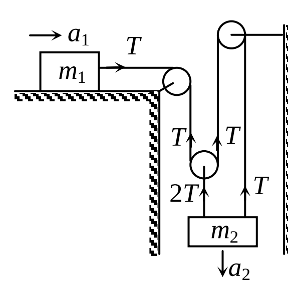
aA = aB, aA = aB = aC = a

$T_A = m_A a = 3a$ and

$T_B = m_B a = a$

$\therefore \quad \frac{T_A}{T_B} = 3:1 \quad \therefore$ **(A)**

56. (C)



$a_1 = 3a_2$ **...(i)**

$T = m_1 a_1$ **...(ii)**

$m_2 g - 3T = m_2 a_2$ **...(iii)**

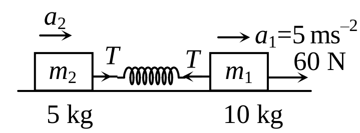
$m_1 = m_2 = m$ **...(iv)**

Solving above equation we get,

$a_1 = \frac{3g}{10}, \; a_2 = \frac{g}{10} \quad \therefore$ **(C)**

57. (A)

$60 - T = 10 \times 5$



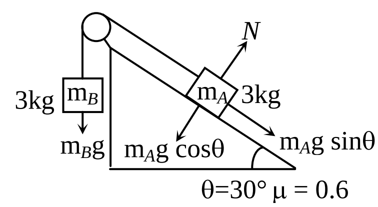
$T = 10$ N

$T = 5a^2$

$a^2 = 2\ ms^{-2}$ $\therefore$ **(A)**

58. (D)

$m_Bg = 30$ N



$m_Ag\sin\theta = 30 \times \frac{1}{2} = 15$ N

Net pulling force = 30 – 15 = 15 N

$f_l = \mu\, m_Ag\cos\theta$

$= 0.6 \times 30 \times \frac{\sqrt{3}}{2}$

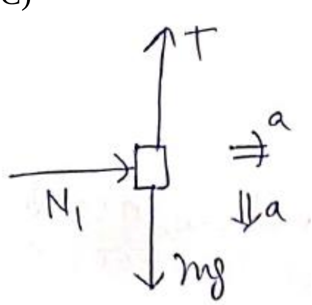
$= 9\sqrt{3} = 9 \times 1.732 = 15.6$ N

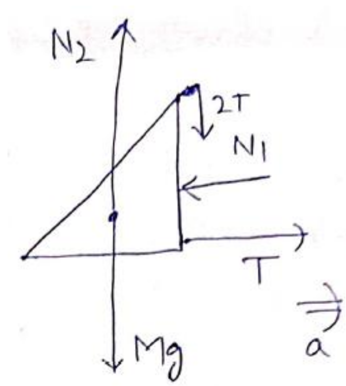
$\because$ $f\ell$ > pulling force

$\therefore$ f = pulling force = 15 N

$\therefore$ **(D)**

59. (C)





$mg - T = ma$ ...(i)

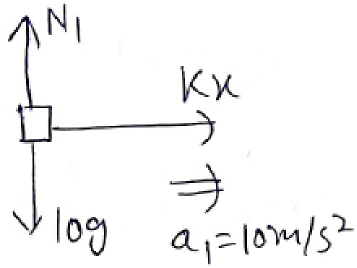
$N_1 = ma$ ...(ii)

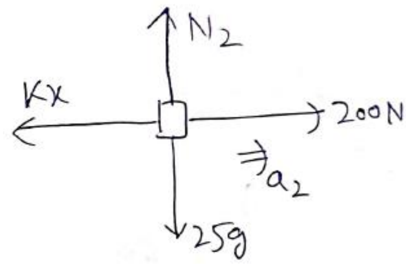
$T - N_1 = Ma$ ...(iii)

Solving (i), (ii), (iii) we get

$a = \frac{g}{5} = 2\ m/s^2$

60. (B)





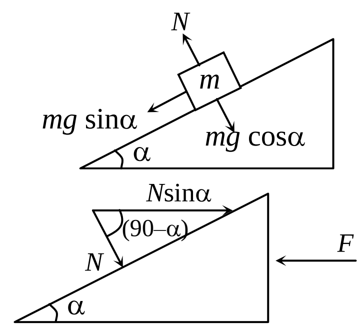
For 10 kg block : $Kx = ma_1$

$\Rightarrow Kx = 10 \times 10 = 100$ N

For 25 kg block : $200 - Kx = 25a_2$

$\Rightarrow 200 - 100 = 25a_2 \Rightarrow a_2 = 4 \text{ m/s}^2$

61. (D)



$N = mg \cos \alpha$

Let F force is required to keep wedge stationary.

From FBD of wedge

$F = N \sin \alpha$

$F = mg \cos \alpha \sin \alpha$

$\therefore$ **(D)**

62. (B)

$f_l = \mu_s m_1 g = 25\text{N}, \ (a_2)_{\max} = \frac{f_l}{m_2} = \frac{5}{6} \text{ ms}^{-2}$

$a_{\text{combined}} = \frac{F}{m_1 + m_2} = 1\text{ms}^{-2}$

$(a_2)_{\max} < a_{\text{combined}}$, $\therefore$ there will be slipping between the blocks.

$\therefore \quad f = \mu_k m_1 g = 12\text{N}$

$a_2 = \frac{f}{m_2} = \frac{12}{30} = 0.4 \text{ ms}^{-2}$

$\therefore$ **(B)**

63. (D)

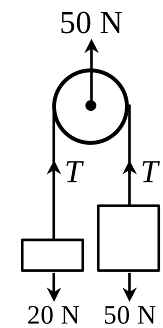
$f = \mu R = \mu mg$, where m is mass of the combination, $f = 0.5 \times 10 \times 10$ N = 50 N.

So, a force of 10 N is unable to start the motion of the system. There is no relative motion between A and B.

$\therefore$ **(D)**

64. (A)

The masses will be lifted if the tension of the string is more than the gravitational pull on masses.



Here weight of 5 kg mass = 5 × 10 = 50 N and 2 kg mass = 2 × 10 = 20 N

From free body diagram 50 – 2T = 0 or T = 25 N

So 5 kg weight can not be lifted (..... acceleration = 0) but 2 kg weight will be lifted.

∴ $25 - 20 = 2a$ or $a = \frac{5}{2} = 2.5\ ms^{-2}$

∴ **(A)**

65. (B)

$F = mg(\sin\theta + \mu\cos\theta)$

$= 10 \times 9.8(\sin 30° + 0.5\cos 30°)$ = 91.4 N

∴ **(B)**

66. (D)

$\tan\theta = \tan 37° = \frac{3}{4} = 0.75$

Here, $\mu_1 > \tan\theta$ so m_1 will not slip.

So, contact force between the blocks is zero.

67. (D)

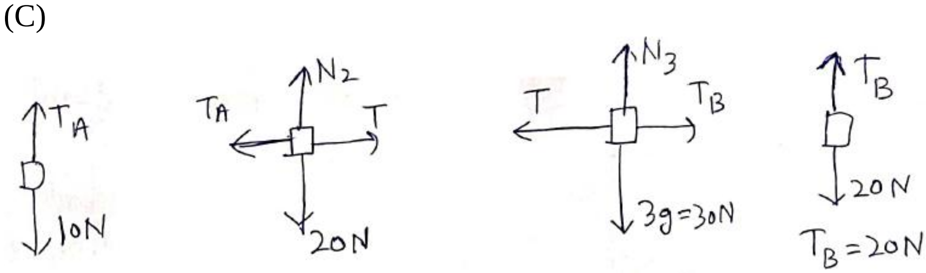
Total length of string is constant.

$x_2 = 7x_1$

$a_2 = 7a_1$

∴ **(D)**

68. (C)



Maximum total friction is $0.5 \times 30 + 0.4 \times 20 = 15 + 8 = 23\,\text{N}$.

Its more than weight difference of blocks $(20 - 10 = 10\,\text{N})$

So option (C) is correct.

69. (B)

$\frac{at}{m_1 + m_2} = \text{acc}$

at sliding pseudo force on m_1 = friction force most

∴ $\frac{m_1 a t_0}{(m_1 + m_2)} = k m_1 g$

70. (B)

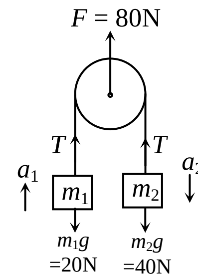
$f_L = \mu N = (0.5)(15) = 7.5N$

driving force i.e. weight of block is less than limiting friction. Hence friction force is equal to weight

$f = mg = 0.5 \times 9.8 = 4.9$ N

71. (B)

Pulley is ideal



$\therefore \quad 2T = 80 \quad \Rightarrow \quad T = 40$ N

$$a_1 = \frac{40-20}{2} = 10 \text{ m/s}^2 \text{ (upwards)}$$

$$a_2 = \frac{40-40}{4} = 0$$

If acceleration of m_1 and m_2 w.r.t. pulley is a_0 and acceleration of pulley is a then,

$a_1 = a + a_0 \Rightarrow a + a_0 = 10$

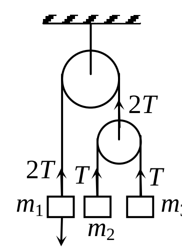
$a_2 = a - a_0 \Rightarrow a = a_0$

$\therefore \quad a_0 = 5$ m/s^2

$\therefore$ **(B)**

72. (B)

If m_1 remains at rest



$2T = m_1 g$ …(i)

$$T = \frac{2m_2 m_3 g}{m_1 + m_2} \quad \text{…(ii)}$$

From (i) and (ii)

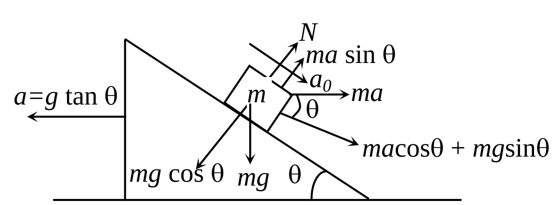
$$\frac{4m_2 m_2 g}{m_1 + m_2} = m_1 g$$

$$\frac{1}{m_1} = \frac{m_1 + m_2}{4m_2 m_3}, \qquad \frac{4}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}$$

$\therefore$ **(B)**

73. (B)

Drawing *FBD* of block *m* from the frame of wedge,
let a_0 is acceleration of block with respect of wedge,



$ma_0 = ma \cos\theta + mg \sin\theta$

$a_0 = g \tan\theta \cos\theta + g \sin\theta$

$a_0 = 2g \sin\theta$

$\therefore$ **(B)**

74. (C)

Friction between rod and bead is less than maximum possible friction.

Exercise – II (One or More than One Option(s) Correct)

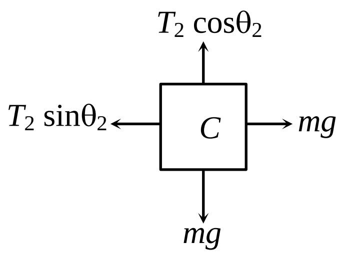
1. (ACD)

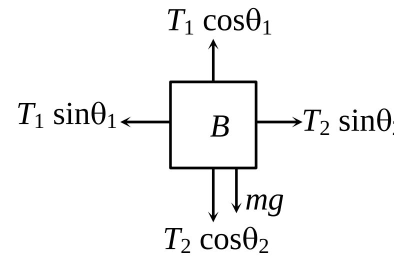
$T_2 \sin\theta_2 = mg$ … (i)

$T_2 \cos\theta_2 = mg$ …(ii)

$T_1 \cos\theta_1 = mg + T_2 \cos\theta_2$

$T_1 \sin\theta_1 = T_2 \sin\theta_2$





∴ **(A) (C) and (D)**

2. (CD)

Resultant force may not be zero for coplanar forces. Hence (a) is not true

Since magnitudes are not equal (b) can not be true. In (c) and (d), net force is zero

∴ **(C, D)**

3. (ABC)

Here $10 - T_2 = 10a$ … (i)

$T_2 - T_1 - 0.3 \times 2g = 3a$ … (ii)

$T_1 - 0.3 \times 2g = 2a$

Summing up $10g - 0.3 \times 4 \times g = 15a$

i.e. $a = 5.86 \text{ ms}^{-2}$

$T_2 = 10 \times 9.8 - 10 \times 5.86 \text{ms}^{-2}$ = 41.4 N

$T_1 = 2 \times 5.86 + 0.6 \times 9.8 = 17.7$ N

∴ **(A) (B) and (C)**

4. (ABC)

$a_1 > 0$ when $\frac{F}{4} > 50$, $F > 200$

$a_2 > 0$ when $\frac{F}{4} > 100$, $F > 400$

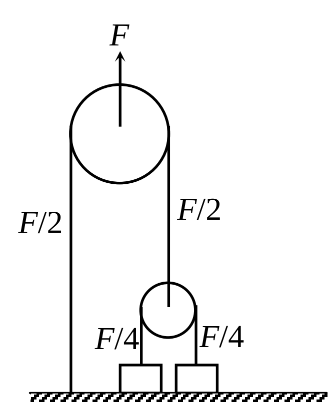
F = 300 N

$a_1 = \frac{F/4 - 50}{5} = \frac{300/4 - 50}{5}$ = 5 m/s^2

$a_2 = 0$

If $F = 500$ N

$a_1 = 15$ m/s^2 , $a_2 = 2.5$ m/s^2



∴ **(A) (B) (C)**

5. (ABC)

Maximum values of friction forces are as follows

Between A and B : $\mu(N_A)$

$= 0.1 \times 10 = 1\text{ N}$

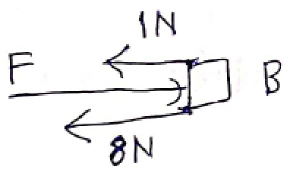
Between B and C : $\mu(N_B)$

$= 0.4 \times 20 = 8\text{ N}$.

Between C and ground : $\mu(N_C) = 0.2 \times 30 = 6\text{ N}$

(A) if $F = 6\text{ N}$, B and C will not move & A will not move.

(B) if $F > 9\text{ N}$,



There will be motion between A and B.

(C) $F > 11\text{ N}$, there will be motion between B and C.

6. (ABC)

Maximum acceleration block $A = \dfrac{0.5mg}{m} = \dfrac{g}{2}$

So, if $M = 2m$, $a_A = a_B = \dfrac{2mg}{4m} = \dfrac{g}{2}$ and friction force is $\dfrac{1}{2}mg$.

$\therefore$ **(A), (B) and (C)**

7. (ABCD)

Friction maximum = 24 N

So net applied force on P is less than f_{max}.

Hence acceleration is zero and $T_A = 20$ N, $T_B = 40$ N

Contact force $= \sqrt{N^2 + (f)^2} = \sqrt{(40)^2 + (20)^2} = 20\sqrt{5}$ N

$\therefore$ **(A) (B) (C) and (D)**

8. (AB)

(A) Frictional force applied by surface on person will be in the direction of motion.

(B) We need to consider all forces acting on object.

(C) & (D) - conceptual.

9. (AB)

If acceleration of the system is

$F = 4ma \Rightarrow a = \dfrac{F}{4m}$ $\therefore$ since acceleration of each block is $\dfrac{F}{4m}$

$\therefore$ net force on each block is $\frac{F}{4}$

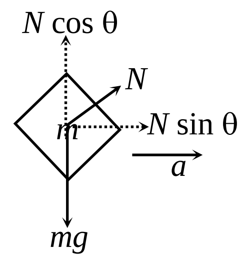
$\therefore$ **(A) and (B)**

10. (BC)

$N\cos\theta = mg$

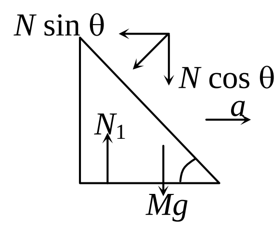
$N\sin\theta = ma$

$a = g\tan\theta$



$N_1 = Mg + N\cos\theta = Mg + mg$

$\therefore$ **(B) and (C)**



11. (BD)

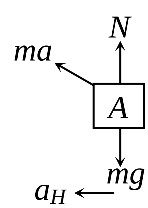
From FBD of A with respect to B

$a_v = 0$

$mg = N - ma\sin\theta$

$\Rightarrow \quad N = mg - ma\sin\theta$

$ma\cos\theta = ma_H \;\Rightarrow\; a_H = a\cos\theta$



If block B is having friction then, for $a_H =$ 0

$ma\cos\theta \le \mu N = \mu(mg - ma\sin\theta)$

$\mu \ge \frac{a\cos\theta}{g - a\sin\theta}$

$\therefore$ **(B) and (D)**

12. (ACD)

$F - T - \mu_2 m_2 g = m_2 a$, $\quad T - \mu_2 m_2 g = m_1 a$

for just equilibrium $a = 0$, $\quad F = 2\mu_2 m_2 g = 4\,\text{N}$

If $F = 6\text{N}$, $a = 1\ \text{m/s}^2 \Rightarrow T = 3\text{N}$

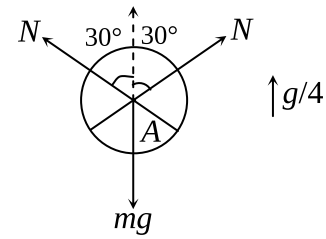
(A), (C) and (D)

13. (BD)

Net upward force on three spheres applied by bottom

$$= 3mg + \frac{3}{4}mg = \frac{15mg}{4}$$

For sphere A, $N\sqrt{3} = mg + \frac{mg}{4}$, $N = \frac{5mg}{4\sqrt{3}}$



$\therefore$ **(B) and (D)**

14. (AC)

Due to symmetry net force on M is zero. Hence its acceleration is also zero and acceleration of B is $\frac{mg}{2m} = \frac{g}{2}$.

$\therefore$ **(A) and (C)**

15. (BCD)

As the tendency of motion is rightward, so the frictional force on B acts leftward, but on A it has any direction i.e. left or right. Also if F_1 and F_2 are less than limiting friction at A and B, then tension is zero.

$\therefore$ **(B), (C) and (D)**

16. (ABC)

(A) Friction force on block A is

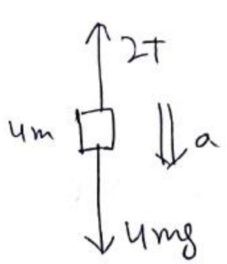
$$mg\sin\theta = 5\times 10\times\frac{3}{5} = 30\text{ N}.$$

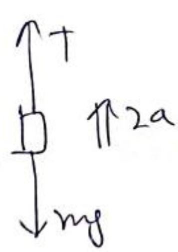
(B) $f_C = \mu mg\cos\theta = 0.1\times 2\times 10\times\frac{4}{5} = 1.6\text{ N}$

(C) tension in rod is : $m_B g\sin 37° + f_C + f_A$

$$= 60 + 30 + 1.6 = 91.6\text{ N}$$

17. (A)





$\Rightarrow 4mg - 2T = 4ma$

$2mg - T = 2ma$ ...(i)

$T - mg = 2ma$ ...(ii)

From (i) & (ii)

$a = \frac{g}{4}$

$\Rightarrow$ acceleration of m block is $2a = \frac{g}{2}$

As A moves 20 cm, B will move 40 cm.

Velocity of B after 40 cm : $V^2 = u^2 + 2as$

$\Rightarrow V^2 = 0 + 2 \times \frac{10}{2} \times \frac{40}{100} = \frac{400}{100}$

After it B will move under gravity.

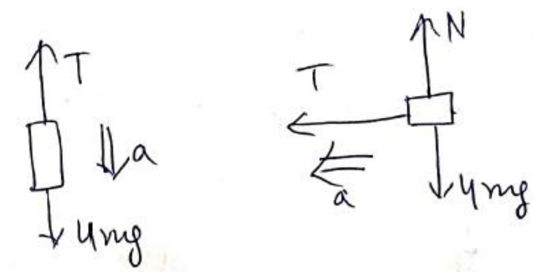
Again $h' = \frac{V^2}{2g} = \frac{400/100}{2 \times 10} = 0.2 \text{ m} = 20 \text{ cm}$

So, max height = 40 + 20 = 60 cm.

18. (C)

There will not be any relative acceleration of B, C, D

So,



$$4mg - T = 4ma \qquad ...(i)$$

$$T = 4ma \qquad ...(ii)$$

Solving we get $a = \frac{g}{2} = 5 \text{ m/s}^2$

Now, $V = u + at = 0 + 5 \times 2 = 10 \text{ m/s}$.

19. (AD)

When $\tan\theta = \mu_2 = \frac{4}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

So, block B will start moving for $\theta > 30°$.

Both will start moving when :

$\mu_1 mg\cos\theta + \mu_2 mg\cos\theta = 2mg\sin\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{(\mu_1 + \mu_2)}{2}$

$\Rightarrow \tan\theta = \frac{\mu_1 + \mu_2}{2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\mu_1 + \mu_2}{2}\right)$

Exercise – III (A) Comprehensions Type

COMPREHENSIONS - 1

1. (B) 2. (A) 3. (D) 4. (C) 5. (D)

To just start sliding

$$mg\sin 30 + F - \mu_s mg\cos 30 = 0$$

$$mg\left(\frac{1}{2}\right) + F - \frac{\mu_s\sqrt{3}}{2}mg = 0 \quad \text{....(i)}$$

To move down the plane with constant speed

$$mg\sin 30 - \mu_k mg\cos 30 - F = 0$$

$$\frac{1}{2}mg - \mu_k\frac{\sqrt{3}}{2}mg - F = 0 \quad \text{....(ii)}$$

and $\frac{\mu_s}{\mu_k} = 2$ ...(iii)

Solving (i) (ii) and (iii) we get

$$\boldsymbol{F = mg/6} \qquad \boldsymbol{\mu_s = \frac{4}{3\sqrt{3}}}$$

If man pushes block continuously by force F down the plane then

$$\mathbf{mg}\sin 30 + F - \mu_k mg\cos 30 = ma$$

$\Rightarrow$ **a = g/3**

Minimum force required to just move block up the incline

$$F_{\min i} = mg\sin 30 + \mu_s mg\cos 30 = \mathbf{\frac{7mg}{6}}$$

Force required to move block up the incline at constant speed

$$F_{up} = mg\sin 30 + \mu_k mg\cos 30 = \frac{mg}{2} + \frac{2}{3\sqrt{3}}mg\frac{\sqrt{3}}{2} = \mathbf{\frac{5mg}{6}}$$

COMPREHENSIONS - 2

6. (C) 7. (D) 8. (A)

When M = 45 acceleration of the system

$$a = \frac{450}{75} = \mathbf{6\,m/s^2}$$

Friction $f_s = (5kg)(6m/s^2)$

$\mathbf{f_s = 30N}$

System will not slip for any value of M.

COMPREHENSIONS - 3

9. (C)

F. B. D of body

$$\Rightarrow mg - kv = ma \quad a = g - \frac{k}{m}v \quad .....(i)$$

Acceleration decreases continuously as a function of velocity. $\therefore$ Ans. (C)

10. (A)

When body attains terminal speed

$$g-\frac{k}{m}v=a=0 \quad v=\frac{mg}{k}$$

$\therefore Ans(A)$

11. (C)

by equation (i) Ans. (C)

COMPREHENSIONS - 4

12. (ABCD)

At a particular value of $a_0(=b)$ pseudo force becomes equal to tension, at this time friction will be zero.

And maximum value of friction depends on normal Reaction between M and trolley and not on a_0 there fore

Ans. (A), (B), (C), (D) all are correct.

13. (ABCD)

$$a_{\max}=\frac{(\mu M+m)g}{M}$$

$$a_{\min}=\frac{(m-\mu M)g}{M}$$

Ans. (A) (B) (C) and (D) are correct.

14. (ABC)

If T < mg 'm' is accelerating downward $\Rightarrow$ friction will be μMg

If $T>mg$ 'm' is accelerating upwards $\Rightarrow$ friction will be μMg

When block m at rest $\Rightarrow$ T = mg

Ans. (A) (B) (C) are correct.

COMPREHENSIONS - 5

15. (C)

Acceleration of system is $a=\frac{mg}{M_{total}} \Rightarrow a\propto m \ \therefore Ans(C)$

16. (D)

Tension is given by

$mg-T=ma$ .....(i)

$T=(M_{Total}-m)a$ ...(ii)

$a=\frac{mg}{M_{Total}}$ ....(iii)

Solving (i) (ii) and (iii) we get

$$T = mg\left[1 - \frac{mg}{M_{Total}}\right] \Rightarrow \text{Ans. (D)}$$

17. (B)

Net force = mass $\times$ acceleration = $m\frac{mg}{M_{Total}}$

$$\Rightarrow \frac{m^2 g}{M_{Total}}$$

Ans. (B)

Exercise – III (B) Matrix-Match Type

1. (A) $\rightarrow$ Q; (B) $\rightarrow$ R; (C) $\rightarrow$ Q; (D) $\rightarrow$ R

$$F_2 - T_A = m_2\frac{(F_2 - F_1)}{m_1 + m_2} \qquad \Rightarrow T_A = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) \text{ Ans.}$$

$$F_2 - T_B = m_2\left(\frac{F_2 + F_1}{m_1 + m_2}\right) = T_B = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right) \text{ Ans.}$$

$$F_2 - N_C = m_2\frac{(F_2 - F_1)}{(m_1 + m_2)} \Rightarrow N_C = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) \text{ Ans.}$$

$$F_2 - N_D = m_2\frac{F_1 + F_2}{(m_1 + m_2)} \Rightarrow N_D = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right) \text{ Ans.}$$

2. (A) $\rightarrow$ R; (B) $\rightarrow$ Q; (C) $\rightarrow$ Q, S; (D) $\rightarrow$ Q
To solve this problem just keep in mind if both the ends of a spring are not free to move spring cannot change its length suddenly

3. (A) $\rightarrow$ Q, S; (B) $\rightarrow$ S
Friction always try to stop slipping therefore it opposes relative motion.

4. (A) $\rightarrow$ P; (B) $\rightarrow$ Q; (C) $\rightarrow$ Q, R; (D) $\rightarrow$ R
If block is at rest friction is $mg\sin\alpha$

Normal reaction always $= mg\cos\alpha$

When block is moving on the plane with constant velocity friction is $= \mu mg\cos\alpha = mg\sin\alpha$

When block is moving on the plane with constant acceleration friction is $= \mu mg\cos\alpha \neq mg\sin\alpha$

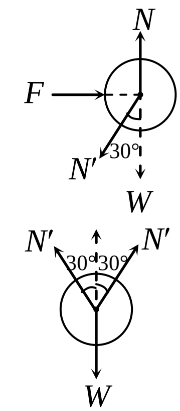
5. (A) $\rightarrow$ P, Q, R, T; (B) $\rightarrow$ P, Q, T; (C) $\rightarrow$ P, Q, R, S, T; (D) $\rightarrow$ P, Q, T

Slope of the curve $\tan\theta = \frac{40}{4} = 10$

Exercise – IV (Subjective Type)

1. 10 N

For the system to remain in equilibrium, the normal force between the lower cylinders must be zero.



Free body diagram of left lower cylinder

i.e. $F = N'\sin 30°$ …(i)

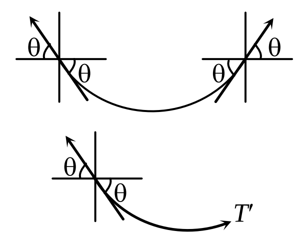
Free body diagram of the upper bar

i.e. $W = 2N'\cos 30°$ …(ii)

From (i) and (ii),

$$F = \frac{W\sin 30°}{2\cos 30°} = \frac{W}{2}\tan 30° = \frac{W}{2\sqrt{3}} = \frac{20\sqrt{3}}{2\sqrt{3}} = 10\text{ N}$$

2. 5 N



$2T\sin\theta = W$ …(i)

$T\cos\theta = T'$ …(ii)

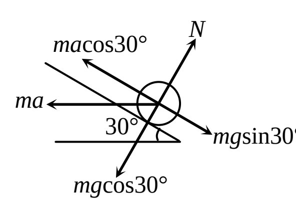
From (i) and (ii)

$$T' = \frac{W\cos\theta}{2\sin\theta} = \frac{W}{2}\cot\theta = 5\text{N}$$

3. 1

When upper spring is cut both the blocks A & B move with same acceleration.

4. 10



$ma\cos 30° = mg\sin 30°$

$$a = g\tan 30° = \frac{g}{\sqrt{3}}$$

$\therefore \quad n = 10$

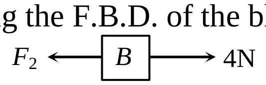
5. 5 m/s

From constraint relation, $v_C = 5\text{m/s}$ towards left

6. 5s

The acceleration of the bracket = $\frac{F_1 - 2F_2}{m} = 4$ m/s^2. (m = 1 kg)

Drawing the F.B.D. of the block B



The acceleration of the block with respect to bracket = $\frac{F_2 - 4}{1} = 4\text{m/s}^2$

Using, the kinematics of uniformly accelerated motion,

$$s = \frac{1}{2}at^2$$

$$50 = \frac{1}{2}\times 4\times t^2 \qquad t = 5\text{ s}$$

7. 5 m/s^2

Let the acceleration of the rope being a downwards. Then the equation of motion for man A is

$$T - mg = m(a + a') \quad \ldots\text{(i)}$$

and for man B being

$$mg - T = ma \quad \ldots\text{(ii)}$$

From (i) and (ii)

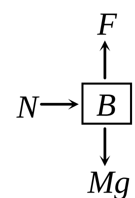
$$ma + ma' = -ma$$

i.e. $2ma = ma'$

$$a = \frac{a'}{2} = 5 \text{ m/s}^2$$

8. 1

The system is moving towards right with acceleration. This is also the acceleration of A. The equation of motion of B, being



$$F - Mg = Ma'$$

$$-\frac{Mg}{2} = Ma'$$

$$a' = \frac{g}{2} \text{ in downward direction.}$$

The net acceleration of B being $\sqrt{a^2 + a'^2}$

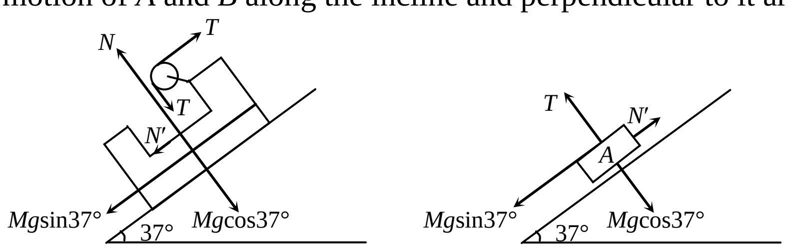
$$a_B = \sqrt{\frac{g^2}{16} + \frac{g^2}{4}} = \frac{\sqrt{5}\,g}{4}$$

Hence, $\frac{a_A}{a_B} = \frac{g}{4} \times \frac{4}{\sqrt{5}\,g} = \frac{1}{\sqrt{5}}$

$\therefore \quad n = 1$

9. 3

The equation of motion of A and B along the incline and perpendicular to it are given by



The equation of motion of A and B along the incline perpendicular to if are given by

$$Mg\sin 37° - N' = Ma_B \quad \ldots\text{(i)}$$

$$T - Mg\cos 37° = Ma_A \quad \ldots\text{(ii)}$$

$$Mg\sin 37° + N' - T = Ma_B \quad \ldots\text{(iii)}$$

By constraint relation, $a_B = a_A$

From (i), (ii) and (iii),

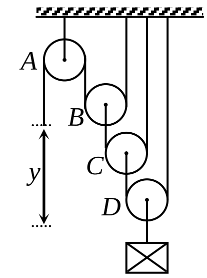
$$a_B = \frac{4}{3} \text{m/s}^2$$

$\therefore \quad n = 3$

10. 3S

By constraint relation,

The acceleration of effort = 8 × the acceleration of load



Given,

$$\frac{dy}{dt} = 24t^2$$

$$\int_0^y dy = 24\int_0^t t^2 dt$$

$$y = \frac{24t^3}{3} = 8t^3 = 72$$

$t = 3$s

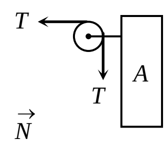
11. 1

In case (i),

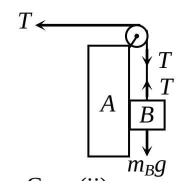
$T - N = 40a$ …(i)

$N = 20a$ …(ii)

$m_B g - T = 20a$ …(iii)



Case (i)



Case (ii)

Solving we get

$$a = \frac{g}{4};$$

Hence acceleration of block B will become $\frac{g}{2\sqrt{2}}$.

In case (ii),

$T = 40a$ ...(iv)

$20g - T = 20a$ …(v)

Solving, we get, $a = \frac{g}{3}$

Hence acceleration of block B will become $\frac{g}{3}$.

The required ratio = $\frac{g/2\sqrt{2}}{g/3} = \frac{3}{2\sqrt{2}}$

$n = 1$

12. 5 m/s^2

$2kx = m_1 g$ …(i)

When the string is cut

$Kx - m_2 g = m_2 a$ …(ii)

$$\frac{m_1 g}{2} - m_2 g = m_2 a$$

$$\frac{15}{2}g - 5g = 5a$$

$$\frac{3g}{2} - g = a$$

$$\Rightarrow \frac{g}{2} = a$$

i..e. $a = 5$

13. 50 cm/s; 1 m/s^2

By constraint relation,

$$v_B = \frac{v_A}{2},\ a_B = \frac{a_A}{2}$$

$$v_B = 50\text{ cm/s},\ a_B = 1\text{m/s}^2$$

14. 175 cm

$$a_2 = \frac{14}{10}\text{ m/s}^2$$

a_1 = 1 m/s^2

$$a_{1/2} = -\frac{4}{10}\text{ m/s}^2$$

$$t^2 = \frac{2s}{a_{1/2}} = \frac{10}{4}\text{s}$$

$$s_2 = \frac{1}{2}\times\frac{14}{10}\times\frac{10}{4} = 175\text{ cm}$$

15. 10s

$$a_{m/p} = 0.2\times10 + \frac{50\times0.2\times10}{10}$$

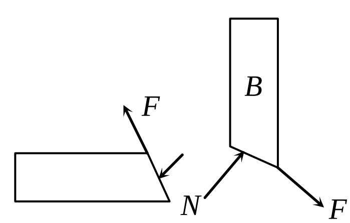
$$= 12\text{ m/s}^2$$

$$v_{\max} = \sqrt{2\times12\times150} = 60\text{ m/s}$$

$$60\frac{t}{2} = 300$$

$$t = 10\text{ seconds}$$

16. 14N



$$\text{N}\cos\theta = mg + f\sin\theta$$

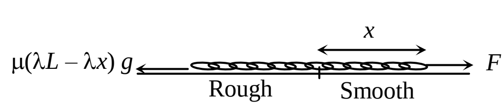
$$\text{N}\cos\theta = \left(\frac{mg}{1-\mu}\right)$$

$$F \geq f\cos45^\circ + N\sin45^\circ$$

$$= \frac{1.4\text{ mg}}{(1-\mu)}$$

$$\frac{=1.4\times0.6\text{ g}}{0.6} = 14\text{ N}$$

17. $\sqrt{F-L}$ m/s



$$F - \mu(\lambda L - \lambda x)g = \lambda L v\frac{dv}{dx}$$

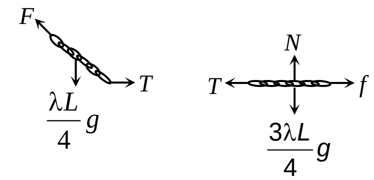
$$F\int_0^L dx - \mu\lambda g\int_0^L (L-x)dx = \lambda L\int_0^v v\,dv$$

$$FL - \mu\lambda g\left(L^2 - \frac{L^2}{2}\right) = \frac{\lambda L v^2}{2}$$

$$v = \sqrt{F-L}$$

18. $\frac{1}{4}$

$F\cos 37° = \frac{\lambda L}{4} g$ (where λ is the mass/length of the chain).



$F\sin 37° = T = f \le \mu N$

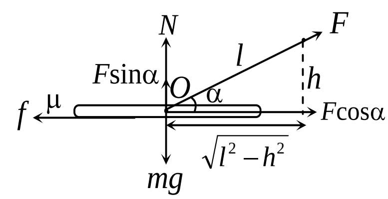
$\Rightarrow \quad \mu \ge \frac{1}{4} \quad \Rightarrow \quad \mu_{\min} = \frac{1}{4}$

19. $\mu = \frac{F\sqrt{l^2 - h^2}}{mgl - Fh}$

$N = mg - F\sin\alpha$

$F\cos\alpha = f = \mu N$

$F\cos\alpha = \mu(mg - F\sin\alpha)$



$$\mu = \frac{F\cos\alpha}{mg - F\sin\alpha} = \frac{F \times \frac{\sqrt{l^2 - h^2}}{l}}{mg - F \times \frac{h}{l}}$$

$$\mu = \frac{F\sqrt{l^2 - h^2}}{mgl - Fh}$$

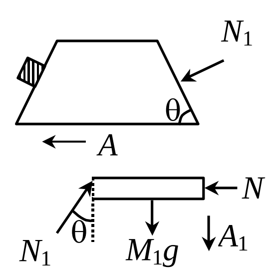
20. 3M

Let M_1 be the mass of the rod.

$M_1 g - N_1 \cos\theta = M_1 A_1 \qquad \ldots$ (i)

$N_1 \sin\theta = (M + M)A \qquad \ldots$ (ii)

$A = g\tan\theta \qquad \ldots$ (iii)



relation between A_1 and A

$A_1 = A\tan\theta$

So by solving these equations $\boldsymbol{M_1 = 3M}$

JEE Advanced : PYQ

1. (B)

Let the length of spring is 3ℓ.

$\Rightarrow K \cdot 3\ell = K' \cdot 2\ell$

$\Rightarrow K' = \frac{3}{2}K$

2. (C)

For equilibrium,

$T_1 = mg; T_2 = mg$

$T_1 \cos\theta_1 + T_2 \cos\theta = \sqrt{2}mg$

$T_1 \sin\theta_1 = T_2 \sin\theta$

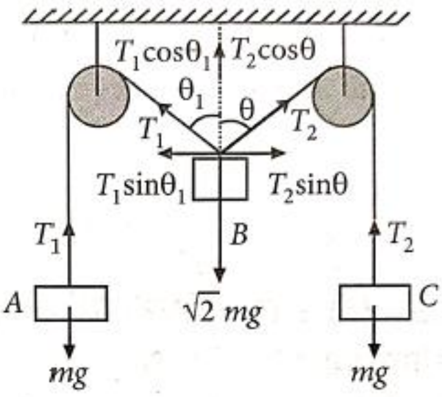
$mg \sin\theta_1 = mg \sin\theta$

$\therefore\ \theta_1 = \theta$ or $T_1 = T_2$

$mg \cos\theta + mg \cos\theta = \sqrt{2}mg$

$2\cos\theta = \sqrt{2}$

or $\theta = 45°$



3. (D)

Weight of the pulley *mg* acts downwards.

Weight of the block *Mg* acts downwards.

Total downward force $F_1 = (m+M)g$

Tension in the sting $T = Mg$

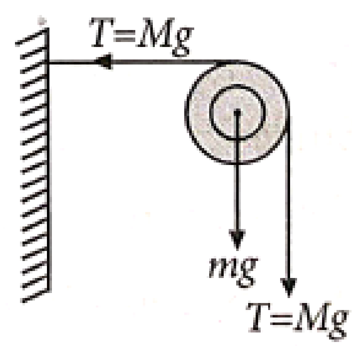
The force on the pulley by reaction of the clamp is equal to the resultant of F_1 and *T*.

Let it be *F*.

$$F^2 = T^2 + F_1^2 \text{ or } F_2 = (Mg)^2 + [(m+M)g]^2$$

or $$F_2 = \left[M^2 + (M+m)^2\right]g^2$$

or $$F = \sqrt{(M+m)^2 + M^2}\ g$$



4. (A)

The insect *I* is under the action of two forces. *R* denotes the normal reaction while *mg* denotes the weight of the insect. The weight *mg* can be resolved into two ⊥ components.

For equilibrium

$R = mg\cos\theta$

$f = mg\sin\alpha$

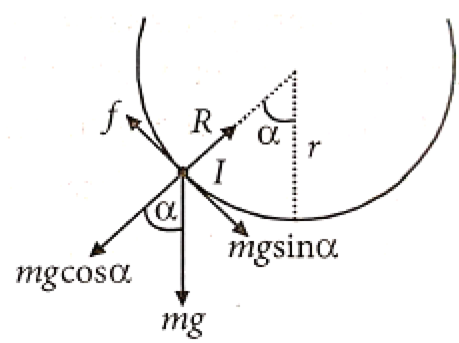
Where *f* denotes force of friction.

or $\mu R = mg\sin\alpha$

or $\mu \times (mg\cos\alpha) = mg\sin\alpha$

or $\mu = \tan\alpha$

or $\frac{1}{3} = \frac{1}{\cot\alpha}$ or $\cot\alpha = 3$.



5. (A)

Just before the string is cut, force on the spring pulling up $= kx = 3mg$

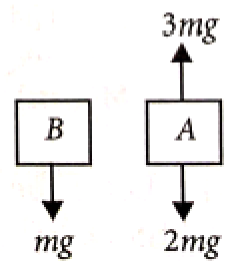
Therefore after string is cut, free body diagram of block A

$\therefore \quad 2ma_A = 3mg - 2mg$

or $a_A = \dfrac{mg}{2m} = \dfrac{g}{2}$

free body diagram of block B

$\therefore \; ma_B = mg$ or, $a_B = g$



6. (B)

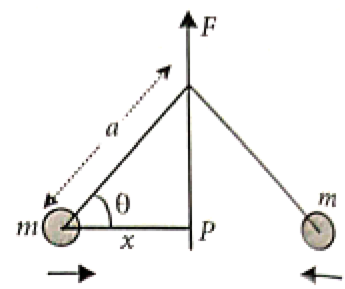
The arrangement is shown in the figure.

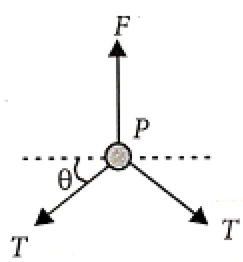
The separation between the two masses is $2x$.

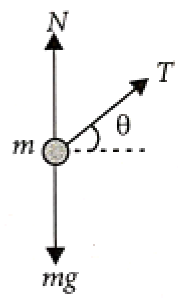
Each mass will move in the horizontal direction as shown in the figure.

Let the tension in the string be T.

The force acting at point P and on one of the masses are shown in the figure.







Net force at point P must equal zero. Why? This condition gives

$$2T\sin\theta = F \qquad ...(i)$$

Also, for the mass m,

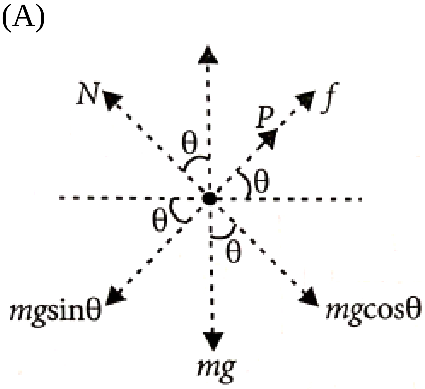
$$N + T\sin\theta - mg = 0 \qquad ...(ii)$$

and $$T\cos\theta = mA \qquad ...(iii)$$

Equaitons (i) and (iii) give

$$A = \frac{F\cot\theta}{2m} = \frac{F}{2m}\left(\frac{x}{\sqrt{a^2 - x^2}}\right).$$

7. (A)



From FBD of block, $N = mg\cos\theta$

$P+f=mg\sin\theta$ or $f=mg\sin\theta-P$

As P varies from $mg(\sin\theta-\mu\cos\theta)$ to $mg(\sin\theta+\mu\cos\theta)$,

f varies from $+\mu mg\cos\theta$ to $-\mu mg\cos\theta$.

This value of friction is always less than or equal to μN in magnitude.

Hence option (A) is correct.

8. (D)

Given : momentum $\vec{p}(t)=A\left[\hat{i}\cos(kt)-\hat{j}\sin(kt)\right]$

And, force, $\vec{F}=\frac{d\vec{p}}{dt}=Ak\left[-\hat{i}\sin(kt)-\hat{j}\cos(kt)\right]$

Here, $\vec{F}.\vec{P}=0$ But $\vec{F}.\vec{p}=Fp\cos\theta$

$\therefore\ \cos\theta=0 \Rightarrow \theta=90^{o}$.

Hence, angle between the force momentum, $\theta=90^{o}$

9. (B)

Given : $m_1=5$ kg; $m_2=10$ κγ; $\mu=0.15$ m

FBD for m_1, $m_1g-T=m_1a$

$\Rightarrow\ 50-T=5\times a$ and $T-0.15(m+10)g$

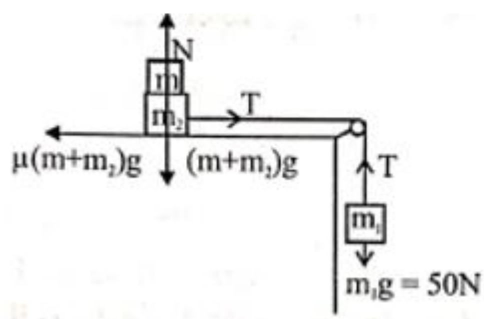
$=(10+m)a$

From rest $a=0$

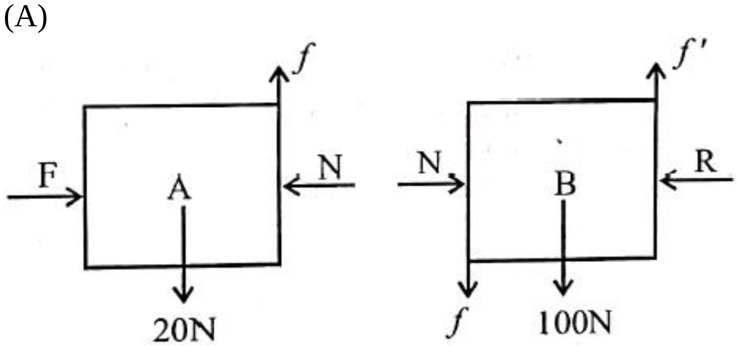
Or, $50=0.15(m+10)10$

$\Rightarrow\ 5=\frac{3}{20}(m+10)$

$\frac{100}{3}=m+10\quad\therefore\ m=23.3$ kg; close to option (B)



10. (A)



Along vertical direction

$A\rightarrow f=20\,\text{N}$

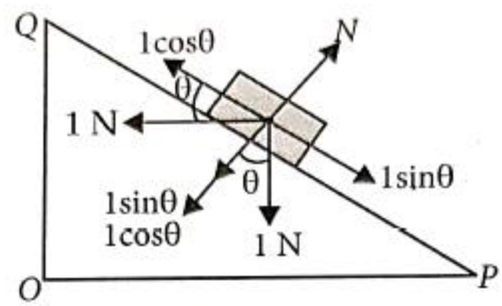
$B\rightarrow f'=f+100=20+100=120\,\text{N}$

11. (B, C)

Here, $m = 0.1$ kg, $g = 10\,ms^{-2}$

$\therefore\ mg = 0.1\times10 = 1N$

The various force acting on the block are as shown in the figure.



If $\theta = 45°$, then $\cos\theta = \sin\theta$

If plane is rough and $\theta > 45°$, then $\sin\theta > \cos\theta$

So frictional force acts on the block towards Q.

If plane is rough and $\theta < 45°$, then $\cos\theta > \sin\theta$

So frictional force acts on the block towards P.

12. (A)

Given, $v = \alpha\left(yx + 2xy\right)$ ………. (i)

$\therefore$ Acceleration of the particle,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[2yX + 2\alpha xY\right]$$

$$= \alpha\frac{dy}{dt}x + 2\alpha\frac{dx}{dt}Y$$

$\Rightarrow\ a = \alpha v_y x + 2\alpha v_x y$ ……… (ii)

And $v_y = 2x\alpha$

Putting these values in eq. (ii),we get

$$a = \alpha(2x\alpha)x + 2\alpha(\alpha y)Y$$

$\Rightarrow\ a = 2x\alpha^2 x + 2\alpha^2 yY$

$\therefore$ Force, $F = ma$

$$= 2m\alpha^2 X + 2m\alpha^2 yY$$

$$= 2m\alpha^2\left(xx + yY\right)$$

13. (i) Zero (ii) $\dfrac{2\sqrt{2}mg}{3}$ (iii) $\dfrac{mg}{3\sqrt{2}}$

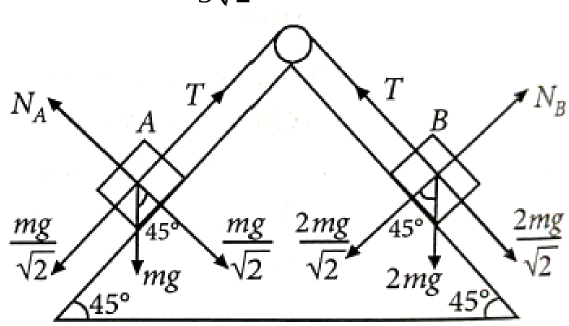
Given :

$\mu_{AW} = \dfrac{2}{3}$

$\mu_{BW} = \dfrac{1}{3}$

Mass of $A = m$

Mass of $B = 2m$



(i) The system consists of the two masses. When A moves upwards, B move downwards. Force causes this motion and friction oppose it.

$$\text{Force} = \frac{2mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}}$$

or Force $= \frac{mg}{\sqrt{2}}$ = This may cause motion.

Force of friction $= f_A + f_{BA} = \mu_A N_A + \mu_B N_B$

$$= \left(\frac{2}{3}\right)\left(\frac{mg}{\sqrt{2}}\right) + \left(\frac{1}{3}\right) = \frac{4mg}{3\sqrt{2}}$$

Since the magnitude of force of friction is greater than the force which may cause motion of A and B, the mass-system will not move.

Hence, acceleration of the system is zero.

(ii) Consider equilibrium of B :

When only B is considered, we have

Force causing motion $= \frac{2mg}{\sqrt{2}}$

Force of friction $= \mu_B N_B = \left(\frac{1}{3}\right) \times \left(\frac{2mg}{\sqrt{2}}\right) = \frac{1}{3}\left(\frac{2mg}{\sqrt{2}}\right)$

T = Difference of above two forces on B

or $T = \frac{2mg}{\sqrt{2}} - \frac{1}{3}\left(\frac{2mg}{\sqrt{2}}\right) = \frac{2}{3} \times \frac{2mg}{\sqrt{2}}$ or $T = \frac{2\sqrt{2}mg}{3}$

(iii) Force of friction on block A :

T acts upwards while weight-component acts downwards on block A.

Force of friction $= T -$ (Weight – Component of A)

$$= \frac{2\sqrt{2}\, mg}{3} - \frac{mg}{\sqrt{2}} = \frac{4mg - 3mg}{3\sqrt{2}}$$

Force of friction of A $= \frac{mg}{3\sqrt{2}}$ down the plane.

Hence (i) Acceleration of A = zero

(ii) Tension in string $= \frac{2\sqrt{2}\, mg}{3}$

(iii) Force of friction on $A = \frac{mg}{3\sqrt{2}}$ down the plane.

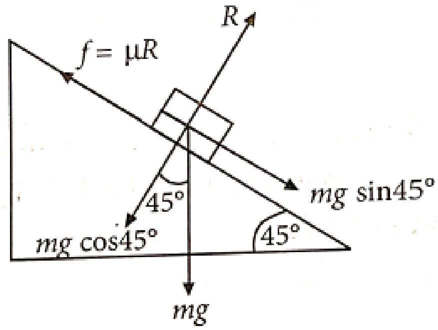
14. $S_A = 8\sqrt{2}m,\ 2s$

For motion of a body, down the inclined plane, we have

Force on body $= mg\sin 45^\circ - \mu R$

$m\times$ acceleration $= mg\left(\sin 45^\circ - \mu mg\cos 45^\circ\right)$

or $ma = mg\left(\sin 45^\circ - \mu\cos 45^\circ\right)$



or $ma = \frac{mg}{\sqrt{2}}(1-\mu)$ or $a = \frac{g(1-\mu)}{\sqrt{2}}$ ...(i)

For body A, $a_A = \frac{g(1-\mu_A)}{\sqrt{2}}$

or $a_A = \frac{10(1-0.2)}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}\ \text{m/s}^2$ ...(ii)

For body B, $a_B = \frac{g(1-\mu_B)}{\sqrt{2}}$

or $a_B = \frac{10(1-0.3)}{\sqrt{2}} = \frac{7}{\sqrt{2}} = 3.5\sqrt{2}\ \text{m/s}^2$ ...(iii)

A and B are released simultaneously. They travel for time t.

Let A slide by a distance S_A along plane and let B slide by a distance S_B along plane till they come on the same line on the inclined plane.

$\therefore$ $S_A = S_B + \sqrt{2}$

or $\frac{1}{2}a_A t^2 = \frac{1}{2}a_B t^2 + \sqrt{2}$, where t is time taken.

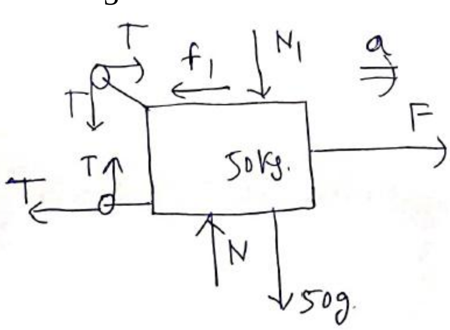
or $\frac{1}{2}\times(4\sqrt{2})\times t^2 = \frac{1}{2}\times(3.5\sqrt{2})\times t^2 + \sqrt{2}$, (from (ii) and (iii))

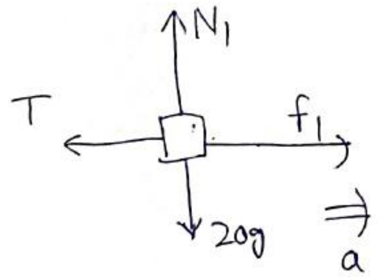
or $2t^2 = \frac{3.5}{2}t^2 + 1 \Rightarrow 0.5t^2 = 2 \Rightarrow t = 2\,\text{sec}$ ...(iv)

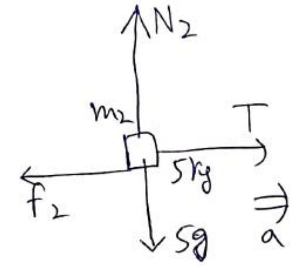
$\therefore$ $S_A = \frac{1}{2}a_A t^2 = \frac{1}{2}\times(4\sqrt{20})\times\sqrt{2}^2 = 8\sqrt{2}\ \text{m}$ ...(v)

Hence, $S_A = 8\sqrt{2}\ \text{m}$, $S_B = 7\sqrt{2}\ \text{m}$, $t = 2\ \text{second}$

15. (B) $a = \frac{3}{5}\text{m/s}^2$, $T = 18\,\text{N}$, $F = 60\,\text{N}$, $f_2 = 15\,\text{N}$, $f_1 = 30\,\text{N}$, $F = 60\,\text{N}$







$N_1 = 200\,\text{N}$

$f_{1\max} = \mu N_1 = 0.3 \times 200 = 60\,\text{N}$

$N_2 = 50\,\text{N}$

$f_{2\max} = \mu N_2 = 0.3 \times 50 = 15\,\text{N}$

Given, $f_1 = 2 f_2$

So, f_1 is static in nature

$f_1 - T = m_1 a$

$T - f_2 = m_2 a$

$\Rightarrow\ f_1 - f_2 = (m_1 + m_2) a \ \Rightarrow\ a = \dfrac{30-15}{25} = 0.6\ \text{m/s}^2$

$T = m_2 a + f_2 = 5 \times 0.6 + 15 = 18\,\text{N}$

Now, M & m_1 are single system.

$F - T = (M + m_1) a \ \Rightarrow\ a = \dfrac{30-15}{25} = 0.6\ \text{m/s}^2$

$T = m_2 a + f_2 = 5 \times 0.6 + 15 = 18\,\text{N}$

Now M & m_1 are single system.

$F - T = (M + m_1) a$

$\Rightarrow\ F = 70 \times 0.6 + 18 = 60\,\text{N}$

So, F = 60 N, T = 18 N, a = 0.6 m/s^2

$f_1 = 30\,\text{N},\ f_2 = 15\,\text{N}$

16. 10 m/s^2

Given : $\cos\theta = \dfrac{4}{5}, \sin\theta = \dfrac{3}{5}$, acceleration of disc $a = 25\ m/s^2$

$$\mu_k = \frac{2}{5}$$

The disc is placed horizontally. The groove gives a constraint mot move in a particular direction. The mass placed on the horizontal disc has a normal reaction $N_1 = mg$. In addition if the disc is moved to the left, the pseudo acceleration is acting on the mass in the horizontal direction to the right. In vertical component is $ma\sin\theta$.

The normal reaction N_2 is $mg\sin\theta$.

$\therefore$ $ma\cos\theta$ is acting down the plane and the force of friction perpendicular to the normal reaction and opposing the motion give the resultant force

$R = ma\cos\theta - \mu_k mg - \mu_k ma\sin\theta$

$\therefore$ Acceleration of the mass with respect to the disc is a'

$$a' = \frac{R}{m} = a\cos\theta - \mu_k g - \mu_k a\sin\theta$$

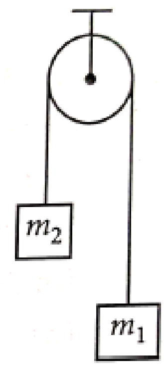
$$a' = 25\left(\frac{4}{5}\right) - \left(\frac{2}{5}\right)10 - \left(\frac{2}{5}\right)(25)\left(\frac{3}{5}\right)$$
$$= 20 - 4 - 6 = 10 \text{ m/s}^2 .$$

17. 8

$m_1 = 0.72$ kg; $m_2 = 0.36$ kg

$m_1 g - T = m_1 a$

$T - m_2 g = m_2 a$



$$a = \frac{(m_1 g - m_2 g)}{m_1 + m_2} = \frac{0.36}{1.08} g = \frac{g}{3}$$

$$T = \frac{2m_1 m_2}{m_1 + m_2} g = \frac{2 \times 0.72 \times 0.36}{1.08} \times 10$$

$\Rightarrow \quad T = 4.8$ N

Distance travelled in the first second is

$$S = ut + \frac{1}{2}at^2 = \frac{a}{2} \text{ as } \mu = 0 \Rightarrow S = \frac{10}{6}$$

Work done by $T = T \cdot S = 4.8 \times \frac{10}{6} = 8$ J

18. 5

Force required to push the block up the inclined plane is

$$F_u = mg \sin\theta + \mu mg \cos\theta \quad \text{...(i)}$$

Force required to just prevent the block from sliding down is

$$F_d = mg \sin\theta - \mu mg \cos\theta \quad \text{...(ii)}$$

According to the problem, $F_u = 3F_d$

$\therefore \quad mg \sin\theta + \mu mg \cos\theta = 3(mg \sin\theta - \mu mg \cos\theta)$

Or $\quad \sin\theta + \mu \cos\theta = 3(\sin\theta - \mu \cos\theta)$

$$\frac{1}{\sqrt{2}} + \frac{\mu}{\sqrt{2}} = 3\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right) \quad (\because \theta = 45^\circ \text{ (Given)})$$

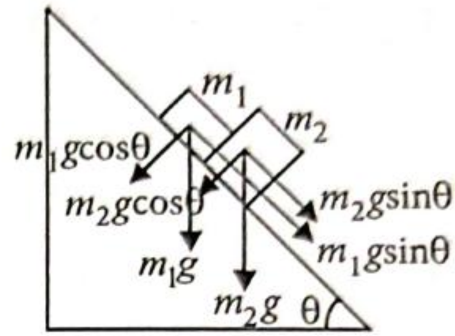
$$1 + \mu = 3 - 3\mu; \; 4\mu = 2 \text{ or } \mu = \frac{1}{2}$$

$N = 10$ m;

$\therefore \quad N = 10 \times \frac{1}{2} = 5$

19. (D)

If the blocks are not slipping then net force on the blocks down the inclined plane is equal to the frictional force on block of mass m_2.



Maximum frictional force on the block m_2,

$f_l = \mu m_2 g \cos\theta$

Blocks will not slip if

$\mu m_2 g \cos\theta \geq (m_1 + m_2) g \sin\theta$

$\Rightarrow \tan\theta \leq \dfrac{\mu m_2}{m_1 + m_2} = \dfrac{0.3 \times 2}{1+2} = 0.2$

Hence, for angles less than $11.5°$, blocks will not be slipping on the inclined plane and friction is static and equal to $(m_1 + m_2) g \sin\theta$ downward force on the blocks.

For angles greater than $11.5°$, blocks will slip on the inclined plane and friction is kinetic and equal to $\mu m_2 g \cos\theta$.

So, **P - 2, Q - 2, R - 3, S - 3**