Circular Motion

JEE Main Exercise

1. (c)

$$\theta = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$\Rightarrow \theta = \left(\frac{20+0}{2}\right)(5) = 50 \text{ rad}$$

Number of revolutions $= \frac{\theta}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi}$

2. (d)

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

$$\because T_1 = T_2 \Rightarrow \frac{\omega_1}{\omega_2} = 1$$

3. (a)

$$a_T = \alpha$$

$$\Rightarrow r\alpha = \alpha$$

$$\Rightarrow r = 1 \text{ m}$$

So, diameter = 2*m*

4. (c)

Centripetal acceleration $(a_c) = \frac{v^2}{R} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2$

Tangential acceleration $(a_T) = \frac{dv}{dt} = 2 \text{ m/s}^2$

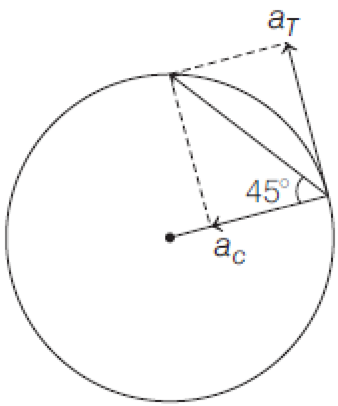
Resultant acceleration $= \sqrt{{a_c}^2 + {a_T}^2}$

$$= \sqrt{(1.8)^2 + 2^2}$$

$$= \sqrt{7.24} = 2.7 \text{ m/s}^2$$

5. (b)

$\tan 45° = \dfrac{a_T}{a_c} = \dfrac{9}{(v^2/4)}$



$\Rightarrow \quad v^2 = 36$

$\Rightarrow \quad v = 6 \text{ m/s}$

and $\quad v = u + at$

$\Rightarrow \quad 6 = 0 + 9t$

$\Rightarrow \quad t = \dfrac{2}{3} \text{ s}$

6. (b)

$a_c = k^2rt^2 \Rightarrow \dfrac{v^2}{r} = k^2rt^2$

$\Rightarrow v = krt$

$\Rightarrow a_T = \dfrac{dv}{dt} = kr$

$P = Fv\cos 0° = (ma_T)v\cos 0°$

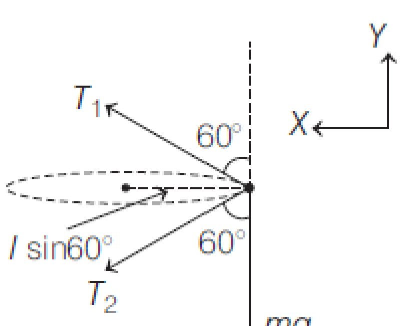
$= mkrkrt = mk^2r^2t$

7. (b)

$v = \omega r$ and $a = \omega^2 r$

If radius becomes half, then both v and a will become half of the original values.

8. (b)



$\sum F_x = ma$

$\Rightarrow \quad T_1 \sin 60° + T_2 \sin 60° = m\omega^2 (l \sin 60°) \qquad \ldots(\text{i})$

$$\sum F_y = 0$$

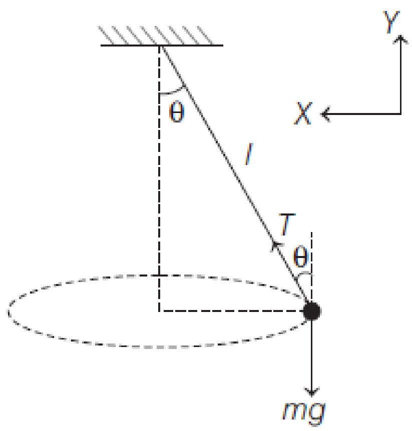
$\Rightarrow \quad T_1 \cos 60° = T_2 \cos 60° + mg$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$\omega = 14 \text{ rad/s}$$

9. (c)

$$\sum F_y = 0$$



$\Rightarrow \quad T \cos\theta = mg$ ...(i)

$$\sum F_x = ma_x$$

$T \sin\theta = m\omega^2 (l \sin\theta)$ ...(ii)

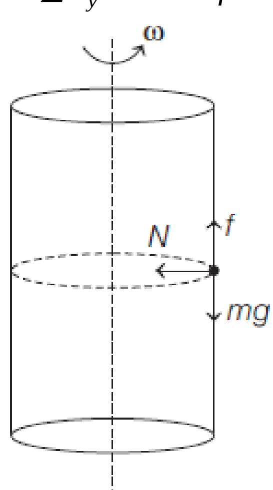
From Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{l\cos\theta}}$$

$$\Rightarrow \quad T = \frac{2\pi}{2\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

10. (a)

$$\sum F_y = 0 \Rightarrow f = mg$$



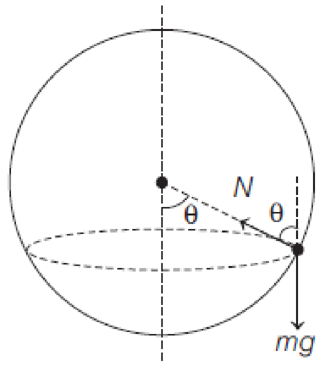
$$\sum F_x = 0 \Rightarrow N = m\omega^2 r$$

and $f \leq f_L$

$$\Rightarrow \quad mg \le \mu\left(m\omega^2 r\right)$$

$$\Rightarrow \quad \omega \ge \sqrt{\frac{g}{\mu r}}$$

11. (b)



$$\sum F_y = 0 \Rightarrow N\cos\theta = mg \qquad ...(i)$$

$$\sum F_x = ma_x \Rightarrow N\sin\theta = m\omega^2\left(\frac{a}{2}\right) \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\omega = \left(\frac{2g}{\sqrt{3}a}\right)^{1/2}$$

12. (c)

$$v = \sqrt{rg\tan\theta} = \sqrt{(10\sqrt{3})(10)\tan 30^\circ}$$

$$= 10 \text{ m/s} = 36 \text{ km/h}$$

13. (b)

$$\mu_{\min} = \frac{v^2}{gr} = \frac{\left(72\times\frac{5}{18}\right)^2}{10(200)} = 0.2$$

14. (a)

$$v_{\max} = \sqrt{\frac{gr(\tan\theta+\mu)}{1-\mu\tan\theta}} = \sqrt{\frac{10\times 20\left(\frac{3}{4}+\frac{1}{4}\right)}{1-\frac{3}{4}\times\frac{1}{4}}}$$

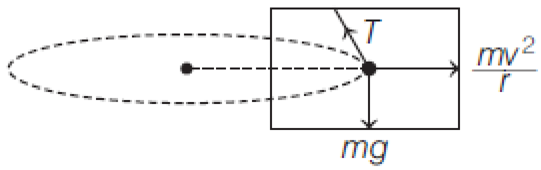
$$= 10\sqrt{\frac{32}{13}} \text{ m/s}$$

15. (b)

$$\tan\theta = \frac{a}{g} = \frac{v^2/r}{g}$$

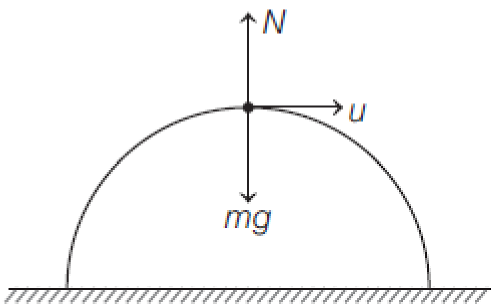
$$\Rightarrow \tan\theta = \frac{v^2}{rg} = \frac{10^2}{10(10)} = 1$$

$$\Rightarrow \theta = 45°$$



16. (a)

$$mg - N = \frac{mu^2}{R}$$



For losing contact, $N = 0$

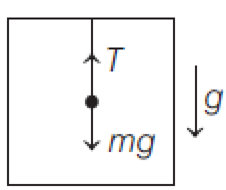
$$\Rightarrow \quad mg = \frac{mu^2}{R}$$

$$\Rightarrow \quad u = \sqrt{gR}$$

17. (d)

In free fall, acceleration of car is g.

For pendulum,



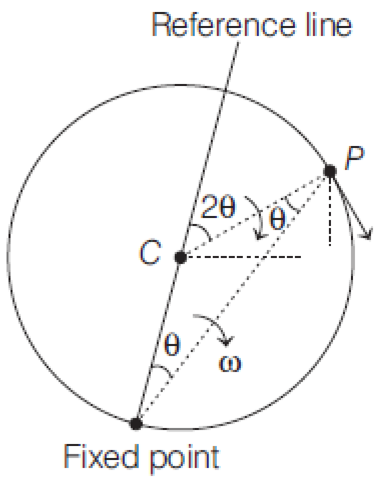
$$\sum F = ma$$

$$\Rightarrow \quad mg - T = m(g)$$

$$\Rightarrow \quad T = 0$$

18. (4)

$$\frac{d\theta}{dt} = 2 \text{ rad/s}$$



Angular velocity of particle w.r.t the centre of circle

$$= \frac{d(2\theta)}{dt}$$

$$= 2\frac{d\theta}{dt} = 4 \text{ rad/s}$$

$$v = \omega r = 4(1) = 4 \text{ m/s}$$

19. (1)

$$\alpha = \frac{\pi}{4} \text{rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \frac{\pi}{2} = 0 + \frac{1}{2} \times \left(\frac{\pi}{4}\right) t^2$$

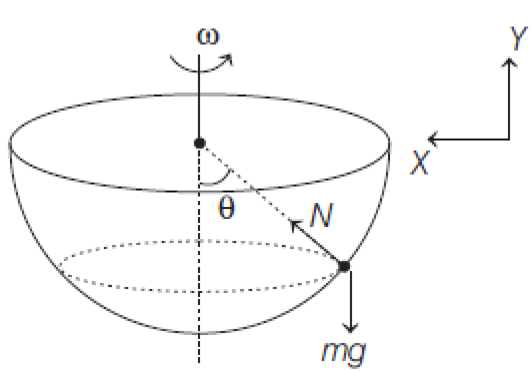
$$\Rightarrow t = 2\text{s}$$

$$v_{\text{avg}} = \frac{\Delta S}{\Delta t} = \frac{\sqrt{2}R}{2} = 1 \text{ m/s}$$

20. (100)

$\sum F_y = 0 \Rightarrow N\cos\theta = mg$ ..... (i)

$\sum F_x = ma_x \Rightarrow N\sin\theta = m\omega^2 R\sin\theta$ ..... (ii)



Solving Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{R\cos\theta}}$$

$$\omega = \sqrt{\frac{10}{0.5\left(\frac{0.3}{0.5}\right)}} = \sqrt{\frac{100}{3}} \text{ rad/s}$$

21. (2)

$a_c = 2t^2$

$\Rightarrow \omega^2(2) = 2t^2$

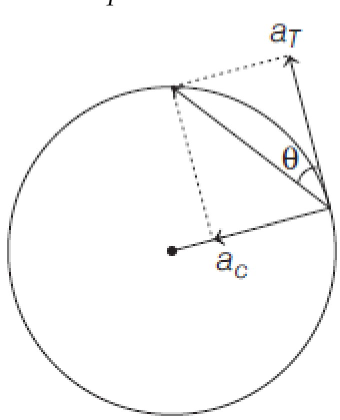
$\Rightarrow \omega = t$

$\Rightarrow \int d\theta = \int_0^2 t\,dt$

$\Rightarrow \theta = \left[\frac{t^2}{2}\right]^2 = 2 \text{ rad}$

22. (2)

$$\tan\theta = \frac{a_c}{a_T} = \frac{\omega^2 R}{R\alpha}$$



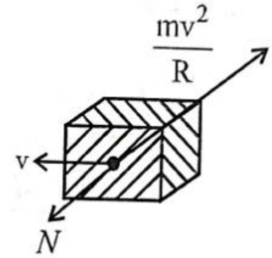
$$\tan\theta = \frac{(\alpha t)^2}{\alpha} = \alpha t^2$$

$$= 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

1. (a)
For observer on block,

$$N=\frac{mv^2}{r} \Rightarrow N \propto v^2$$

So, curve is parabola, symmetric about *N*-axis



2. (b)

$$\Delta\vec{V}=\left|\vec{V}-\vec{u}\right|=\sqrt{v^2+u^2} \qquad [\because \vec{V}\pm\vec{u}\,]$$

Now, $K_1+P_1=K_2+P_2$

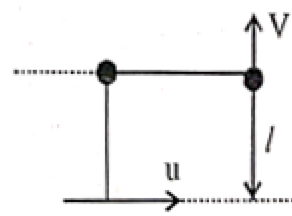
$$\Rightarrow \frac{1}{2}mu^2+0=\frac{1}{2}mV^2+mg\,l$$

$$\Rightarrow V^2=u^2-2g\,l$$

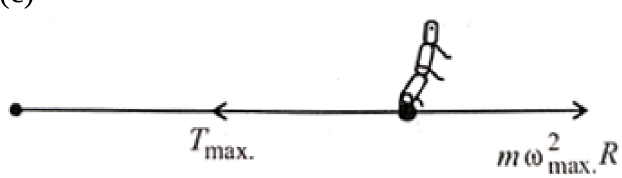
$$\Rightarrow V=\sqrt{u^2-2gl}$$

So, $\Delta V=\sqrt{V^2+u^2}=\sqrt{u^2-2gl+u^2}$

$$\sqrt{2\left(u^2-gl\right)}$$



3. (c)

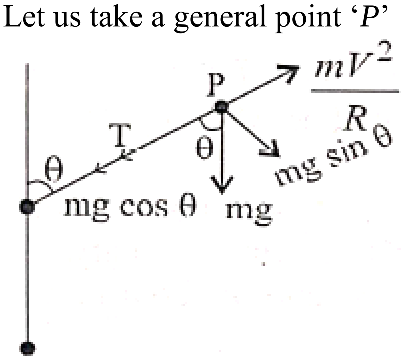


$$T_{max}=m\omega^2{}_{max}R$$

$$80=0.1\times\left(\frac{k}{30}\right)^2\times 2 \qquad \left[\because\ \omega=\frac{k}{\pi}\times\frac{2\pi}{60}=\frac{k}{30}\right]$$

$$\Rightarrow k^2=\frac{30^2\times 80}{2\times 0.1} \Rightarrow k^2=3600000 \Rightarrow k=600$$

4. (b)
Let us take a general point '*P*'



At *P*

$$T + mg\cos\theta = \frac{mv^2}{R} \Rightarrow T = \frac{mv^2}{R} - mg\cos\theta$$

So, T will be minimum when, $mg\cos\theta$ is maximum

i.e., when $\cos\theta$ is maximum

i.e., when $\theta = 0$ and θ is zero when string is at highest point.

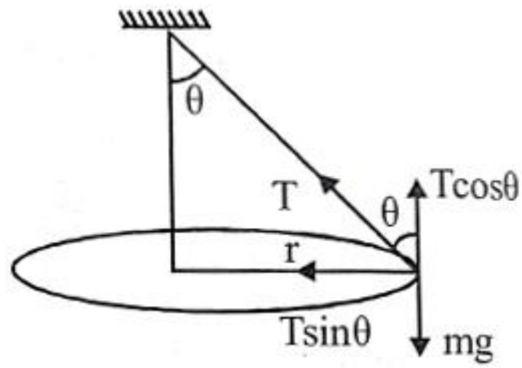
5. (a)

From figure,

$$\sin\theta = \frac{r}{L} = \frac{L}{L\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow q = 45°$$

$$T\sin\theta = \frac{mv^2}{r} \qquad ...(i)$$

$$T\cos\theta = mg \qquad ...(ii)$$



From (i) and (ii)

$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$$

6. (b)

Given, mass of the block, $m = 200\ g = 200 \times 10^{-3}$ Kg

Radius of the circular groove, $r = 20\,\text{cm}$

Time taken to complete one round, $T = 40\,\text{s}$

Here, normal force will provide the necessary centripetal force.

$$N = m\omega^2 r = 200 \times 10^{-3} \times \left(\frac{2\pi}{40}\right)^2 \times 0.2 = 9.859 \times 10^{-4}\ \text{N}.$$

7. (c)

For statement-1

The maximum speed by which cyclist can take a turn one a circular path

$$\Rightarrow \quad v \le \sqrt{\mu r g} \le \sqrt{0.2 \times 2 \times 9.8} \Rightarrow v \le \sqrt{3.92}$$

Speed of cyclist, $7\ \text{kmh}^{-1} = 7 \times \frac{5}{18} = 1.94\ \text{m/s}$

The maximum safe speed on a banked frictional road

$$v_{\text{allowable}} = \sqrt{r \cdot g\,\frac{(\mu + \tan\theta)}{1 - \mu\tan\theta}}$$

$$\Rightarrow \quad v = \sqrt{\frac{2 \times 9.8(0.2 + \tan 45°)}{1 - 0.2 \times \tan 45°}}\sqrt{\frac{2 \times 9.8 \times 1.2}{0.8}}$$

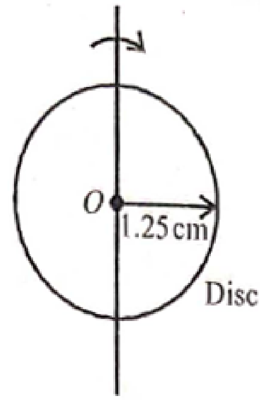
$$= 5.42\ \text{m/s}$$

Speed of cyclist, $18.5\ \text{kmh} = 5.13\ \text{m/s}$

So, both the statement are true.

8. (d)

Using, $\mu mg = \frac{mv^2}{r} = mr\omega^2$



$\omega = 2\pi n = 2\pi \times 3.5 = 7\pi$ rad/sec

Radius, $r = 1.25\text{ cm} = 1.25 \times 10^{-2}$ m

Coefficient of friction, $\mu = ?$

$$\mu mg = \frac{m(r\omega)^2}{r} (\because\ v = r\omega)$$

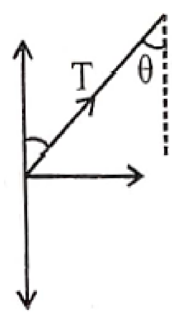
$$\mu mg = mr\omega^2$$

$$\Rightarrow\ \mu = \frac{r\omega^2}{g} = \frac{1.25 \times 10^{-2} \times \left(7 \times \frac{22}{7}\right)^2}{10}$$

$$= \frac{1.25 \times 10^{-2} \times 22^2}{10} = 0.6$$

9. (d)

Given, $\theta = 45°, r = 0.4\text{ m}, g = 10\text{ m/s}^2$



$$T\sin\theta = \frac{mv^2}{r} \qquad ...(i)$$

$$T\cos\theta = mg \qquad ...(ii)$$

From equation (i) & (ii) we have,

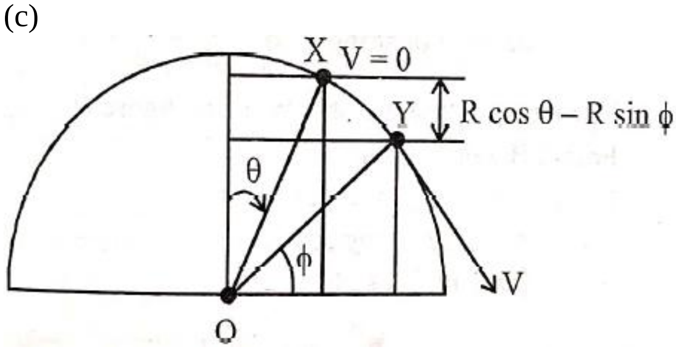
$$\tan\theta = \frac{v^2}{rg}$$

$$v^2 = rg \qquad \because\ \theta = 45^o$$

Hence, speed of the pendulum in its circular path,

$$v = \sqrt{rg} = \sqrt{0.4 \times 10} = 2\text{ m/s}$$

10. (c)



$$K.E._X + P.E._X = K.E._Y + P.E._Y$$

$$\Rightarrow\ \frac{1}{2}mV^2 = mgR(\cos\theta - \sin\phi)$$

$$\Rightarrow\ V^2 = 2gR(\cos\theta - \sin\phi)$$

At Y, $\frac{mv^2}{R} = mg\sin\phi\ [\because\ N_Y = 0]$

$\Rightarrow \; 2mg(\cos\theta - \sin\phi) = mg\sin\phi$

$\Rightarrow \; \sin\phi = \frac{2}{3}\cos\theta$

11. (a)

Initially, $k \times 1 = m\omega^2 R$ ...(i)

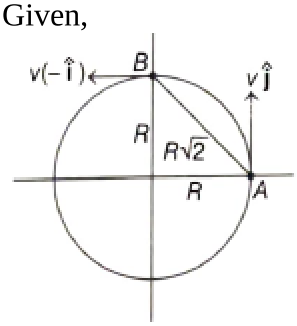
Finally, $k \times 5 = m(2\omega)^2(R+4)$ ...(ii)

From (i) & (ii), we get : $R = 16$

So, original length = (16 – 1) cm = 15 cm.

12. (b)

Given,



$v_A = v_{\hat{j}}, v_B = -v_{\hat{i}}$

Time to reach from A to B

$= \frac{2\pi R}{4} \times \frac{1}{v} = \frac{\pi R}{2v}$

Displacement from A to B $= R\sqrt{2}$

Now, average velocity from A to B

$= \frac{\text{displacement}}{\text{time}}$

$= \frac{R\sqrt{2}}{\frac{\pi R}{2v}} = \frac{R\sqrt{2} \times 2v}{\pi R} = \frac{2\sqrt{2}v}{\pi}$

Instantaneous velocity at B is $v\hat{i}$

According to equation

$= \frac{\text{Instantaneous velocity}}{\text{Avergae velocity}}$

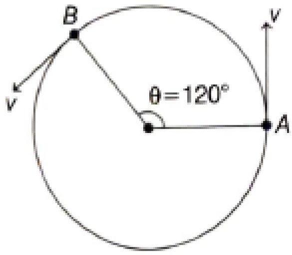
$= \frac{\pi}{x\sqrt{2}}$

$\frac{v}{\frac{2\sqrt{2}v}{\pi}} = \frac{\pi}{x\sqrt{2}} \Rightarrow \frac{\pi}{2\sqrt{2}} = \frac{\pi}{x\sqrt{2}}$

Thus, x = 2

13. (c)

Given,



Given, $v = \pi \, m/s$ or $R\omega = \pi$

$\omega = \frac{\pi}{R} rad/s$

Angular displacement,

$\theta = 120^\circ$ or $\frac{2\pi}{3}$

Using $\theta = \omega t$

$$t = \frac{\theta}{\omega} = \frac{2\pi/3}{\pi/R} = \frac{2R}{3}$$

Linear disbarment,

$$d = 2R\sin(\theta/2)$$

$$d = 2R\sin\left(\frac{120^\circ}{2}\right) = 2R\sin 60^\circ$$

$$= 2R\frac{\sqrt{3}}{2} \Rightarrow R\sqrt{3}$$

Average velocity

$$= \frac{d}{t} = \frac{R\sqrt{3}}{2R/3} = \frac{3\sqrt{3}}{2}$$

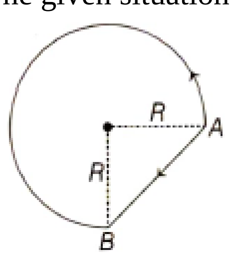
$$= 1.5\sqrt{3} m/s$$

14. (b)

Time period, $T = 4s$

Body will cover $\frac{3}{4}$ th part of circle in 3s.

The given situation can be drawn as



Displacement,

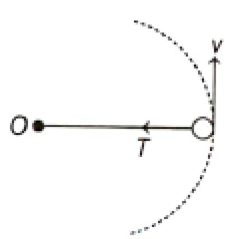
$$AB = \sqrt{R^2 + R^2} = \sqrt{2}R$$

$$= \sqrt{2} \times 10 = 10\sqrt{2}m$$

15. (a)
Given,
$m = 200\text{ kg}, r = 70\text{ m}$
$\omega = 0.2\text{ rad/sec}$
$F_C = m\omega^2 r = 200 \times (0.2)^2 \times 70$
$= 200 \times 0.04 \times 70$
$= 2 \times 4 \times 70 = 560\text{N}$

16. (a)
Given, $m = 5\text{ kg}, r = 2\text{ m}$
Time t for 1 rev = 3.14 sec π s
θ for $1\text{ rev} = 2\pi\text{ rad}$
Therefore $\omega = \frac{\theta}{t} = \frac{2\pi}{\pi} = 2\text{ rad/s}$
Centrifugal force, $F = m\omega^2 r$
$F = 5 \times 2 \times 2^2$
$= 40\text{ N}$

17. (d)
Given, $m = 1\text{ kg}, l = 1\text{m}$
Breaking tension, $T_{max} = 400\text{N}$
(braking tension)
Since, stone rotates horizontally,
Thus, $T_{max} = \frac{mv_{max}^2}{l}$
Or $v_{max}^2 = \frac{T_{max} l}{m}$
or $v_{max} = \sqrt{\frac{T_{max}}{m} l}$
$= \sqrt{\frac{400 \times 1}{1}}$
Or $v_{max} = 20\text{m/s}$



18. (c)
Maximum safe speed of car over a horizontal turn is $v_{max} = \sqrt{\mu r g}$
Here, $r = 50\text{m}, \mu = 0.34, g = 10$
So, $v_{max} = \sqrt{0.34 \times 50 \times 10}$
$= \sqrt{170} \approx 13\text{m/s}$

19. (c)

A bob suspended from the roof of the car by a massless string, then angle (θ) between the string and the vertical,

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20^2}{40\times10}\right)$$

$$= \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$$

20. (c)

Extension in spring due to centripetal

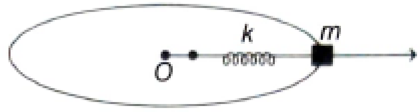
Force, $F = \frac{mv^2}{r} = mr\omega^2$

Given, $m = 200\text{ g} = 0.2\text{ kg}$

$r = x_0 + x$

$=$ natural length + extension

$\because \quad F = kx$



Where, $k = 12.5\text{N}/\text{m}$

Thus, $kx = m(x_0 + x)\omega^2$

$12.5x = 0.2(x_0 + x)5^2$

$12.5x = 5x_0 + 5x$

$\Rightarrow \quad 7.5x = 5x_0$

$\Rightarrow \quad \frac{x}{x_0} = \frac{5}{7.5} = \frac{1}{1.5} = \frac{2}{3}$

21. (b)

Given,

$f_r = m\omega^2 r$

$\Rightarrow \quad \mu mg = m\omega^2 r = \text{constant}$

$\omega^2 r = \text{ constant}$

$\omega_1^2 r_1 = \omega_2^2 r_2$

Given, $r_1 = 1, \omega_2 = \frac{\omega}{2}$ and $\omega_1 = \omega$

$\therefore \quad \omega^2(1) = \left(\frac{\omega}{2}\right)^2 \cdot r_2$

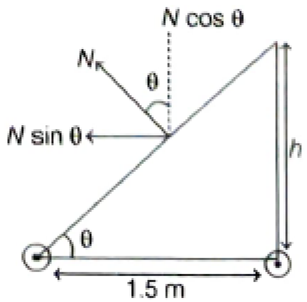
$\Rightarrow \quad r_2 = 4\text{ cm}$

22. (b)

Resolving Normal,

$N \cos\theta = mg$ ... (i)

$N \sin\theta = \frac{mv^2}{R}$ … (ii)



Dividing, $\tan\theta = \frac{v^2}{Rg}$

$\frac{h}{1.5} = \frac{12\times12}{400\times10}$

$h = \frac{1.5\times12\times12}{400\times10} = \frac{54}{1000} = 0.054\ \text{m}$

$= 5.4\ \text{cm}$