Circular Motion

JEE Main Exercise

1. (c)

$$\theta = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$\Rightarrow \theta = \left(\frac{20+0}{2}\right)(5) = 50 \text{ rad}$$

Number of revolutions $= \frac{\theta}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi}$

2. (d)

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

$$\because T_1 = T_2 \Rightarrow \frac{\omega_1}{\omega_2} = 1$$

3. (a)

$$a_T = \alpha$$

$$\Rightarrow r\alpha = \alpha$$

$$\Rightarrow r = 1\,\text{m}$$

So, diameter = 2m

4. (c)

Centripetal acceleration $(a_c) = \frac{v^2}{R} = \frac{(30)^2}{500} = 1.8\,\text{m/s}^2$

Tangential acceleration $(a_T) = \frac{dv}{dt} = 2\,\text{m/s}^2$

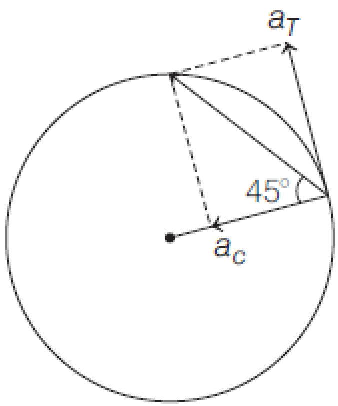
Resultant acceleration $= \sqrt{{a_c}^2 + {a_T}^2}$

$= \sqrt{(1.8)^2 + 2^2}$

$= \sqrt{7.24} = 2.7\,\text{m/s}^2$

5. (b)

$$\tan 45^\circ = \frac{a_T}{a_c} = \frac{9}{(v^2/4)}$$



$\Rightarrow \quad v^2 = 36$

$\Rightarrow \quad v = 6\text{ m/s}$

and $\quad v = u + at$

$\Rightarrow \quad 6 = 0 + 9t$

$\Rightarrow \quad t = \frac{2}{3}\text{ s}$

6. (b)

$a_c = k^2 r t^2 \;\Rightarrow\; \frac{v^2}{r} = k^2 r t^2$

$\Rightarrow v = krt$

$\Rightarrow a_T = \frac{dv}{dt} = kr$

$P = Fv\cos 0^\circ = (ma_T)v\cos 0^\circ$

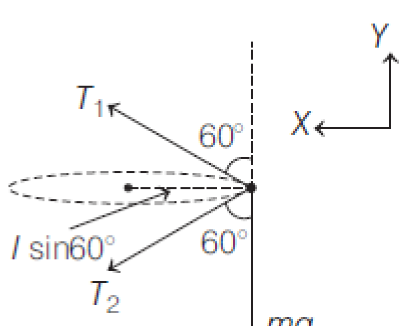
$= mkrkrt = mk^2r^2t$

7. (b)

$v = \omega r$ and $a = \omega^2 r$

If radius becomes half, then both v and a will become half of the original values.

8. (b)



$\sum F_x = ma$

$\Rightarrow \quad T_1\sin 60^\circ + T_2\sin 60^\circ = m\omega^2(l\sin 60^\circ) \quad \ldots(i)$

$$\sum F_y = 0$$

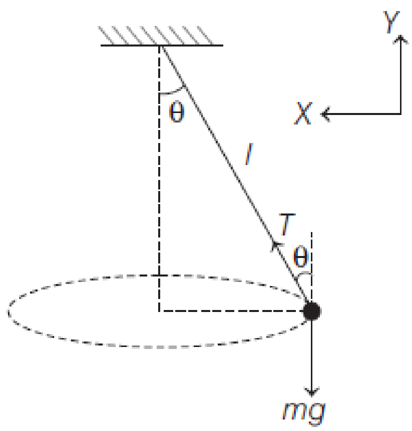
$\Rightarrow \quad T_1 \cos 60° = T_2 \cos 60° + mg$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$\omega = 14 \text{ rad/s}$$

9. (c)

$\sum F_y = 0$



$\Rightarrow \quad T\cos\theta = mg$ ...(i)

$$\sum F_x = ma_x$$

$T\sin\theta = m\omega^2 (l\sin\theta)$ ...(ii)

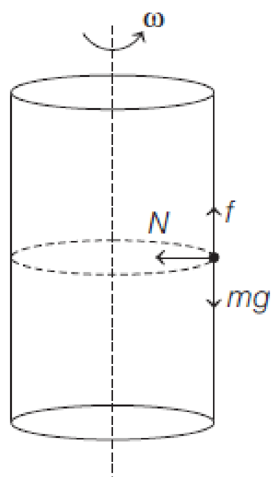
From Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{l\cos\theta}}$$

$$\Rightarrow \quad T = \frac{2\pi}{2\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

10. (a)

$$\sum F_y = 0 \Rightarrow f = mg$$



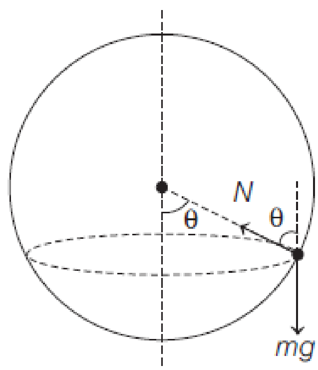
$$\sum F_x = 0 \Rightarrow N = m\omega^2 r$$

and $f \le f_L$

$$\Rightarrow \quad mg \le \mu\left(m\omega^2 r\right)$$

$$\Rightarrow \quad \omega \ge \sqrt{\frac{g}{\mu r}}$$

11. (b)



$$\sum F_y = 0 \Rightarrow N\cos\theta = mg \qquad ...(\text{i})$$

$$\sum F_x = ma_x \Rightarrow N\sin\theta = m\omega^2\left(\frac{a}{2}\right) \qquad ...(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\omega = \left(\frac{2g}{\sqrt{3}a}\right)^{1/2}$$

12. (c)

$$v = \sqrt{rg\tan\theta} = \sqrt{(10\sqrt{3})(10)\tan 30^\circ}$$
$$= 10 \text{ m/s} = 36 \text{ km/h}$$

13. (b)

$$\mu_{\min} = \frac{v^2}{gr} = \frac{\left(72\times\frac{5}{18}\right)^2}{10(200)} = 0.2$$

14. (a)

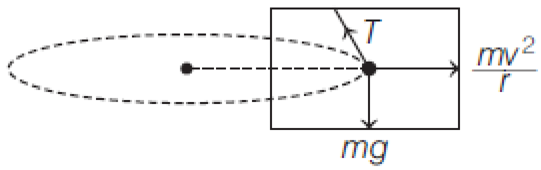
$$v_{\max} = \sqrt{\frac{gr(\tan\theta+\mu)}{1-\mu\tan\theta}} = \sqrt{\frac{10\times 20\left(\frac{3}{4}+\frac{1}{4}\right)}{1-\frac{3}{4}\times\frac{1}{4}}}$$
$$= 10\sqrt{\frac{32}{13}} \text{ m/s}$$

15. (b)

$$\tan\theta = \frac{a}{g} = \frac{v^2/r}{g}$$

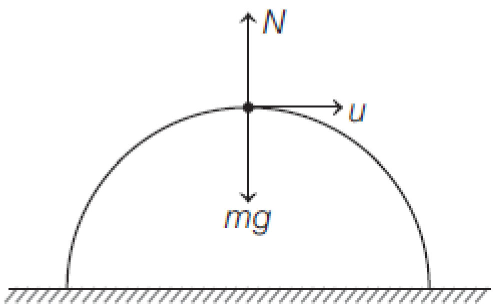
$$\Rightarrow \tan\theta = \frac{v^2}{rg} = \frac{10^2}{10(10)} = 1$$

$$\Rightarrow \theta = 45°$$



16. (a)

$$mg - N = \frac{mu^2}{R}$$



For losing contact, $N = 0$

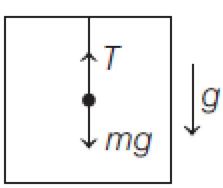
$$\Rightarrow \quad mg = \frac{mu^2}{R}$$

$$\Rightarrow \quad u = \sqrt{gR}$$

17. (d)

In free fall, acceleration of car is g.

For pendulum,



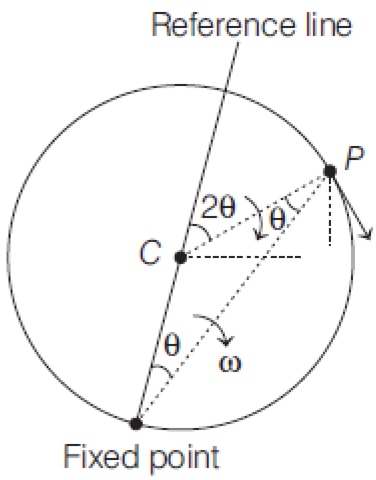
$$\sum F = ma$$

$$\Rightarrow \quad mg - T = m(g)$$

$$\Rightarrow \quad T = 0$$

18. (4)

$$\frac{d\theta}{dt} = 2 \text{ rad/s}$$



Angular velocity of particle w.r.t the centre of circle

$$= \frac{d(2\theta)}{dt}$$

$$= 2\frac{d\theta}{dt} = 4 \text{ rad/s}$$

$$v = \omega r = 4(1) = 4 \text{ m/s}$$

19. (1)

$$\alpha = \frac{\pi}{4} \text{rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \frac{\pi}{2} = 0 + \frac{1}{2} \times \left(\frac{\pi}{4}\right) t^2$$

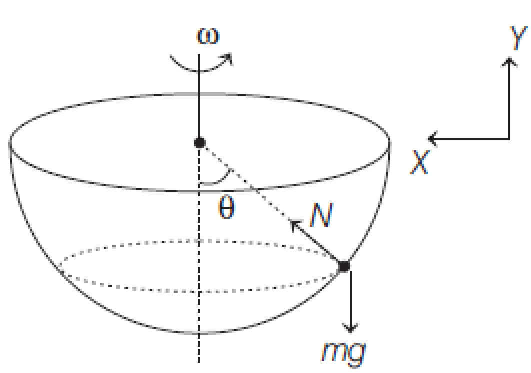
$$\Rightarrow t = 2\text{s}$$

$$v_{\text{avg}} = \frac{\Delta S}{\Delta t} = \frac{\sqrt{2}R}{2} = 1 \text{ m/s}$$

20. (100)

$\sum F_y = 0 \Rightarrow N\cos\theta = mg$ ….. (i)

$\sum F_x = ma_x \Rightarrow N\sin\theta = m\omega^2 R\sin\theta$ ….. (ii)



Solving Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{R\cos\theta}}$$

$$\omega = \sqrt{\frac{10}{0.5\left(\frac{0.3}{0.5}\right)}} = \sqrt{\frac{100}{3}} \text{ rad/s}$$

21. (2)

$a_c = 2t^2$

$\Rightarrow \omega^2(2) = 2t^2$

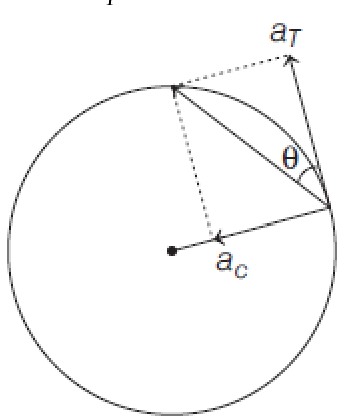
$\Rightarrow \omega = t$

$\Rightarrow \int d\theta = \int_0^2 t\,dt$

$\Rightarrow \theta = \left[\frac{t^2}{2}\right]^2 = 2 \text{ rad}$

22. (2)

$$\tan\theta = \frac{a_c}{a_T} = \frac{\omega^2 R}{R\alpha}$$



$$\tan\theta = \frac{(\alpha t)^2}{\alpha} = \alpha t^2$$

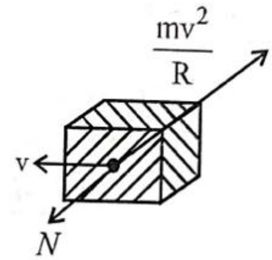
$$= 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

1. (a)

For observer on block,

$N = \frac{mv^2}{r} \Rightarrow N \propto v^2$

So, curve is parabola, symmetric about N-axis



2. (b)

$\Delta\vec{V} = \left|\vec{V} - \vec{u}\right| = \sqrt{v^2 + u^2}$ $[\because \vec{V} \pm \vec{u}]$

Now, $K_1 + P_1 = K_2 + P_2$

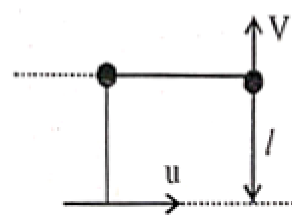
$\Rightarrow \frac{1}{2}mu^2 + 0 = \frac{1}{2}mV^2 + mg\,l$

$\Rightarrow V^2 = u^2 - 2g\,l$

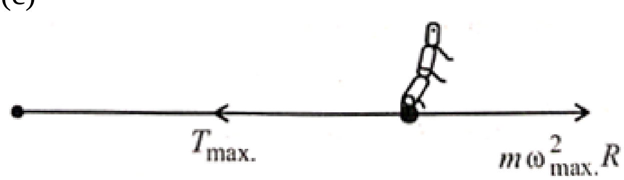
$\Rightarrow V = \sqrt{u^2 - 2gl}$

So, $\Delta V = \sqrt{V^2 + u^2} = \sqrt{u^2 - 2gl + u^2}$

$\sqrt{2\left(u^2 - gl\right)}$



3. (c)



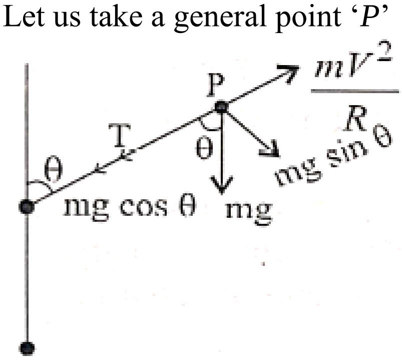
$T_{max} = m\omega^2_{max}R$

$80 = 0.1 \times \left(\frac{k}{30}\right)^2 \times 2$ $\left[\because \omega = \frac{k}{\pi} \times \frac{2\pi}{60} = \frac{k}{30}\right]$

$\Rightarrow k^2 = \frac{30^2 \times 80}{2 \times 0.1} \Rightarrow k^2 = 3600000 \Rightarrow k = 600$

4. (b)

Let us take a general point 'P'



At P

$T + mg\cos\theta = \frac{mv^2}{R} \Rightarrow T = \frac{mv^2}{R} - mg\cos\theta$

So, T will be minimum when, $mg\cos\theta$ is maximum

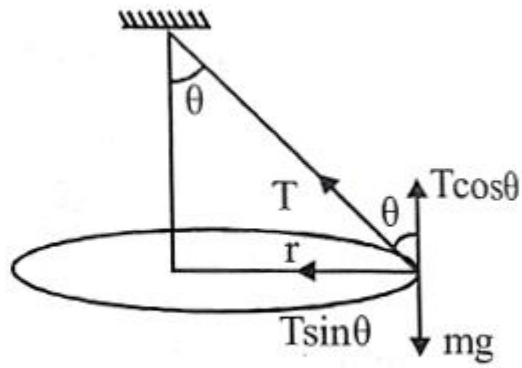
i.e., when $\cos\theta$ is maximum

i.e., when $\theta = 0$ and θ is zero when string is at highest point.

5. (a)

From figure,

$\sin\theta = \frac{r}{L} = \frac{L}{L\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow q = 45°$



$T\sin\theta = \frac{mv^2}{r}$ ...(i)

$T\cos\theta = mg$ ...(ii)

From (i) and (ii)

$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$

6. (b)

Given, mass of the block, $m = 200\ g = 200 \times 10^{-3}$ Kg

Radius of the circular groove, $r = 20$ cm

Time taken to complete one round, $T = 40$ s

Here, normal force will provide the necessary centripetal force.

$N = m\omega^2 r = 200 \times 10^{-3} \times \left(\frac{2\pi}{40}\right)^2 \times 0.2 = 9.859 \times 10^{-4}$ N.

7. (c)

For statement-1

The maximum speed by which cyclist can take a turn one a circular path

$\Rightarrow \quad v \le \sqrt{\mu rg} \le \sqrt{0.2 \times 2 \times 9.8} \Rightarrow v \le \sqrt{3.92}$

Speed of cyclist, $7\ \text{kmh}^{-1} = 7 \times \frac{5}{18} = 1.94$ m/s

The maximum safe speed on a banked frictional road

$$v_{\text{allowable}} = \sqrt{r \cdot g \frac{(\mu + \tan\theta)}{1 - \mu\tan\theta}}$$

$$\Rightarrow \quad v = \sqrt{\frac{2 \times 9.8(0.2 + \tan 45°)}{1 - 0.2 \times \tan 45°}} \sqrt{\frac{2 \times 9.8 \times 1.2}{0.8}}$$

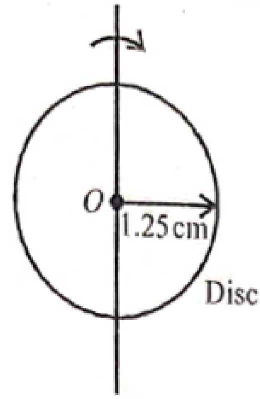
$= 5.42$ m/s

Speed of cyclist, 18.5 kmh $= 5.13$ m/s

So, both the statement are true.

8. (d)

Using, $\mu mg = \frac{mv^2}{r} = mr\omega^2$



$\omega = 2\pi n = 2\pi \times 3.5 = 7\pi$ rad/sec

Radius, $r = 1.25$ cm $= 1.25 \times 10^{-2}$ m

Coefficient of friction, $\mu = ?$

$$\mu mg = \frac{m(r\omega)^2}{r} (\because\ v = r\omega)$$

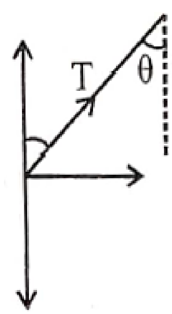
$$\mu mg = mr\omega^2$$

$$\Rightarrow\ \mu = \frac{r\omega^2}{g} = \frac{1.25 \times 10^{-2} \times \left(7 \times \frac{22}{7}\right)^2}{10}$$

$$= \frac{1.25 \times 10^{-2} \times 22^2}{10} = 0.6$$

9. (d)

Given, $\theta = 45°, r = 0.4$ m, $g = 10$ m/s^2



$$T\sin\theta = \frac{mv^2}{r} \qquad ...(i)$$

$$T\cos\theta = mg \qquad ...(ii)$$

From equation (i) & (ii) we have,

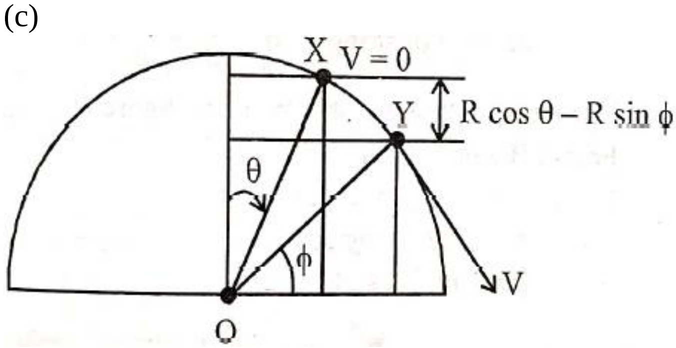
$$\tan\theta = \frac{v^2}{rg}$$

$$v^2 = rg \qquad \because\ \theta = 45^\circ$$

Hence, speed of the pendulum in its circular path,

$$v = \sqrt{rg} = \sqrt{0.4 \times 10} = 2 \text{ m/s}$$

10. (c)



$$K.E._X + P.E._X = K.E._Y + P.E._Y$$

$$\Rightarrow\ \frac{1}{2}mV^2 = mgR(\cos\theta - \sin\phi)$$

$$\Rightarrow\ V^2 = 2gR(\cos\theta - \sin\phi)$$

At Y, $\frac{mv^2}{R} = mg\sin\phi$ $[\because\ N_Y = 0]$

$\Rightarrow \quad 2mg(\cos\theta - \sin\phi) = mg\sin\phi$

$\Rightarrow \quad \sin\phi = \frac{2}{3}\cos\theta$

11. (a)

Initially, $k \times 1 = m\omega^2 R$ ...(i)

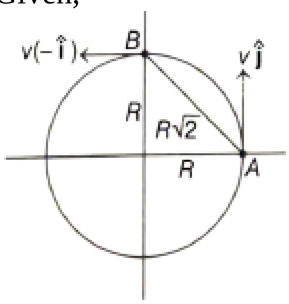
Finally, $k \times 5 = m(2\omega)^2 (R+4)$ ...(ii)

From (i) & (ii), we get : $R = 16$

So, original length = (16 – 1) cm = 15 cm.

12. (b)

Given,



$v_A = v_{\hat{j}}, v_B = -v_{\hat{i}}$

Time to reach from A to B

$= \frac{2\pi R}{4} \times \frac{1}{v} = \frac{\pi R}{2v}$

Displacement from A to B $= R\sqrt{2}$

Now, average velocity from A to B

$= \frac{\text{displacement}}{\text{time}}$

$= \frac{R\sqrt{2}}{\frac{\pi R}{2v}} = \frac{R\sqrt{2} \times 2v}{\pi R} = \frac{2\sqrt{2}v}{\pi}$

Instantaneous velocity at B is $v\hat{i}$

According to equation

$= \frac{\text{Instantaneous velocity}}{\text{Avergae velocity}}$

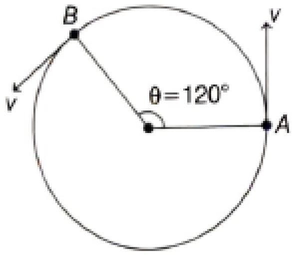
$= \frac{\pi}{x\sqrt{2}}$

$\frac{v}{\frac{2\sqrt{2}v}{\pi}} = \frac{\pi}{x\sqrt{2}} \Rightarrow \frac{\pi}{2\sqrt{2}} = \frac{\pi}{x\sqrt{2}}$

Thus, x = 2

13. (c)

Given,



Given, $v = \pi\, m/s$ or $R\omega = \pi$

$\omega = \frac{\pi}{R} rad/s$

Angular displacement,

$\theta = 120^\circ$ or $\frac{2\pi}{3}$

Using $\theta = \omega t$

$$t = \frac{\theta}{\omega} = \frac{2\pi/3}{\pi/R} = \frac{2R}{3}$$

Linear disbarment,

$$d = 2R\sin(\theta/2)$$

$$d = 2R\sin\left(\frac{120^\circ}{2}\right) = 2R\sin 60^\circ$$

$$= 2R\frac{\sqrt{3}}{2} \Rightarrow R\sqrt{3}$$

Average velocity

$$= \frac{d}{t} = \frac{R\sqrt{3}}{2R/3} = \frac{3\sqrt{3}}{2}$$

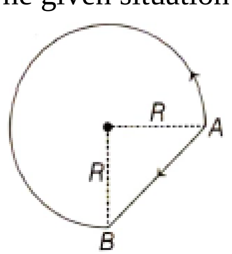
$$= 1.5\sqrt{3} m/s$$

14. (b)

Time period, $T = 4s$

Body will cover $\frac{3}{4}$ th part of circle in 3s.

The given situation can be drawn as



Displacement,

$$AB = \sqrt{R^2 + R^2} = \sqrt{2}R$$

$$= \sqrt{2} \times 10 = 10\sqrt{2}m$$

15. (a)

Given,

$m = 200 \text{ kg}, r = 70 \text{ m}$

$\omega = 0.2 \text{ rad/sec}$

$F_C = m\omega^2 r = 200 \times (0.2)^2 \times 70$

$= 200 \times 0.04 \times 70$

$= 2 \times 4 \times 70 = 560\text{N}$

16. (a)

Given, $m = 5 \text{ kg}, r = 2 \text{ m}$

Time t for 1 rev = 3.14 sec π s

θ for $1 \text{ rev} = 2\pi \text{ rad}$

Therefore $\omega = \frac{\theta}{t} = \frac{2\pi}{\pi} = 2 \text{ rad/s}$

Centrifugal force, $F = m\omega^2 r$

$F = 5 \times 2 \times 2^2$

$= 40 \text{ N}$

17. (d)

Given, $m = 1 \text{ kg}, l = 1\text{m}$

Breaking tension, $T_{max} = 400\text{N}$

(braking tension)

Since, stone rotates horizontally,

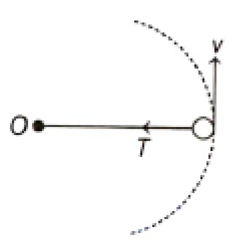
Thus, $T_{max} = \frac{mv_{max}^2}{l}$

Or $v_{max}^2 = \frac{T_{max} l}{m}$

or $v_{max} = \sqrt{\frac{T_{max}}{m} l}$

$= \sqrt{\frac{400 \times 1}{1}}$

Or $v_{max} = 20\text{m/s}$



18. (c)

Maximum safe speed of car over a horizontal turn is $v_{max} = \sqrt{\mu r g}$

Here, $r = 50\text{m}, \mu = 0.34, g = 10$

So, $v_{max} = \sqrt{0.34 \times 50 \times 10}$

$= \sqrt{170} \approx 13\text{m/s}$

19. (c)

A bob suspended from the roof of the car by a massless string, then angle (θ) between the string and the vertical,

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20^2}{40\times 10}\right)$$

$$= \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$$

20. (c)

Extension in spring due to centripetal

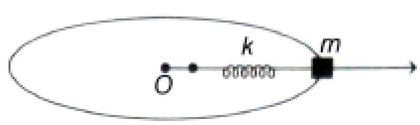
Force, $F = \frac{mv^2}{r} = mr\omega^2$

Given, $m = 200\text{ g} = 0.2\text{ kg}$

$r = x_0 + x$

= natural length + extension

$\because \quad F = kx$



Where, $k = 12.5\text{N/m}$

Thus, $kx = m(x_0 + x)\omega^2$

$12.5x = 0.2(x_0 + x)5^2$

$12.5x = 5x_0 + 5x$

$\Rightarrow \quad 7.5x = 5x_0$

$\Rightarrow \quad \frac{x}{x_0} = \frac{5}{7.5} = \frac{1}{1.5} = \frac{2}{3}$

21. (b)

Given,

$f_r = m\omega^2 r$

$\Rightarrow \quad \mu mg = m\omega^2 r = \text{constant}$

$\omega^2 r = \text{ constant}$

$\omega_1^2 r_1 = \omega_2^2 r_2$

Given, $r_1 = 1, \omega_2 = \frac{\omega}{2}$ and $\omega_1 = \omega$

$\therefore \quad \omega^2(1) = \left(\frac{\omega}{2}\right)^2 \cdot r_2$

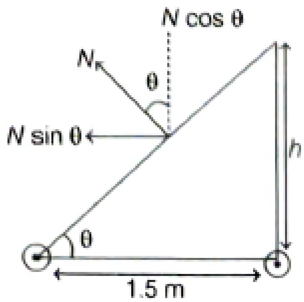
$\Rightarrow \quad r_2 = 4\text{ cm}$

22. (b)

Resolving Normal,

$N\cos\theta = mg$ ... (i)

$N\sin\theta = \dfrac{mv^2}{R}$ ... (ii)



Dividing, $\tan\theta = \dfrac{v^2}{Rg}$

$\dfrac{h}{1.5} = \dfrac{12\times 12}{400\times 10}$

$h = \dfrac{1.5\times 12\times 12}{400\times 10} = \dfrac{54}{1000} = 0.054\text{ m}$

$= 5.4\text{ cm}$

EXERCISE - 1

1. **(D)**

ω = rate of change of angle

$\therefore \quad \frac{\omega_1}{\omega_2} = 1$ as they both complete 2π angle in same time.

2. **(C)**

$$mg - N = \frac{mv^2}{r}$$

$$\Rightarrow \quad N = mg - \frac{mv^2}{r}$$

Since $r_A < r_B$

$\Rightarrow \quad N_A < N_B$

3. **(A)**

Force is always perpendicular to displacement

$\therefore$ work done = 0

4. **(B)**

$$a_{resultant} = \sqrt{a_c^2 + a_{tang.}^2}$$

$$= \sqrt{\left(\frac{30^2}{500}\right)^2 + 2^2} = \frac{\sqrt{181}}{5} = 2.7\ m/s^2$$

5. **(A)**

f = mg

$\Rightarrow \quad \mu N \geq mg$

$\Rightarrow \quad \mu mr\omega^2 \geq mg$

$$\Rightarrow \quad \omega \geq \sqrt{\frac{g}{\mu r}}$$

6. **(C)**

If the coin just slips at a distance of 4r from centre

$\Rightarrow \quad \mu mg = m4r\omega^2 \quad \ldots (1)$

If angular velocity is doubled

$\mu mg = mR(2\omega)^2 \quad \ldots (2)$

From (1) and (2)

$\Rightarrow \quad R = r$

7. **(D)**

$T = mr\,\omega_0^2 \quad \ldots (1)$

$2T = mr\,\omega^2 \quad \ldots (2)$

$\Rightarrow \quad \omega = \sqrt{2}\omega_0 = \sqrt{2} \times 5\ rpm$

8. **(C)**

Since force is always perpendicular to velocity particle moves in circle.
It's speed is constant and velocity variable.

9. **(C)**

$$\mu mg = \frac{mv_{max}^2}{r}$$
$$v_{max} = \sqrt{\mu gr} = \sqrt{0.3 \times 10 \times 300}$$
$$= 30 m/s = 108\, km/hr$$

10. **(C)**

$$F_{net} = ma_{rad.} = \frac{mv^2}{r}$$

11. **(B)**

$T = mr\omega^2 = 0.2 \times 0.5 \times 4^2 = 1.6$ N

12. **(C)**

Centripetal force is provided by friction

13. **(A)**

$$N_A - mg = \frac{mv^2}{r}$$
$$N_A = mg + \frac{mv^2}{r_A}, \quad N_B = mg - \frac{mv^2}{r_B}$$
$$N_C = mg + \frac{mv^2}{r_c}$$

14. **(A)**

Real forces are mg and T only

15. **(A)**

Bead starts slipping, when
$\mu N = mL\omega^2$
$\mu mL\alpha = mL\omega^2$
$\mu\alpha = (0 + \alpha t)^2$
$\Rightarrow \; t = \sqrt{\frac{\mu}{\alpha}}$

EXERCISE - 2

1. **(A, C)**

Time to fall = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times5}{10}} = 1\,s$

Distance covered in y direction = v × t

= 3 × 1

= 3m

Since there is no velocity along x-direction x is always 2m.

2. **(A, B)**

For no wear and tear friction is zero

$\Rightarrow \quad \frac{v^2}{Rg} = \tan 15^o$

$\Rightarrow \quad v = \sqrt{Rg\tan 15^o} = 28.1\,m/s$

$v_{max} = \sqrt{\frac{Rg(\mu+\tan\theta)}{1-\mu\tan\theta}} = 38.1 m/s$

3. **(B, C)**

$\vec{v} = \frac{d\vec{r}}{dt}$; $\therefore$ $\vec{v} \parallel d\vec{r}$ (i.e. along the tangent)

$\vec{a}_{avg} = \frac{\Delta\vec{v}}{t} = 0$

4. **(A, B)**

$\frac{dS}{dt} = v = K\sqrt{S}$

$\Rightarrow \quad \int_0^S \frac{dS}{\sqrt{S}} = \int_0^t Kdt$

$2\sqrt{S} = Kt$

$S = \frac{K^2t^2}{4}$

$v = \frac{dS}{dt} = \frac{K^2t}{2}$

5. **(A, B, C)**

$T = m\ell\omega^2$, $v = \ell\omega$, $F_{vert} = 0$

6. **(A, B, D)**

$\omega = \frac{v}{R} =$ constant, $\theta = \omega t$

$F_y = -F\sin\theta = -F\sin\omega t$

$= -\frac{mv^2}{R}\sin\omega t$

$V_r = -v\sin\theta = -v \sin \omega t$
x-coordinate = R cos ωt

7. **(B, D)**
If μ = 0.1, $f_{max} = 0.1 \times 0.5 \times 10 = 0.5N$
Req. centripetal force = $mr\omega^2 = 0.5 \times 1 \times 0.5^2 = .125N$

$\therefore \quad f = \frac{1}{8} N$, Tension = zero

If $\mu = \frac{1}{20}, f_{max} = \frac{1}{20} \times 0.5 \times 10 = 0.25$ N

$\therefore \quad f = \frac{1}{8} N$, Tension = zero

If $\mu = \frac{1}{40}, f_{max} = \frac{1}{40} \times 0.5 \times 10 = 0.125$ N

$\therefore \quad f = \frac{1}{8} N$, Tension = 0

EXERCISE - 3

1. **(A)-R; (B)-P; (C)-S; (D)-Q**

$\frac{v^2}{r} = K^2rt^2$ (given)

(a) Centripetal force = mK^2rt^2

(b) Tangential force = $m\frac{dv}{dt} = mKr$

(c) Power of centripetal force $= \vec{F}_{centripetal} . \vec{v} = 0$

(d) Power of tangential force $= \vec{F}_t . \vec{v} = F_t\ v$

$= m.K^2r^2t$

2. **(B)**
Assuming no friction between m_1 and m_2

$a_1 = R\omega^2 - \frac{T}{m_1}$

$a_2 = R\omega^2 - \frac{T}{m_2}$

$\because \quad a_1 > a_2$
$\therefore$ Friction on upper block acts towards left and on lower block towards right.

3. **(A)**
Let the required angular velocity be ω
Then
$T + \mu m_1 g = m_1 R\omega^2$ … (1)
$T = \mu m_1 g + m_2 R\omega^2$ … (2)
$\Rightarrow \quad 2\mu m_1 g = (m_1 - m_2)R\omega^2$

$\Rightarrow \quad \omega = \sqrt{\dfrac{2\mu m_1 g}{(m_1 - m_2)R}} = 6.3\ \text{rad/s}$

4. **(B)**

$T = \mu m_1 g + m_2 R\omega^2$

$= 0.5 \times 2 \times 10 + 1 \times 0.5 \times 40 = 30N$

5. **(B)**

$\tan\theta = \dfrac{v_{design}^2}{gR} = \dfrac{1}{2}$;

$f = m\left(g\sin\theta - \dfrac{v^2}{R}\cos\theta\right)$

$= 300\sqrt{5}\ \text{m/s}$

6. **(A)**

$f = m\left(\dfrac{v^2}{R}\cos\theta - g\sin\theta\right)$

$= 500\sqrt{5}\ \text{m/s}$

7. **(A)**

$\theta = \tan^{-1}\dfrac{1}{2}$

8. **(C)**

$mg\sin\theta = m\dfrac{v^2}{L} \qquad \ldots (1)$

$\dfrac{1}{2}m(\sqrt{3gL})^2 = \dfrac{1}{2}mv^2 + mgL(1+\sin\theta) \qquad \ldots (2)$

$\therefore \quad \theta = \sin^{-1}\left(\dfrac{1}{3}\right)$; Also $v^2 = \dfrac{1}{3}gL$

9. **(C)**

$h_{max} = L(1+\sin\theta) + \dfrac{0^2 - v^2\cos^2\theta}{-2g}$

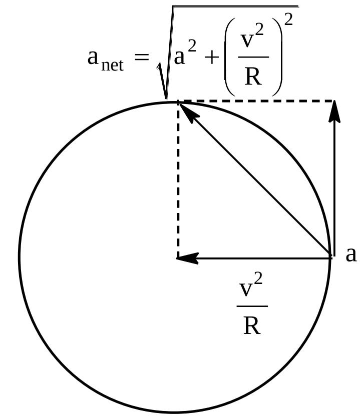
$= \dfrac{40L}{27}$

10. **(B)**

$\dfrac{1}{2}m(\sqrt{3gL})^2 = mg\,h_{max}$

$\Rightarrow \quad h_{max} = \dfrac{3L}{2}$

1. Here frictional force will provide the required tangential and centripetal force for the circular motion of car.
Force of friction will act along the direction of net acceleration.



$$f = m\sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$$

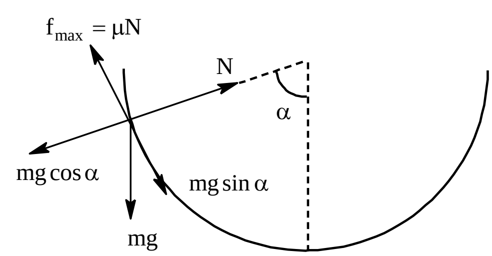
Car will skid when $m\sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$

$$= m\sqrt{a^2 + \frac{a^4 + t^4}{R^2}}$$

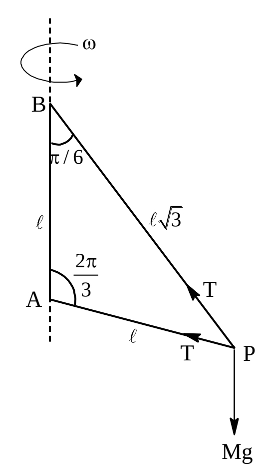
$$\Rightarrow \quad t = \left[\frac{(\mu^2 g^2 - a^2)R^2}{a^4}\right]^{1/4}$$

Till this time distance traveled by car is $D = \frac{1}{2}at^2$. Putting value of t we get $D = \frac{R\sqrt{\mu^2 g^2 - a^2}}{2a}$.

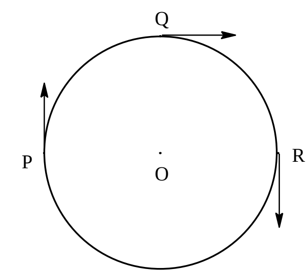
2. The insect will slide when mg sin α becomes equal to limiting friction. At every instant the insect is in equilibrium.
So, $N = mg\cos\alpha$
$\mu N = mg\sin\alpha$
$\Rightarrow \mu\, mg\cos\alpha = mg\sin\alpha \Rightarrow \mu = \tan\alpha \Rightarrow \alpha = \tan^{-1}\mu = \tan^{-1}(1/3)$



3. Applying Newton's law towards the centre of circle we get $N = m\omega^2 R$
Let ω be the minimum angular speed for which man is not falling. At this instant its weight will be balanced by limiting friction acting upwards.
i.e, $\mu N = mg \Rightarrow 0.15 \times 70\, \omega^2 \times 3 = 70 \times 10$
$\Rightarrow \omega = 4.7$ rad/sec.



4. Applying Newton's law along vertical we get



$$T\frac{\cos\pi}{6}+T\frac{\cos\pi}{3}=mg \qquad \dots (1)$$

$$\Rightarrow \quad T=\frac{2mg}{\sqrt{3}+1}$$

Applying Newton's law along horizontal.

We get $T\cos\frac{\pi}{3}+T\cos\frac{\pi}{6}=m\omega^2R=m\omega^2\frac{\ell\sqrt{3}}{2}$

$$\Rightarrow \quad \omega^2=\frac{2g}{\ell\sqrt{3}}$$

5. At position A

$T_A\cos\beta'=mg$

At position B

$T_B-mg\cos\beta=0$

$T_B=mg\cos\beta$

$(\beta'=\beta)$

6. According to question, $a_c=a_t=\frac{v^2}{R}$

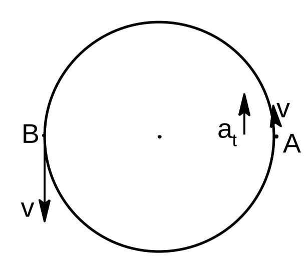
$$\Rightarrow \quad \frac{dv}{dt}=\frac{v^2}{R}\Rightarrow\int_{v_0}^{v}\frac{dv}{v^2}=\int_0^t\frac{dt}{R}$$

$$\Rightarrow \quad \frac{-1}{v}+\frac{1}{v_0}=\frac{t}{R}\Rightarrow V(t)=\frac{Rv_0}{R-tv_0}$$

$$\Rightarrow \quad \int_0^{2\pi R}dx=Rv_0\int_0^T\frac{dt}{R-tv_0}\Rightarrow 2\pi R=\frac{-Rv_0}{v_0}(\ell n(R-tv_0)_0^T$$

$$\Rightarrow \quad T=\frac{R}{v_0}(1-e^{-2\pi})$$

7. Lets assume that the particles meet at time t. Distance traveled by A = distance traveled + πR by B



$$\Rightarrow \quad vt+\frac{1}{2}\times\frac{72v^2t^2}{25\pi R}=vt+\pi R$$

$$\Rightarrow \quad t=\frac{5\pi R}{6v}$$

Angle traced by $A=\frac{\text{distance travelled}}{R}=\frac{11\pi}{6}$

Angular velocity $=\frac{v+at}{R}=\frac{17v}{5R}$

1. **(C, D)**

$\vec{F} \perp \vec{v} \Rightarrow P = 0 \Rightarrow$ kinetic energy = constant; F is constant (given) $\Rightarrow \frac{mv^2}{R} =$ constant

$\Rightarrow R =$ constant.

2. **(A)**

Radius of curvature in (a) is minimum

3. **(A)**

$mg \sin \alpha = \frac{1}{3} mg \cos \alpha \Rightarrow \cot \alpha = 3$

4. **(C)**

$\vec{a} = \vec{a}_{\text{tangential}} + \vec{a}_{\text{normal}}$ and $\vec{a}_{\text{tangential}}$ is downward.

5. $Kx \cos 30° + mg \cos 30° = ma_t$. As $x = \frac{R}{4}$ and $K = \frac{mg}{R}$, $a_t = \frac{5\sqrt{3}}{8} g$;

$N + Kx \cos 60° = mg \cos 60° \Rightarrow N = \frac{3mg}{8}$