Work, Power & Energy

JEE Main Exercise

1. (b)

$$W = \int F\,dx$$

$$W = \int_0^d (a+bx)\,dx$$

$$= \left[ax + \frac{bx^2}{2}\right]_0^d = ad + \frac{db^2}{2}$$

2. (b)

$$W = \Delta K$$

$$W = K_2 - K_1$$

$$W = \frac{1}{2}\times 2\times 0^2 - \frac{1}{2}\times 2\times (20)^2$$

$$W = -400\text{ J}$$

3. (a)

$$W = \mathbf{F}\cdot\mathbf{s}$$

$$W = F\,s\cos\theta$$

$$W = FR$$

4. (a)

$$v = a\sqrt{s}$$

$$\frac{ds}{dt} = a\sqrt{s}$$

$$\Rightarrow \int_0^s \frac{ds}{\sqrt{s}} = a\int_0^t dt$$

$$\Rightarrow 2\sqrt{s} = at$$

$$\Rightarrow s = \frac{1}{4}a^2t^2$$

$$W = \Delta K = \frac{1}{2}m\left(a\sqrt{s}\right)^2 - \frac{1}{2}m(0)^2$$

$$= \frac{1}{2}ma^2s - 0 = \frac{1}{2}ma^2\left(\frac{1}{4}a^2t^2\right)$$

$$= \frac{1}{8}ma^4t^2$$

5. (a)

Applying work-energy theorem,

$$W_{mg} + W_N + W_{\text{friction}} + W_{\text{spring}} = \Delta K$$

$$0 + 0 - (\mu mg)x + \frac{1}{2}K\left(0^2 - x^2\right) = 0 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad -50x - 50x^2 = -\frac{1}{2} \times 50 \times 2^2$$

$$\Rightarrow \quad x^2 + x - 2 = 0$$

$$\Rightarrow \quad x = 1\text{m}$$

6. (b)

From A to B, applying work-energy theorem,

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad -mg(R-h) + 0 = 0 - \frac{1}{2}mu^2$$

$$u = \sqrt{2g(R-h)}$$

7. (d)

$$U = 2x + 5y - xy$$

$$\mathbf{F} = \left(-\frac{\partial U}{\partial x}\right)\hat{\mathbf{i}} + \left(-\frac{\partial U}{\partial y}\right)\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{F} = (-2 + y)\hat{\mathbf{i}} + (-5 + x)\hat{\mathbf{j}}$$

At (2, –2)

$$\mathbf{F} = -4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$$

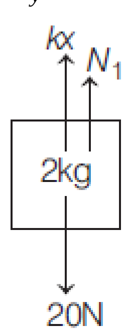
$$\Rightarrow \quad \mathbf{a} = -4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$$

$$\Rightarrow \quad a = 5\ \text{m/s}^2$$

8. (b)

For 2 kg block

$$\sum F_y = 0$$



$$\Rightarrow \quad Kx + N_1 = 20$$

When 2 kg leaves contact $(N_1 = 0)$

$$\Rightarrow \quad Kx = 20 \Rightarrow x = 0.5\ \text{m}$$

Applying work-energy theorem for 5 kg block

$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +5\times10\times0.5+\frac{1}{2}\times40\times\left(0^2-(0.5)^2\right)$$

$$=\frac{1}{2}\times5\times v^2-0$$

$$\Rightarrow \quad v = 2\sqrt{2}\ \text{m/s}$$

9. (a)

Applying work-energy theorem for (2 kg + 1 kg) system

$$W_{mg} + W_T = \Delta K_{\text{system}}$$

$$\Rightarrow \quad (+2\times10\times0.6-1\times10\times0.6)+0=\left(\frac{1}{2}\times2\times v^2+\frac{1}{2}\times1\times v^2\right)-(0+0)$$

$$\Rightarrow \quad v = 2\ \text{m/s}$$

10. (c)

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad v = x\sqrt{\frac{k}{m}} = 0.05\sqrt{\frac{600}{15\times10^{-3}}}$$

$$\Rightarrow \quad v = 10\ \text{m/s}$$

$$R_{\max} = \frac{v^2}{g} = \frac{(10)^2}{10} = 10\ \text{m}$$

11. (c)

$$P = \frac{3t^2}{2}$$

$$\Rightarrow \quad \frac{dK}{dt} = \frac{3t^2}{2}$$

$$\Rightarrow \quad \int_0^K dK = \int_0^t \frac{3t^2}{2}\,dt$$

$$\Rightarrow \quad K = \frac{t^3}{2}$$

At $t = 2$,

$$K = \frac{2^3}{2} = \frac{1}{2}\times2\times v^2$$

$$\Rightarrow \quad v = 2\ \text{m/s}$$

12. (d)

$$U_1 = \frac{1}{2}kx^2$$

$$U_2 = \frac{1}{2}K(x+y)^2$$

$$W_{\text{ext}} = \Delta U + \Delta K = \left(\frac{1}{2}k\left(x+y\right)^2 - \frac{1}{2}kx^2\right) + 0$$

$$= \frac{1}{2}ky\left(2x+y\right)$$

13. (b)

$\mathbf{v} = 2\hat{\mathbf{i}} + 4t\,\hat{\mathbf{j}}$

$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 4\hat{\mathbf{j}}$

$P = \mathbf{F}\cdot\mathbf{v} = \left(ma\right)\cdot\mathbf{v} = 2\left(16t\right)$

At $t=5$, $P = 160\,\text{W}$

14. (a)

$W = \Delta K$

$$W = \frac{1}{2}m\left(v^2 - u^2\right)$$

From rest to speed v,

$$W = \frac{1}{2}m\left(v^2 - 0^2\right) = \frac{1}{2}mv^2$$

From v to $2v$,

$$W = \frac{1}{2}m\left(\left(2v\right)^2 - v^2\right) = \frac{3}{2}mv^2$$

15. (d)

$K_{\text{man}} = \frac{1}{2}k_{\text{boy}}$

$\Rightarrow \quad \frac{1}{2}mv_{\text{man}}^2 = \frac{1}{2}\left(\frac{1}{2}\frac{m}{2}v_{\text{boy}}^2\right)$

$\Rightarrow \quad v_{\text{man}} = \frac{v_{\text{boy}}}{2}$

Now, $\frac{1}{2}m\left(v_{\text{man}}+1\right)^2 = \frac{1}{2}\left(\frac{m}{2}\right)v_{\text{boy}}^2$

$\Rightarrow \quad \left(V_{\text{man}}+1\right)^2 = \frac{\left(2v_{\text{man}}\right)^2}{2}$

$\Rightarrow \quad V_{\text{man}}^2 + 1 + 2v_{\text{man}} = 2v_{\text{man}}^2$

$\Rightarrow \quad v_{\text{man}}^2 - 2v_{\text{man}} - 1 = 0$

$\Rightarrow \quad v_{\text{man}} = \frac{2+\sqrt{8}}{2} = 1+\sqrt{2}$

16. (a)

$P = Fv =$ constant

$\Rightarrow \quad mav =$ constant

$\Rightarrow \quad m\frac{dv}{dt}v =$ constant

$\Rightarrow \int v\,dv \propto \int dt$

$\Rightarrow v^2 \propto t$

$\Rightarrow v \propto \sqrt{t}$

$\frac{ds}{dt} \propto \sqrt{t}$

$\Rightarrow \int ds \propto \int \sqrt{t}\,dt$

$\Rightarrow s \propto t^{3/2}$

$\frac{s}{v} = \frac{t^{3/2}}{t^{1/2}} = t$

17. (c)

Speed of the block is maximum when acceleration of block is zero.

So, $mg\cos\theta = kx$

$\Rightarrow x = \frac{mg\cos\theta}{k}$

18. (a)

$W_{mg} = -mgh$

$W_{\text{friction}} = -\int (\mu mg\cos\theta)\,dl\cos 0^\circ$

$= -\mu mg \int dl\,\cos\theta$

$= -\mu mg \int_0^l dx$

$= -\mu mgl$

$W_N = 0$

$W_F = ?$

$W_{mg} + W_N + W_{\text{friction}} + W_F = \Delta K$

$\Rightarrow -mgh + 0 - \mu mgl + W_F = 0 - 0$

$\Rightarrow W_F = mg(h + \mu l)$

19. (d)

$K = \frac{p^2}{2m}$

$\Rightarrow \log_e K = \log_e\left(\frac{p^2}{2m}\right)$

$\Rightarrow \log_e K = \log_e p^2 - \log_e 2m$

$\Rightarrow \log_e K = 2\log_e p - \log_e 2m$

$\Rightarrow y = 2x - \log_e 2m$

20. (a)

When the object is lowered very slowly to its equilibrium position, then at equilibrium position

$$Kx = mg$$

$$\Rightarrow K(0.1) = 0.5 \times 10$$

$$\Rightarrow K = 50 \text{ N/m}$$

Applying work-energy theorem,

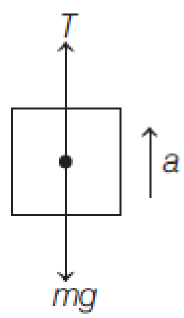
$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow +0.5 \times 10 \times 0.1 + \frac{1}{2} \times 50\left(0^2 - (0.1)^2\right)$$

$$= \frac{1}{2} \times 0.5v^2 - 0$$

$$\Rightarrow v = 1 \text{ m/s}$$

21. (a)



$$\sum F_y = ma_y$$

$$\Rightarrow T - mg = ma$$

$$\Rightarrow T = m(g + a)$$

$$W_T = Fs\cos\theta$$

$$= m(g + a)\frac{1}{2}at^2 \cos 0^\circ$$

$$= \frac{m}{2}(g + a)at^2$$

22. (c)

$P = Fv$ = constant

mav = constant

$$\Rightarrow \frac{dv}{dt}v = \text{constant}$$

$$\Rightarrow \int v\,dv \propto \int dt$$

$$\Rightarrow v^2 \propto t$$

$$\Rightarrow v \propto \sqrt{t}$$

Now, $v = \frac{dx}{dt} \propto \sqrt{t}$

$$\Rightarrow \int dx \propto \int \sqrt{t}\,dt$$

$$\Rightarrow x \propto t^{3/2}$$

23. (d)

Applying work-energy theorem

$$W_{mg} + W_N + +W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +mg\left(\frac{3R}{2}\right) + 0 + \frac{1}{2}k\left(\left(\frac{R}{2}\right)^2 - 0^2\right)$$

$$= \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{4gR}$$

24. (b)

Lets say maximum elongation in the spring is *x*.

$$W_{mg} + W_N + W_F + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad 0 + 0 + Fx + \frac{1}{2}k\left(0^2 - x^2\right) = 0 - 0$$

$$\Rightarrow \quad x = \frac{2F}{k}$$

$$W_F = Fx = F\left(\frac{2F}{k}\right) = \frac{2F^2}{k}$$

25. (a)

$$W_{\text{friction}} = \int -\mu mg\cos\theta dx = \int_0^5 -\mu_0\, x\, mg\cos\theta\, dx$$

$$= \frac{-\mu_0 mg\cos\theta\, s^2}{2}$$

Applying work-energy theorem

$$W_{mg} + W_N + W_{\text{friction}} = \Delta K$$

$$\Rightarrow \quad (mg\sin\theta)s + 0 - \frac{(\mu_0 mg\cos\theta)s^2}{2} = 0 - 0$$

$$\Rightarrow \quad s = \frac{2\tan\theta}{\mu_0}$$

26. (c)

Using work-energy theorem between *A* and *B*.

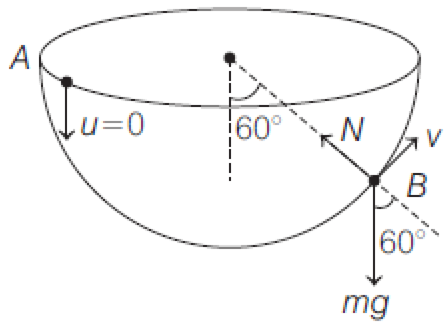
$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mgR\cos 60^\circ + 0 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{gR}$$

Equation for centripetal force at *B*,

$$N - mg\cos 60^\circ = \frac{mv_B^2}{R}$$

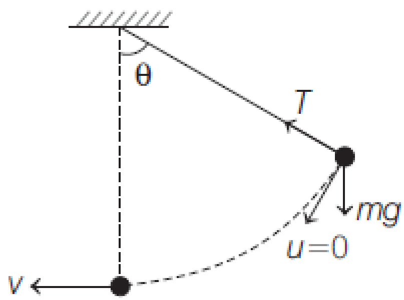


$N = 1.5\,mg$

27. (d)

In extreme position,

$$a_{\text{centripetal}} = 0$$



$$a_{\text{tantential}} = g\sin\theta$$

$$a_1 = \sqrt{a_c^2 + a_T^2} = g\sin\theta$$

Using work-energy theorem,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mgl\left(1-\cos\theta\right) + 0 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{2gl\left(1-\cos\theta\right)}$$

$$a_{\text{centriperal}} = \frac{v^2}{l} = 2g\left(1-\cos\theta\right)$$

$$a_{\text{tantential}} = 0$$

$$a_2 = \sqrt{a_c^2 + a_T^2} = 2g\left(1-\cos\theta\right)$$

$$\because \quad a_1 = a_2$$

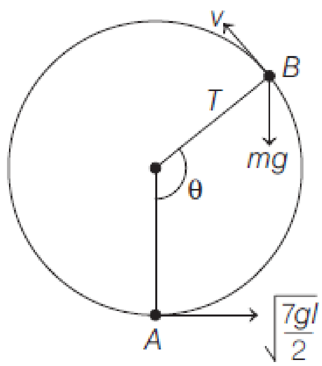
$$\Rightarrow \quad g\sin\theta = 2g\left(1-\cos\theta\right)$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

28. (c)

Using work-energy theorem between A and B,

$$W_{mg} + W_T = \Delta K$$



$$-mgl\left(1+\cos\left(180^\circ-\theta\right)\right)+0=\frac{1}{2}mv^2-\frac{1}{2}m\left(\sqrt{\frac{7gl}{2}}\right)^2$$

$$v^2=\frac{7gl}{2}-2gl\left(1-\cos\theta\right) \qquad \text{...(i)}$$

At *B*,

$$mg\cos\left(180^\circ-\theta\right)+T=\frac{mv^2}{l}$$

$$\Rightarrow \quad -mg\cos\theta+0=\frac{m}{l}\left(\frac{7gl}{2}-2gl\left(1-\cos\theta\right)\right)$$

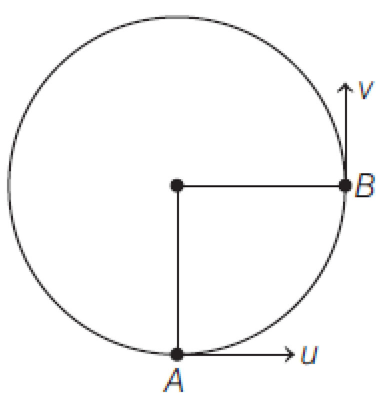
$$\Rightarrow \quad -2\cos\theta=7-4\left(1-\cos\theta\right)$$

$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$

$$\theta=120^\circ$$

29. (d)

$$\mathbf{v}_A=u\,\hat{\mathbf{i}}$$



Using work-energy theorem between *A* and *B*,

$$W_{mg}+W_T=\Delta T$$

$$\Rightarrow \quad -mgL+0=\frac{1}{2}mv^2-\frac{1}{2}mu^2$$

$$\Rightarrow \quad v=\sqrt{u^2-2gL}$$

$$\Rightarrow \quad \mathbf{v}_B=\sqrt{u^2-2gL}\,\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{v}_B - \mathbf{v}_A = \sqrt{u^2 - 2gL}\,\hat{\mathbf{j}} - u\,\hat{\mathbf{i}}$$

$$|\mathbf{v}_B - \mathbf{v}_A| = \sqrt{\left(\sqrt{u^2 - 2gL}\right)^2 + u^2} = \sqrt{2\left(u^2 - gL\right)}$$

30. (a)

Using work - energy theorem between A and B,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mgl\cos\theta + 0 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{2gl\cos\theta}$$

At *B*,

Centripetal acceleration $= \dfrac{v^2}{l} = \dfrac{\left(\sqrt{2gl\cos\theta}\right)^2}{l} = 2g\cos\theta$

Tangential acceleration $= \sqrt{a_c^2 + a_T^2}$

$$= \sqrt{(2g\cos\theta)^2 + (g\sin\theta)^2}$$

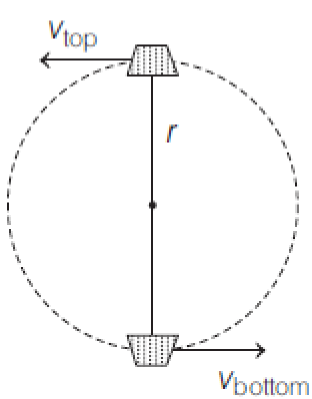
$$= g\left(\sqrt{4\cos^2\theta + \sin^2\theta}\right)$$

$$= g\sqrt{3\cos^2\theta + 1}$$

31. (a)

At the top most point,

$$T + mg = \frac{mv_{\text{top}}^2}{r}$$



For v_{top} to be minimum

$$T = 0$$

$$\Rightarrow \quad v_{\text{top}} = \sqrt{rg}$$

Using work - energy theorem,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mg(2r) + 0 = \frac{1}{2}mv_{\text{bottom}}^2 - \frac{1}{2}m\left(\sqrt{rg}\right)^2$$

$$v_{\text{bottom}} = \sqrt{5rg} \text{ and } T - mg = \frac{mv_{\text{bottom}}^2}{r}$$

$$\Rightarrow \quad T - mg = \frac{m}{r}\left(\sqrt{5rg}\right)^2$$

$$\Rightarrow \quad T = 6\,mg$$

32. (a)

Using work-energy theorem between A and B,

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mg(2R) + 0 = \frac{1}{2}mv_2^2 - 0$$

$$\Rightarrow \quad v_2 = \sqrt{4gR}$$

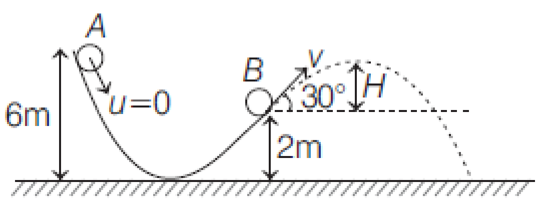
At point 2,

$$N + mg = \frac{mv_2^2}{R}$$

$$\Rightarrow \quad N + mg = 4mg$$

$$\Rightarrow \quad N = 3mg$$

33. (3)



Between A and B

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mg(6-2) + 0 = \frac{1}{2}mv^2 - 0$$

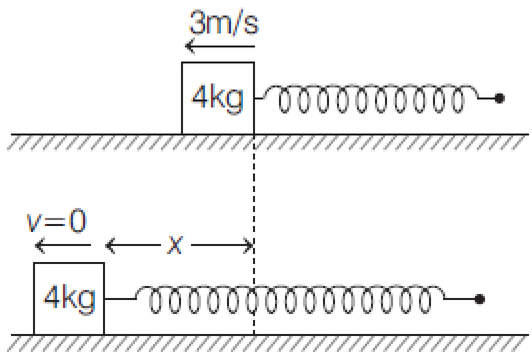
$$\Rightarrow \quad v = \sqrt{8g}$$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{\left(\sqrt{8g}\right)^2 \sin^2 30^\circ}{2g} = 1\text{ m}$$

$$H_{\max} = 2 + 1 = 3\text{ m}$$

34. (6)

With respect to free end, velocity of block is 1 − (−2) = 3 m/s left.



Elongation in the spring will be maximum when velocity of block with respect to free end is zero.

$$W_{mg} + W_N + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad 0 + 0 + \frac{1}{2} \times \frac{100}{10^{-2}}\left(0^2 - x^2\right) = 0 - \frac{1}{2} \times 4 \times 3^2$$

$$x = 0.06\ \text{m} = 6\ \text{cm}$$

35. (8)

$$P = 640 - 16v - 8v^2$$

$$\Rightarrow \quad Fv = 640 - 16v - 8v^2$$

For velocity to be maximum, $a = 0$

$$\Rightarrow \quad F = 0$$

$$\Rightarrow \quad 0 = 640 - 16v - 8v^2$$

$$\Rightarrow \quad v^2 + 2v - 80 = 0$$

$$\Rightarrow \quad v = 8\ \text{m/s}$$

36. (2)

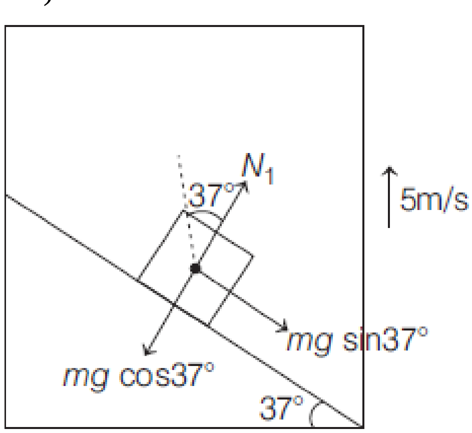
Using work-energy theorem between A and B

$$W_{mg} + W_T + W_F = \Delta K$$

$$\Rightarrow \quad -mgl\left(1 - \cos 37°\right) + 0 + \left(\frac{mg}{2}\right)\left(l \sin 37°\right)$$

$$= \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 \Rightarrow v_B = 2\ \text{m/s}$$

37. (320)



$$\sum F_y = 0$$

$$\Rightarrow \quad N_1 - mg\cos 37° = 0$$

$$\Rightarrow \quad N_1 = mg\cos 37°$$

$$W_{N_1} = Fs\cos\theta$$

$$= (mg\cos 37°)(vt)\cos 37° = 320 \text{ J}$$

38. (2)

Using work-energy theorem for block,

$$W_{mg} + W_N + W_{\text{friction}} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +mg\sin 37° \times 0.5 + 0 - \frac{1}{8}(mg\cos 37°)(0.5) + \frac{1}{2} \times 8\left(0^2 - (0.5)^2\right)$$

$$= \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = 2 \text{ m/s}$$

39. (1.5)

Using work-energy theorem for block,

$$W_{mg} + W_{\text{spring 1}} + W_{\text{spring 2}} + \Delta K$$

$$\Rightarrow \quad +2 \times 10 \times (1+x) + \frac{1}{2} \times 24\left(0^2 - (1+x)^2\right) + \frac{1}{2} \times 24\left(0^2 - x^2\right) = 0 - 0$$

$$\Rightarrow \quad 20 + 20x - 12 - 12x^2 - 24x - 12x^2 = 0$$

$$\Rightarrow \quad 24x^2 + 4x - 8 = 0$$

$$\Rightarrow \quad 6x^2 + x - 2 = 0$$

$$\Rightarrow \quad 6x^2 + 4x - 3x - 2 = 0$$

$$\Rightarrow \quad 2x(3x+2) - (3x+2) = 0$$

$$\Rightarrow \quad x = 0.5$$

So, maximum extension in upper spring

$$= 1 + 0.5 = 1.50 \text{ m}$$

40. (730)

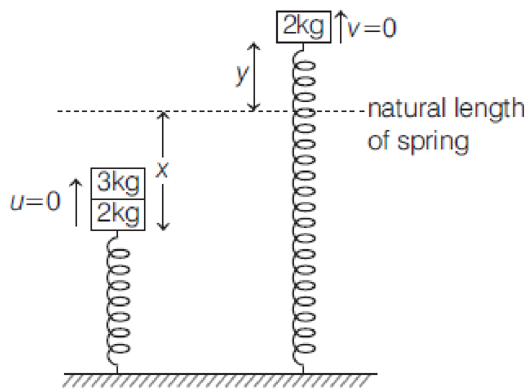
Using work-energy theorem for the object

$$W_{mg} + W_{\text{air}} = \Delta K$$

$$\Rightarrow \quad +5 \times 9.8 \times 20 + W_{\text{air}} = \frac{1}{2} \times 5 \times 10^2 - 0$$

$$W_{\text{air}} = -730 \text{ J}$$

41. (1.50)



Initially lets take compression to be x.

$$\sum F_y = 0$$

$$\Rightarrow \quad 30 + 20 - 40x = 0$$

$$\Rightarrow \quad x = 1.25\,\text{m}$$

After removing 3 kg block, lets take maximum elongation in the spring to be y.

Using work-energy theorem for 2 kg block,

$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad -2\times 10(1.25 + y) + \frac{1}{2}\times 40\left[(1.25)^2 - y^2\right]$$

$$= 0 - 0$$

$$\Rightarrow \quad y = 0.25\,\text{m}$$

So, the maximum height reached by 2 kg block

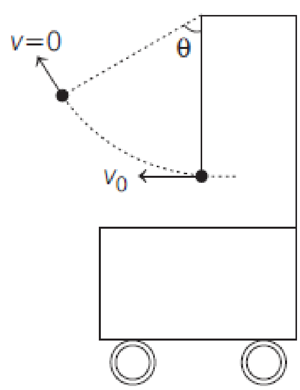
= 1.25 + 0.25

= 1.5 m

42. (7)

Using work - energy theorem,

$$W_{mg} + W_T = \Delta T$$



$$\Rightarrow \quad -mgl(1 - \cos 60^\circ) + 0$$

$$= 0 - \frac{1}{2}mv_0^2$$

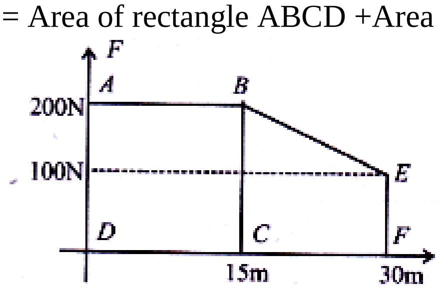
$$\Rightarrow \quad v_0 = \sqrt{gl} = \sqrt{9.8\times 5} = 7\text{ m/s}$$

1. (B)
By work-energy theorem
$W = \Delta K$
$\Rightarrow W = \frac{1}{2}m\left(v_f^2 - v_i^2\right) \Rightarrow W = \frac{1}{2}\times 0.5\times\left(16^2 - 4^2\right)$
$\Rightarrow W = \frac{1}{4}\times 240 \Rightarrow W = 60J$

2. (D)
By work $-$energy theorem
$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}\times 0.5\times\left(b^2.4^5\right)$
$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{4^2}\times 4^5 = 16J$

3. (B)
From work-energy theorem,
$W_{Porter} + W_{mg} = \Delta K.E = 0 \quad (\because \text{velocity constant})$
Or, $W_{Porter} = -W_{mg} = -mgh$
$\therefore W_{Porter} = -80\times 9.8\times\frac{80}{100} = -627.2J$

4. (D)
The given situation can be drawn graphically as shown in figure.
Work done = Area under F-x graph
= Area of rectangle ABCD +Area of trapezium BCFE



$W = (200\times 15) + \frac{1}{2}(100+200)\times 15 = 3000 + 2250$
$\Rightarrow W = 5250J$

5. (C)
Work done, $W = \int \vec{F}.\vec{ds} = \left(-x\hat{i} + y\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right)$
$\Rightarrow W = -\int_1^0 xdx + \int_0^1 ydy = \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1J$

6. (D)

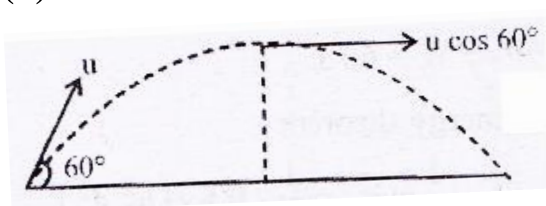
Here, $N - mg = ma = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

N-normal reaction

Now, work done by normal reaction 'N' on block in time, $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}g/2t^2\right)$

Or, $W = \frac{3mg^2t^2}{8}$

7. (C)



At maximum height, we only have horizontal component of velocity . So, Velocity $v = u\cos 60^\circ = \frac{u}{2}$

$\therefore$ K.E. at top most point $= \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{E}{4}$

8. (C)

Momentum of a body is increased by $P' = P + \frac{20}{100}P = 1.2P$

Percentage change in $KE = \frac{K' - K}{K} \times 100$

$$= \left(\frac{\frac{P'^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}}\right) \times 100 = \left[(1.2)^2 - 1\right] \times 100 = 44\%$$

9. (A)

Using $mv = \sqrt{2mk} \Rightarrow v = \frac{1}{m}\sqrt{2mk}$

So, $u = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 90} = 30\,m/s$

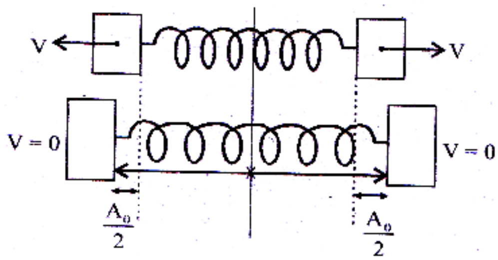
$v = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 40} = 20\,m/s$

$a = \frac{20 - 30}{1} = -10\,m/s^2$ So, $s = \frac{u^2}{2a} = 45\,m$

10. (B)

Given, spring constant of spring , $K = 2\,Nm^{-1}$

Mass of block, $m = 250\,g = \frac{250}{1000}g = \frac{1}{4}kg$



Using energy conservation

$$\frac{1}{2}mv^2 \times 2 = \frac{1}{2}kx^2 \Rightarrow \frac{1}{4}v^2 = \frac{1}{2}\times 2\times x^2$$

$$\therefore x = \frac{v}{2}$$

11. (B)

Kinetic energy, K.E. $\frac{p^2}{2m}$

$$\frac{K.E_1}{K.E_2} = \left(\frac{P_1}{P_2}\right)^2 \times \left(\frac{m_2}{m_1}\right) = \left(\frac{1}{2}\right)^2 \times \frac{8}{5} = \frac{2}{5}$$

12. (D)

By law of conservation of mechanical energy $\Delta k = -\Delta U$

$$\Rightarrow k_f - k_i = U_i - U_f \Rightarrow k_f = mgy - mg\left[y - y_0\right]$$

$$\left[\because k_i = 0, U_i = mgy \text{ and } U_f = mg\left(y - y_0\right)\right]$$

$$\Rightarrow k_f = mgy_0$$

13. (A)

At maximum height , $\nu = 0$

$\Rightarrow m\nu = 0 \Rightarrow P = 0$

14. (C)

By work-energy theorem,

$$\Delta k = W_{\text{all forces}} = \int \vec{F}.d\vec{r}$$

$$= \int\left(4x\hat{i} + 3y^2\hat{j}\right).\left(dx\hat{i} + dy\,\hat{j}\right) = \int_1^2 4x\,dx + \int_2^3 3y^2dy$$

$$= 4\left[\frac{x^2}{2}\right]^2 + 3\left[\frac{y^3}{3}\right]_2^3 = 2\left[2^2 - 1^2\right] + \left[3^3 - 2^3\right]$$

$$= 6 + 19 = 25J$$

15. (A)

Work done by air friction= Final kinetic energy. Initial potential energy $W_{\text{air-friction}} = \frac{1}{2}mv^2 - mgh$

$$= \frac{1}{2}m\left(0.8\sqrt{gh}\right)^2 - mgh$$

$$W_{air-friction} = \frac{64}{2}mgh - mgh = -0.68\,mgh$$

16. (C)

We know area under F-x graph gives the work done by the body

$$\therefore W = \frac{1}{2}\times(3+2)\times(3-2)+2\times2 = 2.5+4 = 6.5J$$

Using work energy theorem,

$w = \Delta KE$ = work done $\therefore \Delta K.E. = 6.5J$

17. (C)

$l_1 + l_2 = l$ and $l_1 = nl_2$ $\quad \therefore l_1 = \frac{nl}{n+1}$ and $l_2 = \frac{l}{n+1}$

As $k \propto \frac{1}{l}$, $\quad \therefore \frac{k_1}{k_2} = \frac{l/(n+1)}{(nl)/(n+1)} = \frac{1}{n}$

18. (E)

Velocity of 1 kg block just before it collides with 3 kg block $= \sqrt{2gh} = \sqrt{2000}\ m/s$

Using principle of conservation of linear momentum just after collision, we get

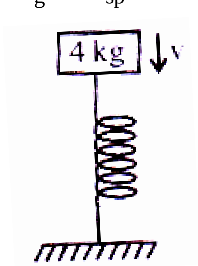
$$1\times\sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4}\ m/s$$

Initial compression of spring

$1.25\times10^6 x_0 = 30 \Rightarrow x_0 \approx 0$

Using work energy theorem,

$W_g + W_{sp} = \Delta KE$



$$\Rightarrow 40\times x + \frac{1}{2}\times1.25\times10^6\left(0^2 - x^2\right) = 0 - \frac{1}{2}\times4\times v^2$$

Solving $x \approx 2cm$

19. (A)

$$W = u_f - u_i = 0 - \left(-\frac{mg}{n}\times\frac{L}{2n}\right) = \frac{MgL}{2n^2}$$

20. (C)

$m\nu = (m+M)V'$

Or $\nu = \frac{m\nu}{m+M} = \frac{m\nu}{m+4m} = \frac{\nu}{5}$

Using conservation of ME, we have

$$\frac{1}{2}mv^2=\frac{1}{2}(m+4m)\left(\frac{v}{5}\right)^2+mgh \text{ or } h=\frac{2}{5}\frac{v^2}{g}$$

21. (D)

When force 'F' is applied, initially $F>F_s$. As F_s will be increase, suppose after x distance $F=F_s$ and there is equilibrium. At this moment block has maximum velocity.

$$F=kx \Rightarrow x=\frac{F}{k}$$

From work-energy theorem,

$$W_F+W_{sp}=\Delta KE$$

$$F(x)-\frac{1}{2}kx^2=\frac{1}{2}mv^2-0 \Rightarrow F\left(\frac{F}{k}\right)-\frac{1}{2}k\left(\frac{F}{k}\right)^2=\frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{K}=\frac{1}{2}mv^2 \text{ or, } v_{max}=\frac{F}{\sqrt{mk}}$$

22. (D)

Position, $x=3t^2+5$

$$\therefore \text{ Velocity, } v=\frac{dx}{dt} \Rightarrow v=\frac{d\left(3t^2+5\right)}{dt}$$

$$\Rightarrow v=6t+0$$

At $t=0$ $\quad v=0$

And, at $t=5\,sec$ $\quad v=30\,m/s$

According to work-energy theorem , $w=\Delta KE$

Or $W=\frac{1}{2}mv^2-0=\frac{1}{2}(2)(30)^2=900J$

23. (C)

$$F=-\frac{\partial u}{\partial r}\hat{r}=\frac{K}{r^3}\hat{r}$$

Since particle is moving in circular path

$$F=\frac{mv^2}{r}=\frac{K}{r^3} \Rightarrow mv^2=\frac{K}{r^2}$$

$$\therefore K.E.=\frac{1}{2}mv^2=\frac{K}{2r^2}$$

Total energy $=P.E.+K.E.$

$$=-\frac{K}{2r^2}+\frac{K}{2r^2}=\text{zero}\left(\because P.E.=-\frac{K}{2r^2}\text{ given}\right)$$

24. (B)

As the particles moving in circular orbits. So $\frac{mv^2}{r}=\frac{16}{r}+r^2$

Kinetic energy, $KE_0=\frac{1}{2}mv^2=\frac{1}{2}\left[16+r^4\right]$

For first particle, $r=1, K_1=\frac{1}{2}m(16+1)$

Similarly, for second particle, $r = 4, K_2 = \frac{1}{2}m(16+256)$

$$\therefore \frac{K_1}{k_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} \simeq \frac{17}{272} \simeq 6\times10^{-2}$$

25. (A)

Let V_f is the final speed of the body. From questions,

$$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \Rightarrow V_f = \frac{V_0}{2} = 5m/s$$

$$F = m\left(\frac{dV}{dt}\right) = -kV^2 \qquad \therefore \left(10^{-2}\right)\frac{dV}{dt} = -kV^2$$

$$\int_{10}^{5}\frac{dV}{V^2} = -100\,K\int_0^{10} dt$$

$$\frac{1}{5} - \frac{1}{10} = 100K(10) \qquad \text{or, } K = 10^{-4}\,kgm^{-1}$$

26. (A)

$$h \propto \frac{V^2}{2g}$$

$h \propto K.E.$

As K.E. becomes half after every collision. So height will also become half.

So, total distance $= h + 2\left(\frac{h}{2} + \frac{h}{4} + ...\right)$

$$= h + 2h\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = 3h$$

27. (C)

Using, $F = ma = m\frac{dV}{dt}$

$$6t = 1.\frac{dV}{dt} \qquad [\because m = 1kg\, given]$$

$$\int_0^v dV = \int 6t\, dtV = 6\left[\frac{t^2}{2}\right]_0^1 = 3ms^{-1}\,[\because t = 1sec\, given]$$

From work energy theorem,

$$W = \Delta KE = \frac{1}{2}m\left(V^2 - u^2\right) = \frac{1}{2}\times1\times9 = 4.5J$$

28. (A)

Work done by friction at $QR = \mu mgx$

In triangle, $\sin 30^\circ = \frac{1}{2} = \frac{2}{PQ} \Rightarrow PQ = 4m$

Work done by friction at $PQ = \mu mg \times \text{Cos}30^\circ \times 4$

$= \mu mg \times \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3}\mu mg$

Since work done by friction on parts PQ and QR are equal, $\mu mgx = 2\sqrt{3}\mu mg \Rightarrow x = 2\sqrt{3} \cong 3.5m$

Using work energy theorem $4mg\sin 30^\circ = 2\sqrt{3}\mu mg + \mu mgx$

$\Rightarrow 2 = 4\sqrt{3}\mu \Rightarrow \mu = 0.29$

29. (B)

$n = \frac{W}{\text{input}} = \frac{mgh \times 1000}{\text{input}} = \frac{10 \times 9.8 \times 1 \times 1000}{\text{input}}$

Input $= \frac{98000}{0.2} = 49 \times 10^4 J$

Fat used $= \frac{49 \times 10^4}{3.8 \times 10^7} = 12.89 \times 10^{-3} kg$

30. (B)

As we know, $dU = F.dr$

$U = \int_0^r \alpha r^2 dr = \frac{\alpha r^3}{3}$ .....(i)

As, $\frac{mv^2}{r} = \alpha r^2 \Rightarrow m^2v^2 = m\alpha r^3$

Or, $2m(KE) = \frac{1}{2}\alpha r^3$ .....(ii)

Total energy $=$ Potential energy + kinetic energy

Now, from equation (i) and (ii)

Total energy $= K.E. + P.E. = \frac{\alpha r^3}{3} + \frac{\alpha r^3}{2} = \frac{5}{6}\alpha r^3$

31. (A)

Let u be the initial velocity of the bullet of mass m. After passing through a plank of width x, its velocity decreases ti v.

$\therefore u - v = \frac{u}{n} \text{ or, } v = u - \frac{u}{n} = \frac{u(n-1)}{n}$

If F be the retarding force applied by each plank, then using work – energy theorem.

$Fx = \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mu^2\frac{(n-1)^2}{n^2}$

$= \frac{1}{2}mu^2\left[\frac{1-(n-1)^2}{n^2}\right]$

$$Fx = \frac{1}{2}mu^2\left(\frac{2n-1}{n^2}\right)$$

Let P be the number of planks required to stop the bullet. Total distance travelled by the bullet before coming to rest $= Px$

Using work-energy theorem again,

$$F(Px) = \frac{1}{2}mu^2 - 0$$

Or, $P(Fx) = P\left[\frac{1}{2}mu^2\frac{(2n-1)}{n^2}\right] = \frac{1}{2}mu^2$

$$\therefore P = \frac{n^2}{2n-1}$$

32. (A)

Given: $k_A = 300\,N/m, k_B = 400\,N/m$

Let when the combination of springs is compressed by force F. Spring A is compressed by x. Therefore compression in spring B

$x_B = (8.75 - x)cm$. In series force is same across both spring

So, $F = 300\times x = 400(8.75 - x)$

Solving we get, $x = 5cm$

$x_B = 8.75 - 5 = 3.75cm$

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300\times(5)^2}{400\times(3.75)^2} = \frac{4}{3}$$

33. (D)

$$F_{thrust} = V_{rel}\frac{dm}{dt} = 5\times 0.5 = 2.5N$$

So, Power $=$ Force $\times$ Velocity $= 2.5\times 5 = 12.5$ watt

34. (B)

We know that

Power, $P = Fv$

But $F = ma = m\frac{dv}{dt}$ $\quad \therefore P = mv\frac{dv}{dt} \Rightarrow P\,dt = mv\,dv$

Integrating both sides $\int_0^t P\,dt = m\int_0^v v\,dv$

$$P.t = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

Distance, $s = \int_0^t v\,dt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt = \sqrt{\frac{2P}{m}}.\frac{t^{3/2}}{3/2}$

$\Rightarrow s=\sqrt{\frac{8P}{9m}}.t^{3/2} \Rightarrow s \propto t^{3/2}$

So, graph (B) is correct.

35. (B)

Total force required to lift maximum load capacity against frictional force = 4000 N

$F_{total} = Mg + friction$

$= 2000 \times 10 + 4000 = 20,000 + 4000 = 4000N$

Using power, $P = F \times v$

$60 \times 746 = 24000 \times v \Rightarrow 1.86 m/s = 1.9 m/s$

Hence speed pf the elevator at full load is close to $1.9\,ms^{-1}$

36. (B)

Centripetal acceleration $a_c = n^2Rt^2$

$a_c = \frac{v^2}{R} = n^2Rt^2$

$v^2 = n^2R^2t^2$

$v = nRt$

Here power is delivered by tangential force only because power by centripetal force is zero. $\left[\text{Since}\,\vec{F}_c \perp \vec{V}\right]$

$a_t = \frac{dv}{dt} = nR$

Power $= ma_t v = m\,nR\,nRt = Mn^2R^2t$

37. (C)

Power, $P = \frac{w}{t} = \frac{E}{t} = \text{constant}$ $\therefore \frac{\frac{1}{2}mv^2}{t} = \text{constant}$

From work-energy theorem, not work done =change in kinetic energy.

$\Rightarrow \frac{v^2}{t} = \text{constant}(k)$ $\therefore kt^{1/2}\text{and}\,\frac{ds}{dt} = kt^{1/2}$

Or, $ds = kt^{1/2}dt$

By integrating, we get $\Rightarrow s = \frac{2kt^{3/2}}{3} + C \Rightarrow s \propto t^{3/2}$

i.e., Distance moved $S \propto t^{3/2}$

38. (2)

Work done by A=Work done by B

$F_A d\cos 45^{\circ} = F_B d\cos 60^{\circ}$

$\Rightarrow F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2} \Rightarrow \frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow x = 2$

39. (450)

Given,

Force, $F = \left(5y + 20\right)\hat{j}$

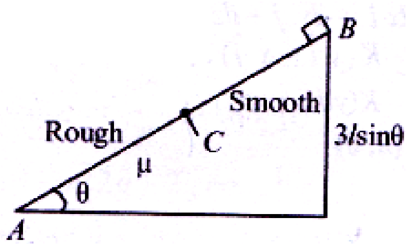
Work done, $E = \int F.dy$

$$\Rightarrow W = \int_0^{10} \left(5y+20\right)dy = \left[\frac{5y^2}{2} + 20y\right]_0^{10}$$

$$= \frac{5}{2} \times 100 + 20 \times 10 = 450J$$

40. (3)

If $AC = l$ then according to question, $BC = 2l$ and $AB = 3l$.



Here, work done by all the forces is zero.

$W_{friction} + W_{mg} = 0$

$mg\left(3l\right)\sin\theta - \mu mg\cos\theta\left(l = 0\right)$

$\Rightarrow \mu mg\cos\theta l = 3mgl\sin\theta \Rightarrow \mu = 3\tan\theta = k\tan\theta$

$\therefore k = 3$

41. (24)

Using work-energy theorem, $W_{net} = \left(K_f - K_i\right)$

$$\Rightarrow \frac{1}{2}Kx^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 = \frac{E}{4} - E$$

$$\Rightarrow \frac{1}{2}Kx^2 = \frac{3E}{4} \Rightarrow K = \frac{3E}{2x^2} \Rightarrow K = \frac{3E}{2\times\left(\frac{1}{4}\right)^2} = 24E$$

So, value of spring constant of used spring is 24 times of kinetic energy

$\therefore n = 24$

42. (2)

Using energy conservation for plane AB

$\frac{1}{2}mu^2 = mgh$ (Here, u= initial velocity of block)

$$\Rightarrow \frac{1}{2}\times m \times u^2 = m \times 10 \times 10 \Rightarrow u = 10\sqrt{2}$$

At point B

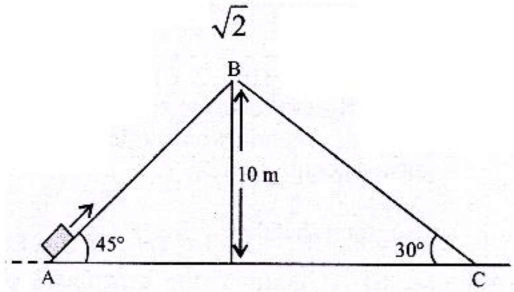
Acceleration, $a = -g\sin 45^\circ = \frac{-10}{\sqrt{2}}$

Using $v = u + at_1$

$\Rightarrow 0 = 10\sqrt{2} - \frac{10}{\sqrt{2}} t_1$

$\Rightarrow t_1 = 2\,\text{sec}$

For plane BC



Using $s = ut_2 + \frac{1}{2}at_2^2$

$\Rightarrow \frac{10}{\sin 30^\circ} = \frac{1}{2}\left(10\sin 30^\circ\right)t_2^2 \quad \left(\because s = \frac{10}{\sin 30^\circ}\right)$

$\Rightarrow t_2 = 2\sqrt{2}$

So total time $T = t_1 + t_2 = 2\sqrt{2} + 2 = 2\left(\sqrt{2}+1\right)\text{sec}$

43. (16)

Mass of engine − wagon system, $m = 40,000\,\text{kg}$ Velocity, $v = 72\times 5/18 = 20\,\text{m/s}$

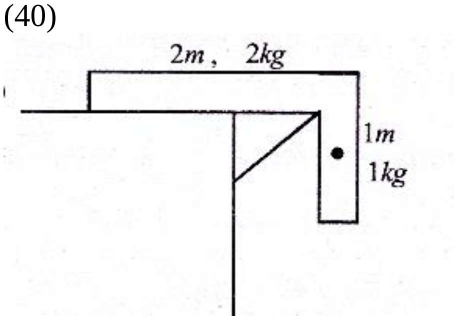
$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\times(40,000)\times(20)^2 = 8000000\text{J}$

As 90% of K.E. of system lost in friction, only 10% is transferred to spring.

$\therefore \frac{1}{2}Kx^2 = \frac{10}{100}\times 8000000 \Rightarrow \frac{1}{2}\times K\times 1\times 1 = 8\times 10^5$

$\Rightarrow K = 16\times 10^5\,\text{N/m}$

44. (40)



Loss in potential energy =gain in kinetic energy

Take zero potential energy at table, initial potential energy

$= -1\times 10\times\frac{1}{2} = -5\text{J}$

Final potential energy $= -3\times 10\times\frac{3}{2} = -45\text{J}$

Change in potential energy $= -5 - (-45)\,\text{J} = 40\text{J}$

$\therefore k = 40$

45. (10)

By mechanical energy conservation,

$T.E_A = T.E_B$

$PE_A + KE_A = PE_B + KE_B$

$mg(10) + 0 = mg(5) + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2 \times g \times 5} = 10\,m/s$

$\therefore x = 10$

46. (6)

Here kinetic energy of ball is equal to P.E. stored in spring i.e., $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$

$\Rightarrow \frac{1}{2} \times 4 \times (10)^2 = \frac{1}{2} \times 100 \times (\Delta x)^2 \Rightarrow \Delta x = 2m$

Therefore length of the compressed spring

$x = 8 - 2 = 6m$

47. (150)

From work energy theorem,

$W = F.s = \Delta KE = \frac{1}{2}mv^2$

Here $V^2 = 2gh$

$\therefore F.s = F \times \frac{2}{10} = \frac{1}{2} \times \frac{15}{100} \times 2 \times 10 \times 20$

$\therefore F = 150N$

48. (10)

Kinetic energy =change in potential energy of the particle.

$KE = mg\Delta h$

Given, $m = 1kg$.

$\Delta h = h_2 - h_1 = 2 - 1 = 1m$

$\therefore KE = 1 \times 10 \times 1 = 10J$

49. (18)

Given, Mass of the body, $m = 2kg$

Power delivered by engine, $P = 1J/s$

Time, t=9 seconds

Power, $P = Fv$

$\Rightarrow P = mav \qquad [\because F = ma]$

$\Rightarrow m\frac{dv}{dt}v = P \qquad \left(\because a = \frac{dv}{dt}\right),$

$\Rightarrow vdv = \frac{P}{m}dt$

Integrating both sides we get

$\Rightarrow \int_0^v vdv = \frac{P}{m}\int_0^t dt \Rightarrow \frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$

$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2P}{m}} t^{1/2} \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt$$

$$\therefore \text{Distance, } x = \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3} t^{3/2}$$

$$\Rightarrow x = \sqrt{\frac{2 \times 1}{2}} \times \frac{2}{3} \times 9^{3/2} = \frac{2}{3} \times 27 = 18\text{m}$$

JEE Advanced Exercise

Exercise – 1

1. **D**

$$W = \int_{x=x_1}^{x=x_2} F dx = \int_{x=0}^{x=5} (7 - 2x + 3x^2) dx = (7x - x^2 + x^3)\Big|_{x=0}^{x=5} y$$

= 135 J

2. **A**

This is the statement of Work – Kinetic Energy Theorem

3. **B**

Since the force acting on the particle is perpendicular to the displacement everywhere, the work done is zero.

4. **B**

Instantaneous power, P = $\vec{F} . \vec{v} = (10\hat{i} + 10\hat{j} + 20\hat{k}) . (5\hat{i} + 3\hat{j} + 6\hat{k})$

= 140 W

5. **D**

Mass of the hanging part = $\frac{M}{3}$; when the hanging part of the chain is paralled on the table, its centre of mass is raised by $\frac{L}{6}$.

The work done = rise in potential energy = $= \left(\frac{M}{3}\right) g \left(\frac{L}{6}\right) = \frac{MgL}{18}$

6. **B**

$$\frac{1}{2} mv^2 = mgR \Rightarrow v = \sqrt{2gR}$$

7. **B**

$K_{longer}\, l_{longer} = K_{original}\, l_{original}$

$$\Rightarrow \quad K_{longer} = \frac{k\ell}{(2\ell/3)} = \frac{3}{2}k$$

8. **C**

W_F = increase in potential energy = mgL (1 – cos θ)

9. **D**

Mean power of gravity = $\frac{\text{work done by gravity}}{\text{time elapsed}} = 0$

10. **B**

Acceleration, a = –kx

$\Rightarrow$ F = –Kx $\therefore$ loss of KE : gain of potential energy $\propto x^2$.

11. C

$$x = \frac{t^3}{3} \Rightarrow v = t^2$$

Now, W = ΔKE = $\frac{1}{2}m(v_f^2 - v_2^2)$ = 16J

12. C

KE_{max} = Maximum loss of KE = Mg*l* (1 – cos θ)

13. A

Centre of mass of the rope is lifted by $\frac{h}{2}$ and the back by h. Therefore,

$$W = Mgh + mg\,\frac{h}{2} = \left(M + \frac{m}{2}\right)gh$$

14. D

$$\mu x\,mg = m\,v\frac{dv}{dx} \Rightarrow \int_{x=0}^{x} \mu\,xg\,dx = \int_{v=0}^{v} mv\,dv$$

$\Rightarrow$ E $\propto x^2$

15. D

$$W = \left(\frac{-3mg}{4}\right)d$$

16. C

$$W = \int_A^B \vec{F}\,.\,d\vec{r} = \frac{F\pi R}{2}$$

17. C

$$P = \frac{dW}{dt} = \frac{3t^2}{2} \Rightarrow W = 4J \Rightarrow v = 2m/s$$

18. C

$W_1 : W_2 : W_3$ = Ratio of corresponding displacements = $1^2 : (2^2 - 1^1) : (3^2 - 2^2)$

= 1 : 3 : 5

19. C

$$\frac{dv}{dx} = \frac{-12a}{x^{13}} + \frac{6h}{x^7} = 0 \Rightarrow x = \left(\frac{2a}{b}\right)^{1/6} \Rightarrow U_{min} = \frac{a}{(2a/b)^2} - \frac{b}{(1a/b)} = -\frac{b^2}{4a}$$

$\therefore$ Minimum energy required = $\frac{b^2}{4a}$

20. D

μ mg v_{max} = P $\Rightarrow$ v_{max} = $\frac{P}{\mu \, mg}$

21. **C**

$$W = \frac{\mu mg}{1+\mu} = 163.3\,J$$

22. **C**

The KE intercepted $\propto v^3$

23. **D**

$$T - mg = m\frac{v^2}{\ell} \Rightarrow T = m\left(g + \frac{5g\ell}{\ell}\right) = 6mg$$

24. **A**

$mg(h + x) = \frac{1}{2}kx^2 \Rightarrow 980x^2 - 2 \times 9.8\,(0.4 + x) = 0$

$\Rightarrow$ $50x^2 - x - 0.4 = 0$

$\Rightarrow$ $(10x - 1)(5x + 0.4) = 0$

$\Rightarrow$ x = 0.1 m = 10 cm

25. **D**

$$W = (kx\,\hat{j}).(a\hat{i}) + \int_{y=0}^{y=a} k(y\hat{i} + a\hat{j}).dy\,\hat{j} = ka^2$$

Exercise – 2

1. **C, D**

Since no work is done by the force speed is constant not velocity. $\vec{a} = \frac{v^2}{r}$ along the centre of circle.

2. **B, C**

W.d by all forces = ΔK.E.

$\Rightarrow$ $W_g + W_N = K.E_f - K.E_i$

$\Rightarrow$ $mgh + 0 = \frac{1}{2}mv^2 - 0$

$\Rightarrow$ $v = \sqrt{2gh} = v_P = v_Q$

where h is the initial height of both blocks from ground.

3. **B, C**

4. **B, C**

w.d = $\vec{F}.\vec{d}$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}).3\hat{j}$

= 6 J

w.d. = $\vec{F}.\vec{d}$

$= (-\hat{i}+2\hat{j}+3\hat{k}).(3\hat{j}+4\hat{k}) = 18J$

5. A, B, C

W.d = Area enclosed by the F-x graph

6. A, D

$a = \frac{F_{net}}{m} = \frac{10-0.2\times2\times10}{2} = 3\,m/s^2$

v(t = 4s) = 0 + 3 × 4 = 12 m/s

s (t = 4s) = $\frac{1}{2}\times3\times4^2 = 24\,m$

w.d by net force = ΔK.E

$= \frac{1}{2}mv^2 = \frac{1}{2}\times2\times12^2$

= 144 J

w.d. by applied force = 10 × 24 = 240 J

w.d by friction = $\vec{F}.\vec{d}$ = – 4 × 24 = –96 J

7. B, C

8. B, C, D

9. A, B, D

w.d by all forces = ΔK.E.

⇒ Fx – mgx = 40 J

⇒ 20x = 40 J

x = 2 m

w.d.$_{gravity}$ = – mgx = –2 × 10 × 2 = – 40 J

w.d.$_{tension}$ = Fx = 40 × 2 = 80 J

10. A, D

Power = $\vec{F}.\vec{v} = Fv = F\times at$ or $F\times\sqrt{2ax}$

Since ‘a’ and ‘F’ are constants

Power varies linearly with time and parabolically with displacement

11. A, C

Hint: Direction of spring force and displacement are same in (a) & (c)

12. B, C

$P_{mg} = \vec{F}.\vec{v}$

$= mg(-\hat{j}).[u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}]$

= – mg (u sin θ – gt)

⇒ $P < 0$ for $t < \frac{u\sin\theta}{g}$ and $P > 0$ for $\frac{2u\sin\theta}{g} > t > \frac{u\sin\theta}{g}$

13. A, C, D

$\vec{a} = \frac{\vec{F}}{m} = -\frac{\delta U}{m\delta x}\hat{i} - \frac{\delta U}{m\delta y}\hat{j} = -3\hat{i} - 4\hat{j}$

$v(\text{at } x = 0) = \sqrt{u^2 + 2as}$

$= \sqrt{0^2 + 2\times 5\times 10}$

= 10 m/s

$\vec{x} = \vec{x}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 6\hat{i} + 4\hat{j} + 0 + \frac{1}{2}(-3\hat{i} + 4\hat{j})1^2$

$= 4.5\hat{i} + 2\hat{j}$

14. C, D

w.d by all force = increase in spring energy

$\Rightarrow \quad Fx_0 + mgx_0 = \frac{1}{2}k(x_0 + \frac{mg}{k})^2 - \frac{1}{2}k\left(\frac{mg}{k}\right)^2$

$\Rightarrow \quad x_0 = \frac{2F}{k}$

15. B, C, D

w.d. by $\vec{F}_2 = 15\times\frac{\pi}{2}\times 6 = 45\pi$ J

w.d. by $\vec{F}_3 = 30\times 6 = 180$ J

w.d. by $\vec{F}_1 = \int F_1 \cos\left(90 - \frac{\theta}{2}\right) rd\theta$

$\vec{F}_1$ is conservative in nature as it is always directed towards P.

16. B, D

At highest point $F_{net} = \frac{mv'^2}{\ell}$

$\Rightarrow \; 2mg + mg = \frac{mv'^2}{\ell}$

$\Rightarrow \; v' = \sqrt{3g\ell}$

Conserving energy velocity at lowest point

$v = \sqrt{7g\ell}$

17. B, D

w.d. = $\vec{F}\cdot\vec{d} = (6\hat{i} - 6\hat{j})\cdot(-3\hat{i} + 4\hat{j})$

$= -18 - 24 = -42$ J

Had there be no initial velocity particle must have moved along straight line making an angle of 45° with x-axis.

18. **A, C, D**

$P = \vec{F}.\vec{v} = +$ ive angle is acute

$= -$ ive angle is obtuse

Area under graph = ΔK.E.

= 20J

19. **A, B**

$\because F = -\frac{dU}{dr} = \frac{2A}{r^3} - \frac{B}{r^2}$ at equilibrium F = or, $r = \frac{2A}{B}$

At infinity U = 0 $r = \frac{2A}{B}$, $U = -\frac{B^2}{4A}$ $\Delta U = \frac{B^2}{4A}$.

20. **A, C**

From conservation of linear momentum (1 + 2)v = (6 x 1) + (2 – 3) v = 4m/s (of both the blocks)

From work energy therom i.e., W_{total} = ΔKE on 1kg block, $W_f = \frac{1}{2} \times 1 \times (4^2 - 6^2) = -10J$ on 2kg block $W_f = \frac{1}{2} \times 2(4^2 - 3^2) = +7J$. ∴ Net work done by friction is –3J.

21. **B, C**

In region OA particle is acccelerated, in region AB particle has uniform velocity while in region BD particle is deceleration., Therefore, work done is positive in region OA , zero in region AB and negative in region BC.

22. **A, C**

At B acceleration of block $= \frac{v^2}{R} = \frac{2gR}{R} = 2g$

23. **A, D**

Exercise – 3

1. **(A - Q), (B - S), (C - R)**

(A – q) Work energy theorem – w.d. by all forces is equal to change in K.e.

(B – s) Negative of work done by conservative force is equal to change in potential energy

(C – r) $W.d_{ext.} + W_{non\ cons.} = \Delta K.E. - W_{cons.}$

$= \Delta K.E. + \Delta U$

$= \Delta T.M.E$

2. **(A - R), (B - Q), (C - P), (D - T)**

(A – R) $\frac{1}{2}mu^2 = mgR + \frac{1}{2}mv_B^2$

$\Rightarrow \quad v_B = \sqrt{7gR}$

(B – Q) $\frac{1}{2}mu^2 = mg \times 2R + \frac{1}{2}mv_C^2$

$v_C = \sqrt{5gR}$

(C – P) $T_B = \frac{mv_B^2}{R} = 7\ mg$

(D – T) $T_C + mg = \frac{mv_C^2}{R}$

$$T_C = \frac{m5gR}{R} - mg = 4mg$$

3. **(A - Q), (B - P), (C - R)**

(A – Q) $w.d = \int_2^4 kx\,dx = \left[\frac{kx^2}{2}\right]_2^4 = \frac{1}{2}k[4^2 - 2^2] = +ive$

(B – P) $w.d = \int_{-4}^{-2} kx\,dx = \left[\frac{kx^2}{2}\right]_{-4}^{-2} = \frac{1}{2}k[2^2 - 4^2] = -ive$

(C – R) $w.d = \int_{-2}^{2} kx\,dx = \left[\frac{kx^2}{2}\right]_{-2}^{2} = 0$

4. **(A – T), (B – P), (C – S), (D – Q)**

$S = \frac{1}{2} \times 2 \times (4)^2 = 16m$

$w.d_{gravity} = -mg \times 16 = -1 \times 10 \times 16 = -160$ J

$w.d_{normal\ reaction} = N \cos\theta \times S = m(g+a)\cos^2\theta \times S = 144J$

$w.d_{friction} = f \times S \times \sin\theta$

$= m(g+a)\sin^2\theta \times S = 48$ J

$w.d._{forces} = \Delta K.E.$

$= \frac{1}{2}m(at)^2 = \frac{1}{2} \times 1 \times (2 \times 4)^2 = 32J$

5. **C**

Work done by both against gravity = mgh

6. **B**

Average Power = $\frac{mgh}{t} = \frac{50 \times 10 \times 15}{30} = 250W$

7. **B**

Chemical energy expended by the physicist ends up increasing the potential energy.

8. **B**

As the physicist falls, gravitational potential gets converted into kinetic energy, increasing his speed. After he hits the cushion, this kinetic energy gets converted into heat.

9. **D**

$\Delta K.E. = K.E_f - K.E_i = K.E_f - 0$

= w.d. by mg

= mgh′

$= 50 \times 10 \times \frac{15}{3} = 2500J$

10. **A**

11. **C**

12. **A**

From conservation of energy at A and B, we have

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgR(1+\sin\theta) \quad \ldots (1)$$

At B the string becomes slack. Therefore

$$mg\sin\theta = \frac{mv_B^2}{R} \quad \ldots (2)$$

After passing through B, the ball goes in a projectile

$\Rightarrow \quad v_B \sin\theta t = R\cos\theta \quad \ldots (3)$

and $-v_B\cos\theta t + \frac{1}{2}gt^2 = R + R\sin\theta \quad \ldots (4)$

On solving 1, 2, 3 & 4

$\theta = 30°$

$v = \sqrt{\frac{7gR}{2}}$ and $v_B = \sqrt{\frac{gR}{2}}$

13. **C**

$\lambda(\ell - x)v + \lambda hg\, dt - \lambda v^2 dt$

$= \lambda(\ell - (x + dx)]\,(v + dv)$, where λ is mass per unit length

$\Rightarrow \quad hg\, dt = (\ell - x)\, dv$

$$\Rightarrow \quad hg\int_{x=0}^{x}\frac{dx}{\ell - x} = \int_{v=0}^{v} v\, dv$$

$$\Rightarrow \quad \frac{v^2}{2} = hg \ln\frac{\ell}{\ell - x}$$

$$\therefore \quad v_{at\,B} = \sqrt{2gh\ln\frac{\ell}{h}}$$

14. **A**

$$KE_x = \lambda(\ell - x)\, gh \ln\frac{\ell}{\ell - x}$$

It is maximum, when $\frac{\ell}{\ell - x} = e$

$$\therefore \quad KE_{max} = \lambda hg\frac{\ell}{e} = \frac{mgh}{e}$$

15. **B**

Heat generated $= \lambda(\ell - h)\, gh - \frac{1}{2}.\lambda h . 2gh \ln\frac{\ell}{h}$

$$= \frac{mgh}{\ell}\left[\ell - h - h\ln\frac{\ell}{h}\right]$$

16. **B**

17. **A**

18. **B**

19. **C**

$U(x) = 20 + (x - 3)^2$
At $x = 0$,
T.M.E = U + K.E
= 20 + 9 + 20 = 49 J
At extreme positions, K.E = 0
$\Rightarrow \quad U = 49J$
$\Rightarrow \quad 20 + (x - 3)^2 = 49$
$\Rightarrow \quad x - 3 = \pm\sqrt{29}$
$\Rightarrow \quad x = 3 \pm \sqrt{29}$ i.e., -3.4 and 7.4 m
$K.E._{max}$ = T.M.E – Min. potential energy
= 49 – 20 (at x = 3)
= 29 J
Body is in equilibrium at min. potential energy
i.e., at x = 3

20. **A**

W_{mg} is path and rate independent

21. **C**

Total energy is conserved when there is no external and no internal non conservative force.

22. **B**

Work done for conservative forces are path independent

23. **D**

$W_{mg} = mgh$

24. **B**

$F.\Delta\vec{r} = \vec{F}.(\vec{r}_2 - \vec{r}_1)$

25. **C**

$t = \frac{2v\sin\theta}{g}$ is time of flight and vertical displacement is zero.

Exercise – 4

1. $W_F = Fh = 80J$; $W_{weight} = -(mg)h = -40$ J.

2. Tension, $T = \frac{2m_1m_2}{m_1+m_2}g$; acceleration $a = \frac{m_2-m_1}{m_1+m_2}g$

$\therefore \quad W = T \cdot \frac{1}{2}at^2 = \frac{m_1m_2(m_2-m_1)}{(m_1+m_2)^2}g^2t^2$

$= \frac{200}{9}J$

3. $W = \int_{x=1}^{x=2}(2+x)\,dx = 3.5J$

4. $\frac{F}{\sqrt{2}} + N = mg$; $\frac{F}{\sqrt{2}} = \mu N$

$\therefore \quad \frac{F}{\sqrt{2}} = \mu\left(mg - \frac{F}{\sqrt{2}}\right) \Rightarrow \frac{F}{\sqrt{2}} = \frac{\mu}{\mu+1}mg = 3.6$

(a) $W_F = \frac{F}{\sqrt{2}}S = 7.2J$

(b) $W_{friction} = -W_F = -7.2J$

(c) $W_{gravity} = 0$

5. W = Area under the curve = $10 \times 2 + \frac{1}{2}\times 2\times 10$

= 30J

6. W_F = increase in potential energy = $mg\,\ell(1-\cos\theta)$

7. $dW = mg(\mu\cos\theta + \sin\theta)\,ds = mg\,(\mu d\ell + dh)$

$\therefore \quad W = mg(\mu\ell + h)$

8. $W = \Delta KE = -\frac{1}{2}(2)\,20^2 = -400J$

9. $W = \Delta KE = \frac{1}{2}m\alpha^2 v$

10. a) $(2m)g\,x_m = \frac{1}{2}k\,x_m^2 \Rightarrow x_m = \frac{4mg}{k}$

b) $\frac{1}{2}(3m)v^2 + \frac{1}{2}K\left(\frac{x_m}{2}\right)^2 = (2m)\,g\left(\frac{x_m}{2}\right)$

$\Rightarrow v = 2g\sqrt{\frac{m}{3k}}$

c) $2mg - k\frac{x_m}{4} = (3m)\,a \Rightarrow a = \frac{g}{3}$

11. $K\left(\frac{2m_A g}{K}\right) = mg \Rightarrow m_A = \frac{m}{2}$

12. Let x be the extension of the spring and θ the angle that the spring makes with the vertical at break off.

K x cos θ = mg ⇒ $40x\frac{0.4}{0.4+x} = 3.2$

⇒ x = 0.1m; The … of B = the slide of $A = \sqrt{(0.5)^2 - (0.4)^2} = 0.3$ metres = h (say)

$\frac{1}{2}(2m)v^2 + \frac{1}{2}Kx^2 = mg\,h$

⇒ v = 1.54 m/s

13. $-\mu\frac{mv^2}{R} = m\frac{dv}{dt} \Rightarrow \int_{v=v_0}^{v}\frac{dv}{v^2} = -\frac{\mu}{R}\int_{t=0}^{t}dt$

$\Rightarrow \frac{1}{v} = \frac{1}{v_0} + \frac{\mu t}{R} \Rightarrow v = v_0\frac{R}{R+\mu v_0 t};\ S = \int_0^t v\,dt = \frac{R}{\mu}\ln\left(1+\frac{\mu v_0}{R}t\right)$

14. $Fb(1^{-\sin\theta}) = 2\times\frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{Fb(1-\sin\theta)}{m}}$, $F_{max} = 2mg$

15. Conserving mechanical energy:

$2 \times 10 \times 1 = 0.5 \times 10 \times (\sqrt{5}-1) + \frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 0.5\left(\frac{2v}{\sqrt{5}}\right)^2$

⇒ v = 3.39 m/s

16. $W = \int_1^2 \vec{F}\,.\,d\vec{s} = \int_{(2,3)}^{(4,6)}(3x^2\hat{i} + 2y\hat{j})\,.\,(dx\hat{i} + dy\,\hat{j} + dz\hat{k})$

$= \int_{(2,3)}^{(4,6)}(3x^2\,dx + 2y\,dy) = x^3\Big|_2^4 + y^2\Big|_3^6$

= 83J

17. (i) $W = \int_{0\,(\text{along}OC)}^{c}(xy\,\hat{i} + xy\,\hat{j})\,.\,(dx\,\hat{i} + dy\,\hat{j})$

$= \int_0^4 x^2(\hat{i}+\hat{j})\,.\,2dx\,\hat{i} = \int_0^1 2x^2\,dx = \frac{2}{3}\,J$

(ii) $W = \int_{0\,(\text{along OA})}^{A} xy\,(\hat{i}+\hat{j})\,.\,dx\,\hat{i} + \int_{A\,(\text{along AC})}^{C} xy\,(\hat{i}+\hat{j})\,.\,dy\,\hat{j}$

$$= 0 + \int_{y=0}^{y=1} y\,dy = \frac{1}{2}\text{ J}$$

(iii) $$W = 0 + \int_{k=0}^{k=1} x\,dx = \frac{1}{2}\text{ J}$$

18. (a) $\int dmg\,h = mgh$

(b) $\int_{x=0}^{x=\ell} \left(\frac{m}{\ell}dx\right) gx = \frac{1}{2} mg\ell$

(c) $\int_{\theta=0}^{\theta=\ell/R} \left(\frac{m}{\ell} Rd\theta\right) g\,(R\cos\theta) = \frac{mgR^2}{\ell}\sin\frac{\ell}{R}$

19. (a) $u\cos\theta = v$

(b) $m \times 10 \times 5 = \frac{1}{2} mv^2 + \frac{1}{2} m\left(\frac{v}{0.8}\right)^2$

$\Rightarrow v = \frac{40}{\sqrt{41}}\text{ m/s}$

20. $mg\left(\frac{3}{4}d\right) + \frac{1}{2}k\left(\frac{D}{4}\right)^2 = \frac{1}{2}mv^2$

$\Rightarrow \quad v = d\sqrt{\frac{3g}{2d} + \frac{k}{16m}}$

21. $mg\,h_{min} = mg\,2r + \frac{1}{2}m\,(\sqrt{gr})^2$

$\Rightarrow \quad h_{min} = \frac{5r}{2};\ mg(5r) - mg\,2r = \frac{1}{2}mv^2$

Now, $F_{resultant} = \frac{mv^2}{r} = 6mg$

22. $mg\,(1 - \cos\theta) = \frac{mv^2}{\ell}$ … (1)

$\frac{1}{2}mg\ell + mg\ell(1 - \cos\theta) = \frac{3}{2}mv^2$ … (2)

From (1) and (2)

$v = \sqrt{\frac{g\ell}{3}}\ ;\ \theta = \cos^{-1}\frac{2}{3}$

23. $\frac{1}{2}mv_0^2 = mg\ell(1 - \cos 60^\circ)$

$\Rightarrow \quad v_0 = \sqrt{g\ell} = \sqrt{9.8 \times 5} = 7\text{ m/s}$

24. $\frac{1}{2}m(\sqrt{5gR})^2 - mgR(1+\cos\alpha) = \frac{1}{2}mv^2$ … (1)

Also, $t_{flight} = \frac{2R\sin\alpha}{v\cos\alpha} = \frac{2v\sin\alpha}{g}$ … (2)

From (1) and (2)

$\alpha = 0$ or $\alpha = 60°$

25. $\frac{1}{2}mv^2 - mg.2R = \frac{1}{2}mv^2$, where v is velocity at the highest point.

$\Rightarrow \quad v = \sqrt{u^2 - 4gR}$

Now, v t_{flight} = 3R

$\Rightarrow \quad \sqrt{u^2 - 4gR}\sqrt{\frac{4R}{g}} = 3R$

$\Rightarrow \quad u = \frac{5}{2}\sqrt{gR}$;

$x_{min} = v_{min}\ t_{flight} = \sqrt{gh}\sqrt{\frac{4R}{g}} = 2R$

26. $\int_{x=0}^{x=\pi R}(\lambda dx)\, g\left[r\sin\frac{x}{r} + x\right] + \frac{1}{2}(\pi r\lambda)\, v^2$

$\Rightarrow \quad v = \sqrt{2gr\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$

27. $mgR\left(\frac{1}{4} + 1 - \cos\theta\right) = \frac{1}{2}mv^2$ … (1)

mg cos θ = $m\frac{v^2}{R}$ … (2)

$\Rightarrow \quad \theta = \cos^{-1}\frac{5}{6}$

28. $mg\sqrt{\left(\frac{n+1}{n+3}\right) - 1} - Mg\left(\sqrt{n^1 - 1} - \sqrt{\left(\frac{n+1}{2}\right)^2 - 1}\right) > 0$

$\Rightarrow \quad \frac{m}{M} > 2\sqrt{\frac{n+1}{n+3}} - 1$

29. $(F R\sqrt{2} - mgR) = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{2R\left(\frac{F\sqrt{2}}{m} - g\right)}$

30. $\frac{1}{2}mv_0^2 = \int_{x=0}^{x=2L}(Gx)(mg)\,dx + \frac{1}{2}k\,L^2$

$\Rightarrow \quad v = \sqrt{4ag + \frac{k}{m}}$

31. Stretch x = 0.4 (sec 30° - 1) = 0.4 $\left(\frac{2}{\sqrt{3}} - 1\right)$; kx sin 30° = μ (mg – kx cos 30°)

$\Rightarrow \quad kx = \frac{\mu mg}{\sin 30^o + \mu\cos 30^o}$

Now, W = ΔU = $\frac{1}{2}kx^2 = 0.09J$

32. a) $mg(1 - \cos\theta) = \frac{1}{2}mv^2$ … (1)

$F + mg\cos\theta = \frac{mv^2}{R}$ … (2)

From (1) and (2)

F = mg (2 – 3 cos θ)

N = Mg – 2F cos θ, = Mg – 2 mg (2 – 3 cos θ) cos θ

Which is minimum when $\theta = \cos^{-1}\frac{1}{3}$

b) $N = 0 \Rightarrow \frac{m}{M} = \frac{3}{2}$

1. **(B)**

The centripetal acceleration

$a_c = k^2rt^2$ or $\frac{v^2}{r} = k^2rt^2$

$\therefore$ $v = krt$

So, tangential acceleration, $a_t = \frac{dv}{dt} = kr$

Work is done by tangential force.

Power $= F_t \cdot v \cdot \cos 0^o$

$= (m\,a_t)(krt)$

$= (mkr)(krt)$

$= mk^2 r^2 t$

2. **(B)**

The force constant of a spring is inversely proportional to the length of the spring.

Let the original length of spring be L and spring constant is K (given)

Therefore,

$K \times L = \frac{2L}{3} \times K'$

$\Rightarrow K' = \frac{3}{2}K$

3. **(D)**

$dU_{(x)} = -Fdx$

$\therefore \quad U_x = -\int_0^x Fdx$

$= \frac{kx^2}{2} - \frac{ax^4}{4}$

U = 0 at x = 0 and at $x = \sqrt{\frac{2k}{a}}$; $\Rightarrow$ we have potential energy zero twice (out of which one is at origin).

Also, when we put x = 0 in the function,

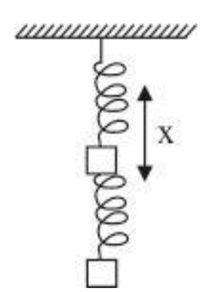
We get F= 0. But $F = -\frac{dU}{dx}$

$\Rightarrow$ At $x = 0; \frac{dU}{dx} = 0$ i.e. the slope of the graph should be zero.

These characteristics are represents by (d).

4. **(B)**

Let the maximum extension of the spring be x as shown in the figure.



Work is done by the gravitational and the spring force.

There is no change in the kinetic energy between the initial and final position of the mass.

From Work-energy theorem;

$W_g + W_s = 0$

Where W_g = work done by gravity
And W_s^g = work done by spring

$$\Rightarrow +Mgx - \frac{1}{2}kx^2 = 0$$

$$\Rightarrow x = \frac{2Mg}{k}$$

5. **(B)**
In a conservative field work done does not depend on the path. The gravitational field is a conservative field.
$\therefore W_1 = W_2 = W_3$

6. **(B)**
We know that

$$\Delta U = -\int_0^x F dx \text{ or } \Delta U = -\int_0^x k\,x dx$$

$$\Rightarrow U_{(x)} - U_{(0)} = -\frac{kx^2}{2}$$

Given $U_{(0)} = 0$

$$U_{(x)} = -\frac{kx^2}{2}$$

7. **(D)**

$$v = \sqrt{5gL} \quad \ldots (1)$$

$$\left(\frac{v}{2}\right)^2 = v^2 - 2gh \quad \ldots (2)$$

$h = L(1 - \cos\theta)$ … (3)
Solving Eqs. (1), (2) and (3), we get

$$\cos\theta = -\frac{7}{8} \text{ or } \theta = \cos^{-1}\left(-\frac{7}{8}\right) = 151^o$$

8. **(C)**
When the block B is displaced towards wall 1, only spring S_1 is compressed and S_2 is in its natural state. This happens because the other end of S_2 is not attached to the wall but is free. Therefore the energy stored in the system $= \frac{1}{2}k_1x^2$. When the block is released, it will come back to the equilibrium position, gain momentum, overshoot to equilibrium position and move towards wall 2. As this happens, the spring S_1 comes to its natural length and S_2 gets compressed. As there are no frictional forces involved, the P.E. stored in the spring S_1 gets stored as the P.E. of spring S_2 when the block B reaches its extreme position after compressing S_2 by y.

$$\therefore \frac{1}{2}k_1x^2 = \frac{1}{2}k_2y^2$$

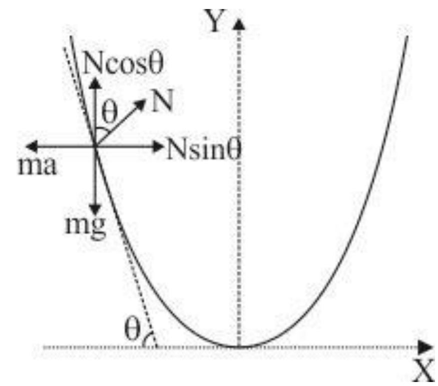
$$\frac{1}{2} \times kx^2 = \frac{1}{2}4ky^2$$

$x^2 = 4y^2$

$\therefore \quad \frac{y}{x} = \frac{1}{2}$

9. **(B)**

The forces acting on the bead as seen by the observer in the accelerated frame are: (a) N; (b) mg; (c) ma (Pseudo force).



Let θ is the angle which the tangent at P makes with the X-axis. As the bead is in equilibrium with respect to the wire, therefore

N sin θ = ma and N cos θ = mg

$\therefore \quad \tan\theta = \frac{a}{g} \qquad ...(i)$

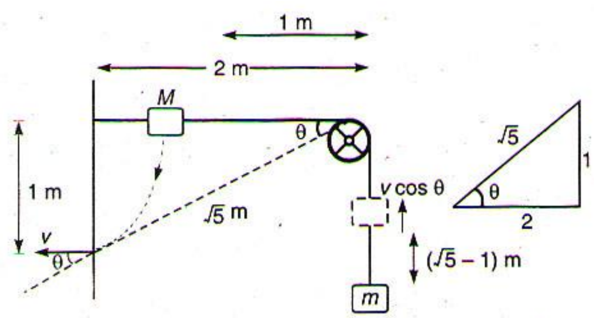
But $y = kx^2$. Therefore,

$\frac{dy}{dx} = 2kx = \tan\theta \qquad ..(ii)$

From (i) & (ii)

$2kx = \frac{a}{g} \Rightarrow x = \frac{a}{2kg}$

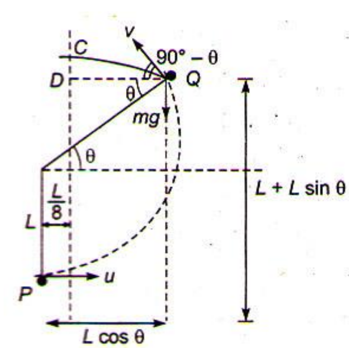
10. Let M strikes with speed v. Then, velocity of m at this instant will be v cos θ or $\frac{2}{\sqrt{5}}v$. Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5}-1)$m. From energy conservation: decrease in potential energy of M = increase in potential energy of m + increase in kinetic energy of both the blocks.



or (2) (9.8) (1) = (0.5) (9.8) $(\sqrt{5}-1) + \frac{1}{2} \times 2 \times v^2 + \frac{1}{2} \times 0.5 \times \left(\frac{2v}{\sqrt{5}}\right)^2$

Solving this equation, we get v = 3.29 m/s

11. Let the string slacks at point Q as shown in figure. From P to Q path is circular and beyond Q path is parabolic. At point C, velocity of particle becomes horizontal, therefore, QD = half the range of the projectile.



Now, we have following equations

(1) $T_Q = 0$. Therefore, $mg \sin\theta = \frac{mv^2}{L}$ … (i)

(2) $v^2 = u^2 - 2gh = u^2 - 2gL(1 + \sin\theta)$ … (ii)

(3) QD = $\frac{1}{2}$ (Range)

$\Rightarrow \quad \left(L\cos\theta - \frac{L}{8}\right) = \frac{v^2 \sin 2(90^\circ - \theta)}{2g} = \frac{v^2 \sin 2\theta}{2g}$ … (iii)

Eq. (iii) canbe written as

$$\left(\cos\theta - \frac{1}{8}\right) = \left(\frac{v^2}{gL}\right)\sin\cos\theta$$

Substituting value of $\left(\frac{v^2}{gL}\right) = \sin\theta$ from eq. (i), we get

$$\left(\cos\theta - \frac{1}{8}\right) = \sin^2\theta - \theta = (1-\cos^2\theta)\cos\theta$$

or $\quad \cos\theta - \frac{1}{8} = \cos\theta - \cos^3\theta$

$\therefore \quad \cos^3\theta = \frac{1}{8}$ or $\cos\theta = \frac{1}{2}$ or $\theta = 60^\circ$

From Eq. (i) $v^2 = gL \sin\theta = gL \sin 60°$

or $\quad v^2 = \frac{\sqrt{3}}{2} gL$

$\therefore \quad$ Substituting this value of v^2 in Eq. (ii)

$$u^2 = v^2 + 2gL(1 + \sin\theta)$$

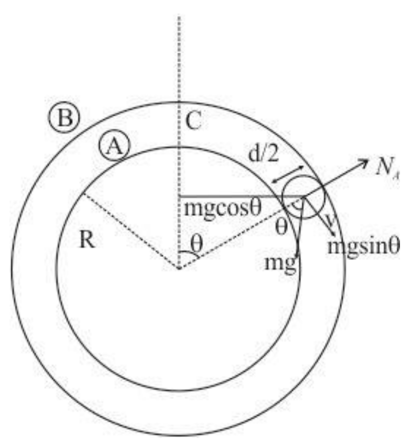
$$= \frac{\sqrt{3}}{2} gL + 2gL\left(1 + \frac{\sqrt{3}}{2}\right)$$

$$= \frac{3\sqrt{3}}{2} gL + 2gL$$

$$= gL\left(2 + \frac{3\sqrt{3}}{2}\right)$$

$$u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$$

12. The ball is moving in a circular motion. The necessary centripetal force is provided by (mg cosθ – N). Therefore,



$$mg \sin\theta - N_A = \frac{mv^2}{\left(R + \frac{d}{2}\right)} \qquad ...(i)$$

According to energy conservation

$$\frac{1}{2}mv^2 = mg\left(R+\frac{d}{2}\right)(1-\cos\theta)...(ii)$$

From (i) and (ii)

$N_A = mg\ (3\cos\theta - 2)$ ...(iii)

The above equation shows that as θ increases N_A decreases. At a particular value of θ, N_A will become zero and the ball will lose contact with sphere A. This condition can be found by putting $N_A = 0$ in eq. (iii)

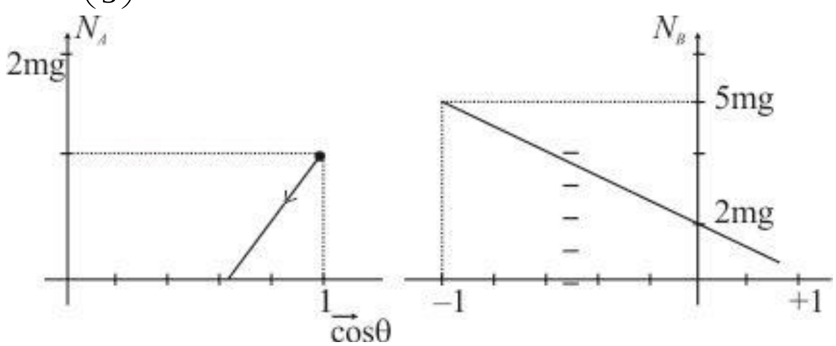
$0 = mg(3\cos\theta - 2)$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

The graph between N_A and cosθ

From equation (iii) when θ = 0, N_A = mg.

When $\therefore \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$



The graph is a straight line as shown

when $\theta > \cos^{-1}\left(\frac{2}{3}\right)$

$$N_B - (mg\cos\theta) = \frac{mv^2}{R+\frac{d}{2}}$$

$$\Rightarrow \quad N_B + mg\cos\theta = \frac{mv^2}{\left(R+\frac{d}{2}\right)} \qquad ...(iv)$$

Using energy conservation

$$\frac{1}{2}mv^2 = mg\left[\left(R+\frac{d}{2}\right) - \left(R+\frac{d}{2}\right)\cos\theta\right]$$

$$\frac{mv^2}{\left(R+\frac{d}{2}\right)} = 2mg[1-\cos\theta] \qquad ...(v)$$

From (iv) and (v), we get

$N_B + mg\cos\theta = 2mg - 2mg\cos\theta$

$N_B = mg(2 - 3\cos\theta)$

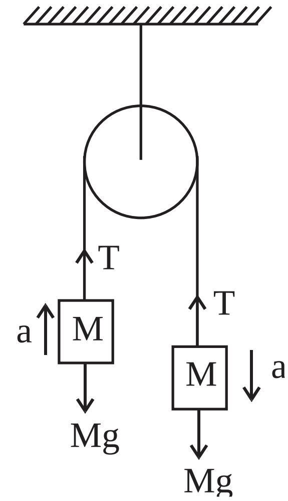
When $\cos\theta = \frac{2}{3}$, $N_B = 0$

When cos θ = –1, N_B = 5 mg

13. **(8)**

Given m = 0.36 kg, M = 0.72 kg.

The figure shows the forces on m and M. When the system is released, let the acceleration be a. Then



T – mg = ma

Mg – T = Ma

$\therefore \quad a=\frac{(M-m)g}{M+m}=g/3$

and T = 4mg/3

For block m:

u = 0, a = g/3, t = 1, s=?

$s=ut+\frac{1}{2}at^2=0+\frac{1}{2}\times\frac{g}{3}\times1^1=g/6$

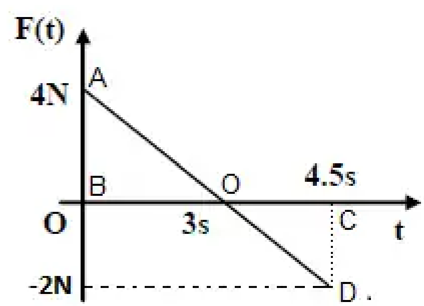
$\therefore$ Work done by the string on m is

$\vec{T}.\vec{s}=Ts=4\frac{mg}{3}\times\frac{9}{6}=\frac{4\times0.36\times10\times10}{3\times6}=8J$

14. **(B)**

Area under the F−t graph gives the change in momentum of the block.

Area A = Area of triangle ABO - Area of triangle DCO



$\therefore \ A=\frac{1}{2}(4)(3)-\frac{1}{2}(2)(1.5)=1.5\text{ Ns}$

Initial momentum of the block $P_i=0$

Using $A=P_f-P_i$

$\Rightarrow \ P_f=4.5\text{ Ns}$

Thus final kinetic energy of the block $K_f=\frac{P_f^2}{2m}$

$\Rightarrow \ K_f=\frac{(4.5)^2}{2(2)}=5.06\text{ J}$

15. **(4)**

Loss in K.E. of the block = Gain in P.E. of the spring + Work done against friction

$\frac{1}{2}mv^2=\frac{1}{2}kx^2+\mu mgx$

or $\frac{1}{2}\times0.18\,v^2=\frac{1}{2}\times2\times(0.06)^2+0.1\times0.18\times10\times0.06$

or $0.09\,v^2=36\times10^{-4}+108\times10^{-4}=144\times10^{-4}$

or $v=\frac{12\times10^{-12}}{0.3}=\frac{4}{10}$

Given $v=\frac{N}{10}$

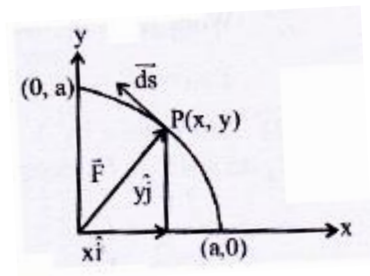
$\therefore \ N=4$

16. **(D)**

Radius of circular path $= a$

The equation of circle is $x^2 + y^2 = a^2$

Given: force



$$\vec{F} = K\left[\frac{x\hat{i}}{(x^2+y^2)^{3/2}} + \frac{y\hat{j}}{(x^2+y^2)^{3/2}}\right]$$

$$\vec{F} = K\left[\frac{x\hat{i}}{(a^2)^{3/2}} + \frac{y\hat{j}}{(a^2)^{3/2}}\right]$$

$$\vec{F} = \frac{K}{a^3}\left[x\hat{i} + y\hat{j}\right]$$

The force acts radially outwards as shown in the figure and the displacement is tangential to the circle path. Here the angle between the force which acts radially outwards and displacement which is tangential to the circular path is 90°

$\therefore$ Work done, $W = FS\cos\theta = 0$

17. **(0.75)**

Given: Force, $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ and $\alpha = -1\,Nm^{-1}$

We know that $dW = \vec{F}.\vec{dr} = (\alpha y\,\hat{i} + 2\alpha x\,\hat{j}).(dx\,\hat{i} + dy\,\hat{j})$

$\therefore dW = \alpha y dx + 2\alpha x dy$

Work done

From $A \to B$ $dy = 0$, as $y = 1$

$$\therefore W_1 = \int_0^1 \alpha y\,dx = \alpha\int_0^1 dx = \alpha$$

From B $\to$**C** $dx = 0, \text{as}\, x = 1$

$$\therefore W_2 = \int_1^{0.5} 2\alpha n\,dy = \int_1^{0.5} 2\alpha dy = 2\alpha(-0.5) = -\alpha$$

From C $\to$**D** dy=0, as y=0.5

$$\therefore W_3 = \int_1^{0.5} \alpha \times 0.5 dx = -\frac{\alpha}{4}$$

From D $\to$**E** $dx = 0, \text{as}\, x = 0.5$

$\therefore W_4 = \int_{0.5}^{0} 2\alpha \times 0.5 dy = -\frac{\alpha}{2}$

From E $\to$F $dy = 0, \text{as } y = 0$

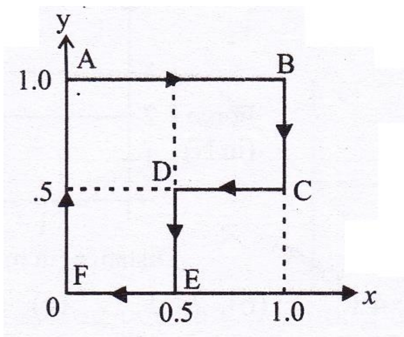
$\therefore W_5 = \int \alpha \times 0 \times dx = 0$

From F $\to$A $dx = 0 \text{ as } x = 0$

$\therefore W_6 = \int 2\alpha \times 0 dx = 0$

$\therefore$ Total work done $W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6$

$= \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2} = -\frac{3\alpha}{4} = \frac{-3(-1)}{4} = 0.75J$



18. **(5)**

Work done = Increase in P.E. + gain in K.E

$F \times d = mgh + \text{gain in K.E}$

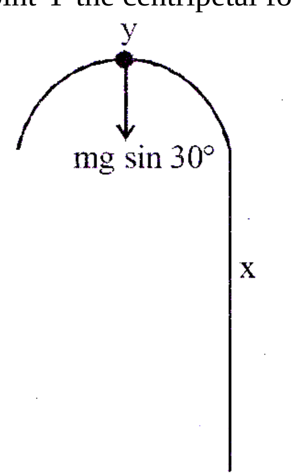
$18 \times 5 = 10 \times 4 + \text{gain in K.E.}$

$\therefore$ Gain in K.E. $= 50J = 10n$

$\therefore n = 5$

19. **(A, D)**

At point Y the centripetal force provided by the component of weight mg



$\therefore mg \sin 30^\circ = \frac{mv^2}{R}$

$\therefore v^2 = \frac{gR}{2}$ ……(ii)

Now by the energy conservation between bottom point and point Y

$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2$$

$\therefore v^2 = v_0^2 - 2gh$ …..(i)

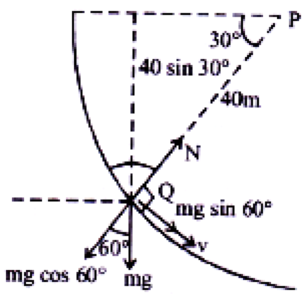
$\therefore$ From eq.(i)

$$\frac{gR}{2} = \frac{v_0^2}{} - 2gh$$

Hence option(A) is correct

At point x and z circular path, the points are at same height but less then h. so the velocity more than at point y. so required centripetal force $= \frac{mv^2}{r}$ is maximum at points x and z.

20. **(B)**



As $W_{\text{all forces}} = \Delta K \Rightarrow W_{mg} + W_{fr} = \frac{1}{2}mv^2 - 0$

$$\Rightarrow mgh - 150 = \frac{1}{2}mv^2$$

$$mg R \sin 30^\circ - 150 = \frac{1}{2}mv^2 \Rightarrow 1 \times 10 \times 40 \times \frac{1}{2} - 150 = \frac{v^2}{2}$$

$$\Rightarrow v = 10\,m/s$$

$N - mg\cos\theta$ will provide required centripetal force

$$N - mg\cos\theta = \frac{mv^2}{R}$$

$$N = mg\cos\theta + \frac{mv^2}{R}$$

$$= 1 \times 10 \times \frac{1}{2} + \frac{1 \times (10)^2}{40} = 7.5N$$

21. **(B)**

As discussed earlier, we get $\nu = 10\,m/s$

22. **(5)**

Using, work − energy theorem, $\Delta K.E. = W = P \times t$

$$\frac{1}{2}mv^2 = P \times t$$

$$\therefore v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5ms^{-1}$$