1(A) Vector - 3D

By PM sir

Q1

Sol Centroid $(0,0,0) \Rightarrow \frac{a-2+4}{3}=0 \Rightarrow a=-2$

& $\frac{1+b+7}{3}=0$ & $b=-8$

& $\frac{3-5+c}{3}=0$ & $c=2$

option (c)

Q2

Sol let $A(0,1,2)$, $B(2,-1,3)$ & $C(1,-3,1)$

$\therefore AB=3$, $BC=3$, $CA=\sqrt{18}$

$\therefore$ Isosceles right angled

Q3

Sol Collinear then equate direction Ratios

i.e of $A(-1,3,2)$ $B(-4,2,-2)$ & $C(5,5,\lambda)$

$\therefore$ AB & AC parallel

$\frac{3}{9}=\frac{1}{3}=\frac{4}{\lambda+2} \Rightarrow \lambda=10$

(4)

Sol^n Apply section formula

$A(1,2,3)$ & $B(3,-5,6)$

Ratio $3:-5$

$$x = \frac{9-5}{-2} = -2, \quad y = \frac{-15-10}{-2} = \frac{-25}{2}$$

$$\& \; z = \frac{18-15}{-2} = -3/2 \qquad \text{option (D)}$$

(5)

$A(1,2,-1)$ & $B(-1,0,1)$

$-1:2$ Ratio

$$\therefore x = \frac{1+2}{1} = 3, \quad y = \frac{4}{1}, \quad z = \frac{-3}{1}$$

(6)

$$\overline{AB} = 6\hat{i} + 6\hat{j} + 4\hat{k}$$

$\therefore$ projection are $(6,6,4)$

(7)

1) $|\bar{a} \times \bar{b} \cdot \bar{c}| = |a||b||c|$

$\Rightarrow$ $\bar{a}, \bar{b}$ & $\bar{c}$ are mutually $\perp^r$

(obvious)

(8)

$\bar{a} + \bar{b} = -\bar{c}$

square $\Rightarrow$ $|a|^2 + |b|^2 + 2\bar{a}\cdot\bar{b} = |c|^2$

$\Rightarrow$ $9 + 25 + 2 \times 3 \times 5 \cos\theta = 49$

$\cos\theta = 1/2 \Rightarrow \theta = \pi/3$

(9)

The diagonals are $\overline{AB} + \overline{BC} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

& $\overline{AB} - \overline{BC} = 4\hat{i} - 2\hat{j}$

$\therefore \cos\theta = \dfrac{8+4}{\sqrt{24}\sqrt{20}} = \dfrac{12}{4\sqrt{30}} = \sqrt{\dfrac{3}{10}}$

(10) $|e_1| = |e_2| = 1$

$\therefore$ $|\bar{e}_1 - \bar{e}_2|^2 = |e_1|^2 + |e_2|^2 - 2\bar{e}_1 \cdot \bar{e}_2$

$= 2(1 - \cos\theta)$

$= 4\sin^2\theta/2$

$\Rightarrow$ $\sin\theta/2 = \dfrac{|e_1 - e_2|}{2}$

(11)

Clearly $\bar{x} = \dfrac{2\bar{b} + \bar{a}}{3}$ $\therefore \overline{xy} = \dfrac{4\bar{b}}{3} - \dfrac{4\bar{a}}{3}$

$\bar{y} = \dfrac{2\bar{b} - \bar{a}}{1}$

option (D)

(12)

$$\left((\vec{a}+3\vec{b}) \times (3\vec{a}-\vec{b}) \right)^2$$

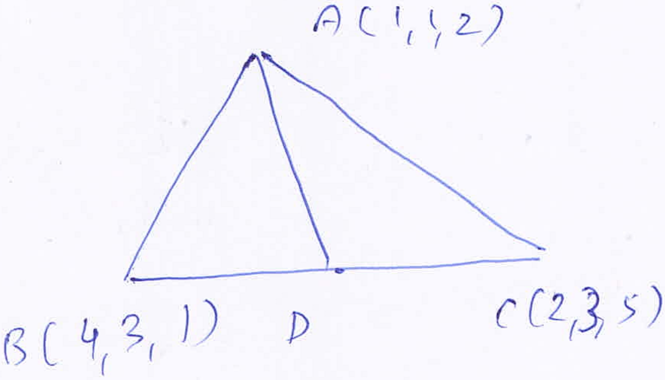
$$\Rightarrow \left(-\vec{a}\times\vec{b} + 9\vec{b}\times\vec{a} \right)^2$$

$$\Rightarrow 100\,|\vec{a}\times\vec{b}|^2$$

$$\Rightarrow 100 \times 4 \times \sin^2\frac{2\pi}{3}$$

$$= 100 \times 3 = 300$$

(13)



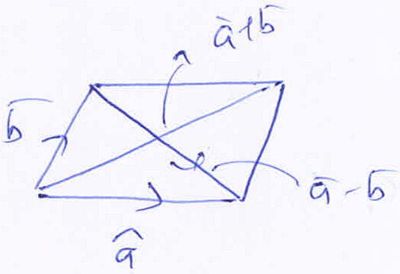
AB = $\sqrt{14}$

AC = $\sqrt{14}$

∴ D is mid pt of BC i.e. D(3, 3, 3)

∴ $\vec{AD} = 2\hat{i} + 2\hat{j} + \hat{k}$

(14)



clearly it is a cute angle

18

(15)

$$\frac{dy}{dx} = \frac{-16}{x^3}$$ $\therefore$ line Eqⁿ is

$\therefore m = \frac{-16}{8} = -2$ $\quad \frac{y-2}{x-2} = -2$

put $y = 0$, $x = 3$ $\quad \therefore B(3,0)$ & $A(2,2)$

$\therefore \overline{AB} \cdot \overline{OB} = (\hat{i} - 2\hat{j}) \cdot (3\hat{i})$

$= 3$

(16) The Bisector has direction along

$$\hat{OA} + \hat{OB} = \left(\frac{-4\hat{i} + 3\hat{k}}{5}\right) + \left(\frac{14\hat{i} + 2\hat{j} - 5\hat{k}}{15}\right)$$

$$= \frac{2}{15}\hat{i} + \frac{2}{15}\hat{j} + \frac{4}{15}\hat{k}$$

clearly (D)

(17)

Vector is $\pm 50 \left(\frac{12\hat{i} - 16\hat{j} - 15\hat{k}}{\sqrt{625}}\right)$

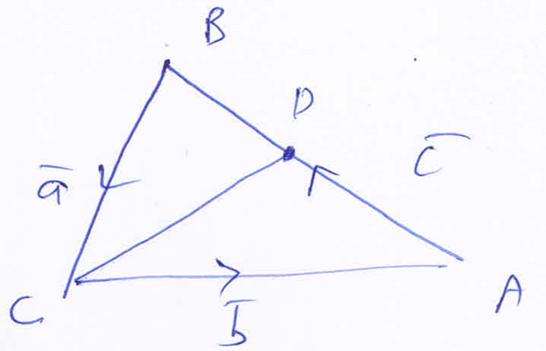
$\pm \cancel{50}^{2} \left(\frac{12\hat{i} - 16\hat{j} - 15\hat{k}}{\cancel{25}}\right)$

But Acute with $+Z$ axis $\Rightarrow$ Coeff of $\hat{k}$ should be (+)ve

$\therefore$ Ans $-2(12\hat{i} - 16\hat{j} - 15\hat{k})$

Hence (B)

18



$\overline{CD} = \dfrac{\bar{b}+\bar{c}}{2} = -\hat{i}+3\hat{j}+4\hat{k}+2\hat{i}-\hat{j}-3\hat{k}$

$= \hat{i}+2\hat{j}+\hat{k}$

$\therefore |CD| = \sqrt{6}$

19

put $\bar{a}\cdot\bar{b}=0 \Rightarrow x-y+2=0$

$\bar{a}\cdot c=4 \Rightarrow x+2y=4$

$\therefore x=0,\ y=2$

$\therefore [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} 0 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 8 = |a|^2$

20

$(\bar{A}+t\bar{B})\cdot\bar{c}=0$

$\Rightarrow \bar{A}\cdot\bar{c}+t(\bar{B}\cdot\bar{c})=0$

$\Rightarrow 5+t(-1) \Rightarrow \boxed{t=5}$

(21) We know $\frac{1}{6}[\bar{a}\ \bar{b}\ \bar{c}] = 3$

$\Rightarrow [\bar{a}\ \bar{b}\ \bar{c}] = 18$

Now $[\bar{a}+\bar{b}\quad \bar{b}+\bar{c}\quad \bar{c}+\bar{a}] = 2[a\ b\ c]$

$= 36$

(22) $\bar{A}+\bar{B}$ is the angle bisector if $|A| = |B|$

$\therefore\ \alpha = 1$

(23) Very easy zero

(24) DR of Normal $\langle 4, -7, 3 \rangle$

$\therefore$ let the plane be $4x - 7y + 3z = \lambda$

$\therefore$ put mid pt as well $\left(1, -\frac{3}{2}, 9/2\right)$

$4 + \frac{21}{2} + \frac{27}{2} = \lambda \Rightarrow \boxed{\lambda = 28}$

option (C)

(25) check that in option (B)

their dot product is zero

$\Rightarrow$ line is ⊥r to normal

or line is parallel to plane

(26) DR are $\langle 2, 3, -6 \rangle$

$\therefore$ DC are $\left\langle \frac{2}{7}, \frac{3}{7}, \frac{-6}{7} \right\rangle$

option (B)

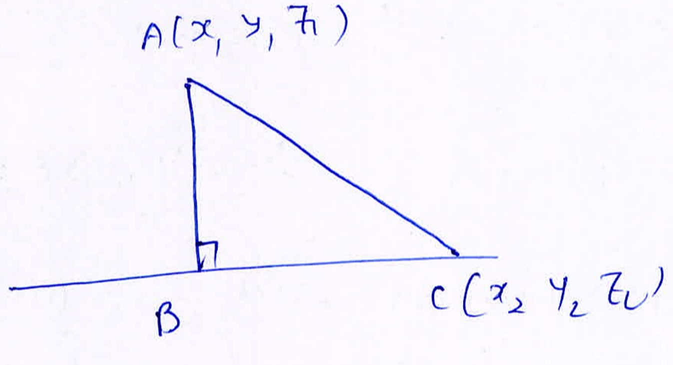
(27) property

(28) We know $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\left(\frac{14}{15}\right)^2 + \left(\frac{1}{3}\right)^2 + \cos^2\gamma = 1$$

$\Rightarrow \cos\gamma = \pm \frac{2}{15}$

(29)



$\longrightarrow l\hat{i} + m\hat{j} + n\hat{k}$

$AC = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

BC is projection of $\overline{AC}$ along $l\hat{i} + m\hat{j} + n\hat{k}$

$\therefore BC = |l(x_1 - x_2) + m(y_1 - y_2) + n(z_1 - z_2)|$

$\therefore$ ~~AB~~ pythagoras theorem

(30) clearly DR are $\langle 2, 2, -2 \rangle$

or $\langle 1, 1, -1 \rangle$

(31)

$$\frac{1}{\sqrt{2}} = \frac{2a - 3 + 10}{3 \cdot \sqrt{34 + a^2}}$$

$\Rightarrow \quad a = 4$

(32) put $x = 5z + 4$

& Also $x = \frac{z-3}{2} + 1$

$$\Rightarrow \quad 5z + 4 = \frac{z - 3 + 2}{2}$$

$$\Rightarrow \quad 10z + 8 = z - 1$$

$9z = -9 \Rightarrow z = -1 \Rightarrow x = -1,$

& $y = -1$

(33) obviously eq^n is

$$\frac{x}{2} + \frac{z}{3} = 1$$

(34) let the plane be

$$2x+3y-4z=\lambda$$

put pt $(1,2,3)$

$$2+6-12=\lambda \Rightarrow \boxed{\lambda=-4}$$

(35) clearly $z=3$

(36) Ans is

$$\frac{3x-6y+2z+5}{7}=+\left(\frac{-4x+12y-3z+3}{13}\right)$$

$$67x-162y+47z+44=0$$

Ans (D)

(37) put dot product as ~~DR~~ DR zero

$$3\times2-6+2k=0$$

$$k=0$$

(38) The line is $\frac{x-2}{1}=\frac{y+3}{-1}=\frac{z-1}{-6}$

let the pt of intersection be $(\lambda+2,-\lambda-3,-6\lambda+1)$

put this in plane

$$2(\lambda+2)-\lambda-3-6\lambda+1=7$$

$$\Rightarrow -5\lambda=5 \Rightarrow \lambda=-1$$ Ans (B)

(39) Assume a pt on line

$(\lambda, 2\lambda+1, 3\lambda-2)$ put this in plane

$2(\lambda) + 3(2\lambda+1) + (3\lambda-2) = 0$

$11\lambda + 1 = 0 \Rightarrow \lambda = \frac{-1}{11}$

Ans (D)

(40) obviously $\perp$r to z axis or // to xy plane

(41) line DR $\langle 2, 3, 2 \rangle$

plane normal DR $\langle 4, -2, -1 \rangle$

dot product = $8 - 6 - 2 = 0$

$\Rightarrow$ line $\perp$r to Normal

or line parallel to plane

(42) The DR of lines are $\langle 1, 2, -5 \rangle$

passing through $(1, 2, 3)$

Eqn is $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{-5}$

(43) $a = b = c$ & $f = g = h = 0$

(44)

$$\sqrt{(x+4)^2+y^2+z^2} + \sqrt{(x-4)^2+y^2+z^2} = 10$$

Rationalize

$$\frac{4 \times 4 \times x}{\sqrt{(x+4)^2+y^2+z^2} - \sqrt{(x-4)^2+y^2+z^2}} = 10$$

$$\therefore \sqrt{(x+4)^2+y^2+z^2} - \sqrt{(x-4)^2+y^2+z^2} = \frac{8}{5}x$$

Add & Square

$$4\left((x+4)^2+y^2+z^2\right) = \left(10+\frac{8}{5}x\right)^2$$

rearrange to get (C)

(45) clearly a sphere

(46) DC are $\left\langle \frac{2}{7}, \frac{-3}{7}, \frac{6}{7} \right\rangle$

option (B)

47) the line is

$$\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$$

put $z=0$

$\Rightarrow$ $x = \frac{13}{5}$, $y = \frac{23}{5}$

option (B)

48) clearly (C)

49) $\frac{x-a}{b} = \frac{y-b}{c} = \frac{z-c}{a}$ option (B)

50) clearly (D)

52) put $l = -(m+n)$

$mn + 2(m+n)^2 = 0$

$2m^2 + 5mn + 2n^2 = 0$

$m = -2n$ or $m = -n/2$

If $m = -2n$
$\Rightarrow l = n$

If $m = -n/2$
$l = -n/2$

DR are

$\langle n, -2n, n \rangle$ or $\left(\frac{-n}{2}, \frac{-n}{2}, n\right)$

or $\langle 1, -2, 1 \rangle$ or $\langle 1, 1, -2 \rangle$

$\therefore \cos\theta = \frac{1-2-2}{\sqrt{6}\cdot\sqrt{6}}$

$= -1/2$

A or B