# Trigonometric Equation

## EXERCISE - 1 [A]

1. **(A)**

$\Rightarrow \tan\frac{2}{3}\theta = \sqrt{3}$

$\Rightarrow \frac{2\theta}{3} = n\pi + \frac{\pi}{3}$

$\Rightarrow 2\theta = 3n\pi + \pi$

$\Rightarrow \theta = 3n\frac{\pi}{2} + \frac{\pi}{2}, n \in I$

2. **(C)**

$\Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$

3. **(C)**

$\Rightarrow \cos\left(\frac{-\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right) = 0$

$\Rightarrow \frac{\theta}{2} = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\pi$

4. **(B)**

$\Rightarrow \cos^2\theta = 1$

$\Rightarrow \cos\theta = \pm 1$

$\Rightarrow \cos\theta = 1$ or $\cos\theta = -1$

$\Rightarrow \theta = 2n\pi$ $\quad\theta = (2n+1)\pi$

$\Rightarrow \therefore \theta = n\pi$

5. **(C)**

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

$\Rightarrow \text{cosec}\,\theta = -\frac{2}{\sqrt{3}} \Rightarrow \sin\theta = \frac{-\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{4\pi}{3} \text{ or } \frac{5\pi}{3}$

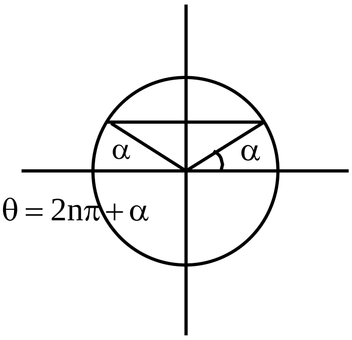
$\Rightarrow \therefore\ \theta = \frac{4\pi}{3}$

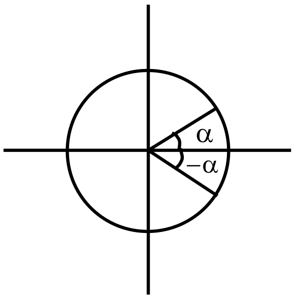
$\Rightarrow \theta = 2n\pi + \frac{4\pi}{3}$

6. **(A)**

$\Rightarrow \sin\theta = \sin\alpha$ $\qquad$ $\cos\theta = \cos\alpha$

$\Rightarrow \theta = n\pi + (-1)^n\ \alpha$ $\qquad$ $\theta = 2n\pi \pm \alpha$




7. **(A)**

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$

$\Rightarrow \frac{\pi}{3} \text{ and } \frac{4\pi}{3}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{3}$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

8. **(A)**

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n\ \frac{\pi}{6}$

P.S. $\frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$

P.S. $= \frac{\pi}{6}, \frac{11\pi}{6}$

Common value $=\frac{\pi}{6}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{6}$

9. **(B)**

$\Rightarrow \sin\theta = \sqrt{3}\cos\theta$ ....$-\pi < \theta < 0$

$\Rightarrow \tan\theta = \sqrt{3}$ .....$\cos\theta \neq 0$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$ .....$\theta \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = -\pi + \frac{\pi}{3}$ .....for $n=-1, \theta \neq \frac{\pi}{2}$

$= -\frac{2\pi}{3}$

$= -\frac{4\pi}{6}$

10. **(B)**

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \theta = n\pi + \frac{5\pi}{6}$

P.S. $=\frac{5\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{5\pi}{6}$

$\Rightarrow$ P.S. $=\frac{5\pi}{6}, -\frac{5\pi}{6}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$

P.S. $=\frac{5\pi}{6}, \frac{11\pi}{6}$

Common solution $=\frac{5\pi}{6}$

11. **(A)**

$\Rightarrow 2\sin x + \tan x = 0$

$\Rightarrow \cos x \neq (2n+1)\pi$

$\Rightarrow 2\sin x + \frac{\sin x}{\cos x} = 0$

$\Rightarrow \sin x\left(\frac{2\cos x + 1}{\cos x}\right) = 0$

$\Rightarrow \sin x = 0$ or $2\cos x + 1 = 0$ .......$\cos x \neq 0$

$\Rightarrow x = n\pi \qquad x = 2n\pi \pm 2\frac{\pi}{3}$

$\Rightarrow \therefore x = (3n \pm 1)\left(\frac{2\pi}{3}\right)$ and $n\pi$

12. **(B)**

$\Rightarrow (2\cos x - 1)(3 + 2\cos x) = 0$

$\Rightarrow 0 \le x \le 2\pi$

$\Rightarrow \cos x = \frac{1}{2}$ or $\cos = -\frac{3}{2}$ ….(not possible)

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3} \text{in} \{0, 2\pi\}$

$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

13. **(B)**

$\Rightarrow (1 + \tan\theta)(1 + \tan\varphi) = 2$

$\Rightarrow (1 + \tan\theta\tan\varphi + \tan\theta + \tan\varphi) = 2$

$\Rightarrow \tan\theta + \tan\varphi = 1 - \tan\theta\tan\varphi$

$\Rightarrow \frac{\tan\theta + \tan\varphi}{1 - \tan\theta\tan\varphi} = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \tan(\theta + \varphi) = \tan\frac{\pi}{4}$

$\Rightarrow \theta + \varphi = 45^{\circ}$

14. **(A)**

$\cos\theta + \cos 2\theta = 2$

$\cos\theta + 2\cos^2\theta - 1 = 2$

$2\cos^2\theta + \cos\theta - 3 = 0$

$\cos\theta = \frac{-3}{2}, 1$

$\cos\theta = 1 \Rightarrow \theta = 2n\pi, n \in I$

15. **(A)**

$\Rightarrow \sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$

$\Rightarrow 1 - \cos^2 - 2\cos\theta + \frac{1}{4} = 0$

$\Rightarrow \frac{5}{4} - \cos^2\theta - 2\cos\theta = 0$

$\Rightarrow \cos^2\theta + 2\cos\theta - \frac{5}{4} = 0$

$\Rightarrow (\cos\theta + 1)^2 - \frac{9}{4} = 0$

$\Rightarrow \left(\cos\theta + 1 - \frac{3}{2}\right)\left(\cos\theta + 1 + \frac{3}{2}\right) = 0$

$$\Rightarrow \left(\cos\theta - \frac{1}{2}\right)\left(\cos\theta + \frac{5}{2}\right) = 0$$

$$\Rightarrow \cos\theta = \frac{1}{2}; \quad \cos\theta = \frac{5}{2} \qquad \text{(not possible)}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$$

16. **(C)**

$$\Rightarrow \tan^2\theta + 2\sqrt{3}\tan\theta = 1$$

$$\Rightarrow \tan^2\theta + 2\sqrt{3}\tan\theta - 1 = 0$$

$$\Rightarrow \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = \frac{-2\sqrt{3} \pm 2\times 2}{2}$$

$$\Rightarrow \tan\theta = -\sqrt{3} \pm 2$$

$$\Rightarrow \tan\theta = 2 - \sqrt{3} \qquad \text{or} \qquad \tan\theta = -2 - \sqrt{3}$$

$$\Rightarrow \tan 15^\circ = 2 - \sqrt{3} \qquad \tan 75^\circ = 2 + \sqrt{3}$$

$$\tan -75^\circ = -2 - \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan 15^\circ = \tan 2 - \sqrt{3}$$

$$= \tan\frac{\pi}{12}$$

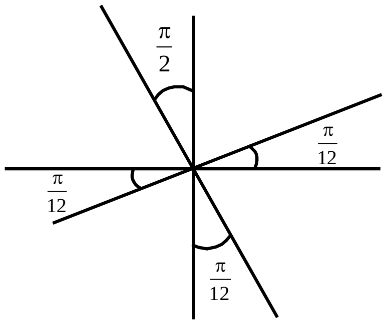
$$\theta = n\pi + \frac{\pi}{12}$$

$$\Rightarrow \tan\theta = \tan\left(-2 - \sqrt{3}\right)$$

$$= \tan - 75$$

$$= \tan - \frac{5\pi}{12}$$

$$\Rightarrow \theta = n\pi - \frac{5\pi}{12}$$




$$\Rightarrow \theta = n\pi + \frac{7\pi}{12}$$

$$= (2n+1)\frac{\pi}{2} + \frac{\pi}{12}$$

$$\Rightarrow \therefore\ \theta = (6n+1)\frac{\pi}{12}$$

17. **(B)**

$$\Rightarrow 25\cos^2\theta + 5\cos\theta - 12 = 0$$

$\Rightarrow \alpha$ is the root then

$$\Rightarrow \cos\alpha = \frac{-5 \pm \sqrt{25+1200}}{50}$$

$= -\frac{4}{5}, \frac{3}{5}$

$\Rightarrow \cos\alpha = -\frac{4}{5}$ ……..II quadrant

$\Rightarrow \sin\alpha = \frac{3}{5}$

$\Rightarrow \sin 2\alpha = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = -\frac{24}{25}$

18. **(A)**

$\Rightarrow \cos x + \sec x = 2$

We know arithmetic mean > Geometric mean

$\Rightarrow \frac{a + \frac{1}{a}}{2} \geq \sqrt{a \times \frac{1}{a}}$

$\Rightarrow a + \frac{1}{a} \geq 2$ (not possible so only equality holds)

Now for $a = \cos x = 1$

$\Rightarrow n = 2n\pi$

19. **(B)**

$\Rightarrow \left(\sin^2\theta\right)\sec\theta + \sqrt{3}\tan\theta = 0$

$\Rightarrow \sec\theta \neq \infty$ ; $\cos\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \sqrt{3}\tan\theta = 0$

$\Rightarrow \tan\theta\left(\sin\theta + \sqrt{3}\right) = 0$

$\Rightarrow \tan\theta = 0$

$\Rightarrow \sin\theta \neq -\sqrt{3}$

$\Rightarrow \theta = n\pi$

20. **(C)**

$\Rightarrow 3\left(\sec^2\theta + \tan^2\theta\right) = 5$

$\Rightarrow 3\left(1 + \tan^2\theta + \tan^2\theta\right) = 5$

$\Rightarrow 3 + 6\tan^2\theta = 5$

$\Rightarrow 6\tan^2\theta = 2$

$\Rightarrow \tan^2\theta = \frac{1}{3} = \tan^2\frac{\pi}{6}$

$\Rightarrow \theta = n \pm \frac{\pi}{6}$

21. **(A)**

$\Rightarrow 2\cot^2\theta = \operatorname{cosec}^2\theta$

$= 1 + \cot^2\theta$

$\Rightarrow \cot^2\theta = 1$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

22. **(A)**

$2\cos^2 x + 3\sin x - 3 = 0$

$2\left(1 - \sin^2 x\right) + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$\sin x = 1$ or $\sin x = \frac{1}{2}$

FQ $0 \le x \le 180^\circ$

$x = 90^\circ, 30^\circ, 150^\circ$

23. **(B)**

$2\sin^2 \theta = 3\cos\theta$

$2\left(1 - \cos^2 \theta\right) = 3\cos\theta$

$2 - 2\cos^2 \theta = 3\cos\theta$

$2\cos^2 \theta + 3\cos\theta - 2 = 0$

$2\cos^2 \theta + 4\cos\theta - \cos\theta - 2 = 0$

$\left(\cos\theta + 2\right)\left(2\cos\theta - 1\right) = 0$

$\cos\theta \ne -2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

24. **(A)**

$\Rightarrow \sin 7\theta + \sin\theta = \sin 4\theta$

$\Rightarrow 2\sin 4\theta \cos 3\theta = \sin 4\theta$

$\Rightarrow \sin 4\theta\left(2\cos 3\theta - 1\right) = 0$

$\Rightarrow \sin 4\theta = 0$ or $2\cos 3\theta - 1 = 0$

$\Rightarrow 4\theta = n\pi$ $\quad \cos 3\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{n\pi}{4}$ $\quad 3\theta = 2m\pi \pm \frac{\pi}{3}$

$\theta = \frac{2m\pi}{3} \pm \frac{\pi}{9}$

between $0 < \theta < \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}$ $\quad \theta = \frac{\pi}{9}$

$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

25. **(B)**

$\cot\theta = \sin 2\theta$

$\frac{\cos\theta}{\sin\theta} = 2\sin\theta\cos\theta$

$\cos\theta = 2\sin^2\theta\cos\theta$

$\cos\theta\left(2\sin^2\theta-1\right)=0$

$-\cos\theta\cdot\cos 2\theta=0$

$\cos\theta=0$ or $\cos 2\theta=0$

$\theta=90^o$ or $\theta=45^o$

26. **(C)**

$\Rightarrow 2\tan^2 a=\sec^2\theta$

$=1+\tan^2\theta$

$\Rightarrow \tan^2\theta=1$

$\Rightarrow \theta=n\pi\pm\frac{\pi}{4}$

27. **(C)**

Snx + cosx = 1

$\frac{1}{\sqrt{2}}\text{Snx}+\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$

$\text{Sn}\left(x+\frac{\pi}{4}\right)=\text{Sn}\frac{\pi}{4}$

$x+\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{4}$

$x=n\pi+(-1)^n\frac{\pi}{4}$

$-\frac{\pi}{4}$

28. **(B)**

$\Rightarrow \cos p\theta=\cos q\theta$

$\Rightarrow p\theta=2n\pi\pm q\theta$

$\Rightarrow p\theta\pm q\theta=2n\pi$

$\Rightarrow \theta(p\pm q)=2n\pi$

$\Rightarrow \theta=\frac{2n\pi}{p\pm q}$

29. **(A)**

$\Rightarrow \tan 5\theta=\cot 2\theta$

$\Rightarrow \tan 5\theta=\tan\left(\frac{\pi}{2}-2\theta\right)$

$\Rightarrow 5\theta=n\pi+\frac{\pi}{2}-2\theta$

$\Rightarrow 7\theta=(2n+1)\frac{\pi}{2}$

$\Rightarrow \theta=(2n+1)\frac{\pi}{14}$

$\Rightarrow \theta=\frac{n\pi}{7}+\frac{\pi}{14}$

$\Rightarrow \tan 5\theta\neq\infty$

$\Rightarrow 5\theta\neq(2n+1)\frac{\pi}{2}$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{14}$

Hence $(2n+1) \neq 7, 21, \text{etc.}$

30. **(B)**

$\Rightarrow \tan\theta + \cot\theta = 2$

$\Rightarrow 2\operatorname{cosec} 2\theta = 2$

$\Rightarrow \sin 2\theta = 1$

$\Rightarrow 2\theta = 2n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = n\pi + \frac{\pi}{4}$

31. **(C)**

$\Rightarrow \cot\theta + \tan\theta = 2\operatorname{cosec}\theta$

$\Rightarrow 2\operatorname{cosec}\theta 2\theta = 2\operatorname{cosec}\theta$

$\Rightarrow \sin\theta = \sin 2\theta$

$\Rightarrow 2\sin\theta\cos\theta = \sin\theta$

$\Rightarrow \sin\theta(2\cos\theta - 1) = 0$

$\Rightarrow \sin\theta \neq 0 \because \theta \neq m\pi$

$\Rightarrow \therefore \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow \theta = 2\pi\pi \pm \frac{\pi}{3}$

$\Rightarrow \tan\theta \neq 0$

$\Rightarrow \theta = n\pi$

$\Rightarrow \cot\theta \neq \infty$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}$

32. **(B)**

$\sin\theta - \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \tan\theta = 1$

If $\theta \in \left(0, \frac{\pi}{2}\right]$ then $\theta = \frac{\pi}{4}$

33. **(B)**

$x \in \left(0, \frac{\pi}{2}\right)$ and $\sin x \cdot \cos x = \frac{1}{4}$

$\Rightarrow \frac{\sin 2x}{2} = \frac{1}{4} \Rightarrow \sin 2x = \frac{1}{2}$

$\therefore 2x = n\pi + (-1)^n \frac{\pi}{6}$

$x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$

$n = 0,\ x = \frac{\pi}{12},\ n = 1,\ x = \frac{\pi}{2} + \frac{-\pi}{12} = \frac{5\pi}{12}$

34. **(C)**

$\Rightarrow 1 + \cot\theta = \operatorname{cosec}\theta$

$\Rightarrow \dfrac{\cos\theta+\sin\theta}{\sin\theta}=\dfrac{1}{\sin\theta}$

$\Rightarrow \sin\theta+\cos\theta=1$

$\Rightarrow \dfrac{1}{\sqrt{2}}\sin\theta+\dfrac{1}{\sqrt{2}}\cos\theta=\dfrac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta+\dfrac{\pi}{4}\right)=\sin\dfrac{\pi}{4}$ ...... $\sin\theta\neq 0;\theta\neq n\pi$

$\Rightarrow \theta+\dfrac{\pi}{4}=n\pi+(-1)^n\dfrac{\pi}{4}$

$\Rightarrow \theta=n\pi+(-1)^n\dfrac{\pi}{4}-\dfrac{\pi}{4}$

$\Rightarrow n=0$ $\theta=0$

$\Rightarrow n=1$ $\pi-\dfrac{\pi}{4}-\dfrac{\pi}{2}=\dfrac{\pi}{2}$

$\Rightarrow n=2$ $2\pi$

$\Rightarrow n=3$ $3\pi-\dfrac{\pi}{2}=\dfrac{5\pi}{2}$

$\Rightarrow \theta=2n\pi+\dfrac{\pi}{2}$

35. **(C)**

$\Rightarrow \sin^2\theta+\sin\theta-2=0$

$\Rightarrow (\sin\theta+2)(\sin\theta-1)=0$

Not possible $\theta=n\pi+(-1)^n\dfrac{\pi}{2}$

$\Rightarrow \theta=2n\pi+\dfrac{\pi}{2}$

36. **(A)**

$\Rightarrow 2\sin^2\theta=4+3\cos\theta$

$\Rightarrow 2-2\cos^2\theta=4+3\cos\theta$

$\Rightarrow 2\cos^2\theta+3.\cos\theta+2=0$

$\Rightarrow 0=b^2-4ac=9-16=-7$

No real roots.

37. **(D)**

$\Rightarrow 3\cos x+4\sin x=6$

$\Rightarrow -5\le 3\cos x+4\sin x\le 5$

For real roots it will never equal 6.

38. **(D)**

$\Rightarrow \cos^2\theta+\sin\theta+1=0$

$\Rightarrow 1-\sin^2\theta+\sin\theta+1=0$

$\Rightarrow (1-\sin\theta)(1+\sin\theta)+(1+\sin\theta)=0$

$\Rightarrow (1+\sin\theta)(1-\sin\theta+1)=0$

$\Rightarrow (1+\sin\theta)(2-\sin\theta)=0$

$\Rightarrow \sin\theta=-1$ or $\sin\theta=2$ (not possible)

$\Rightarrow \theta = \frac{3\pi}{2}$ (principle solution)

39. **(C)**

$\Rightarrow \sin 5x + \sin 3x + \sin x = 0$ .....$0 \le x \le \frac{\pi}{2}$

$\Rightarrow \sin 5x + \sin x + \sin 3x = 0$

$\Rightarrow 2\sin 2x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x\left(2\cos 2x + 1\right) = 0$

$\Rightarrow \sin 3x = 0$

$\Rightarrow 3x = m\pi$

$\Rightarrow x = \frac{m\pi}{3}$

P.S. $= 0, \frac{\pi}{3}, \frac{2\pi}{3}$

$\Rightarrow \cos 2x = -\frac{1}{2}$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = x\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}$

Common value between 0 and $\frac{\pi}{2}$ is $\frac{\pi}{3}$

40. **(A)**

$\Rightarrow \sin 2\theta = \cos 3\theta$

$\Rightarrow \cos 3\theta = \sin 2\theta$

$\Rightarrow \cos 3\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$ or $3\theta = 2n\pi - \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 5\theta = 2n\pi + \frac{\pi}{2}$ $\theta = 2n\pi - \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\left(4n+1\right)}{10}\pi$

$\Rightarrow \theta = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}$ etc.........

$\Rightarrow \therefore$ acute angle $= \frac{\pi}{10}$

$\Rightarrow \theta = 18^o$

$\Rightarrow \sin 18^o = \frac{\sqrt{5}-1}{4}$

41. **(C)**

$\Rightarrow \sqrt{3}\left(\cot\theta + \tan\theta\right) = 4$

As, $\cot\theta + \tan\theta = 2\operatorname{cosec}2\theta$

$\Rightarrow \tan\theta \neq \infty \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \cot\theta \neq \infty \neq n\pi$

$\Rightarrow \therefore 2\sqrt{3}\operatorname{cosec}2\theta = 4$

$\Rightarrow \operatorname{cosec}2\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{3}$

$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$

42. **(D)**

$\Rightarrow \sin x + \frac{1}{\sin x} = \frac{7}{2\sqrt{3}}$

$\Rightarrow 2\sqrt{3}\sin^2 x + 2\sqrt{3} = 7\sin x$

$\Rightarrow 2\sqrt{3}\sin^2 x - 7x + 2\sqrt{3} = 0$

$\Rightarrow \sin x = \frac{7 \pm \sqrt{49 - 4\left(2\sqrt{3}\right)^2}}{4\sqrt{3}}$

$= \frac{7 \pm \sqrt{49-48}}{4\sqrt{3}}$

$= \frac{7 \pm 1}{4\sqrt{3}} = \frac{2}{\sqrt{3}}, \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin = \frac{2}{\sqrt{3}}$ ……..(not possible)

$\Rightarrow \therefore \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = 60^o$

43. **(C)**

$\cos 3x \cdot \cos 7x = \cos^2 2x$

$2\cos 7x + \cos 3x = 2\cos^2 2x$

$\cos 10x + \cos 4x = 2\cos^2 2x$

$\cos 10x + \cos 4x = 1 + \cos 4x$

$\cos 10x = 1 \Rightarrow x = \frac{x\pi}{5}, n \in I$

44. **(D)**

$6\sin^2 x + \sin x - 1 = 0$ …(1)

$6\sin^2 x + 3\sin x - 2\sin x - 1 = 0$

$(2\sin x + 1)(3\sin x - 1) = 0$

$\sin x = \frac{-1}{2}, \sin x = \frac{1}{3}$

Then sum of roots of
Equation 1 in $x \in [0, 2\pi]$
is $4\pi$

45. **(C)**

$2\sin^2 x + 5\sin x + 2 = 0$
$2\sin^2 x + 4\sin x + \sin x + 2 = 0$
$(\sin x + 2)(2\sin x + 1) = 0$

$\sin x = -2$ or $\sin x = \frac{-1}{2}$

Then $x = x\pi + (-1)^n \left(\frac{-\pi}{6}\right), n \in I$

46. **(D)**

$\tan^2\theta = 1 - \sec 2\theta$

$\tan^2\theta = 1 - \left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$

$\tan^2\theta = \frac{1-\tan^2\theta - 1 - \tan^2\theta}{1-\tan^2\theta}$

$\tan^2\theta(1-\tan^2\theta) = -2\tan^2\theta$

$\tan^2\theta - \tan^4\theta = -2\tan^2\theta$
$\tan^4\theta = 3\tan^2\theta$
$\tan^2\theta(\tan^2\theta - 3) = 0$

$\tan^2\theta = 0$

$\tan^2\theta = (\sqrt{3})^2$

$\tan\theta = 0$ $\tan^2\theta = \tan^2\frac{\pi}{3}$

$\theta = n\pi$ $\theta = n\pi \pm \frac{\pi}{3}$

Then $\theta = \frac{n\pi}{3}, n \in I$

47. **(D)**

$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta \cdot \tan 2\theta = \sqrt{3}$
$\tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta \cdot \tan 2\theta)$

$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \cdot \tan 2\theta} = \sqrt{3}$
$\Rightarrow \tan(3\theta) = \sqrt{3}$

$3\theta = n\pi + \frac{\pi}{3}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{9} = (3n+1)\frac{\pi}{9}$

48. **(D)**

$5\cos 2\theta + (1 + \cos\theta) + 1 = 0$

$$5\left(2\cos^2\theta-1\right)+1+\cos\theta+1=0$$
$$10\cos^2\theta+\cos\theta-3=0$$
$$\cos\theta=\frac{1}{2},-\frac{3}{5}$$
$$\theta=\frac{\pi}{3},\quad \pi-\cos\left(\frac{3}{5}\right)$$

49. **(C)**

$$1+\tan^2 x=\sqrt{2}\left(1-\tan^2 x\right)$$
$$\frac{1}{\sqrt{2}}=\cos^2 x$$
$$2x=2n\pi\pm\frac{\pi}{4}$$
$$x=n\pi\pm\frac{\pi}{8}$$

50. **(C)**

$\Rightarrow a\sin x+b\cos x=c$

Will have solution when $|c|<\sqrt{a^2+b^2}$

51. **(D)**

$$\Rightarrow \cos y\cos\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-y\right)\cos x+\sin y\cos\left(\frac{\pi}{2}-x\right)+\cos x\sin\left(\frac{\pi}{2}-y\right)=0$$
$$\Rightarrow \cos y\sin x-\sin y\cos x+\sin y\sin x+\cos x\cos y=0$$
$$\Rightarrow \frac{1}{\sqrt{2}}\sin\left(x-y\right)+\frac{1}{\sqrt{2}}\cos\left(x-y\right)=0$$
$$\Rightarrow \sin\left(x-y+\frac{\pi}{4}\right)=0$$
$$\Rightarrow x-y+\frac{\pi}{4}=n\pi$$
$$\Rightarrow x=n\pi-\frac{\pi}{4}+y$$

52. **(C)**

$$\left(\sin x+\cos x\right)^{(1+\sin 2x)}=2,\ x\in\left[-\pi,\pi\right]$$

$\sin x+\cos x\le\sqrt{2}\ \Rightarrow\ \sin x+\cos x=\sqrt{2}$ and $1+\sin 2x=2$

$\therefore\ \sin 2x=1$

Also, $\sin x+\cos x=\sqrt{2}\ \Rightarrow\ \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2}$

$$\therefore\ \sin\left(x+\frac{\pi}{4}\right)=1$$
$$\therefore\ x=\frac{\pi}{4}$$

53. **(B)**

$$\cos^5 x=1+\left(1-\cos^2 x\right)^2$$

$\cos^5 x = 1 + 1 + \cos^4 x - 2\cos^2 x$
Let $\cos x = t$
$t^5 - t^4 + 2t^2 - 2 = 0$
$(t-1)(t^4 + 2t + 2) = 0$
$t = 1$ or $t^4 + 2t + 2 = 0$
$\cos x = 1$
$x = 2n\pi, n \in I$

54. **(D)**

55. **(B)**
y = Snx – cos x
$y \in \left[-\sqrt{2}, \sqrt{2}\right]$

56. **(D)**
$\sin\left(e^x\right) = 2^x + \frac{1}{2^x}$
As we know that $2^x + \frac{1}{2^x} > 2 \ \forall \ x \in (0, \infty)$ and
$\therefore$ L.H.S $\neq$ R.H.S
Hence no solutions.

57. **(A)**
$\cos x \cos 6x = -1$
Case – 1: $\cos x = 1,$ $\cos 6x = -1$
$x = 2n\pi,$ then $6x = 12n\pi$
cos 6x = 1
Not Possible
Case – 2 : cos x = -1, & cos 6x = 1
$x = (2n+1)\pi$
Then $6x = 6(2n+1)\pi$
$\cos 6x = 1$
$x = (2n+1)\pi$

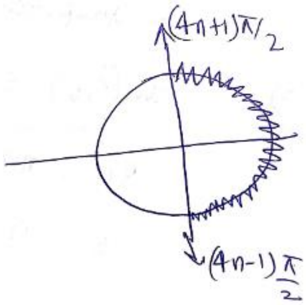
58. **(A)**
$\text{Snx} + \text{Sny} = 2$
$x = \frac{\pi}{2}, \ y = \frac{\pi}{2}$
$n + y = \pi$

59. **(C)**
$\sin x \leq 1$
$\therefore \ x \in R$

60. **(B)**

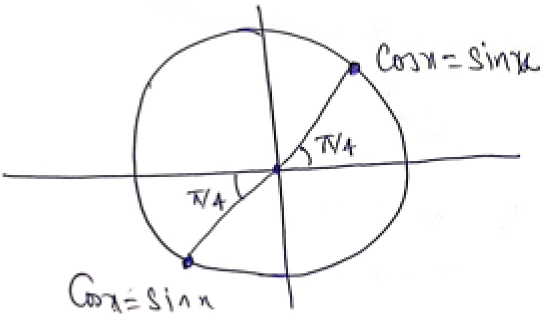
$0 \le \cos x \le 1$

$\therefore \bigcup_{n \in J} \left[ (4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2} \right]$



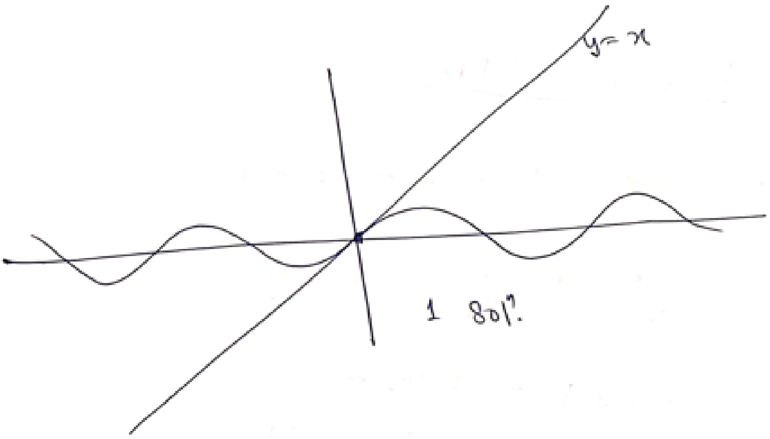

61. **(C)**

$\sin x = \cos x$



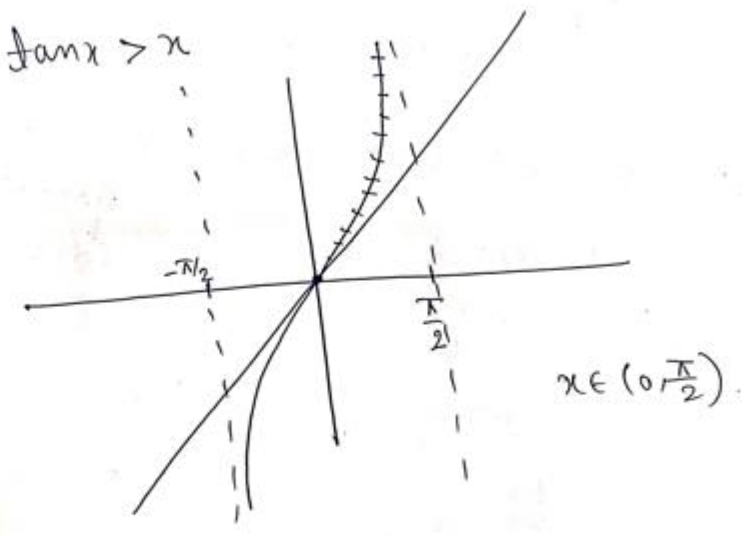

62. **(B)**

$\sin x = x$



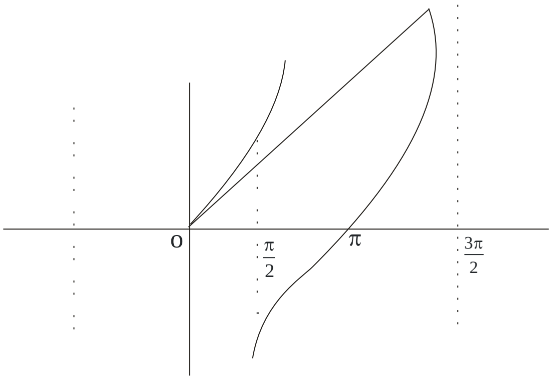

63. **(C)**

$\tan x > x$



64. **(C)**

tan x = x




Soln. lies $\left(\pi, \frac{3\pi}{2}\right)$

65. **(D)**

$x^2 - 4x + 5 = \sin y$

$\underbrace{(x-2)^2 + 1}_{\geq 1} = \underbrace{\sin y}_{\leq 1}$

$\therefore \ \sin y = 1 \quad \& \quad x - 2 = 0$

$y = \frac{\pi}{2} \qquad\qquad x = 2$

# EXERCISE - 1 [B]

1. **(D)**

$\frac{1}{2}(\sin 8\theta + \sin 2\theta) = \frac{1}{2}(\sin 16\theta + \sin 2\theta)$

$\therefore \ \sin 8\theta = \sin 16\theta$

$\sin 16\theta - \sin 8\theta = 0$

$2\sin(4\theta)\cos(12\theta) = 0$

$4\theta = n\pi$

$\theta = \frac{n\pi}{4}, \qquad 12\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{24}$

$0, \frac{\pi}{4}, \frac{\pi}{2}, \ \frac{\pi}{24}, \frac{3\pi}{24}, \frac{5\pi}{24}, \frac{7\pi}{12}, \frac{9\pi}{24}, \frac{11\pi}{12}$

= 9

2. **(B)**

$\lambda = \frac{\sin 4x \cos 4x}{2}$

$= \frac{\sin 8x}{4}$

$\therefore \frac{-1}{4} \leq \lambda \leq \frac{1}{4}$

3. **(D)**

$3\tan^2 x \geq 4\sin^2 x$

$3\sin^2 x \geq 4\sin^2 x\cos^2 x$ [where $x \neq \frac{\lambda}{2}$ ]

$\sin^2 x\left(3-4\cos^2 x\right) \geq 0$

$\sin x = 0$

$x = nz$

$3-4\cos^2 x \geq 0$

$4\cos^2 x - 3 \leq 0$

$\left(\cos x = \frac{\sqrt{3}}{2}\right)\left(\cos x + \frac{\sqrt{3}}{2}\right) \leq 0$

$-\frac{\sqrt{3}}{2} \leq \cos x \leq \frac{\sqrt{3}}{2}$

Where $\cos x \neq 0$

$x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] - \left\{\frac{z}{2}\right\} \cup \{0, z\}$

Hence (D)

4. **(B)**

$\frac{\cos 3x}{4} = \frac{1}{4}$, $\cos 3x = 1$

$3x = 2n\pi$

$x = 2n\frac{\pi}{3}$

$\left\{0 + \frac{2\pi}{3} + \frac{4\pi}{3} + \frac{6\pi}{3} + ...... + \frac{18\pi}{3}\right\}$

$= \frac{10}{2}\left(0 + \frac{18\pi}{3}\right) = 30\pi$

5. **(B)**

$\sin^3 x + \sin x\cos x + \cos^3 x = 1$

$\left(\sin x + \cos x\right)\left(1 - \sin x\cos x\right) + \left(\sin x\cos x - 1\right) = 0$

$\left(\sin x + \cos x\right) = 1$

Or

$\left(\sin x\cos x\right) = 1$

$\sin x\cos x \neq 1$ $\therefore \sin x + \cos x = 1$

$2\sin\frac{\pi}{2}\cos\frac{\pi}{2} = 2\sin^2\frac{\pi}{2}$

$\sin\frac{\pi}{2} = 0$ or $\tan\frac{\pi}{2} = 1$

$x = 2n\pi$ or $\frac{x}{2} = n\pi + \frac{\pi}{4}$

$x = 2n\pi + \frac{\pi}{2}$

6. **(D)**

$7\cos^2 x + \sin x \cos x - 3 = 0$

$x = (2n+1)\frac{\pi}{2}$ is not a solution

Divided by $\cos^2 x$

$7 + \tan x - 3(1 + \tan^2 x) = 0$

$3\tan^2 x - \tan x - 4 = 0$

$\tan = -1$ or $\frac{4}{3}$

$x = n\pi + \frac{3\pi}{4}$ or $k\pi + \tan^{-1}\left(\frac{4}{3}\right)$

7. **(C)**

$4\sin^2 x + 4\sin x + a^2 - 3 = 0$

$(2\sin x + 1)^2 + a^2 - 4 = 0$

$(2\sin x + 1)^2 = 4 - a^2$

$-2 \le 2\sin x \le 2$

$-1 \le 2\sin x + 1 \le 3$

$0 \le (2\sin x + 1)^2 \le 9$

$0 \le (4 - a^2) \le 9$

$-9 \le a^2 - 4 \le 0$

$-5 \le a^2 \le 4$

$a^2 \le 4$

$-2 \le a \le 2$

8. **(A)**

$3\tan(\theta - 15^o) = \tan(\theta + 15^o)$

$\frac{\tan(\theta + 15^o)}{\tan(\theta - 15^o)} = \frac{3}{1}$

$\frac{\sin 2\theta}{\sin 30^o} = \frac{4}{2} = 2$

$\sin 2\theta = 1$

$2\theta = 2n\pi + \frac{\pi}{2}$

$\theta = n\pi + \frac{\pi}{4}$

9. **(B)**

$\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 4$

$\left(\frac{1 + \tan\theta}{1 - \tan\theta}\right) + \left(\frac{1 - \tan\theta}{1 + \tan\theta}\right) = 4$

$2 + 2 + a^2\theta = 4(1 - \tan^2\theta)$

$6 + a^2\theta = 2$

$\tan^2\theta = \frac{1}{3}$, $\theta = n\pi \pm \frac{\pi}{6}$

10. **(A)**

let t = tan θ

$$t + \frac{t+(-1)}{1-(t)(-1)} = 2$$

$$\frac{t(1+t)+t-1}{1+t} = 2$$

$$t^2 + \cancel{2t} - 1 = \cancel{2t} + 2$$

$t^2 = 3$, $\tan^2\theta = \tan^2\frac{\pi}{3}$

$$\theta = n\pi \pm \frac{\pi}{3}$$

11. **(D)**

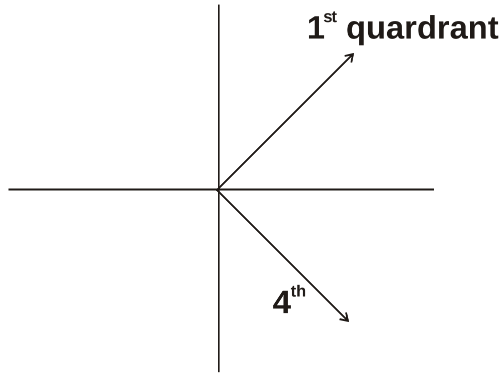
$$2(\sec^2 x - 1) - 5\sec x = 1$$

$$2\sec^2 x - 5\sec x - 3 = 0$$

$$\sec x = \frac{5 \pm \sqrt{25+24}}{4}$$

$$= \frac{12}{4} \text{ or } \frac{-1}{2}$$

$$= 3 \text{ or } \frac{-1}{2}$$




seen = 3, $[0, 6\pi] : 6$ soln.

$\left[6\pi, 6\pi + \frac{3\pi}{2}\right]$: 1 soln.

$\left[0, \frac{15\pi}{2}\right] : 7$ soln.

$n_{max} = 15$

12. **(D)**

$6\tan^2 x - 2\cos^2 x = \cos^2 x$

$6\tan^2 x - (1 + \cos 2x) = \cos 2x$

$$6\left(\frac{1-\cos 2x}{1+\cos 2x}\right) = 2\cos^2 x + 1$$

$$6 - 6\cos^2 x = (1+\cos 2x)(2\cos^2 x + 1)$$

$$6 - 6\cos^2 x = 2\cos^2 2x + 2\cos^2 x + 1 + \cos^2 x$$

$$2\cos^2 2x + 9\cos^2 x - 5 = 0$$

$$\cos^2 x = \frac{1}{2} \text{ or } -5$$

13. **(B)**

Snx – 3Sn2x + Sn3x

= cos x – 3 cos2x + cos3x

2Sn2x cos x – 3 sn 2x

= 2cos 2x cosx – 3 cos2x

Sn 2x (2 cos x – 3 ) – cos 2x (2cos x – 3) = 0

(Sn 2x – cos 2x) (2 cos x – 3 ) = 0

$\tan x = 1 \text{ or } \cos x = \frac{3}{2}$

$2x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{2} + \frac{\pi}{8}$

14. **(C)**

Snx + cosx = 1

$\frac{1}{\sqrt{2}}\text{Snx} + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$

$\text{Sn}\left(x + \frac{\pi}{4}\right) = \text{Sn}\,\frac{\pi}{4}$

$x + \frac{\pi}{4} = n\pi + (-1)^n\,\frac{\pi}{4}$

$x = n\pi + (-1)^n\,\frac{\pi}{4}$

$-\frac{\pi}{4}$

15. **(C)**

$6\,\text{Sn}\,\theta + t\cos\theta = 9$

Let t = $\tan\frac{\theta}{2}$

$6\left(\frac{2t}{1+t^2}\right) + 7\left(\frac{1-t^2}{1+t^2}\right) = 9$

$12t + 7 - 7t^2 = 9 + 9t^2$

$8t^2 - 6t + 1 = 0$

$t = \frac{6 \pm \sqrt{36-32}}{16} = \frac{6 \pm 2}{16} = \frac{1}{2} \text{ or } \frac{1}{4}$

$\tan\frac{\theta}{2} = \frac{1}{2}, \qquad \tan\frac{\theta}{2} = \frac{1}{4}$

$\tan\theta = \frac{2\left(\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{4}{3}, \qquad \tan\theta = \frac{2\left(\frac{1}{4}\right)}{1-\frac{1}{16}}$

$= \frac{1/2}{15/16} = \frac{8}{15}$

16. **(A)**

$\tan x + \sec x = 2\cos x \quad x \in [0, 2\pi)$

$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\sin x + 1 = 2\cos^2 x \qquad 1 + \sin x = 2\left(1 - \sin^2 x\right)$

$(1+\sin x) = 2(1+\sin x)(1-\sin x)$

$\therefore\ 1 = 2(1 - \sin x)$ OR $1 + \sin x = 0$

$1 - \sin x = \frac{1}{2}$ $\sin x = -1$

$\sin x = \frac{1}{2}$ $\cos x = 0$ [Reject]

$\therefore\ \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$. 2 Solutions.

17. **(C)**

$K\cos x = 3Snx = K + 1$

$|K\cos x - 3Snx| \le \sqrt{k^2 + 9}$

$\cancel{k^2} + 2k + 1 \le \cancel{k^2} + 9$

$2k \le 9 - 1$

$k \le 4$

18. **(C)**

$\frac{Sn\,3\theta}{2\cos 2\theta + 1} = \frac{1}{2}, \quad \frac{Sn\,3\theta\ Sn\theta}{2Sn\theta\cos 2\theta + Sn\theta} = \frac{1}{2}$

$\theta \ne n\pi$

$\frac{Sn3\theta\ Sn\theta}{Sn3\theta - Sn\theta + Sn\theta} = \frac{1}{2}$

$Sn\theta = \frac{1}{2}$

$\theta = n\pi + (-1)^n \frac{\pi}{6}$

19. **(C)**

$a^2 - 4a + 6 = (a - 2)^2 + 2 \ge 2$

So, $\min_{a \leftarrow n} \{1, a^2 - 4a + 6\}$

$Sn\,x + asx = 1$

$Sn\left(x + \frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$

$x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

20. **(B)**

$1 + Snx^4 = \cos^2 3x$

$Sn^2 3x + Sn^4 x = 0$

$Sn3x = 0$ & $Snx = 0$

$3x = n\pi$ & $x = n\pi$

$\therefore x = n\pi$

$x = 2\pi$ greatest

21. **(A)**

$$\text{Snx} + \cos x = \sqrt{y + \frac{1}{y}} \geq \sqrt{2}$$

$\therefore \quad \text{Sn}\, x + \cos x = \sqrt{2}$

$x = \frac{\pi}{4}$, & y = 1

22. **(C)**

$\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \tan 4\theta \tan 7\theta$

Refer to (Q. 4) soln.

23. **(C)**

$\cos\theta + \cos 7\theta + \cos 3\theta + \cos 5\theta = 0$

$2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos\theta = 0$

$2\cos 4\theta (\cos 2\theta \cos\theta) = 0$

$\frac{1}{8} \frac{\text{Sns}\,\theta}{\text{Sn}\,\theta} = 0$, $\quad \theta \neq n\pi$

$S\theta = n\pi$

$\theta = n\frac{\pi}{8}, n \neq 8k$

24. **(B)**

$(2\text{Sn}2x \cos x + 3\text{Sn}\, 2x) = (2\cos x \cos^2 x) + 3\cos 2x$

$\text{Sn}\, 2x(2\cos x + 3) - \cos 2x(2\cos x + 3) = 0$

$\tan 2x = 1$ or 2 cos x+ 3 = 0

$2x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{2} + \frac{\pi}{8}$

$\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9}{8}, \frac{13\pi}{8}$

25. **(D)**

$4\text{Sn}\theta \cos\theta - 2\cos\theta - 2\sqrt{3}\text{Sn}\,\theta + \sqrt{3} = 0$

$2\cos\theta(2\text{Sn}\,\theta - 1) - \sqrt{3}(2\text{Sn}\,\theta - 1) = 0$

$(2\cos\theta - \sqrt{3})(2\text{Sn}\theta - 1) = 0$

$\text{Sn}\,\theta = \frac{1}{2}, \quad \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6,} \frac{5\pi}{6}, \frac{11\pi}{6}$

26. **(A)**

$(\text{Sn}\, x + \cos x)(1 - \text{Sn}\, x \cos x0) + 3\sin x \cos x = 1$

$\text{Sn}\, x + \cos x = t$

$\text{Sn}\, x \cos x = \frac{t^2 - 1}{2}$

$$t\left(1-\left(\frac{t^2-1}{2}\right)\right)+3\left(\frac{t^2-1}{2}\right)=1$$
$$t\left(\frac{3-t^2}{2}\right)+\frac{3t^2}{2}-\frac{5}{2}=0$$
$$\frac{t}{2}-\frac{t^3}{2}+\frac{3t^2}{2}-\frac{5}{2}=0$$
$$t^3-3t^2-3t+5=0$$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | -3 | -3 | 5 |
| | $\downarrow$ | 1 | -2 | -5 |
| | 1 | -2 | -5 | $\boxed{0}$ |

$$t^2-2t-5=0$$
$$t=1\pm\sqrt{6}$$
But $\quad t\leftarrow\left[-\sqrt{2},\sqrt{2}\right]$

So, $\quad$ t =1

Sn x + cos x = 1

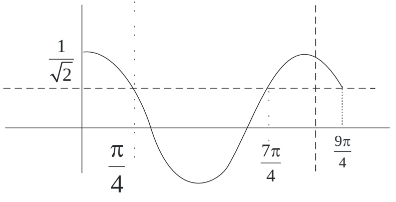
$$x=2n\pi \quad \text{or} \quad 2n+\frac{\pi}{2}$$

27. **(D)**

$$4Sn^2x-8Snx+3\geq 0$$
$$(2Snx-1)(2Snx-3)\leq 0$$
$$\frac{1}{2}\leq Snx\leq\frac{3}{2}$$
$$Snx\geq\frac{1}{2}$$
$$x\leftarrow\left[\frac{\pi}{6},\frac{5}{6}\right]$$

28. **(C)**

$$\cos x-Sn\,x\geq 1 \qquad x\leftarrow[0,2\pi]$$
$$\cos\left(x+\frac{\pi}{4}\right)\geq\frac{1}{\sqrt{2}} \qquad \theta\leftarrow\left[\frac{\pi}{4},\frac{9\pi}{4}\right]$$
$$\cos\theta\geq\frac{1}{\sqrt{2}}$$




$$\frac{7\pi}{4}\leq\theta\leq\frac{9\pi}{4}$$
$$\frac{7\pi}{4}\leq x+\frac{\pi}{4}\leq\frac{9\pi}{4}$$

$$\frac{3\pi}{2} \le x \le 2\pi$$

x = 0 also satisfied

$$x \leftarrow \left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$$

29. **(B)**

$$\frac{2^{Sn\theta} + 2^{-\cos\theta}}{2} \ \ge \ \sqrt{2^{(Sn\theta - \cos\theta)}}$$

$$\ge \sqrt{2^{-\sqrt{2}}}$$

$$\ge 2^{-1/\sqrt{2}}$$

So $2^{Sn\theta} + 2^{-\cos\theta} \ \ge \ 2^{1-\frac{1}{\sqrt{2}}}$

When

$$Sn\theta = -\cos\theta = -\frac{-1}{\sqrt{2}}$$

i.e. $\theta = \dfrac{2n\pi + 7\pi}{4}$

30. **(A)**

$$\left(\sqrt{3}-1\right)Sn\,\theta + \left(\sqrt{3}+1\right)\cos\theta = 2$$

$$\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)Sn\,\theta + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\cos\theta = \frac{1}{\sqrt{2}}$$

$$Sn\,15^o\,Sn\,\theta + \cos 15^o \cos\theta = \cos 45^o$$

$$\cos\left(\theta - 15^o\right) = \cos 45^o$$

$$\theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi K \pm \frac{\pi}{4} + \frac{\pi}{12}$$

31. **(C)**

$$4Sn\theta\ Sn\,2\theta Sn\,4\theta = Sn\,3\theta$$

$$\Rightarrow 2\left(\cos\theta - \cos 3\theta\right)Sn\,4\theta - Sn\,3\theta$$

$$\Rightarrow \left(Sn\,5\theta + Sn\,3\theta - \left(Sn\,7\theta + Sn\,\theta\right)\right) = Sn\,3\theta$$

$$Sn\,\theta + Sn\,7\theta - Sn\,5\theta = 0$$

$$Sn\,\theta + 2Sn\,\theta\cos 6\theta = 0$$

$Sn\theta = 0$ or $\cos 6\theta = \dfrac{-1}{2}$

$\theta = n\pi$ $\qquad 6\theta = 2n\pi \pm \dfrac{2\pi}{3}$

$$\theta = \frac{n\pi}{3} \pm \frac{\pi}{9}$$

$$\theta = \left(3n \pm 1\right)\frac{\pi}{9}$$

32. **(C)**

$$8\cos x\cos 2x\cos 4x = \frac{Sn\,6x}{Snx}$$

$n \neq n\pi$

$$\frac{Sn\,8x}{Sn\,x} = \frac{Sn\,6x}{Sn\,x}$$

$Sn\,8x = Sn\,6x$

$2Sn\,x\ \cos 7x = 0$

$x = n\pi$ or $\quad 7x = (2n+1)\frac{\pi}{2}$

$$x = (2n+1)\frac{\pi}{14}$$

$$x = \frac{n\pi}{7} + \frac{\pi}{14}$$

33. **(B)**

$Sn\,3\alpha = 4Sn\,\alpha\left(Sn^2x - Sn^2\alpha\right)$

$3Sn\alpha - 4Sn^3\alpha = 4Sn\alpha\ \ Sn^2x - 4Sn^3\alpha$

$3Sn\alpha = 4Sn\alpha \qquad Sn^2x$

$$Sn^2x = \frac{3}{4}$$

$$x = n \pm \frac{\pi}{3}$$

34. **(B)**

$\tan(\cot x) = \cot(\tan x)$

$$= \tan\left(\frac{\pi}{2} - \tan x\right)$$

$$\cot x = \frac{\pi}{2} - \tan x + n\pi$$

$$\frac{2}{Sn\,2x} = n\pi + \frac{\pi}{2} = (2n+1)\frac{\pi}{2}$$

$$Sn\,2x = \frac{4}{(2n+1)\pi}$$

35. **(C)**

$12\cos^3 x - 7\cos^2 x + 4\cos x - 9 = 0$

| | | | | |
|---|---|---|---|---|
| 1 | 12 | -7 | 4 | -9 |
| | ↓ | 12 | 5 | 0 |
| | 12 | 5 | 9-5 | |

$(\cos x - 1)\left(12\cos^2 x + 5\cos x + 9\right) = 0$

cos x = 1

$x = (2n\pi)$

Infinite Soln.

36. **(A)**

$\tan 3\theta + \tan\theta = 2\tan 2\theta$

$\frac{\text{Sn } 2\theta}{\cos 3\theta \cos\theta} = \frac{2\,\text{Sn } 2\theta}{\cos 2\theta}$

$\therefore \quad \cos 2\theta = 2\cos 3\theta \cos\theta$

$\cos 2\theta = \cos 4\theta + \cos 2\theta$

$\therefore \quad \cos 4\theta = 0$

$4\theta = (2n+1)\frac{\pi}{2}$

$\theta \neq (2n+1)\frac{\pi}{2}$

$(2n+1)\frac{\pi}{6}$

$(2n+1)\frac{\pi}{4}$

Either

$\text{Sn } 2\theta = 0$

$2 = n\pi$

$\theta = \frac{n\pi}{2}$

But $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

$\theta = m\pi$

$\theta = (2n+1)\frac{\pi}{8}$

37. **(D)**

$\tan\left(\frac{p\pi}{4}\right) = \tan\left(\frac{\pi}{2} - \frac{2}{4}\right)$

$\frac{p\pi}{4} = n\pi + \frac{\pi}{2} - \frac{9\pi}{4}$

$(p+q) = 4n + 2$

$= 2(2n+1)$

38. **(C)**

$\tan(\pi\cos n) = \cot(\pi\,\text{Sn } x)$

$= \tan\left(\frac{\pi}{2} - \pi\,\text{Snx}\right)$

$\pi\cos x = \frac{\pi}{2} - \pi\text{Sn } x + n\pi$

$\sqrt{2} \leq \text{Snx} + \cos x = n + \frac{1}{2} \leq \sqrt{2}$

$\sqrt{2}\cos\left(\frac{\pi}{4} - x\right) = n + \frac{1}{2}, \quad n = 0, -1$

$= \pm\frac{1}{2}$

$$\cos\left(\frac{\pi}{4}-x\right)=\pm\frac{1}{2\sqrt{2}}$$

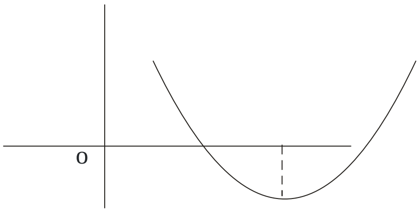
39. **(C)**

$\cos^4 x + a\cos^2 x + 1 = 0$

$D \geq 0 \qquad a^2 - 4 \geq 0$

$\qquad\qquad |a| > 2$

Product of roots = 1




So both roots cannot lie in [0, 1]

Hence one root > 1 & one root lie $\in$ (0, 1)

So $\quad f(1) < 0$

$1 + a + 1 \leq 0$

$a \leq -2$

$\therefore \quad a \leftarrow (-0, -2]$

40. **(C)**

$\tan^4 x - 2\tan^2 x - 2 + a^2 = 0$

$\tan^4 x + 2ta^2x + 1$

$= 3 - a^2$

$\left(ta^2x - 1\right)^2 = 3 - a^2 \geq 0$

$|a| \leq \sqrt{3}$

41. **(A)**

$x^2 + 4 + 3Sn\left(ax+b\right) - 2x = 0$

$\left(x^2 - 2x + 1\right) + 3\left(1 + Sn\left(ax+b\right)\right) = 0$

$\left(x-1\right)^2 + 3\left(1 + Sn\left(ax+b\right)\right) = 0$

$\left(x-1\right) = 0 \;\; \& \;\; Sn\left(ax+b\right) = -1$

$x = 1 \;\&\; Sn\left[a+b\right] = -1$

$a + b = \frac{3\pi}{2}, \frac{7\pi}{2}$ etc.

$\therefore a + b = \frac{7\pi}{2}$

42. **(B)**

$3Sn\,x + 4\cos ax = 7$

Sn x = 1 & cos ax = 1

$x \quad = 2n\pi + \frac{\pi}{2}, ax = 2n\pi$

$\quad = \left(4n+1\right)\frac{\pi}{2} \qquad \therefore \frac{a\pi}{2}\left(4n+1\right) = 2m\pi$

$a=\frac{4m}{4n+1}$

$a=\frac{4m}{4n+1}$ $\quad\quad\quad\quad m(4n+1)K$

$a=\frac{4n(4n+DK)}{4n+1}$

A = 4mk

43. **(A)**

$|\text{Sn}\,x+\cos x|=|\text{Sn}\,x|+|\cos x|$

$\therefore\quad \text{Sn}\,x\ \cos x\geq 0$

I & III Quadrant

44. **(B)**

$\frac{\text{Sin}^3\theta-\cos^3\theta}{\text{Sn}\,\theta-\cos\theta}-\frac{\cos\theta}{\sqrt{1+\cot^2\theta}}$ $\quad\quad\quad\quad \theta\neq\frac{\pi}{4}$

$-2\tan\theta\ \cot\theta=-1$

$1+\text{Sn}\,\theta\ \cos\theta-\cos\theta|\text{Sn}\ \theta|-2=-1$

$\text{Sn}\,\theta\cos\theta=\cos\theta|\text{Sn}\,\theta|$

$|\text{Sn}\,\theta|=\text{Sn}\,\theta$

$\theta\leftarrow\left(\frac{\pi}{2},\pi\right)$

45. **(B)**

$\frac{2^{\text{Sn}\,x}+2^{\text{Cos}\,x}}{2}\geq\sqrt{2^{(\text{Sn}\,x+\cos x)}}$

$\geq\sqrt{2^{-\sqrt{2}}}$

$\geq 2^{-1/2}$

$\therefore 2^{\text{Snx}}+2^{\cos x}\geq 2^{1-\frac{1}{2}}$

Equal if Snx = cosx = $\frac{-1}{2}$

i.e $x=\frac{5\pi}{4}$

$x=2n\pi+\frac{5\pi}{4}$

$x=(2n+1)\pi+\frac{\pi}{4}$

46. **(C)**

$\text{Sn}\left(\pi\left(x^2+x\right)\right)=\text{Sn}\ \pi x^2$

$\pi\left(x^2+x\right)=n\pi+(-1)^n\left(\pi x^2\right)$

$\pi x^2+\pi x=2n\pi+\pi x^2$ or $\pi x^2+\pi x=(2n+1)\pi-\pi x^2$

x = 2n

but $x\neq I$

$2x^2+x-(2n+1)=0$

$$x = \frac{-1 \pm \sqrt{1+8(2n+1)}}{4}$$

$$x = \frac{\sqrt{1+8(2n+1)}-1}{4}$$

$$x = \frac{\sqrt{odd}-1}{}4$$

47. **(C)**

$\cos x = 1 \qquad x = 2n\pi$

$\cos 2\lambda x = 1 \qquad 2\lambda x = 2m\pi$

$4\lambda n\pi = 2m\pi$

$\lambda = \frac{m}{2n}$ will have

Infinite soln. if $\lambda$ is rational

If $\lambda$ is Inatiael the

$\lambda = 0$ is the only solution

$\therefore$ $\lambda$ is inativel

48. **(D)**

$2^{(1-2Sn^2x)} - 3\left(2^{-2Sn^2x}\right) + 1 = 0$

$2\left(2^{-\sin^2 x}\right)^2 - 3\left(2^{-Sn^2x}\right) + 1 = 0$

$2t^2 - 3t + 1 = 0$

t = 1, $\frac{1}{2}$

$2^{Sn^2x} = 1$ or 1

$x = n\pi, \quad n\pi \pm \frac{\pi}{2}$

49. **(C)**

$\sqrt{3}\cos\theta - 3Sn\,\theta = 4Sn\,2\theta\cos 3\theta$

$= 2(Sn\,5\theta) - 2Sn\,\theta$

$\sqrt{3}\cos\theta - Sn\,\theta = 2Sn\,5\theta$

$2\sin\left(\frac{\pi}{3} - \theta\right) = 2\,Sn\,5\theta$

$Sn\left(\frac{\pi}{3} - \theta\right) = m\,5\theta$

$\theta = 2Sn\left(3\theta - \frac{\pi}{6}\right)\cos\left(2\theta + \frac{\pi}{6}\right)$

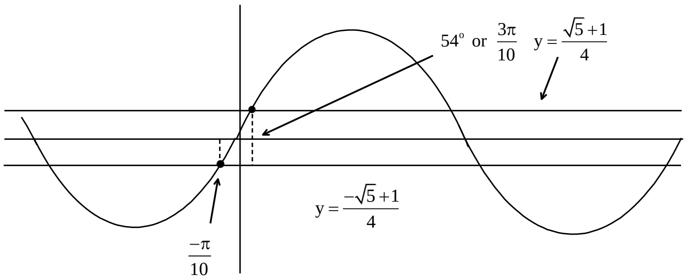
$3\theta - \frac{\pi}{6} = n\pi \quad \& \quad 2\theta + \frac{\pi}{6} = (2n+1)\frac{\pi}{2}$

$3\theta = n\pi + \frac{\pi}{6}, \qquad \theta = n\pi + \frac{\pi}{3}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{18}$

50. **(A)**

$$\frac{-\sqrt{5}+1}{4} < \text{Sn}\,x < \frac{\sqrt{5}+1}{4}$$

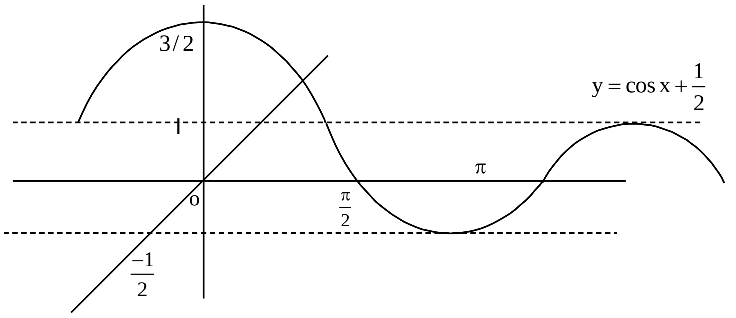



$$x \leftarrow \left(\frac{-\pi}{10}, \frac{3\pi}{10}\right)$$

51. **(A)**

$$\cos x - x + \frac{1}{2} = 0$$

$$x = \cos x + \frac{1}{2} \qquad\qquad y = x$$




Soln. in $\left(0, \frac{\pi}{2}\right)$

52. **(B)**

$$a\,\text{Sn}\,x + 1 - 2\text{Sn}^2x = 2a - 7$$

$$2\text{Sn}^2x - a\,\text{Sn}\,x + 2a - 7 - 1 = 0$$

$$2\text{Sn}^2x - a\,\text{Sn}\,x + 2(a-4) = 0$$

At least one root $\in [-1,1]$

$$D = a^2 - 16(a-4) \geq 0$$

$$a^2 - 16a + 6 \geq 0$$

$$(a-8)^2 \geq 0$$

Snx= $\frac{a \pm (a-8)}{4}$

$$= \frac{8}{4} \text{ or } \frac{2a-8}{4}$$

$$= 2 \text{ or } \frac{a-4}{2}$$

$$-a \leq \frac{a-4}{2} \leq 1$$

$$-2 \leq a-4 \leq 2$$

$$2 \leq a \leq 6$$

53. **(D)**

$\operatorname{Sn} x+\cos x=y^2-y+a$

$y^2-y=\left(y=\frac{1}{2}\right)^2+\frac{3}{4}\geq\frac{3}{4}$

$\therefore\quad y^2-y+a\geq\frac{3}{4}+a$

$\sin x+\cos x\leq\sqrt{2}$

$\therefore$ if $\frac{3}{4}+a>\sqrt{2}$ then no soln.

$a>\sqrt{2}-\frac{3}{4}$

$a>1.414-0.75$

$a\in\left(\sqrt{3},\infty\right)$

54. **(A)**

$4\operatorname{Sn}^2x+\tan^2 x+\operatorname{cos\,ec}^2x+\cot^2 x-6=0$

$\left(2\operatorname{Snx}-\operatorname{cos\,ec}\right)^2+\left(\tan x-\cot x\right)^2=0$

$2\operatorname{Sn} x-\frac{1}{\operatorname{Sn} x}=0\ \&\ \tan-\cot x=0$

$\operatorname{Sn}^2x=\frac{1}{2}\ \&\ \tan^2 x=1$

$x=n\pi k\pm\frac{\pi}{4}$

55. **(C)**

$\tan\left(\pi\cos\theta\right)=\cot\left(\pi\sin\theta\right)$

$\tan\left(\pi\cos\theta\right)=\tan\left(\frac{\pi}{2}-\pi\sin\theta\right)$

$\pi\cos\theta=\frac{\pi}{2}-\pi\sin\theta$

$\pi\left(\sin\theta+\cos\theta\right)=\frac{\pi}{2}$

$\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right]=\frac{1}{2}$

$\sqrt{2}\left[\sin\left(\theta+\frac{\pi}{4}\right)\right]=\frac{1}{2}$

$\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$

56. **(B)**

$\Rightarrow\tan 2\theta\tan\theta=1$

$\Rightarrow\tan 2\theta\tan\theta-1=0$

$\Rightarrow\frac{\sin 2\theta\sin\theta}{\cos 2\theta\cos\theta}-1=0$

$\Rightarrow\frac{\sin 2\theta\sin\theta-\cos 2\theta\cos\theta}{\cos 2\theta\cos\theta}=0$

$\Rightarrow \dfrac{\cos 3\theta}{\cos\theta\cos 2\theta} = 0$

$\Rightarrow 3\theta = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\dfrac{\pi}{6}$

$\Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\Rightarrow \theta = n\pi \pm \dfrac{\pi}{6}$

$\Rightarrow \cos\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \cos 2\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\dfrac{\pi}{4}$

$\Rightarrow \dfrac{\pi}{2}$ and $\dfrac{9\pi}{6}$ are ruled out.

57. **(A)**

$\Rightarrow \sin\left(\dfrac{\pi}{4}\cos\theta\right) = \cos\left(\dfrac{\pi}{4}\tan\theta\right)$

$\Rightarrow \cos\left(\dfrac{\pi}{2} - \dfrac{\pi}{4}\cot\theta\right) = \cos\left(\dfrac{\pi}{4}\tan\theta\right)$

$\Rightarrow \therefore \dfrac{\pi}{2} - \dfrac{\pi}{4}\cot\theta = 2n\pi \pm \dfrac{\pi}{4}\tan\theta$

$\Rightarrow \dfrac{\pi}{2} - 2n\pi = \dfrac{\pi}{4}\cot\theta \pm \dfrac{\pi}{4}\tan\theta$

$\Rightarrow -2n\pi$ can be taken as $2n\pi$ ($\because$ n can be negative integer)

$\Rightarrow \therefore 2n\pi + \dfrac{\pi}{2} = \dfrac{\pi}{4}(\cot\theta \pm \tan\theta)$

$\Rightarrow 8n + 2 = \cot\theta \pm \tan\theta$

$\Rightarrow 2 = \cot\theta \pm \tan\theta$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} \pm \tan\theta$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} + \tan\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = n\pi + \dfrac{\pi}{4}$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} - \tan\theta$

$\Rightarrow 2\tan\theta = 1 - \tan^2\theta$

$\Rightarrow \tan^2\theta + 2\tan\theta - 1 = 0$

$\Rightarrow \tan\theta = \dfrac{-2 \pm \sqrt{8}}{2}$

$\Rightarrow \tan\theta = -1 \pm \sqrt{2}$

58. **(B)**

$\tan\theta = \cot 2\theta$

$\tan\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$

$\theta = n\pi + \frac{\pi}{2} - 2\theta$

$3\theta = n\pi + \frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{6}$, where

$n \in I, n \neq 3m+1,\ m \in I$.

$= \frac{\pi}{2}, \frac{3\pi}{2} \ldots\ldots$ $\qquad$ $\theta = (2n+1)\frac{\pi}{4}$

$\frac{\pi}{4}, \frac{3\pi}{4} \ldots\ldots\ldots$

59. **(D)**

$\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$

$\tan\theta + \tan 2\theta = 1 - \tan\theta\tan 2\theta$

$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = 1$

$\tan(\theta + 2\theta) = 1$

$\tan 3\theta = 1$

$3\theta = n\pi + \frac{\pi}{4}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

Where $\frac{n\pi}{3} + \frac{\pi}{12} \neq \left\{\frac{(2n+1)\pi}{4}(2k+1)\frac{\pi}{2}\right\}$

$m, k \in I$

60. **(A)**

For $x \in \left(0, \frac{\pi}{2}\right)$

$\cos^2 x = 1 - \sin 2x$

$1 - \sin^2 x = 1 - 2\sin x \cos x$

$\sin x(\sin x - 2\cos x)$

$\sin x = 0$ $\qquad$ $\tan x = 2$

$x = \tan^{-1} 2$

61. **(D)**

$|\sin x|^2 + |\sin x| + b = 0$

$t^2 + t + b = 0$ $\qquad$ $\in [0,1]$

The root is negative, other roots must it lie is [0,1] for 2 values of x.

$f(0) \le 0$ $\qquad \Rightarrow \qquad$ $b \le 0$

$f(1) > 0$ $\qquad\qquad$ $b > -2$

$(-2, 0]$

62. **(B)**

$a\cos x + s\sin x = 13$

For no real solution

$\sqrt{a^2 + 25} < 13$

$a^2 + 25 < 169$

$a^2 - 144 < 0$

$(a-12)(a+12) < 0$

$-12 < a < 12$

$a \in (-12, 12)$

63. **(D)**

$|\sin x| < \frac{1}{2}$

$\frac{-1}{2} < \sin x < \frac{1}{2}$

Hence $x \in$

$\left(\frac{-\lambda}{6}, \frac{\lambda}{6}\right) \cup \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

Dig.

$\left(2n\pi - \frac{\pi}{6}, 2n\pi + \frac{\pi}{6}\right) \cup \left(2n\pi + \frac{5\pi}{6}, 2n\pi + \frac{7\pi}{6}\right)$

Where $n \in I$

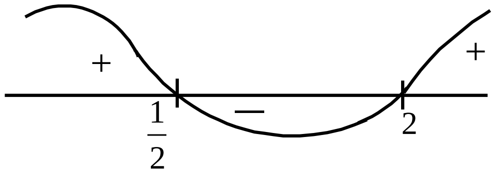
64. **(D)**

$2\sin^2\theta - 5\sin\theta + 2 > 0$

$2\sin^2\theta - 4\sin\theta - \sin\theta + 2 > 0$

$2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) > 0$

$(\sin\theta - 2)(2\sin\theta - 1) > 0$




$\sin\theta < \frac{1}{2}$ or $\sin\theta > 2$ (not possible)

$\theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

65. **(B)**

$|\cos x| = \sin x$

$\Rightarrow |\cos x|$ is always positive

$\therefore$ x must lie in 1st and 2nd quadrant,

So $\sin x$ is positive

and $|\cos x| = \sin x$ at $x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\Rightarrow 0 \le x \le 4\pi$ is 2 periods

So total solution is 4.

# EXERCISE - 1 [C]

1. **(2)**

$\because \cos x = \sqrt{1-\sin 2x}$

$\Rightarrow \cos x = |\sin x - \cos x|$

There are two cases arise

**Case I** $\sin x \le \cos x$

$\Rightarrow \quad \cos x = \cos x - \sin x$

$\Rightarrow \ \sin x = 0$

Where, $x \in \left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

$\Rightarrow x = 2\pi$, neglecting $x = \pi$

**Case II** $\sin x > \cos x$

$\Rightarrow \quad \tan x = 2$

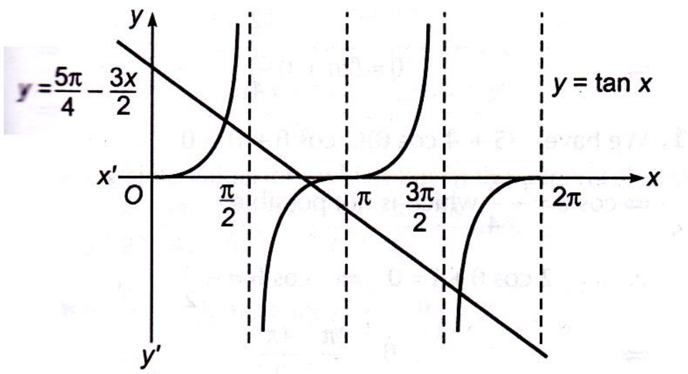
Where, $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\because \ \tan x = 2$

$\Rightarrow x = \tan^{-1}(2)$

Thus, the given equation has two solutions.

2. **(3)**




$\Rightarrow \tan x = \frac{5\pi}{4} - \frac{3x}{2}$

Let $\frac{5\pi}{4} - \frac{3x}{2} = y$

And $y = \tan x$

$\therefore$ Graph of $y = \frac{5\pi}{4} - \frac{3x}{2}$ and $y = \tan x$ meet exactly three times in $[0, 2\pi]$. Thus, the number of solutions of given equation is 3.

3. **(6)**

$3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow 3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$

$\Rightarrow (3\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \sin x = \frac{1}{3} \text{ or } 2$

$\Rightarrow \sin x = \frac{1}{3}$ $\quad (\because \sin x \neq 2)$

Let $\sin^{-1}\frac{1}{3} = \alpha, 0 < \alpha < \frac{\pi}{2}$

Then, $\alpha, \pi-\alpha, 2\pi+\alpha, 3\pi-\alpha, 4\pi+\alpha, 5\pi-\alpha$ are the solutions in $[0, 5\pi]$

$\therefore$ Required number of solutions $= 6$

4. **(0)**

Since, $1+\sin x \sin^2\frac{x}{2} = 0$

$\therefore 1+\sin x\left(\frac{1-\cos x}{2}\right) = 0$

$\Rightarrow 2+\sin x - \sin x\cos x = 0$

$\Rightarrow \sin 2x - 2\sin x = 4$

Which is not possible for any x in $[-\pi, \pi]$

5. **(4)**

Now, $1+|\cos x|+\cos^2 x+|\cos^{-1} x|+.....\infty = \frac{1}{1-|\cos x|}$

$\therefore \quad 8^{\frac{1}{1-|\cos x|}} = 4^3$

$\Rightarrow \frac{3}{2^{1-|\cos x|}} = 2^6 \Rightarrow 1 = 2-2|\cos x|$

$\Rightarrow |\cos x| = \frac{1}{2}$

$\Rightarrow \cos x = \pm\frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, -\frac{2\pi}{3}$

$\therefore$ Number of solutions $= 2$

6. **(0)**

Given, $e^{\sin x} - e^{-\sin x} - 4 = 0$

$\Rightarrow e^{2\sin x} - 4e^{\sin x} - 1 = 0$

$\Rightarrow e^{\sin x} = \frac{4\pm\sqrt{16+4}}{2} = 2+\sqrt{5}$

$\Rightarrow \sin x = \log\left(2+\sqrt{5}\right)$ [ $\because \log\left(2-\sqrt{5}\right)$ is not defined]

Since, $2+\sqrt{5} > e \Rightarrow \log\left(2+\sqrt{5}\right) > 1$

$\Rightarrow \sin x > 1$, which is not possible.

Hence, no solution exist.

7. **(0)**

$2\sin x = 5x^2+2x+3$

$\Rightarrow 2\sin x = 4x^2+(x+1)^2+2$

But $2\sin x \leq 2$

And $4x^2+(x+1)^2+2 > 2$, so it has no solution

8. **(2)**

We have, $(5+4\cos\theta)(2\cos\theta+1)=0$

$\Rightarrow \cos\theta=-\frac{5}{4}$ which is not possible.

$\therefore 2\cos\theta+1=0\Rightarrow \cos\theta=-\frac{1}{2}$

$\Rightarrow \theta=\frac{2\pi}{3},\frac{4\pi}{3}$

$\therefore$ Solution set is $\left\{\frac{2\pi}{3},\frac{4\pi}{3}\right\}\in[0,2\pi]$

9. **(5)**

We have,

$\cos 3x+\cos 2x=\sin\frac{3x}{2}+\sin\frac{x}{2}$

$\Rightarrow 2\cos\frac{5x}{2}\cos\frac{x}{2}=2\sin x\cos\frac{x}{2}$

Either $\cos\frac{x}{2}=0\Rightarrow \frac{x}{2}=(2n+1)\frac{\pi}{2}$

$\Rightarrow x=(2n+1)\pi$ or $\cos\frac{5x}{2}=\sin x$

$\Rightarrow \cos\frac{5x}{2}=\cos\left(\frac{\pi}{2}-x\right)$

$\Rightarrow \frac{5x}{2}=2n\pi\pm\left(\frac{\pi}{2}-x\right)$

Taking the positive sign $\frac{7x}{2}=2n\pi+\frac{\pi}{2}\Rightarrow x=\frac{4n\pi}{7}+\frac{\pi}{7}$

Taking negative sign

$\frac{3x}{2}=2n\pi-\frac{\pi}{2}\Rightarrow x=\frac{4n\pi}{3}-\frac{\pi}{3}$

For $0\le x\le 2\pi$

$x=\frac{\pi}{7},\frac{5\pi}{7},\frac{9\pi}{7},\frac{13\pi}{7},\pi$

Thus, number of solutions $=5$

10. **(0)**

$\because \sin^5 x-\cos^5 x=\frac{\sin x-\cos x}{\sin x\cos x}$

$\Rightarrow \sin x\cos x\left[\frac{\sin^5 x-\cos^5 x}{\sin x-\cos x}\right]=1$

$\Rightarrow \frac{1}{2}\sin 2x\left[\sin^4 x+\sin^3 x\cos x+\sin^2 x\cos^2 x+\sin x\cos^2 x+\cos^4 x\right]=1$

$\Rightarrow \sin 2x\left[\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x+\sin x\cos x\left(\sin^2 x+\cos^2 x\right)+\sin^2 x\cos^2 x\right]=2$

$\Rightarrow \sin 2x\left[1-\sin^2 x\cos^2 x+\sin x\cos x\right]=2$

$\Rightarrow \sin^3 2x-2\sin^2 2x-4\sin 2x+8=0$

$\Rightarrow (\sin 2x-2)^2(2\sin 2x+2)=0$

$\Rightarrow \sin 2x = \pm 2$, which is not possible for any x.

11. **(16)**

$\sqrt{\sin^2 x - \sin x + \frac{1}{2}} = \sqrt{\left(\sin x - \frac{1}{2}\right)^2 + \frac{1}{4}} \geq \frac{1}{2}, \forall x$ and $\sec^2 y \geq t, \forall y$, so $2^{\cos^2 y} \geq 2$.

Hence, the above inequality holds only for those values of x and y for which $\sin x = \frac{1}{2}$ and $\sec^2 y = 1$.

Hence, $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ and $y = 0, \pi, 2\pi, 3\pi$.

Hence, required number of ordered pairs are 16.

12. **(0)**

Since, $2\cos^2 \frac{x}{2} \sin^2 x < 2$

But $x^2 + \frac{1}{x^2} \geq 2$

Thus, the equation has no solution

13. **(2)**

Given, $\sin^4 x + \cos^4 x = \sin x.\cos x$

$\Rightarrow \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x.\cos^2 x = \sin x.\cos x$

$\Rightarrow 1 - \frac{\sin^2 2x}{2} = \frac{\sin 2x}{2}$

$\Rightarrow \sin^2 2x + \sin 2x - 2 = 0$

$\Rightarrow (\sin 2x + 2)(\sin 2x - 1) = 0$

$\Rightarrow \sin 2x = 1 \qquad (\because \sin 2x \geq -1)$

$\therefore 2x = (4n+1)\frac{\pi}{2}$

$\Rightarrow x - (4n+1)\frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

Hence, two solutions exist.

14. **(0.33)**

$\frac{1+\tan\theta}{1-\tan\theta} = 3\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$

On simplification, we get

$3\tan^4\theta - 6\tan^2\theta + 8\tan\theta - 1 = 0$

$\therefore$ Product of roots

$= \tan\alpha.\tan\beta.\tan\gamma.\tan\delta = -\frac{1}{3}$

15. **(1)**

$\because x^3 + x^2 + 4x + 2\sin x = 0$

$\Rightarrow x^3 + (x+2)^2 + 2\sin x = 4$

$x = 0$, satisfies this equation

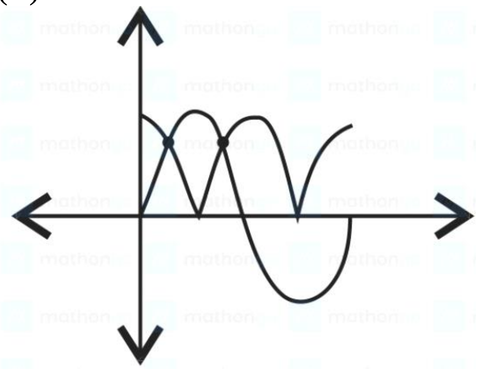
Now, in $0 < x \leq \pi, x^3 + (x+2)^2 + 2\sin x > 4$

And in $\pi < x \leq 2\pi, x^3 + (x+2)^2 + 2\sin x > 27 + 25 - 2 = 50$

Hence, $x = 0$ is the only solution.

## JEE Main : PYQ

1. (C)



2 solutions in $(0, 2\pi)$

So, 8 solutions in $[-4\pi, 4\pi]$

2. (C)

$8^{2\sin^2\theta} + 8^{2-2\sin^2\theta} = 16$

$y + \frac{64}{y} = 16$

$\Rightarrow y = 8$

$\Rightarrow \sin^2\theta = \frac{1}{2}$

$n(S) + \sum_{\theta \in S} \frac{1}{\cos(\pi/4 + 2\theta)\sin(\pi/4 + 2\theta)}$

$= 4 + (-2) \times 4 = -4$

3. (3)

$2\sin^2\theta - \cos 2\theta = 0$

$2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\therefore \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

So, the common solution is

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

Sum $= \frac{l\pi + 11\pi}{6} = 3\pi = k\pi$

$K = 3$

4. (16)

$7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$4\cos^2\theta + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2(1+\cos 2\theta) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2\cos^2 2\theta - 5\cos 2\theta = 0$

$\cos 2\theta(2\cos 2\theta - 5) = 0$

$\cos 2\theta = 0$

$2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{4}$

$S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

For all four values of $\theta$

$x^2 - 2\left(\tan^2\theta + \cot^2\theta\right)x + 6\sin^2\theta = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

Sum of roots of all four equations $= 4 \times 4 = 16$

5. (B)

$\sin\theta\tan\theta + \tan\theta = \sin 2\theta$

$\tan\theta(\sin\theta + 1) = \frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta = 0 \Rightarrow \theta = -\pi, 0, \pi$

$(\sin\theta + 1) = 2.\cos^2\theta = 2(1+\sin\theta)(1-\sin\theta)$

$\sin\theta = -1$ which is not possible

$\sin\theta = \frac{1}{2}$ $\qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$n(s) = 5$

$T = \cos 0 + \cos 2\theta + \cos 2\pi + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$

$T = 4$

$T + n(s) = 9$

6. (4)

$x \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$

$14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$

$= 21 - 4\left(1 - \sin^2 x\right)$

$= 17 + 4\sin^2 x$

$14\operatorname{cosec}^2 x - 6\sin^2 x = 17$

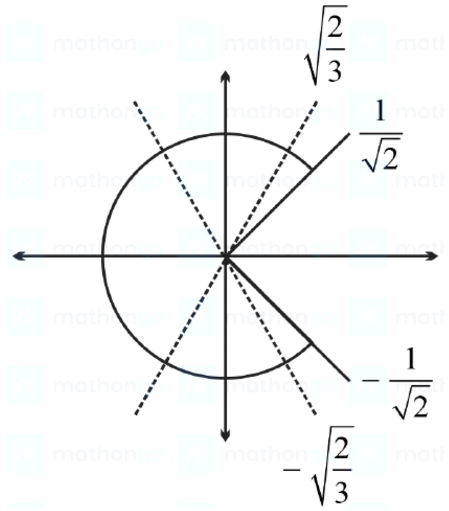
Let $\sin^2 x = p$

$\frac{14}{p} - 6p = 17 \Rightarrow 14 - 6p^2 = 17p$

$6p^2 + 17p - 14 = 0$

$p = -3.5, \frac{2}{3} \Rightarrow \sin^2 x = \frac{2}{3}$

$\Rightarrow \sin x = \pm\sqrt{\frac{2}{3}}$



$\therefore$ Total 4 solutions

7. (32)

$3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$3\cos^2 2\theta + 6\cos 2\theta - 5(1+\cos 2\theta) + 5 = 0$

$3\cos^2 2\theta + \cos 2\theta = 0$

$\cos 2\theta = 0 \text{ OR } \cos 2\theta = -1/3$

$\theta \in [-4\pi, 4\pi]$

$2\theta = (2n+1).\frac{\pi}{2}$

$\therefore \theta = \pm\pi/4. \pm 3\pi/4 ....... \pm 15\pi/4$

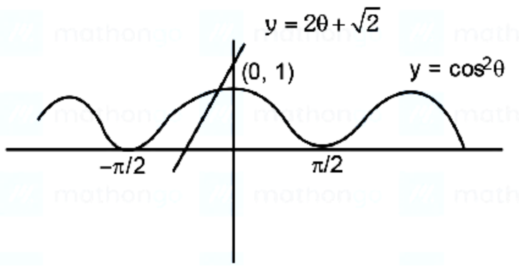
Similarly $\cos 2\theta = -1/3$ gives 16 solution

8. (1)

$2\theta - \cos^2\theta + \sqrt{2} = 0$

$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

$y = 2\theta + \sqrt{2}$




Both graphs intersect at one point.

9. (4)

$\sin^2 x + \sin x - 1 = 0$

$\sin x = \dfrac{-1+\sqrt{5}}{2} = +ve$

Only 4 roots

10. (A)

$2\cos x\left(4\sin\left(\dfrac{\pi}{4}+x\right)\sin\left(\dfrac{\pi}{4}-x\right)-1\right)=1$

$\Rightarrow 2\cos x\left(2\cos(2x)=2\cos\left(\dfrac{\pi}{2}\right)-1\right)=1$

$\Rightarrow 2\cos x\left(4\cos^2 x-3\right)=1$

$\Rightarrow \cos 3x = \dfrac{1}{2}$

$\Rightarrow 3x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3} \Rightarrow x = \dfrac{\pi}{9}, \dfrac{5\pi}{9}, \dfrac{7\pi}{9}$

Number of solutions $= n = 3$

Sum of solutions $= S = \dfrac{13\pi}{9}$

11. (B)

$32^{\tan^2 x} + 32^{\sec^2 x} = 81$

$\Rightarrow 32^{\tan^2 x} + 32^{1+\tan^2 x} = 81$

$\Rightarrow 33 \times 32^{\tan^2 x} = 81$

$\Rightarrow 32^{\tan^2 x} = \dfrac{27}{11}$

$\Rightarrow \tan^2 x = \ln_{32}\left(\dfrac{27}{11}\right)$

$\Rightarrow \tan x = \sqrt{\ln_{32}\left(\dfrac{27}{11}\right)} \in (0,1)$

$\Rightarrow$ One solution in $\left[0, \dfrac{\pi}{4}\right]$

12. (56)

Given, $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\theta \in [0, 4\pi]$

$\Rightarrow 1 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 2 - \sin^2 2\theta - \sin 2\theta = 0$

$\Rightarrow \sin^2 2\theta + \sin 2\theta - 2 = 0$

$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$

$\Rightarrow \sin 2\theta = 1, 2\theta[0, 8\pi]$

$\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$

Sum of solutions $S = \dfrac{28\pi}{4}$

Then, $\frac{8S}{\pi}=\frac{8\pi}{\pi}\times\frac{28\pi}{4}=56$

13. (A)

We have, $\frac{\cos x}{1+\sin x}=|\tan 2x|$

$$\Rightarrow \frac{\cos^2\left(\frac{x}{2}\right)-\sin^2\left(\frac{x}{2}\right)}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}=|\tan 2x|$$

$$\Rightarrow \frac{\left[\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right]\left[\cos\frac{x}{2}+\sin\frac{x}{2}\right]}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}=|\tan 2x|$$

$$\Rightarrow \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}=|\tan 2x|$$

$$\Rightarrow \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=|\tan 2x|$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\tan^2 2x$$

$$\Rightarrow 2x=n\pi\pm\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\Rightarrow 2x=n\pi+\frac{\pi}{4}-\frac{x}{2}$$

Or $2x=n\pi-\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$$\Rightarrow \frac{5x}{2}=\left(n+\frac{1}{4}\right)\pi$$

Or $\frac{3x}{2}=\left(n-\frac{1}{4}\right)\pi$

$$\Rightarrow \frac{-\pi}{2}<\frac{2}{5}\left(n+\frac{1}{4}\right)\pi<\frac{\pi}{2}$$

Or $\frac{-\pi}{2}<\frac{2}{3}\left(n-\frac{1}{4}\right)\pi<\frac{\pi}{2}$

$$\Rightarrow -\frac{5}{4}<n+\frac{1}{4}<\frac{5}{4}$$

Or $\frac{-3}{4}<n-\frac{1}{4}<\frac{3}{4}$

$$\Rightarrow \frac{-6}{4}<n<1$$

Or $\frac{-1}{2}<n<1$

$\Rightarrow n=-1,0$

Or $n=0$

When $n=-1, x=\left(\frac{-3}{10}\right)\pi$

Or when $n=0, x=-\frac{\pi}{6}$

$n=0, x=\left(\frac{1}{10}\right)\pi$

$\therefore$ Required sum

$=\left(\frac{-3}{10}\right)\pi+\left(\frac{1}{10}\right)\pi+\left(\frac{-1}{6}\right)\pi=\left(\frac{-11}{30}\right)\pi$

14. (C)

$\sin^7 x+\cos^7 x=1$ ……(i)

As,

$\sin^2 x+\cos^2 x=1$ ……(ii)

Now, $\sin^7 x \le \sin^2 x$ and $\cos^7 x \le \cos^2 x$.

But according to question, Eqs (i) and (ii), it is only possible, when

$\sin^7 x=\sin^2 x$ and $\cos^7 x=\cos^2 x$

So, $\sin^2 x+\cos^2 x=1$

When $\sin x=0$ and $\cos x=1$

Or $\sin x=1$ and $\cos x=0$

$\Rightarrow x=0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\therefore$ 5 solutions.

15. (A)

Let $\alpha=\max\left(8^{2\sin 3x}.4^{4\cos 3x}\right)$

$=\max\left(2^{6\sin 3x}.2^{8\cos 3x}\right)$

$=\max\left(2^{6\sin x+8\cos 3x}\right)$

and $\beta=\max\left(8^{2\sin 3x}.4^{4\cos 3x}\right)$

$=\max\left(2^{6\sin 3x}.2^{8\cos 3x}\right)$

$=\max\left(2^{6\sin 3x}+8\cos 3x\right)$

Now, determine the range of

$6\sin 3x+8\cos 3x$

( $\because$ Range of $a\sin x+b\cos x$

$$=\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$$

$$=\left[-\sqrt{6^2+8^2}, \sqrt{6^2+8^2}\right]$$

$$=[-10,10]$$

So, $\alpha=2^{10}$ and $\beta=2^{-10}$

Now, $\alpha^{1/5}=\left(2^{10}\right)^{1/5}=4$

$\beta^{1/5}=\left(2^{-10}\right)^{1/5}=\frac{1}{4}$

Quadratic equation with roots 4 and $\frac{1}{4}$ is

$x^2-\left(4+\frac{1}{4}\right)x+4\times\frac{1}{4}=0$

$\Rightarrow x^2-\frac{17}{4}x+1=0$

Multiplying both sides by 8,

$8x^2-34x+8=0$

On comparing, $8x^2+bx+c$, we get

$b=-34$ and $c=8$

So, $c-b=8-(-34)=42$

16. (1)

Given,

$|\cot x|=\cot x+\frac{1}{\sin x}$ …….(i)

If $\cot x>0$, then $|\cot x|=\cot x$

From Eq. (i), $\cot x=\cot x+\frac{1}{\sin x}$

$\Rightarrow \frac{1}{\sin x}=0$ (not possible)

If $\cot x<0$, then $|\cot x|=-\cot x$

From Eq.(ii), $-\cot x=\cot x+\frac{1}{\sin x}$

$\Rightarrow 2\cot x+\frac{1}{\sin x}=0 \Rightarrow 2\cos x=-1$

$\Rightarrow x=\frac{2\pi}{3}\text{ or }\frac{4\pi}{3}$

Here, $x=\frac{4\pi}{3}$ rejected because $\frac{4\pi}{3}\in$ third quadrant and in third quadrant cot x is positive.

Since, we considered $\cot x<0$. $\therefore$ $x=2\pi/3$ is the only one solution.

17. (A)

Given, $x+2\tan x=\frac{\pi}{2}$

$\Rightarrow 2\tan x=\frac{\pi}{2}-x$

$\Rightarrow \tan x=\frac{\pi}{4}-\frac{x}{2}$

$\Rightarrow \tan x=\left(-\frac{1}{2}\right)x+\frac{\pi}{4}$ …….(i)

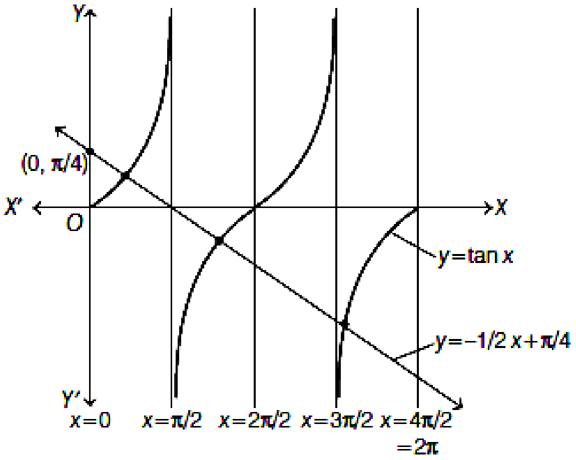
**Approach** In this type of problem solving, graphical approach is best because we have to find only number of solutions, not the solution (i.e. not the value (s) of x).

**Concept:** To find the number of solution(s) for Eq. (i), first of all, let

$y=\tan x$ ….(ii)

and $y=\left(\frac{-1}{2}\right)x+\frac{\pi}{4}$ …….(iii) and then draw the graph of Equation (ii) and (iii)

Now, total number of solution(s) = Total number of point(s) of intersection of the graph(ii) and (iii).




From above, we see that the red line i.e., $y=-\frac{1}{2}x-\frac{\pi}{4}$ intersects the black curve i.e., $y=\tan x$ at three distinct points in $[0,2\pi]$.

$\therefore$ Total number of solutions $=3$

18. (B)

Given, $81^{in^2 x}+81^{\cos^2 x}=30$

$\Rightarrow 81^{\sin^2 x}+81^{(1-\sin^2 x)}=30$

$\Rightarrow 81^{\sin^2 x}+\frac{81}{81^{\sin^2 x}}=30$

Let $81^{\sin^2 x}=y$

$\therefore \quad y+\frac{81}{y}=30$

$\Rightarrow y^2-30y+81=0$

$\Rightarrow (y-27)(y-3)=0$

$\Rightarrow y=3 \text{ or } y=27$

$81^{\sin^2 x}=3 \text{ or } 81^{\sin^2 x}=27$

$3^{4\sin^2 x}=3 \text{ or } 3^{4\sin^2 x}=3^3$

$\Rightarrow 4\sin^2 x=1 \text{ or } 4\sin^2 x=3$

$\Rightarrow \sin^2 x=1/4 \text{ or } \sin^2 x=3/4$

$\Rightarrow \sin^2 x=\sin^2(\pi/6)$

Or $\sin^2 x=\sin^2(\pi/3)$

$\Rightarrow x=n\pi\pm\pi/6 \text{ or } x=n\pi\pm\pi/3$

From $[0,\pi]$

$x=\pi/6, 5\pi/6 \text{ or } =\pi/3, 2\pi/3$

Hence, the total number of solutions =4

19. (1)

Given, $\sqrt{3}\cos^2 x=(\sqrt{3}-1)\cos x+1$,

$x\in[0,\pi/2]$

Let $\cos x = t$, then

$\sqrt{3}t^2 = \left(\sqrt{3}-1\right)t+1$

$\Rightarrow \sqrt{3}t^2 - \sqrt{3}t + t - 1 = 0$

$\Rightarrow \left(\sqrt{3}t^2 - \sqrt{3}t\right) + \left(t-1\right) = 0$

$= \sqrt{3}t\left(t-1\right) + 1\left(t-1\right) = 0$

$\Rightarrow \left(t-1\right)\left(\sqrt{3}t+1\right) = 0$

This gives $t = 1$ and $t = \frac{-1}{\sqrt{3}}$

Put, $t = \cos x$, then

$\cos x = 1$ and $\cos x = \frac{-1}{\sqrt{3}}$

$\cos x = -1/\sqrt{3}$ is rejected as $x \in \left[0, \pi/2\right]$

$\therefore \cos x = 1$

Since, $x \in \left[0, \frac{\pi}{2}\right]$, then $\cos x = \cos 0$

This gives $x = 0$ is only solution. Therefore, number of solution when $x \in \left[0, \pi/2\right]$.

20. (11)

Given, $3\sin x + 4\cos = k+1$ ......(i)

Multiply and divide LHS of Eq. (i) by $\sqrt{3^2+4^2} = 5$

i.e. $5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right) = k+1$

$\Rightarrow 5\left(\cos\alpha \sin x + \sin\alpha \cos x\right) = k+1$

[ Let $\cos\alpha = 3/5$ then $\sin\alpha = \sqrt{1-\left(3/5\right)^2} = \frac{4}{5}$ ]

$5\sin\left(x+\alpha\right) = k+1$

[ Use $\sin\left(a+b\right) = \sin a\cos b + \cos a \sin b$ ]

$\Rightarrow \sin\left(x+\alpha\right) = \frac{k+1}{5}$

Let, $x + \alpha = \theta$

Then, $\sin\theta = \frac{k+1}{5}$

$\because -1 \le \sin\theta \le 1$

$\Rightarrow -1 \le \frac{k+1}{5} \le 1$

$\Rightarrow -5 \le k+1 \le 5$

$\Rightarrow -6 \le k \le 4$

$\therefore$ Possible integral values of k are $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3$ and 4

i.e. Total 11 integral values of k are possible for which Eq. (i) has solution.

21. (B)

The expression, $\cos^4\theta + \sin^4\theta$

$= \left(\cos^2\theta + \sin^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta$

$=1-\frac{1}{2}\sin^2(2\theta)$

$\because \sin^2(2\theta)\in[0,1]$

$\Rightarrow -\frac{1}{2}\sin^2(2\theta)\in\left[-\frac{1}{2},0\right]$

$\Rightarrow 1-\frac{1}{2}\sin^2(2\theta)\in\left[\frac{1}{2},1\right]$

Now, as $\cos^4\theta+\sin^4\theta+\lambda=0$

$\Rightarrow \lambda=-\left(\cos^4\theta+\sin^4\theta\right)$

$\Rightarrow \lambda\in[-1,-1/2]$ for real solution of the given equation $\cos^4\theta+\sin^4\theta+\lambda=0$ for $\theta$.

Hence, option (B) is correct.

22. (8)

Given equation,

$\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|$

$\Rightarrow -\log_2|\sin x|=2+\log_2|\cos x|$

$\Rightarrow \log_2|\sin x|+\log_2|\cos x|+\log_2 4=0$

$\Rightarrow \log_2\left(4|\sin x||\cos x|\right)=0$

$\Rightarrow 4|\sin x||\cos x|=1$

$\Rightarrow \sin 2x=\pm\frac{1}{2}$

$\because x\in[0,2\pi]\Rightarrow 2x\in[0,4\pi]$

$\therefore$ For $2x\in[0,2\pi]\sin 2x=\pm\frac{1}{2}$ has four solutions and $2x\in[2\pi,4\pi],\sin 2x=\pm\frac{1}{2}$ has four more solutions.

$\therefore$ Total number of solutions are 8. Hence, answer is 8

23. (A)

We have $\theta\in[-2\pi,2\pi]$

And $2\cos^2\theta+3\sin\theta=0$

$\Rightarrow 2\left(1-\sin^2\theta\right)+3\sin\theta=0$

$\Rightarrow 2-2\sin^2\theta+3\sin\theta=0$

$\Rightarrow 2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow 2\sin^2\theta-4\sin\theta+\sin\theta=0$

$\Rightarrow 2\sin\theta(\sin\theta-2)+1(\sin\theta-2)=0$

$\Rightarrow (\sin\theta-2)(2\sin\theta+1)=0$

$\Rightarrow \sin\theta=\frac{-1}{2}$ $\quad[\because(\sin\theta-2)\neq 0]$

$\Rightarrow \theta=2\pi-\frac{\pi}{6},-\pi+\frac{\pi}{6},-\frac{\pi}{6},\pi+\frac{\pi}{6}$ $\quad[\because \theta\in[-2\pi,2\pi]]$

Now, sum of all solutions

$$=2\pi-\frac{\pi}{6}-\pi+\frac{\pi}{6}-\frac{\pi}{6}+\pi+\frac{\pi}{6}=2\pi$$

24. (B)

Given equation is $1+\sin^4 x=\cos^2(3x)$

Since, range of $\left(1+\sin^4 x\right)=[1,2]$

And range of $\cos^2(3x)=[0,1]$

So, the given equation holds if

$1+\sin^4 x=1=\cos^2(3x)$

$\Rightarrow \sin^4 x=0 \text{ and } \cos^2 3x=1$

Since, $x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$

$\therefore x=-2\pi,-\pi,0,\pi,2\pi$

Thus, there are five different values of x is possible

25. (D)

The given trigonometric equation is

$\cos 2x+\alpha\sin x=2\alpha-7$

$\Rightarrow 1-2\sin^2 x+\alpha\sin x=2\alpha-7$

$\left[\because \cos 2x=1-2\sin^2 x\right]$

$\Rightarrow 2\sin^2 x-\alpha\sin x+2\alpha-8=0$

$\Rightarrow 2\left(\sin^2 x-4\right)-\alpha(\sin x-2)=0$

$\Rightarrow 2\left(\sin^2 x-2\right)(\sin x+2)-\alpha(\sin x-2)=0$

$\Rightarrow (\sin x-2)(2\sin x+4-\alpha)=0$

$\Rightarrow 2\sin x+4-\alpha=0 \qquad [\because \sin x+2\neq 0]$

$\Rightarrow \sin=\frac{\alpha-4}{2} \qquad \ldots\ldots\text{(i)}$

Now, as we know $-1\le \sin x\le 1$

$\therefore \quad -1\le\frac{\alpha-4}{2}\le 1 \qquad$ [From Eq. (i)]

$\Rightarrow -2\le\alpha-4\le 2$

$\Rightarrow 2\le\alpha\le 6\Rightarrow \alpha\in[2,6]$

26. (A)

We have, $\sin x-\sin 2x+\sin 3x=0$

$\Rightarrow (\sin x+\sin 3x)-\sin 2x=0$

$\Rightarrow 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)-\sin 2x=0$

$\left[\because \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$

$\Rightarrow 2\sin 2x\cos x-\sin 2x=0 \quad [\because \cos(-\theta)=\cos\theta]$

$\Rightarrow \sin 2x(2\cos x-1)=0$

$\Rightarrow \sin 2x=0 \text{ or } 2\cos x-1=0$

$\Rightarrow 2x=0,\pi,\ldots\ldots \text{or } \cos x=\frac{1}{2}$

$\Rightarrow x = 0, \frac{\pi}{2}.....\text{or } x = \frac{\pi}{3}$

In the interval $\left[0, \frac{\pi}{2}\right)$ only two values satisfy, namely $x = 0$ and $x = \frac{\pi}{3}$

27. (C)

Given, $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow \left(1 - \cos^2 2\theta\right) + \cos^4 2\theta = \frac{3}{4}$

$\left(\because \sin^2 x = 1 - \cos^2 x\right)$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow \left(2\cos^2 2\theta - 1\right)^2 = 0$

$\Rightarrow 2\cos^2 2\theta - 1 = 0$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} \Rightarrow \cos 2\theta = \pm\frac{1}{\sqrt{2}}$

If $\theta \in \left(0, \frac{\pi}{2}\right)$, then $2\theta \in (0, \pi)$

$\therefore \cos 2\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow 2\theta = \frac{\pi}{4}, \frac{3\pi}{4},$

$$\left[\begin{aligned} \because \cos\left(\frac{3\pi}{4}\right) &= \cos\left(\pi - \frac{\pi}{4}\right) \\ &= -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}} \end{aligned}\right]$$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8}$

Sum of value of $\theta = \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$

28. (D)

Given expression $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$

$= 3\cos\theta + 5\left(\sin\theta\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\theta\right)$

$= 3\cos\theta + 5\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)$

$= 3\cos\theta - \frac{5}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

$= \frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

$\because$ The maximum value of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2 + b^2}$

So, maximum value of $\frac{1}{2}\cos\theta+\frac{5\sqrt{3}}{2}\sin\theta$ is

$$=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{5\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{75}{4}}$$

$$=\sqrt{\frac{76}{4}}=\sqrt{19}$$

29. (D)

Given, $5\left(\tan^2 x-\cos^2 x\right)=2\cos 2x+9$

$$\Rightarrow 5\left(\frac{1-\cos 2x}{1+\cos 2x}-\frac{1+\cos 2x}{2}\right)=2\cos 2x+9$$

Put $\cos 2x=y$, we have

$$5\left(\frac{1-y}{1+y}-\frac{1+y}{2}\right)=2y+9$$

$$\Rightarrow 5\left(2-2y-1-y^2-2y\right)=2(1+y)(2y+9)$$

$$\Rightarrow 5\left(1-4y-y^2\right)=2\left(2y+9+2y^2+9y\right)$$

$$\Rightarrow 5-20y-5y^2=22y+18+4y^2$$

$$\Rightarrow 9y^2+42y+13=0$$

$$\Rightarrow 9y^2+3y+39y+13=0$$

$$\Rightarrow 3y(3y+1)+13(3y+1)=0$$

$$\Rightarrow (3y+1)(3y+13)=0$$

$$\Rightarrow y=-\frac{1}{3},-\frac{13}{3}$$

$$\therefore \cos 2x=-\frac{1}{3},-\frac{13}{3}$$

$$\Rightarrow \cos 2x=-\frac{1}{3} \qquad \left[\because \cos 2x\neq-\frac{13}{3}\right]$$

Now, $\cos 4x=2\cos^2 2x-1=2\left(-\frac{1}{3}\right)^2-1$

$$=\frac{2}{9}-1=-\frac{7}{9}$$

30. (C)

Given equation is

$$\cos x+\cos 2x+\cos 3x+\cos 4x=0$$

$$\Rightarrow (\cos x+\cos 3x)+(\cos 2x+\cos 4x)=0$$

$$\Rightarrow 2\cos 2x\cos x+2\cos 3x\cos x=0$$

$$\Rightarrow 2\cos x\left(\cos 2x+\cos 3x\right)=0$$

$$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right)=0$$

$$\Rightarrow \cos x.\cos\frac{5x}{2}.\cos\frac{x}{2}=0$$

$\Rightarrow \cos x = 0 \text{ or } \cos\frac{5x}{2} = 0 \text{ or } \cos\frac{x}{2} = 0$

Now, $\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$[\because 0 \le x < 2\pi]$

$\cos\frac{5\pi}{2} = 0 \Rightarrow \frac{5x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}......,$

$\Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5} \qquad [\because 0 \le x < 2\pi]$

And $\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ......$

$\Rightarrow x = \pi \qquad [\because 0 \le x < 2\pi]$

Hence, $x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

31. (D)

$A = \sin^2 x + \cos^4 x$

$\Rightarrow A = 1 - \cos^2 x + \cos^4 x$

$= \cos^4 x - \cos^2 x + \frac{1}{4} + \frac{3}{4}$

$= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \qquad .....(i)$

Where, $0 \le \left(\cos^2 x - \frac{1}{2}\right)^2 \le \frac{1}{4} \qquad .....(ii)$

$\therefore \frac{3}{4} \le A \le 1$

32. (A)

$\sin\theta + \sin 4\theta + \sin 7\theta = 0$

$\Rightarrow \sin 4\theta + (\sin\theta + \sin 7\theta) = 0$

$\Rightarrow \sin 4\theta + 2\sin 4\theta.\cos 3\theta = 0$

$\Rightarrow \sin 4\theta\{1 + 2\cos 3\theta\} = 0$

$\Rightarrow \sin 4\theta = 0, \cos 3\theta = -\frac{1}{2}$

As, $0 < \theta < \pi$

$\therefore 0 < 4\theta < 4\pi$

$\therefore 4\theta = \pi, 2\pi, 3\pi$

$\cos 3\theta = -\frac{1}{2}$

$0 < 3\theta < 3\pi$

$\Rightarrow 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

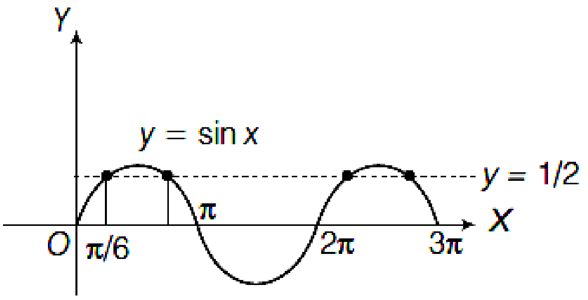
$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

33. (D)

Given equation is $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ $\quad [\because \sin x \neq -3]$




It is clear from figure that the curve intersect the line at four points in the given interval.
Hence, number of solution are 4.

34. (B)

Given, $\cos x + \sin x = \frac{1}{2}$

$\therefore \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1}{2}$

Let $\tan\frac{x}{2} = t$

$\Rightarrow \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2}$

$\Rightarrow 2(1-t^2+2t) = 1+t^2$

$\Rightarrow 3t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{2 \pm \sqrt{7}}{3}$

As $0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$

So, $\tan\frac{x}{2}$ is positive.

$\therefore t = \tan\frac{x}{2} = \frac{2+\sqrt{7}}{3}$

Now, $\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} = \frac{2t}{1-t^2}$

$\Rightarrow \tan x = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2}$

$\Rightarrow \tan x = \frac{-3(2+\sqrt{7})}{1+2\sqrt{7}} \times \frac{1-2\sqrt{7}}{1-2\sqrt{7}}$

$\Rightarrow \tan x = -\left(\frac{4+\sqrt{7}}{3}\right)$

35. (D)

Given that, $f(x) = \sin x - \sqrt{3}\cos x + 1$

$\because -2 \le \sin x - \sqrt{3}\cos x \le 2$

$\left[\because \sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}\right]$

$\Rightarrow -1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$

$\therefore$ Range of $f(x) = [-1, 3]$

36. (B)

Since, $\alpha$ is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$

$\therefore\ 25\cos^2\theta + 5\cos\theta - 12 = 0$

$\Rightarrow (5\cos\alpha - 3)(5\cos\alpha + 4) = 0$

$\Rightarrow \cos\alpha = -\frac{4}{5} \text{ and } \frac{3}{5}$

But $\frac{\pi}{2} < \alpha < \pi$ i.e. in second quadrant

$\therefore \cos\alpha - \frac{4}{5}$

$\Rightarrow \sin\alpha = \frac{3}{5}$

Now, $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{3}{5} \times \left(-\frac{4}{5}\right) = -\frac{24}{25}$

37. (C)

Since, $|c| > \sqrt{a^2+b^2}$

$\Rightarrow c < -\sqrt{a^2+b^2}$

And $c > \sqrt{a^2+b^2}$

But $-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$ ….(i)

And $a\sin x + b\cos x = c$ …..(ii)

From equation (i) and (ii), we see that no solution exists.