P& C Ex. 1(A)

18. (C)

$\Rightarrow$ _ _ _ _ _ unit's place can be filled in 3 different ways.

$\Rightarrow$ Required number of even numbers = ${}^{3}C_{1} \times (4!) = 72$

19. (C)

$\Rightarrow$ Required number of ways = $5! \times {}^{6}C_{5} \times 5! = 5! \times 6!$

20. (A)

$\Rightarrow$ _ ↓ _ ↓ _ ↓ _ ↓ _ ↓ _
B B B B B

$\Rightarrow$ Number of arrangement of boys = 5!

$\Rightarrow$ Number of possible permutations of girls = ${}^{4}P_{3}$.

$\Rightarrow$ Hence, Required number of arrangements = $5! \times {}^{4}P_{3} = 2880$

21. (C)

Arrangements of | L |AU|G|H | will be 4! i.e. 24
Arrangements of |A|U| will be 2! i.e. 2
Total arrangements = 48.

22. (D)

First digit from left may be selected in 3 ways
last digit may be selected in 2 ways,
other 4 digits may be selected in $5 \times 4 \times 3 \times 2$ ways.
Total number of numbers = $5 \times 4 \times 3 \times 2 \times 3 \times 2$ i.e. 720.

23. (D)

In {0, 1, 2, 3, 4, 5} sum of all the digits is 15 which is divisible by 3.
Now we have to choose 5 numbers out of these 6 given numbers such as their sum is also divisible by 3.
Clearly either 0 or three must be discarded.
In the case when 0 is discarded, using {1, 2, 3, 4, 5} we can form 5! numbers.
In the case when 3 is discarded, using {0, 1, 2, 4, 5} we can form $4 \times 4!$ numbers.
Total possible numbers are 216.

24. (A)

1, 3 & 5 we can put at 2^{nd}, 4^{rd}& 6^{th} place in 3! ways.
Now 1^{st} place can be filled by 2 or 4 and 3^{rd}& 5^{th} places can be filled in 2! Ways.
Hence number of all such numbers is $3!\times 2\times 2!$ i.e. 24.

25. (B)

Keeping those three men as one entity we have 4! arrangements.
These three men can be arranged in 3! ways within themselves.
Hence all possible arrangements = $4!\times 3!$ i.e. 144.

26. (B)

We have to keep all the consonants together.

All the consonants as one entity and the 4 vowels may be arranged in $\frac{5!}{2!2!}$

Consonants with each other can be arranged in $\frac{5!}{2!}$ ways.

Hence all possible arrangements are $\frac{5!}{2!2!}\times\frac{5!}{2!}$ i.e. 1800.

27. (A)

For the number to be divisible by four the last two digits must be 12, 16, 32, 36, 52 or 56.
First two digits can be selected from remaining three numbers in 3 ways.
Hence number of such numbers = $6\times 3\times 2$ i.e. 36.

28. (C)

Each of the 9 digits can be selected in 9 ways.
Hence total number of such numbers is 9^5.

29. (A)

Let the sum of any five digits be N.
Now N can be any number from 1 to 45.
In each case the remaining digit can be chosen in two particular ways only so as sum of digits is divisible by 5.
{e.g. If N = 42, then remaining digit can be 3 or 8; if N = 23, then remaining digit is 2 or 7;…etc.}
Hence total possible number of such numbers is $9\times 10^4\times 2$ i.e. 180000.

30. (A)

Digits must be selected from {1, 3, 5, 7, 9}.

As the number must contain each of these so we need to permute 6 objects containing two identical objects and rest all distinct which can be done in $\frac{6!}{2}$ ways.

Also two identical digits can be selected in 5 ways.

Hence total number of such numbers is $5\times\frac{6!}{2}$.

31. (B)

For sum of digits to be even : we can choose first four digits in general in 9×10^3 ways
Now if sum of first four digits is even, then last digit must be even
and if sum of first four digits is odd, then last digit must be odd
Hence in any possibility last digit can be chosen in 5 ways.
Number of all such numbers = $9\times10^3\times5$ or 45000.

32. (A)

D, P, M, L can be arranged in 4! Ways.
Two pairs of vowels can be chosen in 2 ways i.e. {EE, AA}, {EA, AE}
Now these pairs can be put in 5 gaps between D, P, M, L in $2\times{}^5C_2\times2!$ ways.
Total number of arrangements = ${}^5C_2\times2\times2!\times4!$ i.e. 960.

33. (D)

$\Rightarrow$Required number = $(10-1)!\times3!=9!\times3!$

34. (B)

$\Rightarrow$Number of arrangement of men = (7 – 1)! = 6!

$\Rightarrow$Number of permutations of woman on the gaps created among man = 7!

$\Rightarrow$Hence, required number = $6!\times7!$

35. (C)

Number of arrangements of 8 boys around a circular table = 7!.
Number of ways to put the two particular boys in 8 gaps = ${}^8C_2\times2!$.
Hence all possible arrangements = $7!\times{}^8C_2\times2!$ i.e. 7 (8!).

36. (A)

Seats for the two specific persons can be chosen in 5 ways on any of the 2 sides between the master & mistress i.e. 10 ways.
Rest of the 10 places can be filled in $10!\times2$ ways (differentiating between left and right of master and mistress).
Hence total number of ways = $20\times10!$.

37. (C)

A & B can be seated in 1 way on two identical tables.
Rest of the 6 persons can be seated in 6! Ways.
Hence total ways to seat 8 persons = 720.

38. (B)

A & B along with any other two persons can sit on the straight table in ${}^{6}C_{2} \times 4!$ ways.
Remaining four persons can sit around the circular table in 3! Ways.
Total number of arrangements = ${}^{6}C_{2} \times 4! \times 3!$ i.e. 2160.

39. (D)

$\Rightarrow$Number of ways = $({}^{2}C_{1})^{10} = 1024$.

40. (D)

$\Rightarrow$Required number of ways = ${}^{11}C_{3} = 165$.

41. (D)

$\Rightarrow$Number of ways of appointing clerks = ${}^{20}C_{16} = 4845$.

42. (B)

$\Rightarrow$Number of ways ${}^{11}C_{4} = 330$.

43. (D)

$\Rightarrow$Required number of words = $\left({}^{5}C_{3} \times {}^{4}C_{2}\right) \times 5!$

44. (C)

$\Rightarrow$Number of ways of selection = ${}^{15}C_{1} \times {}^{10}C_{1} = 150$

45. (C)

$\Rightarrow {}^{n}C_{2} = 66$

$\Rightarrow n = 13$.

46. (A)

$\Rightarrow$Number of ways = ${}^{8}P_{5} = 6720$.

47. (B)

$\Rightarrow$Number of Greeting card exchanged = ${}^{20}C_{2} \times 2$

48. (C)

$\Rightarrow$The number of times he will go to the garden = ${}^{8}C_{3} = 56$.

49. (B)

$\Rightarrow$The number of ways person can make selection of fruits

$= (4 + 1)(5 + 1)(6 + 1) - 1 = 209$.

50. (C)

$\Rightarrow$Number of ways of selection = (10 + 1) (9 + 1) (7 + 1) – 1 = 879.

51. (B)

$\Rightarrow$Required number of ways = $2^6 - 1 = 63$.

52. (D)

$\Rightarrow$Number of ways in which a student can fail to get all answers correct = $4^3 - 1 = 63$.

53. (B)

To form a rectangle(including squares) we may choose any two vertical lines and any two horizontal lines.

Number of all possible rectangles, r = ${}^9C_2 \times {}^9C_2 = 1296$.

Number of squares of side length 1 = 8×8

Number of squares of side length 2 = 7×7

Number of squares of side length 3 = 6×6

Going by this pattern total number of squares, s = 1 + 4 + 9 + … + 64 = 204.

Hence $\frac{s}{r} = \frac{17}{108}$.

54. (A)

Number of ways to select any 3, 4 or all 5 out of first 5 questions and then to select 7, 6 or 5, respectively, out of the remaining 8 questions = ${}^5C_3 \times {}^8C_7 + {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$ i.e. 276.

55. (D)

To form a triangle we need to chose three non – collinear point.

We can chose 3 points in ${}^{18}C_3$ ways.

As each side of the triangle contains 6 points so we can choose all three points on any one side in $3\times{}^6C_3$ ways.

Total numbers of triangles = ${}^{18}C_3 - 3\times{}^6C_3$ i.e. 711.

56. (A)

$\Rightarrow$Number of ways $= 4\times4\times4\times4\times4\times4 = 4^6$

57. (B)

$\Rightarrow$Hence, required number = 3^4.

58. (A)

$\Rightarrow$Number of ways = $4^3 - 1 = 63$

59. (C)

$\Rightarrow$ Number of ways = 5^4, as each parcel has 5 options.

60. (C)

$\Rightarrow$ Required number of ways is the number of permutations of given 4 grades, taken 3 at a time. i.e., ${}^4P_3$.

61. (B)

$\Rightarrow$ The number of ways in which husbands can be selected = ${}^7C_2$.

$\Rightarrow$ The number of ways in which wives can be selected after selecting husbands = ${}^5C_2$.

$\Rightarrow$ Also, a set of two men and two women can play 2 games.

$\Rightarrow$ Hence, Required number of ways = ${}^7C_2 \times {}^5C_2 \times 2 = 420$.

62. (A)

$\Rightarrow$ Required number is number of circular permutations such that clockwise and anti-clockwise arrangements = $\frac{(5-1)!}{2} = \frac{4!}{2}$

63. (D)

To distribute books in given manner we need to select 5 persons and given one book each to these 5 persons.

Number of ways to do so will be ${}^{10}C_5 \times 5!$.

64. (C)

Sum of internal angles of an n sided polygon = $(n-2)\times 180^o$.

Hence $150^o \times n = (n-2)\times 180^o$ gives n = 12.

Now a diagonal will be formed by joining two of these points but not adjacent two vertices.

Number of diagonals will be ${}^{12}C_2 - 12$ i.e. 54.

65. (A)

Number of ways to choose 4 prizes out of 9 = ${}^9C_4$.

Rest of the 5 prizes can be distributed to remaining 4 students in 4^5 ways.

Total number of ways to distribute 9 prizes among 5 students = ${}^9C_4 \times 4^5$.

66. (C)

Number of ways in which m distinct objects can be permuted at n distinct places = ${}^mP_n$.

67. (D)

If one or more pair is there in the selection, then we can select shoes in the following manner
(i) Selection having at least one pair : can be formed by choosing one pair in 5 ways and then choosing 2 more shoes from remaining 8 in ${}^{8}C_{2}$ ways i.e. $5\times{}^{8}C_{2}$ or 140 ways.

(ii) Selection having two pairs : can be formed by choosing two pairs in ${}^{5}C_{2}$ or 10 ways.

Hence a selection having one or more pairs can be formed in 140 – 10 = 130 ways.

Number of all possible selections = ${}^{10}C_{4}$ or 210 ways.

Thus number of ways to make a selection having no pairs = 210 – 130 = 80.

68. (B)

Only possible way to distribute in given manner is in groups of 1, 4 & 2.

Hence number of ways to distribute $= \frac{7!}{1!\,2!\,4!}\times 3!$ or 630.

69. (A)

Number of ways to distribute m + n + p objects in three groups containing m, n & p objects is given by $\frac{(m+n+p)!}{m!n!p!}$.

Required number of ways $= \frac{10!}{2!3!5!}$.

70. (A)

Number of ways to distribute 'n' identical objects in 'r' distinct groups such that no group is empty $= {}^{n-1}C_{r-1}$.

Hence required number of ways $= {}^{10-1}C_{6-1}$ i.e. 126.

71. (C)

For each question we can have 3 choices namely (i) not selecting, (ii) selecting alternative 1 & (ii) selecting alternative 2.

Hence choices for 10 questions $= 3^{10}$.

But all the question cant be rejected hence required number of ways $= 3^{10} - 1$.

72. (C)

Number of ways to divide 'nr' distinct objects into r groups containing equal number of objects $= \frac{(nr)!}{r!\times(n!)^{r}}$.

Hence 52 cards can be distributed into 4 groups of 13 cards each $= \frac{(52)!}{4!\times(13!)^{4}}$.

73. (B)

Let the subset A contain r elements, then subset B must contain remaining n – r elements so as $A \cap B = \Phi$ & $A \cup B = S$.

Hence number of ordered pairs (A, B) = $\sum_{r=0}^{10} {}^{10}C_r$ i.e. 2^{10}

Total number of unordered pairs of subsets of S = $\frac{2^{10}}{2}$ i.e. 2^9.

74. (B)

Number of positive integers of r–digits not having any digit as 1 = $8 \times 9^{r-1}$.
Number of all the positive integers of r–digits = $9 \times 10^{r-1}$.
Number of numbers having at least one digit as 1 = $9 \times 10^{r-1} - 8 \times 9^{r-1}$.

Hence number of all the numbers up to 9-digit numbers = $\sum_{r=1}^{9} \left(9 \times 10^{r-1} - 8 \times 9^{r-1}\right)$ i.e. $10^9 - 9^9$.

Alternately

If we form a 9 digit number allowing consecutive 0s at places form beginning also, then r consecutive zeros from beginning will account for a number of (9 – r) digits.
Hence all numbers from 1 digit till 9 digit = 10^9.
Hence all numbers from 1 digit till 9 digit not including 1 = 9^9.
Number of numbers having at least one digit as 1 = $10^9 - 9^9$.

75. (C)

Number of outcomes on each die = 6.
Hence all the four dice will show same number I 6 ways.

76. (C)

Each suit can be arranged in 13! Ways and the suits can be arranged with each other in 4! Ways.
Hence total number of arrangements = $(13!)^4 \times 4!$.

77. (C)

12 dice can show any outcome in 12! Ways if all the outcomes could be distinct. Now as there are 2 each 1, 2, 3, 4, 5, 6 hence required number of ways = $\frac{12!}{(2!)^6}$.

78. (C)

If there are n number of B used, then required number of numbers will be $\frac{(a+n)!}{a!n!}$.

Hence total possible number of numbers where $0 \le n \le b$ will be $\sum_{n=0}^{b} \frac{(a+n)!}{a!n!} = \sum_{n=0}^{b} {}^{a+n}C_n$

i.e. ${}^{a+b+1}C_b$.

79. (A)

AAAAA, BBB, D, EE & F can be arranged in $\frac{12!}{5!\,3!\,2!}$ ways.

Now there will be 13 gaps where we can put 3 C in ${}^{13}C_3$ ways.

Hence total number of arrangements = ${}^{13}C_3 \times \frac{12!}{5!\,3!\,2!}$.

Ex. 1(B)

1. (A)

$$\Rightarrow n\cdot\frac{(n-1)!}{(n-r)!}=\frac{n!}{(n-r)!}={}^{n}P_{r}$$

2. (D)

$$\Rightarrow \frac{(k+5)!}{4!}=\frac{11}{2}(k-1)\frac{(k+3)!}{3!}$$

$$\Rightarrow \frac{(k+5)(k+4)}{4}=\frac{11}{2}(k-1)$$

$\Rightarrow k = 6$ or 7.

3. (D)

$\Rightarrow n^2 - n = 2 + 10$

$\Rightarrow n = 4$ or -3

4. (A)

$$\Rightarrow {}^{n}C_3+{}^{n}C_4>{}^{n+1}C_3\Rightarrow {}^{n+1}C_4>{}^{n+1}C_3\Rightarrow \left|\frac{n+1}{2}-4\right|<\left|\frac{n+1}{2}-3\right|$$

$$\Rightarrow |n-7|<|n-5|$$

$\Rightarrow n > 6$

5. (B)

$$\Rightarrow \text{Number of permutations} = \frac{(4+3+2)!}{2!4!3!}=\frac{9!}{2!4!3!}$$

6. (C)

$$\Rightarrow \begin{matrix} - & \underset{\text{E}}{\overset{-}{\downarrow}} & - & \underset{\text{E}}{\overset{-}{\downarrow}} & - & \underset{\text{E}}{\overset{-}{\downarrow}} & - & \underset{\text{E}}{\overset{-}{\downarrow}} & - \end{matrix}$$

$\Rightarrow$ Odd digits can occupy 2^{nd}. 4^{th}, 6^{th} & 8^{th} places.

$$\Rightarrow \text{The number of different nine-digit numbers} = \frac{5!}{2!3!}\times\frac{4!}{2!2!}=60$$

7. (A)

$\Rightarrow \underline{C}\_\_\_\_\_\_\_\underline{Y}$

$\Rightarrow$ Number of words = 6!.

8. (B)

$\Rightarrow \_\_\underline{L}\_\_$

$\Rightarrow$ Required number of ways = 4!

9. (B)

$\Rightarrow$ Number of arrangement of boys = (7!)

$\Rightarrow$ Number of gaps created = 8.

$\Rightarrow$ Hence, Required number of different ways = $7! \times {}^{8}P_{3}$.

10. (B)

To form natural numbers from 1000 to 9999 having all 4 distinct digits

9 choices	9 choices	8 choices	7 choices

Number of such numbers = $9 \times 9 \times 8 \times 7 = 4536$
Number of all the numbers from 1000 to 9999 = 9000.
Hence number of natural numbers from 1000 to 9999 not having all 4 distinct digits
= 9000 – 4536
= 4464.

11. (A)

Two places to put 2s can be chosen in ${}^{7}C_{2}$ ways.
In each of the remaining 5 places we can put 1 or 3 hence these places can be filled in 2^5 ways.
Total number of numbers which can formed = ${}^{7}C_{2} \times 2^{5}$ i.e. 672.

12. (C)

Four consonants may be chosen in ${}^{7}C_{4}$ ways.

Two vowels may be chosen in ${}^{4}C_{2}$ ways.

The selected 6 letters may be arranged in 6! ways.

Total number of words = ${}^{7}C_{4} \times {}^{4}C_{2} \times 6!$ i.e. 151200.

13. (D)

Case I – The required numbers will be of form 57N, where N must be an even number.
Total number of such numbers = 5.
Case II – If 5 is not considered, then
for first digit from left we have 8 choices (not taking 0 & 5),
for second digit we have 9 choices (not taking 5),
for last digit we have 5 choices (only even numbers).
Total number of such numbers = $8\times9\times5$ i.e. 360.
Hence the number of such numbers = 365.

14. (A)

Flag can be designed by having following choices for six strips.

4 choices	3 choices	3 choices	3 choices	3 choices	3 choices

Hence number of designs = 12×81.

15. (D)

All possible arrangements = 4!
Arrangements in which 'ab' kept together = $3!\times2!$
Arrangements in which 'cd' kept together = $3!\times2!$
Arrangements in which 'ab' as well 'cd' kept together = $2!\times2!\times2!$
Arrangements in which neither 'ab' nor 'cd' are together
$= 4! - 3!\times2! - 3!\times2! + 2!\times2!\times2!$
$= 8$.

16. (B)

Two consecutive digits we can have in two ways i.e. first two or last two.
Now if first two digits are identical, then we can chose these in 9 ways (excluding 0)
and then third digit may be selected in 9 ways (excluding the number already placed in first 2 places).
If last two digits are identical, then we can choose first digit in 9 ways (excluding 0)
and last two digits in 9 ways (excluding the number already placed in first place).
Hence number of all such numbers is $9\times9+9\times9$ i.e. 162.

17. (B)

As the number has 9 distinct digits hence the middle digit must be 5, digits in last four places must be 6,7,8,9 and first four digits must be 1,2,3,4.
Now for first four place we have $4!$ ways.
For the last 4 places we have $4!$ ways.
Hence total number of such numbers is $(4!)^2$.

18. (B)

6 digits can be selected in ${}^{9}C_{6}$.

Now as the digits are to be kept in decreasing order so no arrangement required.
Hence total number of such numbers is 84.

19. (D)

If one or more couple is there in the committee, then we can form the committee in the following manner

(i) committee having at least one couple : can be formed by choosing one couple in 4 ways and then choosing 2 more people from remaining 6 in ${}^{6}C_{2}$ ways i.e. $4\times{}^{6}C_{2}$ or 60 ways.

(ii) committee having two couples : can be formed by choosing two couples in ${}^{4}C_{2}$ or 6 ways.

Hence a committee having one or more couples can be formed in 60 – 6 = 54 ways.

Number of all possible committee = ${}^{8}C_{4}$ or 70 ways.

Thus number of ways to form a committee having no couples = 70 – 54 = 16.

20. (C)

Numbers formed using just 1 or 2 are only 2.
All possible numbers using 1 & 2 only = 2^n.
Hence numbers using at least one 1 & one 2 = $2^n - 2$.
Now $2^n - 2 = 510$ gives n = 9.

21. (A)

Number of ways to chose m objects out of (m + n) objects = ${}^{m+n}C_m$

Number of ways to arrange these m and other n on the two tables = $\frac{(m-1)!}{2}\times\frac{(n-1)!}{2}$.

Number of ways of arrangements = ${}^{m+n}C_m\times\frac{(m-1)!}{2}\times\frac{(n-1)!}{2}$ or $\frac{(m+n)!}{4mn}$.

22. (A)

Number of ways to arrange balls keeping balls of same color together $3!\times4!$ i.e. $6\times4!$.

Number of all possible arrangements = $\frac{9!}{2!3!}$ i.e. $6\times7!$.

Number of arrangements in which at least one ball of same color is separated from others of same color = $6(7!-4!)$.

23 (B)

Trials needed for first three digits = 2.
If Last digit is 9, then rest three digits can be tried in 9^3 ways.
If last digit is one of 1, 3, 5 or 7, then one of the remaining three is 9(chosen in 3 ways) and other two digits can be chosen in 9^2 ways, hence all possible ways are $4\times3\times9^2$ ways.

Total number of ways to try the number = $2\times\left(9^3+4\times3\times9^2\right)$ or 3402.

24. (A)

The required number is maximum possible number of points in which the diagonals of an nonagon intersect inside the shape.
Now Each selection of four of the points gives three pair of lines out which exactly one pair intersects in the region enclosed within the four points.
Hence number of required points is ${}^{9}C_{4}$ i.e. 126.

25. (D)

Put one of the distinct objects at 1 place in the circle. Now for remaining objects we can use linear permutations.

Hence 'r' identical & (n – r – 1) distinct objects can be arranged in $\frac{(n-1)!}{r!}$ ways.

26. (B)

Put ladies in (n – 1)! Ways.
Now put gentlemen in (n – 1) gaps between ladies in n! ways.
Total number of ways = $(n-1)! \times n!$.

27. (D)

Place for A can be selected in 2×3 ways (leaving corner seats on both the sides) and then place for B can be selected in 6 ways(leaving places adjacent to A and opposite to A).
Place for A can be selected in 4 ways(seats at corners) and then for place for B in 7 ways (leaving place adjacent to A and opposite to A).
All the other people can sit in 8! Ways.
Hence number of seating arrangements = $(2\times3\times6 + 4\times7)\times8!$ i.e. $64\times8!$.

28.

29. (C)

$\Rightarrow$Total number of different combinations from the letters of word "MISSISSIPPI"
$(M\to1, S\to4, I\to4, P\to2)$ is (1 + 1) (4 + 1) (4 + 1) (2 + 1) – 1 = 149.

30. (A)

$\Rightarrow$Total number of ways of selecting six coins = coefficient of x^6 in $(x^o + x^1 + \ldots\ldots\ldots\ldots x^{20})$ $(x^o + x^1 + \ldots\ldots + x^{10})$ $(x^o + x^1 + \ldots\ldots x^7)$

$$\Rightarrow \text{coefficient of } x^6 \text{ in } \frac{(x^{21}-1)(x^{11}-1)(x^8-1)}{(x-1)(x-1)(x-1)}$$

$$\Rightarrow \text{coefficient of } x^6 \text{ in } (x^{21}-1)(x^{11}-1)(x^8-1)(1-x)^{-3}$$

$$\Rightarrow {}^{6+3-1}C_{3-1} = {}^{8}C_{2} = \frac{8\times7}{2} = 28$$

31. (B)
Each match can result in 3 ways.
forecast for 5 matches can be made in 3^5 ways in which one forecast is completely correct.
Hence there must be at least 243 people.

32. (D)
Let the numbers to fill be a, b, c, d as shown

a	b
c	d

Now a + b + c + d = 10 & a + d = b + c = k, then k = 5.
Now a + d = 5 = b + c gives choices for (a, d) as (1, 4) & (2, 3).
As a & d are interchangeable hence 4 ways to choose (a, d).
Similarly there are 4 ways to choose (b, c).
But as numbers are all distinct therefore for a given selection of (a, d) we can choose (b, c) in 2 ways only.
altogether a, b, c, d can be chosen in $4 \times 2 = 8$ ways.

33. (A)
A student can answer 5 questions in two ways,
(i) one question each from two sections & 3 questions from one section. In this case
number of ways to chose two sections = 3,
number of ways to chose one question out of four from any section = 4,
number of ways to chose three questions out of four from any section = 4.
Hence number of ways to answer five questions = $3 \times 4^2 \times 4 = 192$.
(ii) two questions each from two sections and one question from one section. In this case
number of ways to chose two sections = 3,
number of ways to chose two question out of four from any section = 6,
number of ways to chose one questions out of four from any section = 4.
Hence number of ways to answer five questions = $3 \times 6^2 \times 4 = 432$.
Total number of ways = 192 + 432 = 624.

34. (B)
Any 5 digits from {0, 1, 2,…,9} can be chosen in ${}^{10}C_5$ ways and put in descending order in just one way, hence m = ${}^{10}C_5$.
Any 5 digits from {1, 2, 3,…,9} can be chosen in ${}^{9}C_5$ ways and put in ascending order in just one way, hence n = ${}^{9}C_5$.
Thus m – n = ${}^{10}C_5 - {}^{9}C_5$ or ${}^{9}C_4$.

35. (A)

(i) When A is excluded, to form a triangle we have to choose two points on one line & one point on one line number of triangles = ${}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1$

(ii) When A is included, other than the triangles formed in case (i) we can form more triangles by taking A as one vertex and choosing other two vertices one each on AB & AC.

Hence number of triangles = ${}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1 + {}^{n}C_1 \times {}^{m}C_1$

Ratio of number of triangles in (i) & (ii) = $\dfrac{{}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1}{{}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1 + {}^{n}C_1 \times {}^{m}C_1} = \dfrac{m+n-2}{m+n}$.

36. (C)

9 lines will intersect in ${}^{9}C_2$ points.

9 circles will intersect in $2 \times {}^{9}C_2$ points.

9 lines will intersect with 9 circles in $2 \times {}^{9}C_1 \times {}^{9}C_1$ points.

Total number of points = 270.

37. (D)

a_4	a_3	a_2	a_1

b_4	b_3	b_2	b_1

$a_1 + b_1 \le 9$, $a_2 + b_2 \le 9$, $a_3 + b_3 \le 9$ & $a_4 + b_1 \le 9$, where $a_1, a_2, a_3, b_1, b_2, b_3 \in W$ & $a_4, b_4 \in N$

Now number of solutions of each of the first three equations will be ${}^{9+3-1}C_{3-1}$ or 55 and those of the fourth equation will be ${}^{7+3-1}C_{3-1}$ or 36.

Hence total number of ways = 36×55^3.

38. (C)

Hand shakes by men with each other = ${}^{10}C_2$

Hand shakes by women with each other = ${}^{10}C_2$

Hand shakes by men with women = 9×5.

Total number of hand shakes = 135.

39.

40. (D)

Given $x+1$ divides $P(x)$ hence $P(-1) = 0$ i.e. $a + c = 2b$.

Now if we chose any two odd numbers or any two even numbers from the set of given numbers as ‘a’ & ‘b’, then c gets selected automatically.

Required number of selections = $\left({}^{7}C_2 + {}^{7}C_2\right) \times 2$.

41. (B)

A white square can be selected in 32 ways.

Leaving the 8 black squares in the row and column containing the previously selected white square, we are left with 24 black squares.

Number of ways to selected a white and a black square = 32×24.

42. (D)

Possible way to distribute apples are such that

(i) one boy gets one apple, one boy gets three and remaining one gets four.

Hence number of ways to distribute = ${}^8C_1 \times {}^7C_3 \times 3!$.

(ii) Two boy gets two apples each, remaining one gets four.

Hence number of ways to distribute = $\dfrac{{}^8C_2 \times {}^6C_2 \times 3!}{2!}$.

(iii) two boys get three apples each and remaining one gets two.

Hence number of ways to distribute = $\dfrac{{}^8C_3 \times {}^5C_3 \times 3!}{2!}$.

Total number of ways to distribute apples = ${}^8C_1 \times {}^7C_3 \times 3! + \dfrac{{}^8C_2 \times {}^6C_2 \times 3!}{2!} + \dfrac{{}^8C_3 \times {}^5C_3 \times 3!}{2!}$

i.e. 4620.

Now ${}^7P_3 = 210$ gives $k = 22$.

43. (C)

There is only one way to distribute the toys i.e. 3 to youngest one and 2 each to others.

Number of ways to distribute = $\dfrac{9!}{(2!)^3\, 3!}$.

44. (A)

Marbles are to be distributed in 1 : 2 ratio therefore one child will get 4 & other will bet 8

Required number of ways = $\dfrac{12!}{4!8!} \times 2!$ i.e. 990.

45. (B)

Number of ways to arrange n identical red & r identical green balls = $\dfrac{(n+r)!}{n!\,r!}$ or ${}^{n+r}C_n$.

Now range of r is 0 to m.

Hence number of arrangements = ${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + \ldots + {}^{n+m}C_n = {}^{n+m+1}C_{n+1}$.

$x = n + m + 1$ & $y = n + 1$ or m.

46. (C)

$L = \dfrac{(p+q)!}{p!q!}, M = \dfrac{(p+q)!}{p!q!} \times 2!$ & $N = \dfrac{(p+q)!}{p!q!}$.

47. (C)

For ‘m’ choose 4 persons out of 10 and permute 4 books with 4 persons in ${}^{10}C_4 \times 4!$ ways.

For ‘n’ choose 4 persons out of 10 (no need to permute) in ${}^{10}C_4$ ways.

48. (C)

Standard Results

49. (A)

Number of non negative integral solutions of 3x + y + z = 24 is equal to number of no negative integral solutions of y + z = 24 – 3r, where r = 0, 1,…,8.

i.e. $\sum_{r=0}^{8} {}^{(24-3r)+2-1}C_{2-1}$ or $\sum_{r=0}^{8}(25-3r)$.

Hence required number of solutions are 225 – 3(1 + 2 +…+ 8) i.e. 117.

50. (D)

As $3^3+4^3+5^3=6^3$, hence for every $n \in N$, $(3n)^3+(4n)^3+(5n)^3=(6n)^3$.

There are infinitely many such numbers.

51. (A)

To form a number of r digits , where $1 \le r \le 9$, we have to chose r digits from {1, 2,…,9} in ${}^9C_r$ number of ways and arrange them in ascending order which can be done in just 1 way.

Number of numbers of r digits = ${}^9C_r$.

Hence number of all such numbers = $\sum_{r=1}^{9} {}^9C_r = 2^9-1$.

52. (B)

Red cards can be arranged with each other in 26! Ways.

Black cards can be arranged with each other in 26! Ways.

Ordering of red & black cards can be done in 2 ways.

Number of arrangements = $(26!)^2 \times 2$.

53. (A)

There are m number of ways to distribute each object and k objects are there so required number of ways = m^k.

54. (D)

The word EQUATIONS contains {A, E, I, O, U} which are to be kept in the same order and {Q, T, N, S} which can be kept in any order.

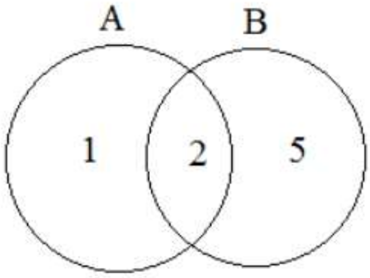
Total number of arrangements are 9! But as vowels can’t be arranged with each other hence required number of arrangements = $\frac{9!}{5!}$.

55. (A)

As targets in same column can be shot only in a certain order hence number of ways to shoot will be $\frac{8!}{3!2!3!}$ i.e. 560.

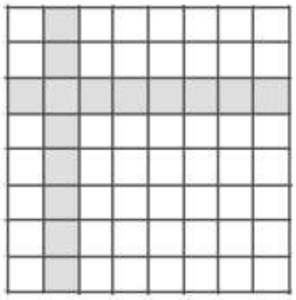
56. (A)

There are two common elements in A & B.
If A has n elements and B has m elements, then $m \times n = 21$.
Hence $\{m, n\} = \{3, 7\}$ or $\{7, 3\}$.
A possible arrangement is as shown in adjoining figure.
Hence $(A \cap B') \cup (A' \cap B) = 6$.



57. (B)

To place two rooks in attacking position they must be placed in the same row or column as shown in the adjoining figure.
As there are eight rows/columns and eight rooks hence each row/column must have exactly one rook.
So eight rooks can be placed in 8! Ways.



Ex. (1-C)

1. (D)

Number of ways to choose superintendent = 3

Number of ways to choose teachers for English school = ${}^{6}C_{2} \times 4!$

Number of ways to choose teachers for Vernacular school = $4!$

Total number of ways = $3 \times {}^{6}C_{2} \times 4! \times 4!$ or 25920.

2. (B)

Trials needed for first four digits = $\frac{4!}{2!}$.

Trials needed for fifth digit = 2.

Trials needed for seventh & eighth digit = 10.

Total number of trials needed = $\frac{4!}{2!} \times 2 \times 10 = 240$.

3. (B)

We can put 10 red balls at a gap of one each in 10! Ways, then green balls can be put in 11 gaps so formed in ${}^{11}C_{9}$ ways.

Hence number of arrangements are ${}^{11}C_{9} \times 10!$.

4. (C)

Keeping volumes of the same together we have 7 objects to arrange.

Also keeping volumes of same book in order we can have two arrangements for each seat – ascending & descending.

Number of ways to arrange = $7! \times 2 \times 2 \times 2$.

5. (C)

Number of words starting with E = $\frac{4!}{2!}$.

Number of words starting with QE = $\frac{3!}{2!}$.

Number of words starting with QUEE = 1.

Rank of QUEUE = 17.

6. (B)

Numbers with 1 at first place = ${}^{8}C_{4}$ or 70.

Numbers with 23 at first two places = ${}^{6}C_{3}$ or 20.

Numbers with 245 at first three places = ${}^{4}C_{2}$ or 6.

As 70 + 20 + 6 = 96, hence 97th number is 24678.

7. (A)

Dashes	5	4	3	2	1	0
Dots	2	3	4	5	6	7
Arrangements	$^7C_2$	$^7C_3$	$^7C_4$	$^7C_5$	$^7C_6$	$^7C_7$

Total Number of arrangements = $2^7 - {}^7C_0 - {}^7C_1$ or 120.

8. (A)

For India to win 5 matches before Pakistan does, there must be a minimum 5 matches and a maximum 9 matches.

In 5 matches – all must be won by India, hence 1 way.

In 6 matches – last match and in first 5 matches 4 must be won by India, hence $^5C_1$ ways.

In 7 matches – last match and in first 6 matches 4 must be won by India, hence $^6C_2$ ways.

In 8 matches – last match and in first 7 matches 4 must be won by India, hence $^7C_3$ ways.

In 9 matches – last match and in first 8 matches 4 must be won by India, hence $^8C_4$ ways.

Total number of ways = $1 + {}^5C_1 + {}^6C_2 + {}^7C_3 + {}^8C_4$ i.e. 126.

9. (D)

Number of ways in which delegates of A & B are together = $8! \times 2!$

Number of ways in which delegates of A & B as well of C & D are together = $7! \times 2! \times 2!$

Hence number of ways in which delegates of A & B are together but those of C & D are not together = $8! \times 2! - 7! \times 2! \times 2!$ i.e. $12 \times 7!$.

10. (D)

Number of ways to select 3 people from n sitting in a row, $P_n = {}^{n-3+1}C_3$

Number of ways to select 3 people from n sitting in a circle, $Q_n = {}^{n-3+1}C_3 - {}^{n-3-1}C_1$.

Hence $^{n-3-1}C_1 = 6$ or n = 10.

Alternately

Number of ways to select 3 people out of n sitting in a circle = $^nC_3$

Number of ways to select two adjacent and one separated = $n \times {}^{n-4}C_1$

Number of ways to select all three adjacent = n.

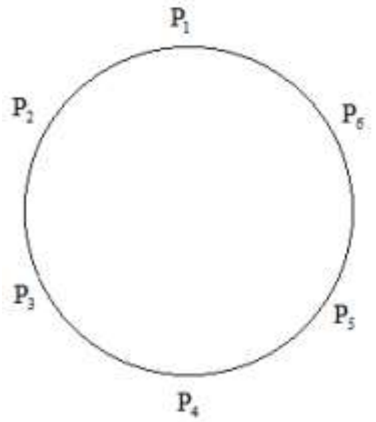
Hence number of ways to select 3 people out of n sitting in a circle such that no two are adjacent

$Q_n = {}^nC_3 - n \times {}^{n-4}C_1 - n$.

$P_n - Q_n = {}^{n-2}C_3 - {}^nC_3 + n \times {}^{n-4}C_1 + n$

$\Rightarrow n - 4 = 6$ i.e. $n = 10$.

11. (D)



Let us put A at P_1.
If B is on right of A i.e. at P_2, then place on right of B i.e. at P_3 can be filled by C or D in 2 ways and remaining 3 places can be filled in 3! Ways. Hence this arrangements can be made in 12 ways.
If C is on right of A i.e. P_2, then we can make B & D sit {P_3, P_4}, {P4, P_5} or {P_5, P_6} and the rest of the two places can be filled in 2 ways. Hence this arrangements can be made in 6 ways.
Total number of arrangements = 18.

12. (C)

Let Miss C & Mr. B be included, then Mr. A cant be selected hence 1 more woman to be selected from 4 remaining and 2 more men to be selected from remaining 4.

Number of combinations = ${}^4C_1 \times {}^4C_2$ i.e. 24.

If Mr. A is included then Mr. B cant be selected hence 2 women to be selected from 5 choices and 2 more men to be selected from remaining 4.

Number of combinations = ${}^5C_2 \times {}^4C_2$ i.e. 60.

In none of A & B are included, then 2 women to be selected from 5 choices and 3 men to be selected from remaining 4.

Number of combinations = ${}^5C_2 \times {}^4C_3$ i.e. 40.

Total possible combinations = 124.

13. (B)

-	AB	-	E	-	F	-

We can place AB, E & F in $3! \times 2!$ i.e. 12 ways.

Now C & D can be placed in gaps in ${}^4C_2 \times 2!$ i.e. 12 ways.

Total number of arrangements = 144.

14. (C)

Let 'r' boxes be filled by red balls and rest '5 – r' with blue balls.
Blue balls must be put in r + 1 gaps between red balls.
Hence number of ways to put green balls = ${}^{r+1}C_{5-r}$.
Also $r+1 \geq 5-r \Rightarrow 2 \leq r \leq 5$.

Number of all possible arrangements = $\sum_{r=2}^{5} {}^{r+1}C_{5-r} = {}^{3}C_{3} + {}^{4}C_{2} + {}^{5}C_{1} + {}^{6}C_{0}$ i.e. 13.

15. (B)

Using 6 points which are not cyclic we can draw ${}^{6}C_{3}$ circles.

Using 2 of the 5 cyclic points and 1 of the 6 noncyclic points we can draw ${}^{5}C_{2} \times {}^{6}C_{1}$ circles.

Using 1 of the 5 cyclic points and 2 of the 6 noncyclic points we can draw ${}^{5}C_{1} \times {}^{6}C_{2}$ circles.

Using the 5 cyclic points only 1 circle can be drawn.
Total number of circles = ${}^{6}C_{3} + {}^{5}C_{2} \times {}^{6}C_{1} + {}^{5}C_{1} \times {}^{6}C_{2} + 1$ i.e. 156.
Alternately
Using 11 points we can draw ${}^{11}C_{3}$ circles, but 5 out of these are cyclic so ${}^{5}C_{3}$ circles will coincide into 1.
Total number of circles = ${}^{11}C_{3} - {}^{5}C_{3} + 1$.

16. (D)

Number of zeros at the end of 50! = exponent of 5 in 50!
i.e. $\left[\frac{50}{5}\right] + \left[\frac{50}{25}\right]$ or 12 , where [x] denotes greatest integer less than or equal to x.

17. (C)

Odd digits – 3,3,5,5.
Even places 2nd, 4th, 6th & 8th.

Number of arrangements of odd digits at even places = $\frac{4!}{2! \times 2!}$.

Number of arrangements of remaining digits at rest of the places = $\frac{5!}{2! \times 3!}$.

Number of all such numbers = $\frac{4!}{2! \times 2!} \times \frac{5!}{2! \times 3!}$ i.e. 60.

18. (A)

mn = 25! Gives mn = $2^{20} \times 3^{10} \times 5^6 \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23$.

Hence mn as 9 prime factors.

For gcd(m, n) = 1, we need to chose m and n from these only.

Number of ways to chose m & n = $\sum_{r=0}^{9} {}^9C_r$ or 2^9.

Now number of selection such that $\frac{m}{n} > 1$ = Now number of selection such that $\frac{m}{n} < 1$

Hence number of ways to select m, n such that $\frac{m}{n} < 1 = \frac{2^9}{2}$ i.e. 2^8.

19. (A)

$${}^{13}C_3 + {}^{13}C_4 = {}^{14}C_4 \ \& \ {}^{13}C_4 + {}^{13}C_5 = {}^{14}C_5$$

$$\Rightarrow {}^{13}C_3 + 2\,{}^{13}C_4 + {}^{13}C_5 = {}^{14}C_4 + {}^{14}C_5$$

$$\Rightarrow {}^{n}C_r = {}^{15}C_5$$

Hence r = 5 or 10.

20. (D)

To form a mixed doubles team we need to select 2 males and two females which can be done in ${}^8C_2 \times {}^8C_2$ ways and then the selected 4 players can be arranged in 2 ways.

Hence total number of ways = $2 \times {}^8C_2 \times {}^8C_2$ i.e. 1568.

21. (D)

Number of pairings = number of ways to distribute 8 distinct objects in 4 groups each containing 2 objects i.e. $\frac{8!}{2!2!2!2!4!} = \frac{8.7.6.5}{16} = 105$.

22. (C)

Let n parabola divide a plane in T_n regions.

1 parabola divides the plane in 2 regions.

If a second parabola is introduced, then 3 parts will increase.

If a third parabola is introduced, then 5 parts will increase.

If a fourth parabola is introduced, then 7 parts will increase.

Now if (n – 1) parabolas are already there and n^{th} parabola is introduced (2n – 1) new regions will be created, hence

$$T_n = T_{n-1} + 2n - 1$$

Also $T_1 = 2$.

Therefore $T_{10} = T_1 + 3 + 5 + ... + 19 = 101$.

23. (D)

We have to find umber of positive integral solutions of $3x + 5y = 283$.

For $y = \frac{283-3x}{5}$, $(283 - 3x)$ must be a multiple of 5, hence last digit of 3x must be 3 or 8.

Now multiples of 3 having last digit 3 and less than 283 are {3, 33, 63, 93,…,273} i.e. 10 and multiples of 3 having last digit 8 and less than 283 are {18, 48, 78, 93,…,258} i.e. 9

Hence total 19 such points are there.

24. (C)

n circles will intersect in $2 \times {}^{n}C_{2}$ ways.

n lines will intersect in ${}^{n}C_{2}$ ways.

n lines will intersect n circles in $2 \times {}^{n}C_{1} \times {}^{n}C_{1}$ ways.

Hence $2 \times {}^{n}C_{2} + {}^{n}C_{2} + 2 \times {}^{n}C_{1} \times {}^{n}C_{1} = 80$ which gives $n = 5$.

25. (B)

Let there be x number of 2s, y number of 5s and z number of 7s are used in forming an n digit number, then the total number of numbers formed will be

$$\sum_{x=0}^{n}\sum_{y=0}^{n}\sum_{z=0}^{n}\frac{n!}{x!y!z!} \geq 900, \text{ where } x+y+z=n.$$

$\Rightarrow$ sum of coefficients in expansion of $(x+y+z)^{n} \geq 900$

$\Rightarrow 3^{n} \geq 900$.

Now $3^6 = 729$, hence least value of $n = 7$.

26. (D)

Given A = {1, 11, 21, …, 551}

There are 28 pairs whose sum is 552 such as (1, 551), (2, 550),…, (271, 281)

So if one number of a pair is taken in B then the other number can't be in B.

Hence B can contain maximum 28 elements.

27. (C)

EARTHQUAKE – {E,E}, {A,A}, {R, T, H, Q, U, K}

A 4 letter permutation can be of

(i) 2 alike & 2 alike = $\frac{4!}{2!\,2!}$ i.e. 6 ways

(ii) 2 alike & two distinct = $2 \times {}^{7}C_{2} \times \frac{4!}{2!}$ i.e. 504 ways

(iii) all four distinct = ${}^{8}C_{4} \times 4!$ i.e. 1680 ways

Hence total number of permutations = 6 + 504 + 1680 = 2190.

1.	**C**	2.	**D**	3.	**B**	4.	**D**	5.	**D**
6.	**A**	7.	**A**	8.	**A**	9.	**B**	10.	**D**
11.	**A**	12.	**A**	13.	**D**	14.	**B**	15.	**A**

16. **B**

$\because$ Each person gets at least one ball.

$\therefore$ 3 Persons can have 5 balls as follow.

Person	No. of balls	No. of balls
I	1	1
II	1	2
III	3	2

The number of ways of distribute balls, 1, 1, 3 in first to three persons

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3$

Also 3, persons having 1, 1 and 3 balls can be arranged in $\frac{3!}{2!}$ ways.

$\therefore$ Total no. of ways of distribute 1, 1, 3 balls to the three persons.

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times \frac{3!}{2!} = 60$

Similarly, total no. of ways to distribute 1, 2, 2, balls to three person

$= {}^5C_1 \times {}^4C_2 \times {}^2C_2 \times \frac{3!}{2!} = 90$

$\therefore$ The required number of ways = 60 + 90 = 150

17. **B**

We know,

$T_n = {}^nC_3,\ T_{n+1} = {}^{n+1}C_3$

ATQ, $T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$

$\Rightarrow {}^nC_2 = 10 \Rightarrow n = 5$.

18. **A**

Number of diagonal $= 54 \Rightarrow \frac{n(n-3)}{2} = 54$

$\Rightarrow n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$

$\Rightarrow n(n-12) + 9(n-12) = 0$

$\Rightarrow n = 12, -9 \Rightarrow n = 12\ (\because n \neq 9)$

19. **A**

(a) Required number of ways

= Total arrangement – number of ways when B_1 and G_1 together = 7! – 6! 2! = 6! (7 – 2) = 5.6!

20. **A**

(a) The thousands place can only be filled with 2, 3 or 4, since the number is greater than 2000.
For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3.
It the sum of digits of the number is divisible by 3, then the number itself is divisible by 3.

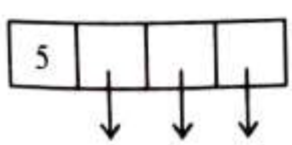
Case I: If we take 2 at thousands place.
The remaining digits can be filled as:
0, 1 and 3 as $2 + 1 + 0 + 3 = 6$ is divisible by 3.
0, 3 and 4 as $2 + 3 + 0 + 4 = 9$ is divisible by 3.
In both the above combinations the remaining three digits can be arranged in 3! ways.
$\therefore$ Total number of numbers in this case $= 2 \times 3! = 12$.

Case II: If we take 3 at thousands place. The remaining digits can be filled as:
0, 1 and 2 as $3 + 1 + 0 + 2 = 6$ is divisible by 3.
0, 2 and 4 as $3 + 2 + 0 + 4 = 9$ is divisible by 3.
In both the above combinations, the remaining three digits can be arranged in 3! ways. Total number of numbers in this case $= 2 \times 3! = 12$.

Case III: If we take 4 at thousands place.
The remaining digits can be filled as:
0, 2 and 3 as $4 + 2 + 0 - 3 = 9$ is divisible by 3.
In the above combination, the remaining three digits can be arranged in 3! ways.
$\therefore$ Total number of numbers in this case $= 3! = 6$.
$\therefore$ Total number of numbers between 2000 and 5000 divisible by 3 are $12 + 12 + 6 = 30$.

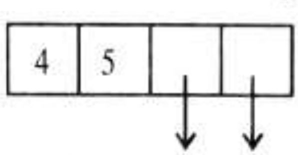
21. **D**

(d) 0, 1, 2, 3, 4, 5
Number of four-digit number starting with 5 is,



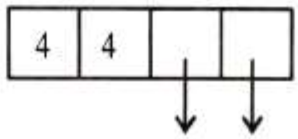
6 6 6 $= 6 \times 6 \times 6 = 216$

Number of four-digit numbers starting with 45 is,



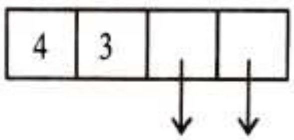
6 6 $= 6 \times 6 = 36$

Number of four-digit numbers starting with 44 is,



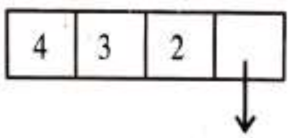
6 6 $= 6 \times 6 = 36$

Number of four-digit numbers starting with 43 and greater than 4321 is,



(5/4/3) 3 6 $= 3 \times 6 = 18$

Number of four-digit numbers starting with 432 and greater than 4321 is,



4 $= 4$

Hence, required numbers $= 216 + 36 + 36 + 18 + 4 = 310$.

22. **C**

(c) Number of ways of selecting 10 objects

$= (10I, 0D)$ or $(9I, 1D)$ or $(8I, 2D)$ or ... $(0I, 10D)$

Here, D signifies distinct object and I indicates identical object

$$= 1 + {}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{10} = \frac{2^{21}}{2} = 2^{20}$$

23. **B**

Number of arrangement $= (3! \times 3! \times 4!) \times 3! = (3!)^3 4!$

24. **C**

(c) We know, $(r+1) \cdot {}^{r}P_{r-1} = (r+1) \cdot \frac{r!}{1!} = (r+1)!$

So, $(2 \cdot {}^{1}P_0 - 3 \cdot {}^{2}P_1 + \dots\dots 51 \text{ terms}) +$
$(1! - 2! + 3! - \dots \text{ upto } 51 \text{ terms})$

$= [2! - 3! + 4! - \dots + 52!] + [1! - 2! + 3! - \dots + 51!]$

$= 52! + 1! = 52! + 1$

25. **D**

(d) $\frac{36}{r+1} \times {}^{35}C_r(k^2-3) = {}^{35}C_r.6$

$\Rightarrow k^2 - 3 = \frac{r+1}{6} \Rightarrow k^2 = 3 + \frac{r+1}{6}$

r can be 5, 35 for $k \in I$

$r = 5, k = \pm 2;\ r = 35, k = \pm 3$

Hence, number of ordered pairs = 4.

26. **D**

(d) Since, a boy plays against a boy.

$\therefore$ Total matches of boys can be arranged in

$7 \times 4 = 28$ ways

Since, a girl plays against a girl.

$\therefore$ Total matches of girls can be arranged in

$n \times 6 = 6n$ ways

$\because\ 28 + 6n = 52 \Rightarrow n = 4$

27. **A**

(a)

Indians	Foreigners	Number of ways
2	4	${}^6C_2 \times {}^8C_4 = 1050$
3	6	${}^6C_3 \times {}^8C_6 = 560$
4	8	${}^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

28. **D**

(d) Required number should be even and should be divisible by 6.

Then, number should also divisible by 2 & 3.

The possible choices for number divisible by 3 are (1, 2, 5, 6, 7) or (1, 2, 3, 5, 7)

Using 1, 2, 5, 6, 7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 2 = 48$

Using 1, 2, 3, 5, 7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 1 = 24$

Required answer is 72.

29. **309**

(309) M O T H E R

3 4 6 2 1 5

$\Rightarrow 2\times 5!+2\times 4!+3\times 3!+2!+1$

$=240+48+18+2+1=309$

... be possible i

30. **17**

(17) No. of ways of selecting 3 boys and 2 girls

$= {}^{b}C_3 \times {}^{g}C_2 = 168$

$b(b-1)(b-2)(g)(g-1) = 8\times 7\times 6\times 3\times 2$

$b+3g=17$

31. **240**

(240) $S \to 2,\ L \to 2,\ A, B, Y, U.$

$\therefore$ Required number of ways $= {}^{2}C_1 \times {}^{5}C_2 \times \dfrac{4!}{2!} = 240.$

32. **135**

(135)

Select any 4 correct questions in ${}^{6}C_4$ ways.

Number of ways of answering wrong question = 3

$\therefore$ Required number of ways $= {}^{6}C_4 (1)^4 \times 3^2 = 135.$

33. **54**

(54)

Let xyz be the three digit number

$x+y+z=10,\ x\ge 1,\ y\ge 0,\ z\ge 0$

$x-1=t \Rightarrow x=1+t$ $\qquad x-1\ge 0,\ t\ge 0$

$t+y+z=10-1=9$ $\qquad 0\le t,\ y,\ z\le 9$

$\therefore$ Total number of non-negative integral solution

$= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \dfrac{11\cdot 10}{2} = 55$

But for $t=9, x=10$, so required number of integers

$=55-1=54.$

34. **136**

(136) ${}^{1}P_1 + 2.\,{}^{2}P_2 + 3.\,{}^{3}P_3 + \ldots + 15\,.\,{}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + \ldots.\ 15 \times 15!$

$= 1! + (3!\,.\,2!) + \ldots.. + (16!\ 15!)$

$$\sum_{r=1}^{15} (r+1)! - (r)! = 16! - 1 = {}^{16}P_{16} - 1 \Rightarrow q = r = 16, s = 1$$

$${}^{q+s}C_{r-s} = {}^{17}C_{15} = \frac{17 \times 16}{2} = 136$$

35. **7744**

(7744) Numbers divisible by 11 will lie from 200 to 500

Now numbers are 209, 220, 231.....

$$\text{Sum} = \frac{27}{2}(209 + 495) = 9504$$

Number containing 1 at unit place

2 3 1
3 4 1
4 5 1

Number containing 1 at 10^{th} place

3 1 9
4 1 8

$\text{Required} = 9504 - (231 + 341 + 451 + 319 + 418) = 7744$

36. **6**

(6) Here 4 digit numbers.

For divisibility by 55, no. should be div. by 5 and 11 both

			5
a	b	c	d

Also, for divisibility by 11

$a + c = b + 5$

for $b = 1$ $a = 2, c = 4$
$\quad\quad\quad\quad$ $a = 4, c = 2$

for $b = 2$ $a = 3, c = 4$
$\quad\quad\quad\quad$ $a = 4, c = 3$

for $b = 3$ $a = 6, c = 2$
$\quad\quad\quad\quad$ $a = 2, c = 6$

$\therefore$ 6 possible four digit no.s are div. by 55 (II) 5 digit number is not possible

(Not possible)

37. **180**

(180) Given a 5 digit number product is 36.

Multiple of $36 = 2^2 \times 3^2$

$= 2 \times 2 \times 3 \times 3$

We need to make five digit number by using the digits 2, 2, 3 & 3 and the fifth digit could be 7.

Case-I: 2, 2, 3, 3, 1

Case-I: 2, 2, 3, 3, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!2!} = 30$

Case-II: 4, 3, 3, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!2!} = 30$

Case-III: 6, 2, 3, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!} = 60$

Case-IV: 9, 2, 2, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!2!} = 30$

Case-V: 4, 9, 1, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{3!} = 20$

Case-VI: 6, 6, 1, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{3!2!} = 10$

Total number of 5 digits numbers

$= 30 + 30 + 60 + 30 + 20 + 10 = 180.$

38. **1492**

(1492)

M	A	N	K	I	N	D

$$\left(\frac{4 \times 6!}{2!}\right) + (5! \times 0) + \left(\frac{4! \times 3}{2!}\right) + (3! \times 2) + (2! \times 1) + (1! \times 1) + (0! \times 0) + 1 = 1492$$

$\Rightarrow \quad 1440 + 36 + 12 + 4 = 1492$

39. **1086**

(1086) Let the number is $abcd$, where a, b, c are divisible by d.

	No. of such numbers
$d=1$,	$9\times10\times10=900$
$d=2$	$4\times5\times5=100$
$d=3$	$3\times4\times4=48$
$d=4$	$2\times3\times3=18$
$d=5$	$1\times2\times2=4$
$d=6,7,8,9$	$4\times4=16$
	$=1086$

40. **150**

(150) $36=2\times2\times3\times3$

We are required such three digit numbers whose G.C.D. with 36 is only 2.

$\Rightarrow$ Number should be odd multiple of 2 and does not having factor 3 and 9

Odd multiple of 2 are e.g. : {(36, 102), (36, 106) .....}

102, 106, 110, 114.....998 (225 no.)

No. of multiples of 3 are

102, 114, 126 .....990 (75 no.)

Which are also included multiple of 9

Hence, required $=225-75=150$

41. **243**

(243)

Total choices of digits are (0, 1, 2, ..., 9).

If 0 taken twice then ways = 9

If 0 taken once then ${}^9C_1\times2=18$

If 0 not taken then ${}^9C_1\,{}^8C_1\times3=216$

Total = 243

42. **40**

(40) Let 'p' and 'q' be the number of correct and incorrect answer is 5.

So, $x+y=5$ and $3x-2y=5$

After solving the equations we have (3, 2).

Therefore, numbers of correct answers are 3 and number of incorrect answers are 2.

So, the number of possibilities are 3, 3, 3, –2, –2.

Number of ways $=\dfrac{5!}{3!2!}\times2\times2=40$

PERMUTATIONS & COMBINATIONS
Exercise 2(A)

Q.1 (A)

<u>Case I</u>:All 3 flowers of first are together = 1 way

<u>Case II</u> : 2 flowers of first kind are together. Keeping these as areference 3rd flower of second kind. No. of ways butanticlockwise arrangements are not different

$\therefore$ Number of ways = $\frac{11+1}{2}$

<u>CaseIII</u> : All 3 flowers of first kind are kept at 12 spaces generated by the flowers of 3rd kind

$\therefore$ Number of ways = 12

Ans. 1 + 6 + 12 = 19.

Q.2 (D)

$504 = 2^3 \cdot 7 \cdot 3^2$

For even divisor selecting at least one 2 & remaining factors inall possible ways, no. of possible divisors = $3 \cdot 2 \cdot 3$=18

Q.3 (D)

For$1 \le k \le p-1, n+k = p! + k + 1$, is clearly divisible by k+1.

$\therefore$ There is no prime no. in the given list…........(Ans.:)

Q.4 (C)

Selecting places for A,B,C,D → ${}^6C_4$

Arranging other persons in 2! Ways : ${}^6C_4$.2! =30.

Q.5 (C)

A={2,3,5,7,11,13,17,19,23,29} $\therefore$ n(A) = 10

Selecting any 2 numbers& arranging them in 2 way = ${}^{10}C_2 \times 2 = 9$

(two numbers a & b give two rational numbers $\frac{a}{b}$ & $\frac{b}{a}$)

Also with repetition (i.e. $\frac{2}{2}$, $\frac{3}{3}$, $\frac{5}{5}$,.......... etc.) only the no.'1'can be formed

$\therefore$ Total number of rational numbers, 90+1=91

Q.6 (B)

Each element in domain can be associated with any of the k elements in codomain

$\therefore$ Total number of mappings = k^k.

Q.7 (C)

Selecting any y form 0 to 9 & arranging them in one way (descending order only)

$= {}^{10}C_4 \times 1 = 210$.

Q.8 (C)

Number of ways to select one point each from three line = $p \times p \times p$ *i.e.* p^3

Number of ways to select 2 points from any one of three lines& one point from any one of the other two lines = ${}^3C_1 \times {}^pC_2 \times {}^2C_1 \times p$ *i.e.* $3p^2(p-1)$

$\therefore$ No. of triangles $= p^3 + 3p^2(p-1) = p^3(4p-3)$.

Q.9 (B)

First round $= {}^6C_2 + {}^6C_2 = 30\, matches$

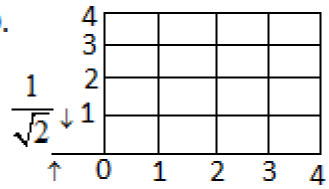
Second round $= {}^6C_2 = 15\, matches$

Third round $= {}^4C_2 = 6\, matches$

Min. no. of matches in best of the 3= 2

Total number of matches = 52.

Q.10.



A side of length $\sqrt{2}$ units vertically can be selected in 3 ways.

A side of length $\sqrt{2}$ units horizontally can be selected in 3 ways.

$\therefore$ no. of squares = 3×3 ....(Ans.: A)

Q.11 (A)

<u>Pattern I</u> : AAB $\rightarrow {}^{3}C_{2} \times 2 = 6$ (two strips of one color and one of different color)

<u>Pattern II</u> : ABB $\rightarrow {}^{3}C_{2} \times 2 = 6$ (two strips of one color and one of different color)

<u>PatternIII</u> : ABA $\rightarrow {}^{3}C_{2} \times 2 = 6$ (two strips of one color and one of different color)

<u>PatternIV</u>: ABC $\rightarrow {}^{3}C_{2} \times 2 = 6$ (One strip each of three colors)

$\therefore$ Total number of flags = 6+6+6+6=24

Q.12 (A)

For $\frac{p}{q}$, p & q both can be selected in $6 \times 6 = 36$ ways

The no. of ways for $\frac{p}{q} = 1$ is 6

The no. of ways for $\frac{p}{q} = \frac{1}{2}$ is 13 i.e. $\left(\frac{1}{2}, \frac{2}{4}, \frac{3}{6}\right)$

The no. of ways for $\frac{p}{q} = \frac{1}{2}$ is 3 i.e. $\left(\frac{2}{1}, \frac{4}{2}, \frac{6}{3}\right)$

The no. of ways for $\frac{p}{q} = \frac{1}{3}$ & 3 are 2 & 2 i.e. $\left(\frac{3}{1}, \frac{6}{2}; \frac{1}{3}, \frac{2}{6}\right)$

The no. of ways for $\frac{p}{q}=\frac{2}{3}$ & $\frac{3}{2}$ are 2 & 2 i.e. $\left(\frac{2}{3},\frac{4}{6};\frac{3}{2},\frac{6}{4}\right)$

$\therefore$ Extra counting = 5+2+2+1+1+1+1=13

$\therefore$ No. of rational nos. = 36 – 13=23

Q.13 (D)

The no. of ways in which only one element be the element of $A \cap B$, is $= {}^nC_1$ *i.e.* n.

For other (n – 1) elements, each have possibilities of being a member of only P or of only Q or of none of P & Q i.e. 3 each

$\therefore$ required no. of ways = n.(3.3.3……….(n – 1)times)= $n \times 3^{n-1}$

Q.14 (B)

The no. of ways to select three numbers = ${}^nC_3$

No. of ways to arrange = 1

$\therefore$ Total number of triplets = ${}^nC_3 \times 1 = {}^nC_3$

Q.15 (D)

For each book number of ways to select (to select 0, 1, 2, …, *l* copies) is $l+1$

$\therefore$ for k books, number of ways to select = $(l+1)^k$

Out of these in one case no book will be selected.

$\therefore$ rejecting the case selection, total number of selections = $(l+1)^k - 1$

Q.16 (A)

The no. of ways to select any 2 stations = ${}^{20}C_2$

For tickets, (A $\longrightarrow$ B and B $\longrightarrow$ A) no. of ways =2

$\therefore$ Total number of tickets required = ${}^{20}C_2$ .2=380.

Q.17 (D)

In MATHEMATICS we have M,A,T twice times each &H,E,I,C,S once each.

No. of words of the form (A,B,C,D i.e. all different)= $^{8}C_{4}.4!$

No. of words of the form (A,A B,C i.e. two alike & two distinct)= $^{3}C_{2} \times {}^{7}C_{2} \times \frac{4!}{2!}$

No. of words of the form (A,A,B,B i.e. two alike & two alike)= $^{3}C_{2} \times \frac{4!}{2!2!}$ =18

Total number of words = 2454.

Q.18 (A)

For y = 1, $x_1 \cdot x_2 \cdot x_3 = y$ has only 1 solution.

For y=2,3 & 5,number of solutions of $x_1 \cdot x_2 \cdot x_3 = y$ is $3 \times 3 = 9$

For y=2.3,2.5,3.5 i.e. for y=6,10 & 15 number of solutionsof $x_1 \cdot x_2 \cdot x_3 = y$ is

$3 \times \left(3! + {}^{3}C_{2}\right) = 27$

Explanation : $\begin{pmatrix} For\ x_1 \neq x_2 \neq x_3 \to 3! \\ For\ x_1 = x_2 \neq x_3 \to {}^{3}C_2 \end{pmatrix}$

For y=30 i.e.2.3.5, the no. of solutions of $x_1 \cdot x_2 \cdot x_3 = y$ is

$\left(3! + {}^{3}C_{2} \times 3! + 3\right) = 27$

Explanation : $\begin{pmatrix} For\ x_1 \neq x_2 \neq x_3 \to 3! \\ For\ x_1 = 1 \neq x_2 \neq x_3 \to {}^{3}C_2.3! \\ For\ x_1 = 1 = x_2, x_3 \to 3 \end{pmatrix}$

Total number of solutions = 1 + 9 + 27 + 27 i.e. 64.

Q.19 (B)

$7! = 2^4 \cdot 3^2 \cdot 5^1 \cdot 7^1$

Factors of the form 3t+1 & odd are 1 & 7.

$\therefore$ sum = 8

Q.20 (B)

No. of ways to select 6 digits form 0,1,2………..,9=$^{10}C_6$

No. of ways to choose a_4 (the smallest)=1

For other 3 the no. of ways to choose =$^5C_3$

$\therefore$ reqd. no. of ways $^{10}C_6 \times {}^5C_3 = 2100$.

Q.21 (A)

60 = $2^2 \cdot 3 \cdot 5$

Let the powers of 2 in x_1, x_2 & x_3 bei , j , k, then i+j+k=2 where i, j, k are whole numbers.

$\therefore$ no. of non-negative integral solution = $^{2+3-1}C_{3-1} = {}^4C_2 = 6$

Also 3 & 5 can be factors of a, b & c in 3 ways each

$\therefore$ no. of non-negative integral solution of $x_1x_2x_3 = 60$ is $^4C_2 \times 3 \times 3 = 54$.

Q.22 (A)

For $2^n \rightarrow$ n=2,3,4,5 & 6 $\therefore$ no. of ways =5

For $3^n \rightarrow$ n=2,3, 4 $\therefore$ no. of ways =3

For $5^n \rightarrow$ n=2 $\therefore$ no. of ways =1

For $7^n \rightarrow$ n=2 $\therefore$ no. of ways =1

For $6^n \rightarrow$ n=2 $\therefore$ no. of ways =1

Also For $10^n \rightarrow$ n=2 $\therefore$ no. of ways =1

Total number of ways =12

Q.23 (C)

First digit can be chosen in 9 ways (let first digit be a, a$\neq$0)

Second digit can be chosen in 9 ways (let second digit be b$\neq$a)

Third digit can be chosen in 9 ways (let third digit be c$\neq$b)

Hence each digit can be chosen in 9 ways.

$\therefore$ number of n digit nos. = 9^n

Q.24 (B)

$3630 = 2 \cdot 3 \cdot 5 \cdot 11^2$

For no. of factors of the form (4n+1) i.e. $3^a \cdot 5^b \cdot 11^c$ the values of a , b & c required are

a= 1, b=0 or 1 c= 2 or a=0, b= 0 or 1& c=2

$\therefore$No. of ways = $1 \times 2 \times 2 + 1 \times 2 \times 1 = 6$.

Q.25 (D)

(3m+1)+(3n+2) $\rightarrow$ is divided by 3 $\therefore$ No. of ways = $4 \times 4 = 16$

3m+3n $\rightarrow$ is div. by 3 $\therefore$ No. of ways = ${}^4C_2 = 6$

$\therefore$ all possible number of ways = 16+6=22.

Q.26 (A)

$30 = 2 \cdot 3 \cdot 5 = x_1 \cdot x_2 \cdot x_3$

Each factor 2, 3&5 has 3 choices for being a factor of x_1, x_2 or x_3

$\therefore$ No. of solution $= 3 \times 3 \times 3$= 27.

PERMUTATIONS & COMBINATIONS

Exercise 2(B)

Q.1 (A)(B)(C)(D)

$^{2n}P_n = n!\left(^{2n}C_n\right)$

$\Rightarrow {}^{2n}P_n = \dfrac{2n(2n-1)(2n-2)...3.2.1}{n(n-1)(n-2)...3.2.1}$

$\Rightarrow {}^{2n}P_n = 2n(2n-1)(2n-2)...(n+1)$

Hence $^{2n}P_n = \dfrac{2^n\left(n(n-1)(n-2)...1\right)\left((2n-1)(2n-3)...3.1\right)}{n(n-1)(n-2)...1}$

or $^{2n}P_n = 2^n\left[1.3.5....(2n-1)\right]$ i.e. $2.6.10....(4n-2)$

Q.2 (A)(C)(D)

Exponent of any prime P in 100! = $\left[\dfrac{100}{P}\right]+\left[\dfrac{100}{P^2}\right]+\left[\dfrac{100}{P^3}\right]+...$, where [x] denotes greatest integer less than or equal to x.

Now exponent of 2, α = 50 + 25 + 12 + 6 + 3 + 1 = 97.
Exponent of 3, β = 33 + 11 + 3 + 1 = 48.
Exponent of 5, γ = 20 + 4 = 24.
Exponent of 7, δ = 14 + 2 = 16.

Q.3 (A)(C)(D)

Number of ways to select any 3, 4 or all 5 out of first 5 questions and then to select 7, 6 or 5, respectively, out of the remaining 8 questions = $^5C_3\times{}^8C_7+{}^5C_4\times{}^8C_6+{}^5C_5\times{}^8C_5$ i.e. 276.

Q.4 (A)(B)

$$\max\left(^nC_r\right)=\begin{cases} ^nC_{n/2}, & \text{if } n \text{ is even} \\ ^nC_{(n-1)/2}, & \text{if } n \text{ is odd} \end{cases}$$

Also $^nC_r = {}^nC_{n-r} \Rightarrow {}^nC_{(n-1)/2} = {}^nC_{(n-1)/2}$.

Q.5 (B)(D)

$x_1+x_2+x_3+x_4 \le n \Rightarrow x_1+x_2+x_3+x_4+x_5 = n$.
Now number of non negative integral solutions = $^{n+5-1}C_{5-1}$ or $^{n+4}C_4$.

Also $^nC_r = {}^nC_{n-r} \Rightarrow {}^{n+4}C_4 = {}^{n+4}C_n$.

Q.6 (A)(B)(C)

Arrangements of 8 balls keeping two particular colors together = $7!\times 2!$
Unrestricted arrangements = 8!
Arrangements of 8 balls keeping two particular colors separated = $8! - 7!\times 2!$.

Q.7 (A)(B)(C)(D)

$$S = 1.1! + 2.2! + 3.3! + ... + n.n!$$
$$\Rightarrow S = (2-1).1! + (3-1).2! + (4-1).3! + ... + (n+1-1).n!$$
$$\Rightarrow S = 2! - 1! + 3! - 2! + 4! - 3! + ... + (n+1)! - n!$$
$$\Rightarrow S = (n+1)! - 1$$

Q.8 (A)(B)(C)(D)

When two vowels are together : $\frac{5!\times 2!}{2!}$.

When vowels occur in alphabetical order : $\frac{6!}{2!\times 2!}$.

When vowels and consonant occupy their respective places : $\frac{4!\times 2!}{2!}$.

When vowels don't occur together : $\frac{6!}{2!} - \frac{5!\times 2!}{2!}$.

Q.9 (A)(B)

Arrangements of 10 objects such that two objects are not be arranged with each other
= $\frac{10!}{2!}$.

Q.10 (B)(C)(D)

Number of ways to select r distinct objects out of n distinct such that no two or more of the selected objects were adjacent = ${}^{n-r+1}C_r$.
Hence required number of arrangements is ${}^{7}C_4 \times 4!$.

Q.11 (B)(C)

Let the number of students be n, then everyone will have to send (n – 1) cards.
Number of greeting cards which were sent will be n(n – 1).
Now n(n – 1) = 1640 gives n = 41.

Q.12 (A)(B)(C)

For sum of digits to be even : we can choose first four digits in general in 9×10^3 ways
Now if sum of first four digits is even, then last digit must be even
and if sum of first four digits is odd, then last digit must be even
Hence in any possibility last digit can be chosen in 5 ways.
Number of all such numbers, x = $9\times10^3\times5$ or 45000.
For sum of digits to be odd : we can choose first four digits in general in 9×10^3 ways
Now if sum of first four digits is even, then last digit must be odd
and if sum of first four digits is odd, then last digit must be even
Hence in any possibility last digit can be chosen in 5 ways.
Number of all such numbers, y = $9\times10^3\times5$ or 45000.
Hence x = y = 45000.

Q.13 (A)(D)

To deal 5 consecutive cards irrespective of any preference for suite we can deal each value card in 4 ways and there are 10 ways to chose a sequence of 5 consecutive values.
Hence total number of ways = 10×4^5.
5 consecutive cards of the same suite can be dealt in 10×4 ways.
Hence number of ways to deal a straight = $10\times\left(4^5-4\right)$ i.e. 10200.

Q.14 (B)

There are two ways to form a triangle –
(i) Choosing one point each on AB, BC & CA.
Number of triangles = ${}^3C_1\times{}^4C_1\times{}^5C_1$ or 60.
(ii) Choosing one point on one side & two points on one side.
Number of triangles = ${}^3C_2\times\left({}^4C_1+{}^5C_1\right)+{}^4C_2\times\left({}^3C_1+{}^5C_1\right)+{}^5C_2\times\left({}^4C_1+{}^3C_1\right)$ or 172.
Total number of triangles = 205.

Q.15 (B)(D)

Number of ways to chose two ice-creams of same flavor & one different = 8×7.
Number of ways to chose all three ice-creams of same flavor = 8.
Hence total number of ways = 64.
Also ${}^{10}C_3-{}^8C_3$= 64.

Q.16 (A)

1 + 2 + 3 + …+ 9 = 45.
Now to chose seven out of these numbers such that their sum is divisible by 3, sum of the two rejected numbers must be divisible by three.
Possible pairs are {1, 2}, {1, 5}, {1, 8}, {2, 4}, {2, 7}, {3, 6}, {3, 9}, {4, 5}, {4, 8}, {5, 7}, {6, 9} & {7, 8}.
Hence 12 selections are possible leaving these pairs out of 1, 2, 3, …,9.
Total number of such possible numbers are $12\times 7!$.

Q.17 (B)(C)

Number of ways to divide 10 students in 3 teams such that one contains 4 students and others 3 each will be $\frac{{}^{10}C_4\times{}^{6}C_3\times{}^{3}C_3}{2!}$ or 2100.

Q.18 (B)(C)

${}^{1000}C_{500}=\frac{1000!}{500!\times 500!}$. Let [x] denote greatest integer less than or equal to x.

Exponent of 7 in 1000! = $\left[\frac{1000}{7}\right]+\left[\frac{1000}{49}\right]+\left[\frac{1000}{243}\right]$ or 164.

Exponent of 7 in 500! = $\left[\frac{500}{7}\right]+\left[\frac{500}{49}\right]+\left[\frac{500}{243}\right]$ or 82.

Hence exponent of 7 in $\frac{1000!}{500!\times 500!}$ = 0.

Exponent of 13 in 1000! = $\left[\frac{1000}{13}\right]+\left[\frac{1000}{169}\right]$ or 81

Exponent of 13 in 500! = $\left[\frac{500}{13}\right]+\left[\frac{500}{169}\right]$ or 40.

Hence exponent of 13 in $\frac{1000!}{500!\times 500!}$ = 1.

Exponent of 191 in 1000! = $\left[\frac{1000}{191}\right]$ or 5

Exponent of 191 in 500! = $\left[\frac{500}{191}\right]$ or 2.

Hence exponent of 13 in $\frac{1000!}{500!\times 500!}$ = 1.

Exponent of 201 in 1000! = $\left[\frac{1000}{201}\right]$ or 4

Exponent of 201 in 500! = $\left[\frac{500}{201}\right]$ or 2.

Hence exponent of 201 in $\frac{1000!}{500!\times 500!}$ = 0.

Hence $^{1000}C_{500}$ is divisible by 13 & 191 but not by 7 &201.

Q.19 (B)(C)

(A) Number of zeros in the end of 125! = exponent of 5 i.e. $\left[\frac{125}{5}\right]+\left[\frac{125}{25}\right]+\left[\frac{125}{125}\right]$ or 31.

[x] denotes greatest integer less than or equal to x.
(B) Total possible combinations of positions of all the arms = 10^{10}.
Now when all the arms are at rest no signal is transmitted hence
Number of signals = $10^{10} – 1$.
(C) A number greater than 400000 will have first digit 4 or 5 and will be of 6 digits as each given choice can be used only once.

If first digit is 4, then rest of the digits can be placed in $\frac{5!}{2!}$.

If first digit is 5, then rest of the digits can be placed in $\frac{5!}{2!2!}$.

Total number of ways = 90.
(D) If there are n players, then number of games played will be $^{n}C_{2}$.

Now $^{n}C_{2}$ = 5050 $\Rightarrow n(n-1)=10100$.

Hence n = 101.

Q.20 (A)(C)(D)

[x] denotes greatest integer less than or equal to x.

(A) Number of zeros at the end of 20! = exponent of 5 in 20! = $\left[\frac{20}{5}\right]$ or 4.

(B) Exponent of 2 in 20! = $\left[\frac{20}{2}\right]+\left[\frac{20}{4}\right]+\left[\frac{20}{8}\right]+\left[\frac{20}{16}\right]$ or 18.

Exponent of 3 in 20! = $\left[\frac{20}{3}\right]+\left[\frac{20}{9}\right]$ or 8.

Exponent of 5 in 20! = $\left[\frac{20}{5}\right]$ or 4.

Exponent of 7 in 20! = $\left[\frac{20}{7}\right]$ or 2.

Exponent of 11, 13, 17 & 19 in 20! = 1 each.

Now $\frac{20!}{10^4}=\frac{2^{18}\times 3^8\times 5^4\times 7^2\times 11\times 13\times 17\times 19}{2^4\times 5^4}$ i.e. $2^{14}\times 3^8\times 7^2\times 11\times 13\times 17\times 19$.

Last digit of 2^{14} = 4, last digit of 3^8 = 1, last digit of 7^2 = 9.

Hence last digit of $\frac{20!}{10^4}$ will be last digit of $4\times 1\times 9\times 1\times 3\times 7\times 9$ or 4.

(C) Exponent of 5 in 20! = 4 & exponent of 5 in 10! = 2.

(D) Exponent of 7 in 20! = 2 & exponent of 7 in 10! = 1.

Passage – 1

In CURRICULUM, there are
3 – U, 2 – C, R, 1 – I, L, M.

Q.21 (B)

Words which can be formed using only three letters can be formed using
(i) 2 identical letters, 2 identical letters & 1 different letter
(ii) 3 identical letters & 2 different letters
In first case, we can choose two pairs of identical letters in 3 ways & a distinct letter in 4 ways and the letters can be arranged in $\frac{5!}{2!2!}$ ways.

Hence total number of words = $3\times 4\times \frac{5!}{2!2!}$ or 360.

In second case three identical letters must be 3 Us and two distinct letters can be chosen in $^5C_2$ ways and the letters can be arranged in $\frac{5!}{3!}$ ways.

Hence total number of words = $^5C_2\times \frac{5!}{3!}=200$.

Number of all such possible words are = 560.

Q.22 (A)

All possible arrangements = $\frac{10!}{3!2!2!}$ or 30.7!

To find arrangements in which vowels are separated –

Keeps consonants at a gap of one each in $\frac{6!}{2!2!}$ or 180 ways,

Now put vowels in 7 gaps in $^7C_4\times \frac{4!}{3!}$ or 140 ways.

Hence number of arrangements = $180\times 140=25200=5(7!)$.

Q.23 (B)

Consonants can be arranged with each other in $\frac{6!}{2!2!}$ or 180 ways.

Vowels can be arranged with each in $\frac{4!}{3!}$ or 4 ways.

Hence all possible arrangements are 180×4 = 720 ways.

Passage – 2

Q.24 (D)

Increasing order : Any selection of 5 digits can be made in ${}^{10}C_5$ ways and arranged in 1 way.
Hence number of numbers = 252.
Decreasing order : Any selection of 5 digits can be made in ${}^{9}C_5$ ways (not including 0 in any selection) and arranged in 1 way.
Hence number of numbers = 126.
Number of all possible numbers = 378.

Q.25 (A)

Two alike & three alike can be chosen in 9×8 or 72 ways (not selecting 0) and arranged in $\frac{5!}{3!2!}$ or 10 ways.
Number of such numbers = 720.
If two ‘0’s and three other identical digits are chosen (in 9 ways) then after placing one nonzero digit at the left most place rest of the digits can be arranged in $\frac{4!}{2!2!}$ or 6 ways.
Number of such numbers = 54.
If three ‘0’s and two other identical digits are chosen (in 9 ways) then after placing one nonzero digit at the left most place rest of the digits can be arranged in $\frac{4!}{3!}$ or 4 ways.
Number of such numbers = 36.
Number of all possible numbers = 810.

Q.26 (A)

Sum of all the digits of S is 45 which is divisible by 3 hence every number made up of all 10 digits will be divisible by 3.
None of the numbers is prime.

Passage – 3

Q.27 (C)

Number of ways to de-arrange 5 objects = $\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)\times 5!$ i.e. 44.

Q.28 (B)

Number of all integers less than or equal to 100 = 100.
Number of integers divisible by 2 = 50
Number of integers divisible by 3 = 33
Number of integers divisible by 5 = 20
Number of integers divisible by 2 & 3 = 16
Number of integers divisible by 2 & 5 = 10
Number of integers divisible by 3 & 5 = 6
Number of integers divisible by all of 2, 3 & 5 = 3.
Number of integers not divisible by any of 2, 3 or 5
= 100 – (50 + 33 + 20) + (16 + 10 + 6) – 3 i.e. 26.

Q.29 (A)

Required number of solutions will be coefficient of x^{30} in $\left(1+x+x^2+...+x^9\right)^4$.

i.e. coefficient of x^{30}in $\left(1-x^{10}\right)^4\left(1-x\right)^{-4}$.

Now general term of $\left(1-x^{10}\right)^4\left(1-x\right)^{-4}$ will be $\left(-1\right)^p\ {}^4C_p\ {}^{q+3}C_3\ x^{10p+q}$.

coefficient of x^{30} = ${}^4C_0\ {}^{33}C_3-{}^4C_1\ {}^{23}C_3+{}^4C_2\ {}^{13}C_3-{}^4C_3\ {}^3C_3$ or 84.

Passage – 4

Q.30 (A)

${}^{1000}C_{500}=\frac{1000!}{500!\times 500!}$. Let [x] denote greatest integer less than or equal to x.

Exponent of 7 in 1000! = $\left[\frac{1000}{7}\right]+\left[\frac{1000}{49}\right]+\left[\frac{1000}{243}\right]$ or 164.

Exponent of 7 in 500! = $\left[\frac{500}{7}\right]+\left[\frac{500}{49}\right]+\left[\frac{500}{243}\right]$ or 82.

Hence exponent of 7 in $\frac{1000!}{500!\times 500!}$ = 0.

Q.31 (C)

Exponent of 5 in 50! = $\left[\frac{50}{5}\right]+\left[\frac{50}{25}\right]$ or 12 .

Hence number of zeros at the end of 50! = 12.

Q.32 (D)

${}^{200}C_{100}=\frac{200!}{100!\times100!}$. Let [x] denote greatest integer less than or equal to x.

Exponent of 59 in 200! = $\left[\frac{200}{49}\right]$ or 4.

Exponent of 59 in 100! = $\left[\frac{100}{49}\right]$ or 2.

Hence exponent of 59 in ${}^{200}C_{100}$ = 0.

Exponent of 53 in 200! = $\left[\frac{200}{53}\right]$ or 3.

Exponent of 53 in 100! = $\left[\frac{100}{53}\right]$ or 1.

Hence exponent of 53 in ${}^{200}C_{100}$ = 1.

Exponent of 59 in 200! = $\left[\frac{200}{59}\right]$ or 3.

Exponent of 59 in 100! = $\left[\frac{100}{59}\right]$ or 1.

Hence exponent of 59 in ${}^{200}C_{100}$ = 1.

Thus 59 divides ${}^{200}C_{100}$. None of the numbers given options is largest divisor of ${}^{200}C_{100}$.

ASSERTION REASONING

Q.33 (A)

${}^{n}C_{r}=\frac{n(n-1)(n-2)...(n-r+1)}{r!}$.

As ${}^{n}C_{r}$ is an integer hence $n(n-1)(n-2)...(n-r+1)$ is divisible by r!.

Q.34 (A)

${}^{40}C_{r}{}^{60}C_{0}+{}^{40}C_{r-1}{}^{60}C_{1}+...+{}^{40}C_{0}{}^{60}C_{r}$ = coefficient of x^r in the expansion of $(1 + x)^{40}(1 + x)^{60}$.

i.e. coefficient of x^r in $(1 + x)^{100}$.
i.e. $^{100}C_r$.
Now ^nC_r is maximum for r = n/2 if n is even, hence r = 50.

Q.35 (D)

Statement – 2 is a standard result.
By the result given in statement – 2, number of solutions of $x_1 + x_2 + x_3 + ... + x_{20} = 100$ is $^{100+20-1}C_{20-1}$ or $^{119}C_{19}$ hence statement – 1 is false.

Q.36 (A)

Statement – 2 is a standard result (Using $^nC_r + {^nC_{r+1}} = {^{n+1}C_{r+1}}$).
For statement – 1

$$S = {^nC_m} + 2\,{^{n-1}C_m} + 3\,{^{n-2}C_m} + ... + (n-m+1)\,{^mC_m}$$

Using $^nC_r = {^{n+1}C_{r+1}} - {^nC_{r+1}}$

$$\Rightarrow S = {^{n+1}C_{m+1}} - {^nC_{m+1}} + 2\,{^nC_{m+1}} - 2\,{^{n-1}C_{m+1}} + 3\,{^{n-1}C_{m+1}} - 3\,{^{n-2}C_{m+1}} + ... + (n-m+1)\,{^{m+1}C_{m+1}} - (n-m+1)\,{^mC_{m+1}}$$

$$\Rightarrow S = {^{n+1}C_{m+1}} + {^nC_{m+1}} + {^{n-1}C_{m+1}} + {^{n-2}C_{m+1}} + ... + {^{m+1}C_{m+1}}$$

$$\Rightarrow S = {^{n+2}C_{m+1}}.$$

Q.37 (D)

Consider the following arrangement.

1		2		3		4		5		6	
	7				8						9

Here f(1) = 7, f(2) = f(3) = 8 & f(4) = f(5) = f(6) = 9
Clearly any such arrangement can be made by putting 1,2,3,4,5,6 at a gap each in on column(domain) and choosing 3 gaps to put 7, 8, 9 in second column(codomain) such that each element of codomain will be the value of function for all elements of domain on left till the previous element of codomain

1		2		3		4		5		6	

Also 9 must be put in last cell only as f(6) has to be the greatest.
Hence we have to chose 2 out of 5 gaps to put 7 & 8.
Total number of functions $^5C_2$ i.e. 10.
Statement 1 is false.
Statement 2 is clearly correct.

MATRIX MATCH TYPE

Q.38 $(A)\to(Q),(B)\to(S),(C)\to(S),(D)\to(P)$

(A) Let the children be denoted as $C_1, C_2, C_3, C_4, C_5, C_6$ in increasing order of heights, where C_1 is the shortest child.
If C_6 is placed at first place, only arrangements possible is descending. Similarly for C_6 standing at last place only ascending order is possible. This can be done in 2 ways.
If C_6 is placed at second place, first place can be filled in 5 ways & rest in only descending order. Similarly for C_6 standing at second last place, last place can be filled in 5 ways & rest only in descending order. This can be done in 2×5 ways.
If C_6 is placed at second place, first & second place can be filled by any two children in 1 way & rest in only descending order. Similarly for C_6 standing at third from last place, last two places can be filled by any two of the remaining five in 1 ways & rest only in descending order. This can be done in $2\times {}^5C_2$ ways.
Hence all possible arrangements are 32.

(B) The children must be arranged such that C_1, C_2, C_3 are put in one order only(not to be arranged with each other.
Hence all possible arrangements are $\frac{6!}{3!}=120$

(C) Let every child get 1 marble. Now remaining 4 marbles must be distributed such that nobody gets more that 3 marbles. Number of ways in which all 4 marbles can be given to one child in 6 ways and in general 4 marbles can be given to 6 boys in ${}^{4+6-1}C_{6-1}$ i.e. 126 ways.
Hence required number of ways = 126 – 6 or 120.

(D) C_6 must be put in first row of one column. Chose two more in ${}^5C_2$ ways to put in the column in which C_6 is in 1 way. The other three can be put in the other column in just 1 way.
Hence all possible arrangements are 20.

Q.39 $(A)\to(R),(B)\to(R),(C)\to(Q),(D)\to(P)$

(A) Number of subsets of set $\{x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ = 2^7 or 128.

(B) Number of subsets of $\{x_1, x_2, x_3, ..., x_{10}\}$ which necessarily contain $\{x_1, x_2, x_3\}$ will also be same as that in (A) as after including $\{x_1, x_2, x_3\}$ rest of the elements have to be chosen from $\{x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ in 2^7 or 128 ways.

(C) Total number of subsets = 2^{10}.

Number of subsets not containing any of $\{x_1, x_2, x_3\}$ = 2^7.

Number of subsets containing at least one of $\{x_1, x_2, x_3\}$ = $2^{10} - 2^7$ or 896.

(D) Number of subset containing exactly one of $\{x_1, x_2, x_3\}$ = ${}^3C_1 \times 2^7$.

Number of subsets not containing any of $\{x_1, x_2, x_3\}$ = 2^7.

Number of subsets containing at the most one of $\{x_1, x_2, x_3\}$ = ${}^3C_1 \times 2^7 + 2^7$ or 512.

Q.40 $(A) \to (P), (B) \to (S), (C) \to (Q), (D) \to (Q)$

(A)

ENDEA	N	O	E	L

Total ways to permute = 5!.

(B)

E								E

To fill remaining 7 places N-2, DAOEL-1 each.

Number of permutations = $\frac{7!}{2!}$ or $21 \times 5!$.

(C)

Arrange DLNN in first 4 places in $\frac{4!}{2!}$ ways and EEEAO in last 5 places in $\frac{5!}{3!}$ ways.

Number of permutations = $\frac{4! \times 5!}{2! \times 3!}$ or $2 \times 5!$.

(D)

AEEEO must be put in 1^{st}, 3^{rd}, 5^{th}, 7^{th}& 9^{th} place only. Number of ways to do so $\frac{5!}{3!}$.

Now DNNL are to be put in rest of the places in $\frac{4!}{2!}$.

Number of permutations = $\frac{4! \times 5!}{2! \times 3!}$ or $2 \times 5!$.

PERMUTATIONS & COMBINATIONS

Exercise 2(C)

Q.1 [8]

The 2 specific persons can be allotted a seat in $2\times2\times2$ ways (2 ways to chose side, 2 to chose adjacent seats & 2 to arrange them).
Now remaining 4 seats can be allotted in ${}^{5}P_{4}$ ways.
Number of possible seating arrangements = $2\times2\times2\times{}^{5}P_{4}$ i.e. $8\times5!$.

Q.2 [8]

Number of ways in which 'r' people can be selected out of 'n' people sitting in a row, if no two of them are consecutive = ${}^{n-r+1}C_{r}$.
Hence $P_{n}={}^{n-2}C_{2}$.
Now ${}^{n-1}C_{3}-{}^{n-2}C_{3}=15\Rightarrow(n-2)(n-3)=45$ or $n=8$.

Q.3 [52]

Possible digits can be{ 3,3,2}, {3,2,2} or {2,3,x} where x can be any of {0,1,4,5,6,7,8,9}.
In first two cases number of numbers will be 3 each.
In third case if 0 is not taken, then number of numbers will be $7\times3!$.
If 0 is taken as one digit then number of numbers will be 2×2.
Total number of numbers = 6 + 42 + 4 = 52.

Q.4 [35]

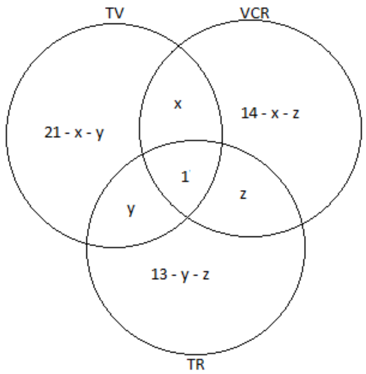
	+		+		+		+		+		+	

Put the 6 '+' signs at a gap each. Any 4 gaps out of these 7 gaps can be selected in ${}^{7}C_{4}$ ways to put the 4 '–' signs.
Hence number of arrangements = ${}^{7}C_{4}$ or 35.

Q.5 [8]

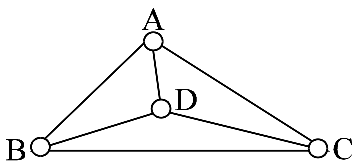
For the first letter we have 3 choices A, B & C and for rest of the places 2 choices each.
Hence an n – lettered word can be formed in $3\times2^{n-1}$ ways.
Now $3\times2^{n-1}$ = 384 gives n = 8.

Q.6 [10]



In adjoining Venn diagram
x + y + z = 9
Number of teachers who own none of three =
50 – (21 – x – y) – (14 – x – z) – (13 – y – z) – x – y – z – 1.
= 50 – (49 – x – y – z) = 10.

Q.7 [12]



To move ahead from A – 3 choices.
To move ahead of B/C/D – 2 choices.
To move ahead from next point – 2 choices.
Hence total number of ways $3\times2\times2$ or 12 ways.

Q.8 [21]

One Green ball (GRRRRR) – 6 ways
Two Green balls (GGRRRR) – 5 ways
Three Green balls (GGGRRR) – 4 ways
Four Green balls (GGGGRR) – 3 ways
Five Green balls (GGGGGR) – 2 ways
Six Green balls (GGGGGG) – 1 way.
All possible ways to fix 6 boxes = 1 + 2 + 3 + …+ 6 = 21.

Q.9 [82]

Exponent of 13 in 1000! = $\left[\frac{1000}{13}\right]+\left[\frac{1000}{13^2}\right]=76+5$ or 81.

Hence if $\frac{1000!}{13^n}$ is not an integer, then n must be at least 82.

Q.10 [36]

Rectangles of size 1×2 = 8,
Rectangles of size 1×3 = 7,
Rectangles of size 1×4 = 6,
⋮
Rectangles of size 1×9 = 1.
Total number of non – congruent rectangles = 1 + 2 + 3 + … + 8 = 36.

Q.11 [23]

7056 = $2^4 \times 3^2 \times 7^2$, hence 7056 has (4 + 1)(2 + 1)(2 + 1) or 45 divisors.
Thus we can write 7056 as a product of two factors in 23 ways.

Q.12 [10]

Number of subsets containing 3 elements in which 3 is the least element = ${}^{n-3}C_2$

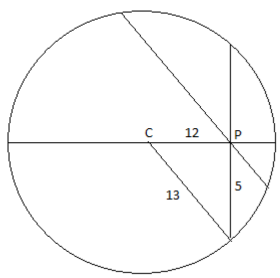
Number of subsets containing 3 elements in which 7 is the greatest element = ${}^{6}C_2$

Number of subsets containing 3 elements in which 3 is the least element as well as 7 is the greatest element = ${}^{3}C_2$

Hence number of subsets containing 3 elements in which 3 is the least element or is the greatest element = ${}^{n-3}C_2 + {}^{6}C_2 - 3$.

Now $\frac{(n-3)(n-4)}{2}+15-3=33 \Rightarrow n^2-7n-30=0$ or $n=10$.

Q.13 [32]



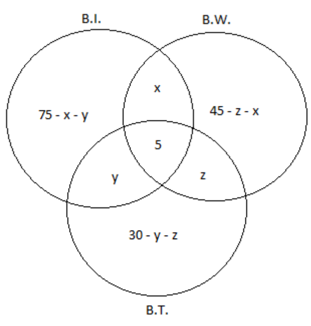
Longest possible chord will be the diameter through P, of length 26.
Shortest possible chord will be perpendicular to diameter through P, of length 10 units.
Possible lengths between 10 & 26 are 11, 12, 13, …, 25.
There will be two chords of each of these lengths and one chord each of length 10 & 26.Total number of chords = $2\times15+2$ or 32.

Q.14 [2]

${}^{20}C_0{}^{20}C_1 + {}^{20}C_1{}^{20}C_2 + \ldots + {}^{20}C_{19}{}^{20}C_{20}$ = coefficient of x^{19} in the expansion of $(1+x)^{20}(x+1)^{20}$.

= coefficient of x^{19} in $(1+x)^{40}$ = ${}^{40}C_{19}$ or ${}^{40}C_{21}$.As $r > 20$ hence $r = 21$.

Q.15 [50]



In adjoining Venn diagram
(75 – x – y) + (45 – z – x) + (30 – y – z) + x + y + z + 5 = 100.
Hence x + y + z + 5 = 50.

Q.16 [50]

Number of ways in which room A can be filled in $A_n = {}^nC_{25} \times 25!$ ways.

$$A_n - A_{n-1} = {}^{49}C_{25} \times 25!$$

$$\Rightarrow {}^nC_{25} - {}^{n-1}C_{25} = {}^{49}C_{25}$$

$$\Rightarrow \frac{n!}{25!(n-25)!} - \frac{(n-1)!}{25!(n-26)!} = \frac{49!}{25!24!}$$

$$\Rightarrow \frac{(n-1)!}{(n-25)!} = \frac{49!}{25!} \text{ or } n = 50$$

Q.17 [41]

108900 = $2^2 \times 3^2 \times 5^2 \times 11^2$.
Now number of divisors of 108900 = $(1 + 2)^4$ or 81.
Hence number of ways to write 108900 as a product of two factors = 41.

Q.18 [56]

Getting more than 25% marks means getting more than 3 marks out of maximum 12 marks.
(i) All 6 questions correct(12 marks) – 1 way (5 correct means all 6 correct).
(iii) 4 correct 2 wrong(6 marks) – ${}^6C_4$ ways.
(iv) 3 correct 3 wrong(3 marks) – ${}^6C_3 \times 2$ ways(3 wrong answers can be marked in 2 ways).
Total number of ways = 1 + 15 + 40 or 56.

Q.19 [9]

To form a 'k' letter palindrome we need to chose only 1^{st}, 2^{nd}& 3^{rd} letter each in k ways.
4^{th}& 5^{th} digits must be same as 1^{st}, 2^{nd} digits.
Hence number of ways to form a palindrome = k^3.
$k^3 = 729 \Rightarrow k = 9$.

Q.20 [15]

Let there be n players.
2 players who withdraw after 3 games will play 6 matches.
Remaining (n – 2) players will play ${}^{n-2}C_2$ matches.
Hence ${}^{n-2}C_2 + 6 = 84$ or $n^2 - 5n - 150 = 0$.
Therefore n = 15.

Only One Option Correct

1. (C)

2. (C)

3. (B)

4. (A)

(a) Total number of ways of arranging the letters of the word BANANA is $\frac{6!}{2!3!} = 60$. Number of words in which 2 N's come together is $\frac{5!}{3!} = 20$

$\therefore$ the required number $= 60 - 20 = 40$

5. (B)

6. (C)

7. (C)

(c) The letter of word *COCHIN* in alphabetic order are C, C, H, I, N, O.

Fixing first and second letter as C, C, rest 4 can be arranged in 4! ways.

Similarly the words starting with each of CH, CI, CN are 4!

Then fixing first two letters as CO and next four places when filled in alphabetic order with remaining 4 letters give the word *COCHIN*.

$\therefore$ Numbers of words coming before *COCHIN*

$= 4 \times 4! = 4 \times 24 = 96$

8. (C)

(c) We have to form 7 digit numbers, using the digits 1, 2 and 3 only, such that the sum of the digits in a number = 10.

This can be done by taking 2, 2, 2, 1, 1, 1, 1, or by taking 2, 3, 1, 1, 1, 1, 1.

$\therefore$ Number of ways $= \frac{7!}{3!4!} + \frac{7!}{5!} = 77.$

9. (C)

(c) $\because$ Card numbered 1 is always placed in envelope numbered 2, we can consider two cases :

Case I: Card numbered 2 is placed in envelope numbered 1. Then it is dearrangement of 4 objects, which can be done in

$$4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) = 9 \text{ ways}$$

Case II: Card numbered 2 is not placed in envelope numbered 1. Then it is dearrangement of 5 objects, which can be done in

$$5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) = 44 \text{ ways}$$

$\therefore$ Total ways = 44 + 9 = 53

10. (A)

(a) Either one boy will be selected or no boy will be selected. Also out of four members one captain is to be selected.

$\therefore$ Required number of ways = $({}^4C_1 \times {}^6C_3 + {}^5C_4) \times {}^4C_1$

$= (80 + 15) \times 4 = 380$

Integer Value Answer / Non-Negative Integer

11. (7)

(7) $\because n_1, n_2, n_3, n_4$ and n_5 are positive integers such that $n_1 < n_2 < n_3 < n_4 < n_5$

Then for $n_1 + n_2 + n_3 + n_4 + n_5 = 20$

If n_1, n_2, n_3, n_4 take minimum values 1, 2, 3, 4 respectively then n_5 will be maximum 10.

$\therefore$ Corresponding to $n_5 = 10$, there is only one solution $n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 4$.

Corresponding to $n_5 = 9$, we can have, only one solution $n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 5$ i.e., one solution

Corresponding to $n_5 = 8$, we can have, only solution

$n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 6$

or $\quad n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 5$

i.e., 2 solution

For $n_5 = 7$, we can have

$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6$

or $\quad n_1 = 1, n_2 = 3, n_3 = 4, n_4 = 5$

i.e. 2 solutions

For $n_5 = 6$, we can have

$n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5$

i.e., one solution

Thus there can be 7 solutions.

12. (5)

(5) Here, __ B_1 __ B_2 __ B_3 __ B_4 __ B_5

Out of 5 girls, 4 girls are together and 1 girl is separate.

Now, to select 2 positions out of 6 positions between boys $= {}^6C_2$ .... (i)

4 girls are to be selected out of 5= ${}^5C_4$ .... (ii)

Now, 2 groups of girls can be arranged in 2! ways .... (iii)

Also, the group of 4 girls and 5 boys is arranged in 4! × 5! ways. .... (iv)

Now, total number of ways $= {}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!$

[from Eqs. (i), (ii), (iii) and (iv)]

$\therefore \quad m = {}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!$

and $n = 5! \times 6!$

$$\Rightarrow \quad \frac{m}{n} = \frac{{}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!}{6! \times 5!} = \frac{15 \times 5 \times 2 \times 4!}{6 \times 5 \times 4!} = 5$$

13. (5)

(5) $\quad x = 10!$ and $y = {}^{10}C_1 \times {}^9C_8 \times \frac{10!}{2!} = 10 \times 9 \times \frac{10!}{2!}$

$$\therefore \quad \frac{y}{9x} = \frac{10 \times 9 \times \frac{10!}{2}}{9 \times 10!} = 5$$

14. (625)

(625) The last 2 digits, in 5-digit number divisible by 4, can be 12, 24, 32, 44 or 52.

Also each of the first three digits can be any of

$\{1, 2, 3, 4, 5\}$

$\therefore$ 5 options for each of the first three digits and total 5 options for last 2-digits

$\therefore$ Required number of 5 digit numbers are

$= 5 \times 5 \times 5 \times 5 = 625$

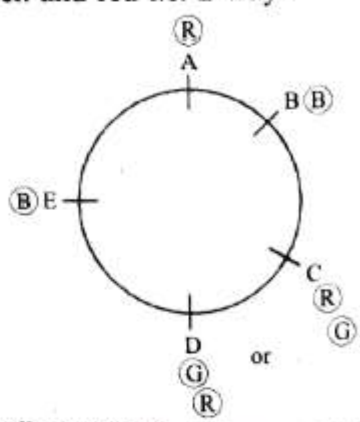
15. (30)

(30) 5 persons A, B, C, D and E are seated in circular arrangement.

Let A be given red hat, then there will be two cases.

Case I : B and E have same coloured hat blue/green. Say B and E have blue hat.

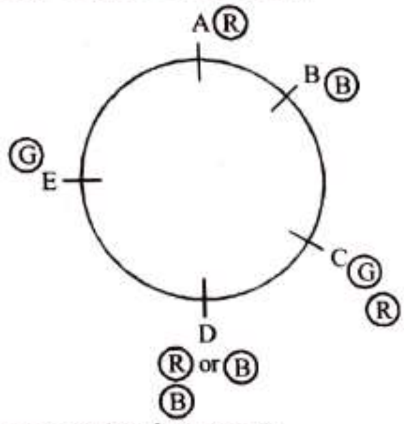
Then C and D can have either red and green or green and red i.e. 2 ways.



Similarly if B & E have green hat, there will be 2 ways for C & D.

Hence there are 2 + 2 = 4 ways.

Case II : B and E have different coloured hats blue and green or green and blue.



Let B has blue and E has green.
If C has green then D can have red or blue.
If C has red then D can have only blue.
$\therefore$ three ways.
Similarly 3 ways will be there when B has green and E has blue.
$\therefore$ there are 3 + 3 = 6 ways
On combining the two cases, there will be 4 + 6 = 10 ways
When similar discussion is repeated with A as blue and green hat, we get 10 ways for each.
Therefore, in all, there will be 10 + 10 + 10 = 30 ways

16. (1080)

$= 240 + 48 + 18 + 2 \ldots$

(1080) Groups can be possible in only 2, 2, 1, 1 way.
Number of ways of dividing persons in group

$$= \frac{6!}{(2!)^2(1!)^2(2!)^2}$$

Number of ways after arranging rooms $= \frac{6!}{(2!)^4} \cdot 4! = 1080$

17. (495.00)

(495.00) We know that total number of ways of selection of r days out of n days such that no two of them are consecutive $= {}^{n-r+1}C_r$.

$\therefore$ Selection of 4days out of 15 days such that no two of them are consecutive $= {}^{15-4+1}C_4 = {}^{12}C_4$

$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2} = 11 \times 5 \times 9 = 495$$

18. (569)

Thus there can be 7 solutions.

(569) Counting integers starting from 2

Case I: At unit's place we can fill 2/3/4/6/7

i.e., 2 0 2 [5] → 5 ways

At unit's place and ten's place we can fill digits as 3/4/6/7 and 0/2/3/4/6/7

or 2 0 [4] [6] → 24 ways

(Numbers except 0 or 2 in 3rd place)

Case II: If non-zero number on 2nd place

i.e., 2 [5] [6] [6] = 180 ways

Counting integers starting from 3

3 [6] [6] [6] = 216 ways

Counting integers starting from 4

Case I: If 0, 2 or 3 on 2nd place

i.e., 4 [3] [6] [6]

= 108 ways

Case II: If 4 on 2nd place

i.e., 4 4 [6] [6] = 36 ways

∴ Total 5 + 24 + 180 + 216 + 108 + 36 = 569 numbers

Match the Following

19. **(A) - P; (B) - S; (C) - Q; (D) - Q**

20. (C)

(c) Given 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5

(i) $\alpha_1 \rightarrow$ Total number of ways of selecting 3 boys and 2 girls from 6 boys and 5 girls

i.e., ${}^6C_3 \times {}^5C_2 = 20 \times 10 = 200 \quad \therefore \ \alpha_1 = 200$

(ii) $\alpha_2 \rightarrow$ Total number of ways selecting at least 2 member and having equal number of boys and girls

i.e., ${}^6C_1\,{}^5C_1 + {}^6C_2\,{}^5C_2 + {}^6C_3\,{}^5C_3 + {}^6C_4\,{}^5C_4 + {}^6C_5\,{}^5C_5$

$= 30 + 150 + 200 + 75 + 6 = 461 \Rightarrow \alpha_2 = 461$

= 30 + 150 + 200 + ...

(iii) $\alpha_3 \to$ Total number of ways of selecting 5 members in which at least 2 of them girls

i.e., ${}^5C_2\,{}^6C_3 + {}^5C_3\,{}^6C_2 + {}^5C_4\,{}^6C_1 + {}^5C_5\,{}^6C_0$

$= 200 + 150 + 30 + 1 = 381 \Rightarrow \alpha_3 = 381$

(iv) $\alpha_4 \to$ Total number of ways for selecting 4 members in which at least two girls such that M_1 and G_1 are not included together.

G_1 is included $\to {}^4C_1 \cdot {}^5C_2 + {}^4C_2 \cdot {}^5C_1 + {}^4C_3$

$= 40 + 30 + 4 = 74$

M_1 is included $\to {}^4C_2 \cdot {}^5C_1 + {}^4C_3 = 30 + 4 = 34$

G_1 and M_1 both are not included

${}^4C_4 + {}^4C_3 \cdot {}^5C_1 + {}^4C_2 \cdot {}^5C_2$

$1 + 20 + 60 = 81$

$\therefore$ Total number $= 74 + 34 + 81 = 189$

$\alpha_4 = 189$

Now, $P \to 4$; $Q \to 6$; $R \to 5$; $S \to 2$