EXERCISE - 1 [A]

1. (B)

Given $f(4)=6, f'(4)=1$

$$\therefore \lim_{x\to 4}\frac{xf(4)-4f(x)}{x-4}=\lim_{x\to 4}\frac{xf(4)-4f(4)+4f(4)-4f(x)}{x-4}$$

$$=\lim_{x\to 4}\frac{(x-4)f(4)}{x-4}-4\lim_{x\to 2}\frac{f(x)-f(4)}{x-4}$$

$$=f(4)-2f'(4)=4$$

2. (D)

$$f(x)=\begin{cases}x^3-1 & , \quad x\ge 1\\ 1-x^3 & , \quad x<1\end{cases} \text{ and } f'(x)=\begin{cases}3x^2 & , \quad x\ge 1\\ -3x^2 & , \quad x<1\end{cases}$$

$$f'(1^+)=3,\ f'(1^-)=-3$$

3. (B)

$$f(x)=\sin 2x.\cos 2x.\cos 3x+\log_2 2^{x+3}\ ,$$

$$\Rightarrow f(x)=\frac{1}{2}\sin 4x\cos 3x+(x+3)\log_2 2\ ,$$

$$\Rightarrow f(x)=\frac{1}{4}[\sin 7x+\sin x]+x+3$$

Differentiate w.r.t. x,

$$f'(x)=\frac{1}{4}[7\cos 7x+\cos x]+1\ ,$$

$$\Rightarrow f'(\pi)=-2+1=-1.$$

4. (B)

(b) In neighborhood of $x=\frac{3\pi}{4}$, $|\cos^3 x|=-\cos^3 x$ and $|\sin^3 x|=\sin^3 x$

$$\therefore\ y=-\cos^3 x+\sin^3 x$$

$$\therefore\ \frac{dy}{dx}=3\cos^2 x\sin x+3\sin^2 x\cos x$$

At $x=\frac{3\pi}{4}$, $\frac{dy}{dx}=3\cos^2\frac{3\pi}{4}\sin\frac{3\pi}{4}+3\sin^2\frac{3\pi}{4}\cos\frac{3\pi}{4}=0$.

5. (D)

$$f(x) = |\log x| = \begin{cases} -\log x, & \text{if } 0 < x < 1 \\ \log x, & \text{if } x \geq 1 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -\dfrac{1}{x}, & \text{if } 0 < x < 1 \\ \dfrac{1}{x}, & \text{if } x > 1 \end{cases}.$$

Clearly $f'(1^-) = -1$ and $f'(1^+) = 1$,

$\therefore$ $f'(x)$ does not exist at x = 1

6. (C)

$$\text{Let } y = \left[\log\left\{e^x\left(\frac{x-1}{x+1}\right)\right\}\right] = \log e^x + \log\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow y = x + [\log(x-1) - \log(x+1)]$$

$$\Rightarrow \frac{dy}{dx} = 1 + \left[\frac{1}{x-1} - \frac{1}{x+1}\right] = 1 + \frac{2}{(x^2-1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+1}{x^2-1}.$$

7. (A)

$$y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{dy}{dx} = 0$$

8. (D)

$$\frac{d}{dx}\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right] = \frac{d}{dx}\tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right] = -1$$

9. (A)

$$y=\frac{(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8)}{1-x}=\frac{1-x^{16}}{1-x}$$

$$\therefore \frac{dy}{dx}=\frac{-16x^{15}(1-x)+1-x^{16}}{(1-x)^2}, \quad \therefore \text{ At } x=0, \frac{dy}{dx}=1.$$

10. (A)

$$f(x)=\frac{2\sin x.\cos x.\cos 2x.\cos 4x}{2\sin x}=\frac{\sin 8x}{8\sin x}$$

$$\therefore f'(x)=\frac{1}{8}.\frac{8\cos 8x.\sin x-\cos x.\sin 8x}{\sin^2 x}$$

$$\therefore f'\left(\frac{\pi}{4}\right)=\sqrt{2}$$

11. (B)

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$$

$$=\frac{a[\cos\theta-\theta(-\sin\theta)-\cos\theta]}{a[-\sin\theta+\theta\cos\theta+\sin\theta]}=\frac{\theta\sin\theta}{\theta\cos\theta}=\tan\theta$$

12. (D)

Obviously $x=\cos^{-1}\frac{1}{\sqrt{1+t^2}}$ and $y=\sin^{-1}\frac{t}{\sqrt{1+t^2}}$

$\Rightarrow x=\tan^{-1}t$ and $y=\tan^{-1}t$

$\Rightarrow y=x \Rightarrow \frac{dy}{dx}=1$.

13. (C)

$x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$

Put $t=\tan\theta$ in both the equations to get

$x=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$ and $y=\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$.

Differentiating both the equations, we get $\frac{dx}{d\theta}=-2\sin 2\theta$ and $\frac{dy}{d\theta}=2\cos 2\theta$.

Therefore $\frac{dy}{dx}=-\frac{\cos 2\theta}{\sin 2\theta}=-\frac{x}{y}$.

14. (D)

$$y=\sqrt{x+1+\sqrt{x+1+\sqrt{x+1\ldots\text{to }\infty}}}\Rightarrow y=\sqrt{x+1+y}$$

$$\Rightarrow y^2=x+y+1\Rightarrow 2y\frac{dy}{dx}=1+\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y-1)=1\Rightarrow \frac{dy}{dx}=\frac{1}{2y-1}$$

15. (B)

$$y=(x+1)^{(x+1)^{(x+1)\ldots\infty}}\Rightarrow y=(x+1)^y$$

$$\Rightarrow \log_e y=y\log_e(x+1)$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\frac{y}{(x+1)}+\ln(x+1)\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y}-\ln(x+1)\right)\frac{dy}{dx}=\frac{y}{x+1}$$

$$\Rightarrow (x+1)(1-\ln y)\frac{dy}{dx}=y^2$$

16. (A)

$$\sqrt{1-x}+\sqrt{1-y}=1\Rightarrow y=x+2\sqrt{1-x}-1$$

$$\Rightarrow \frac{dy}{dx}=1-\frac{1}{\sqrt{1-x}}=\frac{\sqrt{1-x}-1}{\sqrt{1-x}}$$

17. (B)

for $f(x)$, $y=x+\ln x$

then for $f^{-1}(x)$, $x=y+\ln y$

$$\Rightarrow \frac{dx}{dy}=1+\frac{1}{y}\text{ or }\frac{dy}{dx}=\frac{y}{1+y}$$

Further $x+\ln x=1\Rightarrow x=1$, hence for $f^{-1}(x)$, $y=1$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{1+1}=\frac{1}{2}.$$

Ans.[B]

18. (A)

$$x = a\sec\theta \Rightarrow \frac{dx}{d\theta} = a\sec\theta\tan\theta,\; y = b\tan\theta \Rightarrow \frac{dy}{d\theta} = b\sec^2\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} = \frac{b^2x}{a^2y}$$ **Ans.[A]**

19. (C)

$$\frac{dx}{d\theta} = a\,(1+\cos),\; \frac{dx}{d\theta} = a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{1}{2}\theta$$ **Ans.[C]**

20. (A)

$$y = \log e^x - \log(e^x + 1)$$
$$= x - \log(e^x + 1)$$

$$\therefore \frac{dy}{dx} = 1 - \frac{e^x}{e^x+1} = \frac{1}{e^x+1}$$ **Ans.[A]**

21. (B)

$$y = \frac{\sec x - \tan x}{\sec x + \tan x}\cdot\frac{\sec x - \tan x}{\sec x - \tan x}$$
$$= (\sec x - \tan x)^2/\,1$$

$$\therefore \frac{dy}{dx} = 2(\sec x - \tan x)\,(\sec x\tan x - \sec^2 x)$$
$$= -2\sec x\,(\sec x - \tan x)^2$$ **Ans.[B]**

22. (C)

$$y = e^{x+y}$$

$$\Rightarrow \quad \log y = x + y \quad \Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{1-y}$$ **Ans.[C]**

23. (B)

When $x = 0$, $e^y = e \Rightarrow y = 1$

Differentiating w.r.t. x, we get

$$e^y \frac{dy}{dx} + y + x\frac{dy}{dx} = 0 \quad ....(1)$$

$$e^y \frac{d^2y}{dx^2} = e^y\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} = 0 \quad ....(2)$$

When $x = 0, y = 1$ $\quad \therefore$ From (1) $\frac{dy}{dx} = -\frac{1}{e}$

Putting the data in (2), we get

$$e \cdot \frac{d^2y}{dx^2} + e \cdot \frac{1}{e^2} - \frac{2}{e} = 0 \qquad \therefore \frac{d^2y}{dx^2} = \frac{1}{e^2}$$]

24. (A)

$$y = \tan^{-1}\left\{\sqrt{\frac{1+\cos x}{1-\cos x}}\right\}$$

$$= \tan^{-1}\left\{\sqrt{\frac{2\cos^2 x/2}{2\sin^2 x/2}}\right\}$$

$$= \tan^{-1}\left|\cot\frac{x}{2}\right| = \tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right\} = \frac{\pi}{2}-\frac{x}{2}$$

$$\therefore \frac{dy}{dx} = 0 - \frac{1}{2} = -\frac{1}{2}$$

25. (B)

$$f(x) = |x^2-5x+6| = \begin{cases} x^2-5x+6 & \text{if } x \ge 3 \text{ or } x \le 2 \\ -(x^2-5x+6), & \text{if } 2 < x < 3 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} (2x-5), & \text{if } x > 3 \text{ or } x < 2 \\ -(2x-5), & \text{if } 2 < x < 3 \end{cases}$$

26. (C)

$$y'(x) = f'(f(f(f(x))))f'(f(f(x)))f'f(x)f'(x)$$

$$\Rightarrow y'(0) = f'(f(f(f(0))))f'(f(f(0)))f'(f(0))f'(0)$$

$$= f'(f(f(0)))f'(f(0))f'(0)f'(0)$$

$$= f'(f(0))f'(0)f'(0)f'(0)$$

$$= f'(0)f'(0)f'(0)f'(0)$$
$$= (f'(0))^4 = 2^4 = 16$$

27. (C)

$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

$$\Rightarrow \frac{dy}{dx} = 0 + 1 + x + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!}$$

$$\Rightarrow \frac{dy}{dx} + \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}$$

$$\Rightarrow \frac{dy}{dx} = y - \frac{x^n}{n!}$$

28. (D)

7.d. $y = a\sin x + b\cos x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now, $\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2$

$= a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$, and

$y^2 = (a\sin x + b\cos x)^2$

$= a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$

So, $\left(\frac{dy}{dx}\right)^2 + y^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 + y^2 = (a^2 + b^2) = \text{constant.}$$

29. (B)

$$y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$$

$$\Rightarrow \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$$

30. (A)

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(x+\sqrt{x^2+a^2}\right)^n\right]$$

$$=n\left(x+\sqrt{x^2+a^2}\right)^{n-1}\cdot\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$=n(x+\sqrt{x^2+a^2})^{n-1}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a}}\right)$$

$$=\frac{n(x+\sqrt{x^2+a^2})^n}{\sqrt{x^2+a^2}}$$

$$=\frac{ny}{\sqrt{x^2+a^2}}$$

31. (B)

).b. $f(x)=\sqrt{1+\cos^2(x^2)}$

$$\Rightarrow f'(x)=\frac{1}{2\sqrt{1+\cos^2(x)^2}}(2\cos x^2)(-\sin x^2)(2x)$$

$$\Rightarrow f'(x)=\frac{-x\sin 2x^2}{\sqrt{1+\cos^2(x^2)}}$$

$$\Rightarrow f'\left(\frac{\sqrt{\pi}}{2}\right)=\frac{-\frac{\sqrt{\pi}}{2}\sin\frac{2\pi}{4}}{\sqrt{1+\cos^2\frac{\pi}{4}}}=\frac{-\frac{\sqrt{\pi}}{2}\cdot 1}{\sqrt{\frac{3}{2}}}$$

$$\therefore f'\left(\frac{\sqrt{\pi}}{2}\right)=-\sqrt{\frac{\pi}{6}}$$

32. (A)

$$\frac{d}{dx}\cos^{-1}\sqrt{\cos x}=\frac{\sin x}{2\sqrt{\cos x}\sqrt{1-\cos x}}$$

$$=\frac{\sqrt{1-\cos^2 x}}{2\sqrt{\cos x}\sqrt{1-\cos x}}=\frac{1}{2}\sqrt{\frac{1+\cos x}{\cos x}}$$

33. (C)

i.e. $y=\dfrac{\log \tan x}{\log \sin x}$

$$\Rightarrow \frac{dy}{dx}=\frac{(\log \sin x)\left(\dfrac{\sec^2 x}{\tan x}\right)-(\log \tan x)(\cot x)}{(\log \sin x)^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\pi/4}=\frac{-4}{\log 2} \qquad \text{(On simplification)}$$

34. (B)

i.b. $y=\dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$

$$\frac{dy}{dx}=\frac{\sqrt{1-x^2}\,\dfrac{1}{\sqrt{1-x^2}}-\left(\sin^{-1} x\right)\dfrac{1}{2}\dfrac{(-2x)}{\sqrt{1-x^2}}}{1-x^2}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx}=1+x\left(\frac{\sin^{-1} x}{\sqrt{1-x^2}}\right)=1+xy$$

35. (A)

$$y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$$

$$=\cot^{-1}\left[\frac{2+2\cos x}{2\sin x}\right]=\cot^{-1}\left[\frac{1+\cos x}{\sin x}\right]$$

$$=\cot^{-1}\left[\cot\frac{x}{2}\right]=\frac{x}{2}$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}$$

36. (C)

$$y=x^{\left(x^x\right)}$$

$$\Rightarrow \log y=x^x \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{dz}{dx}\log x+\frac{1}{x}z \ \ (\text{where } x^x=z)$$

$$\Rightarrow \frac{dy}{dx}=x^{\left(x^x\right)}\left[x^x(\log ex)\log x+x^{x-1}\right] \qquad \left(\because \frac{dz}{dx}=x^x\log ex\right)$$

38. (C)

$$y = ae^{mx} + be^{-mx}$$

$$\Rightarrow \frac{dy}{dx} = ame^{mx} - mbe^{-mx}$$

Again $\frac{d^2y}{dx^2} = am^2 e^{mx} + m^2 be^{-mx}$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2\left(ae^{mx} + be^{-mx}\right) \Rightarrow \frac{d^2y}{dx^2} = m^2 y$$

$$\Rightarrow \frac{d^2y}{dx^2} - m^2 y = 0$$

EXERCISE - 1 [B]

1. (C)

$f(x+2y)=2f(x)f(y) \Rightarrow 2f'(x+2y)=2f(x)f'(y)$ {partially differentiating w.r.to y}

For x = 5 & y = 0, $f'(5)=f(5)f'(0) \Rightarrow f'(5)=6$

2. (C)

By L'hospital's rule

$$\lim_{x\to 2}\frac{g^2(x)f^2(2)-f^2(x)g^2(2)}{x^2-4}=\lim_{x\to 2}\frac{g(x)g'(x)f^2(2)-f(x)f'(x)g^2(2)}{x}$$

$$=\frac{(-1)\times 4\times 9-3\times(-2)\times 1}{2}=-15$$

3. (B)

Given $5f(2x)+3f\left(\frac{2}{x}\right)=2x+2$ ......(i)

Replacing x by $\frac{1}{x}$ in (i), $5f\left(\frac{2}{x}\right)+3f(2x)=\frac{2}{x}+2$ ......(ii)

On solving equation (i) and (ii), we get, $8f(2x)=5x-\frac{3}{x}+2$,

$$\Rightarrow 8f(x)=\frac{5x}{2}-\frac{6}{x}+2$$

$$\therefore\ 8f'(x)=\frac{5}{2}+\frac{6}{x^2}$$

$$\because\ y=xf(x)\ \Rightarrow \frac{dy}{dx}=f(x)+xf'(x)$$

$$=\frac{1}{8}\left(\frac{5x}{2}-\frac{6}{x}+2\right)+\frac{x}{8}\left(\frac{5}{2}+\frac{6}{x^2}\right)$$

at $x=1$, $\frac{dy}{dx}=\frac{1}{8}\left(\frac{5}{2}-6+2\right)+\frac{1}{8}\left(\frac{5}{2}+6\right)=\frac{7}{8}$

4. (A)

$$x = \exp\left\{\tan^{-1}\left(\frac{y-x}{x}\right)\right\} \Rightarrow \log x = \tan^{-1}\left(\frac{y-x}{x}\right)$$

$$\Rightarrow \frac{y-x}{x} = \tan(\log x) \Rightarrow y = x\tan(\log x) + x$$

$$\Rightarrow \frac{dy}{dx} = \tan(\log x) + x\frac{\sec^2(\log x)}{x} + 1$$

$$\Rightarrow \frac{dy}{dx} = \tan(\log x) + \sec^2(\log x) + 1$$

At $x = 1$, $\frac{dy}{dx} = 2$.

5. (B)

Let $y = \sin^2\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$$\Rightarrow y = \sin^2\left(\cot^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right) = \sin^2\left(\cot^{-1}\left(\tan\frac{\theta}{2}\right)\right)$$

$$\Rightarrow y = \sin^2\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos^2\frac{\theta}{2} = \frac{1}{2}(1+\cos\theta) = \frac{1}{2}(1+x)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}$$

6. (A)

Let $\cos\alpha = \frac{5}{13}$. Then $\sin\alpha = \frac{12}{13}$. So, $y = \cos^{-1}\{\cos\alpha.\cos x - \sin\alpha.\sin x\}$

$\therefore y = \cos^{-1}\{\cos(x+\alpha)\} = x + \alpha$ ($\because$ $x+\alpha$ is in the first or the second quadrant)

$$\therefore \frac{dy}{dx} = 1.$$

7. (C)

$$y\left(\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x\tan^2 x}\right)\cot 3x = \left(\frac{\tan 2x - \tan x}{1 + \tan 2x\tan x}\right)\left(\frac{\tan 2x + \tan x}{1 - \tan 2x\tan x}\right)\cot 3x$$

$$\Rightarrow y = \tan x\tan 3x\cot 3x = \tan x$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 x$$

8. (A)

$$f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$$

Put $x^x=\tan\theta$, $\therefore\ y=f(x)=\cot^{-1}\left(\frac{\tan^2\theta-1}{2\tan\theta}\right)$

$= \cot^{-1}(-\cot 2\theta)\ = \pi-\cot^{-1}(\cot 2\theta)$

$\Rightarrow y\ =\ \pi-2\theta\ \ =\pi-2\tan^{-1}(x^x)$

$\Rightarrow \frac{dy}{dx}=\frac{-2}{1+x^{2x}}.x^x(1+\log x)$

$\Rightarrow f'(1)=-1$.

9. (A)

$xe^{x+y}=y+2\sin x \Rightarrow e^{x+y}+xe^{x+y}(1+y')=y'+2\cos x$

Now $x=0$ gives $y=0$, hence $\frac{dy}{dx}=-1$.

10. (D)

$\sin(3x-2y)=\log(3x-2y)\Rightarrow\left(3-2\frac{dy}{dx}\right)\cos(3x-2y)=\left(3-2\frac{dy}{dx}\right)\frac{1}{3x-2y}$

$\Rightarrow \frac{dy}{dx}=\frac{3}{2}$

11. (C)

$x^4y^5=2(x+y)^9\Rightarrow 4x^3y^5+5x^4y^4\frac{dy}{dx}=18(x+y)^8\left(1+\frac{dy}{dx}\right)$

$\Rightarrow 4\frac{2(x+y)^9}{x}+5\frac{2(x+y)^9}{y}\frac{dy}{dx}=18(x+y)^8\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{4}{x}-\frac{9}{x+y}=\left(\frac{9}{x+y}-\frac{5}{y}\right)\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$

12. (A)

$$y = x^2 + \frac{2}{y} \Rightarrow y^2 = x^2y + 2$$

$$\Rightarrow 2y\frac{dy}{dx} = y.2x + x^2\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy}{2y - x^2}$$

13. (C)

$$x = e^{2y+x}$$

Taking log both sides, $\log x = (2y+x)\log e = 2y+x$

$$\Rightarrow 2y + x = \log x \Rightarrow 2\frac{dy}{dx} + 1 = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{1-x}{2x}$$

14. (C)

$$x = \ln\left(y + \sqrt{1+y^2}\right) \Rightarrow \sqrt{1+y^2} + y = e^x \ \& \ \sqrt{1+y^2} - y = e^{-x}$$

$$\Rightarrow y = \frac{e^x - e^{-x}}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x + e^{-x}}{2} = \frac{e^{2x}+1}{2e^x}$$ **Ans.[C]**

15. (B)

for $x > \frac{1}{2}$, $\sin^{-1}\left(3x - 4x^3\right) = \pi - 3\sin^{-1}x$

Now $y = \pi - 3\sin^{-1}x \Rightarrow \frac{dy}{dx} = -\frac{3}{\sqrt{1-x^2}}$ **Ans.[B]**

16. (C)

$$\frac{dy}{dx} = \frac{-2x}{(x^2-a^2)^2} \Rightarrow \frac{d^2y}{dx^2}$$

$$= -\frac{(x^2-a^2)^2.2 - 2x.2(x^2-a^2).2x}{(x^2-a^2)^4}$$

$$= \frac{2(3x^2+a^2)}{(x^2-a^2)^3}$$ **Ans.[C]**

17. (B)

Let us first express y in terms of x because all alternatives are in terms of x. So

$x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2 (1+y) = y^2 (1+x)$

$\Rightarrow x^2 - y^2 + x^2y - y^2 x = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow x + y + xy = 0 \qquad (\because x \neq y)$

$\Rightarrow y = -\frac{x}{1-x}$

$\therefore \frac{dy}{dx} = -\frac{(1+x)1 - x.1}{(1+x)^2} = -\frac{1}{(1+x)^2}$ **Ans.[B]**

18. (D)

Taking log on both sides, we have

$y \log x + x \log y = 0$

Now using partial derivatives, we have

$\frac{dy}{dx} = -\frac{y/x + \log y}{\log x + x/y} = -\frac{y(y + x\log y)}{x(x + y\log x)}$ **Ans [D]**

19. (B)

Here $y = \sqrt{\sin x + y} \quad \Rightarrow y^2 = \sin x + y$

$\therefore 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$ **Ans.[B]**

20. (C)

$\lim\limits_{x\to 0} x^x = 1$; let $l = x^{x^x}$ hence as $x \to 0, x^x \to 1$

$\therefore \quad L = (0)' - 1 = -1 \quad \Rightarrow \quad (C)$

21. (D)

$f(x) = \frac{\ln(\ln x)}{\ln x}$

Now use Quotient Rule.

22. (D)

$y^4 = x^2 - 6$

$$4y^3\frac{dy}{dx} = 2x \Rightarrow y^3\frac{dy}{dx} = \frac{x}{2} \Rightarrow y^3\frac{d^2y}{dx^2} + 3y^2\left(\frac{dy}{dx}\right)^2 = \frac{1}{2}$$

$$y^3\frac{d^2y}{dx^2} + 3y^2\left(\frac{x}{2y^3}\right)^2 = \frac{1}{2} \Rightarrow y^3\frac{d^2y}{dx^2} + 3y^2\frac{x^2}{4y^6} = \frac{1}{2} \Rightarrow y^3\frac{d^2y}{dx^2} + \frac{3x^2}{4y^4} = \frac{1}{2}$$

$$\Rightarrow y^3\frac{d^2y}{dx^2} = \frac{1}{2} - \frac{3x^2}{4y^4} \Rightarrow y^3\frac{d^2y}{dx^2} = \frac{2y^4 - 3x^2}{4y^4} \Rightarrow \frac{d^2y}{dx^2} = \frac{2y^4 - 3x^2}{4y^7}$$

23. (C)

Let $f(x) = y \Rightarrow x = f^{-1}(y) = g(y) \Rightarrow x = e^{e^y}$

$$\Rightarrow \frac{dx}{dy} = e^{e^y} \cdot e^y = e^{e^y + y} = g'(y)$$

hence $g'(x) = e^{e^x + x}$

24. (D)

Let $y = \log x$

$$\Rightarrow y_1 = \frac{1}{x}, y_2 = \frac{-1}{x^2}, y_3 = \frac{2}{x^3}, \dots, y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}.$$

25. (C)

i.e. $y = \sqrt{\log x + y}$

$$\Rightarrow y^2 = \log x + y$$

$$\Rightarrow 2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{x(2y-1)}$$

26. (C)

$f(\log_e x) = \log_e(\log_e x)$

$$\frac{df(\ln x)}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x}$$

27. (A)

$$y=\sec(\tan^{-1}x)=\sec\left(\sec^{-1}\sqrt{1+x^2}\right)=\sqrt{1+x^2}$$

Differentiating w.r.t. x, we have $\frac{dy}{dx}=\frac{x}{\sqrt{1+x^2}}$

$$\Rightarrow\left(\frac{dy}{dx}\right)_{x=1}=\frac{1}{\sqrt{2}}$$

28. (A)

$$y=f(x^2)\Rightarrow\frac{dy}{dx}=f'(x^2)2x=2x\sqrt{2(x^2)^2-1}$$

At $x=1$, $\frac{dy}{dx}=2\times1\times\sqrt{2-1}=2$

29. (A)

$$\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{f'(x^3)3x^2}{g'(x^2)2x}=\frac{\cos x^3 3x^2}{\sin x^2 2x}$$

$$=\frac{3}{2}x\cos x^3\operatorname{cosec}x^2$$

30. (B)

24.b. $\frac{dx}{dy}=\frac{\frac{dx}{dt}}{\frac{dy}{dt}}=\frac{\cos t-t\sin t}{1+\cos t}$

$$\therefore\ \frac{d^2x}{dy^2}=\frac{\frac{d}{dt}\left(\frac{dx}{dy}\right)}{\frac{dy}{dt}}$$

$$=\frac{\frac{(-2\sin t-t\cos t)(1+\cos t)-(\cos t-t\sin t)(-\sin t)}{(1+\cos t)^2}}{1+\cos t}$$

Now, put $t=\pi/2$

31. (C)

.c. $f(x)=\sqrt{1-\sin 2x}=\sqrt{(\cos x-\sin x)^2}$

$=|\cos x-\sin x|$

$=\begin{cases}\cos x-\sin x, & \text{for } 0\le x\le \pi/4\\ -(\cos x-\sin x), & \text{for } \pi/4<x\le \pi/2\end{cases}$

$\therefore f'(x)=\begin{cases}-(\cos x+\sin x), & \text{for } 0<x<\pi/4\\ (\cos x+\sin x), & \text{for } \pi/4<x<\pi/2.\end{cases}$

32. (D)

!6.d. Let $u=y^2$ and $v=x^2$

$\therefore \frac{du}{dx}=\frac{d}{dx}y^2=\left(\frac{d}{dy}y^2\right)\left(\frac{dy}{dx}\right)$

$=2y(1-2x)=2(x-x^2)(1-2x)=2x(1-x)(1-2x)$ (1)

and $\frac{dv}{dx}=2x$ (2)

Hence, $\frac{du}{dv}=\frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}=\frac{2x(1-x)(1-2x)}{2x}$ (from (1) and (2))

$=(1-x)(1-2x)=1-3x+2x^2$

33. (A)

!.a. $f(x)=\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}\right)\right]+x^x=\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}+x^x$

$\Rightarrow f'(x)=-\frac{1}{\sqrt{2}}\times\frac{1}{2\sqrt{1+x}}+x^x(1+\log x)$

$\Rightarrow f'(1)=-\frac{1}{4}+1=\frac{3}{4}$

34. (B)

$2xf'(x^2)=3x^2 \Rightarrow 4f'(4)=12 \Rightarrow f'(4)=3$

35. (C)

$(a^2-2a-15)e^{ax}+(b^2-2b-15)e^{bx}=0$

$\Rightarrow (a^2-2a-15)=0$ and $b^2-2b-15=0$

$\Rightarrow (a-5)(a+3)=0$ and $(b-5)(b+3)=0$

$\Rightarrow a=5$ or -3 and $b=5$ or -3

$\therefore a\neq b$ hence $a=5$ and $b=-3$

or $a=-3$ and $b=5$

$\Rightarrow ab=-15$

36. (B)

b. $y=\dfrac{(a-x)^{3/2}+(x-b)^{3/2}}{\sqrt{a-x}+\sqrt{x-b}}$

$$=\frac{\left(\sqrt{a-x}+\sqrt{x-b}\right)\left(a-x-\sqrt{a-x}\sqrt{x-b}+x-b\right)}{\sqrt{a-x}+\sqrt{x-b}}$$

$$=a-b-\sqrt{a-x}\sqrt{x-b}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{a-x}}\sqrt{x-b}-\frac{1}{2\sqrt{x-b}}\sqrt{a-x}$$

$$=\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$$

37. (B)

2.b. $f(g(x))=x$

$\Rightarrow f'(g(x))g'(x)=1$

$\Rightarrow (e^{g(x)}+1)g'(x)=1$

$\Rightarrow (e^{g(f(\log 2))}+1)g'(f(\log 2))=1$

$\Rightarrow (e^{\log 2}+1)g'(f(\log 2))=1$

$\Rightarrow g'(f(\log 2))=1/3$

38. (C)

.c. $f'(x)=(kx+e^x)h'(x)+h(x)(k+e^x)$

$f'(0)=h'(0)+h(0)(k+1)$

1. **(B)**

Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

$f'(1) = 30 - 56 + 30 - 63 + 6$

$= 66 - 63 - 56 = -53$

Consider $\lim\limits_{\alpha \to 0} \dfrac{f(1-\alpha) - f(1)}{\alpha^3 + 3\alpha}$

$= \lim\limits_{\alpha \to 0} \dfrac{f'(1-\alpha)(-1) - 0}{3\alpha^2 + 3}$ (By using L'Hospital rule)

$= \dfrac{f'(1-0)(-1)}{3(0)^2 + 3} = \dfrac{-f'(1)}{3} = \dfrac{53}{3}$

2. **(B)**

Let f(x) = $\tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^3}\right)$ where $x \in \left(0, \dfrac{1}{4}\right)$.

$= \tan^{-1}\left(\dfrac{2.\left(3x^{3/2}\right)}{1-\left(3x^{3/2}\right)^2}\right) = 2\tan^{-1}(3x^{3/2})$

As $3x^{3/2} \in \left(0, \dfrac{3}{8}\right)$ $\left[\because 0 < x < \dfrac{1}{4} \Rightarrow 0 < x^{3/2} < \dfrac{1}{8} \Rightarrow 0 < 3x^{3/2} < \dfrac{3}{8}\right]$

So $\dfrac{dx\, f(x)}{dx} = 2 \times \dfrac{1}{1+9x^3} \times 3 \times \dfrac{3}{2} \times x^{1/2} = \dfrac{9}{1+9x^3}\sqrt{x}$

On comparing

$\therefore g(x) = \dfrac{9}{1+9x^3}$

3. **(D)**

$y = \left[x + \sqrt{x^2-1}\right]^{15} + \left[x - \sqrt{x^2-1}\right]^{15}$

Differentiate w.r.t. 'x'

$\dfrac{dy}{dx} = 15\left(x + \sqrt{x^2-1}\right)^{14}\left[1 + \dfrac{x}{\sqrt{x^2-1}}\right]$

$= 15\left(x - \sqrt{x^2-1}\right)^{14}\left[1 - \dfrac{x}{\sqrt{x^2-1}}\right]$

$\Rightarrow \quad \frac{dy}{dx} = \frac{15}{\sqrt{x^2-1}}.y$ ...(i)

$\Rightarrow \quad \sqrt{x^2-1}.\frac{dy}{dx} = 15y$

Again differentiating both sides w.r.t. x

$$\frac{x}{\sqrt{x^2-1}}.\frac{dy}{dx} + \sqrt{x^2-1}\frac{d^2y}{dx^2} = 15\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} + \left(x^2-1\right)\frac{d^2y}{dx^2}$$

$$= \quad 15\sqrt{x^2-1}.\frac{15}{\sqrt{x^2-1}}.y = 225y$$

4. (D)

$\because$ f(x) has extremum values of $x = 1$ and $x = 2$

$\because$ $f'(1) = 0$ and $f'(2) = 0$

As, f(x) is a polynomial of degree 4.

Suppose $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

$$\because \quad \lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{Ax^4+Bx^3+Cx^2+Dx+E}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(Ax^2+Bx+C+\frac{D}{x}+\frac{E}{x^2}+1\right) = 3$$

As limit has finite value, so $D = 0$ and $E = 0$

Now $A(0)^2 + B(0) + C + 0 + 0 + 1 = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$

$f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$f'(1) = 0 \Rightarrow 4A(1) + 3B(1) + 2C(1) + D = 0$

$\Rightarrow 4A + 3B = -4$ ...(i)

$f'(2) = 0 \Rightarrow 4A(8) + 3B(4) + 2C(2) + D = 0$

$\Rightarrow 8A + 3B = -2$ ...(ii)

From equation (i) and (ii) we get

$A = \frac{1}{2}$ and $B = -2$

So, $f(x) = \frac{x^4}{2} - 2x3 + 2x^2$

Therefore, $f(-1)=\frac{(-1)^4}{2}-2(-1)^3+2(-1)^2$

$=\frac{1}{2}+2+2=\frac{9}{2}$. Hence $f(-1)=\frac{9}{2}$

---

5. **(B)**

Here, $\frac{dx}{dt}=\frac{1}{2\sqrt{2^{\operatorname{cosec}^{-1}t}}}2^{\operatorname{cosec}^{-1}t}\log 2.\frac{-1}{x\sqrt{x^2-1}}$

$$\frac{dy}{dt}=\frac{1}{2\sqrt{2^{\sec^{-1}t}}}2^{\sec^{-1}t}\log 2.\frac{-1}{x\sqrt{x^2-1}}$$

$$\therefore \quad \frac{dy}{dt}=\frac{\frac{dy}{dx}}{\frac{dx}{dy}}=\frac{1-\sqrt{2^{\operatorname{cosec}^{-1}t}}}{\sqrt{2^{\sec^{-1}t}}}\frac{2^{\sec^{-1}t}}{2^{\operatorname{cosec}^{-1}t}}$$

$$\frac{dy}{dt}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=-\sqrt{\frac{2^{\sec^{-1}t}}{2^{\operatorname{cosec}^{-1}t}}}=\frac{-y}{x}$$

6. **(C)**

$(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos x)=a^2-b^2$

Differentiating both sides,

$$\left(-\sqrt{2}b\sin x\right)\left(a-\sqrt{2}b\cos y\right)+\left(a+\sqrt{2}b\cos x\right)\ \left(\sqrt{2}b\sin y\right)\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{\left(\sqrt{2}b\sin x\right)\left(a-\sqrt{2}b\cos y\right)}{\left(a+\sqrt{2}b\cos x\right)\left(\sqrt{2}b\sin y\right)}$$

$$\therefore \left[\frac{dy}{dx}\right]_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)}=\frac{a-b}{a+b}\Rightarrow \frac{dx}{dy}=\frac{a+b}{a-b}$$

7. **(A)**

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a} \qquad \left[\frac{0}{0}\text{form}\right]$$

On applying L' Hospital rule, we get

$$= \quad \lim_{x\to a}\frac{f(a)-af'(x)}{1}$$

$f(a)-af'(a)=4-2a$ $\quad\quad\quad\quad$ $[\because f'(a)=2 \text{ and } f(a)=4]$

8. **(A)**

$$y(x)=\cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right]$$

$$y(x)=\cos^{-1}\left(\tan\frac{x}{2}\right)=\frac{\pi}{2}-\frac{x}{2}$$

9. **(B)**

Given function is $y-\tan^{-1}(\sec x^3-\tan x^3)$.

$$=\tan^{-1}\left(\frac{1-\sin x^3}{\cos x^3}\right)=\tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x^3\right)}{\sin\left(\frac{\pi}{2}-x^3\right)}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x^3}{2}\right)\right);\text{ we have , }\frac{\pi}{4}-\frac{x^3}{2}\in\left(\frac{\pi}{4},0\right)$$

Let $y=\left(\frac{\pi}{4}-\frac{x^3}{2}\right)$ ...(i)

Differnetiate w.r.t.x.

$y'=\frac{-3x^2}{2}, y''=-3x$ [from equation (i),]

$$4y=\pi-2x^3\Rightarrow 4y=\pi-2x^2\left(\frac{-y''}{3}\right)$$

$$12y=3\pi+2x^2y''$$

Required equation, $x^2y''-6y+\frac{3\pi}{2}=0$.

10. **(D)**

Given function is

$$\cos^{-1}\left(\frac{y}{2}\right)=\log_e\left(\frac{x}{5}\right)^5\Rightarrow\cos^{-1}\left(\frac{y}{2}\right)=5\log_e\left(\frac{x}{5}\right)$$

Differentiate w.r.t.x.

$$\frac{-1}{\sqrt{1-\frac{y^2}{4}}}\cdot\frac{y'}{2}=5.\frac{1}{\frac{x}{5}}\times\frac{1}{5}\Rightarrow\frac{-y'}{\sqrt{4-y^2}}=\frac{5}{x} \quad ...(i)$$

$$-xy'=5\sqrt{4-y^2}$$

Again, differentiate w.r.t.x.

$$-xy''-y'=5.\frac{1}{2\sqrt{4-y^2}}(-2yy')$$

$$\Rightarrow \quad xy''+y'=\frac{5y'.y}{\sqrt{4-y^2}}\Rightarrow xy''+y'=5.\left(\frac{-5}{x}\right)y \quad \{\text{From (i)}\}$$

$$x^2y''+xy'=-25y$$

Required differential equation is $x^2y''+xy'=-25y$.

11. **(D)**

Take $\log_e 2\frac{d}{dx}(\log_{\cos x}\operatorname{cosec} x)$

Let, $y=\log_{\cos x}\operatorname{cosec} x$

$$y=-\frac{\ln(\sin x)}{\ln(\cos x)}$$

Diff. w.r.t.x both sides,

$$\frac{dy}{dx}=-\frac{[\cot x.\ln(\cos x)+\tan x.\ln(\sin x)]}{(\ln(\cos x))^2}$$

$$\left.\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=\frac{4}{\ln 2}$$

Now, $\Rightarrow \log_e 2.\frac{4}{\ln 2}=4$

12. **(C)**

Since, $f(x)=x^3-x^2f'(1)+xf'(2)-f''(3), x\in R$

Let $f'(1)=a, f''(2)=b, f'''(3)=c$

Now $f(x)=x^3-ax^2+bx-c$

$\Rightarrow f'(x)=3x^2-2ax+b, f''(x)=6x-2a, f'''(x)=6$

So, c = 6, a = 3, b = 6; $f(x) = x^3 - 3x^2 + 6x - 6$

$f(1)=-2, f(2)=2, f(3)=12, f(0)=-6$

thus, $2f(0)-f(1)+f(3)=2=f(2)$

13. (B)

Since, given

$$f(\theta)=3\left(\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^4(3x+\theta)\right)-2\left(1-\sin^2 2\theta\right)$$

and $S=\left\{\theta\in[0,\pi]:f'(\theta)=-\frac{\sqrt{3}}{2}\right\}$

Now $f(\theta)=3\left(\cos^4\theta+\sin^4\theta\right)-2\cos^2 2\theta$

$$\Rightarrow f(\theta)=3\left(1-\frac{1}{2}\sin^2 2\theta\right)-2\cos^2 2\theta$$

$$\Rightarrow f(\theta)=3-\frac{3}{2}\sin^2 2\theta-2\cos^2\theta$$

$$=\frac{3}{2}-\frac{1}{2}\cos^2 2\theta=\frac{3}{2}-\frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right)$$

$$f(\theta)=\frac{5}{4}-\frac{\cos 4\theta}{4}$$

Since $f'(\theta)=\sin 4\theta$

Given, $f'(\theta)=\sin 4\theta=-\frac{\sqrt{3}}{2}$

$$\Rightarrow 4\theta=n\pi+(-1)^n\frac{\pi}{3}\Rightarrow\theta=\frac{n\pi}{4}+(-1)^n\frac{\pi}{12}$$

$$\theta=\frac{\pi}{12},\left(\frac{\pi}{4}-\frac{\pi}{12}\right),\left(\frac{\pi}{2}-\frac{\pi}{12}\right),\left(\frac{3\pi}{4}-\frac{\pi}{12}\right)$$

$$4\beta=\frac{\pi}{4}+\frac{\pi}{2}+\frac{3\pi}{4}=\frac{3\pi}{2}\Rightarrow\beta=\frac{3\pi}{8}$$

$$\Rightarrow f(\beta)=\frac{5}{4}-\frac{\cos\frac{3\pi}{2}}{4}=\frac{5}{4}$$

14. (B)

Let $y\sin^3\left(\pi/3\cos g(x)\right)$

where $g(x)=\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}$

and $g(1)=2\pi/3$

Now, $y' = 3\sin^2\left(\frac{\pi}{3}\cos g(x)\right)\times\left(\frac{\pi}{3}\cos g(x)\right)\times\frac{\pi}{3}\left(-\sin(x)\right)g'(x)$

At $x = 1$

$$y'(1) = 3\sin^2\left(-\frac{\pi}{6}\right).\cos\left(\frac{\pi}{6}\right).\frac{\pi}{3}\left(-\sin\frac{2\pi}{6}\right)g'(1)$$

and $g'(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\left(-12x^2+10x\right)$

$$\Rightarrow g'(1) = \frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2) = -\pi$$

So, $\Rightarrow y'(1) = \frac{\cancel{3}}{4}.\frac{\sqrt{3}}{2}.\frac{\pi}{\cancel{3}}\left(\frac{-\sqrt{3}}{2}\right)(-\pi) = \frac{3\pi^2}{16}$

and $y(1) = \sin^3(\pi/3\cos 2\pi/3) = -\frac{1}{8}$

Thus, $2y'(1) + 3\pi^2 y(1) = 0$

15. **(B)**

Let $x^y = u \Rightarrow y\text{lm}\,x = \ln u$

$\Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \ln x\frac{dy}{dx}\right]$ and $y^x = v \Rightarrow x\ln y = \ln v$

$$\Rightarrow \frac{dv}{dx} = y^x\left[\ln y + \frac{x}{y}\frac{dy}{dx}\right]$$

Now, $2x^y + 3y^x = 20 \Rightarrow 2u + 3v = 20 \Rightarrow \frac{2du}{dx} + \frac{3dv}{dx} = 0$

$$2x^y\left[\frac{y}{x} + \ln x\frac{dy}{dx}\right] + 3y^x\left[\ln y + \frac{x}{y}\frac{dy}{dx}\right] = 0$$

At $(x, y) = (2,2)$; $\frac{dy}{dx} = \frac{-(12\ln 2 + 8)}{12 + 8\ln 2} = -\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

16. **(B)**

$$f(x)=\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)-1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)}$$

$$=\frac{\sin\left(x(x+\pi/4\right)-1}{\sin\left(x-\pi/4\right)}$$

$$f'(x)=\frac{\cos(x+\pi/4)\sin\left(x-\pi/4\right)-\cos\left(x-\pi/4\right)\left(\sin\left(x+\pi/4\right)-1\right)}{\sin^2\left(x-\pi/4\right)}$$

$$f'(x)=\frac{-(1-\cos(x+\pi/4))}{1-\cos^2\left(x-\pi/4\right)}$$

$$f'(x)=\frac{1}{1+\cos\left(x-\pi/4\right)}\Rightarrow\quad f'(x)=\frac{-\sin\left(x-\pi/4\right)}{\left(1+\cos\left(x-\pi/4\right)\right)^2}\Rightarrow$$

$$\Rightarrow\quad f\left(\frac{7\pi}{12}\right)=\frac{-1}{\sqrt{3}},f'(7\pi/12)=-2\sqrt{3}/9$$

$$f\left(\frac{7\pi}{12}\right).f''\left(\frac{7\pi}{12}\right)=\frac{2}{9}$$

17. **(40)**

Put $=0$

$\ln\ y=0\Rightarrow y=1$

$\ln(x+y)=4xy$

Diff. w.r.t.x $\frac{1}{x+y}\left(1+\frac{dy}{x}\right)=4\left(\frac{dy}{dx}+x\frac{dy}{dx}\right)$

at $x=0,\ y=1\ \Rightarrow\frac{dy}{dx}=3$

$$1+\frac{dy}{dx}=4\left(x+y\right)\left(x\frac{dy}{dx}+y\right)\qquad\text{....(ii)}$$

Diff eq (ii) w.r.t. x $\frac{d^2y}{dx^2}=40$

18. **(248)**

Given

$f(x+y)=2^x.\ f(y)+4y.\ f(x).$

Put $y=2$

$2^x\ f(2)+4^2\ f(x)$

$f(x+2) = 2^x . 3 + 16\, f(x)$

$f'(x+2) = 16f'(x) + 3.2^x \ln 2$

$f'(4) = 16f'(2) + 12\ln 2$ ...(i)

$f(y+2) = 4\, f(y) + 3.\, 4^y$

$f'(4) = 4f'(2) + 96\ln 2$ ...(ii)

On solving eqs. (i) and (ii) , we get

$f'(2) = 7\ln 2$

From equation (i), we get

$f'(4) = 2^4.3\ln 2$

Now , $14.\dfrac{f'(4)}{f'(2)}$

$14 \times \dfrac{2^4.31\ln 2}{7\ln 2}$

or $\dfrac{14 \times 124}{7} = 248$

19. **(2)**

Given $f(x) a_0x^2 + a_1x + a^2$

Differentiate w.r.t. x.

$f'(x) = 2a_0x + a_1$; Put $x = 0, 1$

$f'(0) = a_1 = 1; f'(1) = 2a_0 + a_1 = 0$

$2a_0 = -1 \Rightarrow a_0 = \dfrac{-1}{2}$

A G.P series is a , $(a+d)r, (a+2d)r^2, ...(a+(n-1)d)r^{n-1}$

Here, $d = 1, r = 2$ and $a = -\dfrac{1}{2}$

A G.P. series = $\dfrac{-1}{2}, \left(-\dfrac{1}{2}+1\right).2, \left(-\dfrac{1}{2}+2\right)2^2$

$\dfrac{-1}{2}, \left(\dfrac{1}{2}\right).2, \left(\dfrac{3}{2}\right).2^2 = \dfrac{-1}{2}, 1, 6$

So, $f(x) = a_0x^2 + a_1x + a_2 \Rightarrow f(x) = \dfrac{-1}{2}x^2 + x + 6$

Put x = 4 in above f(x).

$f(4) = \dfrac{-1}{2} \times 16 + 4 + 6 \Rightarrow f(4) = 2$

Therefore, the value of f(4) is 2.

20. **(16)**

Given function is

$y(x)=\left(x^{x}\right)^{x}=x^{x^{2}}$

Take log both sides,

$l\,n\,y(x)=x^{2}.l\,n\,x$

Differentiate w.r.t. x both sides,

$\frac{1}{y(x)}y'(x)=\frac{x^{2}}{x}+2x.l\,n\,x$

$y'(x)=y(x)[x+2x l\,n\,x]$

21. **(14)**

Given $f(x)=x^{2}+g'(1)x+g''(2)$ ...(i)

$f'(x)=2x\_g'(1)$

$f'(x)=2$

$g(x)=f(1)x^{2}+xf'(x)+f''(x)$

$g'(x)=2f(1)x+4x+g'(1)\Rightarrow \text{put } x=1, f(1)=-2$

$g''(x)=2f(1)+4\Rightarrow g''(x)=0$ [from (i)]

$g'(1)=-3$

So, $f'(x)=2x-3; f(x)=x^{2}-3x+c; c=0$

$f(x)=x^{2}-3x; g(x)=-3x+2; f(4)-g(4)=14$

22. **(10)**

Given, $f(x)=\sum_{k=1}^{10}kx^{k}$

Now, $f(x)=x+2x^{2}+......+10x^{10}$

$f(x).x=x^{2}+2x^{3}+.....+9x^{10}+10x^{11}$

$f(x)\left(1-x\right)=x+x^{2}+x^{3}+.....+x^{10}-10x^{11}$

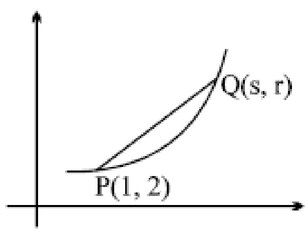
$f(x)=\frac{x\left(1-x^{10}\right)}{\left(1-x\right)^{2}}-\frac{10x^{11}}{\left(1-x\right)}$

EXERCISE - 2 [A]

Only One Option Correct:

1. (C)

I By definition $f'(1)$ is the limit of the slope of the secant line when $s \to 1$.



$$\text{Thus } f'(1) = \lim_{s\to 1}\frac{s^2+2s-3}{s-1}$$

$$= \lim_{s\to 1}\frac{(s-1)(s+3)}{s-1}$$

$$= \lim_{s\to 1}(s+3) = 4 \qquad \Rightarrow \qquad (D)$$

II By substituting $x = s$ into the equation of the secant line, and cancelling by $s - 1$ again, we get $y = s^2 + 2s - 1$. This is f (s), and its derivative is $f'(s) = 2s + 2$, so $f'(1) = 4$.]

2. (C)

Differentiate column wise, where $\Delta_1 = -4$; $\Delta_2 = 0$ and $\Delta_3 = 8$

3. (C)

$D^*f(x) = 2f(x).f'(x)$

$D^*(x\, \ell nx) = 2x\, \ell nx\, (1 + \ell nx)$]

4. (A)

$y = (A + Bx)e^{mx} + (m-1)^{-2} \cdot e^x$

$y \cdot e^{-mx} = (A + Bx) + (m-1)^{-2} \cdot e^{(1-x)x}$

$e^{-mx} \cdot y_1 - my + e^{-mx} = B - (m-1)^{-1} \cdot e^{-(m-1)x}$

$e^{-mx} \cdot y_2 - y_1\, e^{-mx} \cdot m - m[e^{-mx} \cdot y_1 - y\, e^{-mx} \cdot m] = e^{-(m-1)x}$

$e^{-mx} \cdot y_1 - m_2 y_1\, e^{-mx} + my \cdot e^{-mx} = e^{-(m-1)x}$

$y_2 - 2my_1 + my = e^x$ Ans.]

5. (D)

Let $\quad f(x) = px^2 + qx + r$

$\quad f(1) = f(-1)$ gives $p + q + r = p - q + r$

hence $\quad q = 0$

Hence $\quad f(x) = px^2 + r$

$\quad f'(x) = 2px \quad$ ....(1)

Given $\quad$ a, b, c are in A.P.

hence $\quad$ 2pa, 2pb, 2pc will also be in A.P.

or $\quad f'(a), f'(b), f'(c)$ will also be in A.P. $\qquad \Rightarrow \qquad$ (D)]

6. (B)

$2x + 2yy' = 0$

$x + yy' = 0 \quad \Rightarrow \quad y' = -\dfrac{x}{y} \qquad ....(1)$

$1 + yy'' + (y')^2 = 0$

$y'' = -\dfrac{1+(y')^2}{y}$

$$\text{now } k = \frac{y''}{\left(1+(y')^2\right)^{3/2}} = -\frac{1+(y')^2}{y\left(1+(y')^2\right)^{3/2}} = -\frac{1}{y\sqrt{1+(y')^2}} = -\frac{1}{y\sqrt{1+\frac{x^2}{y^2}}} = -\frac{1}{\sqrt{y^2+x^2}} = -\frac{1}{R}$$

7. (C)

Put $\cos\phi = \dfrac{2}{\sqrt{13}}$; $\sin\phi = \dfrac{3}{\sqrt{13}}$; $\tan\phi = \dfrac{3}{2}$

$y = \cos^{-1}\{\cos(x+\phi)\} + \sin^{-1}\{\cos(x-\phi)\}$

$= \cos^{-1}\{\cos(x+\phi) + \dfrac{\pi}{2} - \cos^{-1}\{\cos(\phi - x)\}$ (think !)

$= x + \phi + \dfrac{\pi}{2} - \phi + x$

$y = 2x + \dfrac{\pi}{2}$; $z = \sqrt{1+x^2}$

now compute $\dfrac{dy}{dz}$

8. (D)

We have $\quad f(x) = \dfrac{x^2 - x}{x^2 + 4x}$

f (x) is not defined at x = 0, – 4

$\therefore$ domain of f = R – {0, – 4}

For all $x \in$ domain of f, we have

$f(x) = \dfrac{x^2 - x}{x^2 + 4x} = \dfrac{x-1}{x+4} = 1 - \dfrac{5}{x+4}$

$f\left(f^{-1}(x)\right) = x$

$1 - \dfrac{5}{f^{-1}(x)+4} = x \quad$ or $\quad 1 - x = \dfrac{5}{f^{-1}(x)+4}$

$\therefore \quad f^{-1}(x) = \dfrac{5}{1-x} - 4$

$\dfrac{d}{dx}\left(f^{-1}(x)\right) = \dfrac{5}{(1-x)^2}$; $\quad \therefore \quad \dfrac{d}{dx}\left(f^{-1}(x)\right)_{at\ x=2} = \dfrac{5}{(1-2)^2} = 5$ Ans.]

9. (C)

$f(x) = \sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}$

$\therefore \quad f(x) = \sqrt{\left(\sqrt{x-2}+\sqrt{2}\right)^2} + \sqrt{\left(\sqrt{x-2}-\sqrt{2}\right)^2} = \left|\sqrt{x+2}+\sqrt{2}\right| + \left|\sqrt{x-2}-\sqrt{2}\right|$

for $\sqrt{x-2}$ to exist $x \ge 2$

Also, $\sqrt{x-2} + \sqrt{2} > 0$ (always true, think ! why?)

but $\sqrt{x-2} - \sqrt{2} \ge 0$ only if $x \ge 4$

< 0 only if $x < 4$

$\therefore$ now f (x) becomes

$f(x) = \sqrt{x-2} + \sqrt{2} - \sqrt{x-2} + \sqrt{2}$ for $2 \le x < 4$

$= \sqrt{x-2} + \sqrt{2} + \sqrt{x-2} - \sqrt{2}$ for $x \ge 4$

$\therefore \quad f(x) = 2\sqrt{2}$, for $2 \le x < 4$

$= 2\sqrt{x-2}$, for $4 \le x < \infty$

$\because$ f is continuous $[2, 4) \cup [4, \infty)$(verify)

$\therefore \quad f'(x) = 0, \quad 2 \le x < 4$

$= \dfrac{1}{\sqrt{x-2}}, \quad 4 \le x < \infty$

$\therefore \quad f'(102^+) = \dfrac{1}{\sqrt{102-2}} = \dfrac{1}{10}$

$\therefore \quad 10\, f'(102^+) = 1$

10. (C)

A : $2x + 2yy' = 0 \quad \Rightarrow \quad y' = -\dfrac{x}{y}$

$\therefore \quad y'(\sqrt{2}) = -1 = \mathbf{A}$

B : $\cos y \cdot y' + \cos x = \sin x \cdot \cos y \cdot y' + \sin y \cdot \cos x$

when $x = y = \pi$

$-y' - 1 = 0 + 0 \quad \Rightarrow \quad y'(\pi) = -1$

C : $2e^{xy}(xy' + y) + e^x e^y y' + e^y e^x - e^x - e^y y' = e \cdot e^{xy}(xy' + y)$

at $x = 1, y = 1$

$2e(y' + 1) + e^2 y' + e^2 - e - ey' = e^2(y' + 1)$

$ey' + e = 0 \quad \Rightarrow \quad y' = -1$

hence $A + B + C = -3$

11. (B)

$C_2 \to C_2 - xC_3$

$\Rightarrow f(x) = x^2(\tan x - \cos x)]$

$\Rightarrow f'(x) = (\tan x - \cos x)\,2x - x^2(\sec^2 x + \sin x)$

12. (B)

$\frac{x+a}{2} = b\cot^{-1}(b \ln y)$; $\cot\left(\frac{x+a}{2b}\right) = b\,\ln y$

$\therefore\ -\operatorname{cosec}^2\left(\frac{x+a}{2b}\right)\frac{1}{2b} = \frac{b}{y}y'$; $\therefore\ -\frac{1}{2b^2}\left(1+\cot^2\left(\frac{x+a}{2b}\right)\right) = \frac{y'}{y}$

$\therefore\ -\frac{1}{2b^2}\left((1+(b\,\ln y))^2\right) = \frac{y'}{y}$; $\therefore\ -\frac{1}{2b^2}\left(2(b\,\ln y)\frac{b}{y}y'\right) = \frac{yy''-y'^2}{y^2}$

$\therefore\ -\ln y\, y' = y\,y'' - y'^2$; $\therefore\ y\,y'' = y'^2 - y'\,y\,\ln y$

$\therefore\ y\,y'' + y\,y'\,\ln y = y'^2 - y'\,y\,\ln y + y\,y'\,\ln y = y'^2$

13. (B)

$N^r = \cos 6x + (1+5)\cos 4x + (5+10)\cos 2x + 10$

$= \cos 6x + \cos 4x + 5(\cos 4x + \cos 2x) + 10(1 + \cos 2x)$

$= 2\cos 5x\ \cos x + 10\cos 3x\ \cos x + 20\cos^2 x$

$= 2\cos x\,[\cos 5x + 5\cos 3x + 10\cos x]$

------Denominator------

$\therefore\ y = \frac{N^r}{D^r} = 2\cos x$ $\therefore\ \frac{dy}{dx} = -2\sin x \Rightarrow$ (C)

14. (B)

$\frac{d}{dx}\{[f(x)]^2 - [\phi(x)]^2\} = 2[f(x)\cdot f'(x) - \phi(x)\cdot\phi'(x)] = 2[f(x)\cdot\phi(x) - \phi(x)\cdot f(x)] = 0$

$[\because\ f'(x) = \phi(x)$ and $\phi'(x) = f(x)]$

$\Rightarrow [f(x)]^2 - [\phi(x)]^2 =$ constant

$\therefore\ [f(10)]^2 - [\phi(10)]^2 = [f(3)]^2 - [\phi(3)]^2 = [f(3)]^2 - [f'(3)]^2 = 25 - 16 = 9$

15. (B)

$(x+y)\left(\frac{\cos^4\alpha}{x} + \frac{\sin^4\alpha}{y}\right) = 1 = (\cos^2\alpha + \sin^2\alpha)^4$

$\therefore \quad \frac{y}{x}\cos^4\alpha + \frac{x}{y}\sin^4\alpha - 2\sin^2\alpha\cos^2\alpha = 0$

or $\quad \left(\sqrt{\frac{y}{x}}\cos^2\alpha - \sqrt{\frac{x}{y}}\sin^2\alpha\right)^2 = 0$

$\therefore \quad \tan^2\alpha = \frac{y}{x}$ or $y = x\tan^2\alpha \qquad \therefore \frac{dy}{dx} = \tan^2\alpha$

16. (B)

$y = e^x + x$; diff. w.r.t y,

$1 = (e^x + 1)\frac{dx}{dy}$; $\frac{dx}{dy} = \frac{1}{e^x + 1}$

$\Rightarrow \left.\frac{dx}{dy}\right]_{x=\ell n2} = \frac{1}{e^{\ell n2} + 1} = \frac{1}{3}$

17. (A)

$\frac{dy}{dx} = \frac{x}{1+x^2} + \tan^{-1}x + \frac{1}{\left|\frac{1}{x}\right|\sqrt{\frac{1}{x^2} - 1}}\left(-\frac{1}{x^2}\right)$

$= \frac{x}{1+x^2} + \tan^{-1}x + \frac{|x|^2}{\sqrt{1-x^2}}\left(-\frac{1}{x^2}\right)$

as $x \to 0$, $\frac{dy}{dx} = -1$ Ans.

<u>Alternatively</u> :

$f'(0^+) = \lim_{h\to 0}\frac{h\tan^{-1}(h) + \sec^{-1}(1/h) - \pi/2}{h} = \lim_{h\to 0}\frac{\cos^{-1}(h) - \pi/2}{h}$

$= \lim_{h\to 0}\frac{-\sin^{-1}(h)}{h} = -1$

Similarly $\quad f'(0^-) = -1$

Hence $f'(0) = -1$

18. (B)

$$y = \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \cfrac{\sin x}{1 + \dots\dots\dots}}} \; ; \quad y = \frac{\sin x}{1 + \frac{\cos x}{1+y}} = \frac{\sin x(1+y)}{1+y+\cos x} \; ; \qquad y(0) = 0$$

$y(1 + y + \cos x) = \sin x\,(1 + y)$

$y' + 2yy' + \cos x\, y' - y \sin x = \cos x(1 + y) + \sin xy'$

$y'(0)\,[1 + 2y + \cos x] - 0 = 1 + 0$

$$2y'(0) = 1 \qquad \Rightarrow \qquad y'(0) = \frac{1}{2}$$

19. (C)

Given $\dfrac{d^2y}{dx^2} + y\dfrac{dy}{dx} = 0$

$$\text{now} \quad \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \;\therefore\; \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{\frac{dx}{dy}}\right) = \frac{d}{dy}\left(\frac{1}{\frac{dx}{dy}}\right).\frac{dy}{dx} = -\frac{1}{\left(\frac{dx}{dy}\right)^2}.\frac{d^2x}{dy^2}.\frac{1}{\frac{dx}{dy}}$$

$$\frac{d^2y}{dx^2} = -\frac{\frac{d^2x}{d^2y}}{\left(\frac{dx}{dy}\right)^3} \qquad \text{(putting in (1))}$$

$$\frac{-\frac{d^2x}{d^2y}}{\left(\frac{dx}{dy}\right)^3} + y\frac{dy}{dx} = 0 \;\Rightarrow\; \qquad y\left(\frac{dx}{dy}\right)^2 - \frac{d^2x}{dy^2} = 0 \qquad \Rightarrow C$$

One or More than One Option(s) Correct:

20. (A, B)

$$\Rightarrow\; f(x) = \frac{\sqrt{\left(\sqrt{x-1}\right)^2 + 1 - 2\sqrt{x-1}}}{\sqrt{x-1}-1}\cdot x = \frac{\left|\sqrt{x-1}-1\right|}{\sqrt{x-1}-1}\cdot x = \begin{bmatrix} -x & \text{if } x \in [1,2) \\ x & \text{if } x \in (2,\infty) \end{bmatrix}$$

21. (A, C, D)

$$\Rightarrow y^2 = x + y$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$$

also $y = \frac{x}{y} + 1$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2x+y}$$

make a quadratic in y to get explicit function

22. (A, B, C)
Square both sides, differentiate and rationalize

23. (B, C)

24. (A, C)

$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}} - \frac{e^{-\sqrt{x}}}{2\sqrt{x}} = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$$

$$= \frac{\sqrt{\left(e^{\sqrt{x}} - e^{-\sqrt{x}}\right)^2 - 4}}{2\sqrt{x}} = \frac{\sqrt{y^2-4}}{2\sqrt{x}}$$

25. (B, C, D)

1 is a root of $f(x) = 0, f'(x) = 0$ and $f''(x) = 0$, or

1 is a root of $ax^3 + bx^2 + bx + d = 0$ (1)

$3ax^2 + 2bx + b = 0$ (2)

$$\Rightarrow a + 2b + d = 0$$

$$a + b = 0$$

$$\Rightarrow b + d = 0 \text{ and } a = d.$$

26. (A, C)

$$x^3 - 2x^2y^2 + 5x + y - 5 = 0$$

Differentiating w.r.t. x, we get

$$\Rightarrow 3x^2 - 4xy^2 - 4x^2y\frac{dy}{dx} + 5 + \frac{dy}{dx} = 0$$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{3x^2 - 4xy^2 + 5}{4x^2y - 1}$$

$$y'(1) = \frac{3-4+5}{4-1} = \frac{4}{3}$$

Also, y''

$$= \frac{(6x - 4y^2 - 8xyy')(4x^2y - 1) - (8xy + 4x^2y')(3x^2 - 4xy^2 + 5)}{(4x^2y-1)^2}$$

$$\Rightarrow y''(1) = \frac{(6 - 4 - 8.\frac{4}{3})(4-1) - (8 + 4.\frac{4}{3})(3 - 4 + 5)}{(4-1)^2}$$

$$= -8\frac{22}{27}$$

27. (A, B, C)

$$f(x) = |x^2 - 3|x| + 2|$$

$$= \begin{cases} |x^2 - 3x + 2|, & x \geq 0 \\ |x^2 + 3x + 2|, & x < 0 \end{cases}$$

$$= \begin{cases} x^2 - 3x + 2, & x^2 - 3x + 2 \geq 0, & x \geq 0 \\ -x^2 + 3x - 2, & x^2 - 3x + 2 < 0, & x \geq 0 \\ x^2 + 3x + 2, & x^2 + 3x + 2 \geq 0, & x < 0 \\ -x^2 - 3x - 2, & x^2 + 3x + 2 < 0, & x < 0 \end{cases}$$

$$= \begin{cases} x^2 - 3x + 2, & x \in [0, 1] \cup [2, \infty) \\ -x^2 + 3x - 2, & x \in (1, 2) \\ x^2 + 3x + 2, & x(-\infty, -2] \cup [-1, 0) \\ -x^2 - 3x + 2, & x \in (-2, -1) \end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}2x-3, & x\in(0,1)\cup(2,\infty)\\ -2x+3, & x\in(1,2)\\ 2x+3, & x\in(-\infty,-2)\cup(-1,0)\\ -2x-3, & x\in(-2,-1)\end{cases}$$

28. (B, C)

$$y=\frac{\left(x^2+1\right)^2-3x^2}{x^2+\sqrt{3}x+1}=\frac{\left(x^2+1+\sqrt{3}x\right)\left(x^2+1-\sqrt{3}x\right)}{x^2+1+\sqrt{3}x}$$

$$\frac{dy}{dx}=2x-\sqrt{3}\Rightarrow a=2 \text{ and } b=-\sqrt{3}$$

$$a-b=2+\sqrt{3}=\tan\frac{5\pi}{12}=\cot\frac{\pi}{12}$$

29. (B, D)

$$y=x^{(\log x)^{\log(\log x)}}$$

$$\Rightarrow \log y=(\log x)(\log x)^{\log(\log x)} \qquad (1)$$

Taking log of both sides, we get

$$\Rightarrow \log(\log y)=\log(\log x)+\log(\log x)\log(\log x)$$

Differentiating w.r.t. x, we get

$$\frac{1}{\log y}\cdot\frac{1}{y}\frac{dy}{dx}=\frac{1}{x\log x}+\frac{2\log(\log x)}{\log x}\frac{1}{x}$$

$$=\frac{2\log(\log x)+1}{x\log x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}\cdot\frac{\log y}{\log x}(2\log(\log x)+1)$$

Substituting the value of y from (1), we get

$$\frac{dy}{dx}=\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x)+1)$$

30. (A, B, C)

We have $\sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x)$

$$= \begin{cases} \frac{\pi}{2} - x, & \text{if } 0 < x \le \pi \\ \frac{\pi}{2} - (2\pi - x), & \text{if } \pi < x < 2\pi \end{cases}$$

$$= \begin{cases} \frac{\pi}{2} - x, & \text{if } 0 < x \le \pi \\ x - \frac{3\pi}{2}, & \text{if } \pi < x < 2\pi \end{cases}$$

$$\therefore \frac{d}{dx}\{\sin^{-1}(\cos x)\} = \begin{cases} -1, & \text{if } 0 < x < \pi \\ 1, & \text{if } \pi < x < 2\pi \end{cases}$$

We have $\cos^{-1}(\sin x) = \frac{\pi}{2} - \sin^{-1}(\sin x)$

$$= \begin{cases} \frac{\pi}{2} - x, & \text{if } -\frac{\pi}{2} < x \le \frac{\pi}{2} \\ \frac{\pi}{2} - (\pi - x), & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

$$= \begin{cases} \frac{\pi}{2} - x, & \text{if } -\frac{\pi}{2} < x \le \frac{\pi}{2} \\ x - \frac{\pi}{2}, & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

$$\therefore \frac{d}{dx}(\cos^{-1}(\sin x)) = \begin{cases} -1, & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1, & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

31. (B, C)

$$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}}\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1+x^2}{\sqrt{(1-x^2)^2}}\frac{2(1+x^2)-4x^2}{(1+x^2)^2}$$

$$\Rightarrow \frac{dy}{dx} = -2\frac{\left(1+x^2\right)}{\left|1-x^2\right|}\cdot\frac{\left(1-x^2\right)}{\left(1+x^2\right)^2}$$

32. (A, C)

$$\frac{d}{dx}\{f_n(x)\} = \frac{d}{dx}\{e^{f_{n-1}(x)}\}$$

$$= e^{f_{n-1}(x)}\frac{d}{dx}\{f_{n-1}(x)\} = f_n(x)\frac{d}{dx}\{f_{n-1}(x)\}$$

$$= f_n(x).\frac{d}{dx}\{e^{f_{n-2}(x)}\} = f_n(x).e^{f_{n-2}(x)}\frac{d}{dx}\{f_{n-2}(x)\}$$

$$= f_n(x)\, f_{n-1}(x)\frac{d}{dx}\{f_{n-2}(x)\}$$

...

$$= f_n(x) f_{n-1}(x)\cdots f_2(x)\frac{d}{dx}\{f_1(x)\}$$

$$= f_n(x).f_{n-1}(x)\cdots f_2(x)\frac{d}{dx}\{e^{f_0(x)}\}$$

$$= f_n(x).f_{n-1}(x)\cdots f_2(x)\, e^{f_0(x)}\frac{d}{dx}\{f_0(x)\}$$

Use $e^{f_0(x)} = f_1(x)$ and $f_0(x) = x$

Paragraph Type :

Passage - 1

33. (D)
34. (D)
35. (B)

$\Rightarrow x = f(t) = a^{\ln(b^t)} = a^{t\ln b}$ ....(1)

$\Rightarrow y = g(t) = b^{-\ln(a^t)} = \left(b^{\ln a}\right)^{-t} = \left(a^{\ln b}\right)^{-t} = a^{-t\ln b}$

$\Rightarrow \therefore y = g(t) = a^{\ln(b^{-t})} = f(-t)$ ....(2)

From equation (1) and (2)

$\Rightarrow xy = 1$

(i) $\because y = \frac{1}{x}$

$\Rightarrow \therefore \frac{dy}{dx} = -\frac{1}{x^2} = -\frac{1}{f^2(t)}$ **(A)** is correct

$\Rightarrow$ Also $xy = 1$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x^2} = -\frac{y^2}{1} = -g^2(t)$ **(B)** is correct

$\Rightarrow$ Again $xy = 1$ $\frac{dy}{dx} = -\frac{y}{x} = -\frac{g(t)}{f(t)}$ **(C)** is correct

(D) is incorrect

(ii) $f(t) = g(t) \Rightarrow f(t) = f(-t) \Rightarrow t = 0$

$\{\because f(t)$ is one-one function$\}$

At $t = 0$, $x = y = 1$

$\Rightarrow \therefore \frac{dy}{dx} = \frac{-1}{x^2}$ and $\frac{d^2y}{dx^2} = \frac{2}{x^3}$

$\Rightarrow$ At $x = 1$, $\frac{d^2y}{dx^2} = 2$

(iii) $\therefore xy = 1$ $\therefore fg = 1$ $\therefore fg' + gf' = 0$

$\Rightarrow \therefore fg'' + g'f' + g'f' + gf'' = 0$

$\Rightarrow fg'' + gf'' + 2g'f' = 0$

$$\Rightarrow \frac{fg''}{f'g'} + \frac{gf''}{g'f'} = -2 \qquad \ldots\ldots(3)$$

from equation (2)

$\Rightarrow g(t) = f(-t)$

$\Rightarrow \therefore g'(t) = -f'(-t)$

and $g''(t) = f''(-t)$

substituting in equation (3)

$$\Rightarrow \frac{f(t)}{f'(t)}\cdot\frac{f''(-t)}{-f'(-t)} + \frac{f(-t)}{-f'(-t)}\cdot\frac{f''(t)}{f'(t)} = -2$$

$$\Rightarrow \frac{f(t)}{f'(t)}\cdot\frac{f''(-t)}{f'(-t)} + \frac{f(-t)}{f'(-t)}\cdot\frac{f''(t)}{f'(t)} = 2$$

$\Rightarrow 1$

Passage - 2

36. (D)

37. (C)

38. (C)

$g(x+y) = g(x) + g(y) + 3x^2y + 3xy^2$ (1)

$\Rightarrow g'(x+y) = g'(x) + 6yx + 3y^2$ (differentiating w.r.t. x keeping y as constant)

Put $x = 0$

$\Rightarrow g'(y) = g'(0) + 3y^2$

$\Rightarrow g'(y) = -4 + 3y^2$

$\Rightarrow g'(x) = -4 + 3x^2$

$\Rightarrow g(x) = -4x + x^3 + c$

Now put $x = y = 0$ in (1), we get $g(0) = g(0) + g(0) + 0$

$\Rightarrow g(0) = 0$

$\Rightarrow g(x) = x^3 - 4x$

$g(x) = 0 \Rightarrow x^3 - 4x = 0 \Rightarrow x = 0, 2, -2$. Hence, three roots.

$\sqrt{g(x)} = \sqrt{x^3 - 4x}$ is defined if $x^3 - 4x \geq 0$ or $x \in [-2, 0] \cup [2, \infty)$.

Also, $g'(x) = 3x^2 - 4 \Rightarrow g'(1) = -1$

Passage - 3

39. (B)

40. (C)

$f(0)=0$

So, $\lim_{x\to 0}\frac{f(x)}{x}=f'(0)=1$ (Using L'Hopital's rule)

$f(x+y)=f(x)+f(y)+x^2y+xy^2$

Differentiating w.r.t. x, keeping y constant, we get

$f'(x+y)=f'(x)+2xy+y^2$

Put $x=0$. Then

$f'(y)=y^2+1$ or $f'(x)=x^2+1$

$\therefore$ $f(x)=\frac{x^3}{3}+x+c$

$f(0)=0=c$

$\therefore$ $f(x)=\frac{x^3}{3}+x$

$f'(3)=10$

and $f(9)=243+9=252$

Matrix – Match Type :

41. (A) → P ; (B) → Q, R ; (C) → S, R; (D) → Q, R

a. $f(1-x)=f(1+x)$

$\Rightarrow -f'(1-x)=f'(1+x)$.

Hence, graph of $f(x)$ is symmetrical about point (1, 0)

(as if $f(x)=-f(-x)$, then $f(x)$ is odd and its graph is symmetrical about (0, 0). Now shift the graph at (1, 0)).

b. $f(2-x)+f(x)=0$

Replace x by $1+x$, then $f(2-(1+x))+f(1+x)=0$

$\Rightarrow f(1-x)+f(1+x)=0$

$\Rightarrow -f'(1-x)+f'(1+x)=0$

$\Rightarrow f'(1-x)=f'(1+x)$ (1)

$\Rightarrow$ Graph of $f'(x)$ is symmetrical about line $x=1$.

Also, put $x=2$ in (1), we get $f'(-1)=f'(3)$.

c. $f(x+2)+f(x)=0$ (1)

Replace x by $x+2$, we get $f(x+4)+f(x+2)=0$ (2)

From (1) and (2), we have $f(x)=f(x+4)$

Hence, $f(x)$ is periodic with period 4.

Also, $f'(x)=f'(x+4)$. Hence $f'(x)$ is periodic with period 4.

Put $x=-1$ in $f'(x)=f'(x+4)$, we get $f'(-1)=f'(3)$.

d. Putting $x=0, y=0$, we get $2f(0)+\{f(0)\}^2=1$

$\Rightarrow f(0)=\sqrt{2}-1$ $\{\because f(0)>0\}$

Putting $y=x$, $2f(x)+\{f(x)\}^2=1$

Diff. w.r.t. x, we get

$2f'(x)+2f(x).f'(x)=0$ or $f'(x)\{1+f(x)\}=0$

$\Rightarrow f'(x)=0$, because $f(x)>0$.

42. (A) → Q ; (B) → R ; (C) → S ; (D) → P

a. $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{12t^2-6t-18}{5t^4-15t^2-20}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{t=1}=\frac{12-6-18}{5-15-20}=\frac{2}{5}$

$\Rightarrow 5\left.\frac{dy}{dx}\right|_{t=1}=2$ at $t=1$.

b. Let us take $P(x)=a(x-2)^4+b(x-2)^3+c(x-2)^2+d(x-2)+e$

$-1=P(2)=e$

$0=P'(2)=d$

$2=P''(2)=2c\Rightarrow c=1$

$-12=P'''(2)=6b\Rightarrow b=-2$

$24=p''''(2)=24a \Rightarrow a=1$

Thus, $P''(x)=12(x-2)^2-12(x-2)+2$

$\Rightarrow P''(3)=12-12(1)+2=2$

c. Here $\sqrt{(1+y^4)}=\sqrt{\left(1+\frac{1}{x^4}\right)}=\frac{\sqrt{1+x^4}}{x^2}$ $\left(\because y=\frac{1}{x}\right)$

$\Rightarrow \frac{\sqrt{1+y^4}}{\sqrt{1+x^4}}=\frac{1}{x^2}$ (1)

But $y=\frac{1}{x}$

$\therefore \frac{dy}{dx}=-\frac{1}{x^2}$ (2)

From (1) and (2), $\frac{\sqrt{1+y^4}}{\sqrt{1+x^4}}=-\frac{dy}{dx}$

$$\Rightarrow \frac{\frac{dy}{\sqrt{1+y^4}}}{\frac{dx}{\sqrt{1+x^2}}}=-1$$

d. Obviously, $f(x)$ is a linear function.
Also from $f'(0)=p$ and $f(0)=q, f(x)=px+q$.
$\Rightarrow f''(0)=0$

43. (A) → Q, R ; (B) → P, R, S ; (C) → Q, S ; (D) → Q, R

a. We know that

$$2\tan^{-1}x=\begin{cases}\sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } -1\le x\le 1\\ \pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } x>1\\ -\pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } x<-1\end{cases}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{2}{1+x^2} \text{ if } x<-1 \text{ or } x>1$$

b. $$\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=\begin{cases}\tan^{-1}x, & x\ge 0\\ -\tan^{-1}x, & x<0\end{cases}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{1}{1+x^2} \text{ if } x<0$$

c. $$y=|e^{|x|}-e|=\begin{cases}|e^x-e|, & x\ge 0\\ |e^{-x}-e|, & x<0\end{cases}=\begin{cases}e^x-e, & x\ge 1\\ e-e^x, & 0\le x<1\\ e-e^{-x}, & -1\le x<0\\ e^{-x}-e, & x<-1\end{cases}$$

$$\Rightarrow \frac{dy}{dx}>0 \text{ if } x>1 \text{ of } -1<x<0.$$

d. $u = \log|2x|, v = |\tan^{-1}x|$

$$\Rightarrow \frac{du}{dx} = \frac{1}{x}, \text{ and } \frac{dv}{dx} = \begin{cases} \frac{1}{1+x^2}, & x > 0 \\ -\frac{1}{1+x^2}, & x < 0 \end{cases}$$

$$\Rightarrow \frac{du}{dv} = \begin{cases} \frac{1+x^2}{x}, & x > 0 \\ -\frac{1+x^2}{x}, & x < 0 \end{cases}$$

Now we know that $\frac{1+x^2}{x} = x + \frac{1}{x} > 2$ if $x > 1$ and < -2 if $x < -1$

$\Rightarrow \frac{du}{dv} > 2$ if $x > 1$ or $x < -1$

44. (A) → P, Q, R ; (B) → Q, S ; (C) → Q, R ; (D) → R

a. p, q, r

The graph of $y = |x^2 - 2|x||$

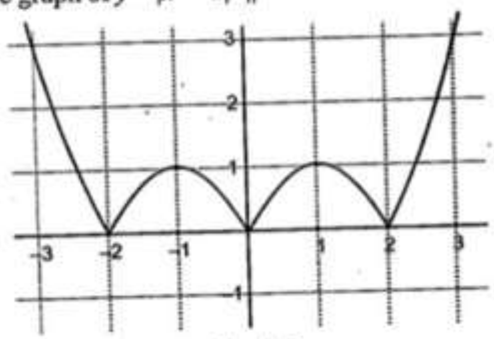


Fig. 4.3

From the graph dy/dx is negative for p, q, r

b. q, s

The graph of $y = |\log|x||$

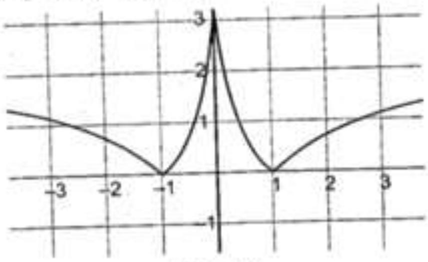


Fig. 4.4

From the graph dy/dx is negative for q, s

c. q, r

$$y = x[x/2] = \begin{cases} -x, & -4 \le x < -2 \\ -x, & -2 \le x < 0 \\ 0, & 0 \le x < 2 \\ x, & 2 \le x < 4 \end{cases}$$

Hence dy/dx is negative for q, r

d. q

The graph of $y = |\sin x|$

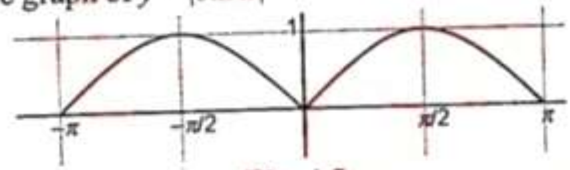


Fig. 4.5

From the graph dy/dx is negative for q

Integer Answer Type :

45. (2)

Since f(x) is odd. Therefore $f(-x) = -f(x) \Rightarrow f'(-x)(-1) = -f'(x)$

$\Rightarrow f'(-x) = f'(x)$

$\therefore f'(-3) = f'(3) = -2$.

46. (5)

Here $x = \alpha$ is a repeated root of the equation f(x) = 0,

hence $x = \alpha$ is also a root of the equation $f'(x) = 0$ i.e.,

$3x^2 + 6x - 9 = 0$ or $x^2 + 2x - 3 = 0$ or $(x+3)(x-1) = 0$ has the root α once which can be either –3, or 1.

If $\alpha = 1$, then $f(x) = 0$ gives c – 5 = 0 or c = 5

If $\alpha = -3$, then $f(x) = 0$ gives –27 + 27 + 27 + c = 0

$\therefore x = -27$

47. (2)

We have g(x) = f(x) sin x ….. (1)

On differentiating equation (1) w.r.t. x, we get

$g'(x) = f(x)\cos x + f'(x)\sin x$ ….. (2)

Again differentiating equation (2) w.r..t x, we get

$g''(x) = f(x)(-\sin x) + f'(x)\cos x + f'(x)\cos x + f''(x)\sin x$ ….. (3)

$\Rightarrow g''(-\pi) = 2f'(-\pi)\cos(-\pi) = 2 \times 1 \times -1 = -2$

Hence $g''(-\pi) = -2$

48. (8)

$\ln(f(x)) = \ln(x-1) + \ln(x-2) + ... + \ln(x-n)$

$\Rightarrow f'(x) = f(x)\left[\frac{1}{x-1} + \frac{1}{x-2} + ... \frac{1}{x-n}\right]$

$\Rightarrow\ f'(x)=(x-2)(x-3)(x-n)+(x-1)(x-3)....(x-n)+...+(x-1)(x-2)....(x(n-1))$

$\Rightarrow\ f'(n)=(n-1)(n-2)(n-3)\cdot 3\cdot 2\cdot 1$ (all other factors except the last vanishes when x = n)

$\Rightarrow\ 5040=(n-1)!$

$\Rightarrow\ n=8$

49. (9)

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x+0)}{h}$

$\Rightarrow\ \lim_{h\to 0}\frac{2f(x)+xf(h)+h\sqrt{f(x)}-2f(x)-xf(0)-0\sqrt{f(x)}}{h}$ as $f(0)=0$

$\Rightarrow\ \lim_{h\to 0}x\left(\frac{f(h)-f(0)}{h-0}\right)+\sqrt{f(x)}=f'(0)+\sqrt{f(x)}$

$\Rightarrow\ f'(x)=\sqrt{f(x)}\qquad (\because f'(0)=0)$

$\Rightarrow\ \int\frac{f'(x)}{\sqrt{f(x)}}dx=\int dx$

$\Rightarrow\ 2\sqrt{f(x)}=x+c$

$\Rightarrow\ f(x)=\frac{x^2}{4}\qquad (\because\ f(0)=0)$

50. (5)

$y=\frac{a+bx^{3/2}}{x^{5/4}}\ \Rightarrow\ y'=\frac{\frac{3}{2}bx^{1/2}x^{5/4}-\frac{5}{4}x^{1/4}\left(a+bx^{3/2}\right)}{x^{5/2}}$

According to the question,

$0=\frac{\frac{3}{2}b5^{1/2}5^{5/4}-\frac{5}{4}5^{1/4}\left(a+b5^{3/2}\right)}{5^{5/2}}$

$\Rightarrow\ \frac{3b}{2}5^{7/4}-a\frac{5^{5/4}}{4}-5b\frac{5^{7/4}}{4}=0$

$\Rightarrow\ b5^{7/4}=a5^{5/4}$

$\Rightarrow\ b\sqrt{5}=a$

$\Rightarrow\ a:b=\sqrt{5}:1$

51. (3)

$y=\frac{x^4-\left(x^2+2x+1\right)}{x^2-x-1}=x^2+x+1$

$\therefore\ \frac{dy}{dx}=2x+1=ax+b$

Hence a = 2 and b = 1

52. (2)

Limit is $f'(e)$ where $f(x)=x^{\ln x}=e^{\ln^2 x}$

$\Rightarrow g'(f(x))f'(x)=e^{\ln^2 x}\cdot\frac{2\ln x}{x}$

$\Rightarrow f'(e)=e\cdot\frac{2}{e}=2$

53. (6)

$g(x)=f(-x+f(f(x)));\ f(0)=0; f'(0)=2$

$g'(x)=f'(-x+f(f(x)))\cdot[-1+f'(f(x))\cdot f'(x)]$

$g'(0)=f'(f(0))\cdot[-1+f'(0)\cdot f'(0)]$

$=f'(0)[-1+(2)(2)]$

$=(2)(3)=6$

54. (1)

$\frac{dx}{dt}=-\frac{3}{t^4}-\frac{2}{t^3}=-\left(\frac{3+2t}{t^4}\right)$

$\frac{dy}{dt}=-\left(\frac{3}{t^3}+\frac{2}{t^2}\right)=-\left(\frac{3+2t}{t^3}\right)$

$\Rightarrow \frac{dy}{dx}=t$

$\Rightarrow \frac{dy}{dx}-x\left(\frac{dy}{dx}\right)^3=t-\left(\frac{1+t}{t^3}\right)\cdot t^3=-1$

55. (5)

$z=(\cos x)^5;\ y=\sin x$

$\frac{dz}{dx}=-5\cos^4 x\cdot\sin x;\ \frac{dy}{dx}=\cos x$

$\therefore \frac{dz}{dy}=-5\cos^3 x\cdot\sin x$

Now, $\frac{d^2z}{dy^2}=\frac{d}{dx}\left(\frac{dz}{dy}\right)\cdot\frac{dx}{dy}$

$\Rightarrow -5\frac{d}{dx}\left[\cos^3 x\cdot\sin x\right]\frac{1}{\cos x}$

$=-5\left[\cos^4 x-3\sin^2 x\cdot\cos^2 x\right]\frac{1}{\cos x}$

$=-5\left(\cos^3 x-3\sin^2 x\cdot\cos x\right)$

$=-5\left(\cos^3 x-3\cos x\left(1-\cos^2 x\right)\right)$

$=-5\left(\cos^3 x-3\cos x\right)$

$=-5\cos 3x$

$$\therefore \left.\frac{d^2z}{dy^2}\right|_{x=\frac{2\pi}{9}} = -5\cos 120^\circ = \frac{5}{2}$$

56. (7)

$$g'(0) = b = \lim_{x\to 0}\frac{x^2 + x\tan x - x\tan 2x}{x(ax+\tan x - \tan 3x)}$$

$$= \lim_{x\to 0}\frac{x + \tan x - \tan 2x}{kax + \tan x - \tan 3x}$$

$$= \lim_{x\to 0}\frac{x + \left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + ...\infty\right) - \left(2x + \frac{8x^3}{3} + \frac{2}{15}\cdot 32x^5 + ...\infty\right)}{ax + \left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + ...\infty\right) - \left(3x + \frac{27x^3}{3} + \frac{2}{15}243x^5 + ...\infty\right)}$$

Only One Option is Correct:

1. **(A)**

Given : $\log(x+y)=2xy$

Clearly, when $x=0$ then $y=1$

On differentiating w.r.t. x, we get

$$\frac{1}{x+y}\left[1+\frac{dy}{dx}\right]=2y+\frac{2xdy}{dx}$$

$$\Rightarrow\frac{dy}{dx}=\frac{\frac{1}{x+y}-2y}{2x-\frac{1}{x+y}}\Rightarrow y'(0)=\frac{1-2}{0-1}=1 \qquad [\because \text{ when } x=0 \text{ then } y=1]$$

2. **(D)**

$$\frac{d^2x}{d^2y}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dx}\left(\frac{dx}{dy}\right)\times\frac{dx}{dy}$$

$$=\left\{\frac{d}{dx}\left[\frac{1}{\left(\frac{dy}{dx}\right)}\right]\right\}\times\frac{1}{\frac{dy}{dx}}=-\frac{1}{\left(\frac{dy}{dx}\right)^2}\times\frac{d^2y}{dx^2}\times\frac{1}{\left(\frac{dy}{dx}\right)}$$

$$=-\left(\frac{dy}{dx}\right)^{-3}\frac{d^2y}{dx^2}$$

3. **(A)**

Given $g(x)=\log f(x)\ \Rightarrow g(x+1)=\log f(x+1)$

$$\Rightarrow g(x+1)=\log xf(x) \qquad [\because f(x+1)=xf(x)]$$

$$\Rightarrow g(x+1)=\log x+\log f(x)\ \ \Rightarrow g(x+1)-g(x)=\log(x)$$

$$\Rightarrow g'(x+1)-g'(x)=\frac{1}{x} \qquad \Rightarrow g''(x+1)-g''(x)=-\frac{1}{x^2}$$

On Putting, $x=x-\frac{1}{2}$, we get

$$\Rightarrow g''\left(x+\frac{1}{2}\right)-g''\left(x-\frac{1}{2}\right)=-\frac{1}{\left(x-\frac{1}{2}\right)^2}=\frac{-2}{(2x-1)2}$$

On putting x = 1,2,3 ...., N; we get

$$g''\left(\frac{3}{2}\right)-g''\left(\frac{1}{2}\right)=-\frac{2^2}{1^2} \qquad ..(i)$$

$$g''\left(\frac{5}{2}\right)-g''\left(\frac{3}{2}\right)=\frac{-2^2}{3^2} \qquad ..(ii)$$

$$g''\left(\frac{7}{2}\right)-g''\left(\frac{5}{2}\right)=\frac{-2^2}{5^2} \qquad ..(iii)$$

$$g''\left(N+\frac{1}{2}\right)-g''\left(N-\frac{1}{2}\right)=-\frac{2^2}{(2N-1)^2} \qquad ...(N)$$

On adding all the above equations, we get

$$g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=-4\left[1+\frac{1}{3^2}+\frac{1}{3^2}+...+\frac{1}{(2N-1)^2}\right]$$

4. **(D)**

$$f_n(x)=\sum_{j=1}^{n}\tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$$

$$=\sum_{j=1}^{n}\tan^{-1}\left(\frac{(x+j)-(x+j-1)}{1+(x+j)(x+j-1)}\right)$$

$$=\sum_{j=1}^{n}\left[\tan^{-1}(x+j)-\tan^{-1}(x+j-1)\right]$$

$$=\tan^{-1}\left(\frac{n}{1+x(n+x)}\right)\Rightarrow f_n'(x)=\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}$$

and $f_n(0)=\tan^{-1}(n)$, $\therefore \tan^2(\tan^{-1}n)=n^2$

Here x = 0 is not in the given domain, i.e., $x\in(0,\propto)$

$\therefore$ Options (a) and (b) are not correct options.

(c) $\lim_{x\to\infty}\tan(f_n(x))=\lim_{x\to\infty}\left(\frac{n}{1+x(n+x}\right)=0$

(d) $\lim_{x\to\infty}\sec^2(f_n(x))=\lim_{n\to\infty}1+\tan^2(f_n(x)$

$$=1+\lim_{x\to\infty}\tan^2(f_n(x))=1$$

One or More than One Option(s) May be Correct:

1. **(B, C)**

Given : $f(x)=x^3+3x+2 \Rightarrow f'(x)=3x^2+3$

$\therefore f(0)=2, f(1)=6, f(2)=16.f(3)=38, f(6)=236$

Also given $gf(x)=x \Rightarrow g(2)=0, g(6)=1, g(16)=2, g(3,8), g(236)=6$

(A) $g(f(x))=x \Rightarrow g'(f(x)).f'(x)=1$

For $g'(2), f(x)=2 \Rightarrow x=0$

On putting x= 0, we get $g'(f(0))f'(0)=1$

$\Rightarrow g'(2)=\dfrac{1}{3}$

(B) $h(g(g(x)))=x \Rightarrow h'(g(g(x))).g'(g(x)).g'(x)=1$

For $h'(1)$, we need $g(g(x))=1$

$\Rightarrow g(x)=6 \Rightarrow x=236$

On putting $x=236$, we get

$h'[g(g(236))]=\dfrac{1}{g'(g(236)).g'(236)}$

$\Rightarrow h'(g(6))=\dfrac{1}{g'(6)g'(236)}$

$\Rightarrow h'(1)=\dfrac{1}{g'(f(1))g'(f(6))}$

$=f'(1)f'(6)=6\times 111=666$

(C) $h[g(g(x))]=x$

For $h(0), f(g(x))=0 \Rightarrow g(x)=2 \Rightarrow x=16$

On Putting $x=16$, we get

$h(g(g(16)))=16 \Rightarrow h(0)=16$

(D) $h[g(f(x))]=x$

For h (g(3)), we need g(x) = 3 $\Rightarrow x=38$

On Putting $x=38$, we get

$h(g(g(38)))=38 \Rightarrow h(g(3))=38$

Integer Answer Type:

1. **(2)**

Given : and $g(x) = f^{-1}(x)$ therefore we should have go $f(x) = x$

$\therefore\ g(f(x)) = x \Rightarrow g(x^3 + e^{x/2}) = x$

On differentiating both sides w.r.t.x, we get

$$g'(x^3.e^{x/2}).\left(3x^2 + e^{x/2}.\frac{1}{2}\right) = 1$$

$$\Rightarrow \qquad g'(x^3.e^{x/2}) = \frac{1}{3x^2 + e^{x/2}.\frac{1}{2}}$$

For $x = 0$, we get $g'(1) = 2$

2. **(1)**

$$f(\theta) = \sin\left(\tan-1\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$$

$$= \sin\left[\sin^{-1}\left(\frac{\sin\theta}{\sqrt{\sin^2\theta + \cos 2\theta}}\right)\right] \qquad \left[\because \tan^{-1}\frac{x}{y} = \sin^{-1}\frac{x}{\sqrt{x^2+y^2}}\right]$$

$$= \sin\left[\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\right] = \tan\theta$$

$$\Rightarrow \frac{df(\theta)}{d\tan\theta} = 1.$$