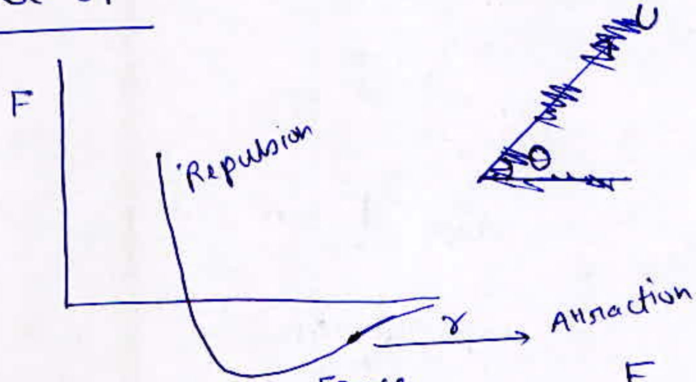
Mechanical properties of solids

Level-01

1.

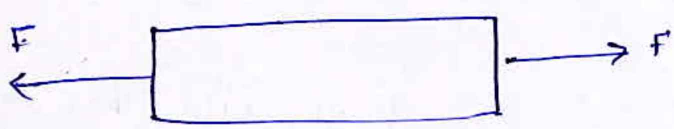


2. $$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2}$$

$$\text{stress} \propto \frac{1}{r^2}$$

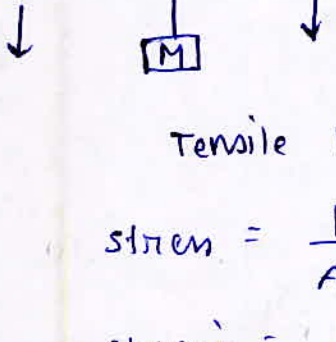
$$\frac{S_1}{S_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

3.



Tensile stress

3.



$$\text{Tensile Force} = Mg + (l-x)mg$$

5. $$\text{stress} = \frac{F}{A} \qquad Y = \frac{\text{stress (Normal)}}{\text{strain (longitudinal)}}$$

$$\text{strain} = \frac{\Delta l}{l} \qquad P = \frac{F}{A}$$

6. B

7. length of spiral spring remains constant but the shape changes.

8. $Y_A = \tan 60 = \sqrt{3}$ = slope of OA

$Y_B = \tan 30 = \frac{1}{\sqrt{3}}$ = stope of OB

$$\frac{Y_A}{Y_B} = 3 \Rightarrow Y_A = 3Y_B$$

9. In part bc, without applying any extra force, length of wire is increasing.

10. For A, The material of the wire has a good plastic Range, so it's a ductile material

For B, ✓

11. $$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l / l}$$

$$Y = \frac{Fl}{A \Delta l}$$

For this graph

$$Y = \frac{80 \times 1}{10^{-6} \times 4 \times 10^{-4}} = 2 \times 10^{11} \ N/m^2$$

12. Yong's modulus is property of material, doesn't depend on shape and size

13. Energy stored per unit volume $= \frac{1}{2} \times \text{stress} \times \text{strain}$

$$= \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain}$$

$$= \frac{1}{2} \times 5 \times 10^{8} \times \left(\frac{5}{10}\right)^{2}$$

$$\frac{E}{V} = \frac{125 \times 10^{6}}{2}$$

$$E = \frac{125}{2} \times 10^{6} \times (25 \times 10^{-6} \times 10 \times 10^{-2})$$

$$E = \frac{125 \times 25}{2} \times 10^{-1}$$

$$\frac{1}{2} \times mv^{2} = \frac{125 \times 2.5}{2}$$

$$v = \sqrt{\frac{125 \times 2.5}{5 \times 10^{-3}}}$$

$$= \sqrt{25 \times 2500}$$

$$= 250 \text{ m/s}$$

14. $$Y = \frac{F/A}{\Delta l / l}$$

$$F = Y \times \frac{\Delta l}{l} \times A$$

$$= 2 \times 10^{11} \times \frac{l}{l} \times 0.1 \times 10^{-4}$$

$$= 2 \times 10^{6} \text{ N}$$

15.

$$\Delta l = l \alpha \Delta t$$

$$\frac{\Delta l}{l} = \alpha \Delta t$$

$$Y = \frac{F/A}{\Delta l / l}$$

$$F = Y \times A \times \frac{\Delta l}{l}$$

$$= Y A \alpha \Delta t$$

16. C

17.

$$Y = \frac{F/A}{\Delta l / l}$$

$$\Delta l = \frac{F l}{A \Delta l Y} = \frac{F l}{\pi R^2 \Delta l Y}$$

$$\frac{\Delta l_1}{\Delta l_2} = \left(\frac{R_2}{R_1}\right)^2 = 4$$

18.

$$Y = \frac{F/A}{\Delta l / l} \Rightarrow \Delta l = \frac{F l}{A Y}$$

$$\Delta l \propto \frac{l}{A}$$

Long thin wire

19. B

20. C is not elastic

$$Y = \frac{F/A}{\Delta l / l} = \frac{F l}{A \Delta l}$$

$$Y \propto \frac{1}{\Delta l}$$

21. $Y = \frac{F/A}{\Delta l / l}$

$\Delta l = \frac{Fl}{AY}$

$\Delta l = \frac{Fl}{\pi R^2 Y}$

$\frac{\Delta l_1}{\Delta l_2} = \left(\frac{R_2}{R_1}\right)^2 = \left(\frac{1}{n}\right)^2$

$\Delta l_2 = n^2 \Delta l_1$

22. Energy per unit volume $= \frac{1}{2} \times$ stress $\times$ strain

$\frac{E}{V} = \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain}$

$E = \frac{1}{2} \times Y \times \left(\frac{x}{L}\right)^2 \times V$

$= \frac{Yx^2}{2L}\left(\frac{V}{L}\right) = \frac{Yx^2 A}{2L}$

23. From Q.22

$E = \frac{Yx^2 A}{2l}$

$E \propto \frac{x^2}{l}$

$Y = \frac{F/A}{\Delta l / l}$

$\Delta l = \frac{Fl}{YA}$

$\Delta l \propto l$

$\left(\frac{\Delta l_1}{\Delta l_2}\right) = \frac{l_1}{l_2} = \frac{1}{2}$

$$\frac{E_1}{E_2} = \left(\frac{\Delta l_1}{\Delta l_2}\right)^2 \times \left(\frac{l_2}{l_1}\right)$$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{1}\right) = \frac{1}{2}$$

24. From Q. 22

$$E = \frac{Y(\Delta l)^2 A}{2l}$$

$$E \propto (\Delta l)^2$$

$$\frac{E_2}{E_1} = \left(\frac{\Delta l_2}{\Delta l_1}\right)^2$$

$$E_2 = V \times \left(\frac{10}{2}\right)^2 = 25V$$

26. stress = Y × strain

$$\frac{F}{A} = 7 \times 10^9 \times \frac{0.2}{100}$$

$$\frac{F}{A} = 14 \times 10^6$$

$$A = \frac{F}{14 \times 10^6} = \frac{10^4}{14 \times 10^6}$$

$$= 7.1 \times 10^{-4}$$

27. wires will break at the same stress

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1 \times \left(\frac{A_2}{A_1}\right) = F_1 \times \left(\frac{R_2^2}{R_1^2}\right) = 4F_1$$

28. Bulk modulus $= -\frac{\Delta P}{\Delta V/V}$

For Isothermal process

$PV = \text{constant}$

$PdV + VdP = 0$

$\frac{dV}{V} = -\frac{dP}{P}$

Bulk modulus $= -\frac{\Delta P}{\Delta V/V} = -\frac{\Delta P}{-\Delta P/P} = P$

$= 1 \text{ atm}$

$= 1.013 \times 10^5 \text{ N/m}^2$

29. Adiabatic elasticity $= \gamma P$

Isothermal elasticity $= P$

$\frac{E_\phi}{E_\theta} = \gamma = \frac{C_P}{C_V}$

30. Volume elasticity $= -\frac{\Delta P}{\Delta V/V}$

$= \frac{-[(\rho g h + P_{atm}) - (P_{atm})]}{\Delta V/V}$

$= -\frac{\rho g h}{\Delta V/V}$

$= \frac{-10^3 \times 10 \times 200}{-0.1 \times 10^{-2}}$

$= 2 \times 10^9$

31. ~~Compre~~ Bulk modulus $= \dfrac{1}{\text{Compressibility}} = -\dfrac{\Delta P}{\Delta V / V}$

$$\frac{\Delta V}{V} = \Delta P \times \text{Compressibility}$$

$$\Delta V = \left(100 \times 4 \times 10^{-5}\right) \times 100$$

$$= 0.4 \text{ cc}$$

Level-02

1. From A to B, potential Energy decreases

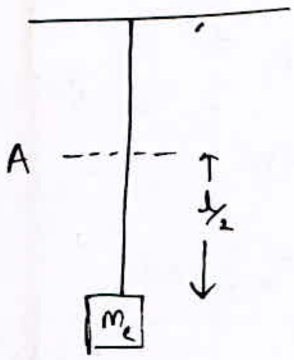
$$F = -\frac{dU}{dx}, \quad \text{Repelling force}$$

From B to C, $\frac{dU}{dx} > 0$

Attractive force.

2. C

3.



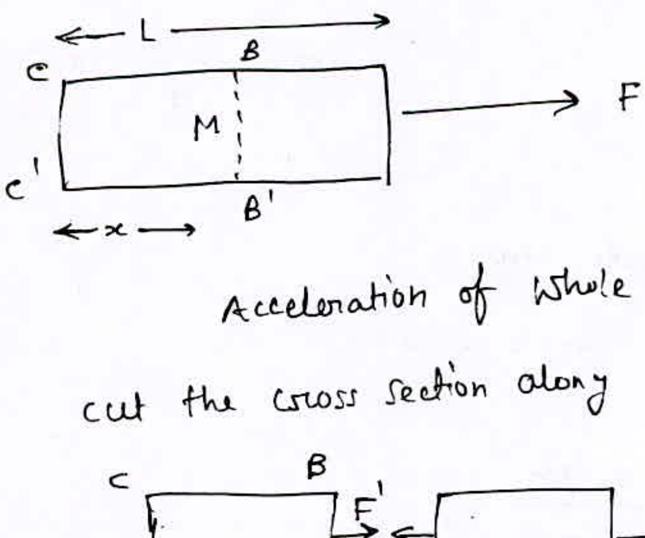
Total Tension force at midpoint A

$$F = \left(\frac{m_1}{l} \times \frac{l}{2} + m_2\right) g$$

$$F = \left(\frac{m_1 + 2m_2}{2}\right) g$$

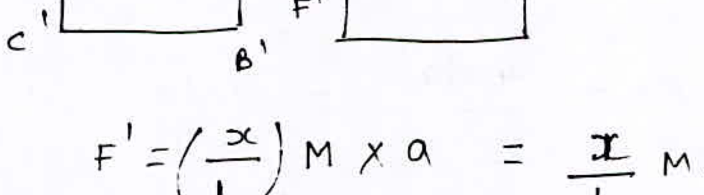
$$\text{stress} = \frac{F}{A}$$

4.



Acceleration of whole body $= \frac{F}{M}$

cut the cross section along BB'



$$F' = \left(\frac{x}{L}\right) M \times a = \frac{x}{L} M \times \frac{F}{M} = \frac{Fx}{L}$$

$$\text{stress} = \frac{Fx}{AL}$$

5. C

6.

$$\text{strain} = \frac{\Delta l}{l}$$

$$= \frac{2\times10^{-3}}{4\times10^{-2}} = 0.05$$

7.

$$Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\Delta l} = \frac{10^5 \times 10 \times 10^{-2}}{(100\times10^{-4})\times 0.5\times10^{-2}}$$

$$Y = 2\times10^8$$

$$Y = \frac{10^5 \times 20\times10^{-2}}{(400\times10^{-4})\times\Delta l} = 2\times10^8$$

$$\Delta l = \frac{10^5\times20\times10^{-2}}{400\times10^{-4}\times2\times10^8}$$

$$= \frac{10^{-2}}{4}\ \text{m}$$

$$= \frac{1}{4}\ \text{cm} = 0.25\ \text{cm}$$

8.

For Hooke's law

stress $\propto$ strain

9.

$$Y = \frac{\text{stress}}{\text{strain}}$$

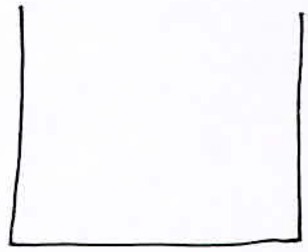
$$Y = \frac{4\times10^7}{2\times10^{-4}} = 2\times10^{11}$$

10. without liquid

$$\text{strain} = \frac{\text{stress}}{Y}$$

$$\frac{\Delta l_1}{l} = \frac{mg}{Y}$$

with liquid



$$\text{stress} = \frac{mg - \text{buoyancy force}}{A}$$

$$= \frac{mg - V \times \rho \times g}{A}$$

$$\frac{\Delta l_2}{l} = \frac{mg - V\rho g}{AY}$$

change in length (decreases)

$$\Delta l_1 - \Delta l_2 = \frac{V\rho g}{\pi r^2 Y} \times l$$

11.

$$\Delta l_{steel} = \frac{FL}{AY}$$

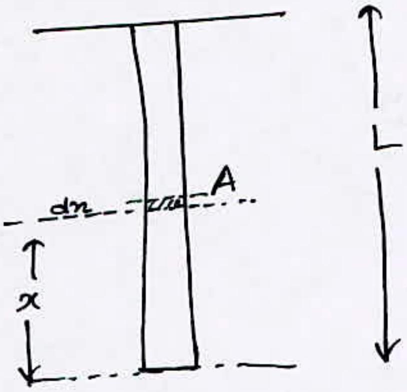
$$\frac{\Delta l_{steel}}{\Delta l_{Brass}} = \left(\frac{F_S L_S}{A_S Y_S}\right) y \left(\frac{A_B Y_B}{F_B L_B}\right)$$

$$= \left(\frac{F_S}{F_B}\right) \times \left(\frac{L_S}{L_B}\right) \times \left(\frac{Y_B}{Y_S}\right) \times \left(\frac{r_B^2}{r_A^2}\right)$$

$$= \left(\frac{3Mg}{2Mg}\right) \times (a) \times \left(\frac{1}{c}\right) \times \left(\frac{1}{b^2}\right)$$

$$= \frac{3a}{2b^2 c}$$

12.



$$F = \frac{x}{L} \times mg$$

$$\text{stress} = \frac{mgx}{AL}$$

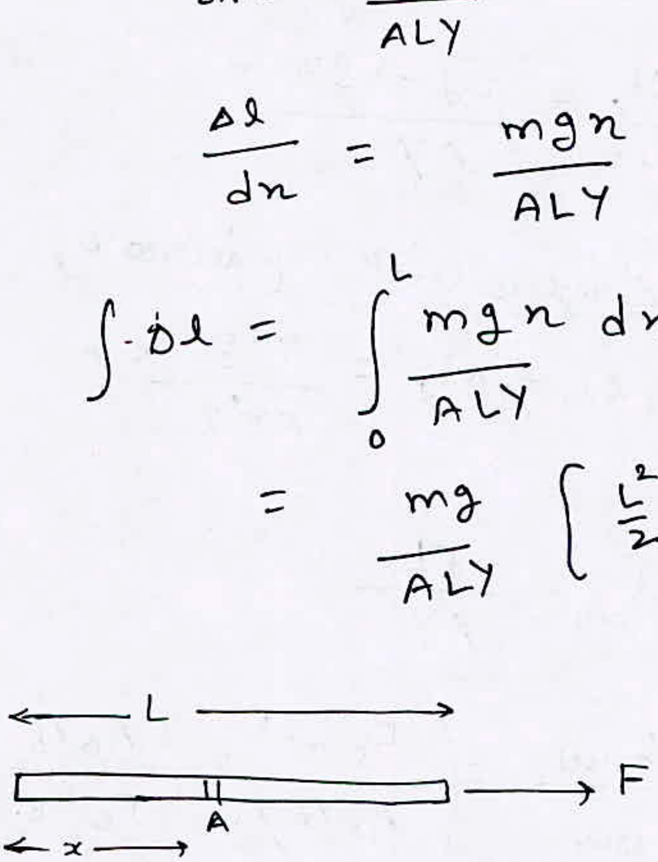
$$\text{strain} = \frac{mgx}{ALY}$$

$$\frac{\Delta l}{dx} = \frac{mgx}{ALY}$$

$$\int \delta l = \int_0^L \frac{mgx}{ALY}\,dx$$

$$= \frac{mg}{ALY}\left[\frac{L^2}{2}\right] = \frac{WL}{2AY}$$

13.



$$a = \frac{F}{M}$$

$$F_A = \frac{x}{L} \times M \times \frac{F}{M}$$

$$F_A = \frac{Fx}{L}$$

$$\text{stress} = \sigma = \frac{Fx}{Al} = Y \times \varepsilon = Y \times \frac{\Delta l}{dx}$$

$$\Delta l = \int_0^L \frac{Fx\,dx}{ALY}$$

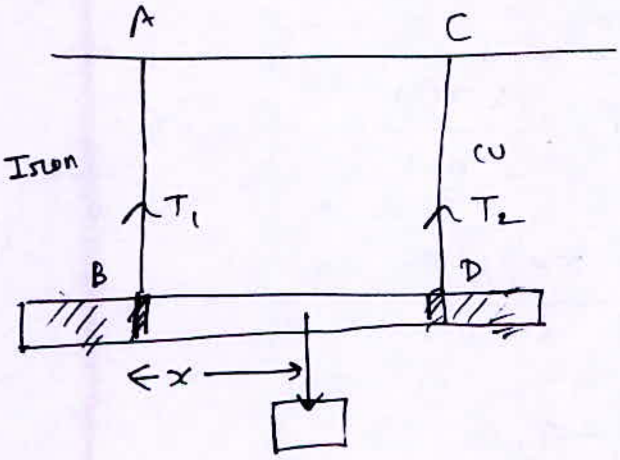
$$= \frac{FL^2}{2ALY} = \frac{FL}{2AY}$$

$$\varepsilon = \frac{\Delta l}{l}$$

$$= \frac{FL}{2AYL}$$

14. Same As Q.12

15.



$$\varepsilon_{Fe} = \varepsilon_{Cu}$$

$$\frac{T_1}{Y_{Fe}} = \frac{T_2}{Y_{Cu}}$$

$$\frac{T_1}{T_2} = \frac{Y_{Fe}}{Y_{Cu}} = \frac{19.6}{11.8} = \frac{5}{3} = \frac{5F}{3F}$$

$$T_1 + T_2 = 20 = 2 \times 10$$

$$5F + 3F = 20$$

$$8F = 20$$

$$F = \frac{5}{2}$$

Torque about B should be zero

$$Wx - T_2 \times L = 0$$

$$T_2 = \frac{Wx}{L}$$

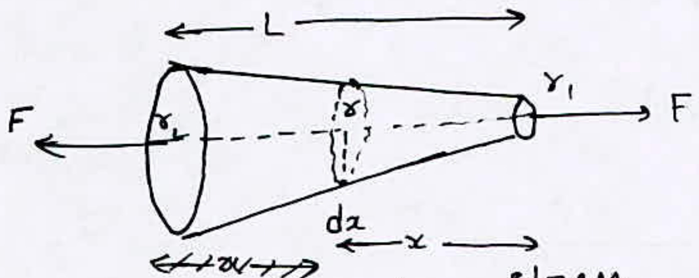
$$T_1 = \frac{5}{3}\frac{Wx}{L}$$

$$\frac{Wx}{L} + \frac{5}{3}\frac{Wx}{L} = W$$

$$\frac{8}{3}\frac{x}{L} = 1$$

$$x = \frac{3}{8}L = \frac{3}{8} \times 0.8 = 0.3\text{ m}$$

16.



$$Y = \frac{\text{stress}}{\text{strain}}$$

$$\frac{r_2 - r_1}{L - 0} = \frac{r - r_1}{x - 0}$$

$$x = \frac{L(r - r_1)}{r_2 - r_1}$$

$$dx = \frac{L\,dr}{r_2 - r_1}$$

$$Y = \frac{\sigma}{\varepsilon}$$

$$\varepsilon = \frac{\sigma}{Y}$$

$$\frac{\Delta l}{dx} = \frac{\frac{F}{\pi r^2}}{Y}$$

strain in dx element $= \Delta l = \dfrac{F\,dx}{\pi Y r^2} = \dfrac{F}{\pi Y r^2} \times \dfrac{L\,dr}{r_2 - r_1}$

$$\Delta l = \frac{FL}{\pi Y (r_2 - r_1)} \times \frac{1}{r^2} dr$$

Total change in length $\Delta L = \frac{FL}{\pi Y (r_2 - r_1)} \times \int_{r_1}^{r_2} \frac{1}{r^2} dr$

$$= \frac{FL}{\pi r_1 r_2 Y}$$

17.

A 

length will increase till $Kx = F$

$$x = \frac{F}{K}$$

x = increase in length

18. $K = -\text{slope}$

$$= +\frac{1}{\sqrt{3}}$$

19. —

20. $Y = \frac{\sigma}{\varepsilon} = \frac{FL}{A\,\Delta x}$

$$F = \frac{A\,\Delta x\,Y}{L}$$

$$\frac{F}{\Delta x} = \frac{AY}{L}$$

21.

$$\eta = \frac{\text{shear force}}{\text{shear strain}} = \frac{F/A}{\theta}$$

$$\eta = \frac{F/A}{y/L}$$

$$F = \frac{\eta y A}{L} = \frac{\eta A}{L} y$$

$$K = \frac{\eta A}{L} = \frac{\eta L^2}{L} = \eta L$$

$$T = 2\pi \sqrt{\frac{M}{K}}$$

$$= 2\pi \sqrt{\frac{M}{\eta L}}$$

22.

$$\text{strain energy} = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

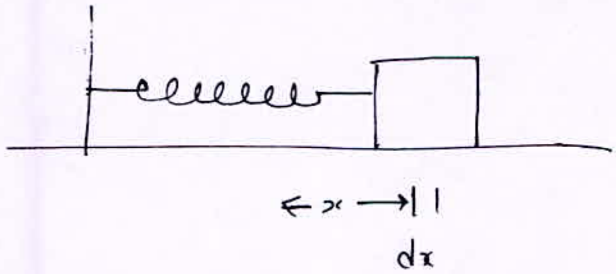
$$\eta = \frac{\text{shear stress}}{\text{shear strain}}$$

$$\text{shear strain} = \frac{S}{\eta}$$

$$\text{strain energy} = \frac{1}{2} \times S \times \frac{S}{\eta} \times V$$

$$= \frac{S^2 V}{2\eta}$$

23.

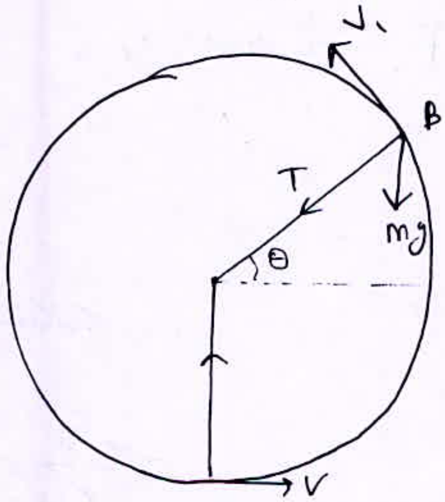


$$dW = F\,dx$$

$$dW = Kx\,dx$$

$$W = \int_{l_1}^{l_2} Kx\,dx = \frac{K}{2}\left(l_2^2 - l_1^2\right)$$

24.



At bottom most point

$$T - mg = \frac{mv^2}{R}$$

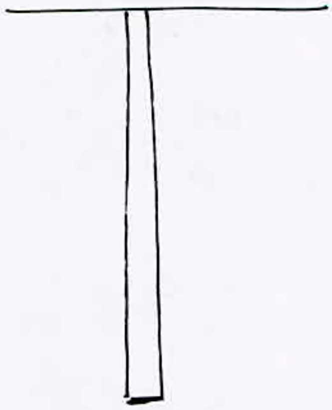
$$T = mg + \frac{mv^2}{R}$$

Tension is maximum at bottom most point.

at B point $\quad T + mg\sin\theta = \frac{mv_1^2}{R}$

$$T = \frac{mv_1^2}{R} - mg\sin\theta$$

25.



Tension is maximum at top most point. ($T = mg$)

Tension is minimum at lowest point ($T = 0$)

longitudinal strain will decrease as you go down.

longitudinal strain ↑, diameter ↓

26.

Isothermal process

$pV = K$

$p\,dv + v\,dp = 0$

$p = -\frac{v\,dp}{dv}$

$K = p = -\frac{dp}{dv/V}$

$p = \frac{K}{V}$

Bulk modulus $= \frac{\text{Normal stress}}{\text{Volumetric strain}}$

$= -\frac{P}{\Delta V / V}$

Bulk modulus $= -\frac{\Delta P}{\Delta V / V}$

$pV = K$

$p\,dv + v\,dp = 0$

$\frac{dv}{V} = -\frac{dp}{p}$

$dp = -p\frac{dv}{V}$

$K = +\frac{p\,dv/V}{dv/V} = +P$

27. $$B = -\frac{\Delta P}{\Delta V / V}$$

Adiabatic process

$$PV^{\gamma} = K$$

$$V^{\gamma}\Delta P + \gamma V^{\gamma-1}\Delta V \times P = 0$$

$$V^{\gamma-1}\left(V\Delta P + \gamma\Delta V P\right) = 0$$

$$V^{\gamma-1} \neq 0$$

$$V\Delta P + \gamma \Delta V P = 0$$

$$\frac{\Delta V}{V} = -\frac{\Delta P}{\gamma P}$$

$$B = -\frac{\Delta P}{\Delta V / V} = -\left(\frac{\Delta P}{-\Delta P / \gamma P}\right) = \gamma P$$

28. $$P = \alpha V$$

$$PV^{-1} = \alpha$$

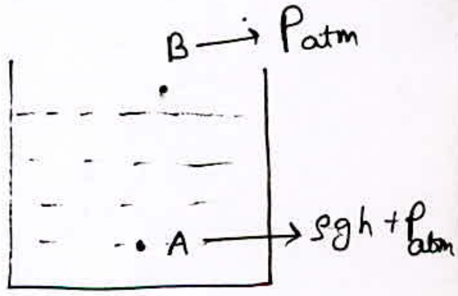
Adiabatic process $\Rightarrow$ PV^{γ} = constant

$$\gamma = -1$$

For Adiabatic process

Bulk modulus $= \gamma P$
$= -P$

29. $$K = \frac{-\Delta P}{\Delta V / V}$$



$$\Delta V = -\frac{\Delta P \times V}{K}$$

$$\Delta P = P_{atm} - \rho g h - P_{atm} = -\rho g h$$

$$\Delta V = -\frac{\Delta P \times V}{K}$$

$$= -\frac{\rho g h \times V}{k}$$

$$= -\frac{10^3 \times 9.8 \times 200 \times 10^{-6}}{2.2 \times 10^9}$$

$$= \frac{19.6}{2.2} \times 10^{-10} \; m^3$$

$$= 8.8 \times 10^{-10} \times 10^6 \; cm^3$$

$$= 8.8 \times 10^{-4} \; cm^3$$

Previous Year's Questions

1. $$Y = \frac{F/A}{\Delta l / l}$$

$$\Delta l = \frac{Fl}{AY} = \frac{Fl^2}{VY} \quad \left(A = \frac{V}{l}\right)$$

$$\Delta l \propto l^2$$

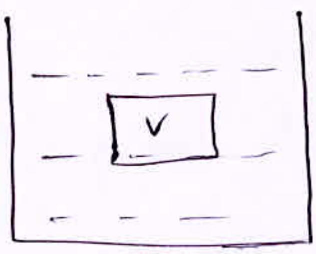
2. $$Y = \frac{F/A}{\Delta l / l} = \frac{Fl}{A \Delta l}$$

$$Y = \frac{Fl}{\pi r^2 \Delta l}$$

$$\Delta l \propto \frac{l}{r^2}$$

3. —

4. $$\frac{\rho_{liquid}}{\rho_W} = 1.2$$



$$\rho_{liquid} = 1.2 \rho_w$$

$$mg - 1.2 \rho_w g V = 44 g$$

$$mg - \rho_w g V = 50 g$$

$$0.2 \rho V = 6$$

$$\rho V = 30$$

$$mg - 30g = 50g$$

$$mg = 80g$$

5. Same as Q.2

Stress And Strain

1. $$Y = \frac{Fl}{A\,\Delta l}$$

$$\Delta l = \frac{Fl}{\pi r^2 Y}$$

$$\Delta l \propto \frac{l}{r^2}$$

2. $$\text{Bulk modulus} = -\frac{\Delta P}{\Delta V/V}$$

$$= -\frac{100}{\dfrac{-0.3\ cm^3}{\frac{4}{3}\pi \times 27\ cm^3}}$$

$$= +\frac{100 \times 3}{0.3 \times 4\pi \times 27} \qquad \frac{100 \times \frac{4}{3}\pi \times 27}{0.3}$$

$$= 4\pi \times 3 \times 10^3\ atm$$

3. Wire will break for same stress

$$\sigma_1 = \sigma_2$$

$$\frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

$$F_2 = F_1 \times \left(\frac{r_2}{r_1}\right)^2$$

$$= 40g \times \left(\frac{2r_1}{r_1}\right)^2$$

$$= 160g$$

$$= 160\ \text{kg weight}$$

5. $$Y = \frac{FL}{A\,\Delta l}$$

$$\Delta l = \frac{Fl}{AY}$$

$$\Delta l_1 = \frac{Fl}{\pi r^2 Y}$$

$$\frac{\Delta l_1}{\Delta l_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2r_1}{r_1}\right)^2 = 4$$

$$\Delta l_2 = \frac{\Delta l_1}{4} = 0.6 \text{ cm}$$

8. Total Energy $= \frac{1}{2} \times$ stress $\times$ strain $\times$ volume

$$= \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{volume}$$

$$= \frac{1}{2Y} \times \left(\frac{F}{\pi r^2}\right)^2 \times \pi r^2 L$$

$$E = \frac{F^2\pi}{2Y} \times \frac{L}{r^2}$$

$$E \propto \frac{L}{r^2}$$ F and Y are same

$$\frac{E_A}{E_B} = \frac{L_A}{L_B} \times \left(\frac{r_B}{r_A}\right)^2$$

$$= 3 \times \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4}$$

9. Wire will break at same stress

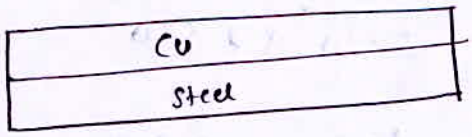
$$\sigma_1 = \sigma_2$$

$$\frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

$$F_2 = F_1 \times \left(\frac{r_2}{r_1}\right)^2$$

$$= F \times \left(\frac{D}{D/2}\right)^2 = 4F$$

10.



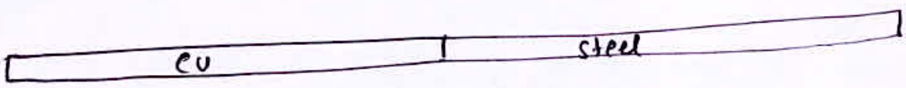
case (i)
↓
Not end to end

$$\text{strain} = \frac{\Delta l}{l} \quad \text{same for both}$$

$$\text{stress} = \text{strain} \times Y$$

$$\text{stress} \propto Y$$ (Y depends on material of body)

11. case (ii)



Here $\text{stress} = \frac{F}{A}$

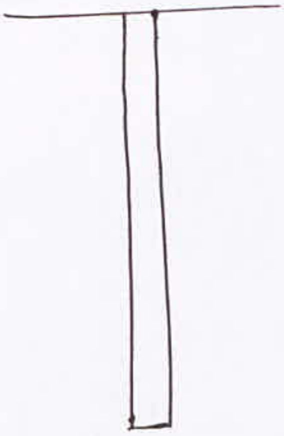
Force and Area is same for both

so same stress

But $\text{strain} = \frac{\text{stress}}{Y}$

$$\text{strain} \propto \frac{1}{Y}$$ Y will be different for material

12.



wire will break at near to topmost point because stress will be maximum at top most point.

$$\text{Stress} = \frac{mg}{A}$$

$$= \frac{\rho A l \times g}{A}$$

$$= \rho l g$$

$$10^6 = 3 \times 10^3 \times l \times 10$$

$$l = \frac{100}{3} = 33.3\ m$$

13.

1. $$Y = \frac{\text{Tensile stress}}{\text{longitudinal strain}}$$

2. Equivalent Force constant $= k_1 + k_2$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{m}{k_1+k_2}}.$$

3. $$Y = \frac{FL}{A\Delta l}$$

$$Y = \frac{FL}{\pi R^2 \Delta l}$$

$$R^2 = \frac{FL}{\pi Y \Delta l}$$

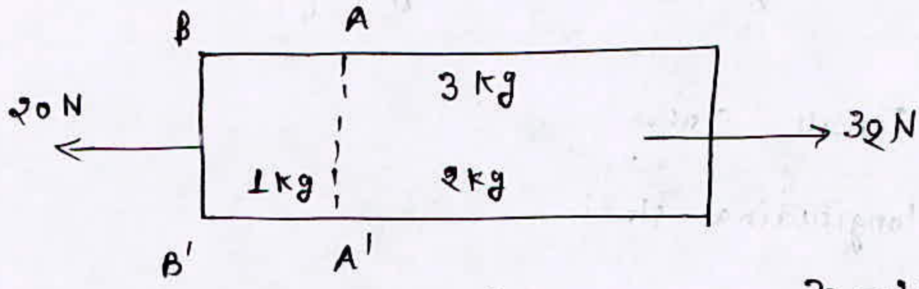
$$\left(\frac{R_s}{R_B}\right)^2 = \left(\frac{F_s}{F_B}\right)\left(\frac{L_s}{L_B}\right)\left(\frac{Y_B}{Y_s}\right)\left(\frac{\Delta l_B}{\Delta l_s}\right)$$

$$= \left(\frac{1}{1}\right)\left(\frac{L}{L}\right)\left(\frac{1}{2}\right)\left(\frac{1}{1}\right)$$

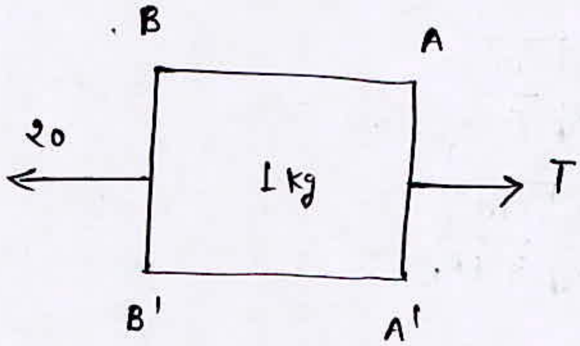
$$\left(\frac{R_s}{R_B}\right)^2 = \frac{1}{2}$$

$$\frac{R_s}{R_B} = \frac{1}{\sqrt{2}}$$

4.



$$\text{Acceleration of system} = \frac{32-20}{3} = 4$$



$$T - 20 = ma$$

$$T - 20 = 1 \times 4$$

$$T = 24$$

5. $Y = 2\eta(1+\sigma)$

Y = Young's Modulus

η = modulus of Rigidity

σ = poisson's Ratio

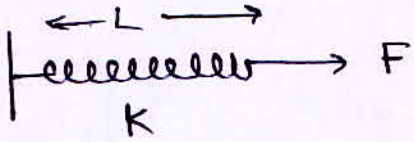
$$2.4\eta = 2\eta(1+\sigma)$$

$$1.2 = 1+\sigma$$

$$\sigma = 0.2$$

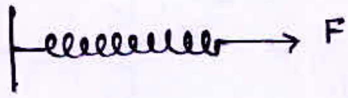
29

6.



For extension x

$$F = Kx$$



For same force F

extension in $\frac{3L}{4}$ length will be $\frac{3x}{4}$.

$$K_1 \times \frac{3x}{4} = F$$

$$K_1 = \frac{4F}{3x}$$

$$K = \frac{F}{x}$$

$$K_1 = \frac{4}{3}K$$

7. K.E. = potential energy of spring

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$x = v\sqrt{\frac{m}{K}}$$

$$= 4\sqrt{\frac{16}{100}}$$

$$= \frac{4 \times 4}{10} = 1.6\ m$$

8. Same as Q.6

9. $$Y = \frac{\sigma}{\varepsilon}$$

$$\varepsilon = 1$$

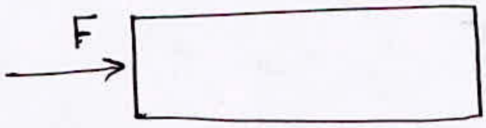
$$Y = \sigma$$

10. $$Y = \frac{\sigma}{\varepsilon}$$

$$Y = \frac{FL}{A\Delta l} \qquad (\Delta l \propto F)$$

$$Y = \frac{20 \times 1}{10^{-6} \times 10^{-4}} = 2 \times 10^{11}\ N/m^2$$

12.



$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta l / l} = \frac{Fl}{A\Delta l}$$

$$\frac{\Delta l}{l} = \frac{F}{AY}$$

14. $$Y = \frac{FL}{A\Delta L}$$

$$\Delta L = \frac{FL}{AY}$$

$$= \left(\frac{F}{A}\right)\frac{L}{Y}$$

$$= \frac{3.18 \times 10^{8} \times 1}{2 \times 10^{11}}\ m = 3.18 \times 10^{-3}\ m$$

$$= 3.18\ mm$$

15. Young's modulus is property of material doesn't depend on shape and size

16.

$$Y = \frac{FL}{A\Delta L}$$

$F = mg$

$$= \frac{(250 \times 10) \times 2}{50 \times 10^{-6} \times 0.5 \times 10^{-3}}$$

$$= 2 \times 10^{11} \ N/m^2$$

Take $g = 9.8$

17.

stress $\propto$ strain

$$Y = \frac{\text{stress}}{\text{strain}} = \text{constant for a material}$$

18.

$$Y = \frac{FL}{A\Delta l}$$

$$\Delta l = \frac{FL}{AY}$$

$$\Delta l \propto F$$

19.

Bulk modulus $K = -\dfrac{\Delta P}{\Delta V/V}$

$$K = -\frac{\rho g h}{\Delta V/V}$$

$$\Delta V = \frac{V \times 0.01}{100} = V \times 10^{-4}$$

$$K = \frac{-\rho g h}{-V \times 10^{-4}/V} = +\rho g h \times 10^{4}$$

$$= 10^{3} \times 9.8 \times 1000 \times 10^{4}$$

$$= 9.8 \times 10^{10} \ N/m^2$$

20. $Y = \frac{\text{stress}}{\text{strain}}$

$\text{stress} = Y \times \text{strain}$

wire will break for same stress

$Y \times \varepsilon_1 = Y \varepsilon_2$

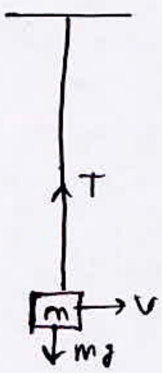
$\varepsilon_1 = \varepsilon_2$

$\frac{\Delta l_1}{l_1} = \frac{\Delta l_2}{l_2}$

2. $\frac{2\text{ mm}}{L} = \frac{\Delta l_2}{L/2}$

21. Same as Q.19

22.



$T - mg = \frac{mv^2}{R}$

$T = mg + m\omega^2 R$

$= 6.5 \times 10 + 6.5 \times \left(\frac{2 \times 2\pi}{1}\right)^2 \times 60 \times 10^{-2}$

$T = 680$

$\Delta l = \frac{FL}{AY} = \frac{680 \times 0.6}{0.05 \times 10^{-4} \times 2 \times 10^{11}}$

$= 4080 \times 10^{-7}$

$= .4 \times 10^{-4}$

23.

$$\Delta V = \frac{200 \times 0.004}{100} \text{ L}$$

$$= 0.008 \text{ L}$$

$$K = -\frac{\Delta P}{\Delta V / V}$$

$$2100 \times 10^6 = -\frac{\Delta P}{-0.008/200}$$

$$\Delta P = \frac{2100 \times 10^6 \times 0.008}{200}$$

$$= 84 \times 10^3 \text{ Pa}$$

$$= 84 \text{ KPa}$$

24.

$$\Delta l = \frac{FL}{AY}$$

$$= \sout{1 \times 10}$$

25.

$$\Delta l = \frac{FL}{AY}$$

F, L, Δl are same

$$A_1 Y_1 = A_2 Y_2$$

$$r_1^2 Y_1 = r_2^2 Y_2$$

$$\frac{r_2}{r_1} = \left(\frac{Y_1}{Y_2}\right)^{1/2} = \left(\frac{7}{12}\right)^{1/2}$$

26.

$$Y = \frac{FL}{A\,\Delta L}$$

$$\Delta L = \frac{FL}{AY} = \frac{FL}{\pi R^2 Y}$$

$$\Delta L \propto \frac{L}{R^2}$$

27.

$$\text{Bulk modulus} = \frac{1}{\text{compressibility}} = -\frac{\Delta P}{\Delta V / V}$$

$$\frac{1}{5\times10^{-10}} = -\frac{15\times10^{6}}{\Delta V / 100}$$

$$\frac{\Delta V}{100} = -15\times10^{6}\times5\times10^{-10}$$

$$\Delta V = -75\times10^{-2}$$

$$= -0.75\ ml$$

28.

$$Y = \frac{Fl}{A\,\Delta l}$$

$$F = \frac{YA\,\Delta l}{l}$$

$$F = \frac{YA\,(l\alpha\,\Delta\theta)}{l}$$

$$F = YA\alpha\,\Delta\theta$$

$F \propto A$ doesn't depend on l

$F \propto l^0$

30. Young's Modulus ~~depend~~ is property of material.

doesn't depend on shape and size.

Work done in stretching wire

1. decrease in potential energy = $Mg\Delta l$

Elastic potential energy per unit volume = $\frac{1}{2} \times$ stress $\times$ strain

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l}$$

$$\frac{E}{V} = \frac{F\Delta l}{2V}$$

$$E = \frac{F\Delta l}{2} = \frac{mg\Delta l}{2}$$

Thermal energy = $Mg\Delta l - Mg\frac{\Delta l}{2} = \frac{Mg\Delta l}{2}$

2.

3.

Increased Energy per unit volume = $\frac{1}{2} \times$ stress $\times$ strain

$$\frac{E}{V} = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l}$$

$$\frac{E}{V} = \frac{F\Delta l}{2V}$$

$$E = \frac{F\Delta l}{2}$$

$$= \frac{20 \times 10^{-3}}{2}$$

$$= 0.01\ J$$

4. Energy per unit volume = $\frac{1}{2} \times$ stress $\times$ strain

$$= \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain}$$

$$= \frac{1}{2} Y X^2$$

5. potential energy per unit volume = $\frac{1}{2}$ × stress × strain

$$\frac{E}{V} = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l}$$

$$\frac{E}{V} = \frac{F\Delta l}{2Al} \qquad (Al = V)$$

$$E = \frac{F\Delta l}{2} = \frac{Fx}{2}$$

6. Energy per unit volume = $\frac{1}{2}$ × stress × strain

$$\frac{E}{V} = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l}$$

$$\frac{E}{\cancel{V}} = \frac{F\Delta l}{2\cancel{V}}$$

$E = \frac{1}{2}$ × load × extension

7. Refer Q.1 and 2

8.
$$Y = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{F/A}{\Delta l/l}$$

$$\frac{\Delta l}{l} = \frac{F}{A}Y$$

F, A, Y is same for both.

$$\Delta l \propto l$$

$$\frac{\Delta l_1}{\Delta l_2} = \frac{2}{5}$$

$W = F\Delta l$ F is same

$W \propto \Delta l$ $\frac{W_1}{W_2} = \frac{\Delta l_1}{\Delta l_2} = \frac{2}{5}$

9.

$$\frac{E}{V} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain}$$

$$= \frac{1}{2} \times 1.1 \times 10^{11} \times 10^{-6}$$

$$= 5.5 \times 10^{4} \ J/m^3$$

$$\Delta l = \frac{l \times 0.1}{100}$$

$$\frac{\Delta l}{l} = 10^{-3}$$

10.

$$\frac{E}{V} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y}$$

$$\frac{E}{Al} = \frac{1}{2} \times \frac{F}{A} \times \frac{F}{A} \times \frac{1}{Y}$$

$$E = \frac{F^2 l}{2AY}$$

$$= \frac{(50)^2 \times 20 \times 10^{-2}}{2 \times 2 \times 10^{-4} \times 1.4 \times 10^{11}}$$

$$= \frac{25 \times 2 \times 10^{-6}}{5.6}$$

(take $g = 9.8$ for 8.57)

$$= 8.57 \times 10^{-6}$$

Poisson's Ration And Thermal stress

1. length will increase and radius will decrease.

mass = density x volume

mass and density remains constant.

2. ~~$V_t = 6$~~

$$mg = 6\pi\eta r v$$

$$m_1 = m_2$$

$$\rho \times \frac{4}{3}\pi r^3 \times 8 = \frac{4}{3}\pi R^3 \times \rho$$

$$R = 2r$$

$$8mg = 6\pi\eta \times 2r \times v'$$

$$v' = \frac{8mg}{6\pi\eta \times 2r}$$

$$v' = 4 \times v = 40\ m/s$$

3. $$\sigma = \frac{-\frac{\Delta r}{r}}{\frac{\Delta l}{l}} = \frac{-\frac{\Delta r}{r} \times 100}{\frac{\Delta l}{l} \times 100}$$

$$= \frac{0.2}{0.4} = 0.5$$

4. $$\Delta l = l \alpha \Delta\theta$$

$$\frac{\Delta l}{l} = \alpha \Delta\theta$$

stress = Y x strain

stress = Y x $\alpha \Delta\theta$

↓ material ↓ temperature

Independent of length

5. From Q. 4

Stress = $Y \times \alpha \Delta\theta$

$Y_1 \alpha_1 \Delta\theta_1 = Y_2 \alpha_2 \Delta\theta_2$

$$\frac{Y_1}{Y_2} = \frac{\alpha_2 \Delta\theta_2}{\alpha_1 \Delta\theta_1}$$

$$= \frac{3}{2} \times \frac{\theta}{\theta} = \frac{3}{2}$$

6. $V = \pi R^2 L$

$$\frac{\Delta V}{V} = 2\frac{\Delta R}{R} + \frac{\Delta L}{L}$$

$$0 = 2\frac{\Delta R}{R} + \frac{\Delta L}{L}$$

$$\frac{\frac{\Delta R}{R}}{\Delta L/L} = -1/2 = -0.5$$

7. Lateral strain = poisson's Ration x longitudinal strain

$= 0.5 \times 0.03$

$= 0.015$

8. poisson Ratio = $\dfrac{\frac{\Delta R}{R}}{\Delta L/L}$

$$\frac{\Delta R}{R} = 0.5 \times 0.1\ \%$$

$$\frac{\Delta R}{R} = 0.05\ \%$$

$$\Delta R = \frac{2 \times 0.05}{100} = 0.001$$

Length increased so diameter will decrease

$$R_1 = R - \Delta R$$

$$= 2 - 0.001$$

$$= 1.999\ mm$$

9. poisson's Ration ≤ 0.5

10. poisson's Ratio $= \dfrac{\text{lateral strain}}{\text{longitudenal strain}}$

$$\frac{\Delta R}{R} = \sigma \times \frac{\Delta l}{l}$$

$$= 0.4 \times 0.025$$

$$= 0.01\ \%$$