Inequations & Equations

EXERCISE - 1 [A]

1. **(c)**

$$\Rightarrow \sqrt{\log_{0.5}^2 4} = \sqrt{[\log_{10} 4]^2} = \sqrt{\left[\log_{\left(\frac{1}{2}\right)} 4\right]^2}$$

$$= \sqrt{\left[\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{2}\right)^{-2}\right]^2} = \sqrt{\left(-2\times\log_{\frac{1}{2}}\frac{1}{2}\right)^2}$$

$$= \sqrt{(-2\times1)} = \sqrt{4}$$

$= 2$ …… (as a square root of a value can't be negative)

2. **(b)**

$$\Rightarrow \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$$

By formula $\log_a b = \dfrac{\log_c b}{\log_c a}$

Given value is $= \left(\dfrac{\log 4}{\log 3}\right)\left(\dfrac{\log 5}{\log 4}\right)\left(\dfrac{\log 6}{\log 5}\right)\left(\dfrac{\log 7}{\log 6}\right)\left(\dfrac{\log 8}{\log 7}\right)\left(\dfrac{\log 9}{\log 8}\right)$

$$= \frac{\log 9}{\log 3} = \log_3 9$$

$= 2$

3. **(c)**

$$\log_7 \log_7\left(\sqrt{7\sqrt{7\sqrt{7}}}\right)$$

$$= \log_7 \log_7\left(7^{1/2}.7^{1/7}.7^{1/8}\right)$$

$$= \log_7 \log_7 7^{7/8} = \log_7(7/8)$$

$$= 1 - 3\log_7 2$$

4. **(c)**

$$\Rightarrow A = \log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$$

$$\Rightarrow \log_2 \log_2 \log_4 (4)^2 + \log_{2^{\frac{1}{2}}}(2)$$

$$\Rightarrow \log_2 \log_2 4 + 2 \times \frac{1}{\left(\frac{1}{2}\right)} \log_2 2$$

$$\Rightarrow \log_2 2 + 4 = 1 + 4$$

$$\Rightarrow 5$$

5. **(d)**

$$\Rightarrow \log_{10} x = y\left(\text{given}\right)$$

$$\Rightarrow \log_{1000} x^2 = 2\log_{10^3} x$$

$$\Rightarrow 2 \times \frac{1}{3}\log_{10} x$$ (by formula $\log_{a^{k^b}} = \frac{1}{k}\log_a b$)

6. **(c)**

$$\Rightarrow a^{mn} = a^{m^n}$$

Take log both side

$$\Rightarrow \log_a a^{mn} = \log_a a^{m^n}$$

$$\Rightarrow mn = m^n$$

$$\Rightarrow m^{n-1} = n$$

$$\Rightarrow m = \left(n^{\frac{1}{n-1}}\right)$$

7. **(b)**

$$\Rightarrow \frac{\left(\log x - \log y\right)\left(\log x^2 + \log y^2\right)}{\left(\log x^2 - \log y^2\right)\left(\log x + \log y\right)}$$

$$\Rightarrow \frac{\left(\log x - \log y\right)2\left(\log x + \log y\right)}{2\left(\log x - \log y\right)x\left(\log x + \log y\right)}$$

$$= 1$$

8. **(c)**

$$\Rightarrow a > 0, b > 0, c > 0$$

$$\Rightarrow \log\left(a^a b^b c^c\right) + \log\left(\frac{1}{abc}\right)$$

$$\Rightarrow \log\left(\frac{a^a b^b c^c}{abc}\right)$$

$$\Rightarrow \log\left(a^{a-1} b^{b-1} c^{c-1}\right)$$

9. **(a)**

$$\log_4\left(\frac{4}{4}\right) - 2\log_4\left(4(-2)^4\right) = \log_4{}^1 - 2\log_4\left(4^3\right)$$

$$= 0 - 2 \times 3 = -6$$

10. (c)

$\Rightarrow \log_x x.\log_5 k = \log_x 5$; given $k \neq 1, k > 0$

$\Rightarrow \frac{\log x}{\log k}.\frac{\log k}{\log 5} = \frac{\log 5}{\log x}$

$\Rightarrow (\log_5 x) = (\log_x 5)$

$\Rightarrow x = 5$ is the only possible solution

11. (c)

$\Rightarrow \log_5 a.\log_a x = 2$

$\Rightarrow \frac{\log a}{\log 5}, \frac{\log x}{\log a} = 2$

$\Rightarrow \log_5 x = 2$

$\Rightarrow x = 5^2$

$= 25$

12. (b)

$\Rightarrow \log(x+1) + \log(x-1) = \log 3$

$\Rightarrow \log(x+1)(x-1) = \log 3$

$\Rightarrow x^2 - 1 = 3$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

But $x+1>0$ & $x-1>0$

$\Rightarrow x > -1 \& x > 1$

So, $x \in (1, \infty)$ is our feasible region

Only $x = 2$ lies in the feasible region.

13. (b)

$\Rightarrow \log_{10}(2x^2 + 7x + 16) = 1$

$\Rightarrow 2x^2 + 7x + 16 = 10^1$

$\Rightarrow 2x^2 + 7x + 6 = 0$

$\Rightarrow 2x^2 + 4x + 3x + 6 = 0$

$\Rightarrow (2x+3)(x+2) = 0$

$\Rightarrow x = -\frac{3}{2}.-2$

14. (b)

$\Rightarrow \log_{10}[\log_{10}(\log_{10} x)] = 0$

$\Rightarrow \log_{10}(\log_{10} x) = 10^0 = 1$

$\Rightarrow \log_{10} x = 10^1 = 10$

$\Rightarrow x = 10^0$

15. **(d)**

$\Rightarrow \log_{16} x + \log_4 x + \log_2 x = 14$

$\Rightarrow \frac{1}{4}\log_2 x + \frac{1}{2}\log_2 x + \log_2 x = 14 \quad \left(\text{by} \log_{a^{k^b}} = \frac{1}{k}\log_a b\right)$

$\Rightarrow \frac{7}{4}\log_2 x = 14$

$\Rightarrow \log_2 x = 8$

$\Rightarrow x = 2^8 = 256$

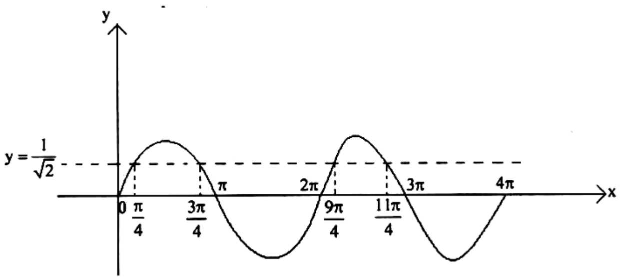
16. **(a)**

$\Rightarrow \log_{\left(\frac{1}{\sqrt{2}}\right)} \sin x > 0; x \in [0, 4\pi]$

As base $\frac{1}{\sqrt{2}}$ lies between 0 to 1satisfy given inequality, $0 < \sin x < 1$

$\Rightarrow x \in (0, \pi) \cup (2\pi, 3\pi)$

As we can see in this interval



We get $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ as integral

Multiples of $\frac{\pi}{4}$

17. **(b)**

$\Rightarrow \log_{\frac{1}{2}}\left(x^2 - 6x + 12\right) \ge -2$

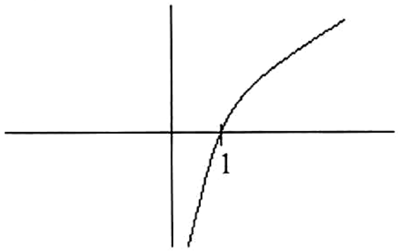
$\Rightarrow \log_{2^{-1}}\left(x^2 - 6x + 12\right) \ge -2$

$\Rightarrow -1 \times \log_2\left(x^2 - 6x + 12\right) \ge -2$

$\Rightarrow \log_2\left(x^2 - 6x + 12\right) \le 2$

$\Rightarrow \log_2\left(x^2 - 6x + 12\right) - \log_2 4 \le 0$

$\Rightarrow \log_2\left(\frac{x^2 - 6x + 12}{4}\right) \le 0$



$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4} \le 1$

Case-1

$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4}$

$\Rightarrow x^2 - 6x + 12 > 0$

$\Rightarrow x \in R$ …… (1) as discriminant if quadratic expression $x^2 - 6x + 12$ is less than zero.

Descriminant $D = (-6)^2 - 4(12)(1)$

$\Rightarrow D = -12$

Case-2

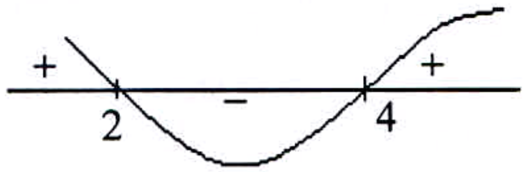
$\Rightarrow \frac{x^2 - 6x + 12}{4} \le 4$

$\Rightarrow x^2 - 6x + 12 \le 4$

$\Rightarrow x^2 - 6x + 8 \le 0$

$\Rightarrow (x-4)(x-2) \le 0$

$\Rightarrow x \in [2, 4]$ ……..(ii)



By taking intersection of (i) & (ii) we get $x \in [2, 4]$

18. **(b)**

$\Rightarrow 2^{\log_{\sqrt{2}}(x-1)} > x + 5$ Here $x - 1 >; x > 1$ ……(1)

$\Rightarrow 2^{\log_{(2)^{\frac{1}{2}}}(x-1)} > x + 5$

$\Rightarrow 2^{2\log_2(x-1)} > x + 5$ (or by formula $\log_{a^k} b = \frac{1}{k} \log \frac{b}{a}$)

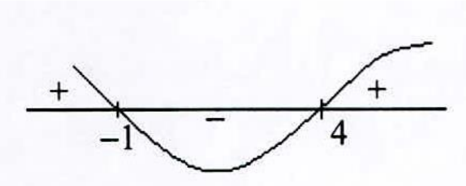
$\Rightarrow 2^{\log_2(x-1)^2} > x + 5$

$\Rightarrow (x-1)^2 > x + 5$

$\Rightarrow x^2 + 1 - 2x > x + 5$

$\Rightarrow x^2 - 3x - 4 > 0$

$\Rightarrow (x-4)(x+1) > 0$



So we get $x \in (-\infty, -1) \cup (4, \infty)$ ....(ii)

By taking intersection of (i) & (ii)

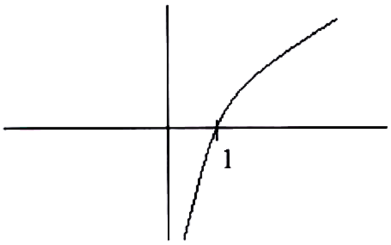
$\Rightarrow x \in (4, \infty)$

19. **(c)**

$\Rightarrow \log_{10}(x^2 - 2x - 2) \leq 0$

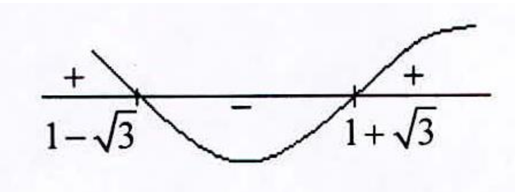
As base is greater than 1 so to hold the inequality true

$\Rightarrow 0 < x^2 - 2x - 2 \leq 1$



So, $0 < x^2 - 2x - 2$ and $x^2 - 2x - 2 \leq 1$

Case-I



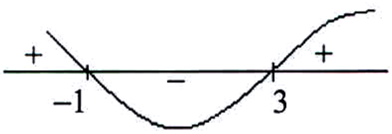
$\Rightarrow x^2 - 2x - 2 > 0$

$\Rightarrow \left[x - (1+\sqrt{3})\right]\left[x - (1-\sqrt{3})\right] > 0$

So, $x \in (-\infty, 1-\sqrt{3}) \cup (1+\sqrt{3}, \infty)$ ....(i)

Case-2

$\Rightarrow x^2 - 2x - 2 \leq 1$



$\Rightarrow x^2 - 2x - 3 \leq 0$

$\Rightarrow (x-3)(x+1) \leq 0$

So we get $x \in [-1, 3]$

By taking intersection of (i) & (ii) we get,

$\Rightarrow x \in \left[-1, 1-\sqrt{3}\right) \cup \left(1+\sqrt{3}, 3\right]$

20. **(a)**

$\Rightarrow \log_{0.2}\dfrac{x+2}{x} \le 1$

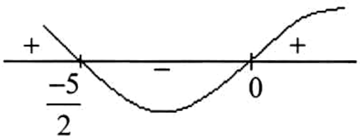
As base of log is less than 1 so hold the inequality true

$\Rightarrow \dfrac{x+2}{x} \ge 0.2$

$\Rightarrow \dfrac{x+2}{x} - 0.2 \ge 0$

$\Rightarrow \dfrac{x+2-0.2x}{x} \ge 0$

$\Rightarrow \dfrac{0.8x+2}{x} \ge 0$



So, $x \in \left(-\infty, \dfrac{-5}{2}\right] \cup [0, \infty)$

$\Rightarrow a^{m\log_a n} \Rightarrow a^{\log_a n^m} \Rightarrow n^m$

21. **(d)**

$\Rightarrow \log_{\frac{1}{2}}\left(x^2-1\right) > 0$

As base of log is less than 1

So, $\log_{\frac{1}{2}}\left(x^2-1\right) > 0$

$\Rightarrow 0 < x^2 - 1 < 1$

$\Rightarrow 1 < x^2 < 2$

$\Rightarrow x^2 > 1$ and $x^2 < 2$

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty) \,\&\, x \in \left(-\sqrt{2}, \sqrt{2}\right) \Rightarrow x \in \left(-\sqrt{2}, -1\right) \cup \left(1, \sqrt{2}\right)$

22. **(a)**

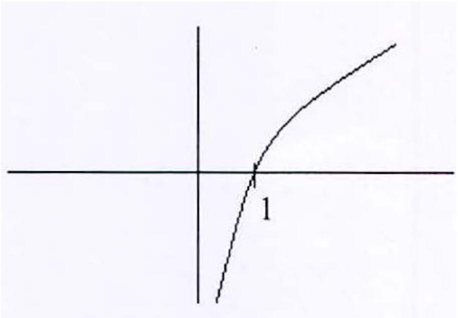
$\Rightarrow \log_{\frac{\sqrt{3}}{2}}\left(x^2-3x+2\right) \ge 2$

$\Rightarrow \log_{\frac{\sqrt{3}}{2}}\left(x^2-3x+2\right) \ge \log_{\frac{\sqrt{3}}{2}}\left(\dfrac{\sqrt{3}}{2}\right)^2$

$\Rightarrow \log_{\frac{\sqrt{3}}{2}}\left(\dfrac{x^2+3x+2}{\left(\dfrac{3}{4}\right)}\right) \ge 0$

$\because$ base is less than 1 so

$$\Rightarrow 0 < \left(\frac{x^2-3x+2}{\frac{3}{4}}\right) \le 1$$



$$\Rightarrow 0 < x^2-3x+2 \,\&\, \left(x^2-3x+2\right) \le \frac{3}{4}$$

$$\Rightarrow (x-2)(x-1) > 0 \,\&\, x^2-3x+\frac{5}{4} \le 0$$

$$\Rightarrow x \in (-\infty,1)\cup(2,\infty) \qquad \ldots\ldots.(1) \;\&\; x^2-\frac{5}{2}x-\frac{1}{2}x+\frac{5}{4} \le 0$$

$$\&\left(x-\frac{5}{2}\right)\left(x-\frac{1}{2}\right) \le 0$$

$$\& x \in \left(\frac{1}{2},\frac{5}{2}\right) \qquad \ldots\ldots.(2)$$

Taking intersection of (i) & (ii)

$$\Rightarrow x \in \left[\frac{1}{2},1\right) \cup \left(2,\frac{5}{2}\right]$$

23. **(a)**

$$\log_{1/3}\left(x^2+x+1\right)+1 < 0$$

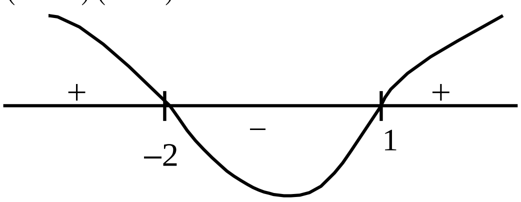
$$\log_{1/3}\left(x^2+x+1\right) < -1$$

$$x^2+x+1 > \left(\frac{1}{3}\right)^{-1}$$

$$x^2+x+1 > 3$$

$$x^2+x-2 > 0$$

$$(x+2)(x-1) > 0$$



$$x \in (-\infty,-2)\cup(1,\infty)$$

24. **(b)**

$$\Rightarrow \left(x^5\right)^{\frac{1}{3}}\left(16x^3\right)^{\frac{2}{3}}\left(\frac{1}{4}x^{\frac{4}{9}}\right)^{-\frac{3}{2}}.(4)^{\frac{1}{6}}$$

$$\Rightarrow x^{\frac{5}{3}}.\left(4^2\right)^{\frac{2}{2}}.\left(x^3\right)^{\frac{2}{2}}.\left(4^{-1}\right)^{\frac{-3}{2}}.\left(x^{\frac{4}{9}}\right)^{\frac{-3}{2}}$$

$$\Rightarrow x^{\frac{5}{3}} \times 4^{\frac{4}{3}} \times x^2 \times 4^{\frac{3}{2}} \times x^{\frac{4}{9}\times\frac{-3}{2}}$$

$$x^{\frac{5}{3}-\frac{2}{3}+2}.4^{\left(\frac{4}{3}+\frac{3}{2}+\frac{1}{6}\right)}$$

$$\Rightarrow 4^3.x^3$$

25. **(c)**

$$\Rightarrow \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}.\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}.\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$$

$$\Rightarrow x^{\left(\frac{1}{c}-\frac{1}{b}\right)} \times x^{\left(\frac{1}{a}-\frac{1}{c}\right)} \times x^{\left(\frac{1}{b}-\frac{1}{a}\right)}$$

$$\Rightarrow x^{\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}+\frac{1}{b}-\frac{1}{a}}$$

$$\Rightarrow x^0 = 1$$

26. **(c)**

$$\Rightarrow a^m.a^n = a^{mn}$$

$$\Rightarrow a^{m+n} = a^{nm}$$

$$\Rightarrow m+n = mn \qquad \ldots\ldots\ldots(1)$$

Then $m(n-2)+n(m-2)=?$

$$\Rightarrow 2mn-2m-2n$$

$$\Rightarrow 2(m+n)-2(m+n)$$

27. **(b)**

$$\Rightarrow \frac{2^{m+3} \times 3^{2m-2n} \times 5^{m+3+n} \times 6^{n+1}}{6^{m+1} \times 10^{n+3} \times 15^m}$$

$$\Rightarrow \frac{\left(2^{m+3}\right)\left(3^{2m-2}\right)\left(5^{m+n+3}\right) \times 2^{n+1} \times 3^{n+1}}{\left(2^{m+1}3^{m+1}\right)\left(2^{n+3}5^{n+3}\right)\left(3^m\right)\left(5^m\right)}$$

$$\Rightarrow 2^{m+3+n+1-(m+1)}3^{2m-n+n+1-(m+1)-m}5^{m+n+3-(n+3)-m}$$

$$\Rightarrow 2^0 3^0 5^0$$

$$\Rightarrow 1$$

28. **(c)**

$$a^{mn} = a^{m^n}$$

$$\Rightarrow mn = m^n$$

$$\Rightarrow n = m^{n-1}$$

$$\Rightarrow m = n^{\frac{1}{n-1}}$$

29. **(d)**

$$\Rightarrow \frac{\left(2^{n+1}\right)^{m}\left(2^{2n}\right)2^{n}}{\left(2^{m+1}\right)^{m}2^{2m}}=1$$

$$\Rightarrow \frac{2^{nm+m}\times 2^{2n+n}}{2^{mn+m}2^{2m}}=1$$

$$\Rightarrow 2^{(nm+m+2n+n)-(mn+n-2m)}=1$$

$$\Rightarrow 2^{2n-m}=1$$

$$\Rightarrow 2n-m=0$$

$$\Rightarrow m=2n$$

30. **(c)**

$$\Rightarrow 5^{x-1}+5(0.2)^{x-2}=26$$

$$\Rightarrow 5^{x-1}+5\left(\frac{1}{5}\right)^{x-2}=26$$

$$\Rightarrow 5^{x-1}+5^{1-x+2}=26$$

$$\Rightarrow 5^{x-1}+5^{3-x}=26$$

At $x=1,3$ above equation satisfy

31. **(c)**

$$\Rightarrow 2^{x}-2^{x-1}=4$$

$$\Rightarrow 2^{x}-\frac{2^{x}}{2}=4$$

$$\Rightarrow 2.2^{x}-2^{x}=8$$

$$\Rightarrow 2^{x}=8$$

$$\Rightarrow x=3$$

So, $x^{x}=3^{x}=27$

32. **(c)**

$$\Rightarrow (25)^{x-2}=(125)^{2x-4}$$

$$\Rightarrow \left(5^{2}\right)^{x-2}=\left(5^{3}\right)^{2x-4}$$

$$\Rightarrow 5^{2x-4}=5^{6x-12}$$

$$\Rightarrow 5^{6x-12-(2x-4)}=1$$

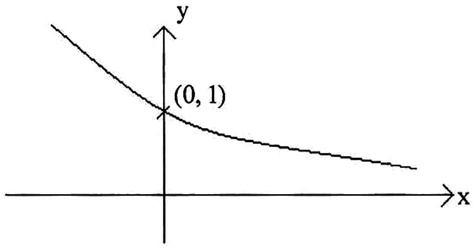
$$\Rightarrow 5^{4x-8}=1=5^{\circ}$$

So, $4x-8=0$

$$\Rightarrow x=2$$

33. **(a)**

$\Rightarrow a^{x^2-x} \ge a^2; 0 < 9 < 1$



$\Rightarrow \frac{a^{x^2}-x}{a^2} \ge 1$

$\Rightarrow a^{x^2-x-2} \ge 1$

$\Rightarrow x^2 - x - 2 \le 0$

$\Rightarrow (x-2)(x+1) \le 0$

$\Rightarrow x \in [-1, 2]$

34. **(a)**

$4^{-x+0.5} - 7.2^{-x} < 4$

$\frac{4^{0.5}}{4^x} - \frac{7}{2^x} < 4$

Let $\frac{1}{2^x} = k$

$\therefore 2.k^2 - 7k < 4$

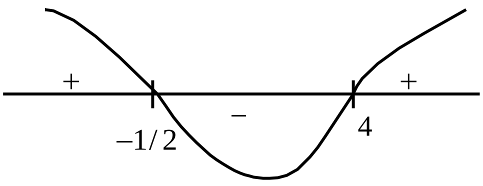
$2k^2 - 7k - 4 < 0$

$2k^2 - 7k - 4 < 0$

$2k^2 - 8k + k - 4 < 0$

$2k(k-4) + (k-4) < 0$

$(2k+1)(k-4) < 0$



$k \in \left(-\frac{1}{2}, 4\right)$

As $k = \frac{1}{2^x}$ it can only be $+ve$

$\therefore 0 < \frac{1}{2^x} < 4$

$\Rightarrow x \in (-2, \infty)$

35. (d)

$\Rightarrow f(x) = 5 - |x-3|$

$\Rightarrow f(x) = 5-(x-3); x \in [3, \infty)$

$\Rightarrow 5+(x-3); x \in (-\infty, 3)$

$\Rightarrow f(x) = 8-x; x \in [3, \infty)$

$= 2+x; x \in (\infty, 3)$

So, greatest value of function occur at $x = 3$

So $f(3) = 8-3 = 5$

36. (b)

Let $f(x) = |x-4|+2$

As we know that $|x| \geq 0$ for every $x \in R$

$\therefore |x-4| \geq 0$

The minimum value of function is attained when $|x-4| = 0$

Hence, minimum value of $f(x) = 0+2 = 2$

Alternate Method

$f(x) = |x-4|+2$

There is one critical point i.e. $x = 4$

$f(4) = |4-4|+2$

$= 0+2$

$= 2$

Hence, 2 is the minimum value of $f(x)$.

37. (b)

Given equation is $x^2 - 3|x| + 2 = 0$

We can write this as

$|x|^2 - 3|x| + 2 = 0$

$\Rightarrow |x|^2 - |x| - 2|x| + 2 = 0$

$\Rightarrow |x|(|x|-1) - 2(|x|-1) = 0$

$\Rightarrow (|x|-1)(|x|-2) = 0$

This is possible if, at least one of the two factors is zero, i.e.

$|x|-1 = 0$ or $|x|-2 = 0$

$\Rightarrow |x| = 1$ or $|x| = 2$

$\Rightarrow x = \pm 1$ or $x = \pm 2$

Clearly, we can see that there is four distinct value of *x*.

38. **(b)**

Given $\left|x^2-12x+32\right|+\left|x^2-9x+20\right|=0$.

Every modulus function is a non-negative function and if two non-negative functions add up to get zero then individual function itself equal to zero simultaneously.

$x^2-12x+32=0$ for $x=$ 4 or 8

$x^2-9x+20=0$ for $x=4$ or 5

Both the equations are zero at $x=4$

So, $x=4$ is the only solution for this equation.

39. **(a)**

Since the modulus function '||' always returns a positive value or 0, its not possible to have –2 as the value of modulus of any expression.

Hence, $|x+2|=-2$ has **no solution**.

40. **(a)**

$\Rightarrow x^2-|x|-6=0$

Case 1: $x \ge 0$

$\Rightarrow x^2-x-6=0$

$\Rightarrow x=3,-2$

But $x \ge 0$ so $x=3$ is the only root.

Case 2: $x<0$

$\Rightarrow x^2-(-x)-6=0$

$\Rightarrow x^2+x-6=0$

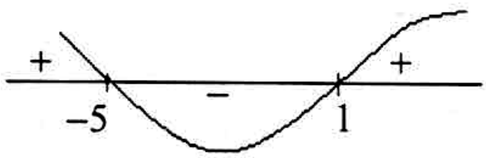
$\Rightarrow x=3,-2$

But $x<0$ so $x=-3$ is the solution.

So multiplication $=3(-3)=-9$

41. **(a)**

$\Rightarrow |x-1|+|x+5|=6$



This is special case as $(x-1)-(x+5)=-6$

So the given expression will hold true if $(x-1)(x+5)\le 0$

$\Rightarrow x \in [-5,1]$

42. **(b)**

$|x-3|+|x+5|=7x$

$2x+2=7x \quad x\ge 3$

$-(x-3)+(x+5)=7x \quad -5<x<3$

$-(x-3)-(x+5)=7x \quad x\le -5$

43. **(c)**

Let $2x+3>5$ and $2x+3<-5$ {by the property of modulus}

So, first take $2x+3>5$

$\Rightarrow 2x+3-3>5-3$

$\Rightarrow 2x>2$

$\Rightarrow x>1$

Hence, $x\in(1,\infty)$

Now take $2x+3<-5$

$\Rightarrow 2x+3-3<-5-3$

$\Rightarrow 2x<-8$

$\Rightarrow x<-4$

Hence, $x\in(-\infty,-4)$

$\therefore x\in(-\infty,-4)\cup(1,\infty)$

44. **(c)**

$\Rightarrow |4-3x|\le\frac{1}{2}$

Case 1: $4-3x\ge0\Rightarrow x\le\frac{4}{3}$ …….(i)

So $4-3x\le\frac{1}{2}$

$\Rightarrow -3x\le-\frac{7}{2}$

$\Rightarrow x\ge\frac{7}{6}$

By intersection of (i) & (ii) $x\in\left[\frac{7}{6},\frac{4}{3}\right]$ ……..(A)

Case 2: $4-3x\le0\Rightarrow x>\frac{4}{3}$ ………(iii)

$\Rightarrow$ So $-(4-3x)\le\frac{1}{2}$

$\Rightarrow -4+3x\le\frac{1}{2}$

$\Rightarrow 3x\le\frac{9}{2}$

$\Rightarrow x\le\frac{3}{2}$ ………(iv)

Taking intersection of (iii) & (iv)

$\Rightarrow x\in\left(\frac{4}{3},\frac{3}{2}\right]$ ……….(B)

So union of A & B is the solution of the given inequality $x\in\left[\frac{7}{6},\frac{3}{2}\right]$ ……..(C)

45. **(c)**

$\Rightarrow \frac{|x|-1}{|x|+2} > 0$

So, $|x| > -2$ or $|x| > 1$

$\because |x| > -2$ holds true for $x \in R$

Now, $|x| > 1$

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

46. **(d)**

$||x|-1| < |1-x|$

Case I $x \geq 1$

$x-1 < -(1-x)$

$x-1 < -(1-x)$

$-1 < -1$

No solution

Case II $0 \leq x \leq 1$

$1-x < 1-x$

No solution

Case III $-1 \leq x < 0$

$|-x-1| < 1-x$

$1+x < 1-x$

$x < 0$

$\Rightarrow x \in [-1, 0)$

Case IV: $x < -1$

$|-x-1| < |1-x|$

$|1+x| < 1-x$

$-(1+x) < 1-x$

$-1-x < 1-x$

$-1 < 1$

True for all x.

$x \in (-\infty, -1)$

$\therefore x \in (-\infty, -1] \cup [-1, 0)$

$\Rightarrow x \in (-\infty, 0)$

47. **(a)**

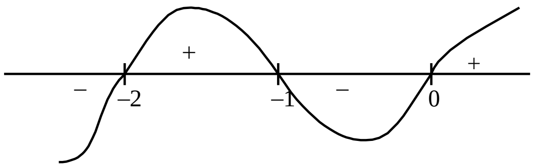
$\left|x+\frac{2}{x}\right|<3$

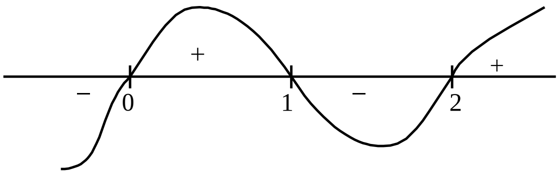
$-3<x+\frac{2}{x}<3$

$x+\frac{2}{x}+3>0$, $x+\frac{2}{x}-3<0$

$\frac{x^2+3x+2}{x}>0$, $\frac{x^2-3x+2}{x}<0$

$\frac{(x+1)(x+2)}{x}>0$, $\frac{(x-1)(x-2)}{x<0}$





$x\in(-2,-1)\cup(0,\infty)$, $x\in(-\infty,0)\cup(1,2)$

$\therefore x\in(-2,-1)\cup(1,2)$

48. **(b)**

$x^2=|x+2|+x>0$

Case I $x\ge -2$

$x^2-(x+2)+x>0$

$x^2-2>0$

$x\in\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$

Case II $x\le -2$

$x^2+(x+2)+x>0$

$x^2+2x+2>0$

$(x+1)^2+1>0$

$x\in R \text{ i.e. } x<-2$

From (1) & (2)

$x\in\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$

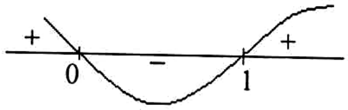
49. **(c)**

$\Rightarrow\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$ here $\frac{x}{1-x}\ge 0;\frac{x}{1-x}\ge 0;\frac{x}{x-1}\le 0$

So, $x\in[0,1)$ is the feasible region for the equation

$\Rightarrow\frac{(x)+(1-x)}{\sqrt{1-x}\sqrt{x6}}=\frac{13}{6}$

$\Rightarrow\frac{1}{\sqrt{x(1-x)}}=\frac{13}{6}$



Taking square both side

$\Rightarrow x(1-x)=\dfrac{36}{169}$

$\Rightarrow x^2-x+\dfrac{36}{169}=0$

$\Rightarrow \left(x-\dfrac{9}{13}\right)\left(x-\dfrac{4}{13}\right)=0$

$\Rightarrow x=\dfrac{9}{13},\dfrac{4}{13}$

Here values lies in the feasible region

So, $x=\dfrac{9}{13},\dfrac{4}{13}$

50. (d)

$\Rightarrow \sqrt{3y+1}=\sqrt{y-1}$ ……..(1)

$\Rightarrow 3y+1\geq 0 \,\&\, y-1\geq 0$

$\Rightarrow y\geq -\dfrac{1}{3} \,\&\, y\geq 0$

$\Rightarrow y\in[0,\infty)$ is our feasible region

By equation (1), taking square of both side,

$\Rightarrow 3y+1=y-1$

$\Rightarrow 2y=-2$

$\Rightarrow y=-1$; which does not lie in feasible range of y.

So no solution of y.

51. (a)

$\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$

$\Rightarrow (x+1)+(x-1)-2\sqrt{x^2-1}$

$=4x-1$

$\Rightarrow 2x+1-4x=2\sqrt{x^2-1}$

$\Rightarrow 1-2x=2\sqrt{x^2-1}$

$\Rightarrow (1-2x)^2=4\left(x^2-1\right)$

$\Rightarrow 1-4x+4x^2=4x^2-4$

$\Rightarrow 4x=5 \Rightarrow x=\dfrac{5}{4}$

Putting $x=\dfrac{5}{4}$ in the original equation,

L.H.S. = 1 & R.H.S. = 2,

Hence, no solution.

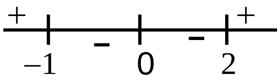
52. **(a)**

$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$

So, $1-\left(\frac{x+2}{x^2}\right) \geq 0$

$\Rightarrow \frac{x^2-x-2}{x^2} \geq 0$

$\Rightarrow \frac{(x-2)(x+1)}{x^2} \geq 0$



So $x \in (-\infty,-1] \cup [2,\infty)$ ……(1)

$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$

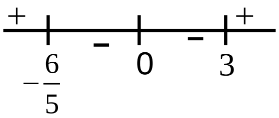
$\Rightarrow 1-\left(\frac{x+2}{x^2}\right) < \frac{4}{9}$

$\Rightarrow \frac{5}{9}-\frac{x+2}{x^2} < 0$

$\Rightarrow \frac{5x^2-9x-18}{9x^2} < 0$

$\Rightarrow \frac{(5x+6)(x-3)}{x^2} < 0$

So, $x \in \left(-\frac{6}{5},0\right) \cup (0,3)$ …….(2)



By taking $(1) \cap (2)$

$\Rightarrow x \in \left(-\frac{6}{5},-1\right] \cup [2,3)$

53. **(c)**

$\Rightarrow (x-1)\sqrt{x^2-x-2} \geq 0$

$\Rightarrow (x^2-x-2) \geq 0$ & $(x-1) \geq 0$

$\Rightarrow (x-2)(x+1) \geq 0$ & $x \geq 1$

$\Rightarrow x \in (-\infty,-1] \cup [2,\infty)$ & $x \in [1,\infty)$

So, $x \in [2,\infty)$

EXERCISE - 1 [B]

1. **(b)**

$\Rightarrow \log(ab) - \log|b|$

We can see that $ab > 0 \Rightarrow a < 0 \& b < 0$ or $a > 0 \& b > 0$

So $\log(ab) - \log|b| = \log|ab| - \log|b|$

$= \log|a|.|b| - \log|b|$

$= \log|a| + \log|b| - \log|b|$

$= \log|a|$

2. **(b)**

$\log_3 M + 9\log_3 N = 3(1 + \log_{0.008} 5)$

$\log_3 MN^9 = 3(\log_{0.008} 5 \times 0.008)$

$\log_3 (MN^9) = 3\log_{0.008} 0.04 = 3 \times \frac{2}{3}$

So $MN^9 = 9$

3. **(b)**

$x = \log_a nc = \frac{\log bc}{\log a}$

$x + 1 = \frac{\log bc}{\log a} + 1$

$\Rightarrow x + 1 = \frac{\log bc + \log x}{\log a}$

$\Rightarrow x + 1 = \frac{\log abc}{\log a}$

Similarly

$y + 1 = \frac{\log abc}{\log b}$ & $z + 1 = \frac{\log abc}{\log c}$

$\therefore \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$\Rightarrow \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$

$= \frac{\log abc}{\log abc} = 1$

4. **(c)**

$N = \frac{4^5 + 4^5 + 4^5 + 4^5}{3^5 + 3^5 + 3^5} \cdot \frac{6^5 + 6^5 + 6^5 + 6^5 + 6^5 + 6^5}{2^5 + 2^5}$

$= N = \frac{4 \times 4^5}{3 \times 3^5} \times \frac{6 \times 6^5}{2 \times 2^5}$

$$= \frac{4^6 \times 6^5}{(2\times 3)^5} = \frac{4^6 \times 6^5}{6^5}$$

$N = 4^6$

$N = \left(2^2\right)^6$

$N = 2^{12}$

$\log_2 N = \log_2 2^{12} = 12$

5. **(c)**

$\log 15 = a \qquad\qquad \log 75 = b$

$\log 5 + \log 3 = a \;\Rightarrow\; \log 15 + \log 5 = b \qquad ...(\text{i})$

$\Rightarrow\; \log 3 + 2\log 5 = b$ (from (i))

$\Rightarrow\; -b + 2a = \log 3$

$$\log_{75} 45 = \frac{\log 15 + \log 3}{\log 75} = \frac{a - b + 2a}{b} = \frac{3a - b}{b}$$

6. **(a)**

$\log_a b = 2, \log_b c = 2, \log_3 c = \log_3 a + 3$

$\log_a b = 2 \;\Rightarrow\; b = a^2$

$\log_b c = 2 \;\Rightarrow\; c = b^2 \qquad \Rightarrow \quad \log_3 c/a = 3 \Rightarrow c = 27a$

Now $a > 0, b > 0, c > 0, a \neq 1, b \neq 1$

If $b = a^2$ & $c = b^2$

$c = a^4 = 27a = 0 \;\Rightarrow\; a\left(a^3 - 27\right) \qquad \Rightarrow$ this gives $a = 0, a = 3$

$a = 3$

$b = a^2 = 9$

$c = b^2 = 81$

7. **(b)**

$$\frac{2}{6\log_4 2000} + \frac{3}{6\log_5 2000}$$

$$= \frac{1}{6}\left[2\log_{2000} 4 + 3\log_{2000} 5\right]$$

$$= \frac{1}{6}\left[\log_{2000} 4^2.5^3\right]$$

$$= \frac{1}{6}\log_{2000} 2000 = \frac{1}{6}$$

8. **(b)**

$\log_{10}{}^2 = \beta$

$$\log_{10}\left(\frac{1025}{1024} \times \frac{4}{4}\right) = \alpha$$

$\Rightarrow\; \log_{10} 4100 - \log_{10} 2^{12} = \alpha$

$\Rightarrow\; \log_{10} 4100 = \alpha + 12\beta$

9. **(d)**

We know that, for any $-1<r<1, a+ar+ar^2+......+\infty=\dfrac{a}{1-r}$ therefore

$$\frac{1}{3}+\frac{1}{3^2}+......+\infty=\frac{1/3}{1-1/3}=\frac{1}{2}$$

Finally we have

$$(0.16)^{\log_{2.5}\left[(1/3)+\left(1/3^2\right)+......+\infty\right]}=\left(\frac{2}{5}\right)^{2\log_{2.5}(1/2)}=\left(\frac{2}{5}\right)^{-2\log_{2.5}(2)}=\left(\frac{5}{2}\right)^{\log_{5/2}(4)}=4$$

10. **(d)**

$$\Rightarrow 81^{\left(\frac{1}{\log_5 3}\right)}+27^{(\log_9 36)}+3^{\frac{4}{\log_7 9}}$$

$$\Rightarrow (81)^{(\log_3 5)}+\left(3^3\right)^{\left(\log_{3^2} 36\right)}+3^4\log_9 7$$

$$\Rightarrow \left(3^4\right)^{\log_3 5}+3^{3\times\left(\frac{1}{2}\log_3 36\right)}+3^{4\times\left(\frac{1}{2}\log_3 7\right)}$$

$$\Rightarrow 3^{\log_3 5^4}+3^{\log_3 (36)^{\frac{3}{2}}}+3^{\log_3 7^2}$$

$$\Rightarrow 5^4+(36)^{\frac{3}{2}}+7^2$$

$$625+36\times6+49=890$$

11. **(b)**

$$\text{A}=12^{300}$$

$$\log_{10}\text{A}=300[\log_{10}12]$$

$$=300[0.6010+0.4771]$$

$$=300\times1.0781=323.43$$

$$\Rightarrow \text{A}=10^{323.43}$$

Hence 324 digits

12. **(a)**

$$\log_{10}2=0.3010$$

$$\log_5 64=\frac{\log_{10}64}{\log_{10}5}=\frac{6\log_{10}2}{\log_{10}10-\log_{10}2}$$

$$=\frac{6\times0.3010}{1-0.3010}$$

$$=\frac{1.8060}{0.6990}$$

$$=\frac{602}{233}$$

13. **(a)**

$$\text{x}=\log_5(1000)=\log_5 125+\log_5 8>4$$

$$\text{y}=\log_{57}(2056)=\log_7 343+\log_7 6<4$$

Hence $\text{x}>4$

14. **(d)**

Note that $x>0, x\neq 1$.

Using $a^{\log_a x}=a$, we get $(1-x)^2=9 \Rightarrow x=4,-2$

Also $x>0$, we get $x=4$.

15. **(b)**

$2^{\log_{10}3\sqrt{3}}=3^{k\log_{10}2}$

$\Rightarrow 2^{\log_{10}3\sqrt{3}}=2^{k}\log_{10}3$

$\Rightarrow \log_{10}3\sqrt{3}=\log_{10}3^{k}$

$\Rightarrow k=3^{3/2}$

$\Rightarrow k=3/2$

16. **(b)**

$\log_{10}(x-1)^3-\log_{10}(x-3)^3=\log_{10}8$

$\Rightarrow \log_{10}\left(\frac{x-1}{x-3}\right)^3=\log_{10}(2)^3 \quad \Rightarrow \frac{x-1}{x-3}=2 \quad \Rightarrow x-1=2x-6$

$\Rightarrow x=5$

So, $\log_x 625=\log_5(5)^4=4$

17. **(d)**

$\log_{x-3}\log_{2x^2-2x+3}\left(x^2+2x\right)=0$

$\Rightarrow \log_{2x^2-2x+3}\left(x^2+2x\right)=1$

$\Rightarrow \left(x^2\right)+2x=2x^2-2x+3$

$\Rightarrow x^2-4x+3=0$

$\Rightarrow (x-3)(x-1)=0$

$\Rightarrow x=1$ or $x=3$

Both do not belong to the domain. So, no solution

18. **(c)**

$\log_2\log x$ is meaningful if $x>1$

Since $4^{\log_2\log x}=2^{2\log_2\log x}=\left(2^{\log_2\log x}\right)^2=(\log x)^2 \qquad \left[a^{\log_a x}=x, a>0, a\neq 1\right]$

So the given equation reduces to

$2(\log x)^2-\log x-1=0$

$\Rightarrow \log x=1, \log x=-\frac{1}{2}$.

But for $x>1$, $\log x>0$

So, $\log x=1$ i.e. $x=e$.

19. **(a)**

The equation is meaningful if $|\sin x|\neq 0,1$ and $1+\cos x\neq 0$

So $x\neq k\pi, k=0,1,....n, x\neq(2k+1)\frac{\pi}{2}, k=0,1,....n-1$.

Now, $\log_{|\sin x|}(1+\cos x)=2$

$\Leftrightarrow \quad 1+\cos x=|\sin x|^2=\sin^2 x=1-\cos^2 x$

$\Rightarrow \quad (1+\cos x)(\cos x)=0$

$\Leftrightarrow \quad \cos x=0$ or $\cos x=-1$

$\Rightarrow \quad \cos x=0 \Rightarrow x=(2k+1)\frac{\pi}{2}.$

So there is no *x* which satisfies the given equation.

20. (d)

$\Rightarrow \log_2 x+\log_x 2=\frac{10}{3}=\log_2 y+\log_y 2$

$\Rightarrow \log_2 x+\log_x 2=\frac{10}{3}$

$\Rightarrow \log_2 x+\frac{1}{\log_2}x=\frac{10}{3}$

Let's take $\log_2 x=a$

So, $a+\frac{1}{a}=\frac{10}{3}$

$\Rightarrow a^2-\frac{10}{3}a+1=0$

$\Rightarrow \left(a-\frac{9}{3}\right)\left(a-\frac{1}{3}\right)=0$

$\Rightarrow a=\frac{9}{3},\frac{1}{3}$

$\Rightarrow \log_2 x=\frac{9}{3},\frac{1}{3}=3,\frac{1}{3}$

$\Rightarrow x=2^3,2^{\frac{1}{3}}$

$\Rightarrow x=8,2^{\frac{1}{3}}$

Similarly, $y=8,2^{\frac{1}{3}}$

If $x\neq y$ then $x+y=8+2^{\frac{1}{3}}$

21. (b)

$x^{\log_3 x^2}+(\log 3x)^2-10=\frac{1}{x^2}$

Clearly one solution is $x=1$

OR

$\log_3 x^2+(\log_3 x)^2-10=-2$

$2\log_3 x+(\log_3 x)^2=8$

Let $\log_3 x=k$

$\therefore 2k+k^2-8=0$

$k^2+2k-8=0$

$(k+4)(k-2)=0$

$\Rightarrow \log_3 x = 2 \text{ or } \log_3 x = -4$

$\Rightarrow x = 0 \text{ or } x = \frac{1}{81}$

$x \in \left\{1, 9, \frac{1}{81}\right\}$

22. **(c)**

$\log_{1/2} x \geq \log_{1/3} x$

$-\log_2 x \geq -\log_3 x$

$\log_2 x \leq \log_3 x$

$\frac{\log x}{\log 2} \leq \frac{\log x}{\log 3}$

$\log^x \left[\frac{1}{\log 2} - \frac{1}{\log 3}\right] \leq 0$

$\log x \leq 0 \left\{\because \frac{1}{\log 2} - \frac{1}{\log 3} > 0\right\}$

$\Rightarrow x \in (0, 1]$

23. **(b)**

$\Rightarrow \log_4\left(\frac{x+1}{x+2}\right) > 0$

$\Rightarrow \frac{x+1}{x+2} > 0$

$\Rightarrow \frac{x+1-x-2}{x+1} > 0$

$\Rightarrow \frac{-1}{x+2} > 0$

$\Rightarrow \frac{1}{x+2} < 0$

$\Rightarrow x \in (-\infty, -2)$

24. **(d)**

$\frac{1}{\log_3\left(2^{2x}-1\right)} > \frac{1}{\log_3\left(2^x+1\right)}$

Domain $x > 0, x \neq \frac{1}{2}$

$\log_3 2^x + 1$is always positive

Hence, $\log_3\left(2^{2x}-1\right) > 0$

$x > \frac{1}{2}$

And $\log_3\left(2^{2x}-1\right) < \log_3\left(2^x+1\right)$

$\Rightarrow 2^{2x} - 1 < 2^x + 1$

$\Rightarrow t^2 - t - 2 < 0 \qquad\qquad (t = 2x)$

$\Rightarrow t \in (-1, 2)$

$\Rightarrow x \in (-\infty, 1)$

Hence solution is $\left(\frac{1}{2}, 1\right)$

25. (b)

For (1) to hold, we must have

$x > 0, x \neq 1$ and $2x^2 + x - 1 > 0$

$\Rightarrow \quad x > 0, x \neq 1$ and $(2x-1)(x+1) > 0$

$\Rightarrow \quad x > \frac{1}{2}, x \neq 1$.

We can write (1) as

$$\log_x\left(\frac{2x^2 + x - 1}{2}\right) > -1 \qquad (2)$$

For $\frac{1}{2} < x < 1$, (2) can be written as

$$\frac{2x^2 + x + 1}{2} < \frac{1}{x}$$

$\Rightarrow \quad 2x^3 + x^2 - x < 2$

$\Rightarrow \quad 2(x^3 - 1) + x(x - 1) < 0$

$\Rightarrow \quad (x-1)(2x^2 + 3x + 2) < 0$

$\Rightarrow \quad x < 1 \qquad\qquad [\because 2x^2 + 3x + 2 > 0 \ \forall \ x > 0]$

For $x > 1$, (2) can be written as

$$\frac{2x^2 + x - 1}{2} > \frac{1}{x}$$

$\Rightarrow \quad (x-1)(2x^2 + 3x + 2) > 0$

This is true for each $x > 1$.

Thus, (1) holds for $\frac{1}{2} < x < 1, x > 1$.

26. (d)

The left hand side of the inequality is defined for x's which satisfy the following.

$1 - x > 0, x - 2 > 0$, $1 - x \neq 1$. Obviously there is no single value for which these inequalities are satisfied. Thus the set of its solutions is empty.

27. (c)

$\left\{x : \log_{1/3}\left(\log_4\left(x^2 - 5\right)\right) > 0\right\}$

$= \left\{x : 0 < \log_4\left(x^2 - 5\right) < 1\right\}$

$= \left\{x : 1 < x^2 - 5 < 4\right\}$

$= \left\{x : 6 < x^2 < 9\right\}$

$=\left(-3,-\sqrt{6}\right)\cup\left(\sqrt{6},3\right)$

28. (b)

$\left(x+2\right)\left(x+4\right)>0,\ x+2>0$

$\Rightarrow\ x>-2$.

Now (1) can be written as

$\log_3\left(x+2\right)\left(x+4\right)-\log_3\left(x+2\right)<\dfrac{\left(\log 7\right)/2}{\left(\log 3\right)/2}$

$\Rightarrow\ \log_3\left(x+4\right)<\log_3 7$

$\Rightarrow\ x+4<7$ or $x<3$.

29. (d)

$\Rightarrow x^{x\sqrt[3]{x}}=\left(x.\sqrt[3]{x}\right)^{x}$; Here $x\neq 0$

$\Rightarrow x^{x+\frac{1}{3}}=\left(x^{1+\frac{1}{3}}\right)^{x}$

$\Rightarrow x^{x^{\frac{4}{3}}}=x^{\frac{4}{3}x}$

Take log both side

$\Rightarrow x^{\frac{4}{3}}\log_x x=\dfrac{4}{3}x\log_x x$

$\Rightarrow x^{\frac{4}{3}}=\dfrac{4}{2}x$

$\Rightarrow x^{\frac{1}{3}}=\dfrac{4}{3}x$

$\Rightarrow x^{\frac{1}{3}}=\dfrac{4}{3}$

$\Rightarrow x=\left(\dfrac{4}{3}\right)^{3}=\dfrac{64}{27}$

30. (b)

$\Rightarrow\sqrt{25-5^{x}}=\sqrt{4^{x}-16}$ ....(1)

Here $25-5^{x}\geq 0$ & $4^{x}-16\geq 0$

$\Rightarrow 5^{x}\leq 25$ & $4^{x}\geq 4^{2}$

$\Rightarrow 5^{x}\leq 5^{2}$ & $x\geq 2$

$\Rightarrow x\in\left(-\infty,2\right]$ & $x\in\left[2,\infty\right)$

So only feasible region for given equation is $x=2$

For $x=2$, gives equation is satisfied

So no of solutions $=1$

31. **(c)**

$\Rightarrow 4^{x^2+2} - 9.2^{x^2+2} + 8 = 0$

$\Rightarrow 2^{2x^2+4} - 9.2^2 2^{x^2+8} = 0$

$\Rightarrow 16.2^{2x^2} - 36.2^{x^2} + 8 = 0$

$\Rightarrow 4.2^{2x^2} - 9.2x^2 + 2 = 0$

Put $2^{x^2} = a$

so, $4a^2 - 9a + 2 = 0$

$\Rightarrow (4a-1)(a-2) = 0$

$\Rightarrow a = \frac{1}{4}, 2$

$\Rightarrow 2^{x^2} = \frac{1}{4}, 2$

$\Rightarrow x^2 = -2, 1$

$\Rightarrow x^2 = -2$ is not possible; $x^2 = 1$

$\Rightarrow x = \pm 1$

32. **(b)**

Let $2^{11x} = t$, given equation reduces to $\frac{t^3}{4} + 4t = 2t^2 + 1$

$\Rightarrow \quad t^3 - 8t^2 + 16t - 4 = 0 \quad \Rightarrow t_1 \cdot t_2 \cdot t_3 = 4$

$\Rightarrow \quad 2^{11x_1} \cdot 2^{11x_2} \cdot 2^{11x_3 = 4} \Rightarrow 2^{11(x_1+x_2+x_3)} = 2^2$

$\Rightarrow \quad 11(x_1 + x_2 + x_3) = 2$

$\therefore \quad x_1 + x_2 + x_3 = \frac{2}{11}$

33. **(a)**

$\Rightarrow \frac{2^{x-1}}{2^{x+1}+1} < 2$

We can cross multiply $(2^{x+1}+1)$ as $2^{x+1}+1 > 0$ for $x \in R$

$\Rightarrow 2^{x-1} - 1 < 2(2^{x+1}+1)$

$\Rightarrow \frac{2^x}{2} - 1 < 4.2^x + 4$

$\Rightarrow \frac{7}{2} 2^x > -5$

$\Rightarrow 2^x > \frac{-10}{7}$

This is true for $x \in R$

34. **(c)**

$\Rightarrow \sqrt{2^{2x}-7} < 2^x - 1$ ……..(1)

Here $2^{2x}-7 \geq 0$

$\Rightarrow 2^{2x} \geq 7$

$\Rightarrow 2x \geq \log_2 7$

$\Rightarrow x \geq \frac{1}{2}\log_2 7$ ……..(2) (feasible region)

From feasible region it is clear that $2^x - 1 > 0$

So by taking square of (1)

$\Rightarrow 2^{2x} - 7 < \left(2^x - 1\right)^2$

$\Rightarrow 2^{2x} - 7 < 2^{2x} + 1 - 2.2^x$

$\Rightarrow 2.2^x < 8$

$\Rightarrow 2^x < 4$

$\Rightarrow x < 2$ ………(2)

By taking intersection of (1) & (2)

We get $x \in \left[\frac{1}{2}\log_2 7, 2\right)$

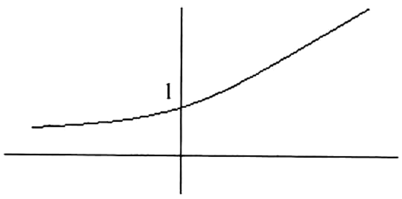
$\Rightarrow x \in \left[\log_4 7, 2\right)$

35. **(c)**

$\Rightarrow 5^{x+2} > \left(\frac{1}{25}\right)^{\frac{1}{x}}$

$\Rightarrow 5^{x+2} > \frac{1}{5^{\frac{2}{x}}}$

$\Rightarrow 5^{x+2}.5^{\frac{2}{x}} > 1$



$\Rightarrow 5^{x+\frac{2}{x}+2} > 5^{\circ}$

So $x + \frac{2}{x} + 2 > 0$

$\Rightarrow \frac{x^2 + 2x + 2}{x} > 0$

Numerator is always > 0

So $\frac{1}{x} > 0$

$\Rightarrow x \in \left(0, \infty\right)$

36. **(c)**

$\Rightarrow 49^{x} + 7^{x+1} - 98 < 0$

$\Rightarrow 7^{2x} + 7.7^{x} - 98 < 0$

$\Rightarrow 7^{x} = a; a^{2} + 7a - 78 < 0$

$\Rightarrow (a+14)(a-7) < 0$ $\quad\quad\quad\quad (\because a + 14 = 7^{x} + 14 > 0 \text{ for } x \in R)$

$\Rightarrow 7^{x} < 7$

So, $x < 1$

37. **(b)**

$|x-p| + |x-15| + |x-p-15| = (x-p) - (x-15) - (x-P-15) = 30 - x$

Minimum = 15

38. **(d)**

$2^{x} + 2^{|x|} \geq 2\sqrt{2}$

Case I $x \geq 0$

$2^{x} + 2^{x} \geq 2\sqrt{2}$

$2^{x} \geq \sqrt{2}$

$\Rightarrow x \geq \frac{1}{2}$

Case II $x < 0$

$2^{x} + \frac{1}{2^{x}} \geq 2\sqrt{2}$

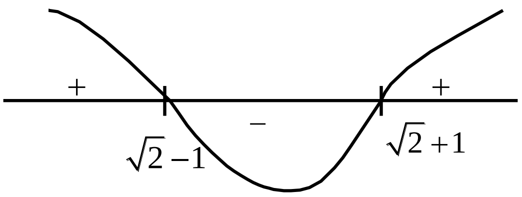
Let $2^{x} = k$ $\quad\quad k + \frac{1}{k} \geq 2\sqrt{2}$

$k^{2} - 2\sqrt{2}k + 1 \geq 1$

$k = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2} = \frac{2\sqrt{2} \pm 2}{2}$

$K = \sqrt{2} \pm 1$

$\left(k - (\sqrt{2}+1)\right)\left(k - \sqrt{2} - 1\right) \geq 0$



$-\infty < 2^{x} \leq \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \leq 2^{x} < \infty$

$\Rightarrow 0 < 2^{x} \leq \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \leq 2^{x} < \infty$

$x \in \left(-\infty, \log_{2}(\sqrt{2}-1)\right] \,\&\, \left[\log_{2}(\sqrt{2}+1), \infty\right)$

From case I & II

$x \in \left(-\infty, \log_{2}(\sqrt{2}-1)\right] \cup \left[\frac{1}{2}, \infty\right)$

39. **(d)**

$\Rightarrow \left|3^{x}-1\right| > \left|3^{x}-9\right|$

Take square both side

$\Rightarrow \left(3^{x}-1\right)^{2}-\left(3^{x}-9\right)^{2} > 0$

$\Rightarrow \left[\left(3^{x}-1\right)+\left(3^{x}-9\right)\right]\left[3^{x}-1-\left(3^{x}-9\right)\right] > 0$

$\Rightarrow \left[2.3^{x}-10\right][8] > 0$

$\Rightarrow 3^{x}-5 > 0 \Rightarrow 3^{x} > 5$

$x > \log_{3} 5$

40. **(a)**

$\Rightarrow \left|x^{3}-1\right| \geq 1-x$

Case I: $x^{3}-1 \geq 0$

$\Rightarrow x \geq 1$

So $\left(x^{3}-1\right) \geq 1-x$

$\Rightarrow x^{3}+x-2 \geq 0$

$\Rightarrow (x-1)\left(x^{2}+x+2\right) \geq 0$

$\Rightarrow x \in [1, \infty)$ ……..(1)

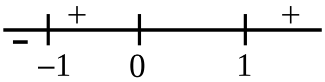
Case 2: $x^{3}-1 < 0$

$\Rightarrow x^{3} < 1 \Rightarrow x < 1$

So, $-\left(x^{3}-1\right) \geq 1-x$

$\Rightarrow -x^{3}+x \geq 0$

$\Rightarrow x^{3}-x \geq 0$



$\Rightarrow x(x-1)(x+1) \leq 0$

$\Rightarrow x \in (-\infty, 1] \cup [0, 1]$ ……….(2)

So take union of (1) & (2)

$\Rightarrow x \in (-\infty, -1] \cup [0, \infty)$

41. **(b)**

$\Rightarrow \dfrac{|x+2|-x}{x} < 2$

Case I: $x+2 \geq 0$

$\Rightarrow x > -2$ ……..(i)

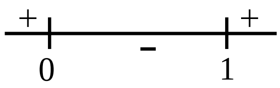
$\Rightarrow \dfrac{(x+2)-x}{x} < 2$

$\Rightarrow \dfrac{2}{x}-2 < 0$

$\Rightarrow \frac{1-x}{x} < 0$

$\Rightarrow \frac{x-1}{x} < 0$

$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$



Taking intersection of (i) & (ii)

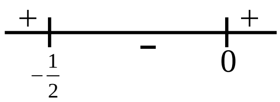
$\Rightarrow x \in [-2, 0) \cup (1, \infty)$ …….(iii)

Case 2: $\Rightarrow x + 2 < 0 \Rightarrow x \in (-\infty, -2)$ ……..(A)

$\Rightarrow \frac{-(x+2)-x}{x} < 2$

$\Rightarrow \frac{-2x-2-2x}{x} < 0$

$\Rightarrow \frac{2x-1}{x} > 0$



$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right) \cup (0, \infty)$ …….(B)

Taking intersection of A & B

$\Rightarrow x \in (-\infty, -2)$ ………(C)

So by taking union of (iii) & (c)

$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

42. **(a)**

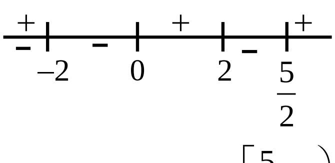
$\Rightarrow \frac{x^2-5x+4}{x^2-4} \le 1$

$\Rightarrow -1 \le \frac{x^2-5x+4}{x^2-4} \le 1$

Case 1: $\frac{x^2-5x+4}{x^2-4} \ge -1$

$\Rightarrow \frac{x^2-5x+4+x^2-4}{x^2-4}$

$\Rightarrow \frac{x(2x-5)}{(x-2)(x+2)} \ge 0$



$\Rightarrow x \in (-\infty, -2) \cup [0, 2) \cup \left[\frac{5}{2}, \infty\right)$ ……..(1)

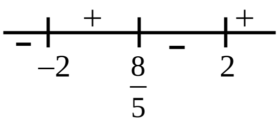
Case 2: $\dfrac{x^2-5x+4}{x^2-4}\le 1$

$\Rightarrow \dfrac{x^2-5x+4-x^{2+4}}{x^2-4}\le 0$

$\Rightarrow \dfrac{8-5x}{(x-2)(x+2)}\le 0$

$\Rightarrow \dfrac{(5x-8)}{(x-2)(x+2)}\ge 0$

$\Rightarrow x\in\left(-2,\dfrac{8}{5}\right]\cup(2,\infty)$



By taking intersection of (1) & (2)

$\Rightarrow x\in\left[0,\dfrac{8}{5}\right]\cup\left[\dfrac{5}{2},\infty\right)$

43. **(c)**

$\Rightarrow \dfrac{x^2-|x|-12}{x-2}\ge 2x$

Case 1: $x\ge 0$

$\Rightarrow \dfrac{x^2-x-12}{x-3}\ge 2x$

$\Rightarrow \dfrac{x^2-x-12}{x-3}-2x\ge 0$

$\Rightarrow \dfrac{x^2-x-12}{x-3}-2x\ge 0$

$\Rightarrow \dfrac{x^2-x-12-2x^2+6x}{x-3}\ge 0$

$\Rightarrow \dfrac{-x^2+5x-12}{x-3}\ge 0$

$\Rightarrow \dfrac{x^2-5x+12}{x-3}\le 0$

as $x^2-5x+12$ is always greater than zero for $x\in R$

so, $\dfrac{1}{x-3}\le .0$

$\Rightarrow x\in(-\infty,3)$

Case 2: $x<0$

$\Rightarrow \dfrac{x^2+x-12}{x-3}-2x\ge 0$

$\Rightarrow \dfrac{x^2+x-12-2x^2+6x}{x-3}\ge 0$

$\Rightarrow \frac{-x^2+7x-12}{x-3} \geq 0$

$\Rightarrow \frac{x^2-7x+12}{x-3} \leq 0$

$\Rightarrow \frac{(x-4)(x-3)}{(x-3)} \leq 0$

$\Rightarrow x-4 \leq 0$

$\Rightarrow x \in (-\infty, 4]$

But for this case $x < 0$

So we get

$\Rightarrow x \in (-\infty, 0)$

Take union of $(1) \& (2)$

$\Rightarrow x \in (-\infty, 3)$

44. **(c)**

$\Rightarrow \left|\frac{x^2-3x-1}{x^2+x+1}\right| < 3$

$\Rightarrow -3 \frac{x^3-3x-1}{x^2+x+1} < 3$

Case 1: $\frac{x^2-3x-1}{x^2+x+1} > -3$

As (x^2+x+1) is always greater than zero

So $x^2-3x-1 > -3(x^2+x+1)$

$\Rightarrow 4x^2+2 > 0$

$\Rightarrow x \in R$ …….(1)

Case 2: $\frac{x^2-3x+1}{x^2+x+1} < 3$

$\Rightarrow x^2-3x-1 < 3x^2+3x+3$

$\Rightarrow 2x^2+6x+4 > 0$

$\Rightarrow x^2+3x+2 => 0$

$\Rightarrow (x+1)(x+2) > 0$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$ …….(2)

Take intersection of $(1) \& (2)$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$

45. **(d)**

$\Rightarrow \frac{|x+3|+x}{x+2} > 1$

$\Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0$

$$\Rightarrow \frac{|x+3|+x-x-2}{x+1>0} > 0$$

$$\Rightarrow \frac{|x+3|-2}{x+2} > 0$$

Case 1: $x+3 \geq x \geq -3$

$$\Rightarrow \frac{x+3-2}{x+2} > 0$$

$$\Rightarrow \frac{x+1}{x+2} > 0$$

$$\Rightarrow x \in (-\infty, -2) \cup (-1, -\infty)$$

But for this case $x \geq -3$

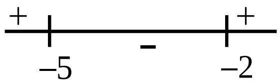
So we get $x \in [-3, -2) \cup (-1, \infty)$

Case 2: $x+3 < 0$

$$\Rightarrow x < -3$$

So, $\frac{-(x+3)}{x+2} > 0$

$$\Rightarrow \frac{x+5}{x+2} < 0$$



$$\Rightarrow x \in (-5, -2)$$

As $x < -3$ so $x \in (-5, -3)$ ……(2)

Take union of $(1)\&(2)$

$$\Rightarrow x \in (-5, -2) \cup (-1, \infty)$$

So least integral value of $x = -4$

46. **(c)**

$\Rightarrow \frac{|x+2|-|x|}{\sqrt{4-x^2}} \geq 0$ here $4-x^3 > 0$

$\Rightarrow x^3 < 4 \Rightarrow x \in \left(-\infty, \sqrt[3]{4}\right)$ ………(1)

As $(4-x^3)$ is greater than zero

$$\Rightarrow |x+2|-|x| \geq 0$$

$$\Rightarrow |x+2| \geq |x|$$

$$\Rightarrow (x+2)^2 \geq x^2$$

$$\Rightarrow (x+2-x)(x+2+x) \geq 0$$

$$\Rightarrow 2(2x+2) \geq 0$$

$\Rightarrow x \ge -1$ ....(2)

So $(1) \cap (2)$

$\Rightarrow x \in \left[-1, \sqrt[3]{4}\right)$

47. **(b)**

$\Rightarrow \frac{x^2 - 5x + 6}{|x| + 7} < 0$

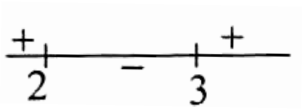
$\Rightarrow \frac{(x-3)(x-2)}{|x| + 7} < 0$

As $7 + |x| \ge 7$

So we can cross multiply $7 + |x|$

$\Rightarrow (x-3)(x-2) < 0$

$\Rightarrow x \in (2, 3)$



48. **(d)**

$\Rightarrow \left|\frac{2x-1}{x-1}\right| > 2$

$\Rightarrow \frac{2x-1}{x-1} < -2$ or $\frac{2x-1}{x-1} > 2$

$\Rightarrow \frac{2x-1+2x-2}{x-1} < 0$ or $\frac{2x-1-2x+2}{x-1} > 0$

$\Rightarrow \frac{4x-3}{x-1}$ or $\frac{1}{x-1} > 0$

$\Rightarrow x \in \left(\frac{3}{4}, 1\right)$ 0r $x > 1$

So, $x \in \left(\frac{3}{4}, \infty\right) - \{1\}$

49. **(a)**

$\Rightarrow \frac{1}{|x| - 3} < \frac{1}{2}$

Case 1: $x \ge 0$

$\Rightarrow \frac{1}{x-3} - \frac{1}{2} < 0$

$\Rightarrow \frac{5-x}{2(x-3)} < 0$

$\Rightarrow \frac{x-5}{(x-3)} > 0$

$\Rightarrow x \in (-\infty, 3) \cup (5, \infty)$

But $x \geq 0$

So, $x \in [0.3) \cup (5, \infty)$ ……..(1)

Case 2: $x < 0$

$\Rightarrow \frac{1}{-x-3} - \frac{1}{2} < 0$

$\Rightarrow \frac{1}{x+3} + \frac{1}{2} > 0$

$\Rightarrow \frac{5+x}{x+3} > 0$

$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$

But $x < 0$

So, $x \in (-\infty, -5) \cap (-3, 0)$ ……..(2)

So (1) $\cup$(2)

$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$

So least positive integer value $= 1$

50. (d)

$|x-1| + |x-2| + |x-3| \geq 6$

Case I : $x \geq 3$

$3x - 6 \geq 6 \Rightarrow x \geq 4$

Case II : $2 < x < 3$

$x \geq 6$ (Not possible)

Case III : $1 \leq x \leq 2$

$4 - x \geq 6$

$\Rightarrow$ $x \leq -2$ (Not possible)

Case IV : $x < 1$

$6 - 3x \geq 6$

$x \leq 0$

$x \in (-\infty, 0] \cup [4, \infty)$

51. (d)

$\Rightarrow \sqrt{3y+1} = \sqrt{y-1}$ ……..(1)

$\Rightarrow 3y + 1 \geq 0 \,\&\, y - 1 \geq 0$

$\Rightarrow y \geq -\frac{1}{3} \,\&\, y \geq 0$

$\Rightarrow y \in [0, \infty)$ is our feasible region

By equation (1), taking square of both side,

$\Rightarrow 3y + 1 = y - 1$

$\Rightarrow 2y = -2$

$\Rightarrow y = -1$; which does not lie in feasible range of y.

So no solution of y.

52. **(d)**

$\sqrt{4x+1}+\sqrt{7-x}=0$

As square root is always positive so given equation is feasible only if

$\Rightarrow 4x+1=0$ & $7-x=0$

$\Rightarrow x=-\frac{1}{4}$ & $x=7$

So no common solution.

53. **(a)**

$\sqrt{x^2-4x+3}+\sqrt{x^2-9}$

$=\sqrt{4x-14x+6}$

$\Rightarrow \sqrt{(x-1)(x-3)}+\sqrt{(x-3)(x+3)}$

$=\sqrt{2(2x-1)(x-3)}$

$\Rightarrow x=3$ or $\sqrt{x-1}+\sqrt{x+3}$

$=\sqrt{2(2x-1)}$

$\Rightarrow (x-1)+(x+3)+2\sqrt{(x-1)(x+3)}$

$=2(2x-1)$

$\Rightarrow 2\sqrt{(x-1)(x+3)}=2x-4$

$\Rightarrow \sqrt{(x-1)(x+3)}=x-2$

$\Rightarrow (x-1)(x+3)=(x-2)^2$

$\Rightarrow x^2+2x-3=x^2-4x+4$

$\Rightarrow 6x=-7$

$x=\frac{-7}{6}$

($\because$ $x^2-9<9$, hence $\sqrt{x^2-9}$ is not defined)

54. **(b)**

We have, $\sqrt{x}+\sqrt{x-\sqrt{(1-x)}}=1$

$\Rightarrow \sqrt{x-\sqrt{1-x}}=1-\sqrt{x}$

On squaring both sides, we get

$x-\sqrt{1-x}=1+x-2\sqrt{x}$

$\Rightarrow -\sqrt{1-x}=1-2\sqrt{x}$

Again, squaring on both sides, we get

$1-x=1+4x-4\sqrt{x}$

$4\sqrt{x}=5x$

$\Rightarrow \sqrt{x}=\frac{4}{5}$ [on squaring both sides]

$\Rightarrow \ x=\dfrac{16}{25}$

Hence, the number of real solutions is 1.

55. (b)

$\Rightarrow \sqrt{4-\sqrt{1-x}}-\sqrt{2-x}>0$

Here $1-x\geq 0$

$\Rightarrow 2-x>0$

$\Rightarrow x\leq 1$ ……..(i)

$\Rightarrow 4-\sqrt{1-x}\geq 0$

$\Rightarrow 4\geq \sqrt{1-x}$

$\Rightarrow 16\geq 1-x$

$\Rightarrow x>-15$ ………(ii)

So, $(i)\cap(ii)$

$\Rightarrow x\in[-15,1]$ …….. (iii) feasible region

Take intersection of $(iv)\&(iii)$

$\Rightarrow x\in[-2,1]$

Now

$\Rightarrow \sqrt{4-\sqrt{1-x}}>\sqrt{2-x}$

Take square both side

$\Rightarrow 4-\sqrt{1-x}>2-x$

$\Rightarrow 2+x>\sqrt{1-x}$

Here $2+x\geq 0\Rightarrow x\geq -2$ ……..(iv)

So, $2+x>\sqrt{1-x}$

Take square $4+x^2+4x>1-x$

$\Rightarrow x^2+5x+3>0$

$\Rightarrow \left(x-\left(\dfrac{-5+\sqrt{13}}{2}\right)\right)\left(x-\left(\dfrac{-5-\sqrt{13}}{2}\right)\right)$

$\Rightarrow x\in\left(-\infty,\dfrac{-5-\sqrt{13}}{2}\right)\cup\left(\dfrac{\sqrt{13}-5}{2},\infty\right)$ ……..(2)

$(1)\cap(2)$

$\Rightarrow x\in\left(\dfrac{\sqrt{13}-5}{2},1\right)$

56. (a)

$\Rightarrow \sqrt{4-x^2}+\dfrac{|x|}{x}\geq 0$

Here $4-x^2\geq 0$

$\Rightarrow x^2\leq 4$

$\Rightarrow x\in[-2,2]$ ……..(i)

Case 1: $x > 0$

$\Rightarrow \sqrt{4-x^2} + \frac{x}{x} \geq 0$ $(x \neq 0)$

$\Rightarrow \sqrt{4-x^2} + 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq -1$ true for $x \in R$

Case 2: $x < 0$

$\Rightarrow \sqrt{4-x^2} + \frac{(-x)}{x} \geq 0$

$\Rightarrow \sqrt{4-x^2} - 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq 1$

Take square $4 - x^2 \geq 1 \Rightarrow x^2 \leq 3$

$\Rightarrow x \in \left[-\sqrt{3}, \sqrt{3}\right]$

But $x < 0$

So $x \in \left[-\sqrt{3}, 0\right)$

So case (1) $\cup$ case (2)

$\Rightarrow x \in R$ ……..(iii)

But our feasible region is $x \in [-2, 2]$

So greatest integral $x = 2$

57. **(b)**

$\Rightarrow \left(x^2-1\right)\sqrt{x^2-x-2} \geq 0$

$\Rightarrow \left(x^2-1\right) \geq 0$ & $x^2 - x - 2 \geq 0$ xh2

$\Rightarrow x \in (-\infty, -1] \cup [1, \infty)$ ……(1) & $(x-2)(x+1) \geq 0$

$x \in (-\infty, -1] \cup [2, \infty)$ ……(2)

Take $(1) \cap (2)$

So, $x \in (-\infty, -1] \cup [2, \infty)$

Least positive integer $= 2$

58. **(c)**

$\sqrt{-x^2+10x-16} < x-2$

For L.H.S. to be defined

$-x^2 - 10x - 16 \geq 0$

$\Rightarrow x^2 - 10x + 16 \leq 0$

$\Rightarrow (x-2)(x-8) \leq 0$

$\Rightarrow x \in [2, 8]$ ...(1)

Now squring, $(x > 2)$

$-x^2 + 10x - 16 < (x-2)^2$

$= x^2 - 4x + 4$

$\Rightarrow\ 2x^2 - 14x + 20 > 0$

$\Rightarrow\ x^2 - 7x + 10 > 0$

$\Rightarrow\ (x-2)(x-5) > 0$

$\Rightarrow\ x \notin [2, 5] \qquad \ldots(2)$

Possible values $\rightarrow$ 6, 7, 8

EXERCISE - 1 [C]

1. **(1)**

$$\frac{1}{\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+......+\frac{1}{\log_{2000} x}}$$

$$\Rightarrow \frac{1}{\log_x 2+\log_x 3+......+\log_x 2000}$$

$$\Rightarrow \frac{1}{\log_x(2.3.......2000)}$$

$$\Rightarrow \frac{1}{\log_x\left(\prod_{n=1}^{2000} n\right)}$$

$$\Rightarrow \frac{1}{\log_x x}=1$$

2. **(2)**

$$9^{\log_3(\log_2 x)}=\log_2 x-(\log_2 x)^2+1$$

$$(\log_2 x)^2=\log_2 x-(\log_2 x)^2+1$$

Let $\log_2 x=t$

$$2t^2-t-1=0$$

$$2t^2-2t+t-1=0$$

$$t=\frac{-1}{2} \qquad t=1$$

$$\log_2 x=\frac{-1}{2} \qquad \log_2 x=1$$

$$x=2$$

$x=\frac{1}{\sqrt{2}}$ (not possible)

3. **(4)**

$$\log_3\left(\frac{4}{3}.\frac{5}{4}...........\frac{243}{242}\right)=\log_3\left(\frac{243}{3}\right)=4.$$

4. **(2)**

$$\left(5^{\log_5 7}+1\right)^{1/3}=(8)^{1/3}=2$$

5. **(4)**

Let $t=\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}...}}}$

$$\Rightarrow t^2-4=-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}...}}$$

$$= -\frac{1}{3\sqrt{2}}t$$

$$\Rightarrow \quad t^2 + \frac{1}{3\sqrt{2}}t - 4 = 0$$

$$\Rightarrow \quad t = \frac{8}{3\sqrt{2}}, \frac{-9}{3\sqrt{2}}$$

As $\quad t > 0, t = \frac{8}{3\sqrt{2}}$

Therefore, the value of $6 + \log_{(3/2)}\left[\frac{1}{3\sqrt{2}}\left(\frac{8}{3\sqrt{2}}\right)\right]$

$$= 6 + \log_{(3/2)}\left(\frac{2}{3}\right)^2 = 6 - 2 = 4$$

6. (9)

$$N = 6^{\log_{10} 40} \cdot 6^{2\log_{10} 5} = 6^{\log_{10} 1000} = 6^3 = 216$$

7. (47)

$$\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 a + \log_5 b}{2}$$

$$\Rightarrow \quad \log_5\left(\frac{a+b}{3}\right)^2 = \log_5(ab)$$

$$\Rightarrow \quad (a+b)^2 = 9ab \Rightarrow a^2 - 7ab + b^2 = 0$$

$$a^4 + b^4 + 2a^2b^2 = 49a^2b^2$$

$$\Rightarrow \quad \frac{a^4 + b^2}{a^2b^2} = 47$$

8. (0)

$$\log_{10}\sqrt{1+x} + 3\log_{10}\sqrt{1-x} = 2 + \log_{10}\sqrt{1-x} + \log_{10}\sqrt{1+x}$$

$$\Rightarrow \quad \log_{10}\sqrt{1-x} = 1$$

$$\sqrt{1-x} = 10 \Rightarrow x = -99 \text{ (not possible)}$$

9. (2)

$$\log_b n = 2$$

$$\log_n(2b) = \log_n 2 + \log_n b = 2$$

$$\log_n 2 + \frac{1}{2} = 2$$

$$\log_n 2 = \frac{3}{2} \Rightarrow n = 2^{2/3}$$

If $\quad \log_b n = 2 \Rightarrow b = n^{1/2} = 2^{1/3}$

$$n \cdot b = 2^{2/3} \cdot 2^{1/3} = 2$$

10. **(9)**

$$\log_y x+\frac{1}{\log_y x}=2$$

$\Rightarrow\ \log_y x=1\ \Rightarrow\ x=y$

$x^2+y=12$

$\Rightarrow\ x^2+x-12=0$

$\Rightarrow\ (x+4)(x-3)=0$

$\Rightarrow\ x=-4$ **or** $x=3$

But $x>0$, then $x=3$

$xy=9$

11. **(1)**

$\left(\log_2 4+\log_2\left(4^x+1\right)\right)\log_2\left(4^x+1\right)=3$

Let $\log_2\left(4^x+1\right)=t$

$t^2+2t-3=0\ \Rightarrow\ t=-3$ or 1

$\log_2\left(4^x+1\right)=1\Rightarrow 4^x=1\Rightarrow\ x=0$

12. **(5)**

$\log_{3^{1/4}}\left(\log_{3\sqrt{5}} x\right)=4\quad\Rightarrow\quad \log_3^{\left(\log_{3\sqrt{5}} x\right)}=1$

$\Rightarrow\ \log_5 x=1\qquad\Rightarrow\quad x=5$

13. **(5)**

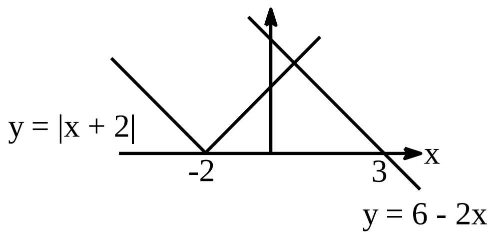
$x^2-10x+16<0$

$\Rightarrow\ (x-3)(x-8)<0$

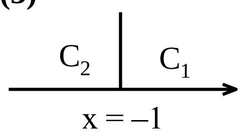
$\Rightarrow\ x\in(2,8)$

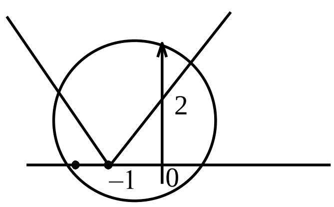
Integers $\rightarrow$ 3, 4, 5, 6, 7

14. **(1)**



15. **(3)**





C_1: If $x\ge 1$ … (a)

Then, 2 (x + 1) > x + 4

x > 2 … (b)

(a) n (b)

$x\in(2,\infty)$ (C – 1)

C_2: If $x < -1$ … (a)
Then, $-2(x + 1) > x + 4$
$\Rightarrow x < -3$ … (b)
(a) n (b)
$x \in (-\infty, -3) \cup (2, \infty)$
$C-1 \cup C-2$
So required x = 3

16. **(4)**
$-5 < x^2 - 4x < 5$
(1) (2)
(1) $x^2 - 4x + 5 > 0 \Rightarrow x \in R\ (\therefore D < 0)$ …(1)
(2) $x^2 - 4x - 5 < 0 \Rightarrow (x-5)(x+1) < 0 \Rightarrow -1 < x < 5$ …(2)
(1) n (2)
$x \in (-1, 5)$
So, m = 0, n = 4 $\Rightarrow$ (n – m) = 4

17. **(4)**
$|x^2 + x| - 5 < 0$
$\Rightarrow |x^2 + x| < 5$
$\Rightarrow |x^2 + x|^2 < 5^2$
$\Rightarrow (x^2 + x)^2 - 5^2 < 0$
$\Rightarrow (x^2 + x - 5)(x^2 + x + 5) < 0$
$\Rightarrow x^2 + x - 5 < 0$
$x \in (\alpha, \beta)$ where α, β are the roots of $x^2 + x - 5$

18. **(7)**
Domain $x \geq 3$
So $x \in (3, 10]$
So number of integer n = 7

19. **(2)**
$0 \leq x^2 + 2x - 3 < 1$
(1) (2)
(1) $(x+3)(x-1) \geq 0 \Rightarrow x \in (-\infty, 3] \cup [1, \infty)$ … (1)
(2) $x^2 + 2x - 4 < 0 \Rightarrow x = -1 \pm \sqrt{5} -1-\sqrt{5} < x < -1+\sqrt{5}$ …(2)
(1) n (2)
$x \in (-1-\sqrt{5}, -3] \cup [1, \sqrt{5}-1)$
So integer $x = \{-3, 1\}$

20. **(9)**

$$\frac{\sqrt{2x^2+15x-17}}{10-x}\geq 0$$

$2x^2+15x-17\geq 0$

$2x^2+17x-2x-17\geq 0$

$x(2x+17)-(2x+17)\geq 0$

$(x-1)(2x+17)\geq 0$

$x\in(-\infty,-8.5]\cup[1,\infty)$

Also x < 10

$\therefore$ No. of integers positive are

{1, 2, 3, 4, 5, 6, 7, 8, 9}

No. of positive integers are 9.

1. (b)

For $x<-2, |x+2| = -(x+2)$

$\therefore x^2 - |x+2| + x > 0$

$\Rightarrow x^2 + x + 2 + x > 0 \Rightarrow (x+1)^2 + 1 > 0$,

Which is valid $\forall x \in R$

But $x<-2, \therefore x \in (-\infty, -2)$ …….(i)

For $x \geq 2, |x+2| = x+2$

$\therefore x^2 - |x+2| + x > 0 \Rightarrow x^2 - x - 2 + x > 0$

$\Rightarrow x^2 > 2 \Rightarrow x > \sqrt{2} \text{ or } x < -\sqrt{2}$

i.e., $x \in \left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

But $x \geq -2 \Rightarrow x \in \left[-2, -\sqrt{2}\right] \cup \left(\sqrt{2}, \infty\right)$

From (i) and (ii),

$x \in \left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

But $x \geq -2 \Rightarrow x \in \left[-2, -\sqrt{2}\right] \cup \left(\sqrt{2}, \infty\right)$

From (i) and (ii)

$x \in (-\infty, -2) \cup \left[-2, -\sqrt{2}\right] \cup \left(\sqrt{2}, \infty\right)$

$\Rightarrow x \in \left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

2. (b)

$\because \left|\sqrt{x} - 3\right| = \begin{cases} \sqrt{x} - 3; & x \geq 9 \\ 3 - \sqrt{x} & x < 9 \end{cases}$

Case-I: $x \in [0, 9)$

$2\left(3 - \sqrt{x}\right) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 6, 2 \Rightarrow x = 36, 4$

Since $x \in [0, 9); \therefore x = 4$

Case-II: $x \in [9, \infty)$

$2\left(\sqrt{x} - 3\right) + x - 6\sqrt{x} + 6 = 0 \Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

Since $x \in [9, \infty); \therefore x = 16$

Hence, $x = 4 \,\&\, 16$

3. (d)

Given inequality is,

$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \leq 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x - 2\sin x + 5} \leq 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \leq 2\sin^2 y$

It is true if $\sin x = 1$ and $|\sin y| = 1$

Therefore, $\sin x = |\sin y|$

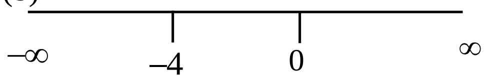
4. (b)

$A = \{x : x \in (-2, 2)\}; N = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$

$A \cap B = \{x : x \in (-2, -1]\}; A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$

$A - B = \{x : x \in (-1, 2)\}; B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

5. (b)



Case I: $x \in (-\infty, -4)$

$(x+3)(x+4) = 6 \Rightarrow x^2 + 7x + 6 = 0 \Rightarrow x = -6$

Case II: $x \in (-4, 0)$

$(x+3)(x+4) = -6 \Rightarrow x^2 + 7x + 18 = 0 \Rightarrow$ No real roots

Case III: $x \in (0, \infty)$

$(x-3)(x+4) = 6 \Rightarrow x^2 + x - 18 = 0 \Rightarrow x = \frac{\sqrt{73} - 1}{2}$

So, the given set contains only 2 elements.

6. (b)

For, S_1 we have

$\Rightarrow \frac{(x+2)(x^2+3x+5)}{x^2-3x+2} \le 0 \Rightarrow x \in (-\infty, -2] \cup (1, 2)$

For S_2, we have $= 3^x(3^x - 3) - 3^2(3^x - 3) \le 0$

For $S_2 x \in [1, 2] \Rightarrow (-\infty, -2] \cup [1, 2]$

7. (d)

$\because S = \{-6, -5, -4, 3\}$

Where $-5, -4, 3$ Satisfy $T \therefore S \cap T = \{-5, -4, 3\}$

8. (256)

$A = \{x \in R : |x-2| > 1\} = (-\infty, 1) \cup (3, \infty)$

$B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\} = (-\infty, -2) \cup (2, \infty)$

$C = \{x \in R : |x-4| \ge 2\} = (-\infty, 2] \cup [6, \infty)$

So, $A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$

$Z \cap (A \cap B \cap C)^C = \{-2, -1, 0, 1, 2, 3, 4, 5\}$

Hence no. of its subsets $= 2^8 = 256$

EXERCISE - 2 [A]

1. **(c)**

$$X = \log_k b = \log_b c = \frac{1}{2}\log_c d \quad \Rightarrow \quad b = (k)^x \quad \Rightarrow \quad c = (b)^x$$

$$\Rightarrow \; d = (c)^{2x} \quad \Rightarrow \quad d = (k)^{2x^3} \qquad \log_k (k)^{2x^3} = 2x^3$$

2. **(a)**

$$\frac{(\log a)^2}{\log b / \log c} - 1 + \frac{(\log b)^2}{\log a \log c} - 1 + \frac{(\log c)^2}{\log a \log b} - 1 = 0$$

$$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3\log a \log b \log c$$

$\because$ a, b, c are distinct

$$\Rightarrow \log a + \log b + \log c = 0$$

$$\Rightarrow abc = 1$$

3. **(b)**

First note that $2x+3>0$ and $2x+3 \neq 1$, that is, $x > -\frac{3}{2}$ and $x \neq -1$.

Also, $3x+7>0$ and $3x+7 \neq 1$, that is, $x > -\frac{7}{3}$ and $x \neq -2$.

Suppose $x > -\frac{3}{2}$, $x \neq 1$. then the given equation can be written as

$$\frac{\log\left[(2x+3)(3x+7)\right]}{\log(2x+3)} = 4 - \frac{2\log(2x+3)}{\log(3x+7)}$$

$$1 + \frac{\log(3x+7)}{\log(2x+3)} = 4 - \frac{2\log(2x+3)}{\log(3x+7)}$$

Put $\frac{\log(3x+7)}{\log(2x+3)} = y$, then $1 + y = 4 - \frac{2}{y}$

Therefore $y = 3 - \frac{2}{y}$

$$y^2 - 3y + 2 = 0$$

$$(y-1)(y-2) = 0$$

This gives $y = 1$ or 2.

Case 1: Suppose that $y = 1$, then

$$\log(3x+7) = \log(2x+3)$$

$$3x+7 = 2x+3$$

$$x = 4$$

This is rejected because $x > -3/2$.

Case 2: Suppose that $y = 2$. Then

$$\log(3x+7) = 2\log(2x+3) = \log(2x+3)^2$$

$$3x+7 = 4x^2 + 12x + 9$$

$$4x^2 + 9x + 2 = 0$$

$x = -\frac{1}{4}$ or -2

Here, $x = -\frac{1}{4}$ (since $x > -\frac{3}{2}$) so, $x = -\frac{1}{4}$

4. **(b)**

$$|x-3|^{\left(x^2+8x+15\right)/(x-2)} = 1$$

$\Rightarrow x \neq 3, x \neq 2$ and $\frac{x^2-8x+15}{x-2}\log|x-3| = 0$

$\Rightarrow x \neq 2, x \neq 3$ and $|x-3| = 1$ or $x^2-8x+15 = 2$

$\Rightarrow x \neq 2, x \neq 3$ and [$x = 3$ or 4 or $(x-3)(x-5) = 0$]

$\Rightarrow x = 4$ or $x = 5$

Therefore, the number of the solutions of the given equation is 2.

5. **(a)**

$\Rightarrow (x)^{x\sqrt{x}} = \left(x\sqrt{x}\right)^{x}$ $\quad$ $(\text{here } x \neq 0)$

$\Rightarrow x^{x^{\frac{3}{2}}} = \left(x^{\frac{3}{2}}\right)^{x}$

$\Rightarrow x^{x^{\frac{3}{2}}} = x^{\frac{3}{2}x}$

Take log both sides

$\Rightarrow x^{\frac{3}{2}} \log_x x = \frac{3}{2} x \log_x x$

$\Rightarrow x^{\frac{3}{2}} = \frac{3}{2} x$

$\Rightarrow x^{\frac{1}{2}} = \frac{3}{2}$

$\Rightarrow x = \frac{9}{4}$

6. **(d)**

$$1 + 2\log_2 x + \log_2 x\left(\log_2 x + 1\right) + \frac{1}{2}\cdot 4\log_2{}^2 x + \log_2{}^3 x = 1$$

$\Rightarrow \log_2{}^3 x + 3\log_2{}^2 x + 3\log_2 x = 0$

$\Rightarrow \log_2 x\left[\log_2{}^2 x + 3\log_2 x + 3\right] = 0$

$D = 9 - 4.3 < 0$

$\Rightarrow x = 1$ but $x \neq 1$

So, no solution.

7. **(c)**

Let $\log_3 x = t \quad \Rightarrow \quad x = 3^t$

$\left(3^t + 1\right)t^2 + 4t.3^t - 16 = 0 \qquad (t+4)\left(t.3^t + t - 4\right) = 0$

$\log_3 x = t = 1, -4 \qquad x = 3, \frac{1}{81}$

or $x\log_3^2 x+\log_3^2 x+4x\log_3 x-16=0$

$x\log_3 x\left(\log_3 x+4\right)+\left(\log_3 x+4\right)\left(\log_3 x-4\right)=0$

$\left(\log_3 x+4\right)\left(x\log_3 x+\log_3 x-4\right)=0$

$c=3^{-4}$ or $\left(x+1\right)\log_3 x=4 \Rightarrow x=3$

$x=\frac{1}{81}, 3$

8. **(b)**

$\log\left(\frac{x}{3}\right)+\log\left(5y\right)=1+\log 2$

$\log\left(\frac{5xy}{3}\right)=\log 20 \quad \Rightarrow \quad xy=\frac{20\times 3}{5}=12$

$xy=12$ & domain $x>0$

$y>0$

$2\log\left(x^2+y^2\right)-\log 5=\log\left\{2\left(x^2+y^2\right)+75\right\}$

Let $x^2+y^2=t$

$\log t^2-\log 5=\log\left(2t+75\right)$

$\frac{t^2}{5}=2t+75 \quad \Rightarrow \quad t^2-10t-375=0$

$\left(t+15\right)\left(t-25\right)=0 \quad \Rightarrow \quad t=25,-15 \quad \Rightarrow \quad x^2+y^2=25$ & $xy=12$

$x^2+y^2=25$ & $xy=12$

On solving $x=3$ & $y=4$

Or $x=4, y=3$

Hence number of ordered pair is 2.

9. **(d)**

Clearly $x>0, y>0$ and $y\neq 1$, so as to make the equations meaningful.

The given equations are equivalent to

$\log_8 x+\log_8 y=3\log_8 x.\log_8 y$

$4\left(\log_8 x-\log_8 y\right)=\frac{\log_8 x}{\log_8 y}$

Put $\log_8 x=m$ and $\log_8 y=n\neq 0$. Then the equivalent system is

$$\left.\begin{aligned} m+n&=3mn \\ 4\left(m-n\right)&=\frac{m}{n}\end{aligned}\right\} \qquad (1)$$

Multiplying both the equations of the equivalent systems we get

$4\left(m^2-n^2\right)=3m^2$

Therefore,

$m^2=4n^2$ or $m=\pm 2n$

Putting $m=2n$ in eq. (1), we get that

$3n=6n^2$ or $n=\frac{1}{2}$ (since $n\neq 0$) and m = 1

Now, $m=1 \Rightarrow \log_8 x=1 \Rightarrow x=8$

$$n=\frac{1}{2} \Rightarrow \log_8 y=\frac{1}{2} \Rightarrow y=2\sqrt{2}$$

Therefore, $x_1=8, y_1=2\sqrt{2}$

Again by taking $m=-2n$, we get that

$$n=6n^2 \text{ or } n=\frac{1}{6} \text{ and } m=-\frac{1}{3}$$

$$-\frac{1}{3}=m=\log_8 x \Rightarrow x=8^{-1/3}=\left(2^3\right)^{-1/3}=\frac{1}{2}$$

$$\frac{1}{6}=n=\log_8 y \Rightarrow y=8^{1/6}=\left(2^3\right)^{1/6}=\sqrt{2}$$

For $x_2=\frac{1}{2}$ and $x_2=\sqrt{2}$.

Therefore, $x_1 x_2+y_1 y_2=8\times\frac{1}{2}+2\sqrt{2}\times\sqrt{2}=4+4=8$

10. (c)

The given number can be written as

$$\log_3(135)\log_3(15)-\log_3 5.\log_3 405$$

$$=(\log_3 5+3)(1+\log_3 5)-(\log_3 5)(\log_3 5+4)=3$$

11. (c)

$$(2x)^{\ln 2}=(3y)^{\ln 3}$$

$$\Rightarrow (\ln 2)(\ln 2+\ln x)=(\ln 3)(\ln 3+\ln y) \quad ...(1)$$

and $3^{\ln x}=2^{\ln y} \Rightarrow (\ln x)(\ln 3)=(\ln y)(\ln 2) \quad ...(2)$

multiply (1) by $\ln 2$ and (2) by $\ln 3$ and subtract to obtain

$$\left[(\ln 2)^2-(\ln 3)^2\right]\ln x=(\ln 2)\left[(\ln 3)^2-(\ln 2)^2\right]$$

$$\Rightarrow \ln x=-\ln 2 \text{ or } x=\frac{1}{2}$$

$$\therefore \quad x_0=\frac{1}{2}$$

12. (b)

For $x = 1$, both parts of the equation vanish, consequently $x = 1$ is root of the equation.

For $x\neq 1$,

$$1=\frac{1}{\log_5 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}=\log_x 5+\log_x 3+\log_x 4$$

$$=\log_x 60$$

$\Rightarrow \quad x=60$. Thus the required set is $\{1, 60\}$.

13. (c)

We have

$$a=\log_{12}18=\frac{\log_2 18}{\log_2 12}=\frac{1+2\log_2 3}{2+\log_2 3} \text{ and } b=\log_{24}54=\frac{\log_2 54}{\log_2 24}=\frac{1+3\log_2 3}{3+\log_2 3}.$$

Putting $x=\log_2 3$, we have

$$ab+5(a-b)=\frac{1+2x}{2+x}\cdot\frac{1+3x}{3+x}+5\left(\frac{1+2x}{2+x}-\frac{1+3x}{3+x}\right)$$
$$=\frac{6x^2+5x+1+5(-x^2+1)}{(x+2)(x+3)}=\frac{x^2+5x+6}{(x+2)(x+3)}=1$$

14. (a)

Set $\log_2 12=a$,

$$\frac{1}{\log_{96}2}=\log_2 96=\log_2 2^3\times 12=3+a,$$

$\log_2 24=1+a, \log_2 192=\log_2(16\times 12)=4+a$ and $\frac{1}{\log_{12}2}=\log_2 12=a$.

Therefore, the given expression

$$=(1+a)(3+a)-(4+a)a=3$$

15. (a)

$x\neq(2n+1)\frac{\pi}{2}, n\pi$ where $n\in\mathbf{I}$. The given inequality can be written as

$$\frac{\log_2(x^2-8x+23)}{\log_2|\sin x|}>\frac{3}{\log_2|\sin x|}$$

As $\log_2|\sin x|<0$, we get

$$\log_2(x^2-8x+23)<3$$

$\Rightarrow\ x^2-8x+23<2^3=8$

$\Rightarrow\ x^2-8x+15<0$

$\Rightarrow\ (x-5)(x-3)<0 \Rightarrow 3<x<5$

For $x\in(3,5), x\neq\pi, \frac{\pi}{2}, \frac{3\pi}{2}$.

Hence, $x\in(3,\pi)\cup\left(\pi,\frac{3\pi}{2}\right)\cup\left(\frac{3\pi}{2},5\right)$.

16. (c)

$$S=\left(a^{\log_3 7}\right)^{\log_3 7}+\left(b^{\log_7 11}\right)^{\log_7 11}+\left(c^{\log_{11}25}\right)^{\log_{11}25}$$
$$=27^{\log_3 7}+49^{\log_7 11}+\sqrt{11}^{\log_{11}25}=469$$

17. (b)

$$x=\frac{z^{1/3}}{2},\ y=\frac{z^{1/6}}{5}$$

If $xy=z^{3/2}$, $\frac{z^{1/3}}{2}\cdot\frac{z^{1/6}}{5}=z^{3/2}\ \Rightarrow\ z=\frac{1}{10}$

18. (c)

$$a=\frac{3}{1+2\log_3 2}\ \Rightarrow\ \log_3=\frac{3-a}{2a};\qquad \log_6 16=\frac{4\log_3 2}{1+\log_3 2}$$

19. **(b)**

Case-I: $2x-3>1$

$3x-4>1$

$x>\frac{5}{3} \Rightarrow x>2$

Case-II: $0<2x-3<1$

$0<3x-4<1$

$x<\frac{5}{3} \Rightarrow \frac{3}{2}<x<\frac{5}{3}$

20. **(d)**

$$\left(2^{x+\frac{1}{3}\left(2x-\frac{3}{x}\right)}\right)^{1/2}=2^{\frac{7}{3}}$$

$$\Rightarrow x+\frac{2}{3}x-\frac{1}{x}=\frac{14}{3} \Rightarrow 5x^2-14x-3=0$$

21. **(d)**

$$25^{\left(2x-x^2+1\right)}+9^{\left(2x-x^2+1\right)}=34\frac{3^{2x-x^2+1}}{3}\cdot\frac{5^{2x-x^2+1}}{5}$$

Let $3^{2x-x^2+1}=a$ and $5^{2x-x^2++1}=b$

$$a^2+b^2=\frac{34}{15}ab$$

$$15a^2-34ab+15b^2=0 \Rightarrow (3a-5b)(5a-3b)=0$$

Case-1 : If $\frac{a}{b}=\frac{5}{3}$

$$\Rightarrow \left(\frac{3}{5}\right)^{2x-x^2+1}=\frac{5}{3}$$

$$\Rightarrow 2x-x^2+1=-1 \Rightarrow x^2-2x-2=0$$

Sum of two values of $x = 2$

Case-2 : If $\frac{a}{b}=\frac{3}{5}$

$$\left(\frac{3}{5}\right)^{2x-x^2+1}=\frac{3}{5}$$

$\Rightarrow 2x-x^2+1=1 \Rightarrow x=0$ and 2

Sum of all values of x is 4.

22. **(c)**

$$a^x=b^y=c^x=d^w$$

$$\Rightarrow b=a^{x/y},\ c=a^{x/z},\ d=a^{x/w}$$

$$\log_a(bcd)=\log_a a^{\left(\frac{x}{y}+\frac{x}{z}+\frac{x}{w}\right)}=\frac{x}{y}+\frac{x}{z}+\frac{x}{w}=x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$$

23. **(b)**

$$\log_x\log_{18}\left(\sqrt{2}+\sqrt{8}\right)=\frac{1}{3}$$

$$\log_x\log_{\left(3\sqrt{2}\right)^2}3\sqrt{2}=\frac{1}{3}$$

$$\log_x\left(\frac{1}{2}\right)=\frac{1}{3} \Rightarrow x=\frac{1}{8}$$

24. **(c)**

$\left(\log_2 x\right)^4 + 16\left(\log_2 x\right)^2 \log_2 \frac{16}{x}$

$\Rightarrow\ t^4 + 16t^2(4-t)$ (where $\log_2 x = t$)

$\Rightarrow\ t^2\left(t^2 + 64 - 16t\right)$

$\Rightarrow\ t^2(t-8)^2$

Since $1 \le x \le 256 \Rightarrow\ 0 \le t \le 8$

$\Rightarrow$ Maximum of $(t-8)^2 t^2$ lies at $t = 4$.

Hence, maximum $(4-8)^2 \cdot 4^2 = 256$

25. **(a)**

$\because\ \log_x 2 \log_{2x} 2 = \log_{4x} 2$

$\therefore\ x > 0, 2x > 0$ **and** $4x > 0$ and $x \ne 1,\ 2x \ne 1,\ 4x \ne 1$

$\Rightarrow x > 0$ and $x \ne 1,\ \frac{1}{2}, \frac{1}{4}$

Then. $\frac{1}{\log_2 x} \cdot \frac{1}{\log_2 2x} = \frac{1}{\log_2 4x}$

$\Rightarrow\ \log_2 x \cdot \log_2 2x = \log_2 4x$

$\Rightarrow\ \log_2 x \cdot (1 + \log_2 x) = (2 + \log_2 x)$

$\Rightarrow\ (\log_2 x)^2 = 2$

$\Rightarrow\ \log_2 x = \pm\sqrt{2}$

$\therefore\ x = 2^{\pm\sqrt{2}}$

$\therefore\ x = \left\{2^{-\sqrt{2}}, 2^{\sqrt{2}}\right\}$

26. **(d)**

$$\left(\frac{1}{\sqrt{27}}\right)^2 \cdot \left(\frac{1}{\sqrt{27}}\right)^{-\left(\frac{\log_5 16}{2\log_5 9}\right)} = \left(\frac{1}{27}\right)\left(\frac{1}{\sqrt{27}}\right)^{-\log_3 2}$$

$$= \left(\frac{1}{27}\right) \cdot 2^{-\log_3 \frac{1}{\sqrt{27}}} = \frac{2\sqrt{2}}{27}$$

27. **(c)**

$\lambda = \log_5(\log_5 3) \Rightarrow\ 5^\lambda = \log_5 3$

$3^{k+5^{-\lambda}} = 3^k \cdot 3^{5^{-\lambda}} = 3^k \cdot 3^{\log_3 5} = 5 \cdot 3^k$

28. **(a)**

$2^x = 3^y = 6^{-x} = k$ (let)

$x = \log_2 k,\ y = \log_3 k,\ z = -\log_6 k$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \log_k 2 + \log_k 3 - \log_k 6 = 0$

29. **(c)**

$$\log_2 x+\frac{1}{2}\log_2 y+\frac{1}{2}\log_2 z=2 \Rightarrow x\sqrt{y}\sqrt{z}=4$$

$$\log_3 y+\frac{1}{2}\log_3 x+\frac{1}{2}\log_3 z=2 \Rightarrow \sqrt{x}\cdot y\cdot\sqrt{z}=9$$

$$\log_4 z+\frac{1}{2}\log_4 x+\frac{1}{2}\log_4 y=2 \Rightarrow \sqrt{x}\cdot\sqrt{y}\cdot z=16 \Rightarrow xyz=24$$

30. **(b)**

$$\log_3 2+\log_3 5=\log_3 10$$

$$\log_3 9<\log_3 10<\log_3 27$$

31. **(a)**

$$\log_{\cos x^2}(3-2x)<\log_{\cos x^2}(2x-1)$$

$$0<\cos x^2<1 \cap 3-2x>2x-1 \cap 3-2x>0 \cap 2x-1>0$$

$$x<1 \qquad x<3/2 \qquad x>1/2$$

32. **(b)**

We have $4x^2-16x+15<0 \Rightarrow \frac{3}{2}<x<\frac{5}{2} \Rightarrow \cot\alpha=2$, the integral solution of the given inequality and $\sin\beta=\tan 45^\circ=1$

$$\therefore \quad \sin(\alpha+\beta)\sin(\alpha-\beta)=\sin^2\alpha-\sin^2\beta=\frac{1}{1+\cot^2\alpha}-1=\frac{1}{1+4}-1=\frac{4}{5}$$

33. **(c)**

Case-1: $x\ge 2$

$3(x-2)-(1-5x)+4(3x+1)=13 \Rightarrow x=\frac{4}{5}$ (Not possible)

Case-2: $\frac{1}{5}\le x<2$

$-3(x-2)-(1-5x)+4(3x+1)=13 \Rightarrow x=\frac{2}{7}$ (Possible)

Case-3: $-\frac{1}{3}\le x<\frac{1}{5}$

$-3(x-2)+(1-5x)+4(3x+1)=13 \Rightarrow x=\frac{1}{2}$ (Not possible)

Case-4: $x<-\frac{1}{3}$

$-3(x-2)+(1-5x)-4(3x+1)=13 \Rightarrow x=-\frac{1}{2}$ (Possible)

34. **(b)**

$$\log_{\cos x}\sin x\ge 2 \Rightarrow \sin x\le\cos^2 x$$

$$\sin^2 x+\sin x-1\le 0$$

$$0<\sin x\le\frac{\sqrt{5}-1}{2} \quad (\sin x>0)$$

35. **(b)**

$\sqrt{3x^2-7x-30}-\sqrt{2x^2-7x+5}$

$=x-5$

Let $A=\sqrt{3x^2-7x-30}$

$B=\sqrt{2x^2-7x-5}$

$\Rightarrow A-B=x-5$ ...(1)

& $A^2-B^2=x^2-25$ ...(2)

$\Rightarrow A+B=x+5$ (or $x=5$) ... (3)

(1) + (3)

$\Rightarrow 2A=2x$

$\Rightarrow A=x\geq 0$ $(\because A\geq 0)$

$\Rightarrow A^2=x^2$

$\Rightarrow 3x^2-7x-30=x^2$

$\Rightarrow 2x^2-7x-30=0$

$\Rightarrow (2x+5)(x-6)=0$

$\Rightarrow x=6$ or $x=-\frac{5}{2}$

$\therefore x>0,\ x=6$

$\Rightarrow x=5$ or $x=6$

(2 solutions)

36. **(a)**

$2^{\frac{x}{2}}+\left(\sqrt{2}+1\right)^x=\left(3+2\sqrt{2}\right)^{x/2}$

$\Rightarrow 2^{x/2}+\left(\sqrt{2}+1\right)^x=\left[\left(\sqrt{2}+1\right)^2\right]^{x/2}$

$\Rightarrow 2^{x/2}=0$

No solutions.

37. **(b)**

$\left(31+8\sqrt{15}\right)^{x^2-3}+1$

$=\left(32+8\sqrt{15}\right)^{x^2-3}$

$\Rightarrow \left(32+8\sqrt{15}\right)^{x^2-3}-3\left(31+8\sqrt{15}\right)^x$

$=1$

We can see that if $x=k$ satisfies the equation, $x=-k$ will also satify the equation.

Hence, sum of values = 0

38. **(c)**

We have, $3^{2x^2} - 2\cdot 3^{x^2+x+6} + 3^{2(x+6)} = 0$

$\Rightarrow (3x^2 - 3^{x+6})^2 = 0$

$\Rightarrow 3x^2 - 3^{x+6} = 0$

$\Rightarrow 3x^2 = 3^{x+6} \Rightarrow x^2 = x + 6$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$

$\therefore x = \{-2, 3\}$

39. **(c)**

We have, $\sqrt{(5x^2-8x+3)} - \sqrt{(5x^2-9x+4)}$

$= \sqrt{(2x^2-2x)} - \sqrt{(2x^2-3x+1)}$

$\Rightarrow \sqrt{(5x-3)(x-1)} - \sqrt{(5x-4)(x-1)} = \sqrt{2x(x-1)} - \sqrt{(2x-1)(x-1)}$

$\Rightarrow \sqrt{x-1}\left(\sqrt{5x-3} - \sqrt{5x-4}\right) = \sqrt{x-1}\left(\sqrt{2x} - \sqrt{2x-1}\right)$

$\Rightarrow \sqrt{x-1} = 0$

$\Rightarrow x = 1$

40. **(b)**

We have, $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right) = a^2 - b = 1$ [given]

$\therefore \left(a+\sqrt{b}\right)^{x^2-15} + \left(a-\sqrt{b}\right)^{x^2-15} = 2a$

$\Rightarrow \left(a+\sqrt{b}\right)^{x^2-15} + \dfrac{1}{\left(a+\sqrt{b}\right)^{x^2-15}} = 2a$

Let $y = \left(a+\sqrt{b}\right)^{x^2-15}$

$\Rightarrow y + \dfrac{1}{y} = 2a \Rightarrow y^2 - 2ay + 1 = 0$

$\Rightarrow y = \dfrac{2a \pm \sqrt{4a^2-4}}{2} = a \pm \sqrt{a^2-1}$

$\therefore y = a \pm \sqrt{b} = \left(a+\sqrt{b}\right)^{\pm 1}$ $\left[\because a^2 - b = 1\right]$

$\Rightarrow \left(a+\sqrt{b}\right)^{x^2-15} = \left(a+\sqrt{b}\right)^{\pm 1}$

$\therefore x^2 - 15 = \pm 1$

$\Rightarrow x^2 = 15 \pm 1 \Rightarrow x^2 = 16, 14$

$\Rightarrow x = \pm 4, \pm\sqrt{14}$

41. **(a)**

We have, $x-\sqrt{1-|x|}<0$

Which is defined only when

$1-|x|\geq 0$

$\Rightarrow\ |x|\leq 1$

$\Rightarrow\ x\in[-1,1]$

Now, from Eq. (i), we get

$x<\sqrt{1-|x|}$

***Case* I** If $x\geq 0$, i.e., $0\leq x\leq 1$

$x-\sqrt{(1-|x|)}<0$

$\Rightarrow\ x<\sqrt{(1-x)}$

On squaring both sides, we get

$x^2+x-1<0$

$\Rightarrow\ \dfrac{-1-\sqrt{5}}{2}<x<\dfrac{-1+\sqrt{5}}{2}$

But $x\geq 0$

$\therefore\ x\in\left[0,\dfrac{-1+\sqrt{5}}{2}\right)$

***Case* II** If $x<0$, i.e., $-1\leq x<0$

$x-\sqrt{(1+x)}<0$

$\Rightarrow\ x<\sqrt{1+x}$ [always true]

$x\in[-1,0)$

Combining both cases, we get

$x\in\left[-1,\dfrac{-1+\sqrt{5}}{2}\right)$

EXERCISE - 2 [B]

One or More Than ONE Option(s) May be Correct:

1. (a, c)

$\Rightarrow 18^{4x-3} = \left(54\sqrt{2}\right)^{3x-4}$

$\Rightarrow 18^{4x-3} = (18)^{3x-4}\left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow 18^{x+1} = \left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow \left[\left(3\sqrt{2}\right)^2\right]^{x+1} = \left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow \left(3\sqrt{2}\right)^{2x+2} \times \left(3\sqrt{2}\right)^{-3x+4} = 1$

$\Rightarrow \left(3\sqrt{2}\right)^{-x+6} = 1$

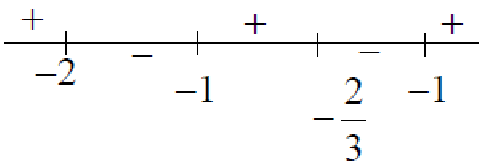
So $-x = 6 \Rightarrow x = 6$

2. (b, c)

$\Rightarrow \frac{2x}{2x^2+5x+2} > \frac{1}{x+1} \Rightarrow \frac{2x}{2x^2+5x+2} - \frac{1}{x+1} > 0$

$\Rightarrow \frac{2x^2+2x-2x^2-5x+2}{\left(2x^2+5x+2\right)(x+1)} > 0$

$\Rightarrow \frac{3x+2}{(2x+1)(x+2)(x+1)} < 0$



So, $x \in (-2,-1) \cup \left(-\frac{2}{3},-1\right)$

3. (a, b, d)

$\Rightarrow x^2 - 6x - 5|x-3| - 5 = 0$

Case – I : $x \geq 3$

$\Rightarrow^2 -6x - 5|x-3| - 5 = 0$

$\Rightarrow x^2 - 11x + 10 = 0$

$\Rightarrow$ x = 1, 10

$\Rightarrow$ x = 1 not a solution for $x \geq 3$

$\Rightarrow$ x = 10

Case – 2: x < 3

$\Rightarrow x^2 - 6x + 5x - 15 - 5 = 0$

$\Rightarrow x^2 - x - 20 = 0$

$\Rightarrow$ (x – 5) (x + 4) = 0

For x < 3, x = 1 – 4

So, l = 1, m = 1

4. (a, d)

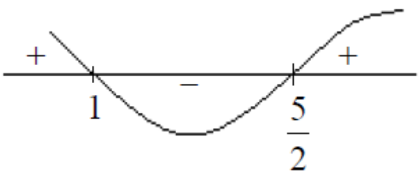
$\Rightarrow |x-1|+|5-2x|=|3x-6|$

As (x – 1) – (5 – 2x) = 3x – 6

So, $(x-1)(5-2x)\le 0$

$\Rightarrow (x-1)(2x-5)\ge 0$

$\Rightarrow x\in(-\infty,1]\cup\left[\frac{5}{2},\infty\right)$



5. (a, c)

$\Rightarrow \log_2 3>1$

$\Rightarrow \log_{12} 5<1$

So, $\log_2 3>\log_{12} 5$

Similarly, $\log_6 5 < 1$

$\Rightarrow \log_7 11 > 1$

$\Rightarrow$ $\log 82 > \log_3 3^4$

$\Rightarrow$ $\log_3 81 > 4$

$\Rightarrow$ $\log_2 15 < \log_2 16$

$\Rightarrow$ $\log_2 15 < \log_2 2^4$

$\Rightarrow$ $\log_2 15 < 4$

$\Rightarrow$ $\log_{16} 15 < 1$

$\Rightarrow$ $\log_{10} 11 > 1$

$\Rightarrow$ $\log_7 6 < 1$

6. (b, d)

$\Rightarrow \log_{x+1}(x-0.5)=\log_{x-0.5}(x+1)$

$\Rightarrow$ x – 5 > 0; x – 0.5 > 0; $x-0.5\ne 1$

$\Rightarrow$ x > 0.5; x > 0.5; $x\ne 1.5$

$\Rightarrow$ x + 1 > 0; x + 1 > 0

$\Rightarrow$ x > –1; x > –1

So, $x\in(0.5,1.5)\cup(1.5,\infty)$ is feasible reason

$\Rightarrow \frac{\log(x-0.5)}{\log(x+1)}=\frac{\log(x+1)}{\log(x-0.5)}$

$\Rightarrow$ $\log^2 (x-0.5) - \log^2 (x+1) = 0$

$\Rightarrow \log\left(\frac{x-0.5}{x+1}\right)\log\{(x-0.5)(x+1)\}=0$

So, $\frac{x-0.5}{x+1}=1$ or (x – 0.5) (x + 1) = 1

$\Rightarrow$ x – 0.5 = x + 1 (no solution)

Or $x^2 + 0.5x - 1.5 = 0$

$\Rightarrow$ $2x^2 + x - 3 = 0$

$\Rightarrow$ $2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow$ x = 1

As $x\ne\frac{-3}{2}$

So x = 1 is the only solution.

7. (b, c)

$\Rightarrow x^{1-og_5 x} = 0.04$

$\Rightarrow x > 0$

$\Rightarrow x.x^{\log\frac{1}{x}} = 0.04$

Only $x = 25, \frac{1}{5}$ satisfy the equation

8. (a, b)

$\Rightarrow 10^{\frac{2}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$

$\Rightarrow 100^{\frac{1}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$

$\Rightarrow 4^{\frac{1}{x}} + 1 = \frac{17}{4}2^{\frac{1}{x}}$

$\Rightarrow 2^{\frac{2}{x}} - \frac{17}{4}2^{\frac{1}{x}} + 1 = 0$

$\Rightarrow \left(2^{\frac{1}{x}} - 4\right)\left(2^{\frac{1}{x}} - \frac{1}{x}\right) = 0$

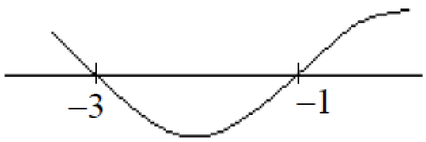
$\Rightarrow 2^{\frac{1}{x}} = 4, \frac{1}{4}$

$\Rightarrow x = \frac{-1}{2}, \frac{1}{2}$

9. (b, c)

$\Rightarrow \left|x^2 + 4x + 3\right| + 2x + 5 = 0$

$\Rightarrow \left|(x+1)(x+3)\right| + 2x + 5 = 0$



Case 1: $x \in (-\infty, -3] \cup [-1, \infty)$

$\Rightarrow x^2 + 4x + 3 + 2x + 5 = 0$

$\Rightarrow x^2 + 6x + 8 = 0$

$\Rightarrow x = -4, -2$

So $x = -4$

Case 2: $x \in (-3, -1)$

$\Rightarrow -(x^2 + 4x + 3) + 2x + 5 = 0$

$\Rightarrow x^2 + 2x - 2 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{12}}{2}$

$\Rightarrow x = -1 + \sqrt{3}, -1 - \sqrt{3}$

So, $x = -1 - \sqrt{3}$

10. (a, b)

$\Rightarrow \log_{a_1 a_2} a_1 = 4$

$\Rightarrow \log a_1 = 4\left[\log_{a_1} + \log_{a_2}\right] 3\log_{a_1} + 4\log_{a_2} = 0$

$\Rightarrow \log_{a_1 a_2} \frac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} \Rightarrow \log_{a_1 a_2}(a_1)^{\frac{1}{3}} - \log_{a_1 a_2}(a_2)^{\frac{1}{2}}$

$\Rightarrow \frac{1}{3}[4] - x$

$\Rightarrow x = \frac{1}{2}\left[\frac{\log(a_2)}{\log a_1 a_2}\right] = \frac{1}{2}\left[\frac{\log a_1 a_2}{\log a_1 a_2}\right]$

$= \frac{1}{2}\left[1 - \log_{a_1 a_2} a_1\right]$

$= \frac{1}{2}[1-4] = -\frac{3}{2}$

So, $\log_{a_1 a_2} \frac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} = \frac{4}{3} + \frac{3}{2} = \frac{17}{6}$

11. (b, c)

$\log_x 2 \,.\, \log_{2x} 2 = \log_{4x} 2$

$\frac{\log 2}{\log x} \cdot \frac{\log 2}{\log^{2x}} = \frac{\log 2}{\log^{4x}}$

log 2 (log 4x) = log x . log 2x

log 2 (2 log 2 + log x) = log x (log 2 + log x)

$2(\log 2)^2 + \log 2 . \log x = \log 2 . \log x + (\log x)^2$

$\log x = \pm\sqrt{2} \log 2$

$x = 2^{\pm\sqrt{2}}$

12. (a)

$\log_{(2x+3)}\left(6x^2 + 23x + 21\right)$

$= 4 - \log_{(3x+7)}\left(4x^2 + 12x + 9\right)$

$\Rightarrow \log_{(2x+3)}\left[(2x+3)(3x+7)\right]$

$= 4 - \log_{(3x+7)}\left[(2x+3)^2\right]$

$\Rightarrow 1 + \frac{\log(3x+7)}{\log(2x+3)} = 4 - \frac{2\log(2x+3)}{\log(3x+7)}$

Let $\frac{\log(3x+7)}{\log(2x+3)} = k$

$\Rightarrow 1 + k = 4 - \frac{2}{k}$

$k + \frac{2}{k} - 3 = 0$

$k^2 - 3k + 2 = 0$

$(k - 1)(k - 2) = 0$

$\frac{\log(3x+7)}{\log(2x+3)} = 1, 2$

$3x + 7 = 2x + 3$ or $3x + 7 = (2x + 3)^2$

$x = -4$ or $3x + 7 = 4x^2 + 9 + 12x$

$4x^2 + 9x + 2 = 0$ $\quad$ $4x^2 + 8x + x + 2 = 0$

$4x(x + 2) + (x + 2) = 0$

$x = -2$ or $x = -1/4$

So, $x = -\frac{1}{4}, -2, -4$

Also $2x + 3 > 0$ & $3x + 7 > 0$

$x > -\frac{3}{4}$ & $x > -\frac{7}{4}$

$\Rightarrow$ only value of $x = -\frac{1}{4}$

13. (b, d)

$2^{\log_{\sqrt{2}}(x-1)} > x+5$

$2^{\log_2 (x-1)^2} > x+5$

$(x - 1)^2 > x + 5$

$x^2 + 1 - 2x > x + 5$

$x^2 - 3x - 4 > 0$

$(x - 4)(x + 1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

14. (a)

$\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$

Also $x \neq y$

$\frac{\log x}{\log^2} + \frac{\log 2}{\log x} = \frac{10}{3}$

Solving we get

$\frac{\log x}{\log 2} = 3, \frac{1}{3}$

x = 8 or $x = 2^{1/3}$

If x = 8 $\Rightarrow$ $y = 2^{1/3}$

& if $x = 2^{1/3} \Rightarrow y = 8$

$\therefore \frac{x}{y} = \frac{2^3}{2^{1/3}}$ or $\frac{2^{1/3}}{2^3}$

$\frac{x}{y} = 2^{8/3}$ or $2^{-8/3}$

15. (a, b)

Let $a = \ln(x), b = kn(y), c = \ln(z)$, then a, b, c > 0, then

$$\alpha = \frac{a+b}{a+b+1} + \frac{b+c}{b+c+1} + \frac{c+a}{c+a+1}$$

$$\beta = \frac{a}{a+1} + \frac{b}{b+1} + \frac{c}{c+1}$$

For $a, b > 0$,

$$\frac{a+b}{a+b+1} - \frac{a}{a+1} = \frac{(a+b)(a+1) - a(a+b+1)}{(a+b+1)(a+1)}$$

$$= \frac{b}{(a+b+1)(a+1)} > 0$$

Similarly for other expressions.

$\therefore \quad \alpha > \beta$.

Also, $\frac{a+b}{a+b+1}-\frac{a}{a+1}-\frac{b}{b+1}$

$$=\frac{E}{(a+b+1)(a+1)(b+1)}$$

Where

$$E=(a+b)(a+1)(b+1)-(a+b+1)(2ab+a+b)$$

$$=ab(a+b)+(a+b)^2+(a+b)-2ab(a+b)-(a+b)^2-2ab-a-b$$

$$=-ab(a+b)-2ab<0$$

$$\Rightarrow \quad \frac{a+b}{a+b+1}<\frac{a}{a+1}+\frac{b}{b+1}$$

Similarly for other two expressions, therefore

$$\sum\frac{a+b}{a+b+1}<2\sum\frac{a}{a+1} \text{ or } \alpha<2\beta.$$

16. (a, b)

$4^x-(a-3)2^x+a-4=0$

Roots are non positive

$x \le 0$

$\Rightarrow 0<2^x \le 1$

$(2^x)^2-(a-3)2^x+a-4=0$

$k^2-(a-3)k+(a-4)=0$

(i) $f(0)>0$

(ii) $f(1)\ge 0$

(iii) $D>0$

(i) $f(0)=a-4>0$

$a>4$

(ii) $f(1)\ge 0$

$1-(a-3)+a-4\ge 0$

(iii) $D>0$

$(a-3)^2-4(a-4)>0$

$a^2+9-6a-4a+16>0$

$a^2-10a+25>0$

$(a-5)^2>0$

$a\in R$

(iv) $0<-\frac{B}{2A}\le 1$

$0<-\frac{(-(a-3))}{2\times 1}\le 1$

$0<\frac{a-3}{2}\le 1$

$3<a\le 5$

From (i), (ii), (iii), (iv)

$a\in(4,5]$

17. (b, c, d)

$27^x + 2\cos 3y + 8 + 6\cos y = 4.3^{x+1}.\cos y$

$3^{3x} + 2\cos 3y + 8 + 6\cos y = 4.3^{x+1}.\cos y$

$\left(3^x\right)^3 + 8\cos^3 y - \cancel{6\cos y} + 8 + \cancel{6\cos y} = 3.2.3^x.2\cos y$

$\left(3^x\right)^3 + \left(2\cos y\right)^3 + 2^3 = 3.\left(3^x\right)\left(2\cos y\right)\left(2\right)$

$3^x + 2\cos y + 2 = 0$

Or $3^x = 2\cos y = 2$

$x = \log_3 2, \cos y = 1$

18. (c, d)

$\log_2 x = \log_4 y + \log_4 \left(4 - x\right)$

$4 - x > 0 \Rightarrow x < 4$

& x, y > 0

$\log_2 x = \frac{1}{2}\log_2\left(y.\left(4-x\right)\right)$

$x^2 = \left(4 - x\right).y$ …(1)

$\log_3\left(x + y\right) = \log_3 x - \log_3 y$

x + y > 0 & x > 0 & y > 0

$x + y = x/y$

$y = \frac{x}{y} - x$

$y = \left(\frac{1}{y} - 1\right)^x$ … (2)

$x = 4/3, y = 2/3$

19. (a, b, c, d)

$f(x) = x^8 - x^5 + x^2 - x + 1$

$f(x) = x^5 (x^3 - 1) + x (x - 1) + 1$

$f(x) = x (x - 1) [x^4 (x^2 + x + 1) + 1] + 1$

$x^4 (x^2 + x + 1) > 0$

Also x (x – 1) > 0 for $n \in \left(-\infty, 0\right) \cup \left(1, \infty\right)$

Now for $x \in \left[0, 1\right]$

$x^8 + x^2 + 1 - x^5 - x > 0$

$\therefore f\left(x\right) > 0 \Rightarrow x \in R$

20. (a, b, c)

$x^2 + y^2 - xy - x - y + 1 \geq k$

$x^2 - \left(y + 1\right)x + y^2 - y + 1 = k \geq 0$

$D \leq 0 \left(y + 1\right)^2 - 4y^2 + 4y - 4 + 4k \leq 0$

$3y2 - 6y + 3 - 4k \geq 0 \left(\forall y \in R\right)$

$\left(D \leq 0\right) 36 - 36 + 48k \leq 0$

Minimum value of expected = 0

$k \leq 0$

Have minimum value is 0
In–equation has solution $x, y \in R$
$\therefore$ All equations with real values if x & y are true.

21. (b, c)
$\log_{(a)} x \le \log_a x^2$
For 0 < a < 1
$x \ge x^2$
$x \in (0,1]$ $\{\text{as } x \ne 0\}$
For 1 < a
$x \le x^2$
$x(x-1) \ge 0$
$x \in [1, \infty)$

22. (a, c)
$\log_3(x+2) > \log_{x+2} 81$
$$\frac{\log(x+2)}{\log 3} > \frac{4\log 3}{\log(x+2)}$$
(i) for x > –1
log(x + 2) > 0
$\therefore [\log(x+2)]^2 > 4(\log 3)^2$
(log (x + 2) – 2 log 3) (log (x + 2) + 2 log 3) > 0
$\Rightarrow$ log (x + 2) < – 2 log 2 & log (x + 2) > 2 log 3
& log (x + 2) > 2 log 3
$x < \frac{1}{9} - 2$ & x > 7
$x < \frac{-17}{9}$
(ii) for x < –1
$\log(x+2)^2 \le 4(\log 3)^2$
$-2\log 3 < \log(x+2) < 2\log 3$
$\Rightarrow \frac{-17}{9} < x < 7$

23. (b, c)
$$\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} \le \frac{1}{\log_4(x+3)}$$
$$\frac{x+1}{x+2} > 1$$
$$\frac{x+1-x-2}{x+2} > 0$$
$$-\frac{1}{x+2} > 0$$
$$\frac{1}{x+2} < 0$$
$\Rightarrow x < -2$ … (1)

$$\frac{x+1}{x+2} > 0$$

$\Rightarrow x > -1$ & $x < -2$ … (2)

$$\Rightarrow 0 < \frac{x+1}{x+2} < 1$$

For $x > -1$

Case I take $x > -1$

$$\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} \le \frac{1}{\log_4(x+3)}$$

$$\log_4(x+3) \ge \log_4\left(\frac{x+1}{x+2}\right)$$

$$\left\{\text{as } \log_4\left(\frac{x+1}{x+2}\right) < 0 \text{ for } x > -1\right\}$$

$$x+3 \ge \frac{x+1}{x+2}$$

$$x+3-\left(\frac{x+1}{x+2}\right) \ge 0$$

$\Rightarrow x > -2$

{we have taken $x > -1$}

i.e. $x > -1$

case II: $-3 < x < -2$

$$x+3 = \frac{x+1}{x+2} \quad \frac{x^2+4x+5}{x+2} \ge 0$$

$x > -2$

no solution

24. (c)

$$N = \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \log_6^2 2$$

$$N = \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \frac{1}{(\log_2 6)^2} \Rightarrow \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \frac{1}{(1+\log_2 3)^2}$$

$$N = \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \frac{(\log_3 2)^2}{(1+\log_3 2)^2}$$

$$N = \frac{(1+\log_3 2)^2}{(1+\log_3 2)^2} = 1$$

25. (a, b, c, d)

$$\frac{1}{\log_{225}^{x}} - \frac{1}{\log_{64}^{y}} = 1$$

$$\Rightarrow \log_{64}^{y} - \log_{225}^{x} = \log_{225}^{x} . \log_{64}^{y}$$

From equation

$$4 - 2\log_{225}^{x} = \log_{225}^{x}\left(4 - \log_{225}^{x}\right)$$

$$\left(\log_{225}^{x}\right)^2 - 6\log_{225}^{x} + 4 = 0$$

So, $\log_{225}^{x_1} + \log_{225}^{x_2} = 6, \quad \log_{225} x_1 . \log_{225} x_2 = 4$

$x_1\, x_2 = (225)^6$

Similarly

$(\log_{64} y)^2 - 2(\log_{64} y) - 4 = 0$

$\log_{64} y_1 + \log_{64} y_2 = 2$

$\log_{64} y_1 . \log_{64} y_2 = -4$

$\therefore \ \left|\log_{64} y_1 - \log_{64} y_2\right| = 2\sqrt{5}$

Also $y_1\, y_2 = (64)^2$

So, $x_1\, x_2\, y_1\, y_2 = (225\times 4)^6 = \log_{30}^{(30)^{12}}$

26. (a, b, c, d)

If a & b are same side of unity then $\log_a b$ is positive and if opposite side of unity then $\log_a b$ is –ve.

27. (a, b, c, d)

$3^{2(x^2-x-6)} - 2.3^{(x^2-x-6)} + 1 = 0$

$\Rightarrow \ 3^{(x^2-x-6)} = 1 \Rightarrow x^2 - x - 6 = 0 \ \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = -2, 3$

28. (b, d)

$$x^{\log_{10}^2 x + \log_{10} x^3 + 3} = \frac{2}{\frac{1}{\sqrt{x+1}-1} - \frac{1}{\sqrt{x+1}+1}}$$

$$x^{\log_{10}^2 x + 3\log_{10} x + 3} = \frac{2\left((x+1)-1\right)}{\left(\sqrt{x+1}+1\right) - \left(\sqrt{x+1}-1\right)} = x$$

So, $x^{\log_{10}^2 x + 3\log_{10} x + 3} = x$

Take log on both sides

$(\log_{10}^2 x + 3\log_{10} x + 3)\log_{10} x = \log_{10} x$

$\log_{10} x(\log_{10}^2 x + 3\log_{10} x + 2) = 0$

Put $\log_{10} x = y$

$y(y+1)(y+2) = 0 \ \Rightarrow \ y = 0, -1, -2$

$\log_{10} x = 0 \ \Rightarrow \ x = 1 \qquad \log_{10} x = -1 \Rightarrow \ x = \frac{1}{10}$

$\log_{10} x = -2 \ \Rightarrow \ x = \frac{1}{100}$

Given $x_1 > x_2 > x_3$

$x_1 = 1 \quad x_2 = \frac{1}{10} \quad x_3 = \frac{1}{100}$

(B) satisfy the values

29. (a, b, d)

(a) $\log_b a . \log_c b . \log_d c . \log_a d = \log_c a . \log_d c . \log_a d$

$= \log_d a . \log_a d = 1$

(b) $2^2 . 2^{-\log_2 5} = 4.2^{\log_2(5^{-1})} = \frac{4}{5}$

(c) $3^{4\log_3 5}+(27)^{\log_9 36}=5^4+(3^3)^{(1/2)\log_3(36)}$

$=625+(36)^{3/2}=625+216=841$

(d) $8^{\log_2 \sqrt[3]{121}+(1/3)}=2^{3\left[\log_2(121)^{1/3}+1/3\right]}$

$=2^{\log_2 121+1}=121\times 2=242$

30. (a, b, c)

First observe that

$\log_2 x=\log_4(x^2)$

$\log_3 y=\log_9(y^2)$

$\log_4 z=\log_{16}(z^2)$

From $\log_2 x+\log_4 y+\log_4 z=2$, we get that

$\log_4 x^2yz=2$ and hence $x^2yz=4^2=16$ ...(1)

Similarly,

$y^2zx=9^2=81$ ...(2)

$z^2xy=16^2=256$ ...(3)

From Eqs. (1)-(3), wet get that $x^4y^4z^4=16\times 81\times 256$

Therefore, $xyz=2\times 3\times 4=24$

Since, $x^2yz=16$ and $xyz=24$, we get that

$x=\frac{16}{24}=\frac{2}{3}$

Similarly, $y=\frac{27}{8}$ and $z=\frac{32}{3}$. Therefore, $xy=\frac{9}{4}$, $yz=36$ and $zx=\frac{64}{9}$.

Comprehensions Type:

Passage - 1

1. (d)

$||x+1|-2|=1$

$\Rightarrow |x-1|=3$ *or* $|x-1|=1$

$\Rightarrow (x-1)^2=9$ $x-1=\pm 1$

$\Rightarrow (x-4)(x+2)=0$ $x=2, 0$

$\Rightarrow x=-2, 4$

2. (b)

$||x-2|-3|=4$

$\Rightarrow |x-2|=7$

$\Rightarrow (x-2)^2=7^2$

$\Rightarrow (x-9)(x-5)=0$

$\Rightarrow x=5, 9$

3. (c)

$|x-2|=10, 0$

$x=2, 12, -8$

Passage - 2

4. (a)

$A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3}$

$\Rightarrow A = 2^2 + 3^{\log_2 4} + 4^1 - 3^{\log_2 4} = 8$

$D = (\log_5 49)(\log_7 125)$

$\Rightarrow D = \frac{\log 49}{\log 5} \times \frac{\log 125}{\log 7} = \frac{2\log 7}{\log 5} \times \frac{3\log 5}{\log 7}$

$\Rightarrow$D = 6

$\Rightarrow a = \log_A D = \log_8 6$

$\Rightarrow a = \frac{\log_2 6}{3}$

Now $\log_6 12 = \log_6 6 + \log_6 2$

$\Rightarrow \log_6 12 = 1 + \frac{1}{\log_2 6}$

$\Rightarrow \log_6 12 = 1 + \frac{1}{3a} = \frac{1+3a}{3a}$

5. (a)

$N = 7^{\log_{49} 900}$

$\Rightarrow N = 7^{\log_{72} 30^2} = 7^{\log_7 30} = 30$

$\log_6 12 = \frac{1+ma}{ma} \Rightarrow m = n = 3$

$\log_N m = \log_{30} 3$

$\log_m N = \log_3 30$

$\log_n N = \log_3 30$

Clearly $\log_{30} 3 < \log_3 30$

$\therefore \log_N m < \log_m N = \log_n N$

6. (b)

$\log_{\left(A-\frac{N}{10}\right)} |N + A + D + m + n| - \log_5 2 = \log_{\left(8-\frac{30}{10}\right)} |30+8+6+3+3| - \log_5 2$

$= \log_5 50 - \log_5 2 = \log_5 25 + \log_5 2 - \log_5 2$

$= \log_5 5^2 = 2$

Passage - 3

7. (a)

$y = 4 - |4x^2 - 9| \Rightarrow |4x^2 - 9| = 4 - y \geq 0$

$\Rightarrow y \leq 4$

8. (a)

$\Rightarrow (4x^2 - 9)^2 = 7^2$

$\Rightarrow (4x^2 - 16)(4x^2 - 2) = 0$

$\Rightarrow x = \pm 2, \pm \frac{1}{\sqrt{2}}$

9. (b)

$|Z| + 4 = 4 - |4x^2 - 9| \Rightarrow |Z| + |4x^2 - 9| = 0$

$\Rightarrow 4x^2 - 9 = 0, Z = 0$

$\Rightarrow x = \pm\frac{3}{2}, Z = 0$

Passage - 4

10. (d)

$y = \log_x(4 - x) + \log_3(2 + x)$

$x > 0, x \neq 1, 4 - x > 0 \,\&\, 2 + x > 0$

$x \in (0,1) \cup (1,4)$

11. (a)

$\Rightarrow \frac{x}{x+3} \geq 1 \Rightarrow \frac{3}{x+3} \leq 0$

$\Rightarrow x < -3$

12. (c)

$y = \log_{\frac{1}{2}}\left(\frac{x+4}{x+1}\right) + \sqrt{x-1}$

$\Rightarrow \frac{x+4}{x+1} > 0 \,\&\, x - 1 \geq 0$

$\Rightarrow x < -4$ or $x > -1$ & $x \geq 1$

$\Rightarrow (-\infty, -4) \cup [1, \infty)$

Passage - 5

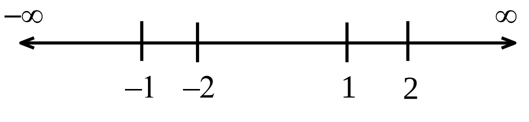
13. (a)

$f(x) = |x-2| + |x-1| + |x+1| + |x+2|$

$f(-x) = f(x)$ then given functions is symmetric about y – axis

14. (b)

$|x-1| + |x-2| + |x+1| + |x+2| = 6$



Case I: x < –2

$\therefore \cancel{1} - x + \cancel{2} - x - n - \cancel{1} - n - \cancel{2} = 6$

– 4x = 6

$x = -3/2$

Case II: $-2 \leq x < -1$

$\cancel{1} - n + \cancel{2} - n - \cancel{n} - \cancel{1} + \cancel{x} + 2 = 6$

– 2x = 2 $\therefore x = -1$

Case III: $-1 \leq x < 1$

$1 - \cancel{x} + 2 - \cancel{x} + \cancel{x} + 1 + \cancel{x} + 2 = 6$

6 = 6

Case IV: $1 \le x < 2$

$\cancel{x} + 1 + 2 - \cancel{x} + n + 1 + n + 2 = 6$

2x = 0 $\quad\therefore$ n = 0

Case V: $x \ge 2$

4x = 6 $\quad \therefore x = 3/2$

$\therefore x \in [-1, 1]$

15. (c)

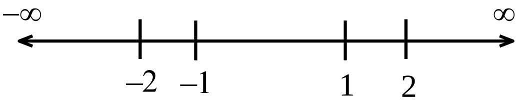
$f(n) = |x-2| + |x-1| + |x+1| + |x+2|$

$f(x)_{\min} = 6$

$\therefore k = 3$

16. (c)

$|x-1| + |x-2| + |x+1| + |x+2| \le 20$



Case I: x < –2

$\cancel{1} - x + \cancel{2} - x - x - \cancel{1} - x - \cancel{2} \le 20$

$-4x \le 20, \quad x \ge -5$

$\therefore x \in [-5, -2]$

Case II: $-2 \le x < -1$

$\cancel{1} - x + \cancel{2} - x - \cancel{x} - \cancel{1} + \cancel{x} + 2 \le 20$

$-2x \le 16 \quad \therefore x \ge -8$

$x \in [-2, -1]$

Case III: $-1 \le x < 1$

$1 - \cancel{x} + 2 - \cancel{x} + \cancel{n} + 1 + \cancel{n} + 2 \le 20$

$6 \le 20$

Case IV: $1 \le x < 2$

$\cancel{x} - \cancel{1} + 2 - \cancel{x} + n + \cancel{1} + x + 2 \le 20$

$2x \le 16 \quad x \le 8$

Case V: $x \ge 2$

$4x \le 20 \quad x \le 5$

$x \in [-5, 5]$

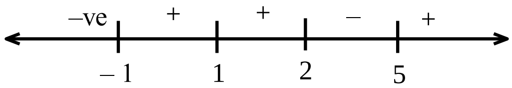
Matrix-Match Type:

1. **(A) → R ; (B) → P ; (C) → S ; (D) → Q**

(a) $\dfrac{(x-1)^2 (x+1)(x-2)}{(x-5)} \ge 0$

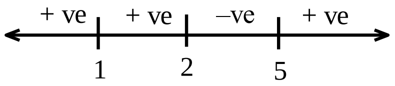
x = 1, 1, –1, 2 (roots)

x = 5 (pole)



$x \in [-1, 2] \cup (5, \infty)$

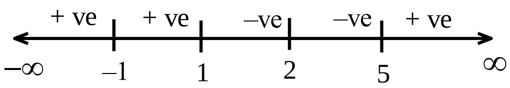
(b) $\frac{|x|(x-5)}{(x-2)} \geq 0$



$x \in (-\infty, 2] \cup [5, \infty)$

(c) $\log_{10} x$
If x is two digit number then Q = [1, 2]
P = $\log_{10}$ x, x is four digit number then $P = [3, 4)$

(d) $\frac{(x-1)(x+1)(x-2)^2}{(x-5)} \leq 0$



$x \in [1, 5]$

2. **(A) → R, S ; (B) → R ; (C) → Q ; (D) → R**
(A) – (r, s)
$\Rightarrow \log_{49} 7 = \frac{1}{2}$
$\Rightarrow \log_3 (5 + 8\log_{49}(5 + 4\log_{49} 7))$
$= \log_3 (5 + 8\log_{49} 7)$
$= \log_3 \left(5 + \frac{8}{2}\right)$
$= \log_3 9 = 2$
$|k| = 2 \Rightarrow k = \pm 2$
(B) $\sqrt{\left(\log_{2^{-1}} .2^2\right)^2} = \sqrt{(-2)^2} = |-2| = 2$
(C) $\log_x \left(x^2 - 1\right) = 0$
$\therefore x^2 - 1 = 1$
$x^2 = 2$
$x = \pm\sqrt{2}$
But $x = -\sqrt{2}$ (rejected)
$\therefore$ only one solution
(D) $\frac{1}{x} = \frac{\sqrt{9-\sqrt{77}}}{\sqrt{81-77}}$ $\quad\therefore\quad$ $\frac{2}{x} = \sqrt{9-\sqrt{77}}$
$x + \frac{2}{x} = \sqrt{9+\sqrt{77}} + \sqrt{9-\sqrt{77}}$
$\left(x + \frac{2}{x}\right)^2 = 18 + 2 \times 2 = 22$
$\frac{1}{11}\left(x + \frac{2}{x}\right)^2 = 2$

3. **(A) → Q, R ; (B) → P, Q, R,S ; (C) → P, Q, R, S ; (D) → (P, Q, R, S**

(A) – (QR)

$1 - x^2 > 0$ $\therefore x^2 < 1$

$2 - x^2 > 2\sqrt{1-x^2}$ $|x| < 1$

$4 + x^4 - \cancel{4x^2} > 4 - \cancel{4x^2}$

$x^4 > 0$ $\therefore x \in (-1, 1)$

(B) – (PQRS)

$x^8 - x^5 + x^2 - x + 1 > 0 \; x \in R$

For $x < 0$ (time)

For $x \in (011)$

$$\frac{x^8}{+\text{ve}} + \frac{(x^2 - x^5)}{+\text{ve}} + \frac{(x)}{+\text{ve}} > 0$$

For $x > 1$

$$\frac{(x^8 - x^5)}{+\text{ve}} + \frac{(x^2 - \alpha)}{+\text{ve}} + \frac{1}{+\text{ve}} > 0$$

(C) – (PQRS)

$x^{12} - x^9 + x^4 - x + 3 > 0$ (same)

(D) – (PQRS)

$2x^2 + 1 > \sqrt{4x^2 + 1}$

$4x^4 + 4x^2 + 1 > 4x^2 + 1$

$x^4 > 0 \;\; n \in (-\infty, \infty) - \{0\}$

4. **(A) → Q, S ; (B) → Q, S ; (C) → P ; (D) → Q, R**

(A) – (QS)

$x^{\log_{10} x} = 100x$

$\Rightarrow (\log_{10} \lambda)^2 = 2(\log_{10} \alpha)$

$\Rightarrow \log_{10} x = 2, -1$

$x = 100, \dfrac{1}{10}$

$x_1.x_2 = 10$

(B) – (QS)

$\log_2 (9 - 2^x) = 3 - x$

$\Rightarrow 9 - 2^x = \dfrac{8}{2^x}$

$\Rightarrow (2^x)^2 - 9.(2^x) + 8 = 0$

$2^x = 1, 2^x = 8$

x = 0, x = 3

$\lambda_1^{\;2} + \alpha_2^{\;2} = 9$

(C) – (P)

$\log_{1/8} \log_{1/4} \log_{1/2} (x) = \dfrac{1}{3}$

$\Rightarrow \log_{1/4} \log_{1/2} x = \dfrac{1}{2}$

$\Rightarrow \log_{1/2} x = \left(\dfrac{1}{4}\right)^{1/2} = 1/2$

$\Rightarrow x = \left(1/2 \right)^{1/2} = \frac{1}{\sqrt{2}}$

(D) – (QR)

$\log_b a = 3 \qquad \log_b c = -4$

a = b^3, $\qquad c = \frac{1}{b^4}$

$b^{9x} = \frac{1}{b^{4x-4}}$

$\Rightarrow b^{9x} = b^{-4x+4}$

$9x = -4x + 4$

$\Rightarrow x = 4/13$

p + q = 17

5. (A) – (R), (B) – (S), (C) – (Q), (D) – (P)

(a) Rewrite as

$$\log_{0.5} \frac{|x^2+2x-8|}{10+3x-x^2} = 1 \Rightarrow \frac{|x^2+2x-8|}{10+3x-x^2} = \frac{1}{2}$$

$\Rightarrow \quad 2|x^2+2x-8| = 10+3x-x^2$

$\Rightarrow \quad 2|(x+4)(x-2)| = (5-x)(2+x)$

$\Rightarrow \quad x = \frac{1}{6}\left(\sqrt{313}-1\right), \frac{1}{2}\left(\sqrt{73}-7\right)$

(b) Put $t = \log_2\left(x^2+7\right) - \log_2 x$ to obtain

$t = 5 - \frac{6}{t} \Rightarrow t^2 - 5t + 6 = 0 \Rightarrow t = 2, 3$

$\Rightarrow \quad \log_2\left(\frac{x^2+7}{x}\right) = 2, 3 \Rightarrow \frac{x^2+7}{x} = 4, 8$

$\Rightarrow \quad x = 1, 7$

(c) $1-2x > 0, 1-2x \neq 1, 1-3x > 0, 1-3x \neq 1, 6x^2-5x+1 > 0, 4x^2-4x+1 > 0$

Rewrite the equations as

$\log_{(1-2x)}\left[(1-2x)(1-3x)\right] - \log_{(1-3x)}(1-2x)^2 = 2$

$\Rightarrow \quad 1 + t - \frac{2}{t} = 2$

When $t = \log(1-3x)/\log(1-2x)$

$\Rightarrow \quad t^2 - t - 2 = 0 \Rightarrow t = -1, 2 \Rightarrow x = \frac{1}{4}$

(d) $\log_{10}(1-x)^2 + 1 - \log_{10}\left(1+x^2\right) = 2\log_{10}(1-x)$

$\Rightarrow \quad \log_{10}\left(1+x^2\right) = 1 \Rightarrow 1+x^2 = 10 \Rightarrow x = \pm 3$.

As $x < 1, x = -3$

6. (A) – (R), (B) – (P), (C) – (Q), (D) – (S)

(a) $\log_6\left(\frac{x^2+x}{x+4}\right) > 1 \Leftrightarrow \frac{x^2+x}{x+4} > 6$

$\Leftrightarrow \quad \dfrac{x^2+x}{x+4}-6>0 \quad \Leftrightarrow \quad \dfrac{(x+3)(x-8)}{x+4}>0$

$\Leftrightarrow \quad -4<x<-3, x>8$

(b) $\log_{1/25}\left(x^2-\dfrac{10}{3}x+1\right)\geq 0$

$\Rightarrow \quad 0<x^2-\dfrac{10}{3}x+1\leq 1$

$\Rightarrow \quad -\infty<x<\dfrac{1}{3}$ or $3<x<\infty$ and $0\leq x\leq\dfrac{10}{3}$.

Thus, $0\leq x<\dfrac{1}{3}$ or $3<x\leq\dfrac{10}{3}$

(c) Rewrite as

$\log_{x^2}(2+x)<\log_{x^2}(x^2)$

Thus, either

$x^2>1, 2+x<x^2, 2+x>0$ or $0<x^2<1, 2+x>x^2, 2+x>0$

$\Rightarrow \quad x\in(-2,-1)\cup(2,\infty)\cup(-1,0)\cup(0,1)$

(d) $\left|\dfrac{x-1}{2x+1}\right|<1 \quad \Rightarrow \quad -1<\dfrac{x-1}{2x+1}<1$

$\Rightarrow \quad \dfrac{x-1}{2x+1}+1>0$ and $\dfrac{x-1}{2x+1}-1<0$

$\Rightarrow \quad \dfrac{3x}{2x+1}>0$ and $\dfrac{x+2}{2x+1}>0$

$\Rightarrow \quad x\in(-\infty,-2)\cup(0,1)\cup(1,\infty)$

7. (A) – (Q, R, S), (B) – (Q, S), (C) – (P, Q), (D) – (S)

Put $\log_5 x=t$

(a) Write the equation as $t^2+\dfrac{1-t}{1+t}=1$

$\Rightarrow \quad t^3+t^2+1-t=1+t \quad \Rightarrow \quad t^3+t^2-2t=0$

$\Rightarrow \quad t=0,1,-2$

$\Rightarrow \quad x=1, 5, \dfrac{1}{25}$

(b) Equation can be written as

$\dfrac{4}{t^2}-\dfrac{3}{t}+\dfrac{1}{t}=2 \quad \Rightarrow \quad \dfrac{2}{t^2}-\dfrac{1}{t}-1=0$

$\Rightarrow \quad t^2+t-2=0 \quad \Rightarrow \quad t=1,-2 \quad \Rightarrow \quad x=5, \dfrac{1}{25}$

(c) $\dfrac{2t}{t-1}-\dfrac{42t}{4+t}+\dfrac{20t}{t+2}=0$

$\Rightarrow \quad t=0, t=2, t=-\dfrac{1}{2}$.

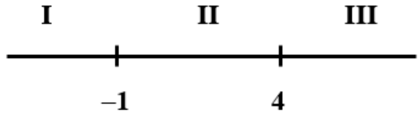
$\Rightarrow \quad x=1, 25, \dfrac{1}{\sqrt{5}}$

(d) $t^2+t+1=\dfrac{-9}{t-1} \quad \Rightarrow \quad t^3-1=-9$

$\Rightarrow \quad t=-2 \quad \Rightarrow \quad x=\dfrac{1}{25}$

1. (2)

$|x+1|+|x-4|>7$



Case – I $(x<-1)$

$\Rightarrow -1-x+4-x>7$

$\Rightarrow 2x<-4$

$\Rightarrow x<-2$

Case – II $(-1\le x<4)$

$x+1+4-x>7$

$\Rightarrow 5>7$ (not possible)

Case – III $(x\ge 4)$

$(x+1)+(x-4)>7$

$\Rightarrow 2x>10 \Rightarrow x>5$

Hence, there are two disjoint sets

$(-\infty,-1)\cup(5,\infty)$

2. (1)

$0\le \dfrac{x^2-x-2}{x^2}<\dfrac{4}{3}$ & solve as above

3. (6)

Domain: $x\in[1,9]$ & on solving $x\in[1,5]$

So (m + n) = 6

4. (5)

Domain (a) $-x^2+2x+2y\ge 0\Rightarrow x^2-2x-2y\le 0$

$\Rightarrow (x-6)(x-4)\le 0\Rightarrow -4\le x\le 6$ … (a)

(b) $8x-x^2\ge 0\Rightarrow x^2 8x\le 0\Rightarrow x\in[0,8]$ …(b)

(a) n (b) $x\in[0,6]$ …(1)

(b) $-x^2+2x+2y\ge 8x-x^2\Rightarrow x\le y$ … (2)

(1) & (2) $x\in[0,4]$

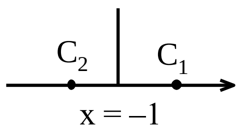
So answer is (5)

5. (1)

As function is increasing so equation change at most one solution

6. (2)

$|2^{x+1}-1|+|2^{x+1}+1|=2^{|x+1|}; x\in R$



C_1 : if $x \ge -1$ …(a)

$2.2^x - 1 + 2.2^x + 1 \Rightarrow 2^x.2$

$\Rightarrow 2^x = 0 \Rightarrow x \in \phi$ … (b)

(a) & (b) $x \in \phi$ … C_1

C_2: If $x < -1$

$1 - 2.2^x + 2.2^x + 1 = \frac{1}{2}.2^{-x}$

$\Rightarrow 2^{-x} = 4 \Rightarrow x = -2$ … (b)

(a) & (b) x = –2 … C_2

$C-1 \cup C-2$

$x = -2 \Rightarrow |x| = 2$

7. (7)

(x – 2) = 0 or 1 $\Rightarrow x = 2, 3$

or $\log_2 x^3 - 3\log_x{}^4 = 3 \Rightarrow 3\log_2{}^x - \frac{6}{\log_2 x} = 3$

$\Rightarrow \log_2^2 x - \log_2 x - 2 = 0 \Rightarrow (\log_2 x - 2)(\log_2 x + 1) = 0$

$\log_2 x = -1, 2 \Rightarrow x = \frac{1}{2}, 4$

So sum = 9.5

8. (2)

$\sqrt{\log_2^x} + \sqrt[3]{\log_2^x} = 2$

x = 2

9. (0)

$(x+1)(x-1) - 2\sqrt{x^2-1} = 4x - 1$

$\Rightarrow -2\sqrt{x^2-1} = (2x-1)$

$\Rightarrow 4(x^2-1) = 4x^2 + 1 - 4x \Rightarrow x = 5/4$

But $x = \frac{5}{4}$. Doesn't satisfy so no solution

10. (2)

$(x-1)^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$

$\Rightarrow (x-1)^{\log_3 x^2 - 2\log_x 9 - 7} = 1$

Case – I

$\log_3 x^2 - 2\log_x 9 - 7 = 0$

$\Rightarrow 2\log_3 x - \frac{2}{\log_9 x} - 7 = 0$

$\Rightarrow 2\log_3 x - \frac{2}{\frac{1}{2}\log_3 x} - 7 = 0$

Let $\log_3 x = y$

$\Rightarrow 2y - \frac{4}{y} - 7 = 0$

$\Rightarrow\ 2y^2-7y-4=0$

$\Rightarrow\ 2y^2-8y+y-4=0$

$\Rightarrow\ 2y(y-4)+(y-4)=0$

$\Rightarrow\ (2y+1)(y-4)=0$

$\Rightarrow\ y=-\dfrac{1}{2}$ or $y=4$

$\Rightarrow\ x=3^{-\frac{1}{2}}$ or $x=3^4$

Case – II

$x-1=1\ \Rightarrow\ x=2$

11. (4)

$D\ge 0\Rightarrow 16-\log_2 n\ge 0\Rightarrow \log_2 n\le 2$

$(0<n\le 4)$

12. (2)

$|x-3|^{\frac{x^2-8x+15}{x-2}}=1$

Case – I

$\dfrac{x^2-8x+15}{x-2}=0$

$\Rightarrow\ x=3$ or $x=5$

Case – II

$|x-3|=1\ \Rightarrow\ x=3\pm 1$

$\Rightarrow\ x=4$ or $x=2$

Note that $x=2$ is rejected as denominator cannot be 0 & $x=3$ is rejected because it will lead to 0° form. Hence $x=5$ & $x=4$ are the two solutions.

13. (7)

(1) Domain: a) $x>0, x\ne\dfrac{1}{2}$ …(a)

b) $x^2-5x+6>0$

$\Rightarrow (x-2)(x-3)>0\Rightarrow x\in(-\infty,2)\cup(3,\infty)$ … (b)

(a) & (b) $x\in(0,2)\cup(3,\infty)-\left\{\dfrac{1}{2}\right\}$ … (1)

(2) C–1: If $x>\dfrac{1}{2}$ … (a)

$x^2-5x+6>2x$

$\Rightarrow x^2-7x+6>0\ \Rightarrow\ (x-6)(x-1)>0$

$\Rightarrow x\in(-\infty,1)\cup(6,\infty)$ …(b)

(a) & (b) $x(6,\infty)$ … (C–1)

C–2 if $x<\dfrac{1}{2}$ … (a)

$x^2-5x+6<2x$

$x\le(1,6)$ … (b)

n $x\in\phi$ … (C–2)

$(C-1)\cup(C-2)$

$x\in(6,\infty)$ ...(2)

(1) n (2) $x\in(6,\infty)$

14. (2)

$y^{(6-y)^2+7(6-y)+12}=1$

$\Rightarrow y^{y^2-19y+90}=1\Rightarrow^{(y-10)(y-9)}=1$

So y = 1, 9, 10, –1 $\Rightarrow$ x = 5, –3, –4, 7

So $(x,y)\equiv(5,1)$ or $(7,-1)$

15. (1)

$3+|x+2|=\sqrt{9-y^2}$

So x = –2, y = 0

So (–2, 0)

16. (2)

$\log(-x)=\sqrt{\log|x|}$

Domain: x < 0

$\log^2(-x)=\log(-x)$

log (–x) = 0, 1

–x = 1, 10 $\Rightarrow$x = –1, –10

i.e. two solutions

17. (2)

Let $\log_a^x=t$

$\frac{2}{t}+\frac{1}{1+t}+\frac{3}{2+t}=0$

$\Rightarrow$ 2(1 + t) (2 + t) + t (2 + t) + 3t (1 + t) = 0

$\Rightarrow$ $6t^2$ + 11t + 2 = 0

$t=\frac{-11\pm\sqrt{121-48}}{12}=\frac{-11\pm\sqrt{73}}{12}$

i.e. two values

18. (3)

$N=\log_3(3^3.5).\log(5.3)-\log_3{}^5.\log_3(3^4.5)$

$=(3+\log_3{}^5)(1+\log_3{}^5)-(\log_3{}^5)(4+\log_3{}^5)$

= 3

19. (0)

log (x – 3) (x – 1)

$\left(\frac{1}{10}\right)(x-3)\ge 1$

log (x – 3) (x + 1)

$\Rightarrow 10(x-3)\le 1$

Domain: $x\in(3,\infty)-\{4\}$

$10^{1+\log_{(x-3)}^{(x-1)}}\le 1$

$\Rightarrow \log_{(x-3)}^{(x-1)} \le -1$

Not possible as x > 3 so no solution

20. (2)

$\sqrt{(x-1)+4-2.\sqrt{4}\sqrt{2-1}}+\sqrt{(x-1)+9-2\sqrt{9}\sqrt{-1}}=1$

$\Rightarrow \left|\sqrt{x-1}-2\right|+\left|\sqrt{x-1}-3\right|=1$

C_1: If $x \ge 4$ … (a)

$\left(\sqrt{x-1}-2\right)+\left(\sqrt{x-1}-3\right)=1 \Rightarrow \sqrt{x-1}=3$

x = 10 … (b)

(a) n (b) x = 10 … (C–1)

C_2: If $x \in (3,4)$ … (a)

$\left(\sqrt{x-1}-2\right)-\left(\sqrt{x-1}-3\right)=1 \Rightarrow 1=1 \Rightarrow x \in R$ …(b)

(a) n (b) $x \in (3,4)$ …(C – 2)

C_3: If $x \le 3$ … (a)

$-\left(\sqrt{x-1}-2\right)-\left(\sqrt{x-1}-3\right)=1$

$\Rightarrow \left(\sqrt{x-1}-2\right)\left(\sqrt{x-1}-3\right)=1$

$\Rightarrow \sqrt{x-1}=2 \Rightarrow x=5$ … (b)

(a) n (b) $x \in \phi$ … (C – 3)

$(C-1)\cup(C-2)\cup(C-3)$

$x \in (3,4)\cup\{10\}$

So, x = 10

21. (0)

$2x^2\left(2^x\right)+4.2^{|x-3|}=16x^2.2^{|x-3|}+\frac{2^x}{2}$

$\Rightarrow 4x^2.\left(2^x\right)+8.2^{|x-3|}=32x^2.2^{|x-3|}+2^x$

$\Rightarrow 4x^2\left(2^x-8.2^{|x-3|}\right)=\left(2^x-8.2^{|x-3|}\right)$

So, $4x^2 = 1$ or $2^x = 2^{|x-3|+3}$

$x=\pm\frac{1}{2}$ or $x=|x-3|+3$

$|x-3|=(x-3)\ x\ge 3$

So negative integral $x=\phi$

22. (0)

Domain: $x \ge -1$

$(x+1)+(x+4)+2\sqrt{(x+1)(x+4)}=(x+2)+(x+3)+2\sqrt{(x+2)(x+3)}$

$\Rightarrow$ (x + 1) (x + 4) = (x + 2) (x + 3)

$\Rightarrow$ y = 6 $\Rightarrow x \in \phi$

23. (2)

$x^{x-y} = y^{x+y}, \left(\sqrt{x}\right) y = 1$

$x^{x-y} = \left[\left(\frac{1}{x}\right)^{1/2}\right]^{x+y}$

$x - y = -\frac{(x+y)}{2}$

$\Rightarrow$ 3x = y

$\Rightarrow (3x).\sqrt{x} = 1$

$\Rightarrow x = \left(\frac{1}{3}\right)^{2/3}$

Also x = y = 1 is a possible solution

So two pairs possible

24. (1)

$2^{x+1} = y^2 + 4$ & $2^{x-1} \leq y$

$\Rightarrow 2^x = \frac{y^2+4}{2}$ & $2^x \leq 2y$

$\Rightarrow \frac{y^2+4}{2} \leq 2y$

$\Rightarrow (y-2)^2 \leq 0$

$\Rightarrow$ y = 2

So no. of values is 1

25. (6)

$|x| + |y| = 1$, k > 0

xy (x + y) = 0

$\Rightarrow$ x = 0 or y = 0 or x + y = 0

(i) If x = 0 $\Rightarrow |y| = k$

$\Rightarrow y = \pm k$

(ii) If y = 0 $\Rightarrow |x| = k$

$\Rightarrow x = \pm k$

(iii) If x + y = 0 $\Rightarrow x = -y$

$\Rightarrow |x| = |y|$

$\Rightarrow |x| + |x| = k$

$\Rightarrow x = \pm\frac{k}{2} = y$

Hence total 6 solutions

26. (2)

$|x^2 + 3x| + x^2 - 2 = 0$

Case – I

$x^2 + 3x \geq 0$

$x(x+3) \geq 0$

$x \in (-\infty, -3) \cup (0, \infty)$

Now, $x^2 + 3x + x^2 - 2 = 0$

$2x^2 + 3x - 2 = 0$

$x = 1/2$ & $x = -2$

$\therefore x = 1/2$ is the solution

Case – II

$x^2 + 3x < 0$

$x(x + 3) < 0$

$x \in (-3, 0)$

Now, $-(x^2 + 3x) + x^2 - 2 = 0$

$-3x - 2 = 0$

$\Rightarrow x = -2/3$

$\therefore x = -2/3$ is the solution

$\Rightarrow$ Total 2 solutions

27. (1)

$4^{\frac{x}{y}+\frac{y}{x}} = 32$

$2^{2\left(x/y + y/x\right)} = 2^5$

$\Rightarrow 2\left(\frac{x}{y} + \frac{y}{x}\right) = 5$ ...(1)

$\log_3(x - y) = 1 - \log_3(x + y)$

$\Rightarrow \log_3\left(x^2 - y^2\right) = 1$

$\Rightarrow x^2 - y^2 = 3$

$\Rightarrow x = \pm\sqrt{3 + y^2}$ ... (2)

Solving (1) & (2) we get

$x = \pm 2$ & $y = \pm 1$

Also $x - y > 0$ & $x + y > 0$

Only solution which satisfy is

$x = 2$ & $y = 1$

So one solution

28. (8)

$\log_b\left(a^{\log_2 b}\right) = \log_a\left(b^{\log_2 b}\right)$

$\Rightarrow \log_b a = \log_a b \quad \Rightarrow \quad a = b$ or $a = \frac{1}{b}$ (not possible)

$\log_a\left(c - (b - a)^2\right) = 3 \Rightarrow c = a^3$

$\Rightarrow$ Minimum value of $c = 8$ at $a = 2$

29. (4)

$\log_b 729 = 6\log_b 3$

If this is an integer, then $b = 3, 3^2, 3^3, 3^6$

30. (0)

Case-1 : If $x + \frac{5}{2} > 1 \Rightarrow x > -\frac{3}{2}$

then $\left(x-5\right)^2<\left(2x-3\right)^2 \Rightarrow 3x^2-2x-16>0 \Rightarrow x\in\left(\frac{8}{3},\infty\right)$

Case-2 : If $0<x+\frac{5}{2}<1 \Rightarrow -\frac{5}{2}<x<-\frac{3}{2}$

then $\left(x-5\right)^2>\left(2x-3\right)^2 \Rightarrow x\in\left(-2,-\frac{3}{2}\right)$

there is not negative integral value of *x*.

31. (4)

$\frac{6}{5}a^{(\log_a x)(\log_{10} a)(\log_a 5)}-3^{(\log_{10} x-1)}=9^{\left(\log_{100} x+\frac{1}{2}\right)}$

$6\cdot 5^{(\log_{10} x-1)}-3^{(\log_{10} x-1)}=3^{(\log_{10} x+1)}$

$6\cdot 5^{(\log_{10} x-1)}=\frac{3^{\log_{10} x}}{3}+3\cdot 3^{\log_{10} x}$

$6\cdot 5^{(\log_{10} x-1)}=\frac{10}{3}\cdot 3^{\log_{10} x}$

$\left(\frac{5}{3}\right)^{\log_{10} x-2}=1$

$\Rightarrow \log_{10} x-2=0$

$\Rightarrow x=100$

Ineger part of $\log_3 100$ is 4.

32. (7)

$x^2=1+6\log_4 y$

$y^2-2^x y-2^{2x+1}=0$

$\Rightarrow y=2^{x+1}$ and $y=-2^x$

If $y=-2^x$ (not possible, because $y>0$)

If $y=2^{x+1}$

$\Rightarrow \log_2 y=x+1$

$x^2=1+3\log_2 y$

$\Rightarrow x^2=1+3\left(x+1\right)$

$x^2-3x-4=0$

$\Rightarrow \left(x-4\right)\left(x+1\right)=0$

$x_1=4$

$\Rightarrow y_1=2^5=32$

$x_2=-1$

$\Rightarrow y_2=2^\circ=1$

$\log_2\left|x_1x_2y_1y_2\right|=\log_2 128=7$

33. (7)

$$\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = \log_7 \log_7 \left(7^{7/8}\right) = \log_7 \left(\frac{7}{8}\right) = 1 - 3\log_7 2$$

$\Rightarrow \quad a = 3$

$$\log_{15} \log_{15} \sqrt{15\sqrt{15\sqrt{15\sqrt{15}}}} = \log_{15} \log_{15} \left(15^{15/16}\right)$$

$$= \log_{15} \left(\frac{15}{16}\right) = 1 - 4\log_{15} 2$$

$\Rightarrow \quad b = 4$

Then $\quad a + b = 7$

34. (20)

$y^x = x^y$

If $\quad x = 2y \quad$ then $\quad y^{2y} = (2y)^y$

$\Rightarrow \quad 2y \log y = y \log(2y)$

If $\quad y \neq 0 \quad$ then $\quad \log y^2 = \log(2y)$

$\Rightarrow \quad y^2 = 2y \Rightarrow y = 2$

$x^2 + y^2 = 5y^2 = 20$

35. (3)

$$\left(e^x - 2\right)\sin\left(x + \frac{\pi}{4}\right)\left(x - \log_e 2\right)$$

$$\left(\sin x - \cos x\right) < 0$$

Note that $\left(e^x - 2\right)\left(x - \log_e 2\right)$ is always positive.

Hence, $\sin\left(x + \frac{\pi}{4}\right)\left(\sin x - \cos x\right) < 0$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\sin x + \cos x\right)\left(\sin x - \cos x\right) < 0$$

$$\Rightarrow \sin^2 x - \cos^2 x < 0$$

$$\Rightarrow \sin^2 x - \left(1 - \sin^2 x\right) < 0$$

$$\Rightarrow 2\sin^2 x < 1$$

$$\Rightarrow \sin^2 x < \frac{1}{2}$$

Clearly $x = 3$ is the least postivie integer value of x satisfying the equation.

1. (a, b, c)

Given equation: $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}}=\sqrt{2}$

For $x>0$, taking log on both sides to the base x, we get

$\frac{3}{4}(\log_2 x)^2+(\log_2 x)-\frac{5}{4}=\log_x\sqrt{2}=\frac{1}{2}\log_x 2$

Let $\log_2 x=y$, then we get , $\frac{3}{4}y^2+y-\frac{5}{4}=\frac{1}{2y}$

$\Rightarrow 3y^3+4y^2-5y-2=0$

$\Rightarrow (y-1)(y+2)(3y+1)=0 \Rightarrow y=1,-2,-1/3$

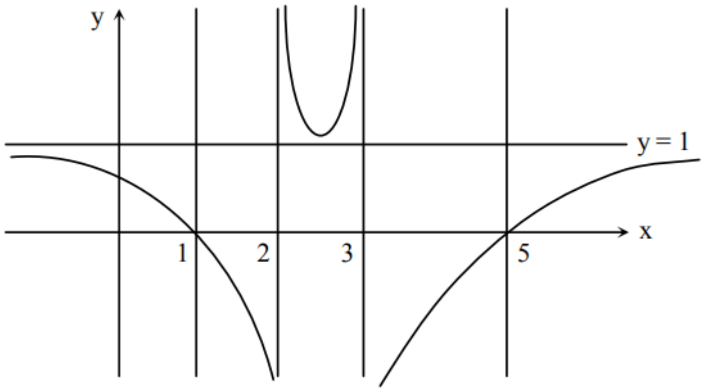
$\Rightarrow \log_2 x=1,-2,-1/3 \Rightarrow x=2.2^{-2},2^{-1/3}$

$\Rightarrow x=2,\frac{1}{4},\frac{1}{2^{1/3}}$, all are possible because they are positive.

2. (A) - P, R, S; (B) - Q, S; (C) - Q, S; (D) - P, R, S

$f(x)=\frac{(x-1)(x-5)}{(x-2)(x-3)}$

The graph of $f(x)$ is shown



(A) If $-1<x<1$

$\Rightarrow 0<f(x)<1$

(B) If $1<x<2 \Rightarrow f(x)<0$

(C) If $3<x<5 \Rightarrow f(x)<0$

(D) If $x>5 \Rightarrow 0<f(x)<1$

3. (c)

$(2x)^{\ln 2}=(3y)^{\ln 3}$

$\Rightarrow \ln 2(\ln 2+\ln x)=\ln 3(\ln 3+\ln y)$ ...(i)

$3^{\ln x}=2^{\ln y}$

$\Rightarrow (\ln x)(\ln 3)=(\ln y)(\ln 2)$ ...(ii)

Using (ii) in (i)

$\Rightarrow \ln 2(\ln 2+\ln x)=\ln 3\left(\ln 3+\frac{(\ln x)(\ln 3)}{\ln 2}\right)$

$\Rightarrow \ln^2 2-\ln^2 3=\ln x\left\{\frac{\ln^2 3}{\ln 2}-\ln 2\right\}$

$\Rightarrow \ln x=-\ln 2$

$\Rightarrow x=\frac{1}{2}$

4. (4)

Let $y=\sqrt{4-\frac{1}{3\sqrt{2}}\cdot\sqrt{4-\frac{1}{3\sqrt{2}}\cdot\sqrt{4-\frac{1}{3\sqrt{2}}.....}}}$

$$\Rightarrow \quad y^2 = 4 - \frac{1}{3\sqrt{2}}y \Rightarrow 3\sqrt{2}y^2 = 12\sqrt{2} - y$$

$$\Rightarrow \quad 3\sqrt{2}y^2 + y - 12\sqrt{2} = 0$$

$$\Rightarrow \quad \left(3y - 4\sqrt{2}\right)\left(\sqrt{2}y + 3\right) = 0$$

$$\Rightarrow \quad y = \frac{4\sqrt{2}}{3}; \; y = -\frac{3}{\sqrt{2}} \text{ (reject)}$$

$$\therefore \quad V = 6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}y\right)$$

$$= 6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \cdot \frac{4\sqrt{2}}{3}\right) = 6 + \log_{3/2}\left(\frac{2}{3}\right)^2$$

$$= 6 - 2 = 4$$

5. (a, b, c)

$$3^x = 4^{x-1} \Rightarrow x\log 3 = 2(x-1)\log 2$$

$$\Rightarrow x = \frac{2\log 2}{2\log 2 - \log 3}$$

$$\Rightarrow x = \frac{2\log_3 2}{2\log_3 2 - 1} = \frac{2}{2 - \log_2 3}$$

$$\text{Also } x = \frac{1}{1 - \frac{1}{2}\log_2 3} = \frac{1}{1 - \log_4 3}$$

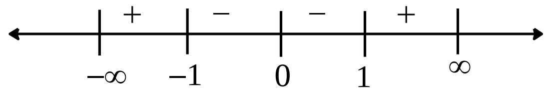
6. (8)

$$\log_2 9^{\frac{2}{\log_2(\log_2 9)}} \times 7^{\frac{1/2}{\log_4 7}}$$

$$= (\log_2 9)^{2\log^2_{\log_2 9}} \times 7^{\frac{1}{2}\log_7 4}$$

$$= 4 \times 2 = 8$$

7. (4)



$$3x^2 + x - 1 = 4\left|x^2 - 1\right|$$

Case I: If $x \in [-1, 1]$

$$3x^2 + x - 1 = -4x^2 + 4$$

$$\Rightarrow 7x^2 + x - 5 = 0$$

$\because D = 141 > 0$ $\therefore$ Equation has two roots

Case 2: If $x \in (-\infty, -1] \cup [1, \infty)$

$$3x^2 + x - 1 = 4x^2 - 4$$

$$\Rightarrow x^2 - x - 3 = 0 \because D = 13 > 0$$

$\therefore$ Equation has two roots

So, total 4 roots.