JEE Main Exercise

1. (A)

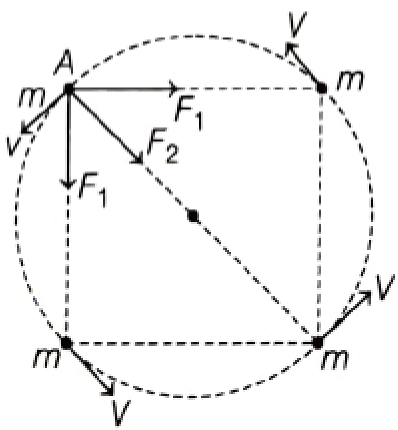
$$F_1 = \frac{GM^2}{a^2} \text{ and } F_2 = \frac{GM^2}{(\sqrt{2}a)^2}$$

Net force on $A = F_2 + 2F_1 \cos 45°$

$$\Rightarrow \left(\frac{1}{2}+\sqrt{2}\right)\frac{GM^2}{a^2}$$

$$\Rightarrow F_{net} = \frac{Mv^2}{r} = \left(\frac{2\sqrt{2}+1}{2}\right)\frac{GM^2}{a^2}, \text{ where } r = \frac{a}{\sqrt{2}}$$

$$\Rightarrow v = \sqrt{\frac{GM(2\sqrt{2}+1)}{2\sqrt{2}a}}$$



2. (D)

$$F = mE = m\left(\frac{\sqrt{2}G\lambda}{R}\right), \text{ where } \lambda = \frac{m}{(\pi R/2)} \Rightarrow F = \frac{2\sqrt{2}Gm^2}{\pi R^2}$$

3. (D)

$$F_1 = \frac{GMm}{(2R)^2}, F_2 = \frac{GMm}{(2R)^2} - \frac{G(M/8)m}{(3R/2^2)} = \frac{7GMm}{36R^2}$$

So, $\frac{F_2}{F_1} = \frac{7}{9}$

4. (C)

Let distance of neutral point from smaller mass be x.

$$E = \frac{GM}{x^2} - \frac{G(4M)}{(6R-x)^2} = 0 \Rightarrow x = 2R$$

5. (B)

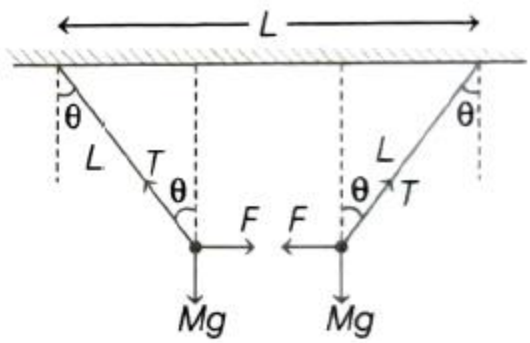
$$F=\frac{GM^2}{(L-2L\sin\theta)^2}=\frac{GM^2}{L^2}\qquad[\because\ (\theta\to 0)]$$

$$T\sin\theta=F \qquad ...(i)$$

$$T\cos\theta=Mg \qquad ...(ii)$$

Dividing Eqs. (i) by (ii)

$$\theta=\tan^{-1}\left(\frac{GM}{gL^2}\right)$$



6. (A)

When body is taken above the earth's surface,

$$\frac{\Delta g}{g}\times 100=-\frac{2h}{R}\times 100$$

$$\Rightarrow\quad -1\%=-\frac{2h}{R}\times 100$$

$$\Rightarrow\quad \frac{h}{R}\times 100=0.5\%$$

When body is taken below the earth's surface,

$$\frac{\Delta g}{g}\times 100=-\frac{h}{g}\times 100=-0.5\%$$

So, weight decreases by 0.5%.

7. (B)

On earth, $T=2\pi\sqrt{\frac{l}{g}}=2$

On planer, $g'=\frac{G(2M)}{(2R)^2}=\frac{g}{2}$

$$T'=2\pi\sqrt{\frac{l}{(g/2)}}=\sqrt{2}T=2\sqrt{2}\text{ s}$$

8. (C)

$$g_{\text{app}}=g-\omega^2R\cos^2\lambda=0$$

$$\Rightarrow \quad \omega = 2\sqrt{\frac{g}{R}}, T = \frac{2\pi}{\omega} = \pi\sqrt{\frac{R}{g}}$$

9. (D)

$$V = V_1 + V_2 + V_3 + ...$$
$$= -\frac{Gm}{1} - \frac{Gm}{2} - \frac{Gm}{4} - \frac{Gm}{8} ....$$
$$= -Gm\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right)$$
$$= -\frac{Gm(1)}{1 - \frac{1}{2}} = -2Gm$$

10. (B)

$$dV = -E\,dr$$
$$\Rightarrow \int_{v_i}^{v} dV = +\int_{d_i}^{r} \frac{k}{r} dr$$
$$\Rightarrow V - V_i = +k\ln\left(\frac{r}{d_i}\right)$$
$$\Rightarrow V = V_i + k\ln\left(\frac{r}{d_i}\right)$$

11. (C)

$$V = -\frac{Gm}{R} = -\frac{G\sigma\left(4\pi R^2\right)}{R} = -G\sigma 4\pi R$$
$$\frac{V_A}{V_B} = \frac{R_A}{R_B} = \frac{3}{4}$$

When shell A and B coalesce into single shell,

$$\sigma 4\pi R_A^2 + \sigma 4\pi R_B^2 = \sigma 4\pi R_C^2$$
$$\Rightarrow R_C = \sqrt{R_A^2 + R_B^2}$$
$$V_C = -G\sigma 4\pi R_C = -G\sigma 4\pi\sqrt{R_A^2 + R_B^2}$$
$$\frac{V_C}{V_A} = \frac{\sqrt{R_A^2 + R_B^2}}{R_A} = \sqrt{1 + \left(\frac{R_B}{R_A}\right)^2} = \frac{5}{3}$$

12. (B)

$$W = \Delta U = -\frac{Gm^2 \times 3}{2a} - \left(-\frac{Gm^2}{a} \times 3\right) = \frac{3Gm^2}{2a}$$

13. (C)

$K_1 + U_1 = K_2 + U_2$

$$\Rightarrow\ 0 - \frac{GMm}{R_0} = \frac{1}{2}mv^2 - \frac{GMm}{R}\ \Rightarrow\ v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)}$$

14. (A)

$$V_A = \frac{-GM}{3R} \qquad V_B = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= -\frac{11GM}{8R}$$

$$0 + M\left(\frac{-GM}{3R}\right) = \frac{1}{2}MV_B^2 - M\left(\frac{11GM}{8R}\right)$$

15. (C)

$K_1 + U_1 = K_2 + U_2$

$$\Rightarrow\ \frac{1}{2}m(kv_e)^2 - \frac{GMm}{R} = 0 - \frac{GMm}{r}$$

$$\Rightarrow\ \frac{1}{2}M\left(K\sqrt{\frac{2GM}{R}}\right)^2 - \frac{GMm}{R} = -\frac{GMm}{r}$$

$$\Rightarrow\ r = \frac{R}{1-k^2}$$

16. (C)

$K_1 + U_1 = K_2 + U_2$

$$\Rightarrow\ \frac{1}{2}m(2v_e)^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow\ v = \sqrt{4v_e^2 - v_e^2} = \sqrt{3}v_e = 11.2\sqrt{3}\ \text{km/s}$$

17. (A)

$v = \omega R$

$$g_{\text{app}} = g - \omega^2 R = \frac{g}{2}\ \Rightarrow\ g = 2\omega^2 R$$

$$v_e = \sqrt{2gR} = \sqrt{2\left(2\omega^2 R\right)R}$$

$$= 2\omega R = 2v$$

18. (D)

$$F = \frac{GMm}{r^m} = m\omega^2 r\ \Rightarrow\ \omega = \frac{1}{r^{(m+1)/2}}$$

$$T = \frac{2\pi}{\omega}\ \Rightarrow\ T \propto r^{(m+1)/2}$$

19. (C)

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{G\rho\left(\frac{4}{3}\pi R^3\right)}} = 2\pi\sqrt{\frac{3}{4G\rho\pi}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{\rho_2}{\rho_1}} \Rightarrow 1 = \frac{\rho_2}{\rho_1} \Rightarrow \frac{\rho_1}{\rho_2} = 1$$

20. (C)

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = \frac{2\pi}{\omega_{\text{earth}}}$$

$$2\pi\sqrt{\frac{r_1^3}{GM}} = \frac{2\pi}{(2\omega_{\text{earth}})}$$

On solving, we get

$$r_1 = \frac{r}{(4)^{1/3}}$$

21. (C)

$$\frac{dA}{dt} = \frac{A}{t_1} = \frac{A}{t_2} \Rightarrow t_1 = t_2$$

22. (D)

Angular momentum is conserved.

So, $L_1 = L_2$

$mv_1r_1 = mv_2r_2 \Rightarrow v_2 > v_1$ $\quad (\because r_2 < r_1)$

$\Rightarrow K_2 > K_1$

23. (A)

$$\frac{dA}{dt} = \frac{A}{T}$$

$$\Rightarrow \frac{L}{2m} = \frac{A}{T} \Rightarrow L = \frac{2mA}{T}$$

24. (A)

$r_p = a(1-e);\ r_a = a(1+e)$

$$mv_pr_p = mv_ar_a \Rightarrow \frac{v_p}{v_a} = \frac{1+e}{1-e}$$

25. (C)

$mv_1r_1 = mv_2r_2$ ….. (i)

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2} \quad \text{..... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1 = \sqrt{\frac{2GMr_2}{r_1(r_1 + r_2)}}$$

So, $E = K_1 + U_1 = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1}$

$$= -\frac{GMm}{2a} = \text{constant}$$

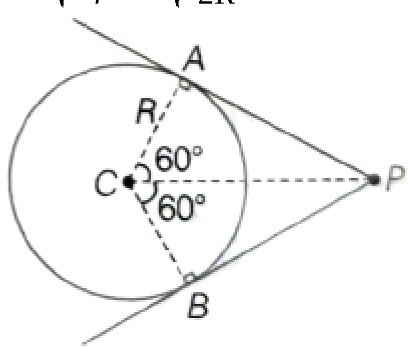
Now, $\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$

$$\Rightarrow \quad v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

26. (2)

$$CP = \frac{R}{\cos 60^\circ} = 2R,$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}}$$

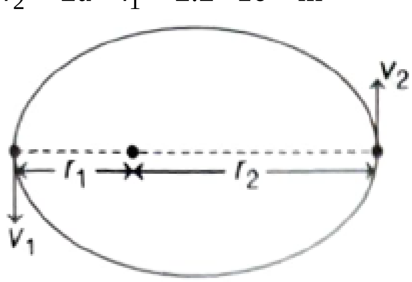


27. (40)

$r_1 = 1.8 \times 10^{12}$ m

$a = 2 \times 10^{12}$ m

$r_2 = 2a - r_1 = 2.2 \times 10^{12}$ m



$$\frac{dA}{dt} = \frac{L}{2m} = \frac{v_2 r_2}{2}$$

$$\Rightarrow \quad 4.4\times10^{16} = \frac{v_2\left(2.2\times10^{12}\right)}{2} \Rightarrow v_2 = 40\,\text{km/s}$$

28. (0.01)

$$g' = g\left(1-\frac{h}{R}\right) = g\left(1-\frac{64}{6400}\right) = 0.99\,g$$

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta g}{g}$$

$$\Rightarrow \quad \frac{\Delta T}{2} = \frac{1}{2}\left(\frac{g-0.99\,g}{g}\right)$$

$$\Rightarrow \quad \Delta T = 0.01\,\text{s}$$

29. (4)

Areal velocity, $= \dfrac{L}{2m} = \dfrac{vr}{2} = \left(\sqrt{\dfrac{Gm}{r}}\right)\dfrac{r}{2} \propto \sqrt{r}$

$$\frac{(\text{Areal velocity})_1}{(\text{Areal velocity})_2} = \frac{\sqrt{r_1}}{\sqrt{r_2}} = 2$$

$$\Rightarrow \quad \frac{r_1}{r_2} = 4$$

30. (5)

$$K_1 + U_1 = K_2 + U_2$$

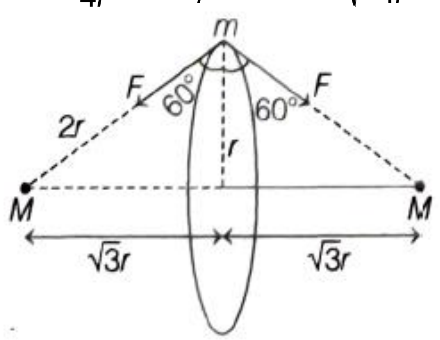
$$\Rightarrow \quad \frac{1}{2}m_0\left(\sqrt{\frac{5GM_e}{4R}}\right)^2 - \frac{GM_e m_0}{R} = 0 - \frac{GM_e m_0}{r}$$

$$\Rightarrow \quad r = \frac{8R}{3}$$

$$h = r - R = \frac{8R}{3} - R = \frac{5R}{3}$$

31. (4)

$$F = \frac{GMm}{4r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{4r}}$$



$$\frac{GMm}{4r^2}=\frac{mv^2}{r} \Rightarrow v=\sqrt{\frac{GM}{4r}}$$

32. (8)

Let C be the midpoint of the line joining C_1 and C_2.

Gravitational field Intensity at C is zero. So, we just need to make the point mass reach C with negligible velocity. Once it crosses C, it will be pulled by the second sphere.

Applying energy conservation between A and C

$$K_1+U_1=K_2+U_2$$

$$\Rightarrow \frac{1}{2}mv_0^2+m\left(\frac{-GM}{R}-\frac{GM}{9R}\right)=0+m\left(\frac{-GM}{5R}-\frac{GM}{5R}\right)$$

$$\Rightarrow v_0=\frac{8}{3}\sqrt{\frac{GM}{5R}}$$

33. (7)

Let speed of the ball just before first collision be v_1.

$$K_1+U_1=K_2+U_2$$

$$\Rightarrow 0-\frac{GMm}{R}=\frac{1}{2}mv_1^2-\frac{3GM}{2R}$$

$$\Rightarrow v_1=\sqrt{\frac{GM}{R}}$$

Speed of ball just after Ist collision,

$$v_2=ev_1=\frac{1}{5}\sqrt{\frac{GM}{R}}$$

Lets take maximum height reached to be h after first collision,

$$K_1+U_1=K_2+U_2$$

$$\frac{1}{2}m\left(\frac{1}{5}\sqrt{\frac{GM}{R}}\right)^2-\frac{3GMm}{2R}=0-\frac{GMm}{2R^3}\left(3R^2-h^2\right)$$

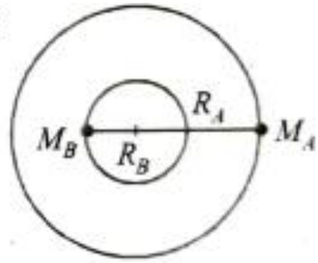
$$\Rightarrow h=\frac{R}{5}$$

Total distance travelled before second collision

$$=R+2h=\frac{7R}{5}$$

1. (D)

(d) Incase of binary star system, the gravitational force of attraction between the stars will provide the necessary centripetal forces.



So angular velocity ω of both stars is the same. Therefore time period $T = \frac{2\pi}{\omega}$ remains the same.

2. (A)

(a) As we know,

Gravitational potential energy = $\frac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2}-1\right) = \frac{-GMm}{6R} \Rightarrow E_i = \frac{-GMm}{R} + K$$

$E_i = E_f$

Therefore minimum required energy, $K = \frac{5GMm}{6R}$

3. (C)

(c) Let mass of smaller sphere (which has to be removed) is m

Radius = $\frac{R}{2}$ (from figure)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3} \Rightarrow m = \frac{M}{8}$$

Mass of the left over part of the sphere

$$M' = M - \frac{M}{8} = \frac{7}{8}M$$

Therefore gravitational field due to the left over part of the sphere

$$E=\left(-\frac{GM}{x^2}+\frac{GM}{8\left(x+\frac{R}{2}\right)^2}\right)$$

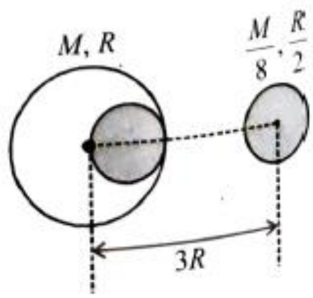
$$=+GM\left[\frac{1}{8\left(x+\frac{R}{2}\right)^2}-\frac{1}{x^2}\right]=GM\left[\frac{x^2-8\left(x+\frac{R}{2}\right)^2}{8\left(x+\frac{R}{2}\right)^2 x^2}\right]$$

$$=GM\left[\frac{x^2-8x^2}{8x^4}\right]=\frac{GM}{8x^2}[-7]=\frac{-7GM}{8x^2}\Rightarrow|\vec{E}|=\frac{7GM}{8x^2}$$

4. (A)

(a) Let M' be the mass of removed section

Then, $\dfrac{M}{\frac{4}{3}\pi R^3}=\dfrac{M'}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}\Rightarrow M'=\dfrac{M}{8}$



$$F=\frac{GM.\frac{M}{8}}{(3R)^2}-\frac{G\frac{M}{8}\frac{M}{8}}{\left(\frac{5R}{2}\right)^2}$$

$$=\frac{41}{3600}\frac{GM^2}{R^2}$$

5. (D)

(d) Gravitational field, $I=\left(5\hat{i}+12\hat{j}\right)$ N/kg

$$I=-\frac{dv}{dr}$$

$$\Delta v=-\left[\int_0^x I_x dx+\int_0^y I_y dy\right]$$

$$=-\left[I_x.x+I_y.y\right]=-\left[5(7-0)+12(-3-0)\right]$$

$$=-\left[35+(-36)\right]=1\,J/kg$$

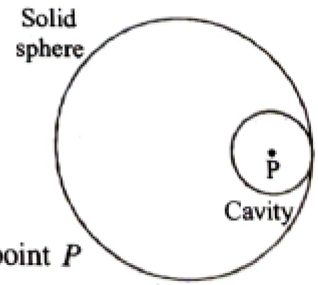
i.e., change in gravitational potential 1 J/kg.

$\Delta U=m\Delta v=1\times1=1$ J

Hence change in gravitational potential energy 1 J

6. (D)

(d) Due to complete solid sphere, potential at point P



$$V_{sphere} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$

Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}} = -\frac{3GM}{8R}$$

So potential at the centre of cavity

$$= V_{sphere} - V_{cavity} = -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

7. (C)

(c) As, $V = -\frac{GM}{2R^3}(3R^2 - r^2)$

Graph (c) most closely depicts the correct variation of $v(r)$.

8. (A)

(a) Due to infinite wire of mass 'm' at 'r' distance

$$E_g = \frac{G\lambda}{r} \quad \text{so } F_g = mE_g$$

So force on star $= \frac{Gm\lambda}{r} = \frac{mv^2}{r} \Rightarrow v = \sqrt{G\lambda}$

as $T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{G\lambda}} \Rightarrow T \propto r$

9. (A)

(a) As we know, Gravitational force of attraction,

$$F = \frac{GMm}{R^2}$$

$$F_1 = \frac{GM_e m}{r_1^2} \text{ and } F_2 = \frac{GM_e M_s}{r_2^2}$$

$$\Delta F_1 = -2\frac{GM_e m}{r_1^3}\Delta r_1 \text{ and } \Delta F_2 = -2\frac{GM_e M_s}{r_2^3}\Delta r_2$$

$$\frac{\Delta F_1}{\Delta F_2} = \frac{m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s \Delta r_2} = \left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$$

Using $\Delta r_1 = \Delta r_2 = 2\,R_{earth}$; $m = 8 \times 10^{22}$ kg;

$M_s = 2 \times 10^{30}$ kg

$r_1 = 0.4 \times 10^6$ km and $r_2 = 150 \times 10^6$ km

$$\frac{\Delta F_1}{\Delta F_2} = \left(\frac{8\times10^{22}}{2\times10^{30}}\right)\left(\frac{150\times10^6}{0.4\times10^6}\right)^3 \times 1 \cong 2$$

10. (D)

(d) With rotation of earth or latitude, acceleration due to gravity vary as $g' = g - \omega^2 R \cos^2 \phi$

Where ϕ is latitude, there will be no change in gravity at poles as $\phi = 90°$

At all other points as ω increases g' will decreases hence, weight, W = mg decreases.

11. (C)

(c) Initial gravitational potential energy, $E_i = -\frac{GMm}{2R}$

Final gravitational potential energy,

$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R}$$

$$= -\frac{4GMm}{6R} = -\frac{2GMm}{3R}$$

$\therefore$ Difference between initial and final energy,

$$E_f - E_i = \frac{GMm}{R}\left(-\frac{2}{3}+\frac{1}{2}\right) = -\frac{GMm}{6R}$$

12. (C)

(c) Areal velocity; $\frac{dA}{dt}$

$$dA = \frac{1}{2}r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

Also, $L = mvr = mr^2\omega$ $\quad \therefore \frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$

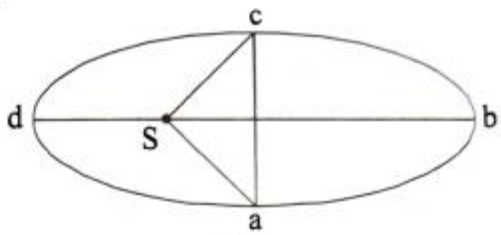
13. (C)

(c) Let area of ellipse abcd = x

Area of SabcS = $\frac{x}{2}+\frac{x}{4}$ (i.e., ar of abca + SacS)

(Area of half ellipse + Area of triangle)

$= \frac{3x}{4}$



Area of SadcS = $x - \frac{3x}{4} = \frac{x}{4}$

$\frac{\text{Area of SabcS}}{\text{Area of SadcS}} = \frac{3x/4}{x/4} = \frac{t_1}{t_2}$

$\frac{t_1}{t_2} = 3$ or, $t_1 = 3t_2$

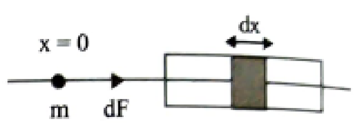
14. (D)

(d) Given $\lambda = (A + Bx^2)$,

Taking small element dm of length dx at a distance x from $x = 0$

so, $dm = \lambda\, dx$

$dm = (A + Bx^2)dx$



$dF = \frac{Gmdm}{x^2}$

$\Rightarrow \quad F = \int_a^{a+L} \frac{Gm}{x^2}(A + Bx^2)dx$

$= Gm\left[-\frac{A}{x} + Bx\right]_a^{a+L}$

$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

15. (D)

(d) $\frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{9}{4}$ or $\frac{g_e}{g_p} = \frac{9}{4}$

or $\frac{GM/R^2}{G(M/9)/R_p^2} = \frac{9}{4}$ $\therefore \quad R_p = \frac{R}{2}$

16. (A)

(a) $g_{\text{eff}} = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$\Rightarrow \dfrac{g}{2} = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} \Rightarrow \sqrt{2} = 1+\dfrac{h}{R} \Rightarrow \dfrac{h}{R} = \sqrt{2}-1$

$\Rightarrow h = (\sqrt{2}-1)\times 6400\times 10^3 \text{ m} = 2.6\times 10^6 \text{ m}$

17. (C)

$E_g = \dfrac{GM}{(3a)^2} + \dfrac{G(2M)}{(3a)^2} = \dfrac{GM}{3a^2}$

18. (B)

(b) $AC = a\sqrt{2} \quad \because r = \dfrac{AC}{2} = \dfrac{a\sqrt{2}}{2} = \dfrac{a}{\sqrt{2}}$

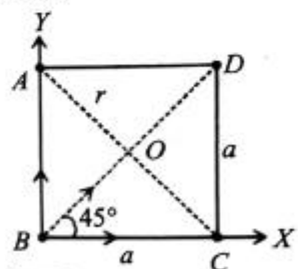
Resultant force on the body

$B = \dfrac{GM^2}{a^2}\hat{i} + \dfrac{GM^2}{a^2}\hat{j} + \dfrac{GM^2}{(a\sqrt{2})^2}(\cos 45°\hat{i} + \sin 45°\hat{j})$

$\Rightarrow |F| = \dfrac{GM^2}{a^2}(\sqrt{2}) + \dfrac{GM^2}{2a^2}$

$\dfrac{Mv^2}{r}$ = Resultant force towards centre

$\therefore \dfrac{Mv^2}{\left(\dfrac{a}{\sqrt{2}}\right)} = \dfrac{GM^2}{a^2}\left(\sqrt{2}+\dfrac{1}{2}\right)$



$\Rightarrow v^2 = \dfrac{GM}{a}\left(1+\dfrac{1}{2\sqrt{2}}\right)$

$\Rightarrow v = \sqrt{\dfrac{GM}{a}\left(1+\dfrac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\dfrac{GM}{a}}$

19. (A)

(a) $F = \dfrac{GMm}{r} = \int a\dfrac{\rho(dV)m}{r^2}$

$= mG\int_0^R \dfrac{k}{r^2}\dfrac{4\pi r^2 dr}{r^2} = -4\pi kGm\left(\dfrac{1}{r}\right)_0^R = -\dfrac{4\pi kGm}{R}$

Using Newton's second law, we have

$\dfrac{mv_0^2}{R} = \dfrac{4\pi kGm}{R}$

or $v_0 = C$ (const.)

Time period, $T = \dfrac{2\pi R}{v_0} = \dfrac{2\pi R}{C}$ or $\dfrac{T}{R}$ = constant.

20. (B)

(c) $U_{surface} + E_1 = U_h$

[$\because$ E_1 is minimum, at height 'h', K.E. = 0]

$$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re+h)}$$

$$\Rightarrow E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right) \Rightarrow E_1 = \frac{GM_e m}{(R_e + h)} \times \frac{h}{R_e}$$

Gravitational attraction

$$F_G = ma_c = \frac{mv^2}{(R_e + h)} = \frac{GM_e m}{(R_e + h)^2} \Rightarrow mv^2 = \frac{GM_e m}{(R_e + h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e + h)}$$

$E_1 = E_2$

Clearly, $\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200\text{km}$

21. (D)

14. (d) $K_E = \frac{1}{2} m \times \frac{2GM_e}{R_e} = \frac{GmM_e}{R_e}$

as, $64\, V_m = V_e \Rightarrow 64 \times \frac{4}{3}\pi R_m^3 = \frac{4}{3}\pi R_e^3 \Rightarrow 4R_m = R_e$

and, $M \propto V$

So, $M_e = 64 M_m$

$$K_{moon} = \frac{1}{2} m \times \frac{2GM_m}{R_m}$$

$$= \frac{GmM_m}{R_m} = \frac{GmM_e/64}{\frac{R_e}{4}} = \frac{GmM_e}{16R_m} = \frac{K_e}{16}$$

22. (B)

(b) Orbital, velocity, $v = \sqrt{\frac{GM}{r}}$

Kinetic energy of satellite A,

$T_A = \frac{1}{2} m_A V_A^2$

Kinetic energy of satellite B,

$$T_B = \frac{1}{2} m_B V_B^2 \Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{GM}{R}}{2m \times \frac{GM}{2R}} = 1$$

23. (D)

(d) For a satellite orbiting close to the earth, orbital velocity is given by

$$v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$$

Escape velocity (v_e) is

$$v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR} \quad [\because h << R]$$

$$\Delta v = v_e - v_0 = (\sqrt{2}-1)\sqrt{gR}$$

24. (B)

(b) At height r from center of earth, orbital velocity

$$v = \sqrt{\frac{GM}{r}}$$

By principle of energy conservation

$$\text{KE of 'm'} + \left(-\frac{GMm}{r}\right) = 0 + 0 \quad (\because \text{At infinity, PE} = \text{KE} = 0)$$

$$\text{or} \quad \text{KE of 'm'} = \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$$

25. (D)

(d) Let M is mass of star m is mass of meteroite

By energy convervation between 0 and ∞.

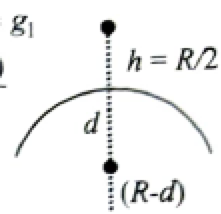
$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV_\infty^2 = 0 + 0$$

$$\therefore \quad v = \sqrt{\frac{4GM}{r}} = \sqrt{\frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}}$$

$$= 2.8 \times 10^5 \text{ m/s}$$

26. (B)

(b) According to question, $g_h = g_d = g_1$



$$g_h = \frac{GM}{\left(R + \frac{R}{2}\right)^2} \text{ and } g_d = \frac{GM(R-d)}{R^3}$$

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3} \Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$$

$$\Rightarrow 4R = 9R - 9d \Rightarrow 5R = 9d$$

$$\therefore \frac{d}{R} = \frac{5}{9}$$

27. (A)

(a) Value of g at equator, $g_A = g \cdot - R\omega^2$

Value of g at height h above the pole,

$$g_B = g \cdot \left(1 - \frac{2h}{R}\right)$$

As object is weighed equally at the equator and poles, it means g is same at these places.

$$g_A = g_B$$

$$\Rightarrow g - R\omega^2 = g\left(1 - \frac{2h}{R}\right)$$

$$\Rightarrow R\omega^2 = \frac{2gh}{R} \Rightarrow h = \frac{R^2\omega^2}{2g}$$

28. (A)

(a) Given : Gravitational field,

$$E_G = \frac{Ax}{(x^2 + a^2)^{3/2}}, V_\infty = 0$$

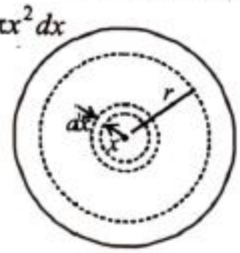
$$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G \cdot \vec{d}_x \Rightarrow V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx$$

$$\therefore V_x = \frac{A}{(x^2 + a^2)^{1/2}} - 0 = \frac{A}{(x^2 + a^2)^{1/2}}$$

29. (D)

(d) Mass of small element of planet of radius x and thickness dx.

$$dm = \rho \times 4\pi x^2 dx = \rho_0 \left(1 - \frac{x^2}{R^2}\right) \times 4\pi x^2 dx$$



Mass of the planet

$$M = 4\pi\rho_0 \int_0^r \left(x^2 - \frac{x^4}{R^2}\right) dx$$

$$\Rightarrow M = 4\pi\rho_0 \left|\frac{r^3}{3} - \frac{r^5}{5R^2}\right|$$

Gravitational field,

$$E = \frac{GM}{r^2} = \frac{G}{r^2} \times 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$\Rightarrow E = 4\pi G\rho_0 \left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$$

E is maximum when $\frac{dE}{dr} = 0$

$$\Rightarrow \frac{dE}{dr} = 4\pi G\rho_0 \left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0 \Rightarrow r = \frac{\sqrt{5}}{3} R$$

30. (B)

(b) Gravitation field at the surface

$$E = \frac{Gm}{r^2}$$

$$\therefore E_1 = \frac{Gm_1}{r_1^2} \text{ and } E_2 = \frac{Gm_2}{r_2^2}$$

From the diagram given in question,

$$\frac{E_1}{E_2} = \frac{2}{3} \; (r_1 = 1\text{m}, R_2 = 2m \text{ given})$$

$$\therefore \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2 \left(\frac{m_1}{m_2}\right) \Rightarrow \frac{2}{3} = \left(\frac{2}{1}\right)^2 \left(\frac{m_1}{m_2}\right)$$

$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \frac{1}{6}$$

31. (A)

(a) Orbital speed of the body when it revolves very close to the surface of planet

$$V_0 = \sqrt{\frac{GM}{R}} \quad \text{...(i)}$$

Here, G = gravitational constant

Escape speed from the surface of planet

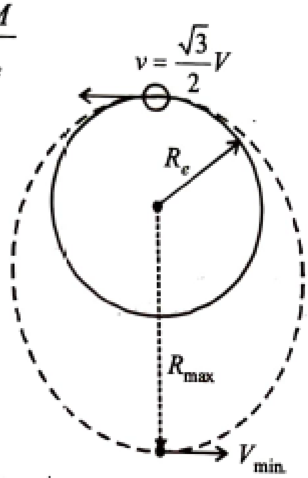
$$V_e = \sqrt{\frac{2GM}{R}} \quad \text{...(ii)}$$

Dividing (i) by (ii), we have

$$\frac{V_0}{V_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$$

32. (C)

(c) Orbital velocity, $V_0 = \sqrt{\frac{GM}{R_e}}$



From energy conversation,

$$-\frac{GMm}{R_e} + \frac{1}{2}m\left(\sqrt{\frac{3}{2}}V\right)^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mV_{min}^2 \quad \text{...(1)}$$

From angular momentum conversation

$$\sqrt{\frac{3}{2}}VR_e = V_{min}R_{max} \quad \text{...(2)}$$

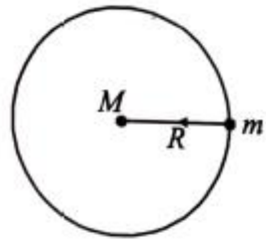
Solving equation (1) and (2) we get,

$$R_{max} = 3R_e$$

33. (A)

(a) According to question, mass density of a spherical galaxy varies as $\frac{k}{r}$.

Mass, $M = \int \rho dV$



$\Rightarrow M = \int_0^{r=R_0} \frac{k}{r} 4\pi r^2 \, dr$

$\Rightarrow M = 4\pi k \int_0^{R_0} r \, dr$

or, $M = \frac{4\pi k R_0^2}{2} = 2\pi k R^2 \Rightarrow F_G = \frac{GMm}{R_0^2} = m\omega_0^2 R \; (= F_C)$

$\Rightarrow \frac{G\frac{4\pi k R^2}{2}}{R^2} = \omega_0^2 R \Rightarrow \omega_0 = \sqrt{\frac{2\pi KG}{R}} \quad \left(\because \omega = \frac{2\pi}{T}\right)$

$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\frac{2\pi R}{KG}} \Rightarrow T^2 = \frac{2\pi R}{KG}$

$\because 2\pi$, K and G are constants $\therefore T^2 \propto R$.

34. (D)

(d) From law of conservation of momentum, $\vec{p}_i = \vec{p}_f$

$m_1 u_1 + m_2 u_2 = MV_f$

$\Rightarrow v_f = \frac{\left(mv + \frac{mv}{4}\right)}{\frac{3m}{2}} = \frac{5v}{6}$

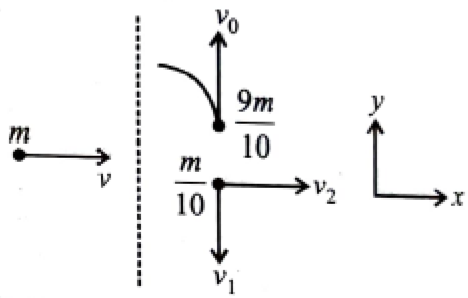
Clearly, $v_f < v_i$ $\therefore$ Path will be elliptical

35. (B)

(b) Let v be the speed of satellite just before ejection of rocket then,

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$\Rightarrow \quad v^2 = u^2 - \frac{GM}{R} \qquad \text{..... (i)}$$



Along x :

$$(P_i)_x = (P_f)_x \Rightarrow mv = \frac{m}{10}v_2$$

$$\Rightarrow \quad v_2 = 10v \Rightarrow v_2^2 = 100\left(u^2 - \frac{GM}{R}\right) \qquad \text{[From (i)]}$$

Along y :

$$(P_i)_y = (P_f)_y \Rightarrow 0 = \frac{9m}{10}v_0 - \frac{m}{10}v_1$$

$$\Rightarrow \quad v_1 = 9v_0 \Rightarrow v_1^2 = 81\frac{GM}{2R}$$

$$\therefore \quad (\text{K.E.})_R = \frac{1}{2}\frac{m}{10}\left(v_1^2 + v_2^2\right) = 5m\left[u^2 - \frac{119}{200}\frac{GM}{R}\right]$$

36. (B)

(b) According to Kepler's law, when a planet revolves around the sun, its areal velocity is constant.

$$\frac{dA}{dt} = \text{constant}$$

37. (C)

(c) We have given $F \propto \frac{1}{R^3} \Rightarrow F = \frac{K}{R^3}$

Here K is a constant. This force will provide the required centripetal force to the particle for revolution.

$$\frac{mv^2}{R} = \frac{K}{R^3} \Rightarrow v \propto \frac{1}{R}$$

Time period of revolution,

$$T = \frac{2\pi R}{v} \quad \therefore T \propto R^2$$

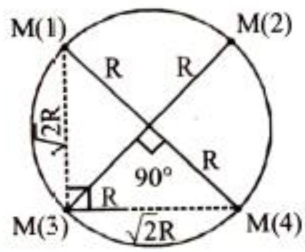
38. (B)

(b) Centripetal Force $F_{net} = \frac{Mv^2}{R}$

Gravitational force between two masses $= \frac{GM_1M_2}{d^2}$

So, $F_{12} = F_{13} = \frac{GM^2}{\left(\sqrt{2}R\right)^2}$

$(\because M_1 = M_2 = M)$



Resultant of these two forces

$= \sqrt{2}\frac{GM^2}{2R^2}$

Combining all forces and equating with centripetal force we get

$$\sqrt{2}\frac{GMM}{\left(\sqrt{2}R\right)^2} + \frac{GMM}{(2R)^2} = \frac{Mv^2}{R} \Rightarrow \frac{GM}{R}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = v^2$$

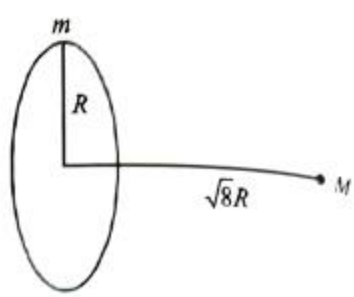
$$\Rightarrow \frac{GM}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right) = v^2 \Rightarrow v = \sqrt{\frac{GM\left(4+\sqrt{2}\right)}{R4\sqrt{2}}}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{GM\left(2\sqrt{2}+1\right)}{R}}$$

39. (A)

(c) Gravitational field of ring

$$E = \frac{-Gmx}{(R^2 + x^2)^{3/2}}$$



Force between sphere and ring

$$F = \frac{GMm\sqrt{8}R}{[R^2 + 8R^2]^{3/2}}$$

$$\Rightarrow F = \frac{\sqrt{8}GMm}{27R^2}$$

40. (C)

(c) As density is same, $2M_E = M_P$

$$\Rightarrow 2\rho \times \frac{4}{3}R_E^3 = \rho \times \frac{4}{3}\pi R_P^3 \Rightarrow R_P = 2^{1/3}R_E$$

Acceleration due to gravity on the surface of planet,

$$g_P = \frac{GM_P}{R_P^2} \Rightarrow g_p = \frac{G2M_E}{\left(2^{1/3}R_E\right)^2} = \frac{G2M_E}{2^{2/3}R_E^2}$$

$\Rightarrow g_P = 2^{1/3} g_e$
Weight on planet = $2^{1/3}$ Weight on earth
$\Rightarrow \quad W_P = 2^{1/3} W$

41. (B)

(b) Weight of body at pole = mg = 49 N
Weight of body at equator due to rotation,

$$g_e = g - R\omega^2$$

so $W_e = mg_e = m(g - R\omega^2)$

$\therefore \; W_P > W_e \qquad W_P = 49$ N

So, $W_e = 48.83$ N. $\qquad W_e < 49$ N.

42. (D)

(d) The gravitational potential at a point A due to mass of the centre is

$$V_1 = -\frac{GM}{r}$$

Gravitational potential at A due to shell is

$$V_2 = -\frac{GM}{R}$$

$$V_A = V_1 + V_2 \Rightarrow V_A = \left[-\frac{GM_1}{r} - \frac{GM_2}{R}\right]$$

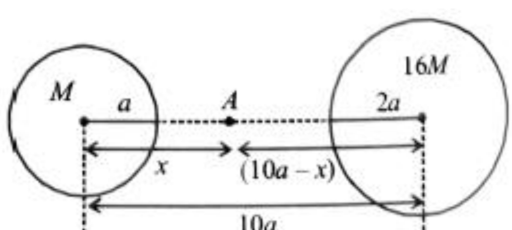
$$= \left[-\frac{50}{25}G - \frac{100}{50}G\right] = -4G$$

43. (A)

Inside uniform spherical shall, field is zero & hence potential is constant same as on surface.

44. (D)

(d)



Let A be the point where gravitation field of both planets cancel each other i.e. zero. After this field due to small mass will dominate and 'm' will easily reach small mass surface.

$$\frac{GM}{x^2} = \frac{G(16M)}{(10a - x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{4}{(10a - x)} \Rightarrow 4x = 10a - x \Rightarrow x = 2a \qquad \text{...(i)}$$

Using conservation of energy, we have

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$KE = GMm\left[\frac{1}{8a}+\frac{16}{2a}-\frac{1}{2a}-\frac{16}{8a}\right]$$

$$\Rightarrow KE = GMm\left[\frac{1+64-4-16}{8a}\right]$$

$$\Rightarrow \frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right] \Rightarrow v=\sqrt{\frac{90GM}{8a}} \quad \Rightarrow v=\frac{3}{2}\sqrt{\frac{5GM}{a}}$$

45. (B)

(b) Total energy at middle point

$= \text{K.E} + \text{P.E of } M_1 \text{ \& m} + \text{P.E of } M_2 \text{ \& m}$

To get escape velocity total energy should be zero.

$$\frac{1}{2}mV^2 - \frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = 0$$

$$\Rightarrow \frac{1}{2}mV^2 = \frac{2Gm}{r}(M_1+M_2) \;\therefore V=\sqrt{\frac{4G(M_1+M_2)}{r}}$$

46. (B)

(b) Speed of satellite, $v=\sqrt{\frac{GM}{r}}$

Time, $T=\frac{2\pi r}{v}=\frac{2\pi r}{\sqrt{\frac{GM}{r}}}=2\pi\sqrt{\frac{r^3}{GM}}$

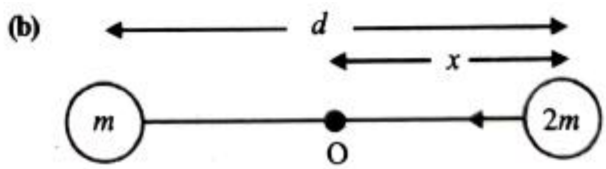
$$\therefore T_B - T_A = \frac{2\pi}{\sqrt{GM}}\left[r_B^{3/2}-r_A^{3/2}\right]$$

$$=\frac{2\pi}{\sqrt{GM}}\left[(8\times10^6)^{3/2}-(7\times10^6)^{3/2}\right]$$

$$=\frac{2\pi}{\sqrt{6.67\times10^{-11}\times6\times10^{24}}}\times10^9[8^{3/2}-7^{3/2}]$$

≈ 1300 s

47. (B)



For point O to be the centre of mass of the system, moment about O should be zero.

$\therefore\ 2mx = m(d-x)$

$\Rightarrow 3mx = md \Rightarrow x = \frac{d}{3}$

For equilibrium,

$F_{gravitational} = F_{centripetal}$

$\therefore\ F = \frac{G(2m)m}{d^2} = (2m)\omega^2\left(\frac{d}{3}\right)$

$\Rightarrow \frac{Gm}{d^2} = \omega^2\frac{d}{3} \Rightarrow \omega^2 = \frac{3Gm}{d^3} \Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$

$\therefore$ Period of revolution, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$

48. (B)

(b) By Kepler's law

$T^2 \propto R^3$

$T_2^2 = T_1^2 \times \left(\frac{R_2}{R_1}\right)^3 \Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{\frac{3}{2}}$

$= 7(3)^{\frac{3}{2}} = 7\times 3\sqrt{3} = 21\sqrt{3}$ hours $\simeq 36$ hours

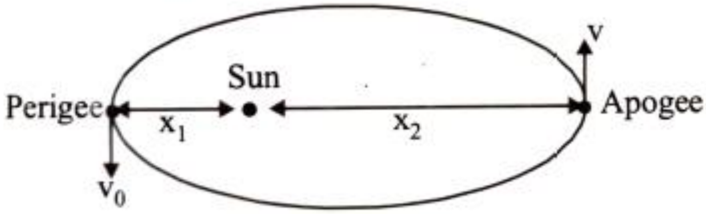
49. (C)

(c) By kepler's law $T^2 \propto r^3$

$\Rightarrow \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow 2^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow r_A^3 = 4r_B^3$

50. (C)

(c) When distance of the planet from the sun is maximum i.e., x at apogee so velocity is minimum and vice-versa.



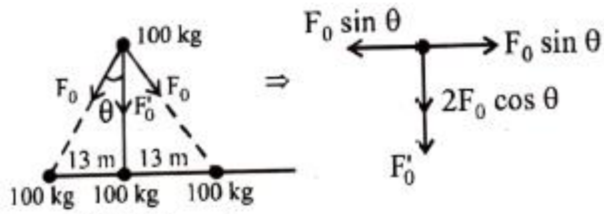
By angular momentum conservation

$$mv_0x_1 = mvx_2$$

$$\Rightarrow \quad v = \frac{v_0x_1}{x_2}$$

51. (B)

(b)



So, $F_{net} = F_o' + 2F_o\cos\theta$

$$= \frac{G\times100^2}{(13)^2} + \frac{2G(100)^2}{(13\sqrt{2})^2}\cdot\frac{13}{13\sqrt{2}}$$

$$= \frac{G100^2}{13^2}\left(1+\frac{1}{\sqrt{2}}\right) = 100\,G$$

52. (A)

(a) Given, radius of earth = 6400 km

We have, $g' = g\left(1-\frac{2d}{R}\right)$

The percentage decrease in the weight,

$$\frac{g'-g}{g} = \frac{-2d}{R} = \frac{2\times32\times100}{6400} = 1\%$$

53. (D)

(d) Since $g = \frac{GM}{R^2} \Rightarrow g + \frac{G}{R^2}\times\rho\times\frac{4}{3}\pi R^3 \Rightarrow g = \rho R$

So, $\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2} - \frac{R_1}{R_2} = \frac{3}{5}\cdot\frac{2}{3} = \frac{2}{5}$

54. (A)

(a) Law of gravitation is universal law so it is valid for any two pair of bodies.

At centre, $d = R$

So, $g = \left(1-\frac{d}{R}\right) = \left(1-\frac{R}{R}\right) = 0$

55. (A)

(a) The expression for acceleration due to gravity is as shown,

$\vec{g} = -\frac{GMr}{R^3}(\hat{r}), \quad r < R$

$= -\frac{GM}{r^2}\hat{r}, \quad r > R \Rightarrow \quad |g| \propto r, \; r < R$

$\propto \frac{1}{r^2}, \quad r > R$

56. (B)

(b) $W_h = \frac{W_{surface}}{3} \Rightarrow Mg' = \frac{Mg}{3}$

$\Rightarrow \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{3} \Rightarrow \left(1+\frac{h}{R}\right) = \sqrt{3}$

$\Rightarrow \frac{h}{R} = \sqrt{3}-1 \Rightarrow h = (\sqrt{3}-1)R$

$\Rightarrow h = 0.732 \times 6400 \simeq 4685 \text{ km}$

57. (C)

(c) From $mg = \frac{GMm}{R^2}$

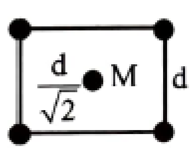
$\Rightarrow \quad g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(0.99R)^2}$

$\because$ Radius of the earth shrinks by 1%

$\therefore \quad \frac{g'}{g} = \left(\frac{R}{0.99R}\right)^2 \Rightarrow g' > g$

58. (A)

(a)



$U_{net} = -\frac{Gmm}{d}\times 4 - \frac{GMm}{d}\times 4\sqrt{2}$

$-\frac{Gmm}{\sqrt{2}d}\times 2 = -\frac{Gm}{d}\left[(4+\sqrt{2})\text{m} + 4\sqrt{2}M\right]$

59. (B)

(b) As, $E = -\frac{GMm}{2r}$

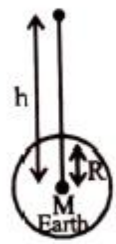
$E \propto \frac{M}{r}$

$\frac{E_A}{E_B} = \frac{M_A}{M_B} \times \frac{r_B}{r_A} = \frac{4}{3} \times \frac{4r}{3r} = \frac{16}{9}$

60. (B)

(b) Initial kinetic energy

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(\lambda v_e)^2$$



Initial potential energy,

$$U_i = \frac{GMm}{R}$$

Final potential energy,

$$U_f = \frac{-GMm}{h}$$

Using law of conservation of energy

$$K_i + U_i = K_f + U_f$$

$$-\frac{GMm}{R} + \frac{1}{2}m\lambda^2 V_e^2 = \frac{GMm}{h} \quad (\because \text{Final kinetic energy} = 0)$$

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}\lambda^2 \frac{2GMm}{R} = -\frac{GMm}{h}$$

$$\Rightarrow \frac{\lambda^2}{R} - \frac{1}{R} = \frac{-1}{h} \Rightarrow \frac{1}{h} = \frac{1-\lambda^2}{R} \Rightarrow h = \frac{R}{1-\lambda^2}$$

61. (A)

(a) Escape velocity, $v_e = \sqrt{\frac{2Gm}{R}}$

from conservation of energy

$$\frac{-GMm}{R} + \frac{1}{2}m\frac{V_e^2}{9} = -\frac{GMm}{R+h}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{V_e^2}{18} \Rightarrow \frac{GM}{R+h} = \frac{GM}{R} - \frac{GM}{9R}$$

$$\frac{GM}{R+h} = \frac{8GM}{9R} \Rightarrow \frac{1}{R+h} = \frac{8}{9R}$$

$$9R = 8R + 8h$$

The maximum height attained by the body,

$$h = \frac{R}{8} \Rightarrow \frac{6400}{8} \Rightarrow 800 \text{ km}$$

62. (A)

(a) We know that, escape velocity is given as

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G \times \rho \times \frac{4}{3}\pi R^3}{R}} \Rightarrow v_e \propto \sqrt{\rho R^2}$$

$$\text{So, } v_{e_2} = v_{e_1}\sqrt{\frac{\rho^2}{\rho_1} \times \left(\frac{R_2}{R_1}\right)^2} = 12\sqrt{4 \times \left(\frac{1}{2}\right)^2} = 12 \text{ km/s.}$$

63. (16.00)

(16.00) Using law of conservation of energy

Total energy at height 10 R = total energy at earth

$$-\frac{GM_E m}{10R} + \frac{1}{2}mV_0^2 = -\frac{GM_E m}{R} + \frac{1}{2}mV^2$$

$$\left[\because \text{Gravitational potential energy} = -\frac{GMm}{r}\right]$$

$$\Rightarrow \frac{GM_E}{R}\left(1 - \frac{1}{10}\right) + \frac{V_0^2}{2} = \frac{V^2}{2} \Rightarrow V^2 = V_0^2 + \frac{9}{5}gR$$

$$\Rightarrow V = \sqrt{V_0^2 + \frac{9}{5}gR} \approx 16\ km/s \quad [\because V_0 = 12 \text{ km/s given}]$$

64. (04.00)

(04.00) At the surface of earth, $g = \dfrac{GM}{R^2}$

At point C $g_c = \dfrac{GM}{\left(R + \frac{R}{2}\right)^2} = \dfrac{4}{9}g$

At point A $g_A = g\left(1 - \dfrac{d}{R}\right)$ or, $g_A = g\left(1 - \dfrac{AB}{R}\right)$

From question,

$$g_A = g_C \Rightarrow \frac{4}{9}g = g\left(1 - \frac{AB}{R}\right) \Rightarrow AB = \frac{5R}{9}$$

$$\therefore \quad OA = OB - AB = R - \frac{5}{9}R = \frac{4R}{9}$$

$$\therefore \quad \frac{OA}{AB} = \frac{x}{y} = \frac{\frac{4R}{9}}{\frac{5R}{9}} = \frac{4}{5} \quad \therefore \quad x = 04.00$$

65. (2)

18. **(2)** $U = -G\left[\frac{(M-m)m}{a}\times 4 + \frac{m^2}{\sqrt{2}a} + \frac{(M-m)^2}{\sqrt{2}a}\right]$

$= -\frac{G}{a}\left[4Mm - 4m^2 + \frac{m^2}{\sqrt{2}} + \frac{(M-m)^2}{\sqrt{2}}\right]$

$\frac{dU}{dm} = 4M - 8m + \frac{2m}{\sqrt{2}} - \frac{2(M-m)}{\sqrt{2}}$

For maximum 'U' $\frac{dU}{dm} = 0$

$\Rightarrow \quad 0 = 4M - 8m + 2\sqrt{2}m - \sqrt{2}M$

$\Rightarrow \quad 0 = M(4-\sqrt{2}) - 2m(4-\sqrt{2}) \Rightarrow M = 2m \Rightarrow \frac{M}{m} = 2$

66. (3)

(3) Binding energy of uniform sphere $= \frac{3}{5}\frac{GM^2}{R}$

Energy given

$E = U_f - U_i = 0 - \left(\frac{3}{5}\frac{GM^2}{R}\right) - \frac{3}{5}\frac{GM^2}{R} \quad \therefore x = 3$

67. (64)

(64) Escape velocity,

$v_e = \sqrt{\frac{2GM}{R}}$

Let R' be the radius so that escape velocity is increased 10 times.

$v' = 10v_e = \sqrt{\frac{2GM}{R'}} \Rightarrow 10\times\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R'}}$

$\therefore \quad R' = \frac{R}{100} = \frac{6400}{100} = 64 \text{ km}$

68. (10)

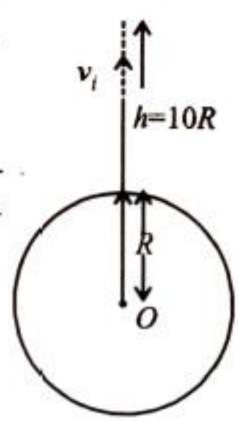
(10) From energy conservation,

$-\frac{GM_e m}{R} + \frac{1}{2}mv_i^2 = -\frac{GM_e m}{11R}$

$v_i = \sqrt{\frac{20}{11}\frac{GM_e}{R}} \quad \because v_e = \sqrt{\frac{2GM_e}{R}}$

$\therefore v_i = \sqrt{\frac{10}{11}}V_e$

$\therefore x = 10.$



69. (2)

(2) At '*h*' height above the ground (h ≪ R)

$$g_h = g\left(1-\frac{2h}{R}\right)$$

At depth '*d*' below the surface of earth

$$g_d = g\left(1-\frac{d}{R}\right)$$

Given, $g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right)$

$$\Rightarrow \frac{2h}{R} = \frac{d}{R} \Rightarrow 2h = d \Rightarrow \alpha = 2$$

70. (6)

(6) Enlongation of wire due to its own weight is given by

$$\Delta l = \frac{mgl}{yA} \text{ or, } \Delta l \propto g$$

$$\Rightarrow \frac{\Delta l_e}{\Delta l_p} = \frac{g_e}{g_p} \Rightarrow g_p = g_e \frac{\Delta l_p}{\Delta l_e}$$

$$\Rightarrow g_p = \frac{10 \times 6 \times 10^{-5}}{10^{-4}} = 6 \text{ m/s}^2$$

71. (2)

(2) We know that orbital velocity is given as

$$V = \sqrt{\frac{GM}{x}} \therefore V \propto \frac{1}{\sqrt{x}} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$$