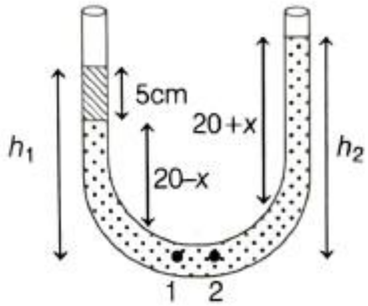
1. (C)



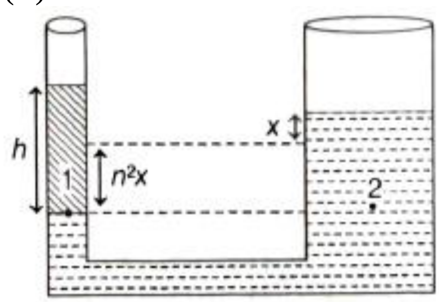
$$p_1 = p_2$$

$$\Rightarrow \quad p_0 + 4g(5) + 1g(20 - x) = p_0 + 1g(20 + x)$$

$$\Rightarrow \quad x = 10\,\text{cm}$$

$$\frac{h_2}{h_1} = \frac{20 + x}{(20 - x) + 5} = \frac{30}{15} = 2$$

2. (B)



$$p_1 = p_2 \;\Rightarrow\; p_0 + \rho_w g h = p_0 + \sigma\rho_w(n^2x + x + x)$$

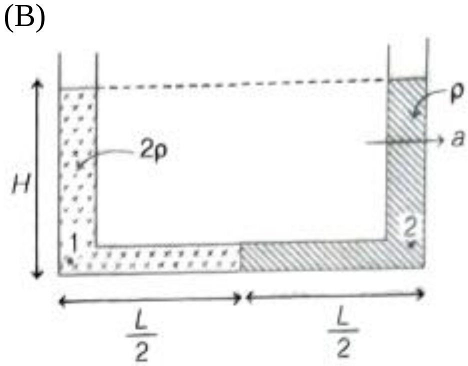
$$\Rightarrow \; x = \frac{h}{(n^2+1)\sigma}$$

3. (C)

$$p_1 = p_2 \Rightarrow p_0 + 1000\,gh = p_0 + \frac{12g}{800 \times 10^{-4}}$$

$$\Rightarrow h = 15\,\text{cm}$$

4. (B)



Along vertical,

$$p_1 = p_0 + (2\rho)gh$$

$$p_2 = p_0 + \rho gh$$

Along horizontal,

$$p_1 = p_2 + \rho a\left(\frac{L}{2}\right) + 2\rho a\left(\frac{L}{2}\right)$$

$$\Rightarrow \;\Rightarrow\; (p_0 + 2\rho gh) = (p_0 + \rho gH) + \frac{3\rho aL}{2}$$

$$\Rightarrow\; H = \frac{3aL}{2g}$$

5. (A)

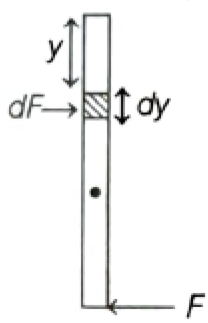
$$\int_{p_1}^{p_2} dp = \int_{r_1}^{r_2} \rho\left(\omega^2 r\right) dr$$

$$\Rightarrow\; p_2 - p_1 = \rho\omega^2\left(\frac{r_2^2 - r_1^2}{2}\right)$$

$$\Rightarrow\; \omega = \sqrt{\frac{2(p_2 - p_1)}{\rho\left(r_2^2 - r_1^2\right)}}$$

6. (C)

The net force acting on the gate element of width dy at a depth y from the surface of the fluid.



$$dF = (p_0 + \rho gy - P_0)(1dy)$$

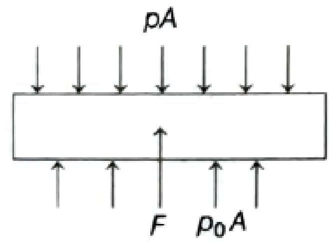
Torque about the hinge $d\tau = \rho gy\, dy\left(\frac{1}{2} - y\right)$

Net torque experienced by the gate.

$$\int d\tau + F\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow\; \int_0^1 \rho gy\, dy\left(\frac{1}{2} - y\right) + \frac{F}{2} = 0 \;\Rightarrow\; F = \frac{\rho g}{6}$$

7. (B)



For equilibrium of piston,

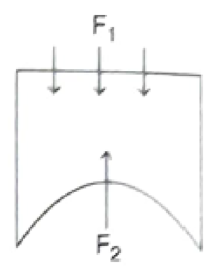
$$pA = p_0A + F$$

$\Rightarrow \left(p_0 + \rho g H\right)A = p_0A + F$

$\Rightarrow F = \rho g H A$

8. (D)

Let force on the bottom of cylinder by liquid be F_2.



$F_1 = pA = (\rho g h)\pi R^2$

Buoyant Force = Net hydrostatic force

$\Rightarrow \rho V g = F_2 - F_1$

$\Rightarrow \rho V g = F_2 - (\rho g h)\pi R^2$

$\Rightarrow F_2 = \rho g\left(V + \pi R^2 h\right)$

9. (B)

Weight removed = Decrease in buoyant force

$\Rightarrow 200g = 1\left(l^2 \times 2\right)g$

$\Rightarrow l = 10$ cm

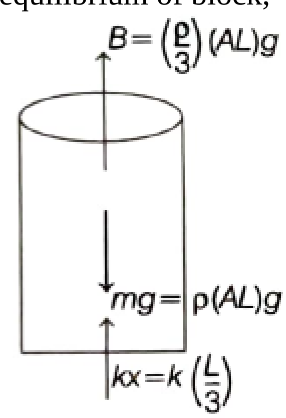
10. (C)

Block experiences a buoyant force due to liquid in upward direction. So, reading of spring balance will be less than 2 kg.

Liquid will experience buoyant force due to block in downward direction. So, reading of balance B will be more than 5 kg.

11. (B)

For equilibrium of block,



$\sum F = 0$

$\Rightarrow \quad B + kx = mg$

$\Rightarrow \quad \frac{\rho}{3} ALg + k\left(\frac{L}{3}\right) = \rho ALg$

$\Rightarrow \quad K = 2\rho Ag$

12. (A)

Fraction of volume immersed $\frac{V_d}{V} = \frac{\rho_S}{\rho_L}$

It depends upon the densities of solid and liquid only. It is independent of acceleration of system. So, no change in fraction of volume immersed.

13. (A)

For equilibrium of cylinder,

$$B_1 + B_2 = mg$$

$\Rightarrow \quad (2d)\left(A\frac{L}{4}\right)g + dA\left(\frac{3L}{4}\right)g = D(AL)g$

$\Rightarrow \quad D = \frac{5d}{4}$

14. (A)

In steady horizontal flow,

$p + \frac{1}{2}\rho v^2 = \text{constant}$

So, pressure is greatest where the speed is least.

15. (A)

Applying Bernoulli's equation for the horizontal pipe,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$\Rightarrow \quad \rho_{Hg} g(1\,\text{cm}) + \frac{1}{2}\rho_w (35\,\text{cm/s})^2$

$= \rho_{Hg} gh + \frac{1}{2}\rho_w (65\,\text{cm/s})^2$

$\Rightarrow \quad 13.6 \times 980 \times 1 + \frac{1}{2} \times 1 \times (35)^2$

$= 13.6 \times 980h + \frac{1}{2} \times 1 \times (65)^2$

$\Rightarrow \quad h = 0.89\,\text{cm of Hg}$

16. (C)

Applying Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$\Rightarrow \quad v_2^2 = v_1^2 + 2g(h_1 - h_2) \qquad \because \{p_1 = p_2\}$

$\Rightarrow \quad v_2^2 = (1)^2 + 2 \times 10 \times (0.15)$

$\Rightarrow \quad v_2 = 2 \text{ m/s}$

Applying equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad 10^{-4} \times 1 = A_2 (2)$$

$$\Rightarrow \quad A_2 = 5 \times 10^{-5} \text{ m}^2$$

17. (B)

If a ball is moving from left to right and also spinning about a horizontal axis in anti-clockwise direction of motion, then relative to the ball air will be moving from right to left. The resultant velocity of air above and below the ball will be $(v + \omega r)$ and $(v - \omega r)$, respectively.

So, according to Bernoulli's principle, due to this differences of pressure, an upward force will act on the ball and hence the ball will get maximum flight.

18. (A)

$$v^2 = \frac{2gh}{1 - \left(\frac{a}{A}\right)^2}$$

$$= \frac{2 \times 10 \times (3 - 0.525)}{1 - (0.1)^2} = 50 \text{ m}^2/\text{s}^2$$

19. (A)

Volume flow rate $= \dfrac{dV}{dt} = Av$

Volume flow rate is same for both holes.

$$\Rightarrow \quad A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad (L^2)\sqrt{2gy} = (\pi R^2)\left(\sqrt{2g(4y)}\right)$$

$$\Rightarrow \quad R = \frac{L}{\sqrt{2\pi}}$$

20. (D)

Applying Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 = \rho_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad (p_0 + 10^3 \times 10 \times 10) + 0 = p_0 + \frac{1}{2} \times 10^3 v_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{200} \text{ m/s}$$

So horizontal range, $R = v_2 T$

$$2R = v'T$$

$$\Rightarrow \quad v_2' = 2v_2 = 2\sqrt{200} \text{ m/s}$$

Applying Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad \left(p_0 + 10^3 \times 10 \times 10 + p_{\text{extra}}\right) + 0$$

$$= p_0 + \frac{1}{2} \times 10^3 \times \left(2\sqrt{200}\right)^2$$

$$\Rightarrow \quad p_{\text{extra}} = 3 \times 10^5 \text{ Pa} = 3 \text{ atm}$$

21. (C)

Let height of p above ground be y.

$v_p = \sqrt{2g(H-y)}$ and $v_Q = \sqrt{2g(H-h)}$

$R_1 = R_2$

$$\Rightarrow \quad \sqrt{2g(H-y)}\sqrt{\frac{2y}{g}} = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad y = H - h$$

22. (A)

Where tube is narrower, liquid is faster and hence according to Bernoulli's principle pressure will be lesser. So, height of liquid in the vertical tubes will be lesser where the tube is narrower.

23. (A)

When air is blown over the tank, atmospheric pressure over the tank decreases. Due to lesser pressure difference at the hole, velocity of efflux decreases.

24. (D)

Initial momentum of liquid $= d\,mv\,\hat{\mathbf{j}}$

Final momentum of liquid $= d\,mv\,\hat{\mathbf{i}}$

Change in momentum, $\mathbf{dp} = dmv\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)$

$$\Rightarrow \quad dp = \sqrt{2}\, dmv$$

$$\Rightarrow \quad \frac{dp}{dt} = \sqrt{2}\frac{dm}{dt}v = \sqrt{2}\frac{d(\rho v)}{dt}v$$

$$\Rightarrow \quad F = \sqrt{2}\,\rho LV$$

25. (A)

$$F = \rho a v^2$$

$$\Rightarrow \quad m\alpha = \rho a\left(\sqrt{2gh}\right)^2$$

$$(\rho Ah)\alpha = \rho a(2gh)$$

$$\Rightarrow \quad \alpha = \frac{2ga}{A}$$

So, acceleration at any instant is independent of h.

Velocity acquired by container,

$$v = u + at \Rightarrow v = \frac{2gat}{A}$$

Hence, v depends on h as t depends on h.

26. (1.9)

Weight of liquid added = Increase in buoyant force

$$\Rightarrow \quad (\rho\rho_w)\left(\frac{1}{3}\pi\left(\frac{r}{3}\right)^2\left(\frac{h}{3}\right)\right)g = (0.8\rho_w)$$

$$\left(\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{r}{2}\right) - \frac{1}{3}\pi\left(\frac{r}{3}\right)^2\left(\frac{h}{3}\right)\right)g$$

$$\Rightarrow \quad \rho = 1.9$$

27. (100)

As water level decreases, buoyant force on the block decreases and tension in the wire increases.
Let water level comes down by y, then wire breaks.

$$T = mg - B$$

$$\Rightarrow \quad 7\times10^6\times10^{-6} = 1500\times(0.1)^3 g - 1000\times(0.1)^2\frac{(10-y)}{100}g$$

$$y = 2\,\text{cm}$$

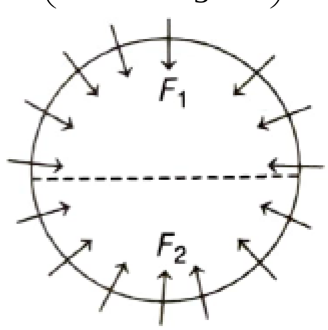
Let the level descends by cm in time it.

$$2t = (200-100)2 \Rightarrow t = 100\,\text{s}$$

28. (3)

$F_1 =$ Hydrostatic force on the upper half of sphere weight of the liquid column above the upper half of sphere

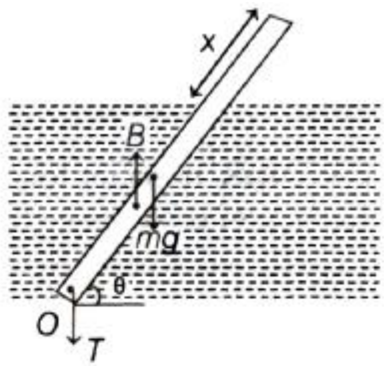
$$= \rho\left(\pi R^2(R) - \frac{2}{3}\pi R^3\right)g = \frac{1}{3}\rho\pi R^3 g \text{ (downward)}$$



$$F_2 - F_1 = B \Rightarrow F_2 - \frac{1}{3}\rho\pi R^3 g = \rho\left(\frac{4}{3}\pi R^3\right)g$$

$$\Rightarrow \quad F_2 = \frac{5}{3}\rho\pi R^3 g$$

29. (2)



Balancing torque about O,

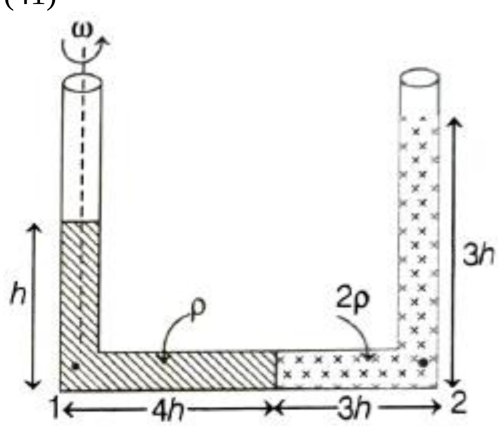
$$\left(\frac{25}{36}\rho_w\right)(12A)\,g\,(6\cos\theta)-\rho_w$$

$$\left[(12-x)A\right]g\left(\frac{12-x}{2}\right)\cos\theta=0$$

$\Rightarrow \quad (12-x)^2=100$

$\Rightarrow \quad x=2\text{ m}$

30. (41)



Along vertical, $p_1=p_0+\rho gh$

$$p_2=p_0+(2\rho)g(3h)=p_0+6\rho gh$$

Along horizontal,

$$\int dp=\int\rho\omega^2x\,dx$$

$$\int_{p_1}^{p_2}dP=\int_0^{4h}\rho\omega^2x\,dx+\int_{4h}^{7h}(2\rho)\omega^2x\,dx$$

$$\Rightarrow \quad p_2-p_1=\rho\omega^2\left(\frac{16h^2}{2}\right)+2\rho\omega^2\left(\frac{33h^2}{2}\right)$$

$$\Rightarrow \quad 5g\rho h=41\rho\omega^2h^2 \ \Rightarrow\ \omega=\sqrt{\frac{5g}{41h}}$$

31. (2)

Equation of continuity, $A_1v_1=A_2v_2$

$$\Rightarrow \quad \frac{\pi(8)^2}{4} \times 0.25 = \frac{\pi(2)^2}{4} v_2$$

$$\Rightarrow \quad v_2 = 4 \text{ m/s}$$

Horizontal range $= v_2 T = v_2 \sqrt{\frac{2h}{g}} = 4\sqrt{\frac{2(1.25)}{10}} = 2 \text{ m}$

32. (3.2)

Let point B at the nozzle.

Applying equation of continuity at A and B,

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad (4A) v_A = A v_B$$

$$\Rightarrow \quad v_A = \frac{v_B}{4}$$

Applying Bernoulli's equation between A and B,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Rightarrow \quad p_A + \frac{1}{2}\rho v_A^2 + 0 = p_B + \frac{1}{2}\rho v_B^2 + \rho g l$$

$$\Rightarrow \quad \left(p_0 + 41 \times 10^3\right) + \frac{1}{2} \times 10^3 \times \left(\frac{V_B}{4}\right)^2$$

$$= p_0 + \frac{1}{2} \times 10^3 \times v_B^2 + 10^3 \times 10 \times 1.1$$

$$\Rightarrow \quad v_B = 8 \text{ m/s}$$

$$\Rightarrow \quad h = \frac{v_B^2}{2g}$$

$$\Rightarrow \quad h = \frac{v_B^2}{2g}$$

$$\Rightarrow \quad h = \frac{(g)^2}{2(10)} = 3.2 \text{ m}$$

33. (80)

In steady state, $A_1 v_1 = A_2 v_2$

$$\Rightarrow \quad \pi(0.02)^2 \times 1 = \pi(0.01)^2 \times \sqrt{2 \times 10 \times h}$$

$$h = 0.8 \text{ m} = 80 \text{cm}$$

34. (2)

Applying Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad \left(p_0 + \rho g h + 2\rho g\left(\frac{h}{2}\right)\right) + 0 = p_0 + \frac{1}{2}(2\rho) v_2^2$$

$\Rightarrow \quad v_2 = \sqrt{2gh}$

Speed just before hitting the ground,

$$v = \sqrt{v_2^2 + \left(2g\left(\frac{h}{2}\right)\right)} = \sqrt{3gh}$$

Applying equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$\Rightarrow \quad \sqrt{6} \times \sqrt{2gh} = A\sqrt{3gh}$

$\Rightarrow \quad A = 2 \text{ cm}^2$

35. (2.78)

$$R = v\sqrt{\frac{2h}{g}}$$

$\Rightarrow \quad 80v\sqrt{\frac{2(20)}{10}}$

$\Rightarrow \quad v = 40 \text{ m/s}$

Applying Bernoulli's equation,

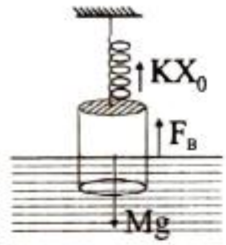
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow (p + 660 \times 10 \times 53) + 0 = 10^5 + \frac{1}{2} \times 660 \times (40)^2$$

$$p = 2.78 \times 10^5 \text{ Pa}$$

1. (C)

(c) From figure, $kx_0 + F_B = Mg$



$kx_0 + \sigma \frac{L}{2} Ag = Mg$

[$\because$ mass = density × volume]

$\Rightarrow kx_0 = Mg - \sigma \frac{L}{2} Ag$

$$\Rightarrow x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

Hence, extension of the spring when it is in equilibrium is,

$$x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

2. (C)

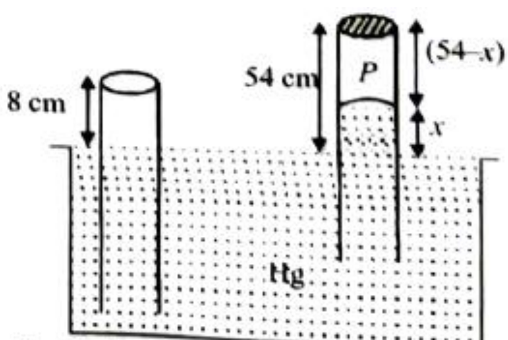
(c) Pressure difference

$$P_2 - P_1 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2} \times 1.2\left((150)^2 - (100)^2\right)$$

$$= \frac{1}{2} \times 1.2(22500 - 10000) \simeq 7500 \text{ Nm}^{-2}$$

3. (A)

(a)



$P_i = 76$ cm Hg

Length of the air column above mercury in the tube is,

$P_f + x = P_0$

$\Rightarrow \quad P_f = (76 - x)$

As T = cons. $\Rightarrow PV$ = cons.

$\Rightarrow P_i V_i = P_f V_f \Rightarrow (8 \times A) \times 76 = (76 - x) \times A \times (54 - x)$

$\therefore \quad x = 38$

Thus, length of air column = 54 – 38 = 16 cm.

4. (B)

(b) Volume flow rate = AV

Mass flow rate = ρAV

Momentum flow rate = ρAV^2

$\therefore \quad F = \rho AV^2 = 1000 \times 10^{-2} \times 1.5^2 = 22.5\text{ N}$

5. (D)

(d) Given: Diameter of water tap = $\dfrac{2}{\sqrt{\pi}}$ cm

$\therefore \quad$ Radius, $r = \dfrac{1}{\sqrt{\pi}} \times 10^{-2}\text{ m}$

Let A_V = Volume flow rate

$$A_V = \frac{3\ell}{\min} = \frac{3 \times 10^{-3}}{60}\frac{m^3}{s} = \frac{1}{20} \times 10^{-3}\text{ m/s}$$

$A_V = VA$

$$V = \frac{\frac{1}{20} \times 10^{-3}}{\pi \times \left(\frac{1}{\sqrt{\pi}}\right)^2 \times 10^{-4}} = 0.5\text{ m/s}$$

Reynold's number, $R_e = \dfrac{\rho V d}{\eta}$

$$= \frac{10^3 \times 0.5 \times \frac{2}{\sqrt{\pi}} 10^{-2}}{10^{-3}} \cong 5500$$

6. (A)

(a) According to Bernoulli's Principle,

$$\frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho v_2^2 \Rightarrow v_1^2 + 2gh = v_2^2$$

$$\Rightarrow 2gH + 2gh = v_2^2 \qquad \text{...(i)}$$

$a_1 v_1 = a_2 v_2$

$$\pi r^2 \sqrt{2gh} = \pi x^2 v_2 \Rightarrow v_2 = \frac{r^2}{x^2}\sqrt{2gh}$$

Substituting the value of v_2 in equation (i)

$$2gH + 2gh = \frac{r^4}{x^4} 2gh \text{ or, } x = r\left[\frac{H}{H+h}\right]^{\frac{1}{4}}$$

7. (D)

The volume of liquid flowing through both the tubes i.e., rate of flow of liquid is same i.e., Q= cons.

$$Q = \frac{\Delta P \pi r^4}{8\eta L} \quad \text{[By Poiseuille equation]}$$

$$\text{i.e.,} \quad \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2} \Rightarrow \frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

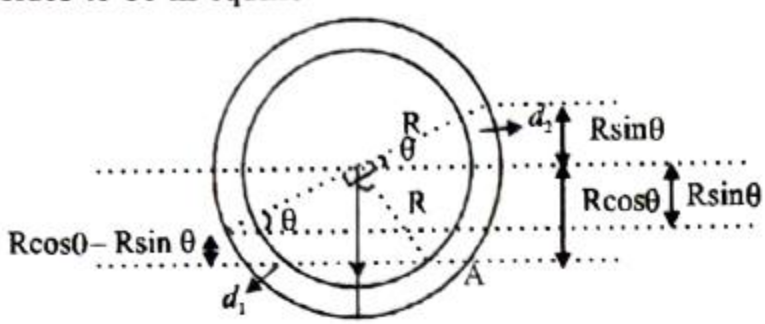
$\because \quad P_2 = 4P_1$ and $l_2 = l_1/4$

$$\frac{P_1 r_1^4}{l_1} = \frac{4P_1 r_2^4}{l_1/4} \Rightarrow r_2^4 = \frac{r_1^4}{16}$$

$$r_2 = r_1/2$$

8. (D)

(d) Pressure at interface A must be same from both the sides to be in equilibrium.



$$\therefore \quad (R\cos\theta + R\sin\theta)\rho_2 g = (R\cos\theta - R\sin\theta)\rho_1 g$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1+\tan\theta}{1-\tan\theta}$$

$$\Rightarrow \quad \rho_1 - \rho_1 \tan\theta = \rho_2 + \rho_2 \tan\theta$$

$$\Rightarrow \quad (\rho_1 + \rho_2)\tan\theta = \rho_1 - \rho_2$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$$

9. (D)

(d) Using $P_1V_1 = P_2V_2$

$$(P_1)\frac{4}{3}\pi r^3 = (P_2)\frac{4}{3}\pi\frac{125r^3}{64}$$

$$\frac{\rho g(10)+\rho gh}{\rho g(10)} = \frac{125}{64} \Rightarrow 640 + 64\,h = 1250$$

On solving we get $h = 9.5$ m

10. (A)

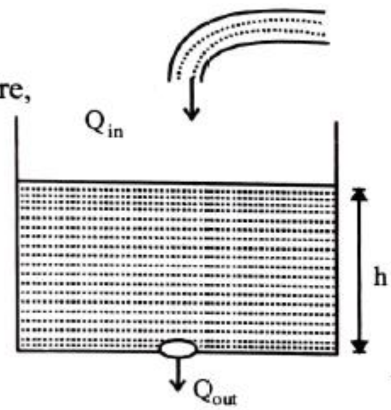
(a) Since height of water column is constant therefore, water inflow rate (Q_{in}) = water outflow rate

$Q_{in} = 10^{-4}\ m^3s^{-1}$

$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$\therefore 10^{-4} = 10^{-4} \times \sqrt{20 \times h}$

$\therefore h = \frac{1}{20}\ m = 5\ cm$



11. (B)

(b) Here, volume tric flow rate

$$= \frac{0.74}{60} = \pi r^2 v = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi} \Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10}\left(\because \pi^2 = 10\right) \Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24} \approx 4.8\,m$$

i.e., The depth of the centre of the opening from the level of water in the tank is close to 4.8 m

12. (B)

(b) When a body floats then the weight of the body = upthrust

$$\therefore (50)^3 \times \frac{30}{100} \times (1) \times g = M_{\text{cube}} g \qquad ...(i)$$

Let m mass should be placed, then

$(50)^3 \times (1) \times g = (M_{\text{cube}} + m)g \qquad ...(ii)$

Subtracting equation (i) from equation (ii), we get

$\Rightarrow mg = (50)^3 \times g(1-0.3) = 125 \times 0.7 \times 10^3 g \Rightarrow m = 87.5\,kg$

13. (A)

(a) $P_1 = P_0 + rgd_1$

$P_2 = P_0 + rgd_2$

$\Delta P = P_2 - P_1 = \rho g \Delta d$

$3.03 \times 10^6 = 10^3 \times 10 \times \Delta d \Rightarrow \Delta d \simeq 300\,m$

14. (C)

(c) $Mg = \left(\frac{4V}{5}\right)\rho_\omega g$ or $\left(\frac{M}{V}\right) = \frac{4\rho_\omega}{5}$ or $\rho = \frac{4\rho_\omega}{5}$

When block floats fully in water and oil, then

$Mg = F_{b_1} + F_{b_2}$

$(\rho V)g = \left(\frac{V}{2}\right)\rho_{oil}\, g + \frac{V}{2}\rho_\omega g$ or $r_{oil} = \frac{3}{5}\rho_\omega = 0.6\rho_\omega$

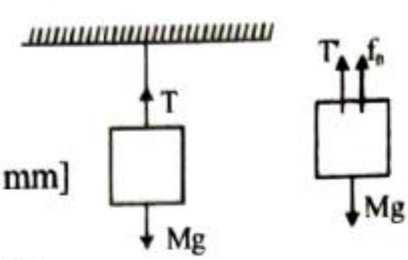
15. (A)

(a) Using $\frac{F}{A} = Y \cdot \frac{\Delta\ell}{\ell} \Rightarrow \Delta\ell \propto F \qquad ...(i)$

$T = Mg$

$T = Mg - f_B = Mg - \frac{M}{\rho_b}\cdot\rho_\ell\cdot g = \left(1 - \frac{\rho_\ell}{\rho_b}\right)Mg = \left(1 - \frac{2}{8}\right)Mg$

$T = \frac{3}{4}Mg$



From eqn (i)

$\frac{\Delta\ell'}{\Delta\ell} = \frac{T'}{T} = \frac{3}{4}$ [Given: $\Delta\ell = 4$ mm]

$\therefore \quad \Delta\ell' = \frac{3}{4}\cdot\Delta\ell = \frac{3}{4}\times 4 = 3$ mm

16. (C)

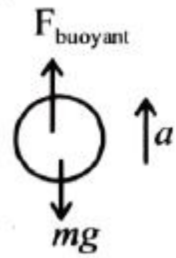
(c) Given :

Radius of air bubble = 1 cm,

Upward acceleration of bubble, $a = 9.8$ cm/s^2,

$\rho_{water} = 1$ g cm^{-3}

Volume $V = \frac{4\pi}{3}r^3 = \frac{4\pi}{3}\times(1)^3 = 4.19$ cm^3



$$F_{\text{buoyant}} - mg = ma \Rightarrow m = \frac{F_{\text{buoyant}}}{g+a}$$

$$\therefore m = \frac{(V\rho_\omega g)}{g+a} = \frac{V\rho_\omega}{1+\frac{a}{g}} = \frac{(4.19)\times 1}{1+\frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15\,\text{g}$$

17. (C)

(c) For minimum density of liquid, solid sphere has to float (completely immersed) in the liquid.

$mg = F_B$ (also $V_{\text{immersed}} = V_{\text{total}}$)

Now, m = $\int \rho dV$ and, $F_B = \frac{4}{3}\pi R^3 \rho_\ell g$

So, $\int_0^R \rho_0 4\pi\left(1-\frac{r^2}{R^2}\right).\,r^2 dr = \frac{4}{3}\pi R^3 \rho_\ell$

$$\Rightarrow 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5R^2}\right]_0^R = \frac{4}{3}\pi R^3\rho_\ell$$

$$\Rightarrow \frac{4\pi\rho_0 R^3}{3}\times\frac{2}{5} = \frac{4}{3}\pi R^3\rho_\ell \qquad \therefore \rho_\ell = \frac{2\rho_0}{5}$$

18. (D)

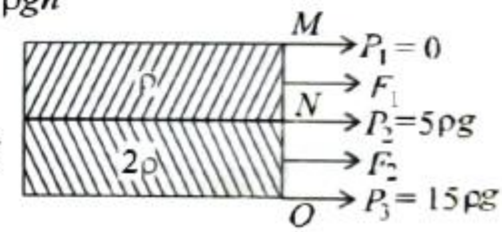
(d) Let P_1, P_2 and P_3 be the pressure at points M, N and O respectively.

Pressure is given by $P = \rho g h$

Now, $P_1 = 0$ ($\because$ $h = 0$)

$P_2 = \rho g(5)$

$P_3 = \rho g(15) = 15\ \rho g$



Force on upper part, $F_1 = \dfrac{(P_1 + P_2)}{2} A$

Force on lower part, $F_2 = \dfrac{(P_2 + P_3)}{2} A$

$\therefore \dfrac{F_1}{F_2} = \dfrac{5\rho g}{20\rho g} = \dfrac{5}{20} = \dfrac{1}{4}$

19. (D)

(d) Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho \upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho \upsilon_2^2 + \rho g h_2$$

For horizontal pipe, $h_1 = 0$ and $h_2 = 0$ and taking

$P_1 = P, P_2 = \dfrac{P}{2}$, we get $\Rightarrow P + \dfrac{1}{2}\rho \upsilon^2 = \dfrac{P}{2} + \dfrac{1}{2}\rho V^2$

$\Rightarrow \dfrac{P}{2} + \dfrac{1}{2}\rho \upsilon^2 = \dfrac{1}{2}\rho V^2 \Rightarrow V = \sqrt{\upsilon^2 + \dfrac{P}{\rho}}$

20. (A)

(a) From the equation of continuity

$A_1 v_1 = A_2 v_2$

Here, v_1 and v_2 are the velocities at two ends of pipe.

A_1 and A_2 are the area of pipe at two ends

$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{A_2}{A_1} = \dfrac{\pi(4.8)^2}{\pi(6.4)^2} = \dfrac{9}{16}$

21. (D)

(d) F = Momentam transferred by water per sec

$= \rho a V \times V$

$F = \rho A v^2 = 10^3 \times 10 \times 10^{-4} \times 20 \times 20$

$F = 400$ N

22. (B)

(b) Let 'x' be the rise (or fall) in water.

Final volume of both vessel will be same

So, $16 \times 10^{-4} \times (150 - x) = 16 \times 10^{-4} \times (100 + x)$

$\Rightarrow 50 = 2x$

$\Rightarrow x = 25$ cm

Work done = Potential Energy of extra water that enters in cylindrical vessel $= m_{\text{extra}} \times g \times x$

$= \rho_w A_w x \times g \times x$

$= \rho_w A_w g \times x^2$

$= 10^3 \times 16 \times 10^{-4} \times 10 \times (25 \times 10^{-2})^2$

= 1 joule

23. (C)

(c) Apply Bernoulli's theorem between Piston and hole

$$P_A + \rho g h = P_0 + \frac{1}{2}\rho v_e^2$$

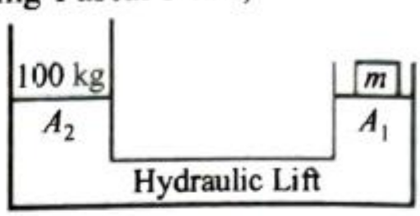
Assuming there is no atmospheric pressure on piston

$$\frac{5 \times 10^5}{\pi} + 10^3 \times 10 \times 10 = 1.01 \times 10^5 + \frac{1}{2} \times 10^3 \times v_e^2$$

$\Rightarrow v_e = 1.78$ m/s

24. (25600)

(25600) Using Pascal's law,



$$\frac{100 \times g}{A_2} = \frac{mg}{A_1} (A_1 < A_2) \quad \text{...(i)}$$

Let m mass can lift M_0 in second case then

$$\frac{M_0 g}{16 A_2} = \frac{mg}{\frac{A_1}{16}} \qquad \left(\because A = \frac{\pi d^2}{4}\right) \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\frac{M_0}{16 \times 100} = 16 \Rightarrow M_0 = 25600 \text{ kg}$$

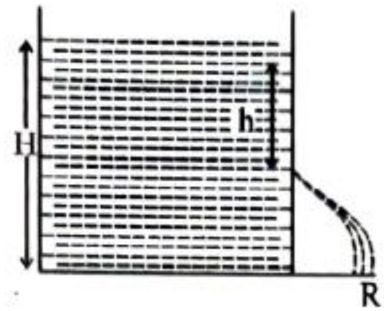
25. (6)

(6) Given,

Height of the water, H = 12 cm

Velocity of water coming out of hole, $v = \sqrt{2gh}$

Range of water, R = vt



$\Rightarrow \quad R = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$

$= 2\sqrt{h(H-h)}$

For maximum range $\frac{dR}{dh} = 0$

$\therefore \quad h = \frac{H}{2}$

Range is maximum when $h = \frac{12}{2} = 6m$

26. (24)

(24) Here, h = constant

So, $P + \frac{1}{2}\rho V^2 =$ constant

$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \Rightarrow P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$

Now, by equation of continuty.

$A_1V_1 = A_2V_2$

$\Rightarrow 2A_2V_1 = A_2V_2 \Rightarrow V_2 = 2V_1$

So, $P_1 - P_2 = \frac{1}{2}\rho 3V_1^2$

$\Rightarrow V_1^2 = \frac{2}{3}\frac{P_1 - P_2}{\rho} = \frac{2}{3} \times \frac{4500}{750} = \frac{9000}{2250} = 4$

So, $V_1 = 2$ m/s.

So, Volume flow rate $= A_1V_1 = 1.2 \times 10^{-2} \times 2$

$= 24 \times 10^{-3}$ m^3/s

27. (300)

(300) By Bernoull's theorem

$$P + \rho gh + \frac{1}{2}\rho V^2 = \text{constant}$$

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho gh = P_0 + \frac{1}{2}\rho V^2$$

[$\because$ Speed of water in tank = 0]

$$\Rightarrow \frac{25 \times 10}{0.5} + 1000 \times 10 \times 0.4 = \frac{1}{2} \times 1000 \times V^2$$

$$\Rightarrow 500 + 4000 = 500V^2 \Rightarrow V^2 = 9$$

$$\Rightarrow V = 3 \text{ m/s} = 300 \text{ cm/sec}$$

28. (363)

(363) By equation of continuity,

$$av_1 = \frac{a}{2}v_2 \Rightarrow v_2 = 2v_1$$

From Bernoulli's theorem,

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) - \rho gh_1$$

$$4100 = \frac{1}{2} \times 800\ (4v_1^2 - v_1^2) - 800 \times 10$$

$$4100 = 400\ \times 3v_1^2 - 8000$$

$$12100 = 1200 \times v_1^2$$

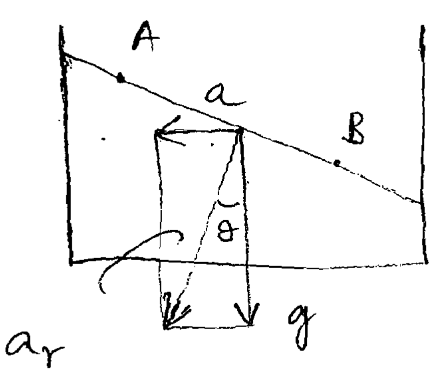
$$v_1^2 = \frac{121}{12}$$

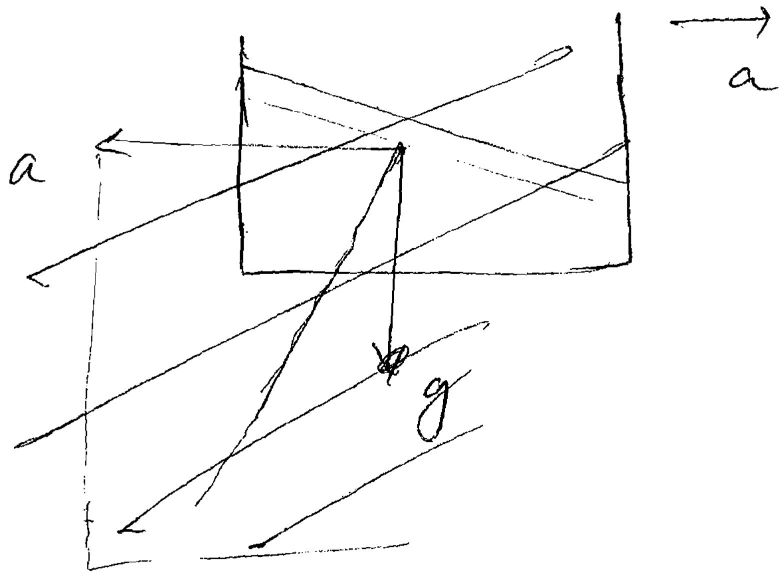
$$v_1^2 = \frac{121 \times 3}{12 \times 3} \Rightarrow v_1 = \frac{\sqrt{363}}{6}\ \text{m/s.}$$

Fluid Mechanics

Inchapter Exercise - I

Q1. (b) If container is accelerating horizontally then, pressure at different levels can be same





$$\tan\theta = \frac{a}{g}$$

$$P_A = P_B$$

Q2 (b) (c) (d)

$$P_N + H\rho g = P_M \qquad \therefore \quad P_N \neq P_M$$

$$P_N = P_0 = \text{atm. pressure}$$

$\therefore$ Gauge Pressure at $N = 0$

Gauge Pressure at $M = H\rho g$

$$= 0.05 \times 13.6 \times 10^3 \times 9.8 =$$

$$= 6.66 \times 10^3 \ N/m^2$$

$P_{\text{absolute bulb}} = P_M = P_0 + P_{gauge}$

Q3 (B) , Before melting, buoyant force

$B =$ wt. of cube + wt. of glass ball (floating)

~~B = wt. of glass ball~~

$B' =$ Volume of fluid displaced by glass ball

Now, density of glass is greater than water. So, it will sink and its weight is balanced by buoyant force and reaction from bottom.

So, $B' < B$.

Hence, water level will fall.

Q4 (B)

$B = V \rho_l g \qquad \rho_l =$ fluid density

Q5 (A) When ice melts, it gives volume of water equal to that displaced by it. So, water level will remain unchanged.

Q6 (A) (B) (C)

Apparent wt. $= mg - B = V(\sigma - \rho) g$

If $\sigma > \rho$, then Apparent wt. > 0

$\sigma = \rho$, Apparent wt $= 0$

Q7. (C) Earlier wt. of balls was supported by buoyant force.

Now, wt. of balls is supported by buoyant force + Reaction at ~~bottom~~ bottom.

So, $B' < B$.

$\therefore$ Volume displaced decreases and hence level of liquid in tank.

Q8. (B)

~~R~~ Force = Pressure x Area

$= h\rho g \cancel{\phi} \times A$

$= 0.4 \times 900 \times \cancel{8}\, 10 \times \cancel{10}\, 2 \times 10^{-3}$

$= 7.2\,N.$

Q9 (A)

$(m_1 + m_2) g = B_1 + B_2$

$\cancel{(V_1\rho_1 + V_2\rho_2) g = (V_1 + V_2)}$

$(V_1\sigma_1 + V_2\cancel{\rho_2}\sigma_2) g = (V_1 + V_2)\rho g.$

$V_1 \frac{\sigma_1}{\rho} + V_2\left(\frac{\sigma_2}{\rho}\right) = V_1 + V_2$

$V_1(2) + V_2(1/2) = V_1 + V_2$

$3V_1 = V_2.$

$\frac{m_1}{} = \frac{\sigma_1 V_1}{} = \cancel{(\sigma_1/\rho_1) V_1} \;/\; \frac{(\sigma_1/\rho) V_1}{} = \frac{2}{}$

Q10 (A) From pascal's law,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = (1500\,g)\left(\frac{\pi r^2}{\pi R^2}\right) = \cancel{1500\,g \times (20)^2}$$

$$F_1 = 1500\,g\left(\frac{1}{20}\right)^2$$

$$F_1 = \frac{1500\,g}{400} \qquad m = 3.75\ kg.$$

Q11 (A)

$g_{eff} = 0$, $B = V\rho g_{eff}$

$\therefore$ $B = 0$

Q12 (C)

When passengers drink water, weight of boat increases and hence it displaces more water equal to wt of water ~~taken~~ ~~by~~ ~~drunk~~ drunk.

Hence, no change in water level of tank.

Fluid - Mechanics

Inchapter - 2

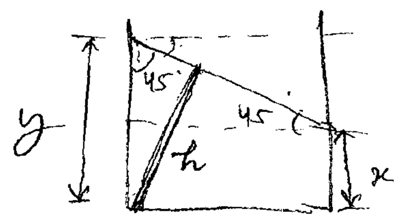
Q1. (C).

Velocity increases with decrease in cross-section, whereas Pressure and hence force decreases with increase in velocity.

Q2 (B).

$\theta = 45° =$ angle made by surface with horizontal.

$$\frac{y-x}{1} = \tan 45° = 1$$



$$y - x = 1$$

$$y = x + 1 \Rightarrow$$

~~Hence~~. Volume of water = const.

$$1^3 = 1 \times \left[x \times 1 + \frac{1}{2} \times (y - x) \times 1\right]$$

~~$2 = 2x + y - x \Rightarrow$ ~~ ~~$y = 2m$~~ ~~$x + y = 2$~~

~~$x = 1m$~~. $x + y = 2$.

$$x = \frac{1}{2}\,m, \quad y = \frac{3}{2}\,m$$

~~tan~~ $\sin 45° = \dfrac{h}{y} \Rightarrow \quad h = \dfrac{y}{\sqrt{2}} = \dfrac{3}{2\sqrt{2}}\,m.$

$\therefore \; V = \sqrt{2 \times 10\sqrt{2} \times \frac{3}{2\sqrt{2}}} = 5.48 \, m/s.$

Q3 (A)(B)(C)(D)

Equation of continuity, is principle of

$\cancel{\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}}$

conservation of mass while Bernoulli's theorem is principle of conservation of energy for ideal fluid.

Q4. (B). As cross-section increases, velocity decreases and pressure increases.

Q5 (A) (D)

$V_B > V_A$ so, $P_B < P_A$

Fluid ~~particle~~ particle at B has to cover more distance as compared to A. in given time.

Q6(C) $A_1V_1 = A_2V_2$

$V_2^2 = V_1^2 + 2gh$ $\quad V_1 = 1 \, m/s$ $\quad h = 0.15 \, m$

solving, $A_2 = 5 \times 10^{-5} \, m^2$

Q7 (C) $V = A_1V_1 = A_2V_2 + A_3V_3$.

solving, we get.

$V_3 = 1 \, m/s$.

Q8. (C). $V_A < V_B$

so, initially level of water decreases in vessel. As a result V_B decreases and hence level starts rising.

Water performs periodic oscillatory motion.

Q9. (B)

loss of weight = Buoyant force

$= V \rho g$.

V = volume of fluid displaced.

Q10 (C)

$$\frac{1}{2} \rho_1 V_1^2 = h_1 \rho_1 g + h_2 \rho_2 g.$$

$$V_1 = \sqrt{2g\left[h_1 + h_2\left(\frac{\rho_2}{\rho_1}\right)\right]}$$

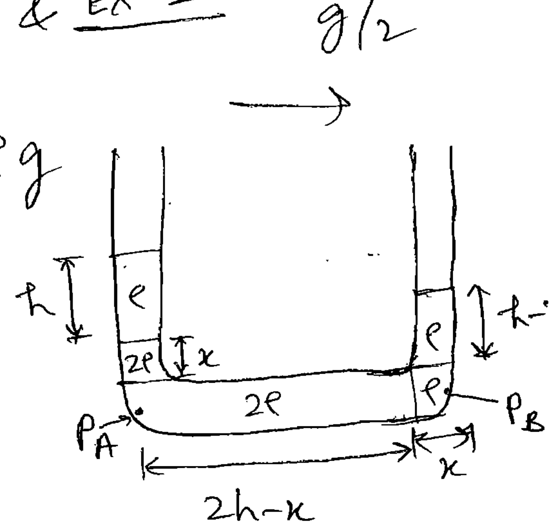
Fluid Mechanics – Ex – I (Solutions)

& Ex – II

Q1 (B) $P_A - P_B =$

$= h\rho g + x(2\rho)g - (h-x)\rho g$

$= 3x\rho g$



Force on horizontal fluid element $= (P_A - P_B) \times A = F$

$A =$ Cross – sectional Area.

$F = ma$ $\qquad a = g/2$

$m =$ mass of horizontal fluid element.

$m = (2h-x)2\rho A + x\rho A$

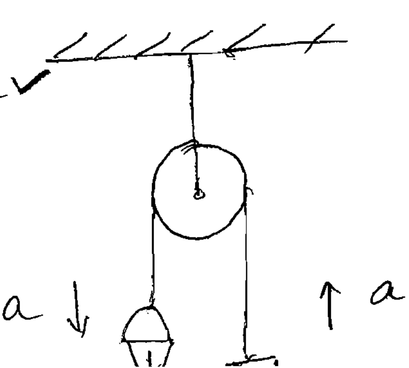
$\therefore (3x\rho g)A = \left((2h-x)2\rho A + x\rho A\right)\frac{g}{2}$

Solving, $x = \frac{4h}{7}$ $\qquad$ Difference in level $= 2x$

$= \frac{8h}{7}$

Q2 (B) acc^n of bucket $= \frac{mg - mg/2}{3m/2}$

$= \frac{g}{3}$



(c) bucket $= h\rho g'$ $\quad g' =$ effective gravity $= g - a = \frac{2g}{3}$

$= 0.15 \times 10^3 \times \frac{2}{3} \times g$

$= 1$ KPa

13 (D) Total force by fluid on cone

$= $ Buoyant Force $= V\rho g = \frac{\pi}{3} R^2 H \rho g$

Force at bottom of cone = Pressure × ~~Ar~~ Area

$= H\rho g \times \pi R^2 = \pi R^2 H \rho g$

∴ Force on slant surface due to liquid

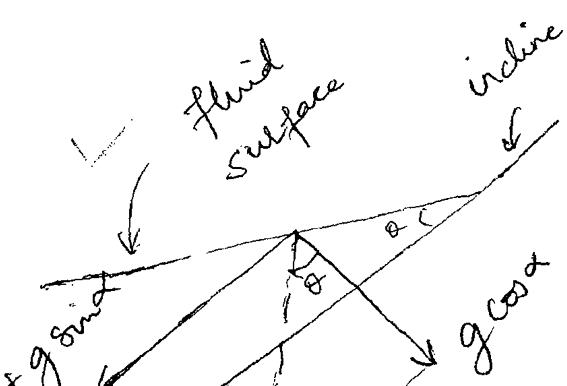
$= \pi R^2 H\rho g - \frac{\pi}{3} R^2 H \rho g = \frac{2\pi}{3} H R^2 \rho g$ (Vertically downward).

14 (C) $P_{wider} = P_{narrow}$ (at same horizontal level).

$\frac{12g}{800 \times 10^{-4}}$ N/m^2 $= h\rho g$

solving, $h = 15$ cm.

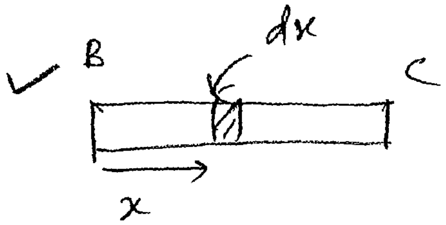
15 (B) $\tan\theta = \frac{a + g\sin\alpha}{\ldots}$



76. (C) New level of liquid in each arm $= l + \frac{l}{2} = \frac{3l}{2}$ (3)

Pressure difference at bottom of B & C

$$= \frac{3l}{2}\rho g.$$



$$dF = (\rho A\, dx)\, x\omega^2$$

$$dP = \frac{dF}{A} = \rho x \omega^2 dx$$

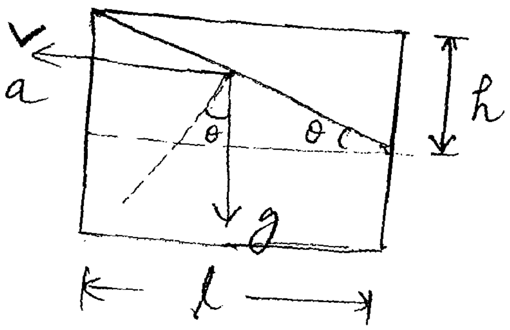
$$P_C - P_B = \int_{x=0}^{l} \rho x \omega^2 dx = \rho\omega^2\left[\frac{x^2}{2}\right]_0^l = \frac{\rho\omega^2 l^2}{2}$$

$$\therefore \quad \frac{\rho\omega^2 l^2}{2} = \frac{3l}{2}\rho g$$

$$\omega = \sqrt{\frac{3g}{l}}$$

07 (B)

Volume spilled $= \frac{l^3}{3}$



$$= \frac{1}{2} \times h \times l \times l$$

$$h = \frac{2l}{3}$$

$$\tan\theta = \frac{a}{g} = \frac{h}{l} = \frac{2}{3} \Rightarrow a = \frac{2g}{3}$$

8(D) (iv) Volume of liquid in two containers

$$V_1 = V_2$$

$$\pi R^2 H_1 = R^2 H_2$$

H_1, H_2 are heights of liquid in two containers

$$F_1 = H_1 \rho g \pi R^2$$

$$F_2 = H_2 \rho g R^2$$

$$\frac{F_1}{F_2} = \frac{H_1 \rho g \pi R^2}{H_2 \rho g R^2} = 1 : 1$$

$$F_1 = F_2$$

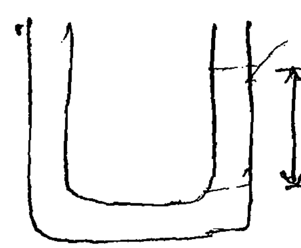
Q9 (A)

$\because$ liquid is in equilibrium. so, total force exerted by flask should be equal ~~to~~ and opposite to weight of liquid.

Q10 (D)

length of water in arm

$$= \frac{60 \text{ cc}}{1 \text{ cm}^2} = 60 \text{ cm}.$$



Volume of liquid in horizontal arm $= 20 \text{ cm} \times 1 \text{ cm}^2 = 20 \text{ cc}$

If h be height of liquid in other arm.

then, from ~~p~~ Pascal's law

$$h \rho_L g = 60 \rho_w g$$

$$h = \frac{60}{4} = 15 \text{ cm}.$$

F_B = Force exerted by bottom of cone on water

F_W = Force by walls on water.

$W = V\rho g = \frac{\pi}{3} R^2 h \rho g$

$F_B = P_B \times A = h\rho g \times \pi R^2 = \pi R^2 h \rho g$

$\therefore$ For equilibrium of water

$F_B = F_W + W \Rightarrow F_W = \frac{2\pi}{3} R^2 h \rho g$

From Newton's third law, force exerted by water on cone walls is $\frac{2\pi}{3} R^2 h \rho g$ upward.

Q14 (B)

Under equilibrium condition

Buoyant Force = weight of cubes

~~$(1^2 \times 1 \times 1.15 \times 10^3 + 1^2 \times h \times 0.6 \times 10^3) g$~~

$(1^2 \times 1 \times 10^3 + 1^2 \times h \times 10^3) g =$

$= (1^2 \times 1 \times 1.15 \times 10^3 + 1^2 \times 1 \times 0.6 \times 10^3) g$

Solving,

$h = 0.75 \text{ m} = 75 \text{ cm}.$

$\therefore$ height above water surface $= 25$ cm.

Q11 (B) If H is height of water
(6)

$2P = H\rho g$

Now, height of water remaining $= H - \frac{H}{5} = \frac{4H}{5}$

$\therefore$ New pressure at bottom $= P + \frac{4}{5} H\rho g$

$= P + \frac{4}{5}(2P) = \frac{13}{5} P$

Q12 (C)

$h_1 - 5 + h_2 = 40$

$h_1 + h_2 = 45$ —— (1)

From Pascal's law

$5\rho g + (h_1 - 5)\rho_w g = h_2 \rho_w g$

$5 \times 4 + (h_1 - 5) = h_2$

$h_2 - h_1 = 15$ —— (2)

Solving (1) & (2) $h_2 = 30$ cm , $h_1 = 15$ cm

$\frac{h_2}{h_1} = \frac{30}{15} = 2 : 1$

Q13 (A) FBD of water

Q15. (D) under equilibrium

$$mg = B$$

$$(abc)dg = (lbc)(1)g \qquad l = \text{length immersed in water}$$

$$\therefore \quad l = ad \quad \text{——— (1)}.$$

If cube is further pushed down by x then restoring force is given by

$$F = B' - mg = (l+x)bc(1)g - abcdg$$

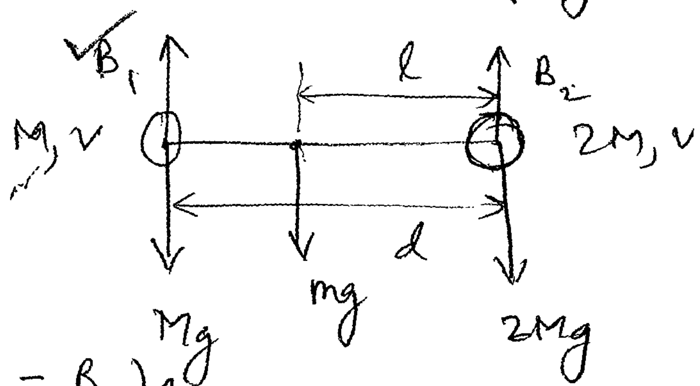
$$(abc)dA = x(bc)g$$

$$A = \frac{x}{ad}g = \frac{g}{ad}x.$$

$$\therefore \quad \omega^2 = \frac{g}{ad}. \qquad \text{or} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ad}{g}}$$

Q16 (B)

Taking torque about point mass (m).



$$(Mg - B_1)(d-l) = (2Mg - B_2)l$$

or

$$\left(Mg - \frac{V}{2}\rho g\right)(d-l) = \left(2Mg - \frac{V}{2}\rho g\right)l$$

$$\frac{d-l}{l} = \frac{2M - V\rho/2}{M - V\rho/2}$$

Q17. (B) Let ρ_1, ρ_2 be the densities of two masses.

$$\therefore V\rho_1 g = 2V\rho_2 g$$

$$\therefore \rho_1 = 2\rho_2 \quad \text{———} \quad (1)$$

When ~~immer~~ immersed in liquid

$$V\rho_1 g - \cancel{V\rho g}\ V\sigma g = 2V\rho_2 g - 2V(0.9)g$$

σ = density of unknown liquid

$$\therefore V\rho_1 g = 2V\rho_2 g.$$

$$\therefore V\sigma g = 2V(0.9)g$$

$$\sigma = 1.8\ gm/cm^3$$

Q18. (C)

$$Mg = \left(a^2 \frac{4a}{5}\right)\rho g = \frac{4a^3}{5}\rho g \quad \text{———} \quad (1)$$

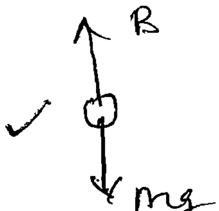
$$(M+m)g = a^3\rho g \quad \text{———} \quad (2)$$

Solving (1) & (2)

$$ma = a^3\rho g$$

$\therefore \; mg = \frac{Mg}{4}$ or $M = 4m$

319. (C) When ~~ship~~ fish is placed in water of bucket then resultant force acting on ~~ship~~ fish is



$F_R = mg - B.$ = Effective weight of fish

So, Effective weight of fish decreases.

This buoyant force is applied by water of bucket on fish.

From Newton's third law, the same buoyant force is acting on water and hence bucket downward.

So, net weight carried by boy remains same.

Q20. Let V be volume of ~~cork~~. In ~~equilibrium~~

$V'(1)\,g = V(0.5)\,g$

Q20. (D)

10

Let V be volume of ~~water~~ cork and V' be volume of cork submerged.

In equilibrium,

$$V(0.5)g = V'(1)g$$

$$\frac{V'}{V} = 0.5 = \frac{1}{2} = \text{fraction submerge}$$

or %.

$$\frac{V'}{V} = \frac{1}{2} \times 100 = 50\%$$

Q21. (A).

Let (A) be cross-section area of cylinders

∴ In equilibrium.

$$(ALd_1)g + (ALd_2)g = \left(A\frac{3L}{2}d\right)g$$

$$d_1 + d_2 = \frac{3}{2}d$$

$$d_1 > d_2$$

$$d_1 + d_1 > d_2 + d_1 = \frac{3}{2}d$$

$$2d_1 > \frac{3}{2}d$$

Q22 (B) (11)

$$W_1 = W - \frac{W}{\rho}\rho_w \quad \text{——} \quad (1)$$

ρ = density of steel

$$W_2 = W - \frac{W}{\rho}\sigma \quad \text{——} \quad (2)$$

σ = density of liquid

$\therefore$ from (1) & (2)

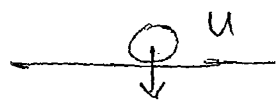
$$\frac{\rho_w}{\rho} = 1 - \frac{W_1}{W}, \quad \frac{\sigma}{\rho} = 1 - \frac{W_2}{W}$$

$$\therefore \frac{\sigma}{\rho_w} = \frac{1 - W_2/W}{1 - W_1/W} = \frac{W - W_2}{W - W_1}$$

= Relative density

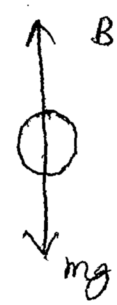
Q23 (A)

velocity of ball just before entering liquid $= \sqrt{2gh} = \sqrt{2g(2)}$,



$u = \sqrt{4g}$

$$ma = B - mg = V\cancel{\sigma}(1)g - V(0.8)g$$



$$V(0.8)a = V(0.2)g$$

$$a = g/4$$

V = volume of ball

(12) At maximum depth velocity of ball is zero.

$u = \sqrt{4g}$ = initial velocity (downward)

$a = g/4$ (upward)

$\therefore$ using $v^2 = u^2 + 2as$.

$$0^2 = (\sqrt{4g})^2 + 2(g/4)\,d.$$

$d = 8\,m$

Q24 (B)

Net ~~to~~ Force acting on ball

$= \cancel{V\rho} \; V\sigma g - V\rho g$ (upward).

$= V(\sigma - \rho)g = ma.$

$\cancel{V\rho g}$ $\quad V(\sigma - \rho)g = V\rho a$

$$a = \left(\frac{\sigma}{\rho} - 1\right) g \text{ upward.}$$

$\therefore$ velocity of ball when it reaches ~~top face~~ surface of water.

$= \sqrt{2ah} = \sqrt{2\left(\frac{\sigma}{\rho} - 1\right) g h}$

When ball comes out of water then, only gravity acts on it.

$\therefore$ initial velocity $= \sqrt{2\left(\frac{\sigma}{\rho} - 1\right) g h}$

Final velocity at height (H) will be zero (13)

$$V^2 = u^2 + 2as$$

$$0^2 = \left(\sqrt{2\left(\frac{\sigma}{\rho}-1\right)gh}\right)^2 - 2gH.$$

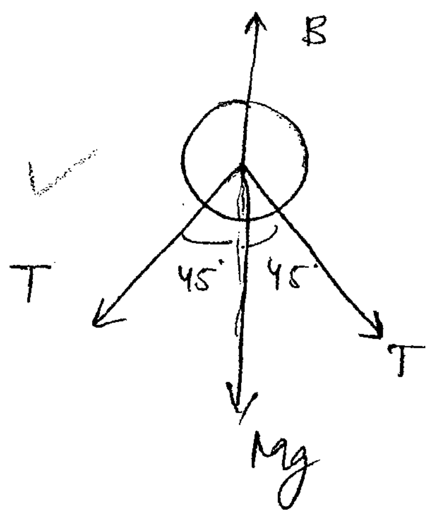
$$\therefore \quad H = \left(\frac{\sigma}{\rho}-1\right)h$$

Q25 (A)

In equilibrium

$$B = Mg + 2T\cos 45^\circ$$

$$T = \frac{\frac{4\pi}{3}R^3 \rho_w g - Mg}{\sqrt{2}}$$



FBD

26. (C)

Let V, V_c be volume of ball and cavities in it.

$$\therefore \quad 9.8\,kg = 7800\,kg/m^3\,(V - V_c)$$

$$1.5\,kg = 1000\,kg/m^3\,V. \quad \text{(due to buoyant force)}$$

$$\frac{V - V_c}{V} = \frac{9.8 \times 1000}{1.5 \times 7800} = \frac{98}{\ldots}$$

(14)

$$\frac{V_c}{V} = \frac{19}{117} = 0.1624 = 16.24\%$$

§27 (A)

Under condition of floating

$$B = mg.$$

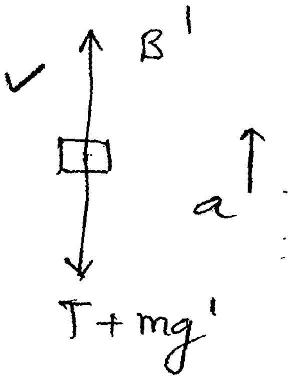
$$\frac{4\pi}{3}R^3(1) = \frac{4\pi}{3}(R^3 - r^3)\,\sigma\, g$$

$$\frac{R^3 - r^3}{R^3} = \frac{1}{\sigma}$$

$$\left(\frac{r}{R}\right)^3 = \frac{\sigma - 1}{\sigma} \quad \text{or} \quad \frac{R}{r} = \left(\frac{\sigma}{\sigma - 1}\right)^{1/3}$$

§28 (B)

$g_{\text{effective}} = g_{eff} = g + a$



$\therefore B' =$ Buoyant force

$$= Vd\,(g_{eff}) = Vd(g+a)$$

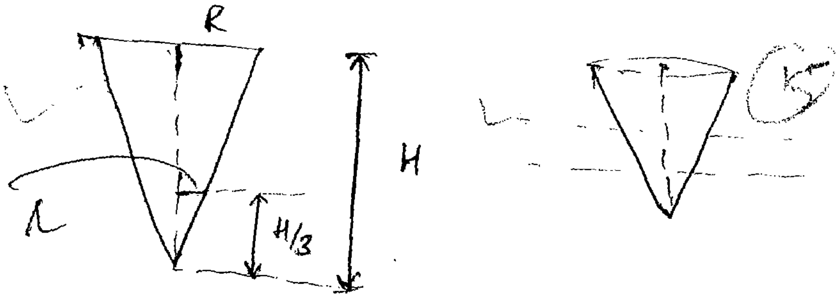
$$mg' = mg_{eff} = V\rho(g+a)$$

$$\therefore \quad T = B' - mg' = V(g+a)(d-\rho)$$

Q29 (D)

$$\frac{r}{R} = \frac{H/3}{H} = \frac{1}{3}$$

$$r = \frac{R}{3}$$



In equilibrium for condition 1.

$$mg = B. \qquad m = \text{mass of cone}$$

$$\frac{\pi}{3}R^2 H \sigma g = \frac{\pi}{3}\left(\frac{R}{3}\right)^2\left(\frac{H}{3}\right)(0.8)\, g$$

$$\therefore \quad \sigma = \frac{0.8}{3^3} = \frac{0.8}{27}$$ = Relative density of material of cone

In equilibrium for condition 2.

$$mg + m_\ell g = \cancel{B}\,\cancel{+}\, B'$$

m_ℓ = mass of liquid

B' = New ~~Extra~~ buoyant force acting on cone.

$$\therefore \quad m_\ell g = B' - B.$$

or. $$\frac{\pi}{3}\left(\frac{R}{3}\right)^2\left(\frac{H}{3}\right)\cancel{\sigma}\,\rho g = \frac{\pi}{3}\left(\frac{R}{2}\right)^2\left(\frac{H}{2}\right)(0.8)\, g$$

16 or

$$\rho = (0.8)\left[\left(\frac{3}{2}\right)^3 - (1)^3\right]$$

solving,

$$\rho = 1.9$$

Q30. (B) Buoyant force acting on stone by water

$$B = V\rho_w g \cancel{= \frac{0.5}{50}}$$

V = volume of stone $= \frac{\text{mass}}{\text{density}} = \frac{0.5}{5000}\ m^3$

$\therefore$

$$B = \frac{0.5}{500} \times 10^3 \times g = 1g. \text{ (vertically upward)}$$

~~The~~ ~~same~~ From Newton's third law stone will apply same amount of force on water but vertically downward.

$\therefore$ Reading of balance = wt. of water + Force acting on it

$= 1.5\ kg + 1\ kg$

$= 2.5\ kg.$

Q31 (B)

Initially, weight of ice + metal were (1)
balanced by buoyant force.

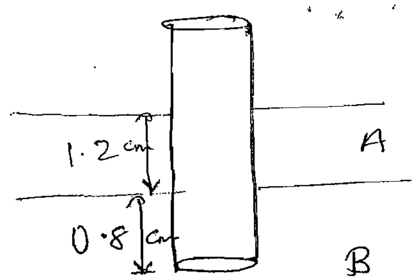
$$\therefore \quad (m_i + m_m) g = V_1 \rho_w g \qquad (1)$$

We know that when ice melts it forms volume of water equal to that displaced by it. So, due to melting of ice there is no change in volume ~~of~~ and hence level of liquid.

But ice cube was displacing more volum of water than it ~~was displacing~~ forms, to support the weight of metal cube.

Q32 (B) If H is total height of cylinder
18

In equilibrium of cylinder



$$B = mg$$

$$\pi R^2 (1.2)(0.7) g + \pi R^2 (0.8)(1.2) g$$

$$= \pi R^2 H (0.8) g.$$

$$\therefore \quad H = \frac{9}{4} = 2.25 \text{ cm.}$$

$\therefore$ length outside liquids = ~~2.25~~ $= 2.25 - 1.2 - 0.8 = 0.25$ cm.

Q33. (B)

In equilibrium of block

$$mg = B + Kx.$$

H = Height of block

ρ = density of of block.

$$AH\rho g = AH\frac{\rho}{3}g + K\frac{H}{3}$$

$$\therefore \quad \frac{2}{3} AH\rho g = K\frac{H}{3}$$

$$K = 2A\rho g$$

Q34 (A) velocity of efflux $= \sqrt{2gh}$

(19) Time taken (t) to hit ground:

$$S = ut + \frac{1}{2}at^2$$

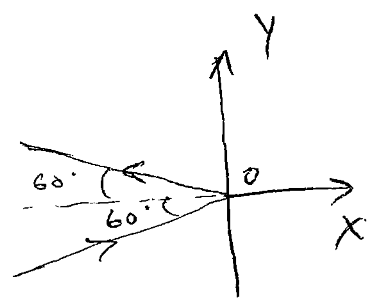
$$H-D = 0 \times t + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(H-D)}{g}}.$$

$$\therefore \text{Range} = x = \sqrt{2gD} \times t = \sqrt{2gD}\sqrt{\frac{2(H-D)}{g}}$$

$$x = 2\sqrt{D(H-D)}$$

Q35 (B)



$$\because F = \frac{\Delta P}{\Delta t}$$

if $\Delta t = 1$ sec.

$$F = \Delta P$$

$\therefore$ Force acting on wall = Momentum transferred to wall per second

$F = \Delta P$ = change in momentum ~~in~~ ~~1 sec~~ of water in one ~~sec~~ second.

Since collision is elastic, so change in

– tum is only along X – axis.

20. $\Delta V = V_f - V_i = -V\cos 60^\circ - V\cos 60^\circ$

$= -2V\cos 60^\circ = -V$

$\therefore \Delta P = m(-V) = -mV$

$m =$ mass of water striking wall in one second.

~~V = velocity of~~

$\Delta P =$ change in momentum in one second.

$\therefore m = \rho(AV) = \rho AV.$

$\therefore \Delta P = -\rho AV^2 = F =$ Force acting on ~~wall~~ water

$\therefore$ Force acting on wall $= \rho AV^2$ (along x-axis)

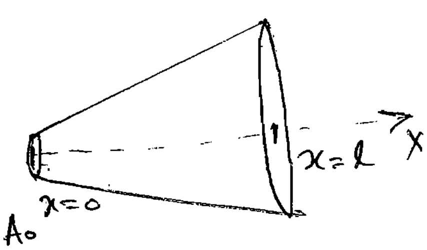
$= 10^3 \times 6 \times 10^{-4} \times (12)^2$

$= 86.4$ N

Q36. (A)

$A - A_0 = m(x - 0)$

$A = A_0 + mx.$ where $m =$ constant



From continuity, $A_0 V_0 = AV.$

$$V = \frac{A_0 V_0}{A_0 + mx}$$

~~Hend~~ Hence, P

From Bernoulli's equation, (21)

$$P_0 + \frac{1}{2}\rho V_0^2 = P + \frac{1}{2}\rho v^2$$

$$P = P_0 + \frac{1}{2}\rho(V_0^2 - V^2)$$

$$P = P_0 + \frac{\rho}{2}\left[V_0^2 - \left(\frac{A_0 V_0}{A_0 + mx}\right)^2\right]$$

$$P = P_0 + \frac{\rho V_0^2}{2}\left[1 - \frac{A_0^2}{(A_0 + mx)^2}\right]$$

$$P = P_0 + \frac{\rho V_0^2}{2}\left[\frac{(mx)^2 + 2A_0 mx}{(A_0 + mx)^2}\right]$$

which is equation of curve.

Also,

$$\frac{dP}{dx} = \frac{\rho V_0^2}{2}\left[\frac{(A_0+mx)^2(2m^2x + 2A_0 m) - 2(A_0+mx)m \times ((mx)^2 + 2A_0 mx)}{(A_0+mx)^4}\right]$$

at $x = 0$

$$\frac{dP}{dx} = \frac{\rho V_0^2}{2}\left[\frac{A_0^2(2A_0 m)}{A_0^4}\right] > 0.$$

$\therefore$ slope at $x = 0$ is positive

Hence, option (A).

37 (c) In equilibrium, ~~water~~

~~At~~ Rate of water flowing in = Rate of water flowing out.

22

$\therefore \quad 1\times10^{-4} \times 2 \ m^3/s = 0.5\times10^{-4} \times \sqrt{2gh}$

$\therefore \quad 4 = \sqrt{2\times10\times h} \Rightarrow \cancel{h = 0.8\ m.} \quad h_{eq} = 0.8\ m.$

From equation of continuity,

$\cancel{A_1V_1 + A_2 \frac{dh}{dt}}$

$$A_1V_1 - A_2V_2 = A_{tank}\frac{dh}{dt}$$

$$2\times10^{-4}\ m^3/s - 0.5\times10^{-4}\sqrt{2gh} = 4000\times10^{-4}\frac{dh}{dt}$$

$$2 - \frac{\sqrt{2gh}}{2} = 4000\frac{dh}{dt}$$

$$\cancel{\frac{dh}{4-\sqrt{2gh}} = \frac{1}{8000}dt}$$

$$\cancel{\frac{dh}{2-\sqrt{5h}} = \frac{1}{4000}dt}$$

$$\frac{dh}{dt} = \frac{2-\sqrt{2gh}/2}{400} \neq \text{constant}$$

So, h-t graph is non linear.

$\therefore$ (D)

$V_0 = \sqrt{2gh}.$

$V' = \sqrt{2gh'} \qquad \cancel{h' = \frac{h}{\sqrt{2}}} \quad h' = h\cos45°$

$h' = h$

Q39 (B) From continuity

$$A_1 V_1 = A_2 V_2$$

$$0.02 \times 2 = 0.01 \times V_2$$

$$V_2 = 4 \text{ m/s}.$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$4\times 10^4 + \frac{1}{2}\rho(V_1^2 - V_2^2) = P_2$$

$$P_2 = 4\times 10^4 + \frac{1}{2}\times 10^3 \times (2^2 - 4^2)$$

$$P_2 = 4\times 10^4 + \frac{1}{2}\times 10^3 \times (-12)$$

$$P_2 = 3.4\times 10^4 \text{ N/m}^2$$

Q40 (C) time taken to reduce the level of water from H to H'.

$$t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right) = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right]$$

$$t_1 = t_2$$

$$\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{\eta}}\right] = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}} - 0\right].$$

Solving, we get $\sqrt{H} = 2\sqrt{\frac{H}{\eta}}$

$$\therefore \eta = 4$$

Q41 (B) (24)

Bernoulli's theorem is based on Principle of conservation of Energy.

P = Pressure Energy per unit volume

$h\rho g$ = gravitational potential energy per unit volume.

$\frac{1}{2}\rho V^2$ = kinetic energy per unit volume.

~~Q42~~. From continuity equation

$$\cancel{\pi\left(\frac{2\times 10^{-2}}{2}\right)^2 \times 3 = 100\times\pi\left(\frac{0.05\times10^{-2}}{2}\right)^2\times V}$$

$$\cancel{V = \frac{3^2\times 2/2}{100\times(0.05)^2} = \frac{3^2\times 2}{(1/2)^2}}$$

$$\cancel{V =}$$

Q42 From equation of continuity

(D)

$$A_1V_1 = n A_2 V_2$$

$$\pi\frac{d_1^2}{4}\times V_1 = n\times \pi\frac{d_2^2}{4}\times V_2$$

$$\frac{\pi}{4}\left(2\times10^{-2}\right)^2\times 3 = 100\times\frac{\pi}{4}\left(0.05\times10^{-2}\right)^2\times V_2$$

$$V_2 = \frac{3\times}{100}\left(\frac{2}{0.05}\right)^2$$

Q43 (A).

From continuity equation

$$A_1 V_1 = A_2 V_2$$

velocity increases, if cross-section decreases.

From Bernoulli's equation

Pressure decreases as velocity increases.

Q44. (D)

time taken to empty the vessel is

$$t_0 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right] = \frac{A}{a}\sqrt{\frac{2H_0}{g}}$$

$H' = 0$.

~~$t' = \frac{A}{a}\sqrt{\frac{2H'}{g}}$~~

$$t' = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{4H} - \sqrt{0}\right]$$

$$t' = \frac{A}{a}\sqrt{\frac{2}{g}}\left(2\sqrt{H}\right) = 2\frac{A}{a}\sqrt{\frac{2H}{g}}$$

$$t' = 2t_0$$

~~Q45~~

Q45 (D) In steady state

water flowing in = water flowing out

$$100\ cm^3/s = \sqrt{2gh} \times A_2 = \sqrt{2gh} \times 1\ cm^2$$

Q46 (D) change in momentum of water

2E

per second $(\Delta P) = mv_2 - mv_1$

m = mass flowing per second

$$\vec{\Delta P} = m(\Delta \vec{V})$$

$m = \rho L$ $\qquad \vec{V_2} = v\hat{i}$ $\qquad \vec{V_1} = v\hat{j}$

$$\Delta\vec{V} = \vec{V_2} - \vec{V_1} = v\hat{i} - v\hat{j}$$

$$|\Delta\vec{V}| = v\sqrt{2}$$

$\therefore\ |\vec{\Delta P}| = \rho L v\sqrt{2}$ = change in momentum per second.

$\sqrt{2}\rho L v$ = Force acting on water

Q47. (C)

Force acting on tank $= -v_r \dfrac{dm}{dt}$

v_r = relative velocity of fluid w.r.t. tank.

$v_r = \sqrt{2gh} = v$

$\therefore$ $\cancel{F = \frac{dm}{dt} = \rho(NA)v\frac{dV}{dt}}$.

$$\frac{dm}{dt} = \rho(A)V$$

$$\therefore F = -\rho A V^2 = -\rho A(2gh)$$

$h = \frac{H}{2}$. $\therefore$ ~~$F = -\rho A(2gh)$~~

$$F = -\rho A\left(2g\frac{H}{2}\right) = Ma$$

M = Total mass of fluid in tank

$$= \rho(NAH)\,a = -\rho A\left(2g\frac{H}{2}\right).$$

$$\therefore a = \frac{g}{N}.$$

Q48 (A).

Volume flow rate in P = Volume flow rate in Q.

$$\therefore A_P V_P = A_Q V_Q$$

$$\pi\left(\frac{2\times10^{-2}}{2}\right)^2 V_P = \pi\left(\frac{4\times10^{-2}}{2}\right)^2 \times V_Q$$

$$V_P = 4V_Q$$

Q49. (B) In steady state

Volume flowing = volume flowing out.

(28) $10^{-4}\ m^3/s = 10^{-4}\ m^2 \times V$

$V = \sqrt{2gh}$ for small hole.

$\therefore \quad V = \sqrt{2gh} = 1\ m/s.$

$2 \times 9.8 \times h = 1 \Rightarrow h = 0.051\ m$

Q50 (C)

~~Time velocity of efflux~~ $\cancel{= \sqrt{2gx}}$

Velocity of efflux $= \sqrt{2g(3H-x)}$

time taken by fluid to reach ground

$$\cancel{= \sqrt{\frac{2(3H-x)}{g}}} = \sqrt{\frac{2x}{g}}$$

$\therefore$ ~~Range~~ $\cancel{= \sqrt{2gx} \times \sqrt{\frac{2(3H-x)}{g}}}$

$$R = 2\sqrt{x(3H-x)} = 2\sqrt{3Hx - x^2}$$

$$\frac{dR}{dx} = 2 \cdot \frac{1}{2\sqrt{3Hx-x^2}} \cdot (3H - 2x) = 0.$$

$$\therefore \quad x = \frac{3}{2}H = 1.5H.$$

51. (D)

$\therefore$ ... fluid ... $R = \sqrt{\ldots}$

$$R_1 = R_2$$

$$\cancel{\sqrt{2g h_1 \; 2h_2}}$$

$$\sqrt{2g(H-h_1)}\sqrt{\frac{2h_1}{g}} = \sqrt{2g(H-h_2)}\sqrt{\frac{2h_2}{g}}$$

$$\therefore \quad (H-h_1)(h_1) = (H-h_2)h_2$$

$$Hh_1 - Hh_2 = h_1^2 - h_2^2$$

$$H(h_1-h_2) = h_1^2 - h_2^2$$

$$H = h_1 + h_2$$

For maximum range, $h = \frac{H}{2} = \frac{h_1+h_2}{2}$

Q52. (D)

$$V = \sqrt{2gh} = \sqrt{2\times 10\times 10} = 10\sqrt{2} \text{ m/s}.$$

For Range to be twice, the velocity of efflux must be twice.

$$\therefore \quad V' = 2V = 20\sqrt{2} \text{ m/s}$$

From Bernoulli's equation,

$$P + h\rho g = \frac{1}{2}\rho V'^2$$

$$P + 10\times 10^3\times 10 = \frac{1}{2}\times 10^3\times 400\times 2$$

5 [illegible] $/m^2 = 3$ atm.

Q53. (B)
(30) Let d be depth of water in barrel
so, velocity of efflux is given by

$$V = \sqrt{2gd}$$

time taken to reach ground $= \sqrt{\frac{2h}{g}}$

$$\therefore R = V \times t = \sqrt{2gd} \times \sqrt{\frac{2h}{g}}$$

$$d = \frac{R^2}{4h}$$

Q54. (B). From Bernoulli's theorem

$$P_0 + \frac{mg}{A} + h\rho g = P_0 + \frac{1}{2}\rho V^2$$

$$\frac{10 \times 9.8}{10^{-1} m^2} + \frac{1}{2} \times 10^3 \times 9.8 = \frac{1}{2} \times 10^3 \times V^2$$

$$V^2 = 11.76 \quad \text{or} \quad V = 3.4 \text{ m/s}.$$

Q55. From continuity equation,
(B)

$$A_1 V_1 = A_2 V_2$$

$$1 \times V_1 = \frac{1}{2} \times V_2 \Rightarrow 2V_1 = V_2$$

From 3rd equation of motion

$$v^2 = u^2 + 2aS$$

$$\therefore \quad (2V_1)^2 = V_1^2 + 2gh$$

$$3V_1^2 = 2 \times 980 \times 10$$

$$V_1^2 = \frac{19600}{3} \Rightarrow V_1 = \sqrt{\frac{19600}{3}} \text{ cm/s.}$$

$$V_1 = \sqrt{\frac{196}{3}} \times 10 \text{ cm/s.} = \frac{140}{\sqrt{3}} \text{ cm/s.}$$

volume rate of flow $= A_1 V_1$

$$= 1 \text{ cm}^2 \times \frac{140}{\sqrt{3}} \text{ cm/s} = \frac{140}{\sqrt{3}} \text{ cm}^3/\text{s.}$$

$$= \frac{140}{\sqrt{3} \times 1000} \times 60 \frac{\text{lit}}{\text{min}} = 4.8496 \frac{\text{lit}}{\text{min}}$$

356. (A)
Force acting on pipe = momentum transfered by water to pipe per second.

$$F = \frac{\Delta P}{\Delta t}, \quad \Delta t = 1 \text{ sec.}$$

$$\therefore \quad F = \frac{\Delta P}{\Delta t} = \frac{m}{\Delta t}(\Delta V)$$

ΔV = change in velocity of water

$\frac{m}{\Delta t}$ = mass of water flowing per second.

$$\frac{m}{\Delta t} = (AV)\rho \qquad \Delta V = V\sqrt{2}$$

(37) $\therefore$ $F = (\rho A V)\, V\sqrt{2} = \sqrt{2}\, \rho A V^2$

$F = \sqrt{2} \times 10^3 \times 10 \times 10^{-4} \times (20)^2$

$F = 4\sqrt{2} \times 100 = 565.6$ N.

Q57 (B)

Force acting on plate = Momentum transfered to plate per second

= − change in momentum of ~~plate~~ water per second.

$= -\dfrac{\Delta P}{\Delta t}$.

Initial velocity of water $(u) = 10$ m/s.

Final velocity of water $(v) = 0$

$\therefore$ Momentum transfered $= \dfrac{m}{\Delta t} \cdot (\Delta V)$

ΔV = change in velocity $= v - u = 0 - 10$

$= -10$ m/s.

$\dfrac{m}{\Delta t}$ = mass of water striking per second.

$\therefore$ ~~Force~~ $= \rho A V$.

Force acting on plate $= (\rho A V)\, V = \rho A V^2$

$= 10^3 \times 2 \times 10^{-4} \times (10)^2 = 20$ N

Q58. (D) From Bernoulli's theorem

$$P_0 + \frac{h}{2}\rho g = P_0 + \frac{1}{2}\rho V_1^2$$

$$\therefore \quad V_1 = \sqrt{gh}.$$

$$P_0 + h\rho g + \frac{h}{2}(2\rho)g = P_0 + \frac{1}{2}(2\rho)V_2^2$$

$$V_2 = \sqrt{2gh}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{1}{\sqrt{2}}$$

Q59 (C) $A_1V_1 = A_2V_2$

$$\therefore \quad 10^{-2} \times 2 = 0.5 \times 10^{-2} \times V_2$$

$$V_2 = 4\ m/s.$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2.$$

$$8000 + \frac{1}{2} \times \rho(V_1^2 - V_2^2) = P_2$$

$$P_2 = 8000 + \frac{1}{2} \times 10^3 (2^2 - 4^2).$$

$$= 8000 + \frac{1}{2} \times 10^3 (-12)$$

$$P_2 = 2000\ Pa$$

$\frac{360}{34}$. (D)

same as Q 45.

Q61. (C)

V = volume rate of flow = $A_1 V_1$

$10^{-1}\ m^3/s = (10^{-2}\ m^2)\ V_1$

$V_1 = 10\ m/s.$

mass of water flowing per second = ρV

$(m) = \rho A_1 V_1 = 10^3 \times 10^{-1} = 100\ kg/sec.$

~~Kinetic~~ ~~ex~~ From Work - Energy theorem

$W_g + W_{ext} = \Delta K.$

$(-mgh) + W_{ext} = \frac{1}{2} \times m \times V_1^2$

$W_{ext} = mgh + \frac{1}{2} m V_1^2$

$= 100 \frac{kg}{sec} \times 10 \frac{m}{s^2} \times 10\ m +$

$\frac{1}{2} \times 100 \frac{kg}{sec} \times \left(10 \frac{m}{s}\right)^2$

$W_{ext} = \cancel{15}\ 15000\ \cancel{Watt}\ \frac{J}{sec}$

... agent

Q62. (A)

(5)

Divide the containers into two equal halves.

The first container will have greater than half of volume in upper half while the second one will have ~~least to~~ less than half of total volume of fluid.

For (i) more than half of volume will come out with greater speed as compared to the other two.

So, (i) will take the least time to empty.

36

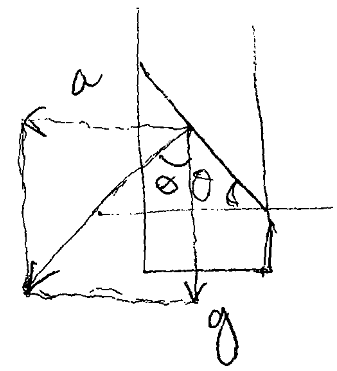
~~963~~ Q1. (A) (C)

Ex - II Multiple choice

$\tan\theta = \frac{a}{g} = \frac{1}{\cot\theta}$

$\theta = \tan^{-1}\left(\frac{a}{g}\right) = \cot^{-1}\left(\frac{g}{a}\right)$

backwards

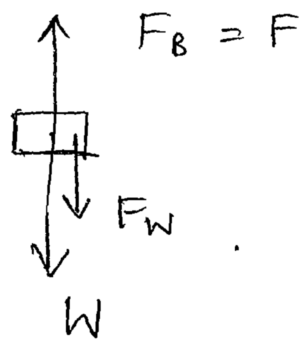


~~964~~ Q2. (D)

Pressure at bottom of vessel $= 2h\rho g$

Force exerted by fluid at bottom $= (2h\rho g) A$

FBD fluid



F_B = Force exerted by bottom of vessel on fluid = F

F_W = Force exerted ~~by~~ by wall of vessel on fluid

~~= F_W =~~

Force exerted by fluid on wall is due to pressure of fluid acting normal

Since fluid is in equilibrium, so

$$F_B = F_W + W$$

or $$W = F_B - F_W$$

$$W < F_B = F$$

$$F_W = F_B - W = F - W$$

Q65 (A) (C)

38

$W_1 = m_B g$ m_b = mass of ~~balloon~~ empty balloon.

$W_2 = m_B g - B + m_a g$ B = Buoyant force on filled balloon

Now, B = weight of air displaced

wt. of Air displaced = wt. of ~~air~~ air filled in balloon

= W (given)

~~Q53~~ (A) (C)

Q3 :

$W_1 = m_B g$ = wt. of empty balloon.

$W_2 = m_B g + m_a g - B$ = wt. of filled balloon

$W_2 = W_1 + W - B$.

m_a = mass of air inside balloon.

B = Buoyant Force = Weight of air displaced.

Air inside balloon is identical to outside air.

$\therefore$ $B = W$ = weight of ~~air~~ air inside balloon.

$\therefore$ $W_2 = W_1 + W - W$

$W_2 = W_1$ Hence (A)

$W_2 < W_1 + W$ (C)

~~Q66~~ Q4.

(C) (D) If x cm of block is below surface common to oil and water. In equilibrium,

$ma - B$

(40) $0.92\,g = B_o + B_w$

B_o = Buoyant force due to oil

B_w = Buoyant force due to water

$$B_o = (0.1-x)\,\rho_o g = (0.1-x) \times 0.6 \times 10^3\, g$$

$$B_w = x\,\rho_w g = x \times 1 \times 10^3\, g.$$

$$\therefore \quad 0.92g = (0.1-x) \times 0.6 \times 10^3 g + x \times 1 \times 10^3 g$$

Solving,

$$x = 0.08\text{ m} = 8\text{ cm}.$$

$\therefore$ 8 cm below oil-water interface.

Q5.

(B) (C)

Due to ~~bouy~~ buoyant force acting on mass m, its effective ~~wet~~ weight decreases.

$$W = mg - B.$$

Hence, A will read less than 2kg.

This buoyant force (B) was exerted by liquid

block

buoyant force is applied by ~~block~~ mass (m) ~~of~~ liquid.

Hence, net force on liquid acting downwar increases. so, reading of spring balance B increas

~~Q5~~ Q6. (A)(C)

Velocity of efflux $= V = \cancel{\sqrt{2gh}}\ \sqrt{2gy}$

~~For~~ If t is the time taken by liquid to reach the ~~bottom of ff~~ ground, then,

$$\cancel{2h = 0 \times t}$$

$$2h - y = 0 \times t + \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2(2h-y)}{g}}$$

$$\therefore \text{Range} = x = V \times t = \sqrt{2gy}\ \sqrt{\frac{2(2h-y)}{g}}$$

$$= 2\sqrt{y(2h-y)} = 2\sqrt{2hy - y^2}$$

$$x = \cancel{\$}\ 2\sqrt{h^2 - (h-y)^2}$$

x will be maximum when

(112) $\therefore$ Maximum Range $(x_m) = 2\sqrt{h(h)} = 2\sqrt{h^2}$

$= 2h$

Q69 (B) (C)

Initial velocity of water $= V$

Final velocity of water $= 0$

Change in velocity $(\Delta V) = 0 - V = -V$

change in momentum $= \text{mass} \times (\Delta V) = -mV$.

~~change in momentum (per second)~~

Force acting on water $= \dfrac{dP}{dt}$

$= \dfrac{d(mV)}{dt} = V\dfrac{dm}{dt}$

$\dfrac{dm}{dt}$ = mass of water striking per second.

44 $\therefore$ Maximum Range $(x_m) = 2\sqrt{h(h)} = 2h$

Q7 (B) (~~c~~)(d)

Initial velocity of water = V

Final velocity of water = 0

ΔV = change in velocity = $0 - V = -V$

ΔP = change in momentum = mass $\times \Delta V$

Change in momentum (per second)

= mass of striking per second $\times$ change in velocity

mass of water striking per second

= $\rho A V$ = mass of water in element having length = V and area of cross-section ~~of~~ of tube ~~e~~ (~~say~~ (say A)

$\therefore \Delta P$ (per second) = ~~ρA~~ $(\rho A V)(-V)$

$= -\rho A V^2$

... water = F = dP

If $dt = 1$ second

$$F = \frac{\text{change in momentum}}{1 \text{ sec}}$$

F = change in momentum per second

$\therefore$ $F = -\rho A V^2$

$\therefore$ From Newton's ~~that~~ third law

$$F_{wall} = -F = \rho A V^2$$

$\therefore$ If velocity ~~becoming~~ becomes ~~four~~ two times, force acting on ~~to~~ wall becomes four times.

Energy lost per second = ~~KE~~ kinetic energy lost by water in 1 second

$$= \frac{1}{2} \times (\rho A V) \times V^2 = \frac{1}{2} \rho A V^3$$

$\therefore$ Energy lost becomes eight times.

~~370~~ (B)
~~Q8~~. Velocity of efflux $= V = \sqrt{2gy}$

(116) $= \sqrt{\frac{2(H-y)}{g}}$

$\therefore$ Range $(x) = \sqrt{2gy} \times \sqrt{\frac{2(H-y)}{g}}$

$= 2\sqrt{y(H-y)} = 2\sqrt{\left(\frac{H}{2}\right)^2 - \left(y-\frac{H}{2}\right)^2}$

$\therefore$ ~~For~~ $x_{max} = H$ at $y = \frac{H}{2}$

If y is increased from 0 to H, then, x first increases and becomes maximum at $y = \frac{H}{2}$ and then decreases.

& x ~~depends~~ is independent of density of the liquid.

~~Ans~~ (D)

Q9. $A_P V_P = A_Q V_Q$ (Equation of continuity

$A_P > A_Q$

$\therefore V_P < V_Q$

From Bernoulli's theorem, greater velocity implies less pressure.

~~$P_P + \frac{1}{2}\rho V_P^2 = P_Q + \frac{1}{2}$~~

$$P_P + \frac{1}{2}\rho V_P^2 = P_Q + \frac{1}{2}\rho V_Q^2$$ (47)

$$V_P < V_Q$$

$$\therefore \quad P_P > P_Q$$

$$\text{K.E at } X = \frac{1}{2}\rho V_P^2$$

$$\text{K.E at } Y = \frac{1}{2}\rho V_Q^2$$

$$(\text{K.E.})_X < (\text{K.E.})_Y$$

Ex-3 - ~~comprehension~~

Passage — Fluid Mechanics

Q1. (C) Pressure at P is less than atmos-
-pheric pressure. So, mercury will not ~~flow~~ come out from hole.

Q2 (B) Pressure at P is atmospheric pressure. Pressure at P' is less than atmospheric pressure. ~~due~~ So, mercury may come out after some time.

Q3 (C) $P_0 = h \rho g$

$\rho_m > \rho_w$ so, $h_m < h_w$

Q4 (C) ~~B~~
If all of air is pulled out.

$$P_{inside} = 0 = h \rho_m g \Rightarrow h = 0$$

Q5 (B)

Total length $= 1$ m

Free length $= 100 - 76 = 24$ cm.

Q6 (A)

Buoyant force arises due to the pressure difference.

Q7 (B)

Mass flowing per second at every cross-section ~~is~~ constant, is basis for equation of continuity.

Q8 (A)

Buoyant force arises due to pressure difference.

When container is accelerated horizontally Pressure ~~varies~~ ~~or~~ varies horizontally.

$$\therefore \quad \text{Buoyant force} = V\rho g_{eff}.$$

$$\text{where, } g_{eff.} = \sqrt{g^2 + a_H^2}$$

a_H = horizontal acceleration.

~~Q10~~
Q9. 2

If y be height of water at any time above orifice. then,

velocity of efflux $= V = \sqrt{2gy}$

$R = V \times t = \sqrt{2gy} \times 2$

Speed of block $= -\frac{dR}{dt} = -\frac{d}{dt}\left(2\sqrt{2gy}\right)$

$= -2\sqrt{2g}\ \frac{1}{2\sqrt{y}}\ \frac{dy}{dt} = -\cancel{\sqrt{}}\sqrt{\frac{2g}{y}}\ \frac{dy}{dt}$

From continuity eqn:

$$\cancel{A\,dx}\quad -A\frac{dy}{dt} = a\sqrt{2gy}$$

$\Rightarrow \quad \frac{dy}{dt} = \frac{a}{A}\sqrt{2gy}$

So, speed of block

$= -\sqrt{\frac{2g}{y}} \times \frac{a}{A}\sqrt{2gy} = -\frac{a}{A}(2g)$

$= -20\ \frac{a}{A}\ m/s = -0.02\ m/s = -2\ cm/s.$

Q10 $\quad K = 9$

Net Force acting on ball at any instant can be given as

$F = 6\pi\eta R V_t - 6\pi\eta R V$

or $$a = \frac{6\pi\eta R(V_t - V)}{\cancel{m\rho/2}\ V(\rho/2)}$$

$$a = V\frac{dV}{ds} = \frac{9\eta}{R^2\rho}(V_t - V)$$

$$\frac{V\,dV}{V_t - V} = \frac{9\eta}{R^2\rho}\,ds.$$

$$\int_{V=0}^{V=V_t/2} \frac{V}{V_t - V}\,dV = \frac{9\eta}{R^2\rho}\int_0^s ds.$$

Solving,

$$\cancel{S = \rho R^2}$$

$$\boxed{S = \frac{\rho R^2 V_t}{9\eta}\left[\ln 2 - \frac{1}{2}\right]}$$

$\therefore \quad k = 9$

Fluid Mechanics – Ex – IV – Solutions

Q1 Let m_1, m_2 be masses of liquids

$\therefore$ Volume of liquids V_1, V_2 is given by

$$V_1 = \frac{m_1}{\rho_1}, \quad V_2 = \frac{m_2}{\rho_2}$$

Total volume of mixture $= V_1 + V_2 = V$ (given)

$$\therefore \quad \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} = V \quad \text{——— (1)}$$

Total mass of mixture $= m_1 + m_2$

$\therefore$ Density of mixture $= \dfrac{m_1 + m_2}{V} = \sigma$ (given)

$$\therefore \quad m_1 + m_2 = \sigma V \quad \text{——— (2)}$$

Solving equations (1) & (2)

$$m_1 = \frac{(\sigma - \rho_2) V}{(1 - \rho_2/\rho_1)}, \quad m_2 = \frac{(\sigma - \rho_1) V}{(1 - \rho_1/\rho_2)}$$

JP Page 1–15.

92. Let ρ_1, ρ_2 be specific gravity of two metals respectively. 2.

$\therefore$ When equal volumes (say V) of two metals are mixed together

Total volume of mixture $= V + V = 2V$

Total mass of mixture $= \rho_1 V + \rho_2 V = (\rho_1 + \rho_2) V$

$\therefore$ Density of mixture $= \dfrac{(\rho_1 + \rho_2) V}{2V} = \dfrac{\rho_1 + \rho_2}{2} =$

$\cancel{\dfrac{\rho_1 + \rho_2}{2}} = 4$ (given)

$\therefore \quad \dfrac{\rho_1 + \rho_2}{2} = 4 \Rightarrow \rho_1 + \rho_2 = 8$ ——— (1)

When equal masses (say m) of two metals mixed together.

Total mass of mixture $= m + m = 2m$

Total volume of mixture $= \dfrac{m}{\rho_1} + \dfrac{m}{\rho_2}$

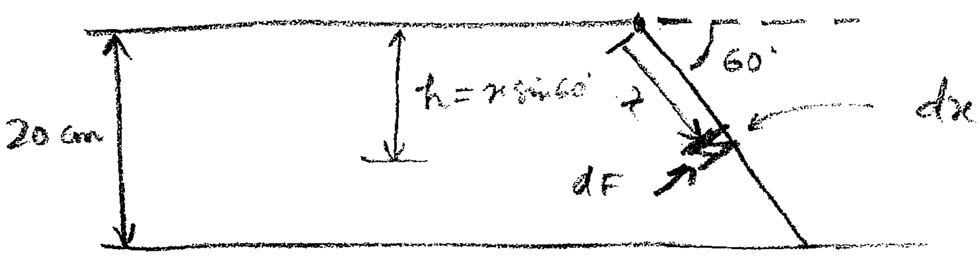
$\therefore$ Density of mixture $= \dfrac{2m}{\frac{m}{\rho} + \frac{m}{\rho}} = \dfrac{2}{\frac{1}{\rho_1} + \frac{1}{\rho_2}}$

$$\frac{2\rho_1\rho_2}{\rho_1+\rho_2} = 3 \quad \text{———} \quad (2).$$

solving (1) & (2).

$\rho_1 = 2$ or 6 $\qquad$ $\rho_2 = 6$ or 2

Q3.



Consider a small element of thickness dx at distance x measured along the wall from free surface. The pressure at this element is

$$P = h\rho g = (x \sin 60°)\rho g = x\rho g \sin 60°$$

Force on this element is given by

$$dF = P(dA) = (x\rho g \sin 60°)(b\,dx)$$

where b = width of flap. $= 0.4$ m.

$$\therefore \text{Total force on flap} = \int_{x=0}^{x=H/\sin 60°} dF$$

$$= \int^{x=H/\sin 60°} \rho g x \sin 60° \; b\,dx$$

4.

$$= \rho g b \sin 60^\circ \left[\frac{x^2}{2} \right]_0^{H/\sin 60^\circ} = \rho g b \sin 60^\circ \cdot \frac{H^2}{2 \sin^2 60^\circ}$$

$$= \rho g b \frac{H^2}{2 \sin 60^\circ} = \frac{10^3 \times 9.8 \times (0.4)(0.2)^2}{2(\sqrt{3}/2)}$$

$F = 90.5312$ N

24.

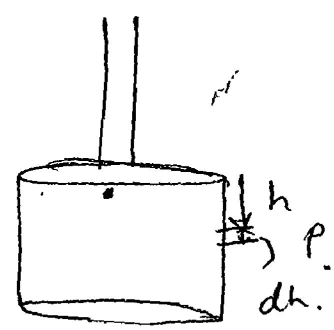
(a) Pressure at bottom = length of water column above it

$$= h\rho g = 0.3 \times 10^3 \times 10 = 3000 \text{ N/m}^2$$

thrust $= h\rho g A = 3000 \times \frac{22}{7} \times (0.1)^2$

$= 94.29$ N

(b) Pressure at point P



$= (0.2 + h)\rho_w g$

$= (0.2 + h) \times 10^3 \times 10$

$= (0.2 + h) \times 10^4$ N/m²

∴ Force ~~on~~ at Point P due to hydro-static pressure $= P \times \cancel{dV}\, dA$

"Total force acting on vertical sides

$$= \int_0^{0.1} dF$$

$$= \int_0^{0.1} (0.2 + h) \times 10^4 \times 2 \times \frac{22}{7} \times (0.1)\, dh$$

$$= \frac{44}{7} \times 10^3 \left[0.2h + \frac{h^2}{2} \right]_0^{0.1}$$

$$= \frac{44}{7} \times 10^3 \left[(0.2)(0.1) + \frac{(0.1)^2}{2} \right]$$

$$= \frac{1100}{7}\ N = 157.14\ N.$$

(c). Pressure at top face

$$= h \rho_w g = 0.2 \times 10^3 \times 10 = 2000\ N/m^2$$

$\therefore$ Force at top face

$$= 2000 \times \frac{22}{7} \left((0.1)^2 - \right.$$

$$= \frac{44000}{7} \left((0.1)^2 - (0.02)^2 \right)$$

$$= 60.34\ N.$$

Solutions

Q1. Let m_1, m_2 be masses of two liquids

$\therefore$ $m_1 = \rho_1 V_1$ $\quad m_2 = \rho_2 V_2$

Total mass $= m_1 + m_2 = (\rho_1 + \rho_2) V$

of mixture

35.

Pressure at Base due to water column $= h \rho_w g = 0.5 \times 10^3 \times 10$

$P_{BW} = 5 \times 10^3 \ N/m^2$

Force at Bottom $= P_{BW} \times$ Area of cross section $\left(\frac{\pi d^2}{4}\right)$

$= 5 \times 10^3 \times \frac{22}{7} \times \frac{(0.1)^2}{4}$

$= 39.286$ N. (downwards)

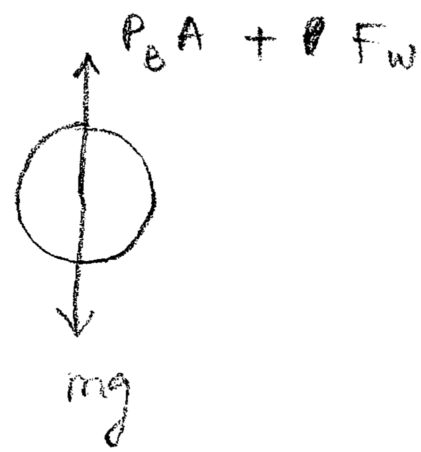
Weight of water $= V \rho g =$ ~~(10 lit) (1kg/lit)(10m~~

~~= (10 lit) (1kg/lit)(g) =~~ 10

$= (10 \text{ lit}) \ (1 \text{ kg/lit}) \ (10 \text{ m/s}^2)$

$= 100$ N.

FBD water



mg = weight of water

$P_B A$ = Pressure force extented by bottom face on water

From Newton's third law

Force by water at bottom of pot

= Force by bottom of pot on water

$\therefore \quad P_B A = P_w A = 39.286 \text{ N}$ upwards

Since, water is in equilibrium. So net force on water must be zero.

This will be possible only when ~~resultant~~ net resultant force exerted by walls on water ~~but~~ acts vertically upward.

$$\therefore \quad P_B A + F_w = mg$$

$$F_w = 100 \text{ N} - 39.286 \text{ N}$$

$$F_w = 60.714 \text{ N} \text{ (vertically upwards)}.$$

From Newton's third law

Force by walls on ~~walls~~ water

= Force by water on walls

26/ Let the ~~densities~~ densities of three liquids be $a+d,\ a,\ a-d$

~~This will be~~
In given situation equilibrium is possible only when liquid c is lightest liquid.

$\therefore\ \rho_c = a-d$

$\rho_b = a \qquad \rho_a = a+d.$

Let x be fraction of liquid A in right ~~arm~~ arm.

If l be length of each arm then, Pressure due to both arms at the bottom will be same.

$$1 \times a \times g = x(a-d)g + (1-x)(a+d)g$$

$$\cancel{\rho_c l g = \rho_a (xl) g + \rho_b (1-x) l g}$$

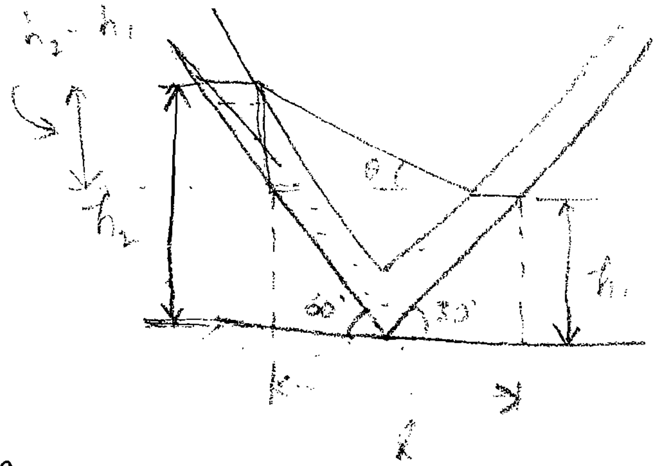
$$\cancel{(a-d) l g = (a+d) x l g + a(1-x) l g}$$

Solving, we get

$x = \frac{1}{2}$ or 0.5

Q7.

$$\tan\theta = \frac{h_2 - h_1}{l}$$



~~t=~~

$$\tan 60^\circ = \frac{h_2}{l_2}$$

$$l_2 = \frac{h_2}{\tan 60^\circ} = \frac{h_2}{\sqrt{3}}$$

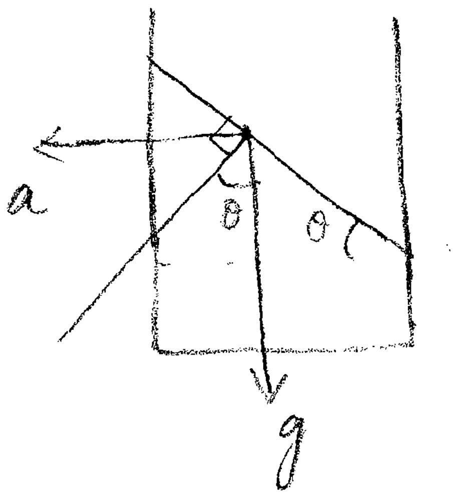
$$\tan 30^\circ = \frac{h_1}{l_1} \Rightarrow l_1 = \frac{h_1}{\tan 30^\circ} = \sqrt{3}\,h_1$$

$$l = l_1 + l_2 = \sqrt{3}\,h_1 + \frac{h_2}{\sqrt{3}}$$

$$\therefore \tan\theta = \frac{h_2 - h_1}{\sqrt{3}\,h_1 + h_2/\sqrt{3}} = \frac{\sqrt{3}\,(h_2 - h_1)}{(3h_1 + h_2)} \quad \text{——— (1)}$$

If tube were continuous container, then

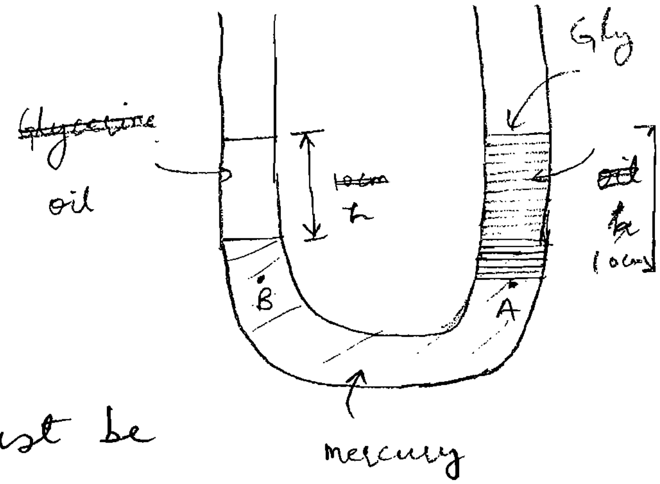
$$\tan\theta = \frac{a}{g} \quad \text{——— (2)}$$



from eqn (1) & (2).

$$a = \frac{\sqrt{3}\,(h_2 - h_1)}{(3h_1 + h_2)}\,g$$

~~08~~. since oil has lower density than glycerine, so, in this case to balance pressure in same horizontal level height of oil column must be less than 10 cm.



Taking two points A & B at same horizontal level.

$$P_A = P_B$$

$$P_0 + \frac{h}{100}\rho_0 g + \frac{(10-h)}{100}\rho_m g = P_0 + \frac{10}{100}\rho_{gly} g$$

$$\therefore \quad h\rho_0 + (10-h)\rho_m = \frac{10}{\cancel{100}}\rho_{gly}$$

$$h(\overset{0.8}{\cancel{1.3}}) + (10-h)(13.6) = \frac{10}{\cancel{100}}(1.3)$$

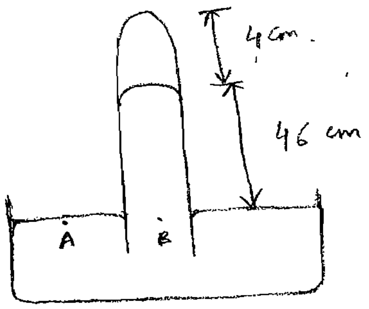
$$0.8\cancel{1.3}h + 136 - 13.6h = 13$$

$$123 = \cancel{12.3h}.\ 12.8h$$

$$h = \frac{123}{12.8}\text{ cm} = 9.6094$$

$$h = 9.6094 \text{ cm}$$

Q9 Let P_1 be pressure of air trapped in tube (in cm Hg).



$\therefore$ $P_{atm} = P_A = P_B = P_1 + 46$

$P_{atm} = 76$ cm Hg.

$\therefore$ $76 = P_1 + 46$ or $P_1 = 30$ cm Hg

Now, when 20 cm of more tube is released then, suppose height of mercury be h.

$\therefore$ length of air column trapped

$= 50 + 20 - h = 70 - h.$

$P_{atm} = P_2 + h$ (2)

P_2 = pressure of air trapped

or $76 = P_2 + h$

$P_2 = (76 - h)$ cm Hg.

Let (A) be cross-section of tube then, ~~Len~~ if temperature of air is constant.

$$P_i V_i = P_f V_f$$

$$30 \times (A \times 4) = (76-h)\,(A \times (70-h))$$

or

$$120 = (76-h)(70-h)$$

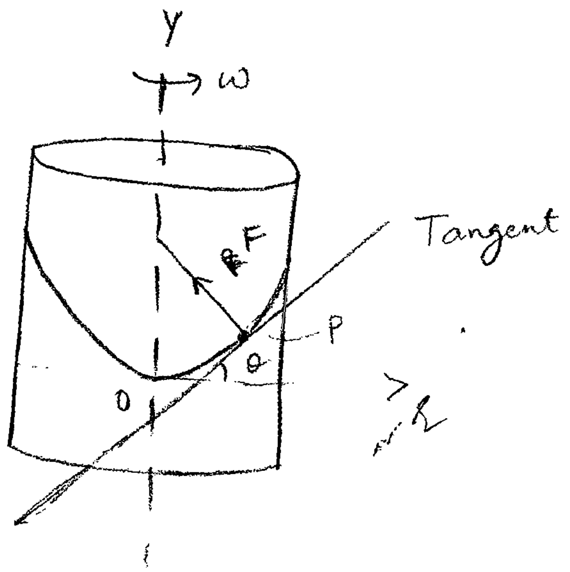
$$h^2 - 146h + 5200 = 0$$

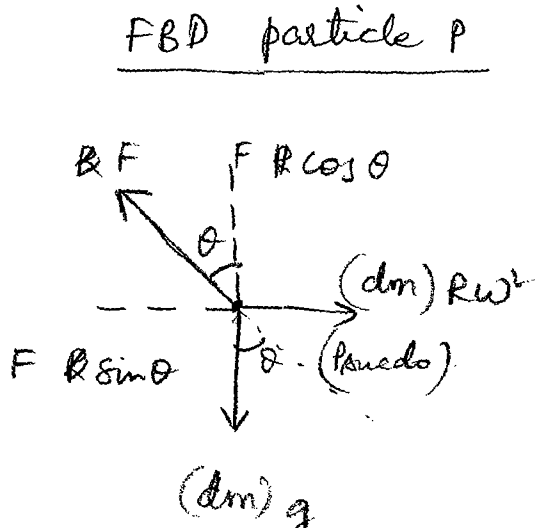
$$h = 61.642 \text{ or } 84.36$$

$\because$ $h < 70$ cm.

$\therefore$ $h = 61.642$ cm

<u>Q10.</u>





Take point P on the surface of rotating liquid.

If F is a force exerted by neighbouring particles on P then,

$$F \sin\theta = (dm)\cancel{R}\cancel{\omega^2}\, r\omega^2$$

$$\tan\theta = \frac{r\omega^2}{g} \quad \text{———} \quad (1)$$

Also, $\tan\theta$ = slope of tangent at P

$$\tan\theta = \left(\frac{dy}{dr}\right)_{at\ P} \quad \text{———} \quad (2)$$

from (1) & (2).

$$\frac{dy}{dr} = \frac{r\omega^2}{g}$$

$$\int_{y_{min}}^{y} dy = \int_{r=0}^{r} \frac{r\omega^2}{g}\, dr$$

$$\left[y \right]_{y_{min}}^{y} = \frac{\omega^2}{g}\left[\frac{r^2}{2}\right]_0^r$$

$$y - y_{min} = \frac{\omega^2}{2g}\left[r^2 - 0^2 \right]$$

$$y = y_{min} + \frac{\omega^2 r^2}{2g}$$

<u>Given</u>

$d = 10$ cm $= 2R$, $r = 5$ cm.

$y_{max} - y_{min} = ?$ $\qquad \omega = 120$ rpm $= 120 \times \frac{2\pi}{60}$

$\omega = 4\pi$ rad/sec

At $r = R$ $\quad y = y_{max}$.

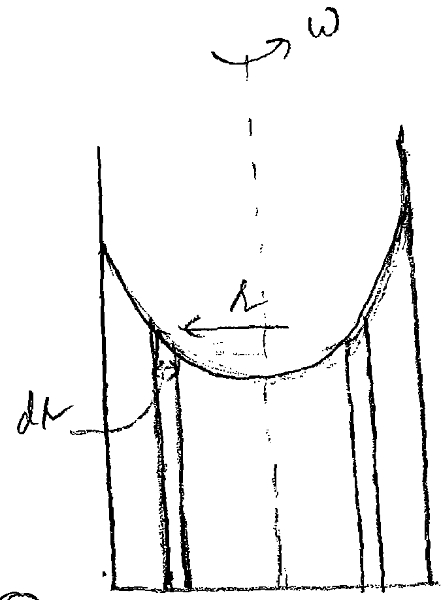
$$\therefore \quad y_{max} - y_{min} = \frac{(R\omega)^2}{2g}$$

$$= \frac{(5 \times 4\pi)^2}{2 \times 980} \text{ cm} = 2.012 \text{ cm.}$$

Q14. Given $\quad y_{min} = 25$ cm.

$R = 10$ cm

Equation for surface of water



$$y = y_{min} + \frac{(r\omega)^2}{2g}$$

$$y = 25 + \frac{(r\omega)^2}{2g} \quad \text{——— (1)}$$

consider a hollow cylinderical water surface of radius (x) thickness (dx) and height (y).

Volume of water in surface is given by (dV)

$$dV = (2\pi x)(dx)(y)$$

$$V = \int_{r=0}^{R} 2\pi r \left(25 + \frac{(r\omega)^2}{2g}\right) dr$$

$$= 2\pi \left[\frac{25r^2}{2} + \frac{\omega^2}{2g} \cdot \frac{r^{4}}{4} \right]_0^R$$

$$V = 2\pi \left[\frac{25R^2}{2} + \frac{\omega^2 R^3}{8\cancel{4}g} \right]$$

Volume of water initially present

$$= \pi R^2 h = \pi R^2 (50)$$

$$50\pi R^2 = 2\pi \left[\frac{25}{2} R^2 + \frac{\omega^2 R^4}{8\cancel{4}g} \right]$$

$$25 = \frac{25}{2} + \frac{\omega^2 R^2}{8\cancel{4}g}$$

$$\omega^2 = \frac{25 \times 8g}{2R^2} = \frac{100\,g}{R^2} = 980$$

$$\omega = 31.32 \text{ rad/sec}$$

substituting $\omega^2 = 980$ $R = 10$ cm. $g = 980$ cm/s^2

$$y = y_{max} = 75 \text{ cm}$$

Equation of water surface

$y = 25 + (0.5)\cancel{t}x^2$ in cgs units

$y = 0.25 + 50\cancel{t}x^2$ in S.I. units

Q12 (V) Volume of cube $= (10)^3 = 1000\ cm^3$.

Let ~~(t)~~ V' be volume of mercury displaced by cube.

$\therefore$ ~~to~~ In equilibrium

$$V' \rho_m g = V \rho_{iron} g$$

$$V' = \frac{\rho_{iron}}{\rho_m} \times V = \frac{7.8}{13.6} \times 1000$$

$V' = 573.529\ cm^3$ = volume of mercury displaced.

Q13. Since, block is in equilibrium. So, net force on block is zero.

OR Buoyant Force = Weight of block.

$$(10)^2 \cancel{\ell}(2) \rho_w g + (10)^2 (8) \rho_o g = \cancel{(10)^2(10)}\rho\ mg$$

$\therefore \quad m = (10)^2 (2) \rho_w + (10)^2 (8) \rho_o$

$= (200) \times (0.8) + (800) \times (1)$

$m = 160 + 800 = 960 \text{ gm}$

$\cancel{M = 960 \text{ gm or } 0.96 \text{ kg}}$

$m = 960 \text{ gm} \quad \text{or} \quad 0.96 \text{ kg}.$

(b) Gauge pressure = Pressure excess to atmospheric pressure

or Excess pressure relative to atmospheric pressure

$\cancel{P_{gauge} = P + P_{atmosph}}$

$P_{gauge} = P_{absolute} - P_{atm}$

$P_{gauge} = (10) \rho_o g + (2) \rho_w g$

$= (10)(0.8)(980) + (2)(1)(980)$

$= 9800 \text{ dyne/cm}^2$

214 (a) ~~ρ~~ Let ρ_s = density of solid

ρ_o = density of oil

$\therefore \; mg = V\rho_s g = 60$ ——— (1).

where, ~~where~~ V = volume of solid

~~∀~~ $mg - B_w = 40$

$V\rho_s g - V\rho_w g = 40$ ——— (2).

from ~~⊕~~ (1) & (2).

$V\rho_w g = 20$

$\therefore \; \frac{V\rho_s g}{V\rho_w g} = \cancel{\frac{2060}{60} = \frac{1}{3}} \; \frac{60}{20} = 3$

$\cancel{\rho_s = \frac{1}{3}\rho_w}$

$\rho_s = 3\rho_w = 3 \times 10^3 \, kg/m^3$

When immersed in oil.

$mg - B_{oil} = 45$

$V\rho_s g - V\rho_o g = 45$ ——— (3).

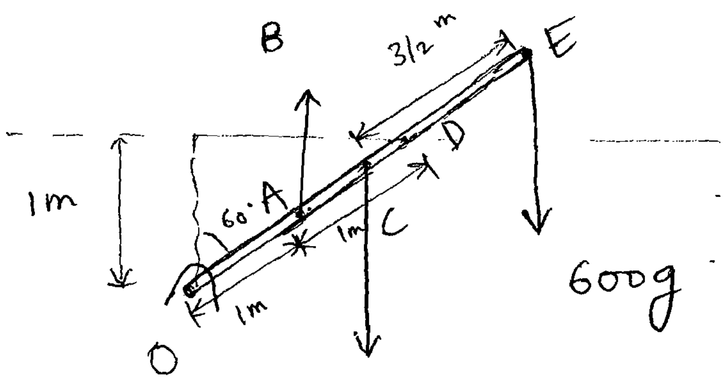
from (1) & (3).

$$\frac{V\rho_0 g}{V\rho_w g} = \frac{15}{20} \quad \text{or} \quad \rho_0 = \frac{3}{4}\rho_w.$$

$$\rho_0 = 0.75 \times 10^3 \text{ kg/m}^3 = 750 \text{ kg/m}^3$$

Q15.

For Rotational equilibrium of Rod.



$$mg\,\frac{300}{2}\sin 60° + 600g\,(300)\sin 60° - B\,(100)\sin 60° = 0$$

$m = (300\,A)(0.4)$ gm.

$$\text{or.} \quad (300\,A)(0.4)\,g\left(\frac{300}{2}\right) + (600g)(300) = (200\,A)(1)\,g\,(100)$$

$$(3)(0.2)(A)(3) + 18 = 2A.$$

$$18 = 2A - 1.8A = 0.2A$$

$$A = \frac{18}{0.2} = 90 \text{ cm}^2$$

Q18.

Upthrust B acting on stick

$= V \rho_w g$

$= A(200)(1)\ 980$

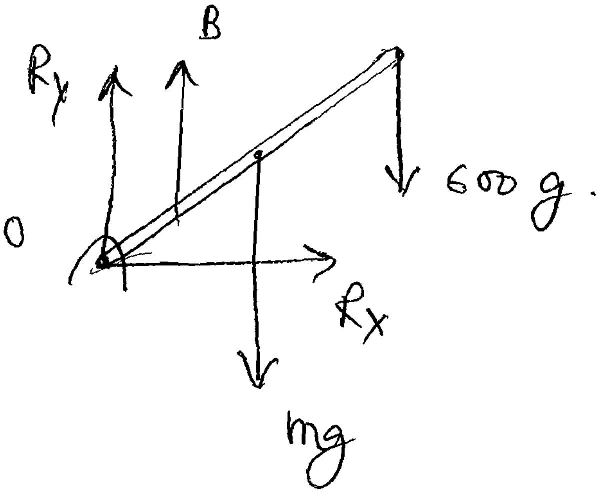
$= 90 \times 200 \times 1 \times 980$ dynes

$= 1.764 \times 10^7$ dynes.

$= 176.4$ N

Reaction at hinge

In vertical direction



$R_y + B - mg - 600g = 0$

$R_y = 600g + mg - B$

$= 600g + 300 \times 0.4\, gA - 1.764 \times 10^7$

$= 5.88 \times 10^5 + 1.0584 \times 10^7 - 1.764 \times 10^7$

$= -6.468 \times 10^6$ dynes.

$= -64.68$ N

$R_x = 0$

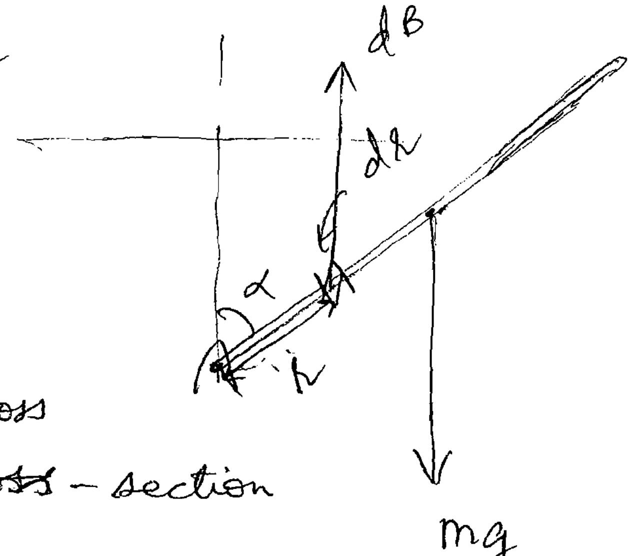
Q17.

Let (σ) be density of material of rod.

$\therefore$ mass of rod

$= \sigma (A l)$

where A = Area of ~~cross~~ cross-section



$\therefore$ Torque due to weight $= mg \frac{l}{2} \cos\alpha$.

$= (\sigma A g l^2/2) \cos\alpha$.

~~Torq~~ Now, ~~the~~ buoyant force will be variable. and hence the torque

Consider a small element ~~at~~ ~~at~~ of length (dr) at distance (r) from hinge along the length of rod.

Small torque on this element due to buoyant force about point O ~~will~~ is given by

$$d\tau = (dB)\, r \cos\alpha.$$

$$dB = \rho_0 (k - r\cos\alpha)\, A\, g\, dr$$

∴ Net torque about point O is given by

$$= \int d\tau = \int_{r=0}^{r=H/\cos\alpha} \rho_0 A \, (k - r\cos\alpha)\, r\cos\alpha \, g \, dr$$

For rotational equilibrium of rod.

$$\sigma A g \frac{l^2}{2} \cos\alpha = \rho_0 A g \cos\alpha \int_0^{H/\cos\alpha} (k - r\cos\alpha)\, r \, dr$$

$$\sigma = \frac{2\rho_0}{l^2} \left[\frac{k r^2}{2} - \frac{r^3}{3}\cos\alpha \right]_0^{H/\cos\alpha}$$

$$= \frac{2\rho_0}{l^2} \left[\frac{k H^2}{2\cos^2\alpha} - \frac{H^3}{3\cos^2\alpha} \right]$$

$$\sigma = \frac{2\rho_0 H^2}{l^2 \cos^2\alpha} \left[\frac{k}{2} - \frac{H}{3} \right]$$

$$\sigma = \frac{\rho_0 H^2 (3k - 2H)}{3 l^2 \cos^2\alpha}$$

Q18. Rate of flow of liquid $= 7200 \frac{\text{lit}}{\text{min}}$

$$V = 7200 \times \frac{10^{-3} m^3}{60 \text{ sec.}} = 0.12 \, m^3/\text{sec.}$$

Velocity at $d_1 = 25 \text{ cm} = 0.25 m = \frac{1}{4} m.$

$$V = A_1 V_1 = \frac{\pi d_1^2}{4} V_1$$

$$\therefore \quad 0.12 = \frac{22}{7} \times \frac{(1/4)^2}{4} \times V_1$$

$$V_1 = 2.44 \text{ m/s.}$$

From Bernoulli's theorem

$$P_1 + \frac{1}{2} \rho_w V_1^2 = \frac{1}{2} \rho_w V_2^2$$

$$6 \rho_w g + \frac{1}{2} \rho_w V_1^2 = \frac{1}{2} \rho_w V_2^2$$

$$12 g + V_1^2 = V_2^2$$

$$V_2 = 11.13 \text{ m/s}$$

$$V = A_2 V_2 = \frac{\pi d_2^2}{4} \times V_2$$

$$d_2^2 = \frac{4 \times 0.12}{22 \times 11.13}$$

~~$d_2 = 11.7$~~

$d_2 = 0.1172\ m = 11.72\ cm$

Q19. Let P_1, P_2 be pressures at entrance and throat.

$\therefore \quad P_1 - P_2 = h \rho_m g = (0.12\ m)(13.6\times10^3)(9.8)$

$= 1.6\times10^4\ N/m^2$

From ~~Berno~~ Bernoulli's theorem

$P_1 - P_2 = \frac{1}{2}\rho_w (V_2^2 - V_1^2)$

V_1, V_2 are velocities at entrance and throat.

$\therefore \quad 1.6\times10^4\ N/m^2 = \frac{1}{2}\times 0.9\times10^3\,(V_2^2 - V_1^2)$

$V_2^2 - V_1^2 = \frac{320}{9} \quad \text{——— (1)}$

From ~~A~~ equation of continuity

$A_1 V_1 = A_2 V_2$

$$\therefore \quad \frac{\pi d_1^2}{4} \cdot V_1 = \frac{\pi d_2^2}{4} V_2$$

$$V_1 = \left(\frac{d_2}{d_1}\right)^2 V_2 = \left(\frac{3}{9.5}\right)^2 V_2$$

$$V_1 = \left(\frac{6}{19}\right)^2 V_2 \qquad \text{——————} \quad (2)$$

$\therefore$ from (1) & (2)

$$V_2^2 = 35.91 \quad \text{or} \quad V_2 = 5.99 \text{ m/s.}$$

$\therefore$ Volume rate of flow $= A_2 V_2$

$$= \frac{\pi d_2^2}{4} V_2$$

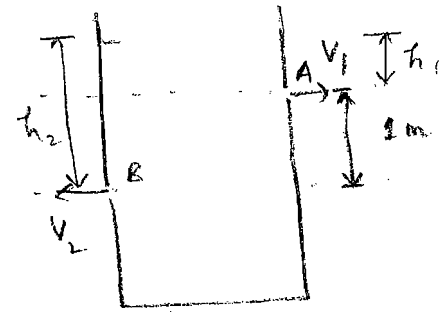
$$= \frac{22}{7} \times \frac{(3 \times 10^{-2})^2}{4} \times V_2$$

$$= 42.36 \times 10^{-4} \text{ m}^3/\text{sec.}$$

Volume rate of flow in lit/sec

$= 4.236$ lit/sec.

Q20 V_1 = velocity of efflux at hole A



$V_1 = \sqrt{2gh_1}$

$V_2 = \sqrt{2gh_2}$ = velocity of efflux at B.

If a system of mass (m) is releasing mass at the rate $\left(\frac{dm}{dt}\right)$ ~~then~~ with velocity (V_r) relative to it then, force acting on system is given by

$$\vec{F} = -\vec{V}\frac{dm}{dt}.$$

(From ~~reduced~~ variable - mass concept).

If A is cross-section of hole ~~then~~, and (ρ) be density of fluid then, ~~dm~~ rate of reducing mass is given by

$$\therefore \frac{dm}{dt} = \rho A V$$

$$\therefore F = \rho A V^2$$

$$\therefore F_1 = \rho A V_1^2 \qquad F_2 = \rho A V_2^2$$

$$F_{net} = |F_1 - F_2| = \rho A (V_2^2 - V_1^2) = \rho A (2gh_2 - 2gh_1$$

$$F_{net} = \rho A(2g)(h_2 - h_1)$$

$$= 10^3 \times 4 \times 10^{-4} \times 2 \times 10 \times 1$$

$$F_{net} = 8\ N$$

Only One Option Correct

1. (D)

2. (B)

Pressure will be same at the same horizontal level.

Therefore, $P_A = P_B$

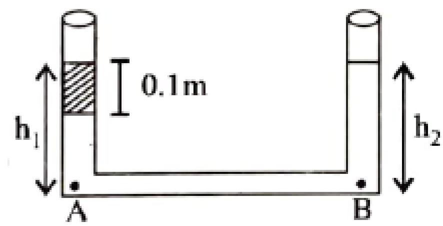
$\Rightarrow P_0 + \rho_k g \times 0.1 + \rho_w g \times (h_1 - 0.1) = \rho_w g h_2 + \rho_0$

$\Rightarrow 80 + 1000(h_1 - 0.1) = 1000 h_2$

$\Rightarrow 80 + 1000h_1 - 100 = 1000 h_2$

$\Rightarrow h_1 - h_2 = \dfrac{20}{1000}$

$\Rightarrow h_1 - h_2 = 0.02 \qquad ...(i)$



Also, $h_1 - 0.1 + h_2 = 2 \times 0.29$

$\Rightarrow h_1 + h_2 = 0.68 \qquad ...(ii)$

Solving (i) and (ii), we get

$2h_1 = 0.70$

$\Rightarrow h_1 = 0.35\text{ m}$ and $h_2 = 0.33\text{ m}$

So, $\dfrac{h_1}{h_2} = \dfrac{35}{33}$

One or More than One Option Correct

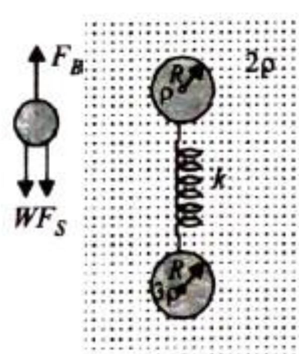
1. (ABD)

2. (A, D)

The complete system is as shown in figure.

Let x be the net elongation of the spring.

At equilibrium, for upper sphere



$W + F_S = F_B$

$\dfrac{4}{3}\pi R^3 \rho g + kx = \dfrac{4}{3}\pi R^3 (2\rho) g$

$\Rightarrow kx = \dfrac{4}{3}\pi R^3 2\rho g - \dfrac{4}{3}\pi R^3 \rho g$

$\Rightarrow kx = \dfrac{4\pi R^3 \rho g}{3}$ or $x = \dfrac{4\pi R^3 \rho g}{3k}$

3. (B, C)

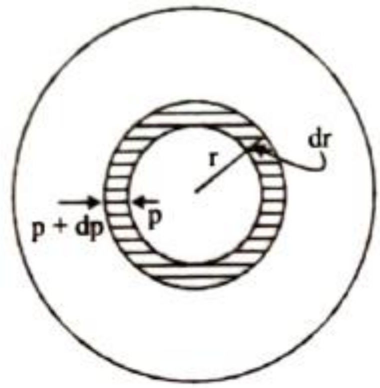
Let us consider an elemental mass dm shown in the shaded portion of the figure.

Here, $P4\pi r^2 - (P + dP)4\pi r^2 = \dfrac{GMr}{R^3}\rho\left(4\pi r^2\right)dr$

$\therefore -\int_0^P dP = \dfrac{GM\rho}{R^3}\int_R^r r\,dr$

$\therefore\ P = \frac{GM\rho}{2R^3}\left[R^2 - r^2\right]$

$\therefore\ \frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{\left[R^2 - \frac{9R^2}{16}\right]}{\left[R^2 - \frac{4R^2}{9}\right]} = \frac{\frac{7R^2}{16}}{\frac{5R^2}{9}} = \frac{63}{80}$



and $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{\left[R^2 - \frac{9R^2}{25}\right]}{\left[R^2 - \frac{4R^2}{25}\right]} = \frac{16}{21}$

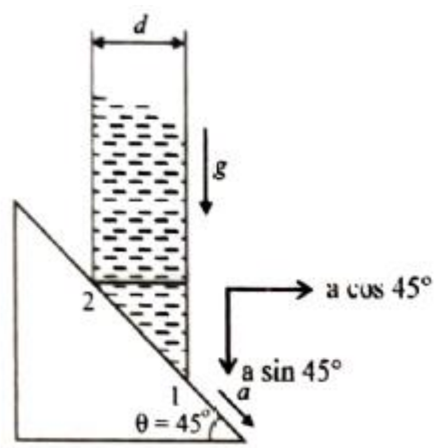
4. (A, C)

$P_1 = P_2 - \rho a \cos 45° d + \rho(g - a \sin 45°) d$

$\Rightarrow \frac{P_1 - P_2}{\rho g d} = 1 - \frac{\sqrt{2}a}{g}$

$\therefore\ \beta = 0$ for $a = \frac{g}{\sqrt{2}}$

and $\beta = \frac{\sqrt{2} - 1}{\sqrt{2}}$ for $a = \frac{g}{2}$



Comprehensions Type

Comprehensions-I

1. (D)
2. (B)
3. (B)

4. (C)

Her, piston is pushed at a speed, $v_1 = 5$ m/s

Let air comes out of nozzle with a speed v_2

From principle of continuity,

$a_1 v_1 = a_2 v_2$

$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 \ \Rightarrow\ r_1^2 v_1 = r_2^2 v_2$

$\Rightarrow (20)^2 \times 5 = (1)^2 \times v_2$

$\therefore\ v_2 = 2000\ \text{mms}^{-1} = 2\ \text{ms}^{-1}$

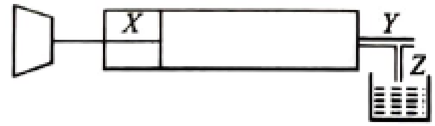
5. (A)

$P_X = P_Y = \frac{1}{2}\rho_a v_a^2$

$P_Z - P_Y = \frac{1}{2}\rho_l v_l^2$

But $P_Z = P_X$

$\therefore\ \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow v_l \sqrt{\frac{\rho_a}{\rho_l}} \times v_a$



$\therefore$ Volume flow rate $\propto \sqrt{\frac{\rho_a}{\rho_l}}$

Matrix Match Type

1. (C)

Integer Answer Type

1. (0.24)

As we know, bulk modulus $B = \frac{P}{\left(-\frac{dv}{v}\right)}$

$\Rightarrow \frac{dv}{v} = -\frac{P}{B} \Rightarrow 3\left(\frac{\Delta\ell}{\ell}\right) = -\frac{P}{B}$

$\ell\Delta = \left(\frac{-P}{B}\right)\frac{\ell}{3} = \left(\frac{\rho g h}{B}\right)\frac{\ell}{3}$

$= \frac{10^3 \times 10 \times 5 \times 10^3}{70 \times 10^9} \times \frac{1}{3} = 0.238 \times 10^{-3}$

$\therefore\ \Delta\ell = 0.238\,\text{mm} \simeq 0.24\,\text{mm}$