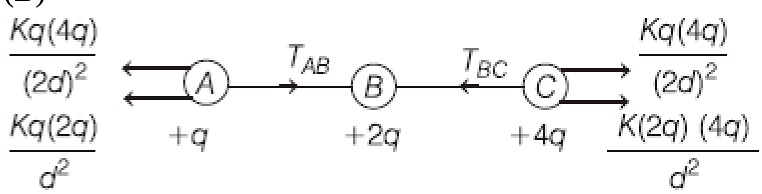
ELECTROSTATICS

1. (D)

$$\mathbf{F}_{21} = \frac{Kq_1q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3}(\mathbf{r}_2 - \mathbf{r}_1) = \frac{1 \times q_1 \times q_2}{4\pi\varepsilon_0 |-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}|^3}(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

$$= \frac{q_1q_2(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})}{56\sqrt{14}\,\pi\,\varepsilon_0}$$

2. (B)



Since, A is in equilibrium, $\Sigma F = 0$

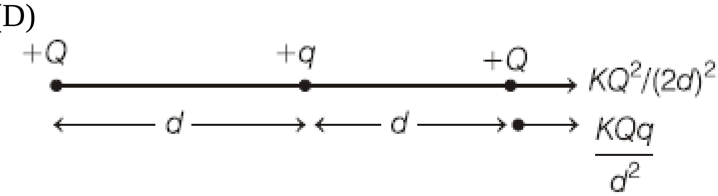
$$\Rightarrow \quad T_{AB} = \frac{Kq(4q)}{(2d)^2} + \frac{Kq(2q)}{d^2} = \frac{3Kq^2}{d^2}$$

Since, C is in equilibrium $\Sigma F = 0$

$$\Rightarrow \quad T_{BC} = \frac{Kq(4q)}{(2d)^2} + \frac{K(2q)(4q)}{d^2} = \frac{9Kq^2}{d^2}$$

$$\frac{T_{AB}}{T_{BC}} = \frac{3Kq^2/d^2}{9Kq^2/d^2} = \frac{1}{3}$$

3. (D)

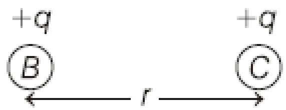


Central particle is already in equilibrium.

For equilibrium of $+Q$; $\Sigma F = 0$

$$\Rightarrow \quad \frac{KQ^2}{(2d)^2} + \frac{KQq}{d^2} = 0 \quad \Rightarrow \quad q = -\frac{Q}{4}$$

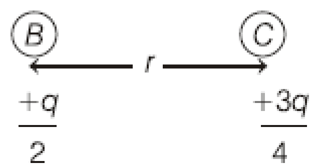
4. (D)



$$F = \frac{Kq^2}{r^2}$$

When A and B are touched, charges on each of them would be $\frac{q+0}{2} = \frac{q}{2}$.

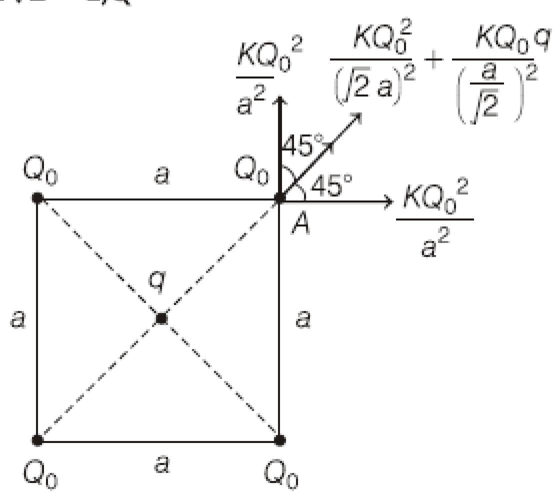
When A and C are touched, charges on each of them would be $\frac{\frac{q}{2}+q}{2} = \frac{3q}{4}$



$$F' = \frac{K\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right)}{r^2} = \frac{3F}{8}$$

5. (A)

$Q_0 = (2\sqrt{2} - 1)Q$



Net force on

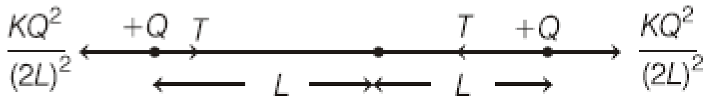
$$Q_0 = \frac{KQ_0^2}{(\sqrt{2}a)^2} + \frac{KQ_0 q}{\left(\frac{a}{\sqrt{2}}\right)^2} + \left(\frac{KQ_0^2}{a^2}\cos 45°\right) \times 2 = 0$$

$$\Rightarrow \qquad q = -\left(\frac{2\sqrt{2}+1}{4}\right)Q_0$$

$$\Rightarrow \qquad q = -\left(\frac{2\sqrt{2}+1}{4}\right)(2\sqrt{2}-1)Q = -\frac{7Q}{4}$$

6. (A)

In gravity free space, angle between the two threads will be 180°.

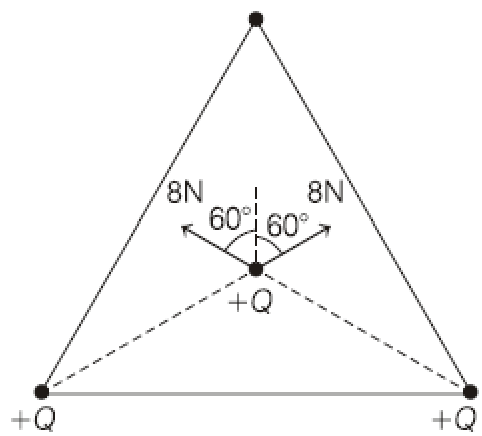


and $\quad T = \frac{KQ^2}{(2L)^2} = \frac{1\,Q^2}{4\pi\varepsilon_0(2L)^2}$

7. (D)

8. (D)

Net force on the central particle



$= 8\cos 60° + 8\cos 60° = 8\text{N}$

9. (A)

$qE = mg$

$\Rightarrow \quad q(10^9) = (1.6 \times 10^{-3})(9.8)$

$q = 1.6 \times 9.8 \times 10^{-12}\text{ C}$ and $q = ne$

$1.6 \times 9.8 \times 10^{-12} = n(1.6 \times 10^{-19})$

$n = 9.8 \times 10^7$

10. (B)

Electric force on $q = qE$

Work done by electric force $= qEy$

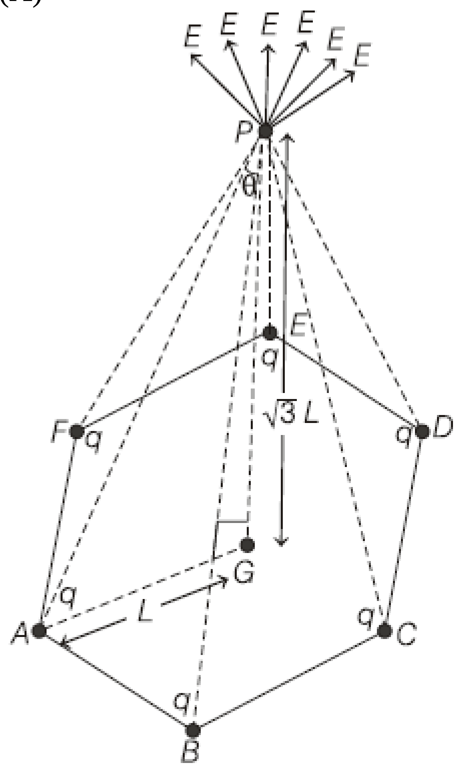
Work-energy theorem, $W = \Delta K$

$\Rightarrow \quad qEy = K_2 - 0 \Rightarrow K_2 = qEy$

11. (A)

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$
$$= \frac{9\times10^9 \times 5\times10^{-6}}{(0.1)^2}\hat{\mathbf{i}} + \frac{9\times10^9\times10\times10^{-6}}{(0.1)^2}\hat{\mathbf{i}}$$
$$= 13.5\times10^6 \text{ N/C, towards } -10\mu\text{C}.$$

12. (A)



$$AP = \sqrt{AG^2 + GP^2} = \sqrt{L^2 + (\sqrt{3}L)^2} = 2L$$
$$E = \frac{Kq}{(2L)^2} = \frac{Kq}{4L^2} = \frac{q}{16\pi\varepsilon_0 L^2}$$
Net electric field at $P = 6E\cos\theta$
$$= 6\left(\frac{q}{16\pi\varepsilon_0 L^2}\right)\left(\frac{\sqrt{3}L}{2L}\right) = \frac{3\sqrt{3}q}{16\pi\varepsilon_0 L^2}$$

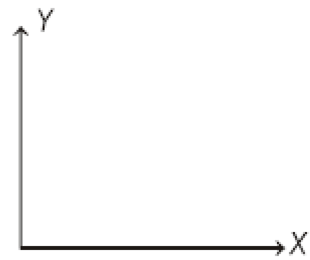
13. (C)

Electric field between the two charges is negative. So, q_2 will be positive and q_1 will be negative.

Since, electric field is zero at a point closer to q_2, $|q_2| < |q_1|$.

14. (D)

$\mathbf{F} = (-q)(-E\,\mathbf{j})$



$$m\mathbf{a} = qE\hat{\mathbf{j}} \quad \Rightarrow \quad \mathbf{a} = \frac{qE}{m}\hat{\mathbf{j}}$$

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$L = vt + 0 \quad \Rightarrow \quad t = \frac{L}{v}$$

For droplet to not hit the plate,

$$s_y < \frac{d}{2}$$

$$\Rightarrow \quad u_y t + \frac{1}{2}a_y t^2 < \frac{d}{2} \quad \Rightarrow \quad 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v}\right)^2 < \frac{d}{2}$$

$$\Rightarrow \qquad q < \frac{mv^2 d}{EL^2} \quad \Rightarrow \quad q_{\max} = \frac{mv^2 d}{EL^2}$$

15. (B)

$$E = \frac{KQx}{(r^2 + x^2)^{3/2}}$$

Electric field is maximum at $x = \dfrac{r}{\sqrt{2}}$.

$$E_{\max} = \frac{KQ(r/\sqrt{2})}{\left(r^2 + \left(\frac{r}{\sqrt{2}}\right)^2\right)^{3/2}} = \frac{KQ\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{3\sqrt{3}}{2\sqrt{2}}\right)r^2} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 r^2}$$

16. (C)

$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2}$

$$E_A = 0 + 0 = 0$$

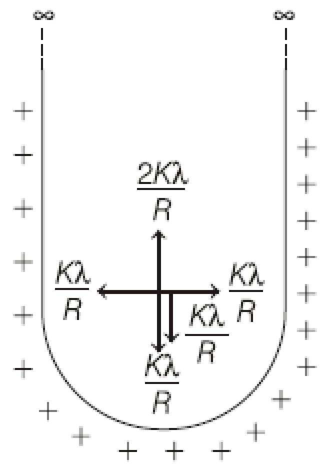
$$E_B = \frac{KQ}{\left(\frac{3r}{2}\right)^2} + 0 = \frac{4KQ}{9r^2}$$

$$E_C = \frac{KQ}{\left(\frac{5r}{2}\right)^2} + \frac{K(-Q)}{\left(\frac{5r}{2}\right)^2} = 0$$

$$\Rightarrow \qquad E_B > E_A = E_C$$

17. (D)

Net electric field = 0



18. (B)

Charge on the gap element $= \frac{1}{2\pi(0.5)} \times 0.002\,\pi$

$= 0.002$ C

Electric field due to gap element at the centre $= \frac{Kdq}{R^2}$

$$= \frac{9 \times 10^9 \times 0.002}{(0.5)^2}$$

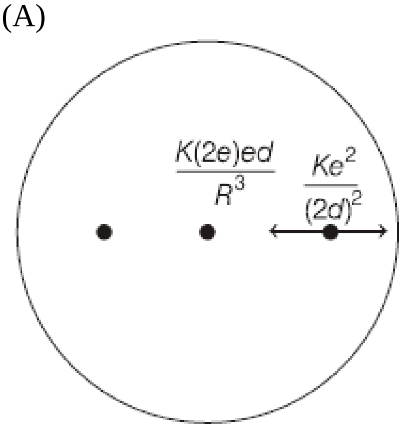
$$= 7.2 \times 10^7 \text{ N/C}$$

| Electric field due to gap element | = | Electric field due to remaining ring |

19. (C)

If another identical hemispherical shell is kept upside down over it, net electric field at any point inside the spherical shell should be zero.

20. (A)



For equilibrium of electron,

$$\frac{K(2e)ed}{R^3} = \frac{Ke^2}{(2d)^2}$$

$$d = R/2$$

Separation between the electrons $= 2d = 2\left(\frac{R}{2}\right) = R$

21. (B)

$F = qE$

$\Rightarrow \quad F = (-Q)\left(\frac{K(\lambda 2\pi a)\,x}{(a^2 + x^2)^{3/2}}\right)$

$\Rightarrow \quad F = -\frac{Qa\lambda}{2a^3\varepsilon_0}x \qquad (\because x << a)$

$\Rightarrow \quad F = -\left(\frac{Q\lambda}{2\varepsilon_0 a^2}\right)x$

This is equation of SHM.

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{\frac{\lambda Q}{2\varepsilon_0 a^2}}} = 2\pi\sqrt{\frac{2m\varepsilon_0 a^2}{\lambda Q}}$$

22. (C)

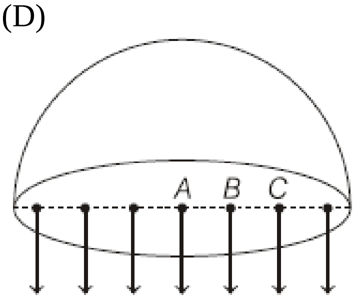
$V = V_1 + V_2 + V_3$

$$= \frac{k(\lambda\pi R)}{R} + \int_R^{3R}\frac{k(\lambda dx)}{x} + \int_R^{3R}\frac{k\lambda dx}{x}$$

$$= k\lambda\pi + k\lambda \ln 3 + k\lambda \ln 3 = k\lambda(\pi + 2\ln 3)$$

$$= \frac{\lambda}{4\pi\varepsilon_0}(\pi + 2\ln 3)$$

23. (D)



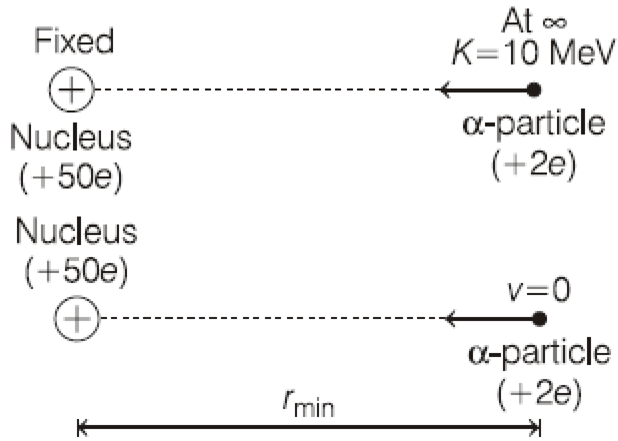
Electric field due to hemisphere at points lying on the diameter is perpendicular to the diameter. So, all points on the diameter are equipotential.

24. (C)

A negative charge released in an electric field will go against the direction of electric field.

So, it will move toward a position of higher electric potential and lower potential energy.

25. (A)



At the minimum distance of approach, speed of α-particle would be zero.
Applying mechanical conservation for α-particle,

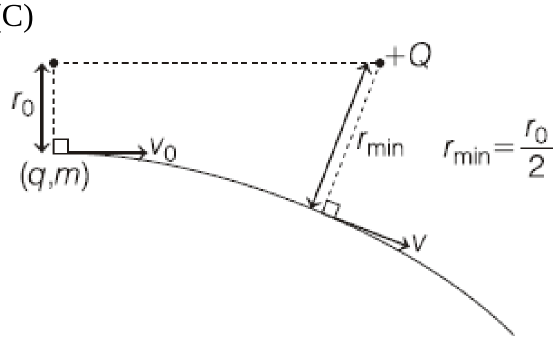
$$K_1 + U_1 = K_2 + U_2$$

$$10MeV + 0 = 0 + \frac{k(+50e)(+2e)}{r_{\min}}$$

$$\Rightarrow \quad r_{\min} = \frac{9\times10^9\times100\times e\times e}{10\times10^6\times e}$$

$$= 14.4\times10^{-15} \text{ or } 1.44\times10^{-14} \text{ m}$$

26. (C)



Using angular momentum conservation of q about Q,

$$mv_0r_0 = mvr_{\min} \qquad ...(i)$$

Using energy conservation,

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv^2 + \frac{kQq}{r_{\min}} \qquad ...(ii)$$

Solving Eqs. (i) and (ii), we get

$$q = -\frac{3Q}{4}$$

27. (C)

Using energy conservation, $K_1 + U_1 = K_2 + U_2$

$$\Rightarrow \quad 0 + \left[mg\frac{4R}{3} + q\left(\frac{kQ}{\sqrt{R^2 + \left(\frac{4R}{3}\right)^2}} \right) \right]$$

$$= 0 + \left[0 + q\left(\frac{kQ}{R} \right) \right]$$

$$\Rightarrow \quad \frac{4}{3}mgR + \frac{3kQq}{5R} = \frac{kQq}{R}$$

$$\Rightarrow \quad \frac{4}{3}mgR = \frac{2}{5}\frac{kQq}{R}$$

$$\Rightarrow \quad \frac{4}{3}\left(\frac{Q^2}{4\pi\varepsilon_0 R^2} \right)R = \frac{2}{5}\left(\frac{Qq}{4\pi\varepsilon_0 R} \right) \Rightarrow q = \frac{10Q}{3}$$

28. (A)

$F = qE$

$$\Rightarrow \quad 3000 = 3E \Rightarrow E = 1000 \text{ N/C}$$

$$\Delta V = Ed = (1000\text{N/C})\,(1 \times 10^{-2}\text{m}) = 10 \text{ V}$$

29. (D)

$W = (qE)s\cos\theta$

$$\Rightarrow \quad 4 = (0.2E)\,(2)\cos 60°$$

$$\Rightarrow \quad E = 20 \text{ N/C}$$

30. (A)

$dV = -\mathbf{E} \cdot d\mathbf{r}$

$$\Rightarrow \quad \int dV = -\int (E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

$$\Rightarrow \quad \int_{V_0}^{V} dV = -\int_0^x E_x dx - \int_0^y E_y dy$$

$$\Rightarrow \quad V - V_0 = -E_x[x]_0^x - E_y[y]_0^y$$

$$\Rightarrow \quad V = V_0 - xE_x - yE_y$$

31. (A)

$V = -xy^2\sqrt{z}$

$$\mathbf{E} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$$

$$= -(-y^2\sqrt{z})\hat{\mathbf{i}} - (-2xy\sqrt{z})\hat{\mathbf{j}} - \left(-\frac{xy^2}{2\sqrt{z}}\right)\hat{\mathbf{k}}$$

$$= y^2\sqrt{z}\hat{\mathbf{i}} + 2\,xy\sqrt{z}\,\hat{\mathbf{j}} + \frac{xy^2}{2\sqrt{z}}\hat{\mathbf{k}}$$

At (2, 1, 1),

$$\mathbf{E} = (1^2\sqrt{1})\hat{\mathbf{i}} + [2(2)(1)\sqrt{1}]\ \hat{\mathbf{j}} + \left(\frac{2(1)^2}{2\sqrt{1}}\right)\hat{\mathbf{k}}$$

$$= \hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

32. (A)

$E_y = -\dfrac{dV}{dy}$

At $y = 2.5$ m,

$$E_y = -\frac{dV}{dy} = -\text{ slope of } V\text{-}y \text{ graph}$$

$$= -\left(\frac{5-2}{1}\right) = -3$$

At $y = 5.5$ m,

$$E_y = -\frac{dV}{dy} = -\text{ slope of } V\text{-}y \text{ graph}$$

$$= -\left[-\frac{(5-0)}{(1-0)}\right] = 5$$

33. (C)

Loss in KE = Gain in PE

$\Rightarrow \quad -\Delta K = \Delta U$

$\Rightarrow \quad -\left(0-\frac{1}{2}mv^2\right) = q\Delta V$

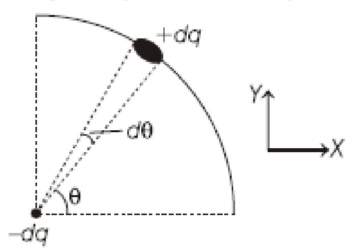
$\Rightarrow \quad \frac{1}{2}mv^2 = q\left(\frac{\sigma}{2\varepsilon_0}d\right)$

$\Rightarrow \quad v = \sqrt{\frac{q\sigma d}{m\varepsilon_0}}$

34. (A)

$dq = \frac{q}{\pi R/2}(Rd\theta)$

$\mathbf{dp} = dp\cos\theta\hat{\mathbf{i}} + dp\sin\theta\hat{\mathbf{j}}$



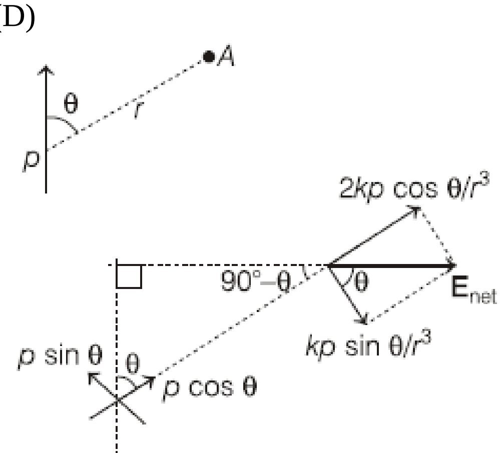
$\Rightarrow \quad \mathbf{dp} = \left(\frac{q}{\pi R/2}Rd\theta\right)R\cos\theta\hat{\mathbf{i}} + \left(\frac{q}{\pi R/2}Rd\theta\right)R\sin\theta\hat{\mathbf{j}}$

$\Rightarrow \int \mathbf{dp} = \frac{2qR}{\pi}\left(\int_0^{\pi/2}\cos\theta d\theta\,\hat{\mathbf{i}} + \int_0^{\pi/2}\sin\theta\, d\theta\,\hat{\mathbf{j}}\right)$

$\Rightarrow \quad \mathbf{p} = \frac{2qR}{\pi}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

$\Rightarrow \quad |\mathbf{p}| = \frac{2\sqrt{2}qR}{\pi}$

35. (D)



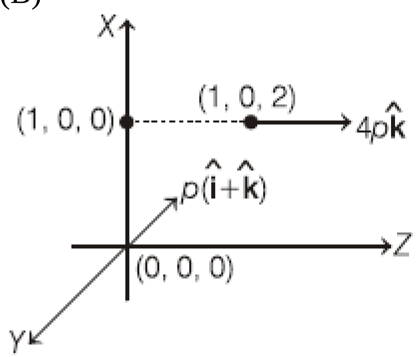
$$\tan\theta = \frac{\dfrac{2kp\cos\theta}{r^3}}{\dfrac{kp\sin\theta}{r^3}}$$

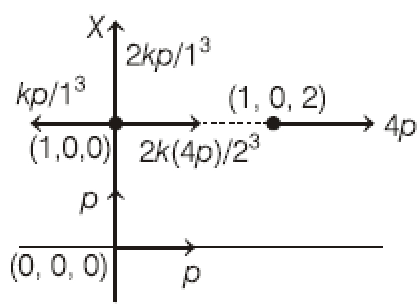
$$\Rightarrow \quad \tan\theta = 2\cot\theta$$

$$\Rightarrow \quad \tan^2\theta = 2$$

$$\Rightarrow \quad \theta = \tan^{-1}(\sqrt{2})$$

36. (B)

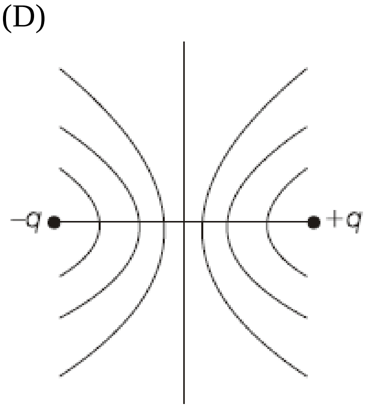




$$\mathbf{E} = \left(\frac{2k(4p)}{2^3} - \frac{kp}{1^3}\right)\hat{\mathbf{i}} + \left(\frac{2kp}{1^3}\right)\hat{\mathbf{j}}$$

$$\mathbf{E} = \frac{p}{2\pi\varepsilon_0}\hat{\mathbf{j}}$$

37. (D)



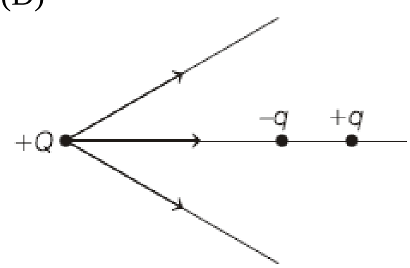
38. (C)

$$\tau = pE\sin\theta$$
$$\tau_{max} = pE, \text{ when } \theta = 90°$$
$$= (10^{-6} \times 2 \times 10^{-2})(1.0 \times 10^5)$$
$$= 2.0 \times 10^{-3} \text{ N-m}$$

39. (C)

$$W_{ext} = \Delta U + \Delta K$$
$$= (U_2 - U_1) + 0$$
$$= -\mathbf{p}_2 \cdot \mathbf{E} + \mathbf{p}_1 \cdot \mathbf{E}$$
$$= (\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{E}$$
$$= [(7\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times 10^{-30})] \cdot (4000\hat{\mathbf{i}})$$
$$= 2.8 \times 10^{-26} \text{ J}$$

40. (D)



Force on the dipole will be non-zero as electric field produced by a point charge is non-uniform. Torque will be zero, if the dipole is aligned along one of electric field lines as shown in figure. For any other orientation of dipole, torque will be non-zero.

41. (D)

Since, the charge enclosed is zero, flux through Gaussian sphere is zero. Electric field is not zero anywhere on the sphere.

42. (C)

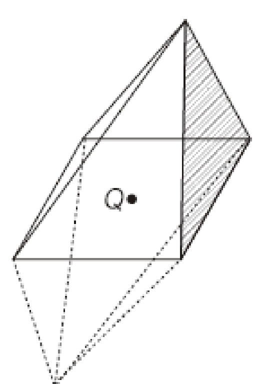
Put another identical pyramid as shown in figure.

Total flux emanating from q will be equally divided between the two pyramids.

Flux through upper pyramid

$$= \frac{1}{2}\left(\frac{Q}{\varepsilon_0}\right)$$

Flux through each face of upper pyramid $= \frac{1}{4}\left(\frac{Q}{2\varepsilon_0}\right) = \frac{Q}{8\,\varepsilon_0}$



43. (B)

$$\phi = \phi_q + \phi_{2q} + \phi_{3q} + \phi_{4q}$$

$$= \frac{q}{\varepsilon_0} + \frac{1}{4}\left(\frac{2q}{\varepsilon_0}\right) + \frac{1}{2}\left(\frac{3q}{\varepsilon_0}\right) + \frac{1}{8}\left(\frac{4q}{\varepsilon_0}\right) = \frac{7q}{2\varepsilon_0}$$

44. (D)

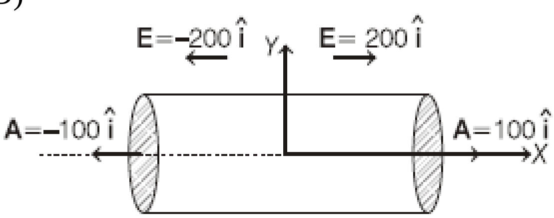
$$\phi = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \phi_{max} = \frac{q_{max}}{\varepsilon_0} \Rightarrow \phi_{max} = \frac{\lambda l_{max}}{\varepsilon_0}$$

The longest wire which is inside the cube is of length $\sqrt{3}a$.

$$\Rightarrow \quad \phi_{max} = \frac{\sqrt{3}\lambda a}{\varepsilon_0}$$

45. (D)



Flux through right flat face $= \mathbf{E} \cdot \mathbf{A}$

$$= (200\,\hat{\mathbf{i}}) \cdot (100\hat{\mathbf{i}}) = 2 \times 10^4$$

Flux through left flat face $= \mathbf{E} \cdot \mathbf{A}$

$$= (-200\,\hat{\mathbf{i}}) \cdot (-100\hat{\mathbf{i}}) = 2 \times 10^4$$

Flux through curved surface $= EA \cos 90° = 0$

Net flux through cylinder

$$= 2 \times 10^4 + 2 \times 10^4 + 0 = 4 \times 10^4$$

Using Gauss's law,

$$\phi = \frac{q_{inside}}{\varepsilon_0} \Rightarrow 4 \times 10^4 = \frac{q_{inside}}{8.85 \times 10^{-12}}$$

$$\Rightarrow \quad q_{inside} = 35.4 \times 10^{-8}\text{ C}$$

46. (C)

Using Gauss's law, $\phi = \frac{q_{\text{enclosed}}}{\varepsilon_0}$

$$\Rightarrow \quad E(4\pi x^2) = \frac{\int_0^x (\rho_0 r^2)\, 4\pi r^2 dr}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho_0 x^3}{5\varepsilon_0}$$

$$\Rightarrow \quad E \propto x^3$$

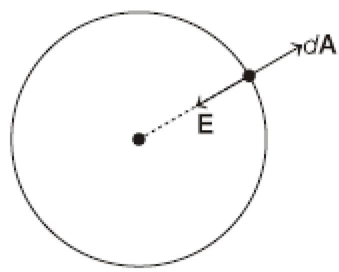
47. (B)

$$d\phi = \mathbf{E} \cdot d\mathbf{A}$$

$$\Rightarrow \quad d\phi = [90r(-\hat{\mathbf{r}})] \cdot (dA\hat{\mathbf{r}})$$

$$\Rightarrow \quad \int d\phi = -90r \int dA$$

$$\Rightarrow \quad \phi = -90r\,(4\pi r^2)$$



Using Gauss's law, $\phi = \frac{q_{\text{enclosed}}}{\varepsilon_0}$

$$\Rightarrow \quad -90r(4\pi r^2) = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

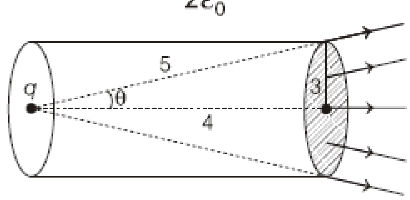
$$\Rightarrow \quad q_{\text{enclosed}} = -90\,(4\pi\varepsilon_0)\, r^3$$

$$= \frac{-90 \times 2^3}{9 \times 10^9}$$

$$= -80 \text{ nC}$$

48. (B)

Flux through cylinder $= \frac{q}{2\varepsilon_0}$

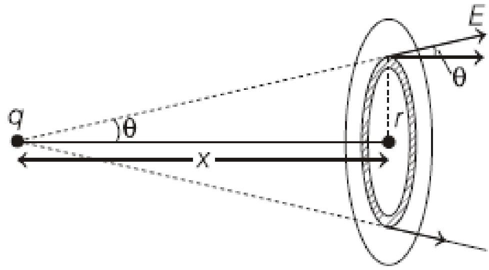


Flux through the right flat face $= \frac{q/\varepsilon_0}{4\pi}[2\pi(1-\cos\theta)]$

$$= \frac{q}{2\varepsilon_0}\left(1 - \frac{4}{5}\right) = \frac{q}{10\,\varepsilon_0}$$

Flux through curved surface $= \frac{q}{2\varepsilon_0} - \frac{q}{10\,\varepsilon_0} = \frac{2q}{5\varepsilon_0}$

49. (D)



Lets take a ring of radius r and thickness dr on the disc.

Flux through the ring $= d\phi = E\,dA\cos\theta$

$\Rightarrow \quad d\phi = E(2\pi r dr)\cos\theta$

$\Rightarrow \quad d\phi = (2\pi r dr)(E\cos\theta)$

Multiplying by σ on both sides

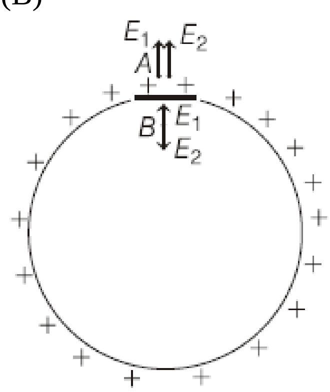
$\Rightarrow \quad \sigma\,d\phi = (\sigma 2\pi\,r dr)(E\cos\theta)$

$\Rightarrow \quad \sigma\,d\phi = dq\,(E\cos\theta)$

$\Rightarrow \quad \sigma d\phi = dF \Rightarrow \sigma\int d\phi = \int dF$

$\Rightarrow \quad \phi = \dfrac{F}{\sigma}$

50. (B)



E_1 = Electric field due to shell with hole

E_2 = Electric field due to part of shell within the hole.

Point A is just outside the shell,

$$E_A = E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \qquad \text{... (i)}$$

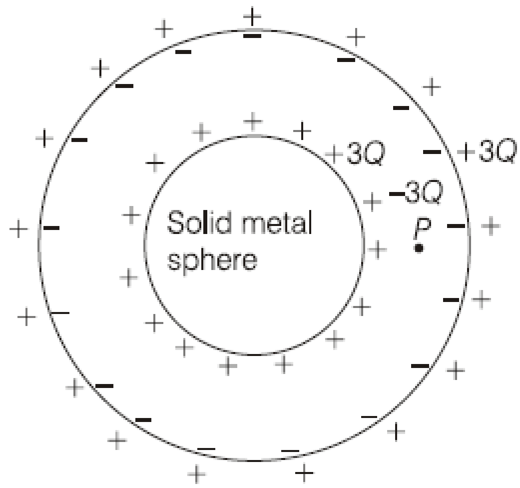
Point B is just inside the shell,

$$E_B = E_1 - E_2 = 0 \qquad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

51. (C)



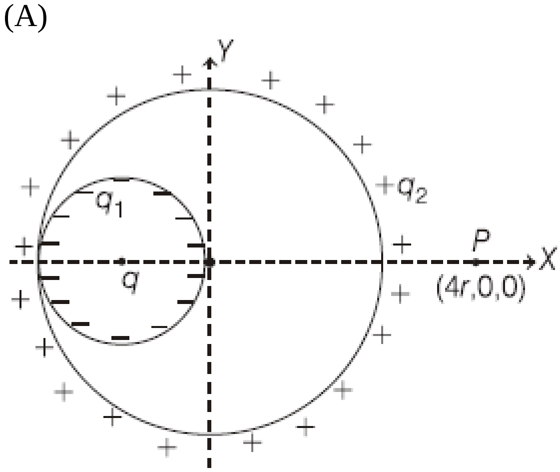
At P,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$$

$\Rightarrow$ $$E = \frac{k(3Q)}{r^2} + 0 + 0$$

$\Rightarrow$ $$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{3Q}{r^2}$$

52. (A)



$\because$ $q_1 = -q$ and $q_2 = +q$

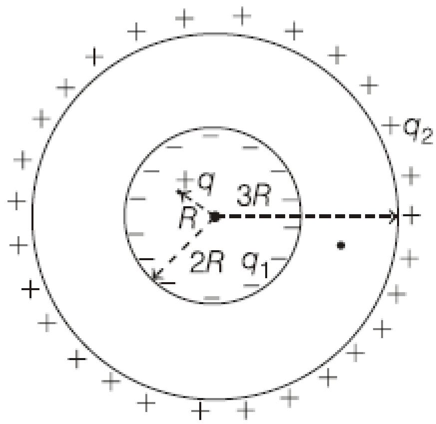
At point P,

$$\mathbf{E} = (\mathbf{E}_{\text{due to } q} + \mathbf{E}_{\text{due to } q_1}) + \mathbf{E}_{\text{due to } q_2}$$

$\Rightarrow$ $$E = 0 + \frac{kq}{(4r)^2}$$

$\Rightarrow$ $$E = \frac{kq}{16r^2}$$

53. (C)



$q_1 = -q$ (non-uniform distribution)

$q_2 = +q$ (uniform distribution)

Potential at the centre,

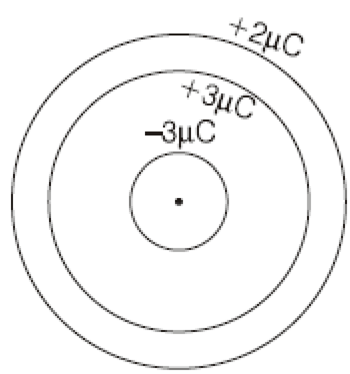
$$V = V_{\text{due to } q} + V_{\text{due to } q_1} + V_{\text{due to } q_2}$$

$$= \frac{kq}{R} + \frac{k(-q)}{2R} + \frac{kq}{3R}$$

$$= \frac{5}{6}\frac{kq}{R}$$

54. (A)

Potential of the inner surface of the spherical shell,



$$V = V_1 + V_2 + V_3$$

$$= \frac{k(-3\mu C)}{(9\text{ cm})} + \frac{k(+3\mu C)}{(9\text{ cm})} + \frac{k(+2\mu C)}{(10\text{ cm})}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6}}{10 \times 10^{-2}}$$

$$= 180\text{ kV}$$

55. (B)

At point P,

$$V_{\text{due to } q} + V_{\text{due to induced charges on the inner surface}} = 0$$

$$\Rightarrow V_{\text{due to induced charges on the inner surface}} = -V_{\text{due to } q} = \frac{-kq}{5a}$$

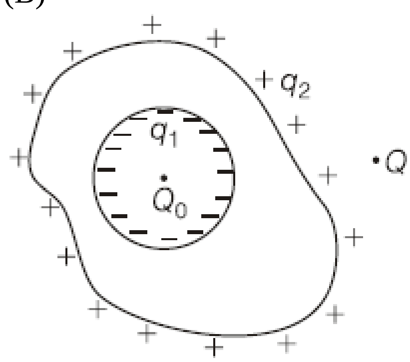
56. (B)

Potential at A due to charges outside the conductor is zero, since the shell is grounded.

$$V_A = V_{\text{due to } q} + V_{\text{due to charges on the outer surface of shell}} = 0$$

$$\Rightarrow V_{\text{due to charges on the outer surface of shell}} = -V_{\text{due to } q} = -\frac{kq}{(2.5R)}$$

57. (B)



$q_1 = -Q_0$ (uniform) and $q_2 = +Q$

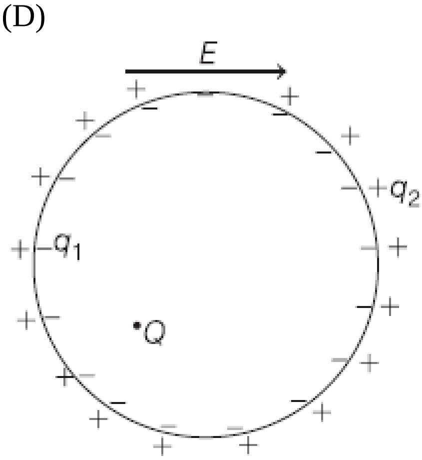
Force on Q_0,

$$\mathbf{F} = \mathbf{F}_{\text{due to } q_1} + (\mathbf{F}_{\text{due to } q_2} + \mathbf{F}_{\text{due to } Q}) = 0 + 0 = 0$$

Force on Q,

$$\mathbf{F} = (\mathbf{F}_{\text{due to } Q_0} + \mathbf{F}_{\text{due to } q_1}) + \mathbf{F}_{\text{due to } q_2} = 0 + \mathbf{F}_{\text{due to } q_2} \neq 0$$

58. (D)



$q_1 = -Q$ (non-uniform); $q_2 = +Q$ (non-uniform)

(a) Force on Q due to $E = QE$

(b) Net force on $Q = \mathbf{F}_{\text{due to } q_1} + (\mathbf{F}_{\text{due to } q_2} + \mathbf{F}_{\text{due to } E})$

$$= \mathbf{F}_{\text{due to } q_1} + 0$$
$$= \mathbf{F}_{\text{due to } q_1}$$
$$\neq 0$$

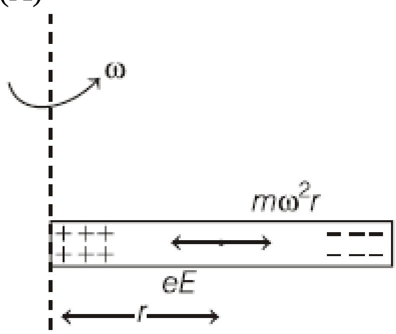
(c) Net force on Q and conducting shell considered as a system $= q_2E + q_1E + QE$

$$= QE - QE + QE = QE$$

(d) Net force on the shell due to $E = q_1E + q_2E$

$$= -QE + QE = 0$$

59. (A)

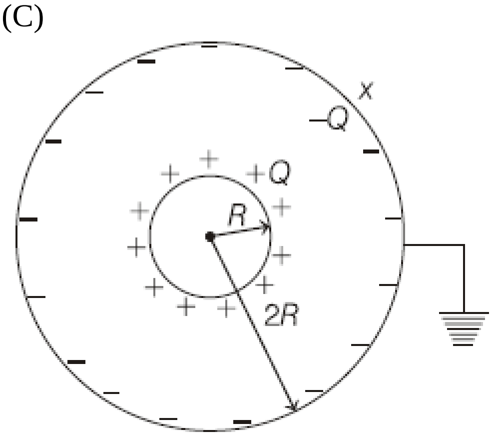


Due to centrifugal on free electrons, electrons will accumulate at the right end of the rod and there will be deficiency of electrons at the left end. These induced charges will give rise to an electric field.

Electrons will stop accumulating at the right end when net force on free electron is zero.

$$\Delta V = \int E\,dr = \int_0^l \frac{m\omega^2 r}{e}\,dr = \frac{m\omega^2 l^2}{2e}$$

60. (C)



Since, B is earth, therefore $V_B = 0$

$$\Rightarrow \quad \frac{kQ}{2R} + \frac{k(x-Q)}{2R} = 0$$

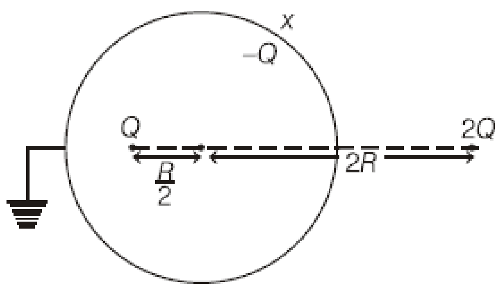
$$\Rightarrow \quad x = 0$$

$$V_A = \frac{kQ}{R} + \frac{k(x-Q)}{2R}$$

$$= \frac{kQ}{2R} = \frac{Q}{8\pi\varepsilon_0 R}$$

61. (A)

Lets take charge on outer surface of shell to be x after it has been earthed.



Potential due to outside charges inside the shell = 0

At the centre, $\frac{kx}{R} + \frac{k(2Q)}{2R} = 0$

$$x = -Q$$

Total charge on the shell $= x - Q$

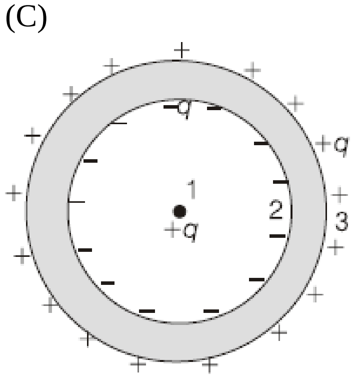
$$= -Q - Q = -2Q$$

Charge flown to earth $= Q_1 - Q_2$

$$= Q - (-2Q)$$

$$= 3Q$$

62. (C)

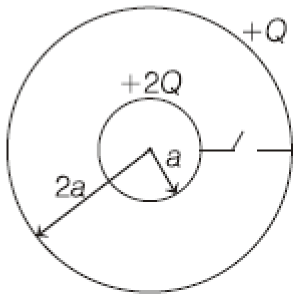


$$U = U_{12} + U_{23} + U_{31} + U_2 + U_3$$

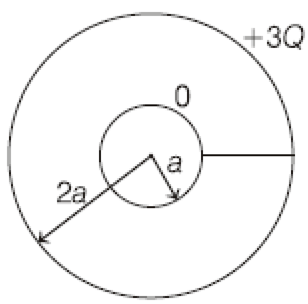
$$\Rightarrow \quad U = (-q)\left(\frac{kq}{a}\right) + q\left[\frac{k(-q)}{b}\right] + q\left(\frac{kq}{b}\right) + \frac{k(-q)^2}{2a} + \frac{kq^2}{2b}$$

$$\Rightarrow \quad U = -\frac{kq^2}{a} - \frac{kq^2}{b} + \frac{kq^2}{b} + \frac{kq^2}{2a} + \frac{kq^2}{2b} = \frac{kq^2}{2b} - \frac{kq^2}{2a}$$

63. (B)



$$U_1 = \frac{k(2Q)^2}{2a} + \frac{kQ^2}{2(2a)} + Q\left[\frac{k(2Q)}{2a}\right] = \frac{13kQ^2}{4a}$$



After closing the switch, the entire charge of inner shell will be transferred to the outer shell.

$$U_2 = \frac{k(3Q)^2}{2(2a)} = \frac{9kQ^2}{4a}$$

Heat dissipated $= U_1 - U_2 = \frac{13kQ^2}{4a} - \frac{9kQ^2}{4a} = \frac{kQ^2}{a}$

64. (234)

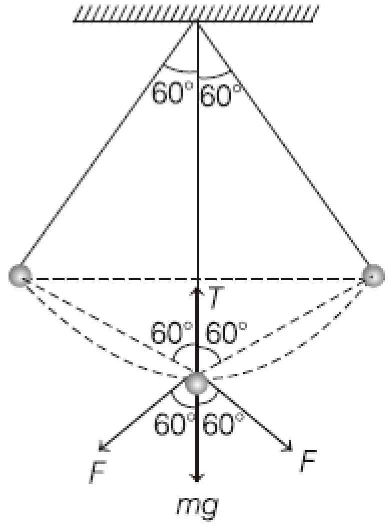
$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$E = \frac{2KQ_1}{R^2}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right) + \frac{KQ_2 x}{(R^2 + x^2)^{3/2}}$$

$$E = \frac{2 \times 9 \times 10^9 \times 1 \times 10^{-6}}{4^2}\left(1 - \frac{3}{5}\right) + \frac{9 \times 10^9 \times (-1 \times 10^{-6}) \times 3}{(5^2)^{3/2}}$$

$E = 450 - 216 = 234$

65. (3)



$\Sigma F_y = 0$

$\Rightarrow \quad T = mg + 2F\cos 60°$

$$T = (20\times 10^{-3}\times 10) + 2\left(\frac{9\times 10^9\times (10\times 10^{-6})^2}{(2\times 3\times \sin 30°)^2}\right)\times \frac{1}{2}$$

$T = 0.3\,\text{N}$

66. (4)

$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$E = \frac{KQx}{(R^2+x^2)^{3/2}} - \frac{KQ}{x^2} = \frac{KQx}{x^3\left(1+\frac{R^2}{x^2}\right)^{3/2}} - \frac{KQ}{x^2}$$

$$= \frac{KQ}{x^2}\left(1+\frac{R^2}{x^2}\right)^{-3/2} - \frac{KQ}{x^2}$$

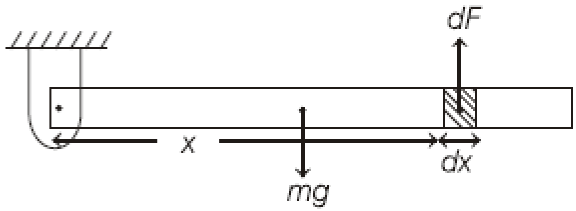
$$= \frac{KQ}{x^2}\left(1-\frac{3}{2}\frac{R^2}{x^2}\right) - \frac{KQ}{x^2}$$

$$= \frac{-3}{2}\frac{KQR^2}{x^4}$$

$\Rightarrow \quad E \propto \frac{1}{x^4}$

So, $n = 4$

67. (3)



$$dF = \lambda dx = \lambda_0 x dx$$
$$d\tau = dFx$$
$$\int d\tau = \int_0^l (\lambda_0 x dx E)\, x$$
$$\tau = \frac{\lambda_0 l^3 E}{3}$$

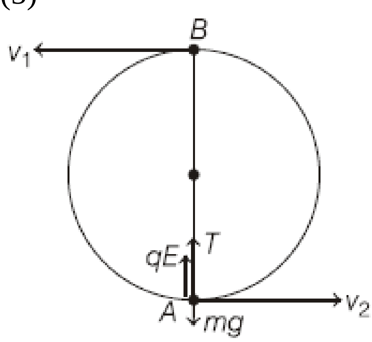
For equilibrium of rod, $\Sigma\tau_{net} = 0$

$\Rightarrow \qquad mg\left(\frac{l}{2}\right) - \frac{\lambda_0 l^3 E}{3} = 0$

$\Rightarrow \qquad E = \frac{3mg}{2\lambda_0 l^2}$

So, $n = 3$

68. (5)



At the lowest position,

$$T + qE - mg = \frac{mv_2^2}{l}$$
$$15mg + 3mg - mg = \frac{mv_2^2}{l}$$

$\Rightarrow \qquad v_2 = \sqrt{17gl}$

Applying work-energy theorem between A and B,

$$W_{mg} + W_T + W_{electric} = \Delta K - mg(2l) + 0 + qE(2l)$$
$$= \frac{1}{2}mv_1^2 - \frac{1}{2}m(\sqrt{17gl})^2$$
$$v_1 = \sqrt{25gl} = \sqrt{25 \times 10 \times 0.1} = 5$$

69. (144)

Electric field inside a solid sphere,

$$E = \frac{KQ}{R^3} r$$

$\Rightarrow$ Slope of E-r graph

$$= \tan 53° = \frac{KQ}{R^3}$$

$$\Rightarrow \quad \frac{4}{3} = \frac{Q}{4\pi\varepsilon_0 (3)^3}$$

$$\Rightarrow \quad Q = 144\,\pi\,\varepsilon_0$$

$$Q = 144\pi\varepsilon_0 = n\pi\varepsilon_0$$

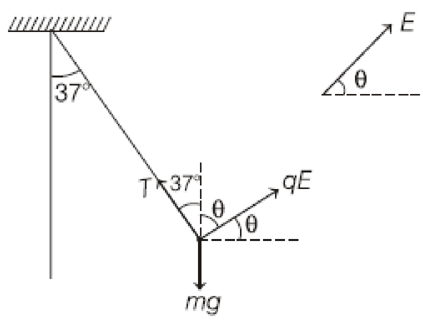
$$\Rightarrow \quad n = 144$$

70. (6)

For equilibrium,

$$qE\cos\theta = T\sin 37°$$

and $$qE\sin\theta = mg - T\cos 37°$$



Dividing Eq. (ii) by Eq. (i), we get

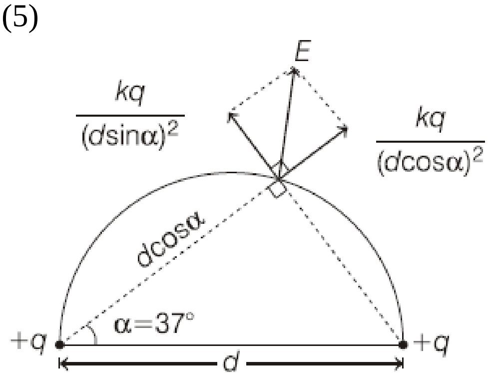
$$\tan\theta = \frac{mg - 2mg\cos 37°}{2mg\sin 37°} = \frac{-1}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \quad \sqrt{5}E\left(\frac{2}{\sqrt{5}}\right) = 2 \times 1 \times 10 \times \frac{3}{5}$$

$$\Rightarrow \quad E = 6\text{ N/C}$$

71. (5)



$$E=\sqrt{\left[\frac{kq}{(d\cos\alpha)^2}\right]^2+\left[\frac{kq}{(d\sin\alpha)^2}\right]^2}$$

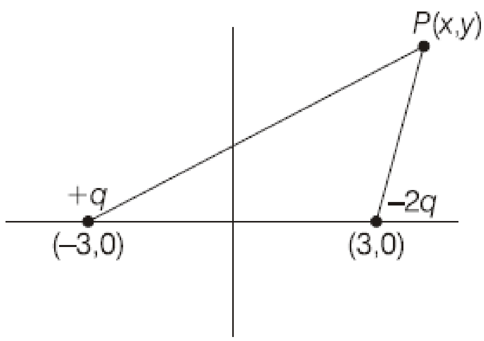
$$=\frac{kq}{d^2}\sqrt{\frac{\sin^4\alpha+\cos^4\alpha}{\sin^4\alpha\cos^4\alpha}}$$

$$V=V_1+V_2=\frac{kq}{d\cos\alpha}+\frac{kq}{d\sin\alpha}$$

$$=\frac{kq}{d}\frac{(\sin\alpha+\cos\alpha)}{\sin\alpha\cos\alpha}$$

$$\frac{E}{V}=\frac{1}{d}\frac{\sqrt{\sin^4\alpha+\cos^4\alpha}}{(\sin\alpha+\cos\alpha)\sin\alpha\cos\alpha}=5$$

72. (7)



At point P,

$$V=V_1+V_2$$

$$V=\frac{k(-2q)}{\sqrt{(x-3)^2+y^2}}+\frac{k(+q)}{\sqrt{(x+3)^2+y^2}}=0$$

$\Rightarrow$ $2\sqrt{(x+3)^2+y^2}=\sqrt{(x-3)^2+y^2}$

$\Rightarrow$ $4(x+3)^2+4y^2=(x-3)^2+y^2$

$\Rightarrow$ $3x^2+3y^2+30x+27=0$

$\Rightarrow$ $x^2+y^2+10x+9=0$

$\Rightarrow$ $(x+5)^2+y^2=16$

$\Rightarrow$ $\alpha=5$ and $\beta=2$

$$\alpha+\beta=5+2=7$$

73. (4)

$$V=\frac{kQ}{R}=\frac{k\int dq}{R}=\frac{k}{R}\int \lambda R d\phi$$

$$=\frac{k}{R}\int_0^{2\pi}\lambda_0 \cos^2\phi R d\phi$$

$$=\frac{\lambda_0}{4\pi\varepsilon_0}\int_0^{2\pi}\cos^2\phi d\phi$$

$$=\frac{\lambda_0}{4\pi\varepsilon_0}\int_0^{2\pi}\left(\frac{1+\cos 2\phi}{2}\right)d\phi$$

$$=\frac{\lambda_0}{8\pi\varepsilon_0}\left(\phi+\frac{\sin 2\phi}{2}\right)_0^{2\pi}=\frac{\lambda_0}{4\varepsilon_0}$$

So, $n=4$

74. (6)

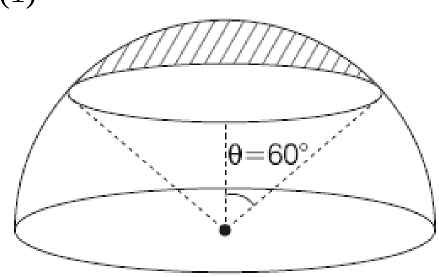
$V=1.2V_{\text{surface}}$

$$\frac{kQ}{2R^3}(3R^2-r^2)=1.2\frac{kQ}{R}$$

$$\Rightarrow \quad 15R^2-5r^2=12R^2$$

$$\Rightarrow \quad r=\sqrt{\frac{3}{5}}R=\sqrt{\frac{3}{5}}\times\sqrt{60}=6\text{ cm}$$

75. (1)



Solid angle subtended by the shaded part

$=2\pi(1-\cos\theta)=2\pi(1-\cos 60^\circ)=\pi$

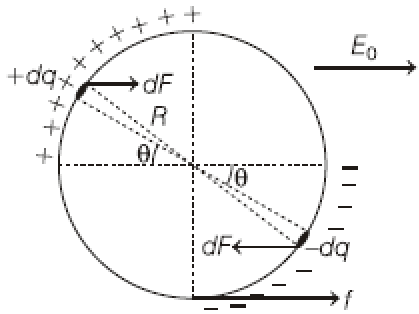
Solid angle subtended by the remaining hemisphere

$$=2\pi-\pi=\pi$$

$$V_{\text{centre}}=\frac{kQ}{R}=\frac{k}{R}\left[\sigma\left(\frac{4\pi R^2}{4\pi}\times\pi\right)\right]$$

$$=\frac{\sigma R}{4\varepsilon_0}=\frac{(2\varepsilon_0)(2)}{4\varepsilon_0}=1\text{ V}$$

76. (1)



Net torque about the centre $= \left(\int dF(2R\sin\theta)\right) - fR$

$\Rightarrow \quad \tau = \left[\int dq E_0(2R\sin\theta)\right] - fR$

$\Rightarrow \quad \tau = \int_0^{\pi/2} \lambda R d\theta\, E_0(2R\sin\theta) - fR$

$\Rightarrow \quad \tau = 2\lambda E_0 R^2 - fR$

$\Rightarrow \quad I\alpha = 2\lambda R^2 E_0 - fR$

$\Rightarrow \quad \alpha = \dfrac{2\lambda R^2 E_0 - fR}{mR^2}$ ... (i)

For pure rolling, $a_{CM} = R\alpha$

$\Rightarrow \quad \dfrac{f}{m} = R\alpha$

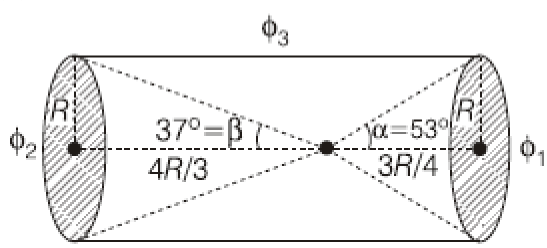
$\Rightarrow \quad \alpha = \dfrac{f}{mR}$ ... (ii)

From Eqs. (i) and (ii), we get

$$\frac{f}{mR} = \frac{2\lambda R^2 E_0 - fR}{mR^2}$$

$\Rightarrow \quad f = \lambda R E_0$

77. (7)



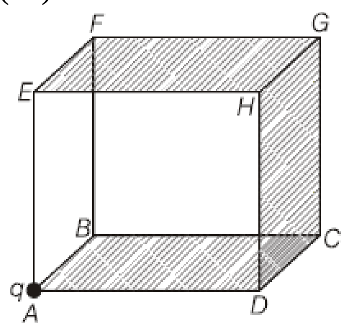
$$\phi_1 = \frac{q}{2\varepsilon_0}(1-\cos\alpha) = \frac{q}{2\varepsilon_0}\left(1-\frac{3}{5}\right) = \frac{q}{5\varepsilon_0}$$

$$\phi_2 = \frac{q}{2\varepsilon_0}(1-\cos\beta) = \frac{q}{2\varepsilon_0}\left(1-\frac{4}{5}\right) = \frac{q}{10\varepsilon_0}$$

$$\phi_1 + \phi_2 + \phi_3 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \frac{q}{5\varepsilon_0} + \frac{q}{10\varepsilon_0} + \phi_3 = \frac{q}{\varepsilon_0} \Rightarrow \phi_3 = \frac{7q}{10\varepsilon_0}$$

78. (12)



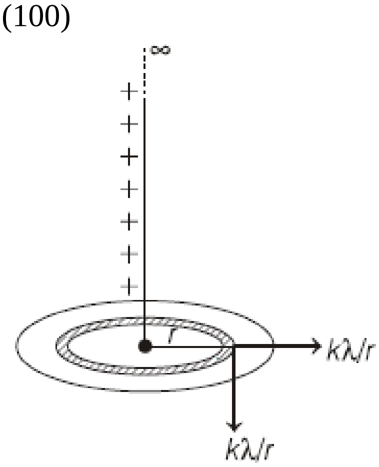
$$\phi_{ABCD} = 0$$

$$\phi_{CDGH} = \phi_{EFGH} = \frac{q}{24\varepsilon_0}$$

$$\phi = \phi_{ABCD} + \phi_{CDGH} + \phi_{EFGH}$$

$$= 0 + \frac{q}{24\varepsilon_0} + \frac{q}{24\varepsilon_0} = \frac{q}{12\varepsilon_0}, \text{ so } n = 12$$

79. (100)



Lets take a ring of radius r and thickness dr flux through the ring $= d\phi$

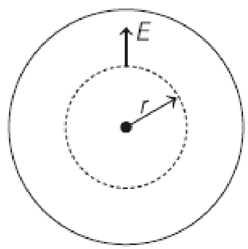
$$\Rightarrow \qquad d\phi = \left(\frac{k\lambda}{r}\right)(2\pi r dr) \;\Rightarrow\; \int d\phi = \frac{\lambda}{2\varepsilon_0}\int_0^R dr$$

$$\Rightarrow \qquad \phi = \frac{\lambda R}{2\varepsilon_0} = \frac{8.85\times10^{-10}\times 2}{2\times 8.85\times10^{-12}} = 100 \text{ V-m}$$

80. (8)

$\rho \propto h$

$\Rightarrow \quad \rho = Kh = k(R-r)$



$$d\phi = \mathbf{E}\cdot d\mathbf{A} = EdA\cos 0^\circ$$

$$\Rightarrow \int d\phi = E\int dA \quad \Rightarrow \phi = E(4\pi r^2)$$

Using Gauss's law,

$$\phi = E(4\pi r^2) = \frac{\int_0^r K(R-r)\,4\pi r^2 dr}{\varepsilon_0}$$

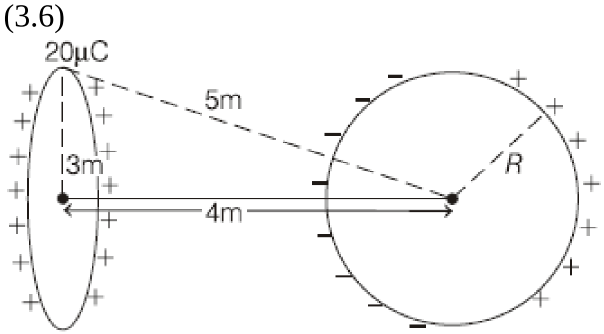
$$\Rightarrow \quad E = \frac{K\left(\frac{Rr^3}{3} - \frac{r^4}{4}\right)}{\varepsilon_0 r^2} \;\Rightarrow\; E = \frac{K}{\varepsilon_0}\left(\frac{Rr}{3} - \frac{r^2}{4}\right)$$

For maximizing electric field,

$$\frac{dE}{dr} = \frac{K}{\varepsilon_0}\left(\frac{R}{3} - \frac{2r}{4}\right) = 0 \;\Rightarrow\; r = \frac{2R}{3}$$

$$\text{Depth} = R - r = R - \frac{2R}{3} = \frac{R}{3} = \frac{24}{3} = 8 \text{ m}$$

81. (3.6)



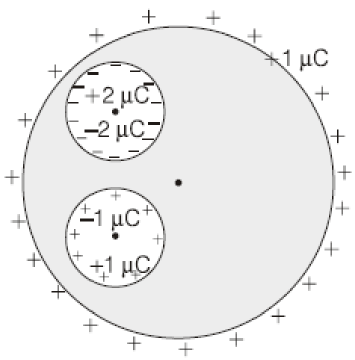
Potential of sphere,

Potential at the centre $= V_{\text{due to ring}} + V_{\text{due to induced charges}}$

$$= \frac{9\times 10^{9}\times 20\times 10^{-6}}{\sqrt{3^2+4^2}} + \frac{k(0)}{R}$$

$$= 3.6\times 10^{4}\text{ V}$$

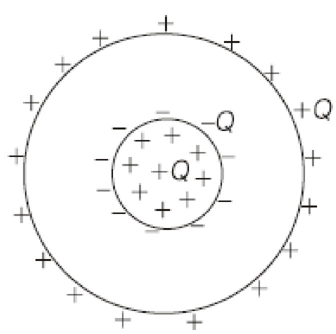
82. (9)



Electric potential at centre of conductor

$$= \frac{k(1\,\mu\text{C})}{(10\text{ cm})} = \frac{9\times 10^{9}\times 10^{-6}}{0.1} = 9\times 10^{4}\text{V}$$

83. (3)



$$U_1 = \frac{3kQ^2}{5R}$$

$$U_2 = \frac{3kQ^2}{5R} + \frac{k(-Q)^2}{2R} + \frac{kQ^2}{2(5R)} + \left[\frac{k(-Q)}{5R}\right]Q + \frac{(kQ)Q}{5R} + \frac{(kQ)}{R}(-Q)$$

$$\Rightarrow \quad U_2 = \frac{kQ^2}{5R} \quad \Rightarrow \quad \frac{U_1}{U_2} = \frac{\frac{3kQ^2}{5R}}{\frac{kQ^2}{5R}} = 3$$

84. (96)

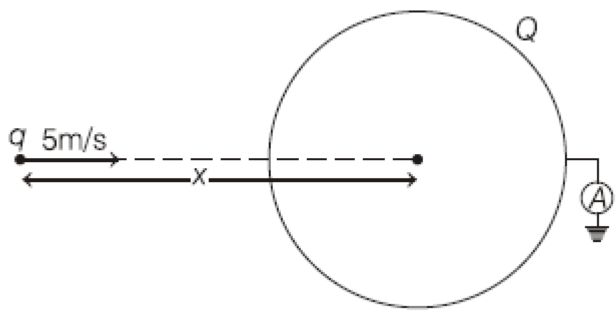
$$\frac{\sigma^2}{2\varepsilon_0} = \frac{4S}{R}$$

$$\Rightarrow \quad \frac{(Q/4\pi R^2)^2}{2\varepsilon_0} = \frac{4S}{R} \quad \Rightarrow \quad \frac{Q^2}{(16\pi^2 R^4)2\varepsilon_0} = \frac{4S}{R}$$

$$\Rightarrow \quad Q^2 = 96\pi\varepsilon_0 S\left(\frac{4}{3}\pi R^3\right) \Rightarrow \quad Q^2 = 96\pi\varepsilon_0 SV$$

So, $n = 96$

85. (800)



$$V_{\text{sphere}} = \frac{kq}{x} + \frac{kQ}{2} = 0$$

$$\Rightarrow \quad Q = -\frac{2q}{x}$$

$$\Rightarrow \quad \frac{dQ}{dt} = -2q\left(\frac{-1}{x^2}\right)\frac{dx}{dt}$$

$$= \frac{2 \times 2 \times 10^{-3}}{5^2} \times 5$$

$$= 0.8 \times 10^{-3}\ \text{A} = 800 \times 10^{-6}\ \text{A} = 800\ \mu\text{A}$$

CAPACITORS

1. (A)

$V = \frac{q}{C}$

$\Rightarrow \quad V = \frac{i \times t}{C}$

$\Rightarrow \quad 10 = \frac{50 \times 10^{-6} \times t}{800 \times 10^{-6}}$

$\Rightarrow \quad t = 160 \text{ s}$

2. (B)

$C = \frac{A\varepsilon_0}{d}$

$\frac{dC}{dt} = \frac{-A\varepsilon_0}{d^2} \frac{d(d)}{dt}$

$\Rightarrow \quad \frac{dC}{dt} = \frac{-A\varepsilon_0}{d^2} v$

$\Rightarrow \quad \frac{dC}{dt} \propto \frac{1}{d^2}$

3. (B)

Capacitance will get halved. So, charge on capacitor will also get halved as voltage across the capacitor is constant.

$E = \frac{V}{d}$

Since, V is constant and d is doubled, electric field will become half of its initial value.

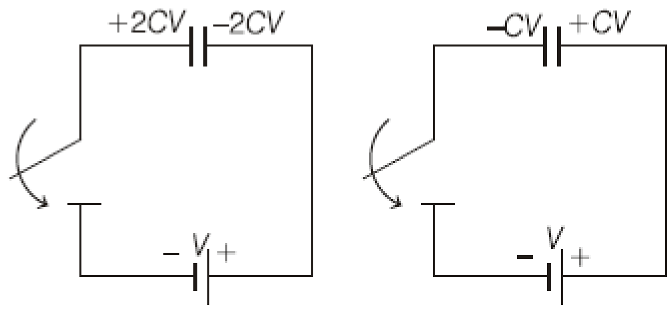
4. (C)

$U_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 12 \times 10^{-6} \times (10)^2$

$= 600 \,\mu\text{J}$

$U_2 = \frac{q^2}{2C_2} = \frac{(C_1 V)^2}{2C_2}$

5. (A)



Charge flown through battery $= CV - (-2CV)$

$= 3CV$

Work done by battery $= (3CV)V = 3CV^2$

$$\Delta U = U_2 - U_1$$

$$= \frac{1}{2}CV^2 - \frac{(2CV)^2}{2C}$$

$$= \frac{-3}{2}CV^2$$

Heat produced $= W_{\text{battery}} - \Delta U$

$= 3CV^2 - (-1.5\,CV^2)$

$= 4.5\,CV^2$

6. (C)

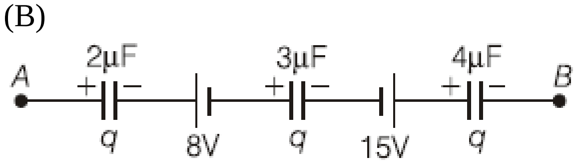
Energy loss $= \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$

$$= \frac{1}{2} \times \frac{4\times10^{-6}\times2\times10^{-6}}{6\times10^{-6}}(200-0)^2$$

$$= \frac{8}{3}\times10^{-2}$$

$$\%\ \text{loss} = \frac{8/3\times10^{-2}}{\frac{1}{2}\times4\times10^{-6}\times(200)^2}\times100 = 33.33\%$$

7. (B)



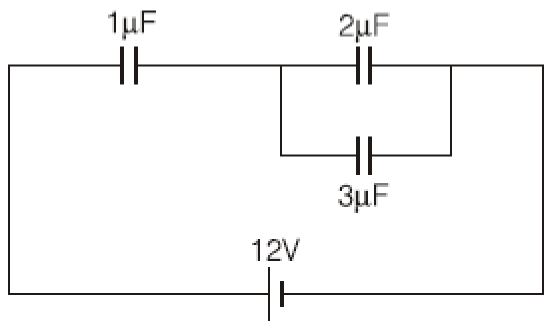
$$V_A - \frac{q}{2} - 8 - \frac{q}{3} + 15 - \frac{q}{4} = V_B$$

$$\Rightarrow \quad V_A - V_B = \left(\frac{q}{2} + \frac{q}{3} + \frac{q}{4}\right) - 7$$

$$\Rightarrow \quad 19 = \frac{13q}{12} - 7$$

$$\Rightarrow \quad q = 24\,\mu C$$

8. (B)



$$C_{eq} = \frac{5}{6}\mu F$$

Charge flown through battery $= \frac{5}{6} \times 12$

$= 10\,\mu C$

Charge on $2\,\mu F$ capacitor $= \frac{2}{2+3} \times 10$

$= 4\,\mu C$

9. (C)

Equivalent capacitance across battery $= 2\,\mu F$

Charge flown through battery $= 72 \times 2 = 144\,\mu C$

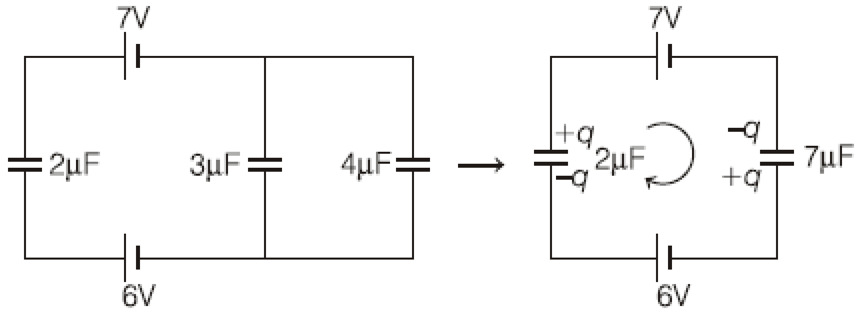
Charge on $1\,\mu F$ capacitor $= \left(\frac{1}{1+5}\right)\left(\frac{2}{2+4}\right)144$

$= 8\,\mu C$

Energy stored in $1\,\mu F$ capacitor $= \frac{q^2}{2C}$

$$= \frac{(8 \times 10^{-6})^2}{2 \times 1 \times 10^{-6}} = 32\,\mu J$$

10. (B)

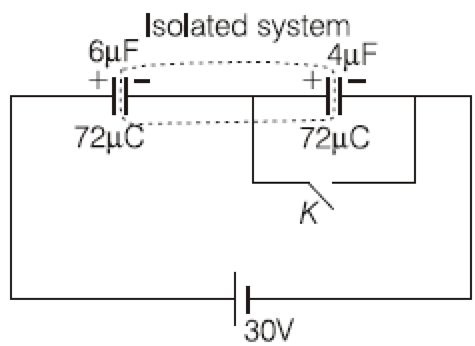


Applying KVL in the loop,

$$+\frac{q}{2}-7+\frac{q}{7}+6=0 \Rightarrow q=\frac{14}{9}\mu C$$

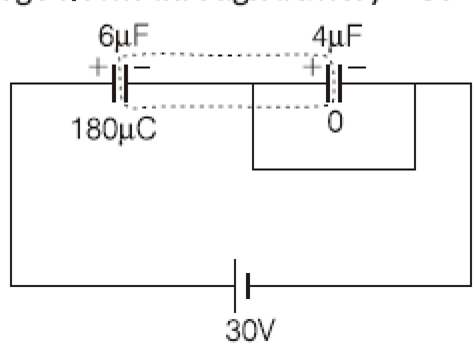
Charge on 3 μF capacitor $=\frac{3}{3+4}\times\frac{14}{9}=\frac{2}{3}\mu C$

11. (C)



$$C_{eq}=\left(\frac{1}{6}+\frac{1}{4}\right)^{-1}=2.4\,\mu F$$

Charge flown through battery $=30\times2.4=72\,\mu C$



When key is closed, 4 μF capacitor gets short-circuited.
Charge on 6 μF capacitor will be 180 μC.

Charge flown through the key = Change in charge of isolated system

$$=(180+0)-(-72+72)=180\,\mu C$$

12. (B)

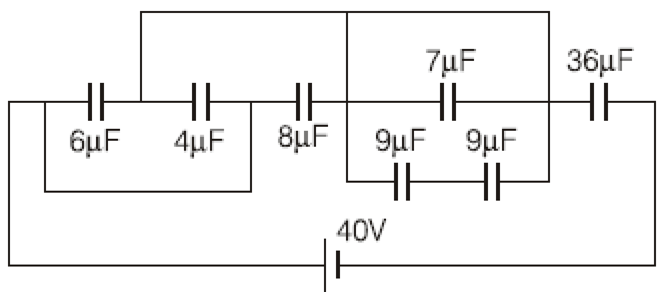
V_1 = Potential drop across $C_1 = 6V$

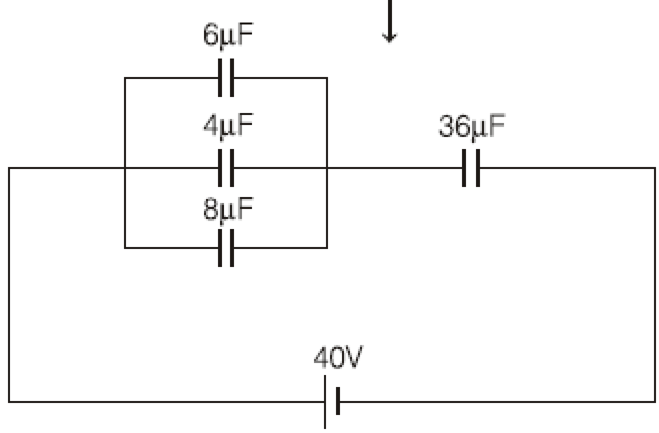
V_2 = Potential drop across $C_2 = 4V$

In series combination,

$$\frac{C_1}{C_2} = \frac{V_2}{V_1} \Rightarrow \frac{C_1}{C_2} = \frac{4}{6} = \frac{2}{3}$$

13. (C)





$$\frac{1}{C_{eq}} = \frac{1}{36} + \frac{1}{(6+4+8)}$$

$$C_{eq} = 12\,\mu F$$

Charge flown through battery $= 12 \times 40 = 480\,\mu C$

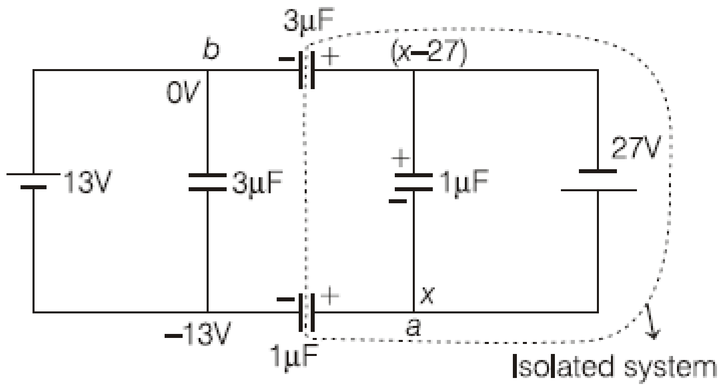
Charge on $8\,\mu F$ capacitor $= \frac{8}{18} \times 480 = 213.33\,\mu C$

$$\simeq 214\,\mu C$$

Potential difference across $8\mu F$ capacitor

$$= \frac{q}{C} = \frac{214}{8} \simeq 27V$$

14. (C)



Let $V_b = 0$ and $V_a = x$

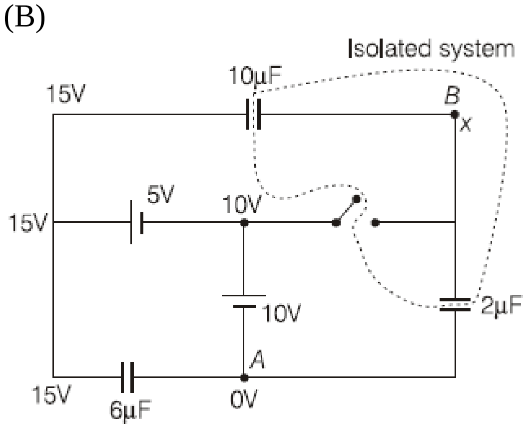
Applying charge conservation for the isolated system,

$\Rightarrow \quad 3[(x-27)-0]+1[x-(-13)]=0$

$\Rightarrow \quad x = 17\text{V}$

$V_a - V_b = 17 - 0 = 17\text{ V}$

15. (B)

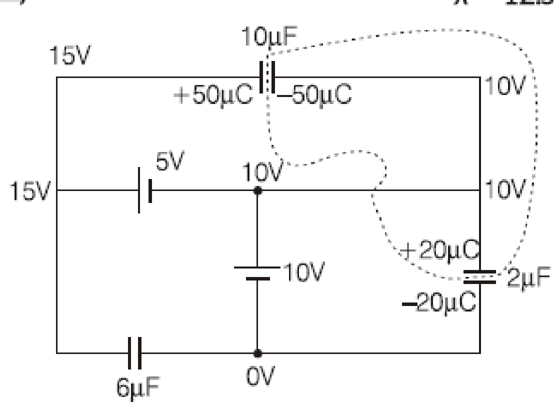


Let $V_A = 0$ and $V_B = x$

Total charge on the isolated system = 0

$\Rightarrow \quad 10(x-15)+2(x-0)=0$

$\Rightarrow \quad x = 12.5\text{ V}$



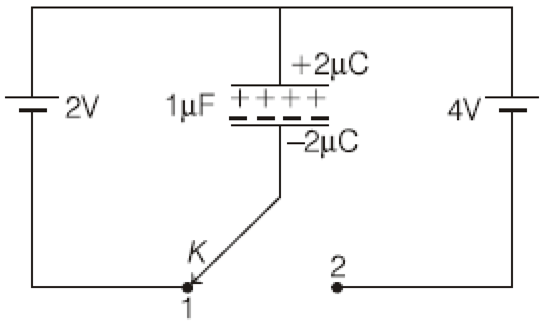
Change in charge of the isolated system

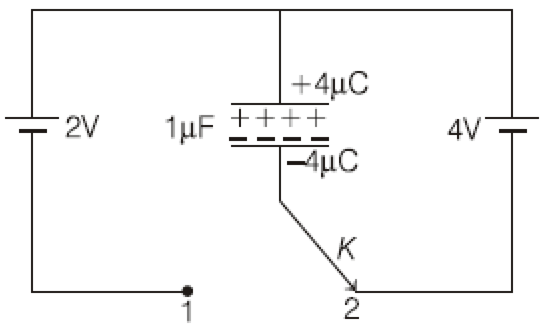
$$= (-50 + 20) - 0$$

$$= -30\,\mu C$$

Charge flown through switch $= 30\mu$ C

16. (A)





Charge flown through 4V cell $= 4 - 2 = 2\,\mu C$

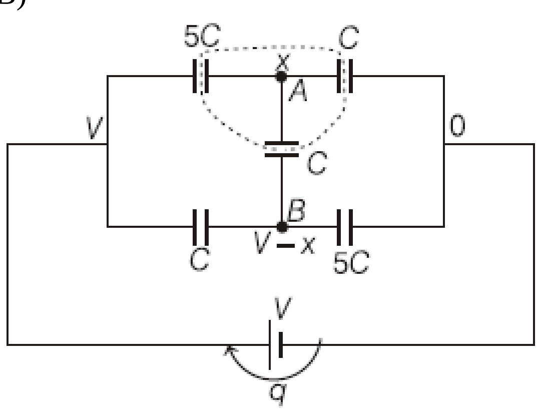
Work done by 4V cell $= 4 \times 2 = 8\,\mu J$

$$\Delta U = \frac{(4\times10^{-6})^2}{2\times1\times10^{-6}} - \frac{(2\times10^{-6})^2}{2\times1\times10^{-6}} = 6\,\mu J$$

Heat produced $= W_{cell} - \Delta U$

$$= 8 - 6 = 2\,\mu J$$

17. (B)



Applying KCL at junction A,

$5C(x-V)+C[x-(V-x)]+C(x-0)=0$

$x=\frac{3V}{4}$

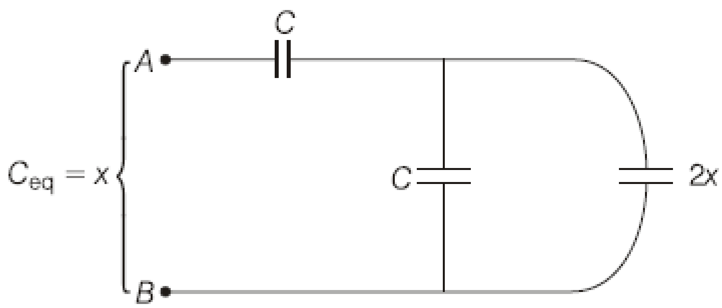
$q=\text{Charge flown the battery}=5C\left(V-\frac{3V}{4}\right)+C\left(\frac{3V}{4}\right)$

$=2CV$

$C_{eq}=\frac{q}{V}=\frac{2CV}{V}=2C$

18. (A)

Let equivalent capacitance between A and B be x.



$$\frac{1}{x}=\frac{1}{C}+\frac{1}{(2x+C)}$$

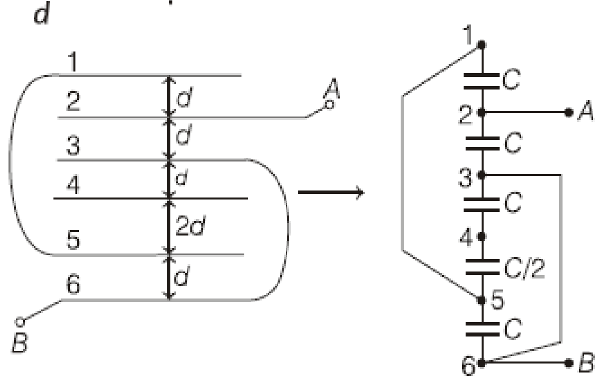
$$\Rightarrow \quad \frac{1}{x}=\frac{2x+2C}{C(2x+C)}$$

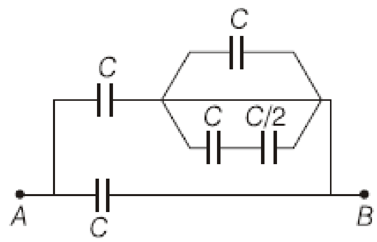
$$\Rightarrow \quad 2x^2+2Cx=2Cx+C^2$$

$$\Rightarrow \quad x=\frac{C}{\sqrt{2}}$$

19. (B)

Let $\frac{A\varepsilon_0}{d}=C=7\,\mu F$





$$C_{eq} = \frac{11C}{7} = \frac{11}{7}(7\,\mu F) = 11\,\mu F$$

20. (D)

Charge will remain same as the capacitor is isolated.

$$E = \frac{E_0}{K} \text{ and } V = \left(\frac{E_0}{K}\right)d = \frac{V_0}{K}$$

As dielectric constant becomes three times,

$$E = \frac{E_0}{3} \text{ and } V = \frac{V_0}{3}$$

21. (A)

$$C' = \frac{A\varepsilon_0}{(d-t)+\frac{t}{K}} \Rightarrow \frac{7}{6}\frac{A\varepsilon_0}{d} = \frac{A\varepsilon_0}{\frac{d}{3}+\frac{2d}{3K}}$$

$$\Rightarrow \frac{7}{6} = \frac{3K}{K+2} \Rightarrow K = \frac{14}{11}$$

22. (C)

$$V = Ed \Rightarrow 25 = E(5\,\text{mm})$$

$$\Rightarrow E = 5\ \text{V/mm}$$

$$V' = E(2\,\text{mm}) + \left(\frac{E}{K}\right)(3\,\text{mm})$$

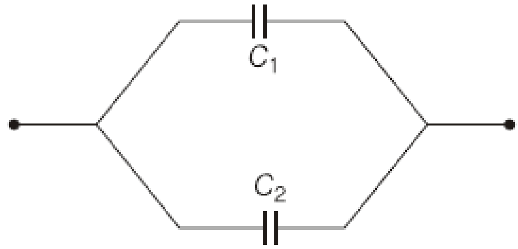
$$= 5\times 2 + \frac{5}{10}\times 3 = 11.5\ \text{V}$$

23. (A)

$$C_2 = \frac{C_1}{2} \Rightarrow \frac{A\varepsilon_0}{\frac{3}{4}+\frac{5}{\varepsilon_r}} = \frac{1}{2}\left(\frac{A\varepsilon_0}{\frac{3}{4}}\right)$$

$$\Rightarrow \varepsilon_r = \frac{20}{3}$$

24. (C)



$$C_1 = K\frac{A}{2}\frac{\varepsilon_0}{d}, \quad C_2 = \left(\frac{A}{2}\right)\frac{\varepsilon_0}{d}$$

$$C = C_1 + C_2$$

$$= K\frac{A}{2}\frac{\varepsilon_0}{d} + \frac{A}{2}\frac{\varepsilon_0}{d}$$

$$= (K+1)\frac{A\varepsilon_0}{2d}$$

25. (D)

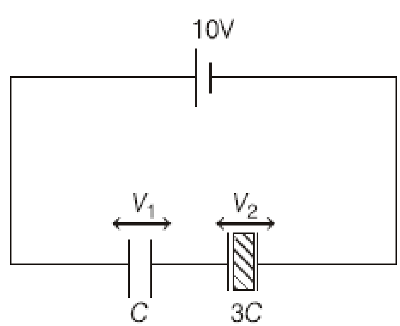
$$C = C_1 + C_2$$

$$= \frac{6\left[\pi\left(\frac{r}{2}\right)^2\right]\varepsilon_0}{d} + \frac{1\left[\pi r^2 - \pi\left(\frac{r}{2}\right)^2\right]\varepsilon_0}{d}$$

$$= \frac{9}{4}\frac{(\pi r^2)\varepsilon_0}{d}$$

$$= \frac{9C}{4}$$

26. (C)



$$V_1 + V_2 = 10 \qquad \text{...(i)}$$

and $$\frac{V_1}{V_2} = \frac{3}{1} \qquad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$V_1 = 7.5\text{ V}$$

and $$V_2 = 2.5\text{ V}$$

27. (C)

Let the final common potential drop be V.

$$q_1 + q_2 = q_1' + q_2'$$

$\Rightarrow$ $1 \times 100 - 2 \times 20 = (1 \times 4)\, V + 2V$

$\Rightarrow$ $V = 10\, V$

$$U_1 = \frac{(1 \times 100 \times 10^{-6})^2}{2 \times 4 \times 10^{-6}} + \frac{1}{2} \times 2 \times 10^{-6} \times (20)^2$$

$$= 1.65 \times 10^{-3}\, J$$

$$U_2 = \frac{1}{2} \times (4 \times 10^{-6})\,(10)^2 + \frac{1}{2} \times (2 \times 10^{-6})\,(10)^2$$

$$= 0.3 \times 10^{-3}\, J$$

Heat dissipated $= U_1 - U_2$

$$= 1.65 \times 10^{-3} - 0.3 \times 10^{-3}$$

$$= 1.35 \times 10^{-3}\, J$$

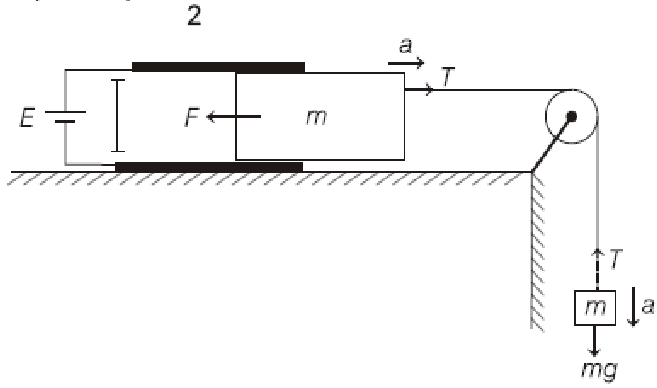
28. (C)

Force on dielectric slab due to capacitor

$$= \frac{1}{2} \frac{b\varepsilon_0 V^2}{d} (K - 1)$$

$\Rightarrow$ $$F = \frac{1}{2} \frac{b\varepsilon_0}{d} \left(\sqrt{\frac{mgd}{b(K-1)\varepsilon_0}} \right)^2 (K - 1)$$

$\Rightarrow$ $$F = \frac{mg}{2}$$



$$mg - T = ma \qquad ...(i)$$

$$T - F = ma \qquad ...(ii)$$

Solving equations (i) and (ii), we get

$$a = \frac{mg - F}{2m} = \frac{mg - \frac{mg}{2}}{2m} = \frac{g}{4}$$

29. (46)

Equivalent capacitance across battery $= C_{eq}$

$$\frac{1}{C_{eq}} = \frac{1}{12} + \frac{1}{10} + \frac{1}{5} \Rightarrow C_{eq} = \frac{60}{23}\,\mu F$$

Charge flown through battery $= C_{eq}E = \left(\frac{60}{23}\right)E$

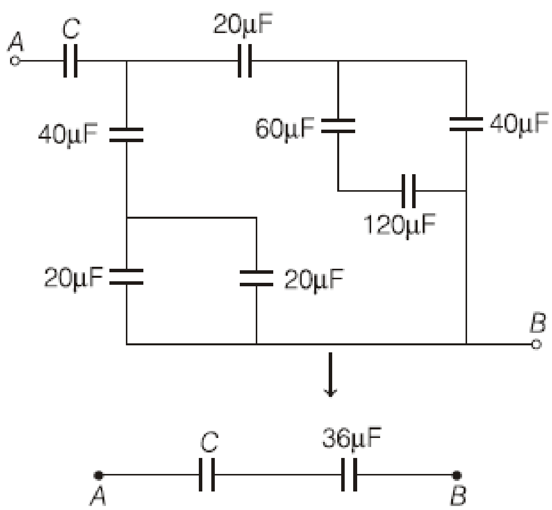
Charge on $12\,\mu F$ capacitor $= \frac{4}{4+6} \times \frac{60}{23}E = \frac{24E}{23}$

Potential difference across $12\,\mu F$ capacitor

$$= \frac{q}{C} = \frac{24E/23}{12} = \frac{2E}{23}$$

$$\frac{2E}{23} = 4 \Rightarrow E = 46V$$

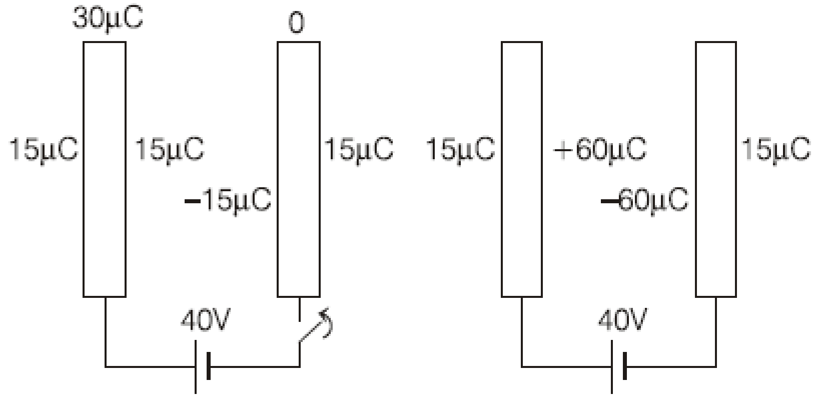
30. (18)



$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{36} \Rightarrow \frac{1}{12} = \frac{1}{C} + \frac{1}{36}$$

$\Rightarrow$ $C = 18\mu F$

31. (1800)



Charge flown through battery $= 60 - 15 = 45\,\mu C$

Work done by battery $= (45\,\mu C)(40\,V) = 1800\,\mu J$

32. (42)

$q_1 + q_2 = q_1' + q_2'$

$\Rightarrow \quad 8 \times 125 + 0 = (8 + C_0)20$

$\Rightarrow \quad C_0 = 42\,\mu F$

33. (24)

Charge on capacitor $q = CV$

$= 5 \times 120 \times 10^{-12} \times 50$

$= 30 \times 10^{-9}\,C = 30\,nC$

Induced charges on mica $= q\left(1 - \frac{1}{K}\right) = (30\,nC)\left(1 - \frac{1}{5}\right)$

$= 24\,nC$

34. (3)

For $x < 2$; $E_0 = -\frac{dV}{dx} = \frac{10-4}{2-0} = 3\,V/mm$

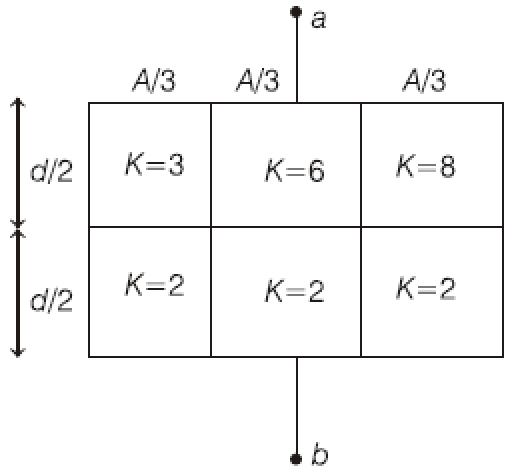
For $2 < x < 3$; $E = -\frac{dV}{dx} = \frac{4-3}{3-2} = 1\,V/mm$

$E = \frac{E_0}{K}$

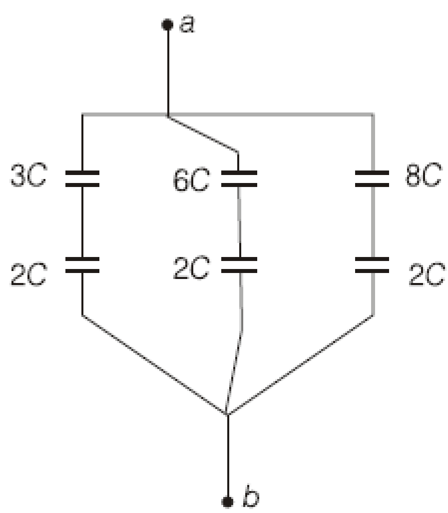
$\Rightarrow \quad 1\,V/mm = \frac{3\,V/mm}{K}$

$\Rightarrow \quad K = 3$

35. (43)



Let $$\frac{\frac{A}{3}\varepsilon_0}{\frac{d}{2}} = C \Rightarrow C = \frac{2A\varepsilon_0}{3d}$$



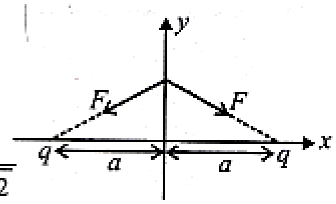
$$C_{eq} = \frac{(3C)(2C)}{3C+2C} + \frac{(6C)(2C)}{6C+2C} + \frac{(8C)(2C)}{8C+2C}$$

$$= \frac{43C}{10} = \frac{43}{10} \times \frac{2A\varepsilon_0}{3d}$$

$$= \frac{43}{15}\frac{A\varepsilon_0}{d}$$

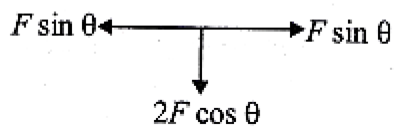
1. (A)

$$F_{net} = 2F\cos\theta$$



$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \quad (\because y << a)$$



$$\Rightarrow \frac{kq^2 y}{a^3} \quad \text{So, } F \propto y$$

2. (B)

$$F = qE = mg \;(q = 6e = 6\times 1.6\times 10^{-19})$$

$$\text{Density (d)} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi r^3} \quad \text{or } r^3 = \frac{m}{\frac{4}{3}\pi d}$$

Putting the value of d and m $m\left(=\frac{qE}{g}\right)$ and solving we get r = 7.8×10^{-7} m

3. (C)

(c) $q = 1\mu C = 1\times10^{-6}C, r = 4\text{ cm} = 4\times10^{-2}$ m

There will be no potential due to induced charges.

Potential $V = \frac{kq}{r} = \frac{9\times10^9\times10^{-6}}{4\times10^{-2}} = 2.25\times10^5$ V.

Induced electric field $E = -\frac{kq}{r^2}$

$[\because \vec{E}_i + \vec{E}_{1\mu c} = 0 \Rightarrow \vec{E}_i = -\vec{E}_{1\mu c}]$

$$= \frac{9\times10^9\times1\times10^{-6}}{16\times10^{-4}} = -5.625\times10^6 \text{ V/m}$$

4. (C)

The work done in moving a charge along an equipotential surface is always zero.
The direction of electric field is perpendicular to the equipotential surface or lines.

5. (C)

(c) As, $C = \frac{Q}{V} = \frac{It}{V}$

$$\Rightarrow \frac{V}{t} = \frac{I}{C} = \frac{2}{1\times10^{-6}} = 2\times10^6 \text{ V/s}$$

6. (C)

(c) Potential difference between any two points in electric field is given by,

$$dV = -\vec{E}\cdot\overrightarrow{dx} \Rightarrow \int_{V_O}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_O = -[10x^3]_0^2 = -80\,J/C$$

7. (A)

(a) Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by, $E = \dfrac{\sigma}{K\varepsilon_0}$

Then, charge density, $\sigma = K\varepsilon_0 E$
$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4 \approx 6 \times 10^{-7}\ C/m^2$

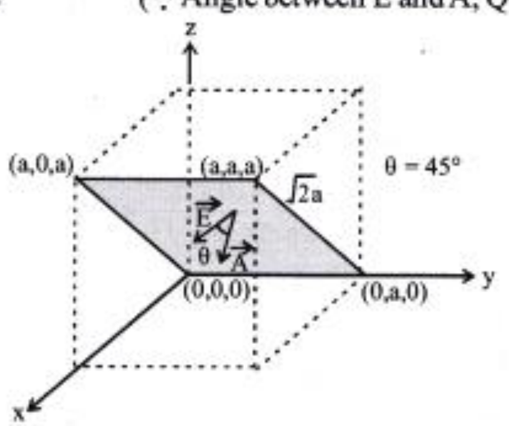
8. (C)

(c) Given $\vec{E} = E_0\,\hat{x}$

i.e, electric field $\vec{E}$ acts along $+x$ direction and is a constant. Therefore the electric flux through the shaded portion whose area $\vec{A} = a \times \sqrt{2}\,a = \sqrt{2}\,a^2$

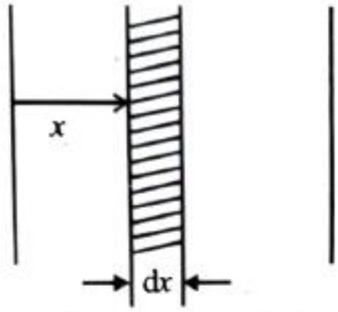
$$\phi = \vec{E}.\vec{A} = EA\cos\theta = E_0(\sqrt{2}a^2)\cos\ 45° = E_0(\sqrt{2}a^2)\times\frac{1}{\sqrt{2}}$$

$= E_0a^2$ $\quad$ ($\because$ Angle between E and A, Q = 45°)



9. (C)

(c)



Let us take an elemental capacitor of width 'dx'. Then,

$$C_{el} = \frac{(K_0 + \lambda x) A \in_0 d}{dx \ d} = \frac{(K_0 + \lambda x) C_0 d}{dx}, C_0 = \frac{A \in_0}{d}$$

$$\int \frac{1}{c_{el}} = \int_0^d \frac{dx}{(K + \lambda x) C_0 d} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{\lambda C_0 d} [\ln(K + \lambda x)]_0^d$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{\lambda C_0 d}\left[\ln\left(\frac{K + \lambda d}{K}\right)\right] \Rightarrow C_{eq} = \frac{\lambda C_0 d}{\ln\left(1 + \frac{\lambda d}{K}\right)}$$

10. (C)
Field line originate perpendicular from positive charge and terminate perpendicular from at negative charge. Further this system can be treated as an electric dipole.

11. (A)

(a) $\frac{KQ}{R} = V_0$

Now, $V = \frac{KQ}{r}$, $r \geq R$

$V = \frac{KQ}{2R^3}(3R^2 - r^2)$, $r < R$

when $r = R_1$, $V = \frac{3V_0}{2}$

$\therefore \frac{3V_0}{2} = \frac{3}{2}\frac{KQ}{R} - \frac{KQR_1^2}{R^3} \Rightarrow \frac{3V_0}{2} = \frac{3}{2}V_0 - \frac{KQ}{R^3} \cdot R_1^2$

$\Rightarrow R_1 = 0$

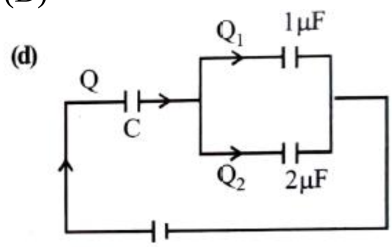
Similarly, $R_2 = \frac{R}{\sqrt{2}}, R_3 = \frac{4R}{3}, R_4 = 4R$

So, $R_4 - R_3 = \frac{8R}{3} > \frac{R}{\sqrt{2}}$

i.e. $R_2 < R_4 - R_3$

So, option (a) is correct.

12. (D)

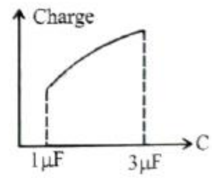


From figure, $Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q$

$Q = E\left(\frac{C\times 3}{C+3}\right)$

Therefore graph *d* correctly dipicts.

$\therefore Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$



13. (D)

Given: Length of wire $L = 20$ cm

charge $Q = 10^3\varepsilon_0$

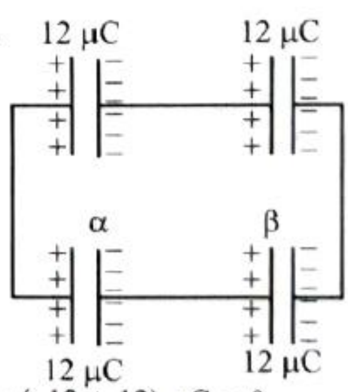
We know, electric field at the centre of the semicircular arc

$$E = \frac{2K\lambda}{r} \Rightarrow E = \frac{2K\left(\frac{2Q}{\pi r}\right)}{r}\left[As\,\lambda = \frac{2Q}{\pi r}\right]$$

$$= \frac{4KQ}{\pi r^2} = \frac{4KQ\pi^2}{\pi L^2} = \frac{4\pi KQ}{L^2} = 25\times 10^3\, N/C\,\hat{i}$$

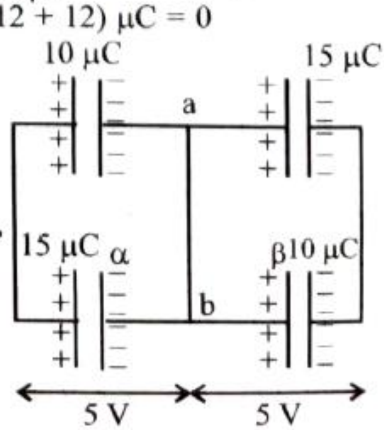
14. (A)

(a) Initially,



$(Q_\alpha + Q_\beta)_i = (-12 + 12)\ \mu C = 0$

Finally,



$(Q_\alpha + Q_\beta)_f = -15 + 10 = -5\ \mu C$
So, 5 μC charge from $b \to a$
so that not charge on plate α and β is zero.

15. (C)

Inside the cavity net charge is zero.
$\therefore\ Q_1 = 0$ and $\sigma_1 = 0$
There is no effect of point charges $+Q$, $-Q$ and induced charge on inner surface on the outer surface.
$\therefore\ Q_2 = 0$ and $\sigma_2 = 0$

16. (C)

(c) Applying Gauss's law

$$\oint_S \vec{E}\cdot\overrightarrow{ds} = \frac{Q}{\epsilon_0};\ Q = \int_a^r \frac{A}{r}4\pi r^2 dr = 2\pi A[r^2 - a^2]$$

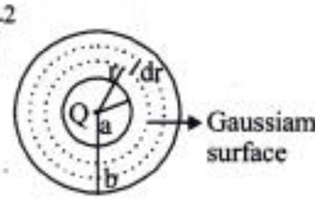
$$\therefore E \times 4\pi r^2 = \frac{Q + 2\pi A r^2 - 2\pi A a^2}{\epsilon_0}$$

$$E \times 4\pi r^2 = (Q - 2\pi A a^2) + 2\pi A r^2$$

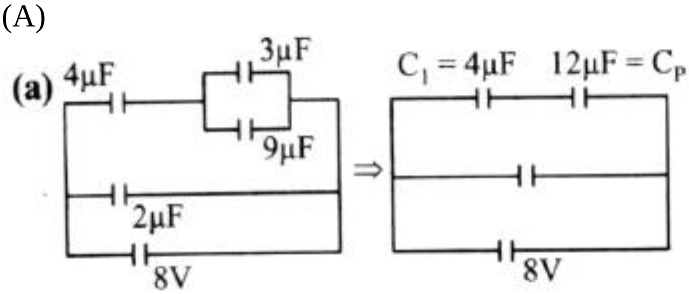
For E to be independent of 'r'

$$Q - 2\pi A a^2 = 0$$

$$\Rightarrow A = \frac{Q}{2\pi a^2}$$



17. (A)



By voltage division rule,

$$\text{Charge on } C_1 \text{ is } q_1 = \left[\left(\frac{12}{4+12}\right)\times 8\right]\times 4 = 24\mu C$$

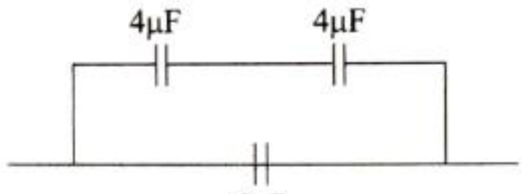
$$\text{The voltage across } C_P \text{ is } V_P = \frac{4}{4+12}\times 8 = 2V$$

$\therefore$ Voltage across 9μF is also 2V
$\therefore$ Charge on 9μF capacitor $= 9 \times 2 = 18\mu C$
$\therefore$ Total charge on 4 μF and 9μF $= 42\mu C$

$$\therefore E = \frac{KQ}{r^2} = 9\times 10^9 \times \frac{42\times 10^{-6}}{30\times 30} = 420\ NC^{-1}$$

18. (D)

(d) To get effective capacitance of 6 μF two capacitors of 4 μF each connected in sereies and one of 4 μF capacitor in parallel with them.



Two capacitances in series

$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

1 capacitor in parallel

$\therefore C_{eq} = C_3 + C = 4 + 2 = 6\,\mu F$

19. (A)

(a) $T = PE \sin\theta$ Torque experienced by the dipole in an electric field, $\vec{T} = \vec{P} \times \vec{E}$

$\vec{p} = p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$; $\vec{E}_1 = E\vec{i}$

$\vec{T}_1 = \vec{p} \times \vec{E}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E(\hat{i})$

$\tau\hat{k} = pE\sin\theta(-\hat{k})$ ...(i)

$\vec{E}_2 = \sqrt{3}E_1\hat{j}$; $\vec{T}_2 = p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times \sqrt{3}E_1\hat{j}$

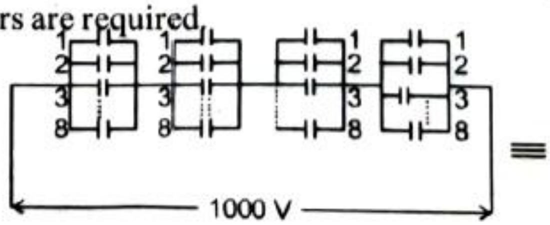
$\tau\hat{k} = \sqrt{3}\,pE_1\cos\theta\hat{k}$ ...(ii)

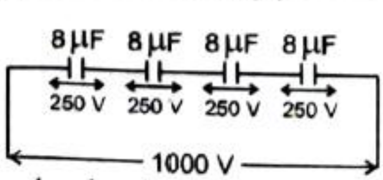
From eqns. (i) and (ii)

$pE\sin\theta = \sqrt{3}pE\cos\theta$; $\tan\theta = \sqrt{3}$ $\therefore$ $\theta = 60°$

20. (B)

(b) To get a capacitance of 2μF arrangement of capacitors of capacitance 1μF as shown in figure 8 capacitors of 1 μF in parallel with four such branches in series i.e., 32 such capacitors are required.





$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ $\therefore$ $C_{eq} = 2\,\mu F$

21. (C)

(c) The net flux linked with closed surfaces S_1, S_2, S_3 & S_4 are

For surface S_1, $\phi_1 = \frac{1}{\varepsilon_0}(2q)$

For surface S_2, $\phi_2 = \frac{1}{\varepsilon_0}(q+q+q-q) = \frac{1}{\varepsilon_0}2q$

For surface S_3, $\phi_3 = \frac{1}{\varepsilon_0}(q+q) = \frac{1}{\varepsilon_0}(2q)$

For surface S_4, $\phi_4 = \frac{1}{\varepsilon_0}(8q-2q-4q) = \frac{1}{\varepsilon_0}(2q)$

Hence, $\phi_1 = \phi_2 = \phi_3 = \phi_4$ i.e. net electric flux is same for all surfaces.

Keep in mind, the electric field due to a charge outside (S_3 and S_4), the Gaussian surface contributes zero net flux through the surface, because as many lines due to that charge enter the surface as leave it.

22. (C)

(c) Potential gradient is given by,

$\Delta V = E.d \Rightarrow 0.8 = Ed\,(\text{max})$

$\Delta V = Ed\cos\theta = 0.8 \times \cos 60 = 0.4$

Hence, maximum potential at a point on the sphere = 589.4 V

23. (D)

(d) Energy of sphere $= \frac{Q^2}{2C}$

$\Rightarrow 4.5 = \frac{16 \times 10^{-12}}{2C} \Rightarrow C = \frac{16 \times 10^{-12}}{9} = 4\pi\varepsilon_0 R$

(capacity of spherical conductor)

$R = \frac{16 \times 10^{-12}}{9} \times \frac{1}{4\pi\varepsilon_0} \qquad \because \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

$= 9 \times 10^9 \times \frac{16}{9} \times 10^{-12} = 16\text{ mm}$

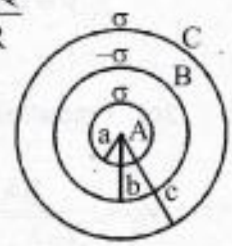
24. (B)

(b) Potential outside the shell, $V_{outside} = \frac{KQ}{r}$

where r is distance of point from the centre of shell

Potential inside the shell, $V_{inside} = \frac{KQ}{R}$

where 'R" is radius of the shell



$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

$$V_B = \frac{1}{4\pi \in_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\in_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

25. (A)

(a) $Q_i = Q_{Plate}\left(1 - \frac{1}{K}\right)$

$$= KCV\left(1 - \frac{1}{K}\right) = CV(K-1) = 90\,pF \times 2 \times \left(\frac{5}{3} - 1\right) = 1.2\,nC$$

26. (A)

Equilibrium position will shift to point where resultant force = 0

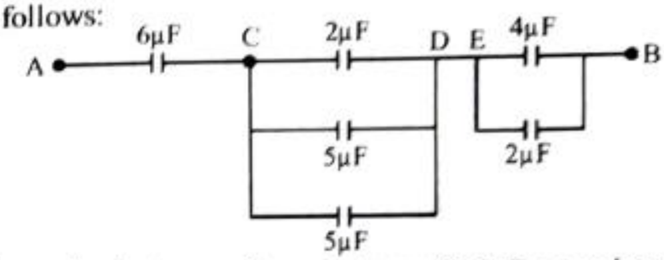
$$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$$

Total energy $= \frac{1}{2}m\omega^2A^2 + \frac{1}{2}kx_{eq}^2$

Total energy $= \frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$

27. (D)

(d) The simplified circuit of the circuit given in question as follows:



The equivalent capacitance between C & D capacitors of 2μF, 5μF and 5 μF are in parallel.

$\therefore$ $C_{CD} = 2 + 5 + 5 = 12$ μF ($\because$ In parallel grouping $C_{eq} = C_1 + C_2 + ... + C_n$)

Similarly equivalent capacitance between E & B $C_{EB} = 4 + 2 = 6$μF

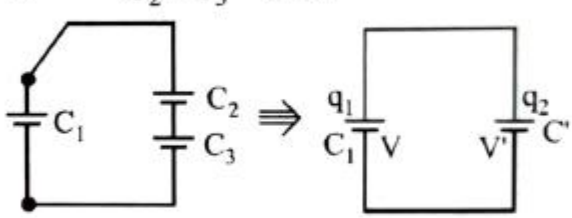
Now equivalent capacitance between A & B

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$$

$$\Rightarrow C_{eq} = \frac{12}{5} = 2.4\ \mu F \quad (\because \text{In series grouping, } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n})$$

28. (D)

(d) $C' = \frac{C_2C_3}{C_2 + C_3} = \frac{3 \times 6}{3 + 6} = 2\mu F$



As $V = V' \Rightarrow \frac{q_1}{C_1} = \frac{q_2}{C'} \Rightarrow \frac{q_1}{1} = \frac{q_2}{2} \Rightarrow q_1 = \frac{q_2}{2}$

and, $q_1 + q_2 = (60 \times 1)$

$\Rightarrow q_1 + 2q_1 = 60 \Rightarrow 3q_1 = 60 \Rightarrow q_1 = 20\mu C$

and, $q_2 = 60 - 20 = 40\ \mu C$.

29. (B)

(b) By Gauss law,

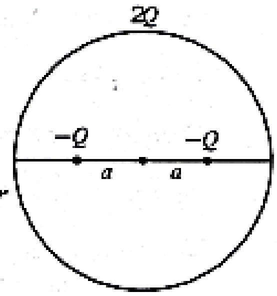
$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\epsilon_o} = \frac{Q + q - q}{\epsilon_o} = \frac{Q}{\epsilon_o}$$

or $E \times 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, r \geq R$

So, electric field outside is same as point charge inside the shell.

30. (A)

(a) $\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$



or $E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int S(4\pi r^2)dr$

$\Rightarrow E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r (kr)(4\pi r^2)dr$

or $E \times 4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\left(\frac{r^4}{4}\right)$

$\therefore \quad E = \frac{k}{4\varepsilon_0}r^2$ ...(i)

Also $2Q = \int_0^R (kr)\ (4\pi r^2)dr = 4\pi k\left|\frac{r^4}{4}\right|_0^R$

$Q = \frac{\pi k R^4}{2}$ ....(ii)

From above equations,

$E = \frac{Qr^2}{2\pi\varepsilon_0 R^4}$. For $r = a$, $E = \frac{Qa^2}{2\pi \epsilon_o R^4}$ ....(iii)

According to given condition

$QE = \frac{1}{4\pi \epsilon_o}\frac{Q^2}{(2a)^2} = Q\cdot\frac{Qa^2}{2\pi \epsilon_o R^4} = \frac{Q^2}{4\pi \epsilon_o \cdot 4a^2}$

$\Rightarrow R = a8^{1/4} \Rightarrow a = 8R^{-1/4}$

(b) $\tau = -PE\sin\theta$

$I\alpha = -PE\,(\theta) \Rightarrow \alpha = \frac{PE}{I}(-\theta)$

On comparing with

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$

31. (B)

(b) $\tau = -PE\sin\theta$

$I\alpha = -PE\,(\theta) \Rightarrow \alpha = \frac{PE}{I}(-\theta)$

On comparing with

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$

32. (B)

(b) Potential energy of a dipole is given by

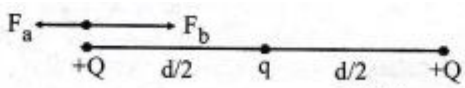
$U = -\vec{P}.\vec{E} = -PE\cos\theta$

[Where θ = angle between dipole and perpendicular to the field]

$= -(10^{-29})(10^{3})\cos 45^\circ$

$= -0.707 \times 10^{-26}\ J = -7 \times 10^{-27} J$

33. (A)



Force due to charge + Q, $F_a = \frac{KQQ}{d^2}$

Force due to charge q, $F_b = \frac{KQq}{\left(\frac{d}{2}\right)^2}$

For equilibrium, $\vec{F}_a + \vec{F}_b = 0$

$\Rightarrow \frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0 \qquad \therefore q = -\frac{Q}{4}$

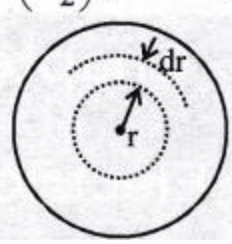
34. (B)

$$Q = \int \rho dv = \int_0^R \frac{A}{r^2} e^{-2r/a}\left(4\pi r^2 dr\right)$$

$$= 4\pi A \int_0^R e^{-2r/a} dr = 4\pi A \left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R = 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a} - 1\right)$$

$Q = 2\pi a A(1 - e^{-2R/a})$

$R = \frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi a A}}\right)$



35. (D)

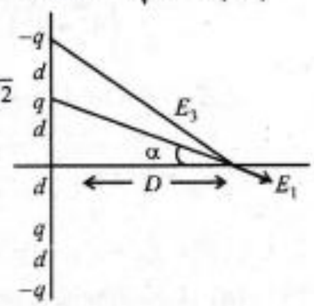
$\vec{E} = (\vec{E}_1 + \vec{E}_2) + (\vec{E}_3 + \vec{E}_4)$ or $E = 2E\cos\alpha - 2E\cos\beta$

$$= \frac{2kq}{(D^2 + d^2)} \times \frac{D}{\sqrt{D^2 + d^2}} - \frac{2kq}{(D^2 + (2d)^2} \times \frac{D}{\sqrt{D^2 + (2d)^2}}$$

$$= \frac{2kqD}{(D^2 + d^2)^{3/2}} - \frac{2kqD}{[D^2 + (2d)^2]^{3/2}}$$

For d << D

$E \propto \frac{D}{D^3} \propto \frac{1}{D^2}$

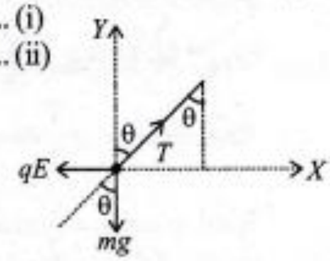


36. (D)

At equilibrium resultant force on bob must be zero, so

$T\cos\theta = mg$ ..... (i)

$T\sin\theta = qE$ ..... (ii)



Solving (i) and (ii) we get

$\tan\theta = \frac{qE}{mg}$

$\tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2}$

[Here, $q = 5\times10^{-6}$ C, E = 2000 v/m, m = 2×10^{-3} kg]

$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$

37. (B)

Electric field on the axis of a ring of radius R at a distance h from the centre,

$E = \frac{kQh}{(h^2+R^2)^{3/2}}$

Condition: for maximum electric field $\frac{dE}{dh} = 0$

$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{(R^2+h^2)^{3/2}}\right] = 0$

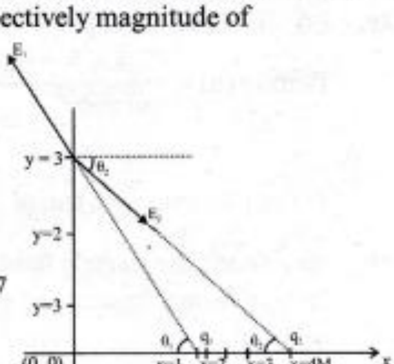
By using the concept of maxima and minima we get,

$h = \frac{R}{\sqrt{2}}$

38. (A)

Let $\vec{E}_1$ and $\vec{E}_2$ are the vaues of electric field due to charge, q_1 and q_2 respectively magnitude of

$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2}$



$= \frac{1}{4\pi\epsilon_0}\frac{\sqrt{10}\times10^{-6}}{(1^2+3^2)}$

$= (9\times10^9)\times\sqrt{10}\times10^{-7}$

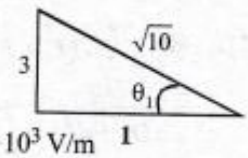
$= 9\sqrt{10}\times10^2$

$\therefore \vec{E}_1 = 9\sqrt{10}\times10^2\left[\cos\theta_1(-\vec{i}) + \sin\theta_1\vec{j}\right]$

$\Rightarrow E_1 = 9\times\sqrt{10}\times10^2\left[\frac{1}{\sqrt{10}}(-\hat{i}) + \frac{3}{\sqrt{10}}\hat{j}\right]$

$\Rightarrow E_1 = 9\times10^2\left[-\hat{i}+3\hat{j}\right] = \left[-9\hat{i}+27\hat{j}\right]10^2$

Similarly, $E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}$



$E_2 = \frac{9\times10^9\times(25)\times10^{-6}}{(4^2+3^2)}$ $E_2 = 9\times10^3$ V/m

$\therefore \vec{E}_2 = 9\times10^3(\cos\theta_2\hat{i} - \sin\theta_2\hat{j}) \because \tan\theta_2 = \frac{3}{4}$

$\therefore \vec{E}_2 = 9\times10^3\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right) = (72\hat{i} - 54\hat{j})\times10^2$

$\therefore \vec{E} = \vec{E}_1 + \vec{E}_2 = (63\hat{i} - 27\hat{j})\times10^2 \, V/m$

39. (B)

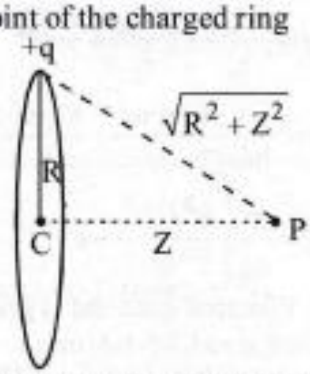
(b) The electric potential at the bisector is zero and electric field is antiparallel to the dipole moment.

$$\therefore V = 0 \text{ and } \vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-\vec{P}}{d^3}\right)$$

40. (C)

(c) Potential at any point of the charged ring

$$V_p = \frac{Kq}{\sqrt{R^2+Z^2}}$$



R = 3a, Z = 4a

$\ell = \sqrt{R^2+Z^2} = 5a$

The minimum velocity (v_0) should just sufficient to reach the point charge at the center, therefore

$$\frac{1}{2}mv_0^2 = q[V_C - V_P] = q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

$$\Rightarrow v_0^2 = \frac{4Kq^2}{15ma} = \frac{4}{15}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{ma}$$

$$\Rightarrow v_0 = \sqrt{\frac{2}{m}}\left(\frac{2q^2}{15\times4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$$

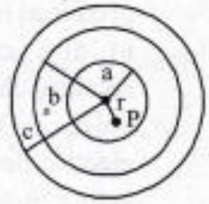
41. (B)

Electric potential is constant inside a charged spherical shell and outside it varsely with distance.

42. (D)

(d) Potential at point P, $V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$

Since surface charge densities are equal to one another i.e., $\sigma_a = \sigma_b = \sigma_c$



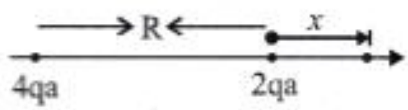
$\therefore Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$

$\therefore Q_a = \left[\frac{a^2}{a^2+b^2+c^2}\right]Q$, $Q_b = \left[\frac{b^2}{a^2+b^2+c^2}\right]Q$

$Q_c = \left[\frac{c^2}{a^2+b^2+c^2}\right]Q \quad \therefore V = \frac{Q}{4\pi\epsilon_0}\left[\frac{(a+b+c)}{a^2+b^2+c^2}\right]$

43. (D)

(d) Let at a distance 'x' from point B, both the dipoles produce same potential



$\therefore \frac{4qa}{(R+x)} = \frac{2qa}{(x^2)}$

$\Rightarrow \sqrt{2}x = R + x \Rightarrow x = \frac{R}{\sqrt{2}-1}$

Therefore distance from A at which both of them produce the same potential

$= \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}$

44. (C)

(c) Using conservation of energy

$U_i = U_F + \frac{1}{2}mv^2$; $\frac{kq_1q_2}{r_1} = \frac{kq_1q_2}{r_2} + \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}mv^2 = kq_1q_2\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \Rightarrow v^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= \frac{2\times 9\times 10^9 \times 10^{-12}}{4\times 10^{-6}\times 10^{-3}}\left[1 - \frac{1}{9}\right] = 4\times 10^{+6}$

$v = 2\times 10^3$ m/s

45. (D)

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D+\frac{d}{2}\right)} + \frac{(-q)Q}{\left(D-\frac{d}{2}\right)}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right], \text{Ignoring } \frac{d^2}{4}$$

46. (B)

(b) Using, $[K+U]_i = [K+U]_f$

or $0 + Vq = mv^2 + v'q$ or $mv^2 = (V - V')q$

$$= -q\int_{r_0}^{r} E dr = q\int_{r_0}^{r} \frac{\lambda}{2\pi \in_0 r} dr = \frac{\lambda q}{2\pi \in_0} \ln\left(\frac{r}{r_0}\right)$$

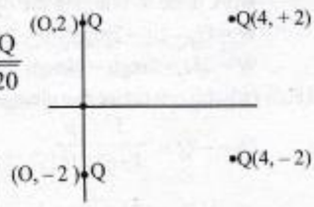
$$\Rightarrow v \propto \sqrt{l\,n\frac{r}{r_0}}$$

47. (B)

(b) Potential at origin

$$v = \frac{KQ}{2} + \frac{KQ}{2} + \frac{KQ}{\sqrt{20}} + \frac{KQ}{\sqrt{20}}$$

and potential at $\infty = 0$



$$= KQ\left(1 + \frac{1}{\sqrt{5}}\right)$$

$\therefore$ Work required to put a fifth charge Q at origin W = VQ

$$= \frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right)$$

48. (A)

(a) For parallel combination

$q = 10(C_1 + C_2)$

$q_1 = 500\ \mu C$

$500 = 10(C_1 + C_2)$

$C_1 + C_2 = 50 \mu F$ ....(i)

For Series Combination–

$$q_2 = 10\frac{C_1C_2}{(C_1 + C_2)}$$

$80 = 10\frac{C_1C_2}{50}$ From equation ....(ii)

$C_1C_2 = 400$ ....(iii)

From equation (i) and (ii)

$C_1 = 10\mu F$ $C_2 = 40\mu F$

49. (B)

(b) $U = U_f - U_i = \frac{q}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right)$

$$= \frac{(5\times 10)^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right)\times 10^6 = 3.75 \times 10^{-6} J$$

50. (B)

(b) Capacitance of a capacitor with a dielectric of dielectric constant k is given by

$$C = \frac{k \in_0 A}{d}$$

$\because\ E = \frac{V}{d} \qquad \therefore\ C = \frac{k \in_0 AE}{V}$

$$15 \times 10^{-12} = \frac{k \times 8.86 \times 10^{-12} \times 10^{-4} \times 10^{6}}{500}$$

$k = 8.5$

51. (B)

(b) Energy stored in the system initially

$$U_i = \frac{1}{2} CE^2$$

$$U_f = \frac{1}{2}\frac{Q^2}{C_{eq}} = \frac{(CE)^2}{2 \times 4C} = \frac{1}{2}\frac{CE^2}{4}$$

[As $Q = CE$, and $C_{eq} = 4C$]

$$\Delta U = \frac{1}{2} CE^2 \times \frac{3}{4} = \frac{3}{8} CE^2 = \frac{3}{8}\frac{Q^2}{C}$$

52. (C)

(c) $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

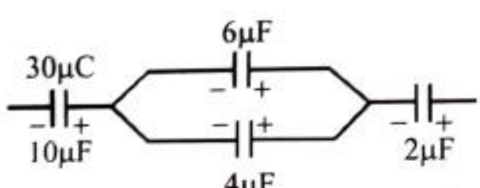
$\therefore Q = \varepsilon_0 . E. A$

$= 8.85 \times 10^{-12} \times 100 \times 1$

$= 8.85 \times 10^{-10} C$

53. (D)

(d)



As given in the figure, 6μF and 4μF are in parallel. Now using charge conservation.

Charge on 6μF capacitor $= \frac{6}{6+4} \times 30 = 18\mu C$

Since charge is asked on right plate therefore is $+18\mu C$

54. (B)

(b) $W = -\Delta U$

$= U_i - U_f$

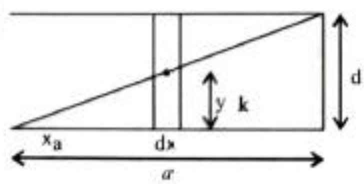
$$= U_i - \frac{U_i}{K} = U_i\left(1 - \frac{1}{K}\right) = \frac{1}{2} CV^2 \left(1 - \frac{1}{K}\right) = 508 PJ$$

55. (B)

(b) From figure, $\frac{y}{x} = \frac{d}{a} \Rightarrow y = \frac{d}{a}x$

$$dy = \frac{d}{a}(dx) \Rightarrow \frac{1}{dc} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$$

$$\frac{1}{dc} = \frac{1}{(dx)a\varepsilon_0}\left(\frac{y}{k} + d - y\right)$$



$$\int dc = \int \frac{\varepsilon_0 adx}{\frac{y}{k} + d - y}$$

$$\Rightarrow c = \varepsilon_0 a.\frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)}$$

$$= \frac{\varepsilon_0 a^2}{\left(\frac{1}{k} - 1\right)d}\left[\ell n\left(d + y\left(\frac{1}{k} - 1\right)\right)\right]_0^d$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right)$$

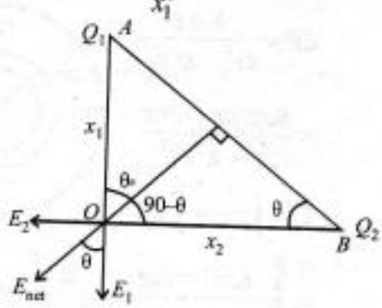
$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k \in_0 a^2 \ell nk}{(k-1)d}$$

56. (C)

Electric field due charge Q_2, $E_2 = \frac{kQ_2}{x_2^2}$

Electric field due charge Q_1, $E_1 = \frac{kQ_1}{x_1^2}$

From figure,



$$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2}$$

$$\Rightarrow \frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$$

$$\Rightarrow \frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{x_2}{x_1}$$

or, $\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$.

57. (C)

For spherical shell

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad (\text{if } r \geq R)$$

$$= 0 \qquad (\text{if } r < R)$$

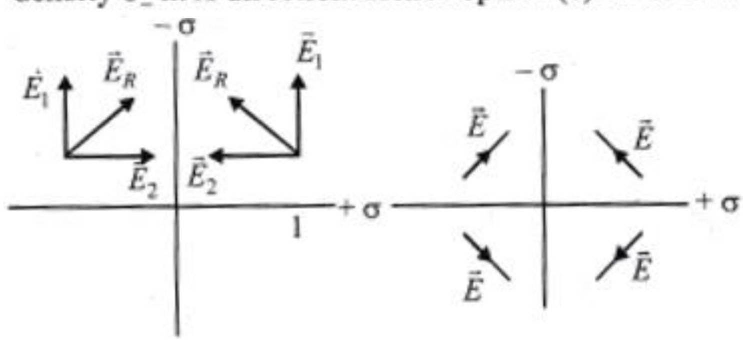
Force on charge in electried field, $F = qE$

$\therefore F = 0 \qquad$ (For $r < R$)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_q}{r^2} \qquad (\text{For } r > R)$$

58. (C)

The electric field produced due to uniformly charged infinite plane is uniform. So option (b) and (d) are wrong. And +ve charge density σ_+ is bigger in magnitude so its field along Y direction will be bigger than field of –ve charge density σ_- in X direction. Hence option (c) is correct.



59. (C)

Given, Electric field, $E = E_0(1-x^2)$

$\therefore$ Force, $F = qE = qE_0(1-x^2)$

Also, $F = ma = mv\frac{dv}{dx} \qquad \left(\because a = v\frac{dv}{dx}\right)$

$\therefore mv\frac{dv}{dx} = qE_0(1-x^2) \Rightarrow v\,dv = \frac{qE_0(1-x^2)dx}{m}$

Integrating both sides we get,

$\Rightarrow \int_0^v v\,dv = \int_0^x \frac{qE_0(1-x^2)dx}{m} \Rightarrow \frac{v^2}{2} = \frac{qE_0}{m}\left(x - \frac{9x^3}{3}\right) = 0$

$\Rightarrow x = \sqrt{\frac{3}{a}}$

60. (B)

$F_x = 0,\ a_x = 0,\ (v)_x = \text{constant}$

Time taken to reach at $'P' = \frac{d}{v_0} = t_0$ (let) ...(i)

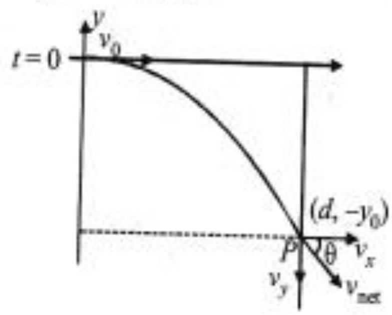
(Along – y), $y_0 = 0 + \frac{1}{2}\cdot\frac{qE}{m}\cdot t_0^2$ ...(ii)

$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m\cdot v_0}, \left(t = \frac{d}{v_0}\right)$

$\tan\theta = \frac{qEd}{m\cdot v_0^2}$, Slope $= \frac{-qEd}{mv_0^2}$

No electric field $\Rightarrow F_{net} = 0, \vec{v} =$ const.

$y = mx + c, \begin{Bmatrix} m = \frac{qEd}{mv_0^2} \\ (d, -y_0) \end{Bmatrix}$



$-y_0 = \frac{-qEd}{mv_0^2}, d + c$

$\Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$

$y = \frac{-qEd}{mv_0^2}x - y_0 + \frac{qEd^2}{mv_0^2} \Rightarrow y_0 = \frac{1}{2}\cdot\frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2}\frac{qEd^2}{mv_0^2}$

$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2}\frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$

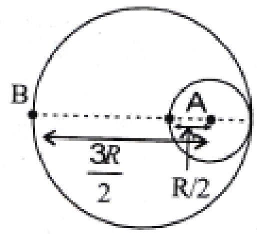
$y = \frac{-qEd}{mv_0^2} + \frac{1}{2}\frac{qEd^2}{mv_0^2} \Rightarrow y = \frac{\text{qEd}}{\text{mv}_0^2}\left(\frac{d}{2} - x\right)$

61. (B)

Electric field at A $\left(R'=\frac{R}{2}\right)$

$$E_A.ds=\frac{q}{\varepsilon_0}$$

$$\Rightarrow \vec{E}_A=\frac{\rho\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\varepsilon_0\cdot 4\pi\left(\frac{R}{2}\right)^2}$$



$$\Rightarrow \vec{E}_A=\frac{\sigma(R/2)}{3\varepsilon_0}=\left(\frac{\sigma R}{6\varepsilon_0}\right)$$

Electric fields at 'B'

$$\vec{E}_B=\frac{k\times\rho\times\frac{4}{3}\pi R^3}{R^2}-\frac{k\times\rho\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$\Rightarrow \vec{E}_B=\frac{\sigma R}{3\varepsilon_0}-\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(\sigma)}{\left(\frac{3R}{2}\right)^2}\frac{4\pi}{3}\left(\frac{R}{2}\right)^3$$

$$\Rightarrow \vec{E}_B=\frac{\sigma R}{3\varepsilon_0}-\frac{\sigma R}{54\varepsilon_0}\Rightarrow E_B=\frac{17}{54}\left(\frac{\sigma R}{\varepsilon_0}\right)$$

$$\left|\frac{E_A}{E_B}\right|=\frac{1\times 54}{6\times 17}=\left(\frac{9}{17}\right)=\frac{9}{17}\times\frac{2}{2}=\frac{18}{34}$$

62. (C)

Since $\vec{r}\cdot\vec{p}=0$

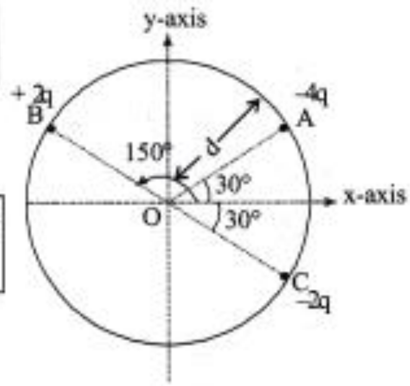
$\vec{E}$ must be antiparallel to $\vec{p}$

$\therefore\ \hat{E}$ is parallel to $\left(\hat{i}+3\hat{j}-2\hat{k}\right)$

63. (A)

Electric field due to charge +2q at centre O

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{d^2}\left[\frac{\sqrt{3}\hat{i} - \hat{j}}{2}\right]$$



Electric field due to charge $-2q$ at centre O

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{d^2}\left[\frac{\sqrt{3}\hat{i} - \hat{j}}{2}\right]$$

Electric field due to charge $-4q$ at centre O

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0} \times \frac{4q}{d^2}\left[\frac{\sqrt{3}\hat{i} + \hat{j}}{2}\right]$$

∴ Net electric field at point O

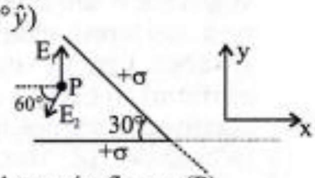
$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\sqrt{3}\,q}{\pi\varepsilon_0 d^2}\hat{i}$$

64. (D)

From figure, $\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{y}$ and

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}(-\cos 60°\hat{x} - \sin 60°\hat{y})$$

$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$



Electric field in the region shown in figure (P)

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\left[-\frac{1}{2}\hat{x} + \left(1 - \frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

$$\text{or, } \vec{E}_P = \frac{\sigma}{2\varepsilon_0}\left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$$

65. (B)

(b) Let v be the speed of dipole.
Using energy conservation

$K_i + U_i = K_f + U_f$

$\Rightarrow 0 - \frac{2k \cdot p_1}{r^3} p_2 \cos(180°) = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 0$

$\left(\because \text{Potential energy of interaction between dipole} \right.$

$\left. -\vec{P_1} \cdot \vec{E_2} = -\vec{P_2} \cdot \vec{E_1} = \frac{-2p_1p_2\cos\theta}{4\pi \epsilon_0 r^3} \right)$

$\Rightarrow mv^2 = \frac{2kp_1p_2}{r^3} \Rightarrow v = \sqrt{\frac{2kp_1p_2}{mr^3}}$

When $p_1 = p_2 = p$ and $r = a$

$v = \frac{p}{a}\sqrt{\frac{1}{2\pi \epsilon_0 ma}}$

66. (B)

(b) By Gauss law

$\oint \vec{E} \cdot \vec{da} = \frac{q_{in}}{\epsilon_o}$

If E = cons. and $\vec{E} \parallel_r \vec{A}$, then $|E||A| = \frac{q_{in}}{\epsilon_o}$

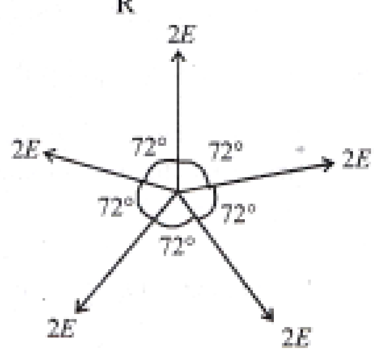
$\Rightarrow |E| = \frac{q_{in}}{\epsilon_o A}$ · And $\vec{E}$ is always $\perp$r to equipotential surface so, gaussian surface is equipotential.

67. (C)

(c) Potential at the centre, $V_C = \frac{KQ_{net}}{R}$

$\because Q_{net} = 0 \quad \therefore V_C = 0$

Let E be electric field produced by each charge at the centre, then resultant electric field will be $E_C = 0$, since equal electric field vectors are acting at equal angle so their resultant is equal to zero.



68. (A)

(a) We have given two metallic hollow spheres of radii R and $4R$ having charges Q_1 and Q_2 respectively.

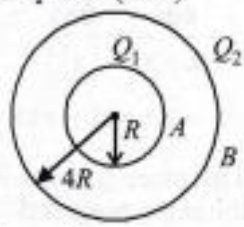
Potential on the surface of inner sphere (at A)

$$V_A = \frac{kQ_1}{R} + \frac{kQ_2}{4R}$$

Potential on the surface of outer sphere (at B)

$$V_B = \frac{kQ_1}{4R} + \frac{kQ_2}{4R}$$

$$\left(\text{Here, } k = \frac{1}{4\pi\varepsilon_0}\right)$$



Potential difference,

$$\Delta V = V_A - V_B = \frac{3}{4}\cdot\frac{kQ_1}{R} = \frac{3}{16\pi\epsilon_0}\cdot\frac{Q_1}{R}$$

69. (D)

(d) By using energy conservation,

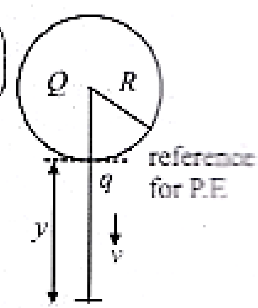
$$\Delta KE + (\Delta PE)_{\text{Electro}} + (\Delta PE)_{\text{gravitational}} = 0$$

$$\frac{1}{2}mV^2 + \left(k\frac{Qq}{R+y} - k\frac{Qq}{R}\right) + (-mgy) = 0$$

$$\Rightarrow \frac{1}{2}mV^2 = mgy + kQq\left(\frac{1}{R} - \frac{1}{R+y}\right)$$

$$\Rightarrow V^2 = 2gy + \frac{2kQq}{m}\frac{y}{R(R+y)}$$

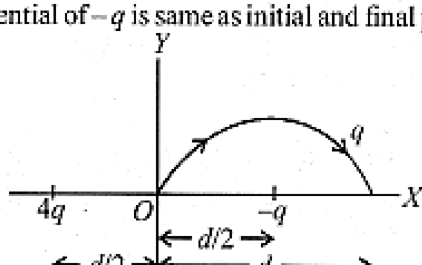
$$\text{or, } V^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$$



70. (D)

(d) Change in potential energy, $\Delta u = q(V_f - V_i)$

Potential of $-q$ is same as initial and final point of the path.

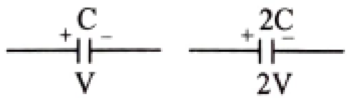


$$\Delta u = q\left(\frac{k4q}{3d/2} - \frac{k4q}{d/2}\right) = -\frac{4q^2}{3\pi\varepsilon_0 d}$$

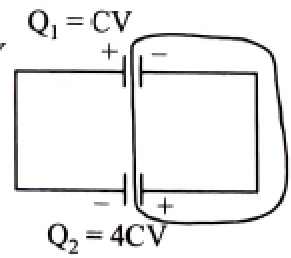
–ve sign shows the energy of the charge is decreasing.

71. (B)

(b) When capacitors C and $2C$ capacitance are charged to V and 2V respectively.



$Q_1 = CV \quad Q_2 = 2C \times 2V = 4CV$



When connected in parallel

By conservation of charge

$4CV - CV = (C + 2C)V_{common}$

$\Rightarrow V_{common} = \frac{3CV}{3C} = V$

Therefore final energy of this configuration,

$U_f = \left(\frac{1}{2}CV^2 + \frac{1}{2} \times 2CV^2\right) = \frac{3}{2}CV^2$

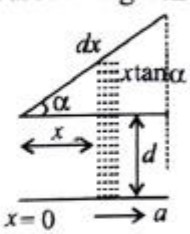
72. (A)

(a) Consider an infinitesimal strip of capacitor of thickness dx at a distance x as shown.

Capacitance of parallel plate capacitor of area A is given by $C = \frac{\varepsilon_0 A}{t}$

[Here t = seperation between plates]

So, capacitance of thickness dx will be



$\therefore dC = \frac{\varepsilon_0 a dx}{d + x\tan\alpha}$

Total capacitance of system can be obtained by integrating with limits x = 0 to x = a

$$\therefore C_{eq} = \int dC = a\varepsilon_0 \int_{x=0}^{x=a} \frac{dx}{x\tan\alpha + d}$$

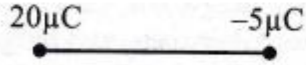
$$= a\varepsilon_0 \int_{x=0}^{x=a} \frac{dx}{d\left(1 + \frac{x}{d}\tan\alpha\right)}$$

[By Binomial expansion]

$$\Rightarrow C_{eq} = \frac{a\varepsilon_0}{d}\int_0^a \left(1 - \frac{x\tan\alpha}{d}\right)dx = \frac{a\varepsilon_0}{d}\left(x - \frac{x^2\tan\alpha}{2d}\right)_0^a$$

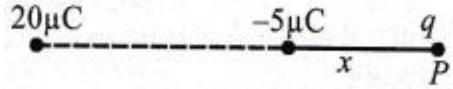
$$\Rightarrow C_{eq} = \frac{a^2\varepsilon_0}{d} = \left(1 - \frac{a\tan\alpha}{2d}\right) = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

73. (B)



Let, charge q be placed at P.

At point P forces due to 20 μC & $-5\mu C$ should be in opposite direction



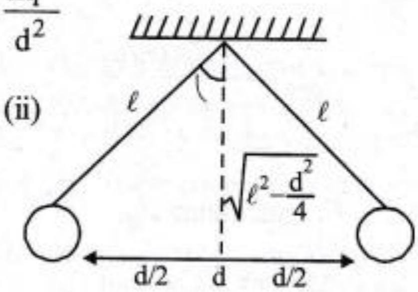
For net force $\vec{F} = 0$ & from coulomb's law force

$$F = \frac{k\, q_1\, q_2}{r^2} \Rightarrow k\frac{20q}{(5+x)^2} = \frac{k5q}{x^2} \Rightarrow x = 5\,\text{cm}$$

74. (B)

$$T\cos\theta = m\,g \quad \text{....... (i)}$$

Force due to charges $= \dfrac{kq^2}{d^2}$

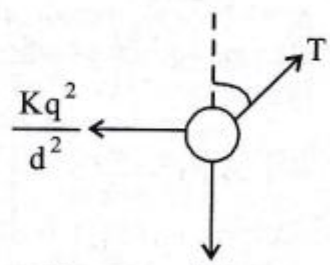


$$T\sin\theta = \frac{kq^2}{d^2} \quad \text{....... (ii)}$$

From (i) and (ii) we get

$$\tan\theta = \frac{\frac{kq^2}{d^2}}{m\,g}$$

as $\tan\theta \approx \sin\theta \approx \dfrac{d}{2\ell}$



$$\frac{kq^2}{m\,g\,d^2} = \frac{d}{2\ell}$$

$$\Rightarrow d^3 = \frac{2kq^2\ell}{m\,g}$$

$$\Rightarrow \quad d = \left(\frac{2kq^2\ell}{m\,g}\right)^{1/3} = \left(\frac{q^2\ell}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

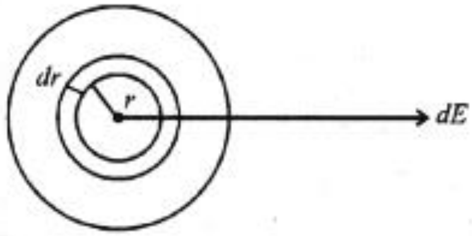
d

75. (A)

Consider a small ring of radius r and thickness dr on disc.

$dq = \sigma 2r\pi\, dr$

$dE = \frac{Kdqz}{(r^2+z^2)^{3/2}}$



$= \frac{K\sigma(2\pi r)drz}{(r^2+z^2)^{3/2}}$

$E = \int dE = \int dE$

$= \int_0^R \frac{K\sigma 2\pi r dr z}{\left(z^2+r^2\right)^{3/2}} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]$

76. (B)

Charge per unit length of the rod,

$\lambda = \left(\frac{-Q}{R\theta}\right) = \left(\frac{-Q}{R\frac{2\pi}{3}}\right) = \frac{-3Q}{2\pi R}$

$E = \int dE \cos\theta \Rightarrow E = \int_{-\pi/3}^{\pi/3} \frac{K\times(+\theta)}{\frac{2\pi}{3}R} \times \frac{Rd\theta}{R^2}\cos\theta$

$\Rightarrow E = \frac{3}{2\pi}\frac{K\theta}{R^2}[\sin\theta]_{-\pi/3}^{\pi/3} = \frac{3}{2\pi}\frac{K\theta}{R} \times \frac{2\sqrt{3}}{2}$

$\Rightarrow E = \frac{3\sqrt{3}Q}{8\pi^2 \epsilon_0 R^2}\left(+\hat{i}\right)$

77. (B)

The electrostatic field will balance the weight of oil drop,

$m_{oil} \times g_{oil} = qE$

$\Rightarrow \frac{4}{3}\pi r^3 \times \rho \times g = neE$ (n = no. of excess electrons)

$\Rightarrow n = \frac{\frac{4}{3}\pi r^3 \rho g}{eE} = \frac{\frac{4}{3}\times\pi\times(2\times10^{-3})^3\times(3\times10^3)\times 9.81}{1.6\times10^{-19}\times3.55\times10^5}$

$= 173.65\times10^8;\ 1.73\times10^{10}$

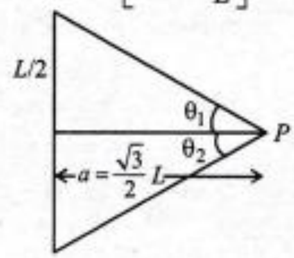
78. (D)

Electric field at point P

$$E = \frac{k\lambda}{a}(\sin\theta_1 + \sin\theta_2)$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{L} \times \frac{1}{\left(\frac{\sqrt{3}L}{2}\right)} \times 2\sin\theta \qquad \left[\because \lambda = \frac{Q}{L}\right]$$

From figure,



$$\tan\theta = \frac{\frac{L}{2}}{\frac{\sqrt{3}L}{2}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{2Q}{\sqrt{3}L^2} \times \left(\frac{2\times 1}{2}\right) \qquad \left[\because \sin 30^\circ = \frac{1}{2}\right]$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 L \times \sqrt{3}\frac{L}{2}} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

79. (B)

(b) At bottom surface, electric field is zero as $y = 0$

$\therefore$ Electric flux, $\phi_1 = 0$; At top surface, $y = 0.5$

$\therefore$ Electric flux, $\phi_2 = EA = (150y^2)(0.5)^2$

$= 150 \times (0.5)^2 \times (0.5)^2$

And flux through all other surface is zero because $\vec{E} \perp \vec{A}$ for each of them

$= \frac{150}{4}(0.5)^2 = \frac{150}{16} \quad \therefore \phi_{total} = \phi_1 + \phi_2 = \frac{150}{16}$

Using Gauss's law $\phi = \frac{Q_{in}}{\epsilon_0} \Rightarrow \frac{150}{16} = \frac{Q_{in}}{\epsilon_0}$

$\Rightarrow Q_{in} = \frac{150}{16} \times 8.85 \times 10^{-12} = 8.3 \times 10^{-11}\,C$

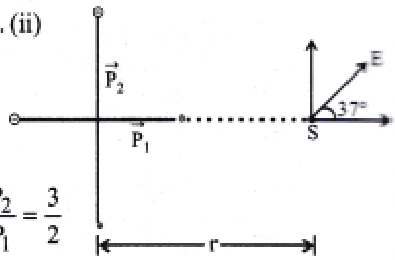
80. (C)

(c) Electric field due to $\vec{P}_1$ at axis point S

$$E_{axis} = \frac{2kP_1}{r^3} \quad \text{... (i)}$$

Electric field due to $\vec{P}_2$ at perpendicular bisector at point S.

$$E_{equator} = \frac{kP_2}{r^3} \quad \text{... (ii)}$$



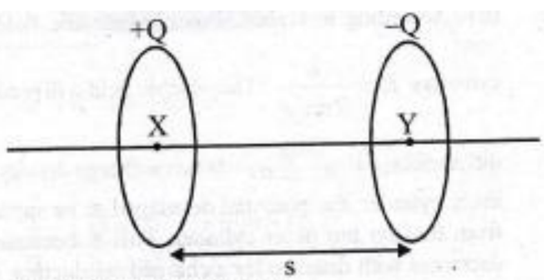
$$\tan 37° = \frac{E_{equator}}{E_{axis}}$$

$$\Rightarrow \frac{3}{4} = \frac{\frac{KP_2}{r^3}}{K\frac{2P_1}{r^3}} \Rightarrow \frac{P_2}{P_1} = \frac{3}{2}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{2}{3}$$

81. (D)

(d)



Potential at the centre of ring X

$$V_X = \frac{Q}{4\pi \epsilon_0 a} - \frac{Q}{4\pi \epsilon_0 \sqrt{a^2+s^2}}$$

Potential at the centre of ring Y

$$V_Y = \frac{-Q}{4\pi \epsilon_0 a} + \frac{Q}{4\pi \epsilon_0 \sqrt{a^2+s^2}}$$

$$V_X - V_Y = \frac{2Q}{4\pi \epsilon_0 a} - \frac{2Q}{4\pi \epsilon_0 \sqrt{a^2+s^2}}$$

$$= \frac{Q}{2\pi \epsilon_0}\left(\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right)$$

82. (C)

(c) $V = \dfrac{qd}{A \epsilon_0}$

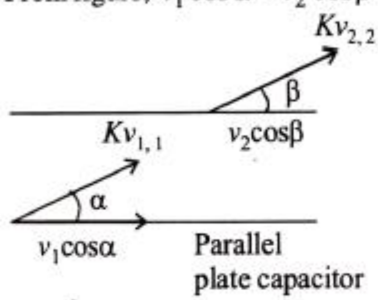
$i = \dfrac{V}{R} = \dfrac{qd}{A \epsilon_0 \rho \dfrac{d}{A}}$ $\quad \left(\because R = \rho \dfrac{d}{A}\right)$

$i_{max} = \dfrac{q_{max}}{\rho K \epsilon_0} \Rightarrow i_{max} = \dfrac{2 \times 10^{-12} \times 40}{200 \times 50 \times 8.85 \times 10^{-12}}$

$= 0.9\ mA$

83. (C)

(c) From figure, $v_1 \cos\alpha = v_2 \cos\beta$

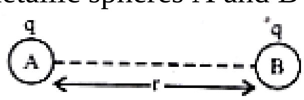


$\therefore \dfrac{K_1}{K_2} = \dfrac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \dfrac{v_1^2}{v_2^2} = \dfrac{\cos^2\beta}{\cos^2\alpha}$

84. (B)

Consider two metallic spheres A and B both have charge q.

$F = \dfrac{kq^2}{r^2}$

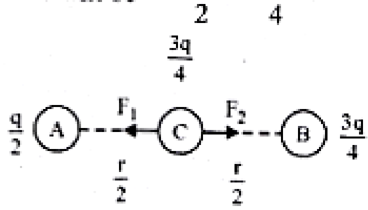


When sphere C is placed in contact with sphere A, charge on sphere A and sphere C will be $= \dfrac{q}{2}$

Now sphere C is placed in contact with sphere B, charge on sphere B and sphere C will be $= \dfrac{q + \frac{q}{2}}{2} = \dfrac{3q}{4}$

Now,

Then $F_1 = \dfrac{\frac{kq}{2} \cdot \frac{3q}{4}}{\frac{r^2}{4}}$



$F_2 = \dfrac{\frac{k3q}{4} \cdot \frac{3q}{4}}{\frac{r^2}{4}}$

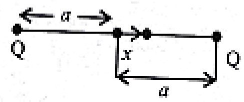
The force experienced by sphere C,

$F' = F_2 - F_1 = \dfrac{\left(k\frac{3q}{4} - k\frac{q}{2}\right)}{\frac{r^2}{4}} \cdot \dfrac{3q}{4} = \dfrac{3kq^2}{4r^2} = \dfrac{3F}{4}$

85. (A)

Force on charge 'Q'

$K = \frac{KQq_0}{(a-x)^2} - \frac{KQq_0}{(a+x)^2}$



$= KQq_0\left(\frac{4ax}{(a^2-x^2)^2}\right)$ as $a << x$

So, $F = \frac{KQq_0 \times 4ax}{a^4} = \frac{4Kxq_0Q}{a^3}$

As, $F = m\omega^2 x$

So, $m\omega^2 = \frac{4KQq_0}{a^3} \Rightarrow \omega = \sqrt{\frac{4KQq_0}{ma^3}} \Rightarrow T = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0Q}}$

86. (B)

Let distance between the two divided charges be r.

From Coulomb's law, force between two charge,

$$F = \frac{Kq(4-q)}{r^2}$$

For F to be maximum,

$$\frac{dF}{dq} = \frac{K}{r^2}[4-2q] = 0 \Rightarrow q = 2$$

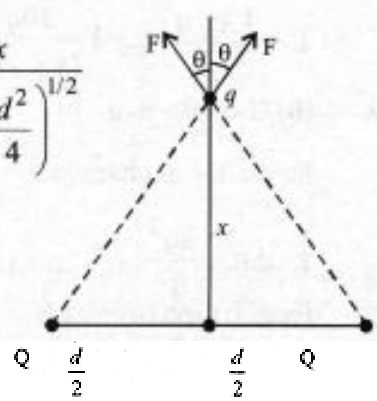
87. (D)

We have, from the given figure

$F_{net} = 2F\cos\theta$

$F_{net} = 2\frac{KQq}{x^2+\frac{d^2}{4}} \cdot \frac{x}{\left(x^2+\frac{d^2}{4}\right)^{1/2}}$

$F_{net} = \frac{2KQqx}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}$



For maximum F_{net}

$\frac{dF_{net}}{dx} = 0$

$\Rightarrow x \times -\frac{3}{2}\left(x^2+\frac{d^2}{4}\right)^{-5/2} .2x + \left(x^2+\frac{d^2}{4}\right)^{-3/2} = 0$

$\Rightarrow \left(x^2+\frac{d^2}{4}\right)^{-5/2}\left[-3x^2+x^2+\frac{d^2}{4}\right] = 0$

$\Rightarrow 2x^2 = \frac{d^2}{4} \Rightarrow x^2 = \frac{d^2}{8} \Rightarrow x = \frac{d}{2\sqrt{2}}$

88. (D)

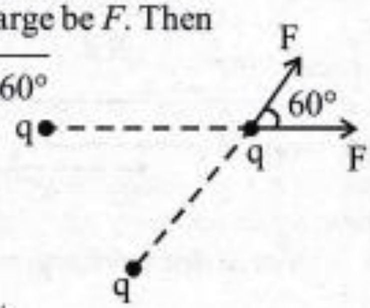
Let force between two charge be F. Then

$F_{net} = \sqrt{F^2 + F^2 + 2.F.F\cos 60^\circ}$

$= \sqrt{3}F$

so, required ratio

$= \frac{F_{net}}{F} = \frac{\sqrt{3}F}{F} = \sqrt{3}$



89. (B)

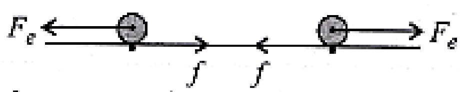
From law of conservation of charge $q_i = q_f$

$\Rightarrow 64q = Q \Rightarrow \frac{Q}{q} = 64$ and, $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$

$\Rightarrow R = 4r \Rightarrow \frac{r}{R} = \frac{1}{4}$

So, $\frac{\sigma_{bigger}}{\sigma_{smaller}} = \frac{\frac{Q}{4\pi R^2}}{\frac{q}{4\pi r^2}} = \frac{Q}{q} \times \left(\frac{r}{R}\right)^2 = 64 \times \frac{1}{16} = \frac{4}{1}$

90. (A)

(a) 

For charge to stay in equilibrium $F_e = f$

$\frac{kq^2}{L^2} = \mu mg$

$L = \sqrt{\frac{kq^2}{\mu mg}} = \sqrt{\frac{k}{\mu mg}}.q = \sqrt{\frac{9 \times 10^9}{0.25 \times 10^{-2} \times 10}} \times 2 \times 10^{-7}$

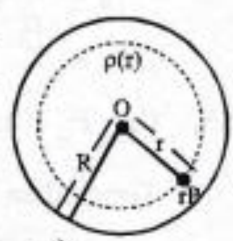
$= 12$ cm

91. (A)

(a) Here, $E.4\pi r^2 = \frac{\int_0^r \rho_0 \left(\frac{3}{4} - \frac{r}{R}\right) 4\pi r^2 dr}{\varepsilon_0}$

$\Rightarrow \quad E.4\pi r^2 = \frac{\rho_0 4\pi}{\varepsilon_0}\left(\frac{3}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right)$

$\Rightarrow \quad Er^2 = \frac{\rho_0 r^3}{4\varepsilon_0}\left(1 - \frac{r}{R}\right) \quad \Rightarrow E = \frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)$



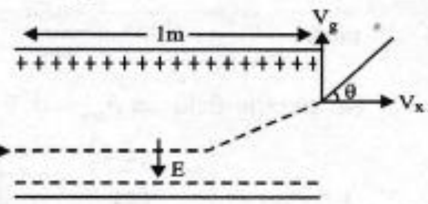
92. (B)

(b) $a_y = \frac{F_y}{m} = \frac{e(E)}{m} = \frac{e\left(\frac{8m}{e}\right)}{m} = 8\ m/s^2$

$s_x = u_x t$

$\Rightarrow 1 = 2 \times t$

$\Rightarrow t = \frac{1}{2}$ sec



and, $v_y = u_y + a_y t$

$\Rightarrow v_y = 0 + 8 \times \frac{1}{2} \Rightarrow v_y = 4$ m/s

$\tan\theta = \frac{V_y}{V_x} = \frac{4}{2} = 2$

$\Rightarrow \theta = \tan^{-1}(2)$

93. (D)

Here $a = \frac{qE}{m} = \frac{40 \times 10^{-6} \times 10^5}{100 \times 10^{-6}} = 40000\ m/s^2$

As charge particle is moving opposite to direction of $\vec{E}$ So $\vec{E}$ will declerate the charge and charge will come to rest.

Now, $v^2 = u^2 + 2as$

$0 = 200^2 + 2 \times (-40 \times 10^3) \times s$

$80 \times 10^3 s = 4 \times 10^4$

$\therefore s = \frac{4 \times 10^4}{80 \times 10^3} = \frac{40}{80} = 0.5$ m.

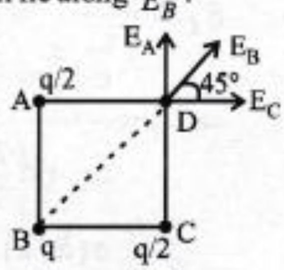
94. (A)

(a) Here, sum of $\vec{E}_C$ and $\vec{E}_A$ will lie along $\vec{E}_B$.

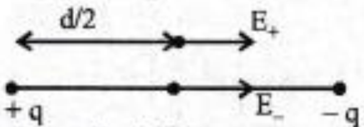
So, $\left|\vec{E}_{net}\right| = \left|\vec{E}_B\right| + \left|\vec{E}_C + \vec{E}_B\right|$

$= \frac{Kq}{(\sqrt{2}a)^2} + \left[\sqrt{2}\frac{kq}{2a^2}\right]$

$= \frac{kq}{a^2}\left[\frac{1}{2} + \frac{1}{\sqrt{2}}\right]$



95. (B)



So, $E_{net} = E_+ + E_-$

$$= 2\frac{kq}{\left(\frac{d}{2}\right)^2} = \frac{8kq}{d^2} \Rightarrow 6.4 \times 10^4 = 8 \times 9 \times 10^9 \times \frac{8 \times 10^{-6}}{d^2}$$

$$\Rightarrow d^2 = \frac{72 \times 8 \times 10^{+3}}{6.4 \times 10^4} \Rightarrow d^2 = 9 \Rightarrow d = 3\text{ m}$$

96. (B)

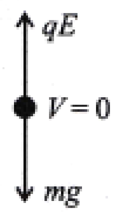
As water droplet is at rest

So, $\overline{F}_{\text{net}} = 0$

$$\Rightarrow mg = qE \Rightarrow q = \frac{mg}{E}$$

$$\Rightarrow q = \frac{0.1 \times 10^{-3} \times 9.8}{4.9 \times 10^5}$$

$$\Rightarrow q = 2 \times 10^{-9}\,C$$



97. (None)

(None) Total flux through complete spherical surface is $\frac{q}{\epsilon_0}$.

So flux, through curved surface will be $\frac{q}{2\epsilon_0}$.

The flux through flat surface will be zero.

98. (C)

Electric field due to infinite sheet is given by $E = \frac{\bar{0}}{2\epsilon_0}$,

clearly $|\vec{E}|$ is independent of distance

So, $E_1 = E_2 = \frac{\bar{0}}{2\epsilon_0}$

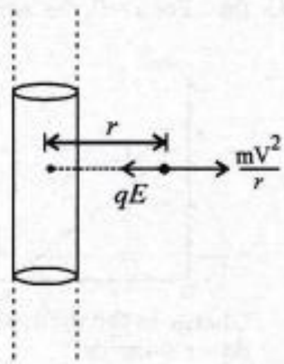
99. (A)

(a) As charge particle is moving in circular path. So,

$$qE = \frac{mv^2}{r}$$

$$\Rightarrow mv^2 = qEr$$

$$\Rightarrow \frac{mv^2}{2} = \frac{1}{2}qEr \quad ...(i)$$

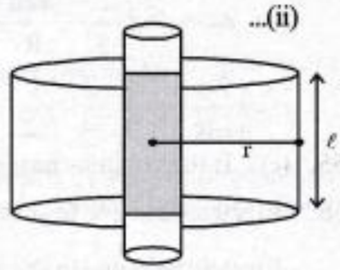


Now, by Gauss's law

$$E \times 2\pi rl = \frac{q_{in}}{\epsilon_0}$$

$$E \times 2\pi rl = \frac{\rho \times \pi R^2 l}{\epsilon_0}$$

$$E = \frac{\rho R^2}{2\epsilon_0 r} \quad ...(ii)$$



From (i) and (ii), we get

$$\frac{1}{2}mv^2 = \frac{1}{2}q \times \frac{\rho R^2}{2\epsilon_0 r} \times r$$

$$\Rightarrow K.E. = \frac{\rho q R^2}{4\epsilon_0}$$

100. (B)

(b) Since, $V_A = V_B$

$$\Rightarrow \frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}.$$

Here, $K = \frac{1}{4\pi\epsilon_o} \Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B} = \frac{1}{2}$

$$E_A = \frac{kQ_A}{R_A^2}; E_B = \frac{kQ_B}{R_B^2} \Rightarrow \frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{R_B^2}{R_A^2} = \frac{R_B}{R_A} = \frac{2}{1}$$

101. (D)

(d) We have

$V = 3x^2$ Volt

$$E_x = \frac{-\partial V}{\partial x} = -6x \Rightarrow E_{x/x=1} = -6$$

$$E_y = -\frac{\partial V}{\partial y} = 0$$

$$E_z = -\frac{\partial V}{\partial z} = 0$$

So, $|\vec{E}| = \sqrt{-6^2 + 0^2 + 0^2} = 6$

In vector form, $\vec{E} = -6\hat{i} V/m$

102. (A)

(a) Equivalent capacitance in series,

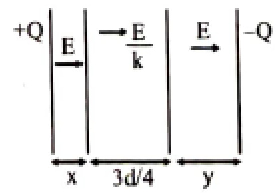
$$C_{eq} = \frac{C(KC)}{C + KC} = \frac{KC}{K+1}$$

$$24 = \frac{K40}{K+1} \qquad \left[\because C_{eq} = 24\mu F\right]$$

$$24(K+1) = 40K \therefore K = 1.5$$

103. (A)

(a) We have, $x + y + \frac{3d}{4} = d$



$$\Rightarrow x + y = \frac{d}{4}$$

$$\Delta V = Ex + \frac{E}{k} \times \frac{3d}{4} + Ey$$

$$\Rightarrow \Delta V = \frac{3Ed}{4k} + E(x + y) \Rightarrow \Delta V = E\left[\frac{3d}{4k} + \frac{d}{4}\right]$$

$$\Delta V = \frac{\sigma}{\epsilon_0}\left[\frac{3d + dk}{4k}\right] = \frac{Qd}{A\epsilon_0}\left[\frac{3 + k}{4k}\right]$$

$$\frac{Q}{\Delta V} = C = \frac{A\epsilon_0}{d}\left[\frac{4k}{3 + k}\right] = \frac{4kC_0}{k + 3} \qquad \left[\because C_0 = \frac{A\epsilon_0}{d}\right]$$

104. (A)

(a) Given that all capacitors are connected in parallel so, the equivalent capacitance will be $C_{eq} = C_1 + C_2 + C_3 + C_4$ $= 1 + 2 + 4 + 3 = 10\ \mu F$

Voltage of battery $V = 20$ V

We have, $Q = CV = 10\ \mu F \times 20 = 200\ \mu C$

105. (A)

(a) As $Q = CV$

$\Rightarrow Q \propto V$. so, graph will be straight line between V and Q.

Now, $Q_{max} = CV_{max}$

$$\Rightarrow V_{max} = \frac{Q_{max}}{C} = \frac{5}{2 \times 10^{-6}} = 2.5 \times 10^6 \text{ V}$$

So, most suitable option is (A).

106. (D)

(d) We have

$$U = \frac{Q^2}{2C} = \frac{Q_0^2 e - 2t/\tau}{2C} = U_0 e^{-2t/\tau}, U = e^{-2t/\tau}$$

as, $U_{t=t_1} = \frac{U_0}{2}$

$$U_0 e^{-2t_1/\tau} = \frac{U_0}{2}$$

$$e^{-t_1/\tau} = \frac{1}{2} \Rightarrow t_1 = \frac{\tau}{2} ln2 \text{ and, } Q_{t=t_2} = \frac{Q_0}{8}$$

$$Q_0 e^{-t_2/\tau} = \frac{Q_0}{8} \Rightarrow e^{-t_2/\tau} = \frac{1}{8} \Rightarrow t_2 = \tau\, ln8 \Rightarrow t_2 = 3\tau ln2$$

So, $\frac{t_1}{t_2} = \frac{\frac{\tau}{2} ln2}{3\tau ln2} = \frac{1}{6}$

107. (A)

(a) $\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20} = \frac{6+4+3}{60} = \frac{60}{13} \mu F$

So, $Q = C_{eq} V = \frac{60}{13} \times 13 \mu C = 60 \mu C$

In series, charge across each capacitor is same and is equal to net charge.

So, $Q_{15\mu F} = 60 \mu C$

108. (C)

(c) We have, $C_i = \frac{A\epsilon_0}{d} = 4\mu F$

$$C_f = \frac{A\epsilon_0}{d - t + \frac{t}{k}} = \frac{A\epsilon_0}{d - \frac{d}{2} + \frac{d}{2 \times 3}} = \frac{A\epsilon_0}{d\left(1 - \frac{1}{2} + \frac{1}{6}\right)}$$

$$= \frac{4\mu F}{\frac{2}{3}} = 6\mu F$$

109. (D)

(d) $q = CV$

$$\Rightarrow q = \frac{kA\epsilon_0}{d}.Ed \Rightarrow k = \frac{q}{A\epsilon_0 E}$$

$$= \frac{7 \times 10^{-6}}{30\pi \times 10^{-4} \times \frac{1}{4\pi \times 9 \times 10^9} \times 3.6 \times 10^7} = \frac{7}{3} = 2.33$$

110. (A)

(a) $U = \frac{q^2}{2c}$. As C = constant, $U \propto q^2$

So, $\frac{U_2}{U_1} = \left(\frac{q_2}{q_1}\right)^2 \Rightarrow q_1^2 = \frac{U_1}{U_2} q_2^2$

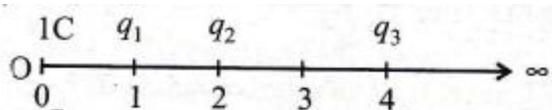
$\Rightarrow q_1^2 = \frac{U}{1.44U} \times (q_1 + 2)^2 . \left(\frac{q_1}{q+2}\right)^2 = \frac{1}{1.44}$

$\Rightarrow \frac{q_1}{q_1 + 2} = \frac{1}{1.2}$

$\Rightarrow 1.2q_1 = q_1 + 2 \Rightarrow 0.2\,q_1 = 2$

$\Rightarrow q_1 = 10\,C$

111. (12)



$F_{\text{total}} = \left[\frac{kq_1q_2}{r_1^2} + \frac{kq_1q_3}{r_2^2} + \frac{kq_1q_4}{r_3^2} + \ldots\right]$

$\Rightarrow \quad F = (k)(10^{-6})\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \ldots\right]$

$= \frac{(k)10^{-6}}{1 - \frac{1}{4}} = \frac{(9 \times 10^9) \times 4 \times 10^{-6} N}{3} = 12 \times 10^3 N$

112. (36)

When two spheres charges $q'_1 = 2.1$ nC and $q'_2 = -0.1$ nC are brought into contact and then separated by a distance $r = 0.5$ m then,

$q_1' = q_2' = \frac{Q_1 + Q_2}{2} = 1$ nC

Electrostatic force between the two charged sphere,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{r^2} = 9 \times 10^9 \times \frac{10^{-9} \times 10^{-9}}{(0.5)^2} = 36 \times 10^{-9}\,N$$

$\therefore\ x = 36$

113. (640)

(640) Flux, $\phi = \vec{E} \cdot \vec{A} = \left[\frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}\right] \cdot \vec{A}$

$= \frac{E_0}{5}(2\hat{i} + 3\hat{j}) \cdot (0.4\hat{i}) \quad = \frac{4000}{5}(2 \times 0.4) = 640\ \text{Nm}^2\text{C}^{-1}$

114. (226)

(226) $\phi_{Total} = \frac{q}{\varepsilon_0}$,

From symmetry electric flux

$$\phi_{Sq} = \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{12\times10^{-6}}{6\times8.85\times10^{-12}} = 225.98\times10^3$$

$$= 226\times10^3 \frac{Nm^2}{C}$$

115. (128)

(128) Potential, $V = \frac{Kq}{r}$

If charge on each drop $= q$ and radius $= r$

$$\therefore V_0 = \frac{Kq}{r} = 2\text{ V}$$

$$\frac{4}{3}\pi R^3 = 512\times\frac{4}{3}\pi r^3$$

$$\Rightarrow R = (512\times r^3)^{1/3} = 8r$$

Potential of bigger drop,

$$V = \frac{K(512)q}{8r} = \frac{512}{8}\frac{Kq}{r} = \frac{512}{8}\times 2\text{ V} = 128\text{ V}.$$

116. (16)

(16) When battery is connected,

$$Q_1 = C_1V = 2\times10 = 20\ \mu C$$

When battery is removed and the capacitor is connected

$$Q_1 = C_1V + C_2V \Rightarrow 20 = (2+8)V \qquad \therefore V = 2V$$

Therefore charge in $C_2 = C_2V = 16\mu C$

117. (6)

(6) Maximum torque,

$|\tau|_{max} = PE$ or, $\tau_1 = P_1E_1$ and $\tau_2 = P_2E_2$

$$\therefore \frac{\tau_1}{\tau_2} = \frac{P_1E_1}{P_2E_2} = \frac{1.2\times10^{-30}\times5\times10^4}{2.4\times10^{-30}\times15\times10^4} = \frac{1}{6} = \frac{1}{x}$$

$\therefore$ x=6

118. (45)

(45) Here, $\phi_e = \int \vec{E}.dA \Rightarrow \frac{\phi_e}{A} = E$

$$\Rightarrow \frac{\phi_e}{A} = \frac{\rho R}{3\epsilon_0} = \frac{2\times10^0\times6}{3\times8.85\times10^{-12}} = 45\times10^{10}\ NC^{-1}$$

119. (60)

(60) Capacitances of the capacitors,

$$C_1 = \frac{\epsilon_0\epsilon_{r_1}A}{t_1},\ C_2 = \frac{\epsilon_0\epsilon_{r_2}A}{t_2}$$

Let V be the voltage of conducting foil. As the capacitors are connected in series, so charge on the capacitors should be same

$Q_1 = Q_2$

$\Rightarrow\ C_1(100-V) = C_2V\quad (\because\ Q = CV)$

$$\Rightarrow \frac{\epsilon_0\epsilon_{r_1}A}{t_1}(100-V) = \frac{\epsilon_0\epsilon_{r_2}AV}{t_2}$$

$$\Rightarrow \frac{3\times(100-V)}{0.5\times10^{-3}} = \frac{4\times V}{1\times10^{-3}}$$

$\Rightarrow\ 600-6V = 4V \Rightarrow V = 60\ V$