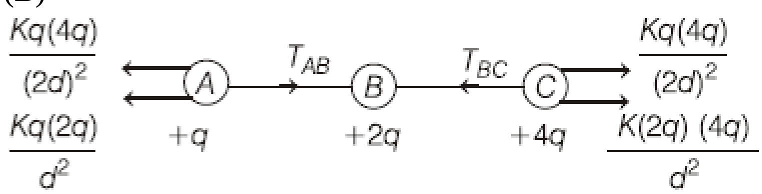
ELECTROSTATICS

1. (D)

$$\mathbf{F}_{21} = \frac{Kq_1q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3}(\mathbf{r}_2 - \mathbf{r}_1) = \frac{1 \times q_1 \times q_2}{4\pi\varepsilon_0 |-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}|^3}(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

$$= \frac{q_1q_2(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})}{56\sqrt{14}\,\pi\,\varepsilon_0}$$

2. (B)



Since, A is in equilibrium, $\Sigma F = 0$

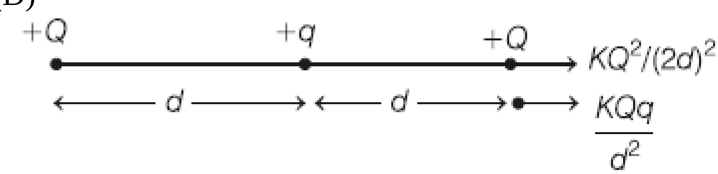
$$\Rightarrow \quad T_{AB} = \frac{Kq(4q)}{(2d)^2} + \frac{Kq(2q)}{d^2} = \frac{3Kq^2}{d^2}$$

Since, C is in equilibrium $\Sigma F = 0$

$$\Rightarrow \quad T_{BC} = \frac{Kq(4q)}{(2d)^2} + \frac{K(2q)(4q)}{d^2} = \frac{9Kq^2}{d^2}$$

$$\frac{T_{AB}}{T_{BC}} = \frac{3Kq^2/d^2}{9Kq^2/d^2} = \frac{1}{3}$$

3. (D)



Central particle is already in equilibrium.

For equilibrium of $+Q$; $\Sigma F = 0$

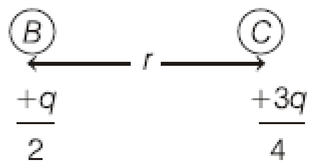
$$\Rightarrow \quad \frac{KQ^2}{(2d)^2} + \frac{KQq}{d^2} = 0 \quad \Rightarrow \quad q = -\frac{Q}{4}$$

4. (D)

$$F = \frac{Kq^2}{r^2}$$

When A and B are touched, charges on each of them would be $\frac{q+0}{2} = \frac{q}{2}$.

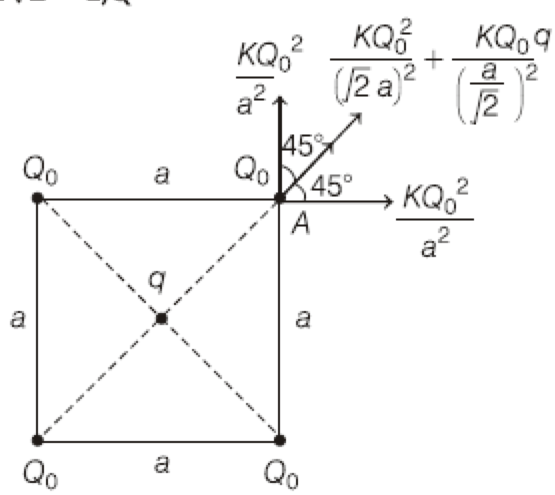
When A and C are touched, charges on each of them would be $\frac{\frac{q}{2}+q}{2} = \frac{3q}{4}$



$$F' = \frac{K\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right)}{r^2} = \frac{3F}{8}$$

5. (A)

$Q_0 = (2\sqrt{2} - 1)Q$



Net force on

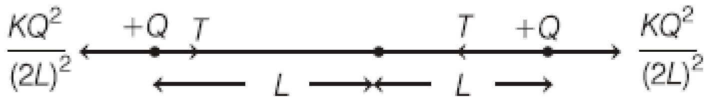
$$Q_0 = \frac{KQ_0^2}{(\sqrt{2}a)^2} + \frac{KQ_0 q}{\left(\frac{a}{\sqrt{2}}\right)^2} + \left(\frac{KQ_0^2}{a^2}\cos 45^\circ\right) \times 2 = 0$$

$$\Rightarrow \qquad q = -\left(\frac{2\sqrt{2}+1}{4}\right)Q_0$$

$$\Rightarrow \qquad q = -\left(\frac{2\sqrt{2}+1}{4}\right)(2\sqrt{2}-1)Q = -\frac{7Q}{4}$$

6. (A)

In gravity free space, angle between the two threads will be 180°.

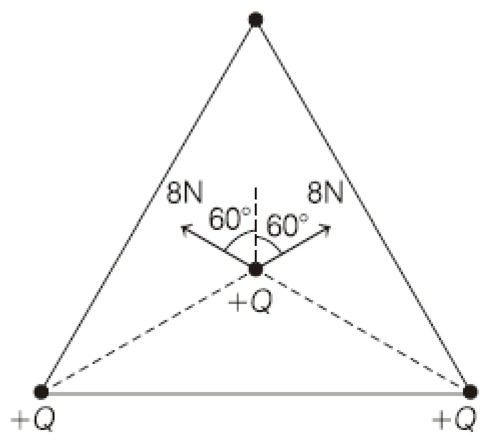


and $T = \frac{KQ^2}{(2L)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2L)^2}$

7. (D)

8. (D)

Net force on the central particle



$= 8\cos 60° + 8\cos 60° = 8\text{N}$

9. (A)

$qE = mg$

$\Rightarrow \quad q(10^9) = (1.6 \times 10^{-3})(9.8)$

$q = 1.6 \times 9.8 \times 10^{-12}$ C and $q = ne$

$1.6 \times 9.8 \times 10^{-12} = n(1.6 \times 10^{-19})$

$n = 9.8 \times 10^7$

10. (B)

Electric force on $q = qE$

Work done by electric force $= qEy$

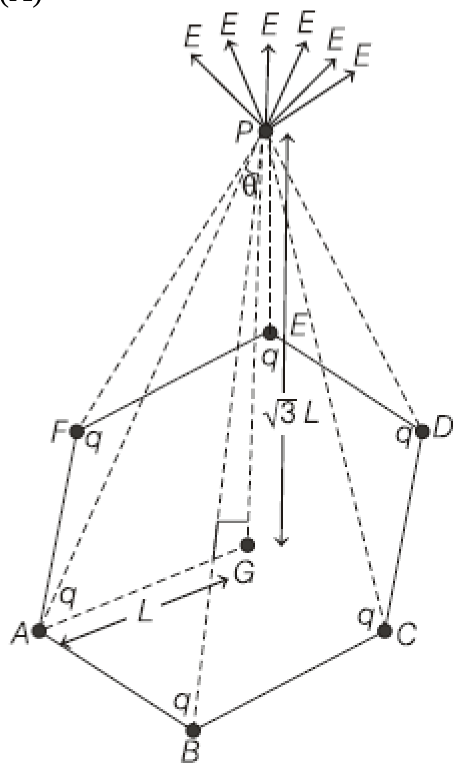
Work-energy theorem, $W = \Delta K$

$\Rightarrow \quad qEy = K_2 - 0 \Rightarrow K_2 = qEy$

11. (A)

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$
$$= \frac{9 \times 10^9 \times 5 \times 10^{-6}}{(0.1)^2}\hat{\mathbf{i}} + \frac{9 \times 10^9 \times 10 \times 10^{-6}}{(0.1)^2}\hat{\mathbf{i}}$$
$$= 13.5 \times 10^6 \text{ N/C, towards } -10\mu\text{C}.$$

12. (A)



$$AP = \sqrt{AG^2 + GP^2} = \sqrt{L^2 + (\sqrt{3}L)^2} = 2L$$
$$E = \frac{Kq}{(2L)^2} = \frac{Kq}{4L^2} = \frac{q}{16\pi\varepsilon_0 L^2}$$

Net electric field at $P = 6E\cos\theta$
$$= 6\left(\frac{q}{16\pi\varepsilon_0 L^2}\right)\left(\frac{\sqrt{3}L}{2L}\right) = \frac{3\sqrt{3}q}{16\pi\varepsilon_0 L^2}$$

13. (C)

Electric field between the two charges is negative. So, q_2 will be positive and q_1 will be negative.

Since, electric field is zero at a point closer to q_2, $|q_2| < |q_1|$.

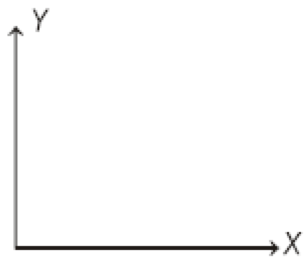
14. (D)

$\mathbf{F} = (-q)(-E\,\mathbf{j})$

$$m\mathbf{a} = qE\hat{\mathbf{j}} \quad \Rightarrow \quad \mathbf{a} = \frac{qE}{m}\hat{\mathbf{j}}$$

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$L = vt + 0 \quad \Rightarrow \quad t = \frac{L}{v}$$



For droplet to not hit the plate,

$$s_y < \frac{d}{2}$$

$$\Rightarrow \quad u_y t + \frac{1}{2}a_y t^2 < \frac{d}{2} \quad \Rightarrow \quad 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v}\right)^2 < \frac{d}{2}$$

$$\Rightarrow \qquad q < \frac{mv^2 d}{EL^2} \quad \Rightarrow \quad q_{\max} = \frac{mv^2 d}{EL^2}$$

15. (B)

$$E = \frac{KQx}{(r^2 + x^2)^{3/2}}$$

Electric field is maximum at $x = \frac{r}{\sqrt{2}}$.

$$E_{\max} = \frac{KQ(r/\sqrt{2})}{\left(r^2 + \left(\frac{r}{\sqrt{2}}\right)^2\right)^{3/2}} = \frac{KQ\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{3\sqrt{3}}{2\sqrt{2}}\right)r^2} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 r^2}$$

16. (C)

$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$E_A = 0 + 0 = 0$$

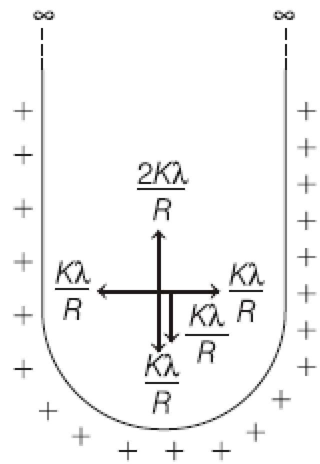
$$E_B = \frac{KQ}{\left(\frac{3r}{2}\right)^2} + 0 = \frac{4KQ}{9r^2}$$

$$E_C = \frac{KQ}{\left(\frac{5r}{2}\right)^2} + \frac{K(-Q)}{\left(\frac{5r}{2}\right)^2} = 0$$

$$\Rightarrow \qquad E_B > E_A = E_C$$

17. (D)

Net electric field = 0



18. (B)

Charge on the gap element $= \frac{1}{2\pi(0.5)} \times 0.002\,\pi$

$= 0.002$ C

Electric field due to gap element at the centre $= \frac{Kdq}{R^2}$

$= \frac{9 \times 10^9 \times 0.002}{(0.5)^2}$

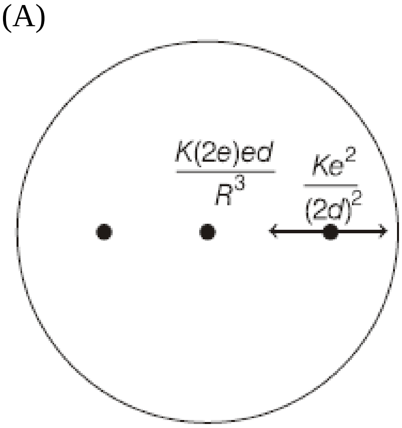
$= 7.2 \times 10^7$ N/C

| Electric field due to gap element | = | Electric field due to remaining ring |

19. (C)

If another identical hemispherical shell is kept upside down over it, net electric field at any point inside the spherical shell should be zero.

20. (A)



For equilibrium of electron,

$$\frac{K(2e)ed}{R^3} = \frac{Ke^2}{(2d)^2}$$

$$d = R/2$$

Separation between the electrons $= 2d = 2\left(\frac{R}{2}\right) = R$

21. (B)

$F = qE$

$\Rightarrow \quad F = (-Q)\left(\frac{K(\lambda 2\pi a)\,x}{(a^2 + x^2)^{3/2}}\right)$

$\Rightarrow \quad F = -\frac{Qa\lambda}{2a^3\varepsilon_0}x \qquad (\because x << a)$

$\Rightarrow \quad F = -\left(\frac{Q\lambda}{2\varepsilon_0 a^2}\right)x$

This is equation of SHM.

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{\frac{\lambda Q}{2\varepsilon_0 a^2}}} = 2\pi\sqrt{\frac{2m\varepsilon_0 a^2}{\lambda Q}}$$

22. (C)

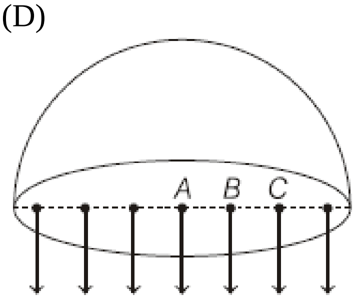
$V = V_1 + V_2 + V_3$

$$= \frac{k(\lambda\pi R)}{R} + \int_R^{3R}\frac{k(\lambda dx)}{x} + \int_R^{3R}\frac{k\lambda dx}{x}$$

$$= k\lambda\pi + k\lambda\ln 3 + k\lambda\ln 3 = k\lambda(\pi + 2\ln 3)$$

$$= \frac{\lambda}{4\pi\varepsilon_0}(\pi + 2\ln 3)$$

23. (D)



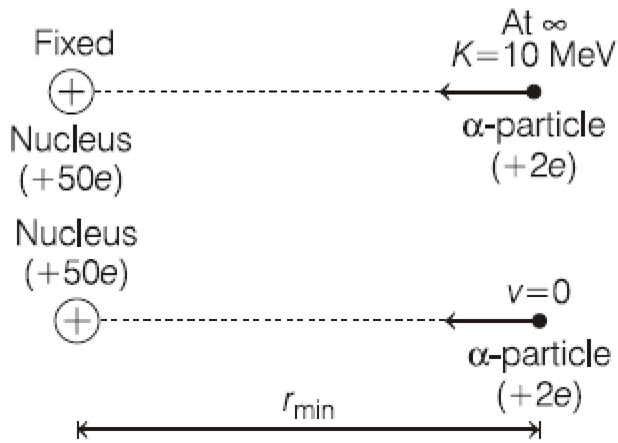
Electric field due to hemisphere at points lying on the diameter is perpendicular to the diameter. So, all points on the diameter are equipotential.

24. (C)

A negative charge released in an electric field will go against the direction of electric field.

So, it will move toward a position of higher electric potential and lower potential energy.

25. (A)



At the minimum distance of approach, speed of α-particle would be zero.
Applying mechanical conservation for α-particle,

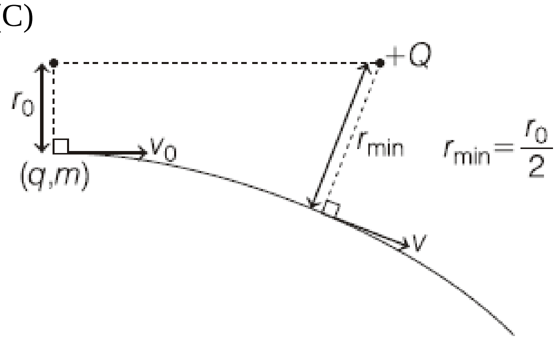
$$K_1 + U_1 = K_2 + U_2$$

$$10MeV + 0 = 0 + \frac{k(+50e)(+2e)}{r_{\min}}$$

$$\Rightarrow \quad r_{\min} = \frac{9\times10^9 \times 100 \times e \times e}{10 \times 10^6 \times e}$$

$$= 14.4\times10^{-15} \text{ or } 1.44\times10^{-14} \text{ m}$$

26. (C)



Using angular momentum conservation of q about Q,

$$mv_0r_0 = mvr_{\min} \qquad ...(i)$$

Using energy conservation,

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv^2 + \frac{kQq}{r_{\min}} \qquad ...(ii)$$

Solving Eqs. (i) and (ii), we get

$$q = -\frac{3Q}{4}$$

27. (C)

Using energy conservation, $K_1 + U_1 = K_2 + U_2$

$$\Rightarrow \quad 0 + \left[mg\frac{4R}{3} + q\left(\frac{kQ}{\sqrt{R^2 + \left(\frac{4R}{3}\right)^2}} \right) \right]$$

$$= 0 + \left[0 + q\left(\frac{kQ}{R}\right) \right]$$

$$\Rightarrow \quad \frac{4}{3}mgR + \frac{3kQq}{5R} = \frac{kQq}{R}$$

$$\Rightarrow \quad \frac{4}{3}mgR = \frac{2}{5}\frac{kQq}{R}$$

$$\Rightarrow \quad \frac{4}{3}\left(\frac{Q^2}{4\pi\varepsilon_0 R^2}\right)R = \frac{2}{5}\left(\frac{Qq}{4\pi\varepsilon_0 R}\right) \Rightarrow q = \frac{10Q}{3}$$

28. (A)

$F = qE$

$$\Rightarrow \quad 3000 = 3E \;\Rightarrow\; E = 1000 \text{ N/C}$$

$$\Delta V = Ed = (1000\text{N/C})\,(1 \times 10^{-2}\text{m}) = 10 \text{ V}$$

29. (D)

$W = (qE)s\cos\theta$

$$\Rightarrow \quad 4 = (0.2E)\,(2)\cos 60^\circ$$

$$\Rightarrow \quad E = 20 \text{ N/C}$$

30. (A)

$dV = -\mathbf{E}\cdot d\mathbf{r}$

$$\Rightarrow \quad \int dV = -\int (E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}})\cdot(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

$$\Rightarrow \quad \int_{V_0}^{V} dV = -\int_0^x E_x dx - \int_0^y E_y dy$$

$$\Rightarrow \quad V - V_0 = -E_x[x]_0^x - E_y[y]_0^y$$

$$\Rightarrow \quad V = V_0 - xE_x - yE_y$$

31. (A)

$V = -xy^2\sqrt{z}$

$$\mathbf{E} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$$

$$= -(-y^2\sqrt{z})\hat{\mathbf{i}} - (-2xy\sqrt{z})\hat{\mathbf{j}} - \left(-\frac{xy^2}{2\sqrt{z}}\right)\hat{\mathbf{k}}$$

$$= y^2\sqrt{z}\hat{\mathbf{i}} + 2\,xy\sqrt{z}\;\hat{\mathbf{j}} + \frac{xy^2}{2\sqrt{z}}\hat{\mathbf{k}}$$

At (2, 1, 1),

$$\mathbf{E} = (1^2\sqrt{1})\hat{\mathbf{i}} + [2(2)(1)\sqrt{1}]\;\hat{\mathbf{j}} + \left(\frac{2(1)^2}{2\sqrt{1}}\right)\hat{\mathbf{k}}$$

$$= \hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

32. (A)

$E_y = -\dfrac{dV}{dy}$

At $y = 2.5$ m,

$$E_y = -\frac{dV}{dy} = -\text{ slope of } V\text{-}y \text{ graph}$$

$$= -\left(\frac{5-2}{1}\right) = -3$$

At $y = 5.5$ m,

$$E_y = -\frac{dV}{dy} = -\text{ slope of } V\text{-}y \text{ graph}$$

$$= -\left[-\frac{(5-0)}{(1-0)}\right] = 5$$

33. (C)

Loss in KE = Gain in PE

$\Rightarrow \quad -\Delta K = \Delta U$

$\Rightarrow \quad -\left(0-\frac{1}{2}mv^2\right) = q\Delta V$

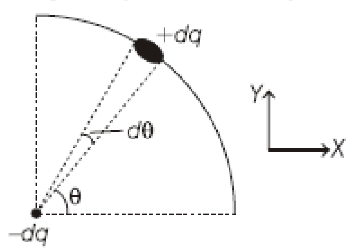
$\Rightarrow \quad \frac{1}{2}mv^2 = q\left(\frac{\sigma}{2\varepsilon_0}d\right)$

$\Rightarrow \quad v = \sqrt{\frac{q\sigma d}{m\varepsilon_0}}$

34. (A)

$dq = \frac{q}{\pi R/2}(Rd\theta)$

$\mathbf{dp} = dp\cos\theta\,\hat{\mathbf{i}} + dp\sin\theta\,\hat{\mathbf{j}}$



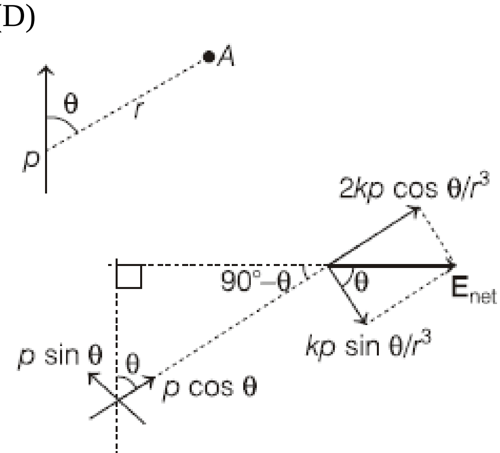
$\Rightarrow \quad \mathbf{dp} = \left(\frac{q}{\pi R/2}Rd\theta\right)R\cos\theta\,\hat{\mathbf{i}} + \left(\frac{q}{\pi R/2}Rd\theta\right)R\sin\theta\,\hat{\mathbf{j}}$

$\Rightarrow \quad \int \mathbf{dp} = \frac{2qR}{\pi}\left(\int_0^{\pi/2}\cos\theta d\theta\,\hat{\mathbf{i}} + \int_0^{\pi/2}\sin\theta\, d\theta\,\hat{\mathbf{j}}\right)$

$\Rightarrow \quad \mathbf{p} = \frac{2qR}{\pi}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$\Rightarrow \quad |\mathbf{p}| = \frac{2\sqrt{2}qR}{\pi}$

35. (D)



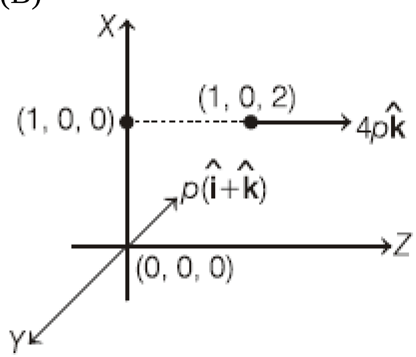
$$\tan\theta = \frac{\dfrac{2kp\cos\theta}{r^3}}{\dfrac{kp\sin\theta}{r^3}}$$

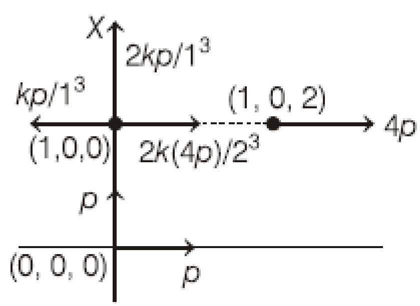
$\Rightarrow \quad \tan\theta = 2\cot\theta$

$\Rightarrow \quad \tan^2\theta = 2$

$\Rightarrow \quad \theta = \tan^{-1}(\sqrt{2})$

36. (B)

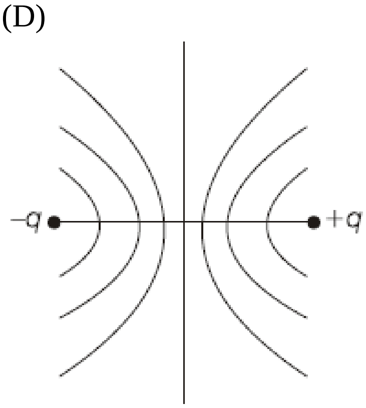




$$\mathbf{E} = \left(\frac{2k(4p)}{2^3} - \frac{kp}{1^3}\right)\hat{\mathbf{i}} + \left(\frac{2kp}{1^3}\right)\hat{\mathbf{j}}$$

$$\mathbf{E} = \frac{p}{2\pi\varepsilon_0}\hat{\mathbf{j}}$$

37. (D)



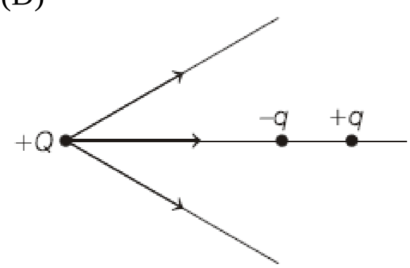
38. (C)

$$\tau = pE\sin\theta$$
$$\tau_{max} = pE, \text{ when } \theta = 90°$$
$$= (10^{-6} \times 2 \times 10^{-2})(1.0 \times 10^{5})$$
$$= 2.0 \times 10^{-3} \text{ N-m}$$

39. (C)

$$W_{ext} = \Delta U + \Delta K$$
$$= (U_2 - U_1) + 0$$
$$= -\mathbf{p}_2 \cdot \mathbf{E} + \mathbf{p}_1 \cdot \mathbf{E}$$
$$= (\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{E}$$
$$= [(7\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times 10^{-30})] \cdot (4000\hat{\mathbf{i}})$$
$$= 2.8 \times 10^{-26} \text{ J}$$

40. (D)



Force on the dipole will be non-zero as electric field produced by a point charge is non-uniform. Torque will be zero, if the dipole is aligned along one of electric field lines as shown in figure. For any other orientation of dipole, torque will be non-zero.

41. (D)

Since, the charge enclosed is zero, flux through Gaussian sphere is zero. Electric field is not zero anywhere on the sphere.

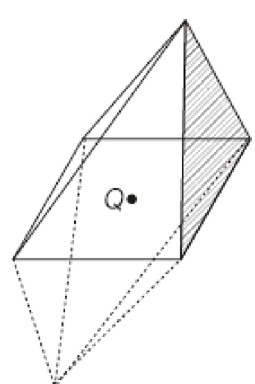
42. (C)

Put another identical pyramid as shown in figure.

Total flux emanating from q will be equally divided between the two pyramids.

Flux through upper pyramid $= \frac{1}{2}\left(\frac{Q}{\varepsilon_0}\right)$

Flux through each face of upper pyramid $= \frac{1}{4}\left(\frac{Q}{2\varepsilon_0}\right) = \frac{Q}{8\,\varepsilon_0}$



43. (B)

$$\phi = \phi_q + \phi_{2q} + \phi_{3q} + \phi_{4q}$$

$$= \frac{q}{\varepsilon_0} + \frac{1}{4}\left(\frac{2q}{\varepsilon_0}\right) + \frac{1}{2}\left(\frac{3q}{\varepsilon_0}\right) + \frac{1}{8}\left(\frac{4q}{\varepsilon_0}\right) = \frac{7q}{2\varepsilon_0}$$

44. (D)

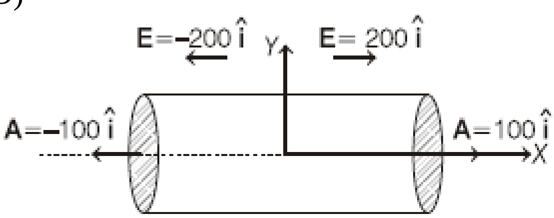
$$\phi = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \phi_{max} = \frac{q_{max}}{\varepsilon_0} \Rightarrow \phi_{max} = \frac{\lambda l_{max}}{\varepsilon_0}$$

The longest wire which is inside the cube is of length $\sqrt{3}a$.

$$\Rightarrow \quad \phi_{max} = \frac{\sqrt{3}\lambda a}{\varepsilon_0}$$

45. (D)



Flux through right flat face $= \mathbf{E} \cdot \mathbf{A}$

$$= (200\,\hat{\mathbf{i}}) \cdot (100\hat{\mathbf{i}}) = 2 \times 10^4$$

Flux through left flat face $= \mathbf{E} \cdot \mathbf{A}$

$$= (-200\,\hat{\mathbf{i}}) \cdot (-100\hat{\mathbf{i}}) = 2 \times 10^4$$

Flux through curved surface $= EA \cos 90° = 0$

Net flux through cylinder

$$= 2 \times 10^4 + 2 \times 10^4 + 0 = 4 \times 10^4$$

Using Gauss's law,

$$\phi = \frac{q_{inside}}{\varepsilon_0} \Rightarrow 4 \times 10^4 = \frac{q_{inside}}{8.85 \times 10^{-12}}$$

$$\Rightarrow \quad q_{inside} = 35.4 \times 10^{-8}\ \text{C}$$

46. (C)

Using Gauss's law, $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

$$\Rightarrow \quad E(4\pi x^2) = \frac{\int_0^x (\rho_0 r^2)\, 4\pi r^2 dr}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho_0 x^3}{5\varepsilon_0}$$

$$\Rightarrow \quad E \propto x^3$$

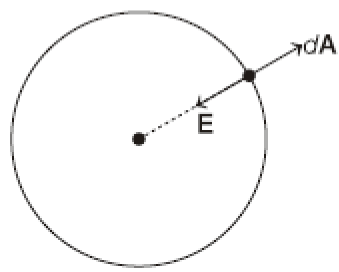
47. (B)

$$d\phi = \mathbf{E} \cdot d\mathbf{A}$$

$$\Rightarrow \quad d\phi = [90r(-\hat{\mathbf{r}})] \cdot (dA\hat{\mathbf{r}})$$

$$\Rightarrow \quad \int d\phi = -90r \int dA$$

$$\Rightarrow \quad \phi = -90r\,(4\pi r^2)$$



Using Gauss's law, $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

$$\Rightarrow \quad -90r(4\pi r^2) = \frac{q_{enclosed}}{\varepsilon_0}$$

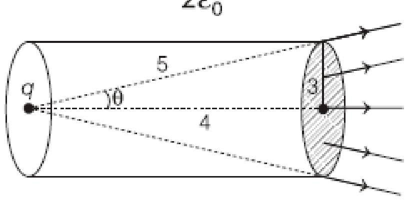
$$\Rightarrow \quad q_{enclosed} = -90\,(4\pi\varepsilon_0)\, r^3$$

$$= \frac{-90 \times 2^3}{9 \times 10^9}$$

$$= -80 \text{ nC}$$

48. (B)

Flux through cylinder $= \frac{q}{2\varepsilon_0}$

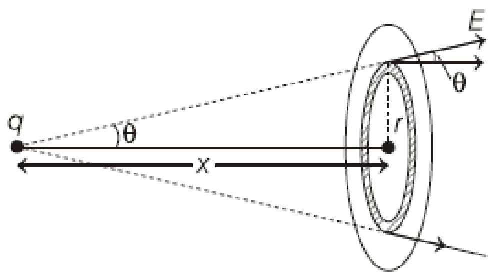


Flux through the right flat face $= \frac{q/\varepsilon_0}{4\pi}[2\pi(1-\cos\theta)]$

$$= \frac{q}{2\varepsilon_0}\left(1 - \frac{4}{5}\right) = \frac{q}{10\,\varepsilon_0}$$

Flux through curved surface $= \frac{q}{2\varepsilon_0} - \frac{q}{10\,\varepsilon_0} = \frac{2q}{5\varepsilon_0}$

49. (D)



Lets take a ring of radius r and thickness dr on the disc.

Flux through the ring $= d\phi = E\,dA\cos\theta$

$\Rightarrow \quad d\phi = E\,(2\pi r dr)\cos\theta$

$\Rightarrow \quad d\phi = (2\pi r dr)\,(E\cos\theta)$

Multiplying by σ on both sides

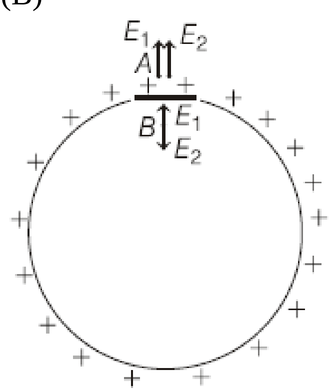
$\Rightarrow \quad \sigma\, d\phi = (\sigma 2\pi\, r dr)\,(E\cos\theta)$

$\Rightarrow \quad \sigma\, d\phi = dq\,(E\cos\theta)$

$\Rightarrow \quad \sigma d\phi = dF \Rightarrow \sigma\int d\phi = \int dF$

$\Rightarrow \quad \phi = \dfrac{F}{\sigma}$

50. (B)



E_1 = Electric field due to shell with hole

E_2 = Electric field due to part of shell within the hole.

Point A is just outside the shell,

$$E_A = E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \quad \text{... (i)}$$

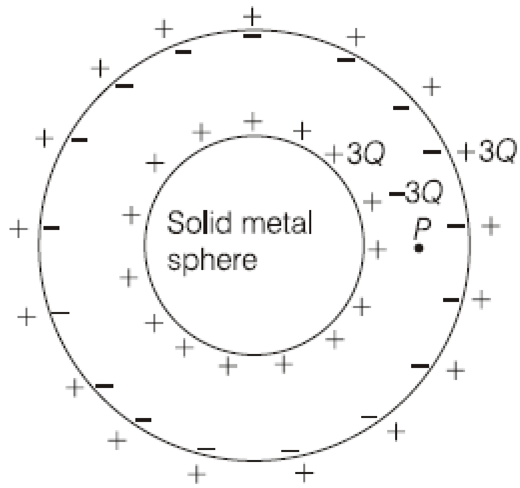
Point B is just inside the shell,

$$E_B = E_1 - E_2 = 0 \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

51. (C)



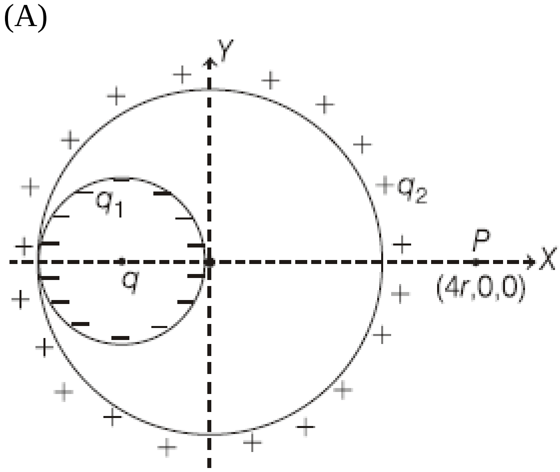
At P,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$$

$\Rightarrow$ $$E = \frac{k(3Q)}{r^2} + 0 + 0$$

$\Rightarrow$ $$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{3Q}{r^2}$$

52. (A)



$\because$ $q_1 = -q$ and $q_2 = +q$

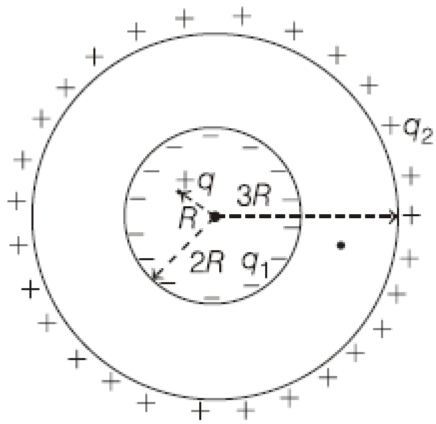
At point P,

$$\mathbf{E} = (\mathbf{E}_{\text{due to } q} + \mathbf{E}_{\text{due to } q_1}) + \mathbf{E}_{\text{due to } q_2}$$

$\Rightarrow$ $$E = 0 + \frac{kq}{(4r)^2}$$

$\Rightarrow$ $$E = \frac{kq}{16r^2}$$

53. (C)



$q_1 = -q$ (non-uniform distribution)

$q_2 = +q$ (uniform distribution)

Potential at the centre,

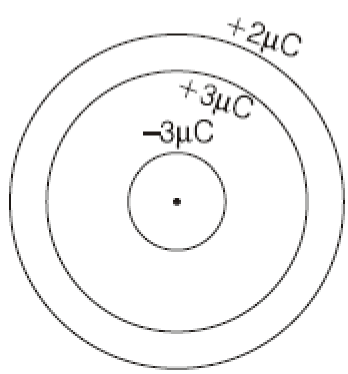
$$V = V_{\text{due to } q} + V_{\text{due to } q_1} + V_{\text{due to } q_2}$$

$$= \frac{kq}{R} + \frac{k(-q)}{2R} + \frac{kq}{3R}$$

$$= \frac{5}{6}\frac{kq}{R}$$

54. (A)

Potential of the inner surface of the spherical shell,



$$V = V_1 + V_2 + V_3$$

$$= \frac{k(-3\mu C)}{(9 \text{ cm})} + \frac{k(+3\mu C)}{(9 \text{ cm})} + \frac{k(+2\mu C)}{(10 \text{ cm})}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6}}{10 \times 10^{-2}}$$

$$= 180 \text{ kV}$$

55. (B)

At point P,

$$V_{\text{due to } q} + V_{\text{due to induced charges on the inner surface}} = 0$$

$$\Rightarrow V_{\text{due to induced charges on the inner surface}} = -V_{\text{due to } q} = \frac{-kq}{5a}$$

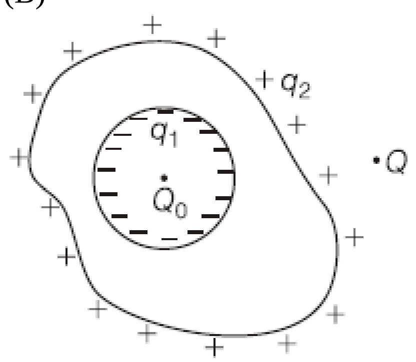
56. (B)

Potential at A due to charges outside the conductor is zero, since the shell is grounded.

$$V_A = V_{\text{due to } q} + V_{\text{due to charges on the outer surface of shell}} = 0$$

$$\Rightarrow \quad V_{\text{due to charges on the outer surface of shell}} = -V_{\text{due to } q} = -\frac{kq}{(2.5R)}$$

57. (B)



$$q_1 = -Q_0 \text{ (uniform) and } q_2 = +Q$$

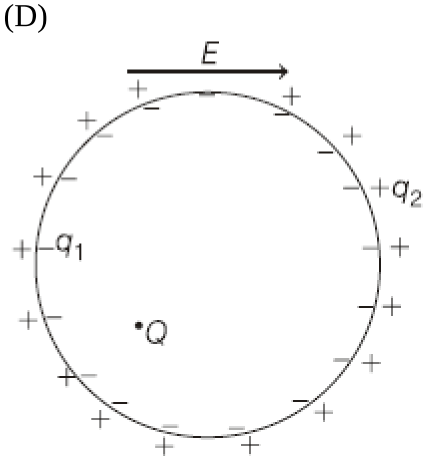
Force on Q_0,

$$\mathbf{F} = \mathbf{F}_{\text{due to } q_1} + (\mathbf{F}_{\text{due to } q_2} + \mathbf{F}_{\text{due to } Q}) = 0 + 0 = 0$$

Force on Q,

$$\mathbf{F} = (\mathbf{F}_{\text{due to } Q_0} + \mathbf{F}_{\text{due to } q_1}) + \mathbf{F}_{\text{due to } q_2} = 0 + \mathbf{F}_{\text{due to } q_2}$$
$$\neq 0$$

58. (D)



$q_1 = -Q$ (non-uniform); $q_2 = +Q$ (non-uniform)

(a) Force on Q due to $E = QE$

(b) Net force on $Q = \mathbf{F}_{\text{due to } q_1} + (\mathbf{F}_{\text{due to } q_2} + \mathbf{F}_{\text{due to } E})$

$$= \mathbf{F}_{\text{due to } q_1} + 0$$
$$= \mathbf{F}_{\text{due to } q_1}$$
$$\neq 0$$

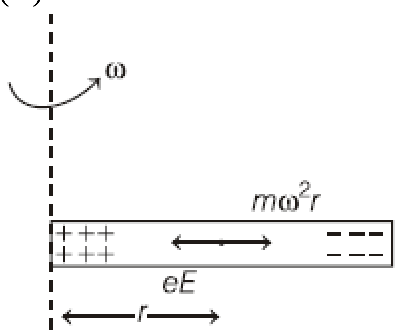
(c) Net force on Q and conducting shell considered as a system $= q_2E + q_1E + QE$

$$= QE - QE + QE = QE$$

(d) Net force on the shell due to $E = q_1E + q_2E$

$$= -QE + QE = 0$$

59. (A)

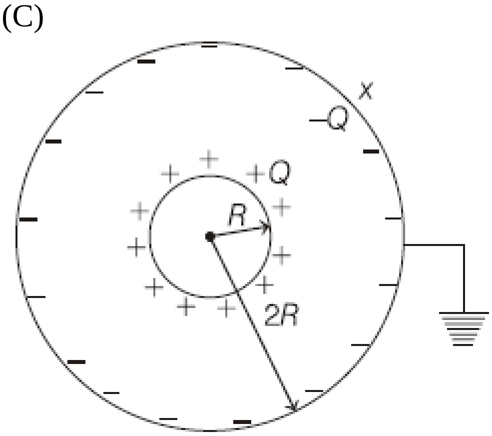


Due to centrifugal on free electrons, electrons will accumulate at the right end of the rod and there will be deficiency of electrons at the left end. These induced charges will give rise to an electric field.

Electrons will stop accumulating at the right end when net force on free electron is zero.

$$\Delta V = \int E\,dr = \int_0^l \frac{m\omega^2 r}{e}\,dr = \frac{m\omega^2 l^2}{2e}$$

60. (C)



Since, B is earth, therefore $V_B = 0$

$$\Rightarrow \quad \frac{kQ}{2R} + \frac{k(x-Q)}{2R} = 0$$

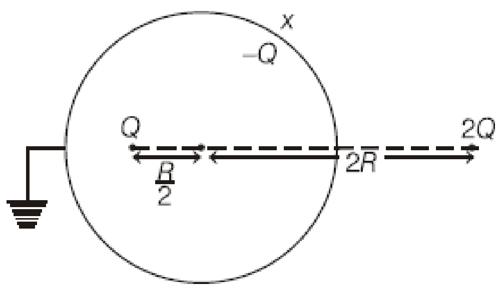
$$\Rightarrow \quad x = 0$$

$$V_A = \frac{kQ}{R} + \frac{k(x-Q)}{2R}$$

$$= \frac{kQ}{2R} = \frac{Q}{8\pi\varepsilon_0 R}$$

61. (A)

Lets take charge on outer surface of shell to be x after it has been earthed.



Potential due to outside charges inside the shell = 0

At the centre, $\frac{kx}{R} + \frac{k(2Q)}{2R} = 0$

$$x = -Q$$

Total charge on the shell $= x - Q$

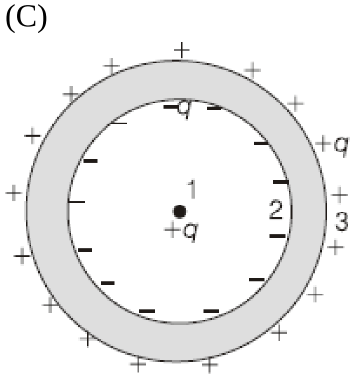
$$= -Q - Q = -2Q$$

Charge flown to earth $= Q_1 - Q_2$

$$= Q - (-2Q)$$

$$= 3Q$$

62. (C)

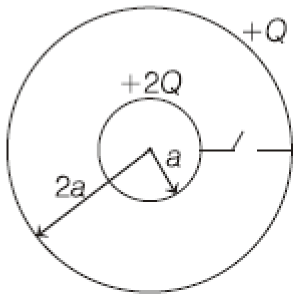


$$U = U_{12} + U_{23} + U_{31} + U_2 + U_3$$

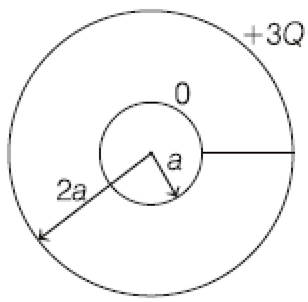
$$\Rightarrow \quad U = (-q)\left(\frac{kq}{a}\right) + q\left[\frac{k(-q)}{b}\right] + q\left(\frac{kq}{b}\right) + \frac{k(-q)^2}{2a} + \frac{kq^2}{2b}$$

$$\Rightarrow \quad U = -\frac{kq^2}{a} - \frac{kq^2}{b} + \frac{kq^2}{b} + \frac{kq^2}{2a} + \frac{kq^2}{2b} = \frac{kq^2}{2b} - \frac{kq^2}{2a}$$

63. (B)



$$U_1 = \frac{k(2Q)^2}{2a} + \frac{kQ^2}{2(2a)} + Q\left[\frac{k(2Q)}{2a}\right] = \frac{13kQ^2}{4a}$$



After closing the switch, the entire charge of inner shell will be transferred to the outer shell.

$$U_2 = \frac{k(3Q)^2}{2(2a)} = \frac{9kQ^2}{4a}$$

Heat dissipated $= U_1 - U_2 = \frac{13kQ^2}{4a} - \frac{9kQ^2}{4a} = \frac{kQ^2}{a}$

64. (234)

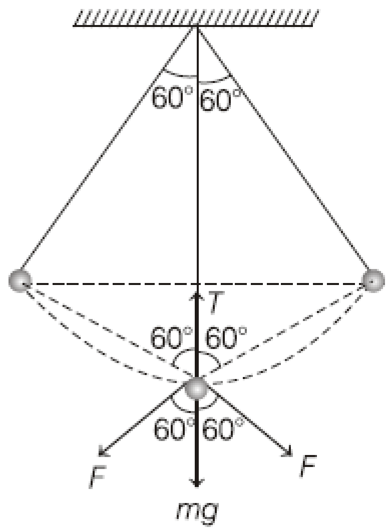
$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$E = \frac{2KQ_1}{R^2}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right) + \frac{KQ_2 x}{(R^2 + x^2)^{3/2}}$$

$$E = \frac{2 \times 9 \times 10^9 \times 1 \times 10^{-6}}{4^2}\left(1 - \frac{3}{5}\right) + \frac{9 \times 10^9 \times (-1 \times 10^{-6}) \times 3}{(5^2)^{3/2}}$$

$E = 450 - 216 = 234$

65. (3)



$$\Sigma F_y = 0$$

$$\Rightarrow \quad T = mg + 2F\cos 60^\circ$$

$$T = (20\times 10^{-3}\times 10) + 2\left(\frac{9\times 10^9\times (10\times 10^{-6})^2}{(2\times 3\times \sin 30^\circ)^2}\right)\times \frac{1}{2}$$

$$T = 0.3\,\text{N}$$

66. (4)

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$E = \frac{KQx}{(R^2+x^2)^{3/2}} - \frac{KQ}{x^2} = \frac{KQx}{x^3\left(1+\frac{R^2}{x^2}\right)^{3/2}} - \frac{KQ}{x^2}$$

$$= \frac{KQ}{x^2}\left(1+\frac{R^2}{x^2}\right)^{-3/2} - \frac{KQ}{x^2}$$

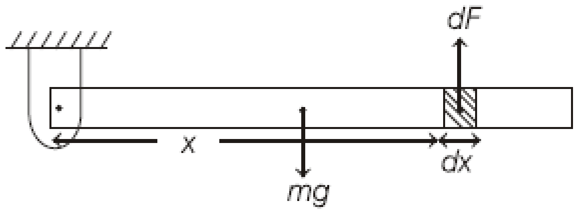
$$= \frac{KQ}{x^2}\left(1-\frac{3}{2}\frac{R^2}{x^2}\right) - \frac{KQ}{x^2}$$

$$= \frac{-3}{2}\frac{KQR^2}{x^4}$$

$$\Rightarrow \quad E \propto \frac{1}{x^4}$$

So, $n = 4$

67. (3)



$$dF = \lambda dx = \lambda_0 x dx$$
$$d\tau = dFx$$
$$\int d\tau = \int_0^l (\lambda_0 x dx E)\, x$$
$$\tau = \frac{\lambda_0 l^3 E}{3}$$

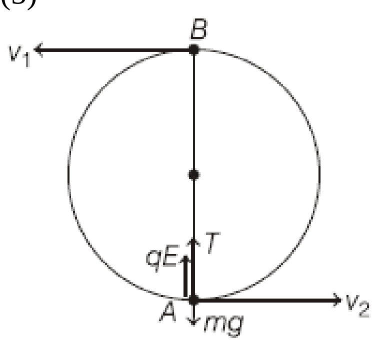
For equilibrium of rod, $\Sigma\tau_{net} = 0$

$$\Rightarrow \quad mg\left(\frac{l}{2}\right) - \frac{\lambda_0 l^3 E}{3} = 0$$

$$\Rightarrow \quad E = \frac{3mg}{2\lambda_0 l^2}$$

So, $n = 3$

68. (5)



At the lowest position,

$$T + qE - mg = \frac{mv_2^2}{l}$$

$$15mg + 3mg - mg = \frac{mv_2^2}{l}$$

$$\Rightarrow \quad v_2 = \sqrt{17gl}$$

Applying work-energy theorem between A and B,

$$W_{mg} + W_T + W_{electric} = \Delta K - mg(2l) + 0 + qE(2l)$$
$$= \frac{1}{2}mv_1^2 - \frac{1}{2}m(\sqrt{17gl})^2$$
$$v_1 = \sqrt{25gl} = \sqrt{25 \times 10 \times 0.1} = 5$$

69. (144)

Electric field inside a solid sphere,

$$E = \frac{KQ}{R^3} r$$

$\Rightarrow$ Slope of E-r graph

$$= \tan 53° = \frac{KQ}{R^3}$$

$$\Rightarrow \quad \frac{4}{3} = \frac{Q}{4\pi\varepsilon_0 (3)^3}$$

$$\Rightarrow \quad Q = 144\,\pi\,\varepsilon_0$$

$$Q = 144\pi\varepsilon_0 = n\pi\varepsilon_0$$

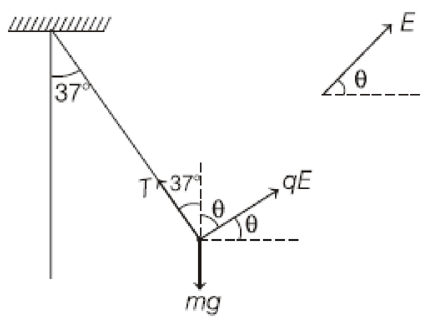
$$\Rightarrow \quad n = 144$$

70. (6)

For equilibrium,

$$qE\cos\theta = T\sin 37°$$

and $$qE\sin\theta = mg - T\cos 37°$$



Dividing Eq. (ii) by Eq. (i), we get

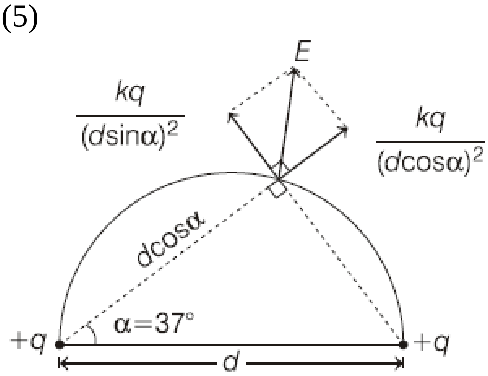
$$\tan\theta = \frac{mg - 2mg\cos 37°}{2mg\sin 37°} = \frac{-1}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \quad \sqrt{5}E\left(\frac{2}{\sqrt{5}}\right) = 2 \times 1 \times 10 \times \frac{3}{5}$$

$$\Rightarrow \quad E = 6 \text{ N/C}$$

71. (5)



$$E=\sqrt{\left[\frac{kq}{(d\cos\alpha)^2}\right]^2+\left[\frac{kq}{(d\sin\alpha)^2}\right]^2}$$

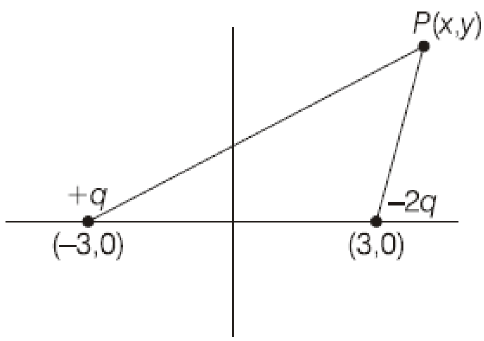
$$=\frac{kq}{d^2}\sqrt{\frac{\sin^4\alpha+\cos^4\alpha}{\sin^4\alpha\cos^4\alpha}}$$

$$V=V_1+V_2=\frac{kq}{d\cos\alpha}+\frac{kq}{d\sin\alpha}$$

$$=\frac{kq}{d}\frac{(\sin\alpha+\cos\alpha)}{\sin\alpha\cos\alpha}$$

$$\frac{E}{V}=\frac{1}{d}\frac{\sqrt{\sin^4\alpha+\cos^4\alpha}}{(\sin\alpha+\cos\alpha)\sin\alpha\cos\alpha}=5$$

72. (7)



At point P,

$$V=V_1+V_2$$

$$V=\frac{k(-2q)}{\sqrt{(x-3)^2+y^2}}+\frac{k(+q)}{\sqrt{(x+3)^2+y^2}}=0$$

$\Rightarrow$ $2\sqrt{(x+3)^2+y^2}=\sqrt{(x-3)^2+y^2}$

$\Rightarrow$ $4(x+3)^2+4y^2=(x-3)^2+y^2$

$\Rightarrow$ $3x^2+3y^2+30x+27=0$

$\Rightarrow$ $x^2+y^2+10x+9=0$

$\Rightarrow$ $(x+5)^2+y^2=16$

$\Rightarrow$ $\alpha=5$ and $\beta=2$

$$\alpha+\beta=5+2=7$$

73. (4)

$$V = \frac{kQ}{R} = \frac{k\int dq}{R} = \frac{k}{R}\int \lambda R d\phi$$

$$= \frac{k}{R}\int_0^{2\pi} \lambda_0 \cos^2 \phi R d\phi$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0}\int_0^{2\pi} \cos^2 \phi d\phi$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0}\int_0^{2\pi} \left(\frac{1+\cos 2\phi}{2}\right) d\phi$$

$$= \frac{\lambda_0}{8\pi\varepsilon_0}\left(\phi + \frac{\sin 2\phi}{2}\right)_0^{2\pi} = \frac{\lambda_0}{4\varepsilon_0}$$

So, $n = 4$

74. (6)

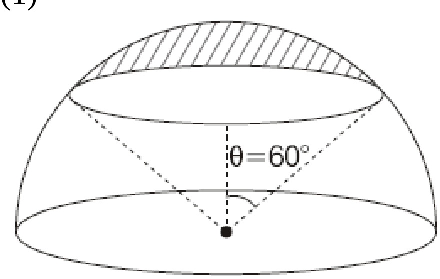
$V = 1.2\, V_{\text{surface}}$

$$\frac{kQ}{2R^3}(3R^2 - r^2) = 1.2\frac{kQ}{R}$$

$$\Rightarrow \quad 15R^2 - 5r^2 = 12R^2$$

$$\Rightarrow \quad r = \sqrt{\frac{3}{5}}R = \sqrt{\frac{3}{5}} \times \sqrt{60} = 6 \text{ cm}$$

75. (1)



Solid angle subtended by the shaded part

$= 2\pi(1 - \cos\theta) = 2\pi(1 - \cos 60°) = \pi$

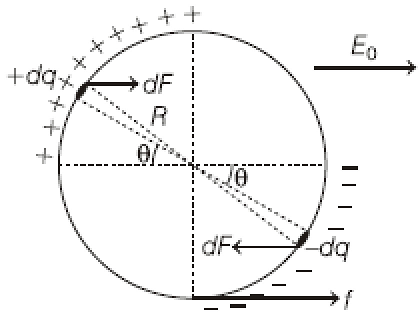
Solid angle subtended by the remaining hemisphere

$$= 2\pi - \pi = \pi$$

$$V_{\text{centre}} = \frac{kQ}{R} = \frac{k}{R}\left[\sigma\left(\frac{4\pi R^2}{4\pi} \times \pi\right)\right]$$

$$= \frac{\sigma R}{4\varepsilon_0} = \frac{(2\varepsilon_0)(2)}{4\varepsilon_0} = 1 \text{ V}$$

76. (1)



Net torque about the centre $= (\int dF(2R\sin\theta)) - fR$

$\Rightarrow \quad \tau = [\int dqE_0(2R\sin\theta)] - fR$

$\Rightarrow \quad \tau = \int_0^{\pi/2} \lambda R d\theta\, E_0(2R\sin\theta) - fR$

$\Rightarrow \quad \tau = 2\lambda E_0 R^2 - fR$

$\Rightarrow \quad I\alpha = 2\lambda R^2 E_0 - fR$

$\Rightarrow \quad \alpha = \dfrac{2\lambda R^2 E_0 - fR}{mR^2}$ ... (i)

For pure rolling, $a_{CM} = R\alpha$

$\Rightarrow \quad \dfrac{f}{m} = R\alpha$

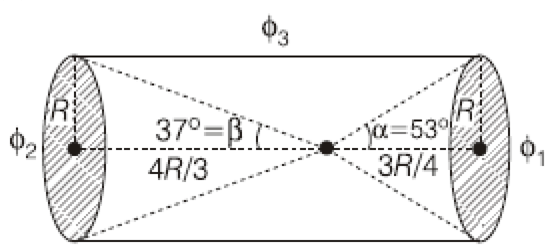
$\Rightarrow \quad \alpha = \dfrac{f}{mR}$ ... (ii)

From Eqs. (i) and (ii), we get

$$\frac{f}{mR} = \frac{2\lambda R^2 E_0 - fR}{mR^2}$$

$\Rightarrow \quad f = \lambda R E_0$

77. (7)



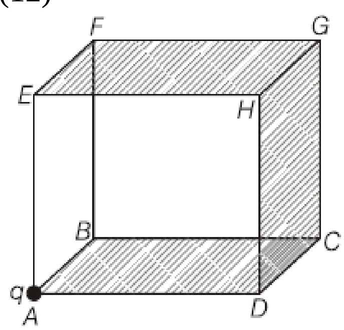
$$\phi_1 = \frac{q}{2\varepsilon_0}(1-\cos\alpha) = \frac{q}{2\varepsilon_0}\left(1-\frac{3}{5}\right) = \frac{q}{5\varepsilon_0}$$

$$\phi_2 = \frac{q}{2\varepsilon_0}(1-\cos\beta) = \frac{q}{2\varepsilon_0}\left(1-\frac{4}{5}\right) = \frac{q}{10\varepsilon_0}$$

$$\phi_1 + \phi_2 + \phi_3 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \frac{q}{5\varepsilon_0} + \frac{q}{10\varepsilon_0} + \phi_3 = \frac{q}{\varepsilon_0} \Rightarrow \phi_3 = \frac{7q}{10\varepsilon_0}$$

78. (12)



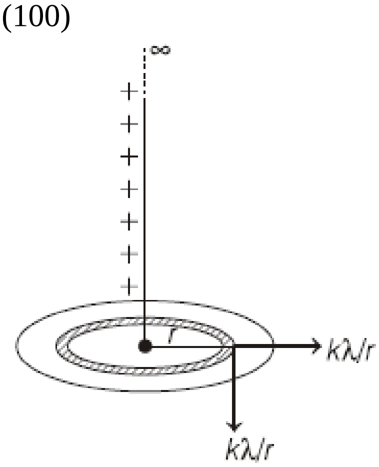
$$\phi_{ABCD} = 0$$

$$\phi_{CDGH} = \phi_{EFGH} = \frac{q}{24\varepsilon_0}$$

$$\phi = \phi_{ABCD} + \phi_{CDGH} + \phi_{EFGH}$$

$$= 0 + \frac{q}{24\varepsilon_0} + \frac{q}{24\varepsilon_0} = \frac{q}{12\varepsilon_0}, \text{ so } n = 12$$

79. (100)



Lets take a ring of radius r and thickness dr flux through the ring $= d\phi$

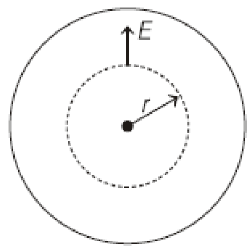
$$\Rightarrow \qquad d\phi = \left(\frac{k\lambda}{r}\right)(2\pi r dr) \Rightarrow \int d\phi = \frac{\lambda}{2\varepsilon_0}\int_0^R dr$$

$$\Rightarrow \qquad \phi = \frac{\lambda R}{2\varepsilon_0} = \frac{8.85\times10^{-10}\times 2}{2\times 8.85\times10^{-12}} = 100 \text{ V-m}$$

80. (8)

$\rho \propto h$

$\Rightarrow \quad \rho = Kh = k(R-r)$



$$d\phi = \mathbf{E}\cdot d\mathbf{A} = EdA\cos 0^\circ$$

$$\Rightarrow \int d\phi = E\int dA \quad \Rightarrow \phi = E(4\pi r^2)$$

Using Gauss's law,

$$\phi = E(4\pi r^2) = \frac{\int_0^r K(R-r)\,4\pi r^2 dr}{\varepsilon_0}$$

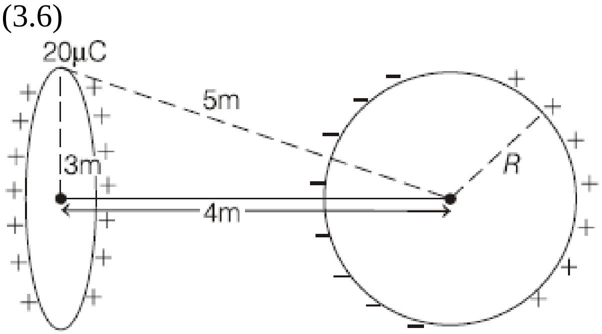
$$\Rightarrow \quad E = \frac{K\left(\frac{Rr^3}{3} - \frac{r^4}{4}\right)}{\varepsilon_0 r^2} \Rightarrow E = \frac{K}{\varepsilon_0}\left(\frac{Rr}{3} - \frac{r^2}{4}\right)$$

For maximizing electric field,

$$\frac{dE}{dr} = \frac{K}{\varepsilon_0}\left(\frac{R}{3} - \frac{2r}{4}\right) = 0 \Rightarrow \quad r = \frac{2R}{3}$$

$$\text{Depth} = R - r = R - \frac{2R}{3} = \frac{R}{3} = \frac{24}{3} = 8 \text{ m}$$

81. (3.6)



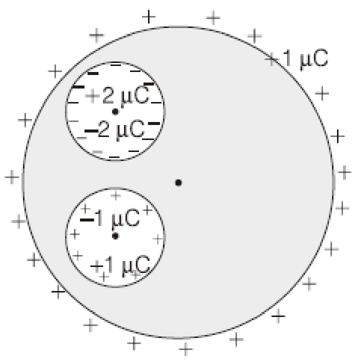
Potential of sphere,

Potential at the centre $= V_{\text{due to ring}} + V_{\text{due to induced charges}}$

$$= \frac{9\times10^{9}\times20\times10^{-6}}{\sqrt{3^2+4^2}} + \frac{k(0)}{R}$$

$$= 3.6\times10^{4}\text{ V}$$

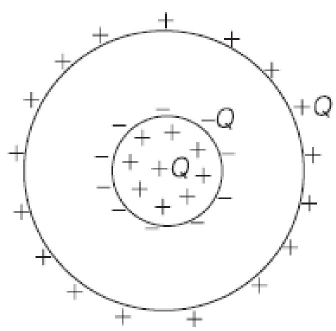
82. (9)



Electric potential at centre of conductor

$$= \frac{k(1\,\mu\text{C})}{(10\text{ cm})} = \frac{9\times10^{9}\times10^{-6}}{0.1} = 9\times10^{4}\text{V}$$

83. (3)



$$U_1 = \frac{3kQ^2}{5R}$$

$$U_2 = \frac{3kQ^2}{5R} + \frac{k(-Q)^2}{2R} + \frac{kQ^2}{2(5R)} + \left[\frac{k(-Q)}{5R}\right]Q + \frac{(kQ)Q}{5R} + \frac{(kQ)}{R}(-Q)$$

$$\Rightarrow \quad U_2 = \frac{kQ^2}{5R} \quad \Rightarrow \quad \frac{U_1}{U_2} = \frac{\frac{3kQ^2}{5R}}{\frac{kQ^2}{5R}} = 3$$

84. (96)

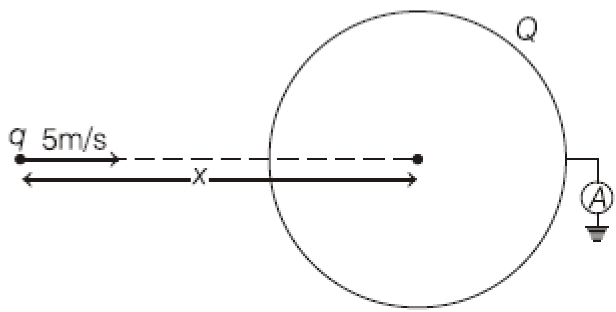
$$\frac{\sigma^2}{2\varepsilon_0} = \frac{4S}{R}$$

$$\Rightarrow \quad \frac{(Q/4\pi R^2)^2}{2\varepsilon_0} = \frac{4S}{R} \quad \Rightarrow \quad \frac{Q^2}{(16\pi^2 R^4)2\varepsilon_0} = \frac{4S}{R}$$

$$\Rightarrow \quad Q^2 = 96\pi\varepsilon_0 S\left(\frac{4}{3}\pi R^3\right) \Rightarrow \quad Q^2 = 96\pi\varepsilon_0 SV$$

So, $n = 96$

85. (800)



$$V_{\text{sphere}} = \frac{kq}{x} + \frac{kQ}{2} = 0$$

$$\Rightarrow \quad Q = -\frac{2q}{x}$$

$$\Rightarrow \quad \frac{dQ}{dt} = -2q\left(\frac{-1}{x^2}\right)\frac{dx}{dt}$$

$$= \frac{2 \times 2 \times 10^{-3}}{5^2} \times 5$$

$$= 0.8 \times 10^{-3}\ \text{A} = 800 \times 10^{-6}\ \text{A} = 800\ \mu\text{A}$$

CAPACITORS

1. (A)

$V = \frac{q}{C}$

$\Rightarrow \quad V = \frac{i \times t}{C}$

$\Rightarrow \quad 10 = \frac{50 \times 10^{-6} \times t}{800 \times 10^{-6}}$

$\Rightarrow \quad t = 160 \text{ s}$

2. (B)

$C = \frac{A\varepsilon_0}{d}$

$\frac{dC}{dt} = \frac{-A\varepsilon_0}{d^2} \frac{d(d)}{dt}$

$\Rightarrow \quad \frac{dC}{dt} = \frac{-A\varepsilon_0}{d^2} v$

$\Rightarrow \quad \frac{dC}{dt} \propto \frac{1}{d^2}$

3. (B)

Capacitance will get halved. So, charge on capacitor will also get halved as voltage across the capacitor is constant.

$E = \frac{V}{d}$

Since, V is constant and d is doubled, electric field will become half of its initial value.

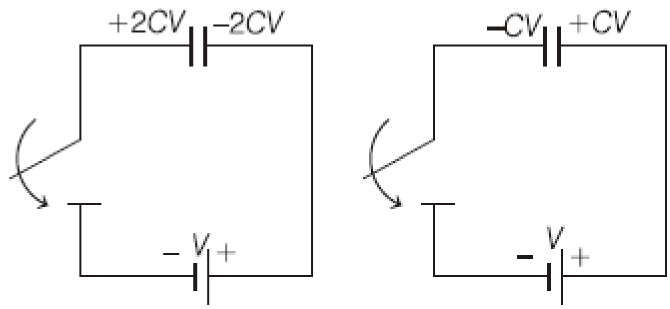
4. (C)

$U_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 12 \times 10^{-6} \times (10)^2$

$= 600 \, \mu\text{J}$

$U_2 = \frac{q^2}{2C_2} = \frac{(C_1 V)^2}{2C_2}$

5. (A)



Charge flown through battery $= CV - (-2CV)$

$= 3CV$

Work done by battery $= (3CV)V = 3CV^2$

$$\Delta U = U_2 - U_1$$

$$= \frac{1}{2}CV^2 - \frac{(2CV)^2}{2C}$$

$$= \frac{-3}{2}CV^2$$

Heat produced $= W_{\text{battery}} - \Delta U$

$$= 3CV^2 - (-1.5\,CV^2)$$

$$= 4.5\,CV^2$$

6. (C)

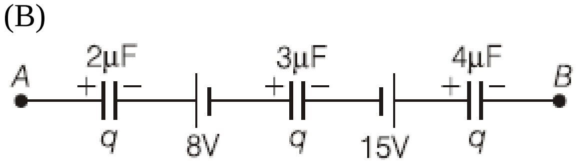
Energy loss $= \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

$$= \frac{1}{2} \times \frac{4 \times 10^{-6} \times 2 \times 10^{-6}}{6 \times 10^{-6}}(200 - 0)^2$$

$$= \frac{8}{3} \times 10^{-2}$$

$$\%\text{ loss} = \frac{8/3 \times 10^{-2}}{\frac{1}{2} \times 4 \times 10^{-6} \times (200)^2} \times 100 = 33.33\%$$

7. (B)



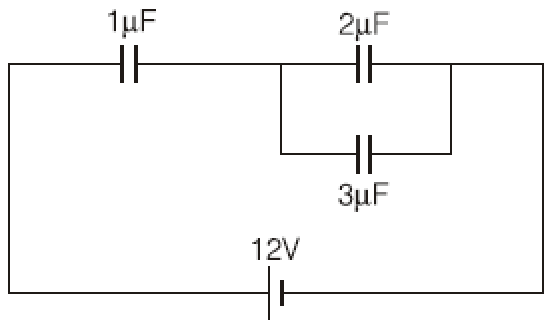
$$V_A - \frac{q}{2} - 8 - \frac{q}{3} + 15 - \frac{q}{4} = V_B$$

$$\Rightarrow \quad V_A - V_B = \left(\frac{q}{2} + \frac{q}{3} + \frac{q}{4}\right) - 7$$

$$\Rightarrow \quad 19 = \frac{13q}{12} - 7$$

$$\Rightarrow \quad q = 24\,\mu C$$

8. (B)



$$C_{eq} = \frac{5}{6}\mu F$$

Charge flown through battery $= \frac{5}{6} \times 12$

$= 10\,\mu C$

Charge on $2\,\mu F$ capacitor $= \frac{2}{2+3} \times 10$

$= 4\,\mu C$

9. (C)

Equivalent capacitance across battery $= 2\,\mu F$

Charge flown through battery $= 72 \times 2 = 144\,\mu C$

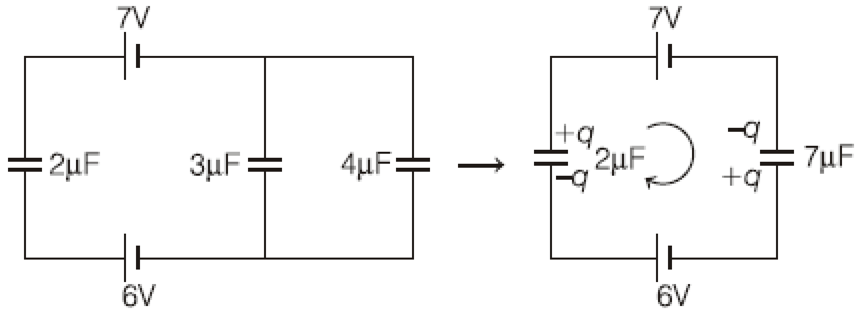
Charge on $1\,\mu F$ capacitor $= \left(\frac{1}{1+5}\right)\left(\frac{2}{2+4}\right)144$

$= 8\,\mu C$

Energy stored in $1\,\mu F$ capacitor $= \frac{q^2}{2C}$

$$= \frac{(8 \times 10^{-6})^2}{2 \times 1 \times 10^{-6}} = 32\,\mu J$$

10. (B)

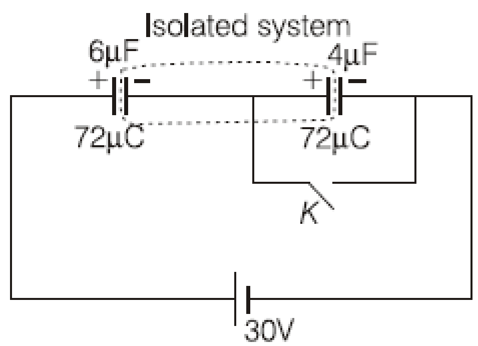


Applying KVL in the loop,

$$+\frac{q}{2}-7+\frac{q}{7}+6=0 \Rightarrow q=\frac{14}{9}\mu C$$

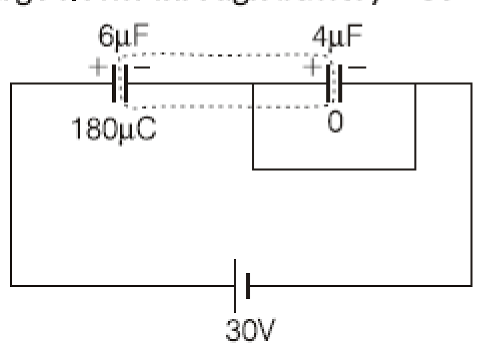
Charge on 3 μF capacitor $=\frac{3}{3+4}\times\frac{14}{9}=\frac{2}{3}\mu C$

11. (C)



$$C_{eq}=\left(\frac{1}{6}+\frac{1}{4}\right)^{-1}=2.4\,\mu F$$

Charge flown through battery $=30\times 2.4=72\,\mu C$



When key is closed, 4 μF capacitor gets short-circuited.
Charge on 6 μF capacitor will be 180 μC.

Charge flown through the key = Change in charge of isolated system

$$=(180+0)-(-72+72)=180\,\mu C$$

12. (B)

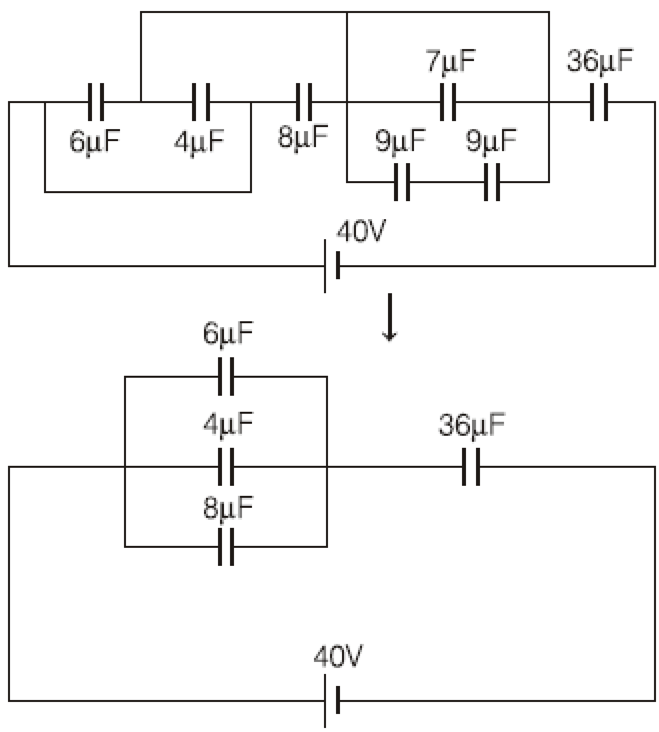
V_1 = Potential drop across $C_1 = 6V$

V_2 = Potential drop across $C_2 = 4V$

In series combination,

$$\frac{C_1}{C_2} = \frac{V_2}{V_1} \Rightarrow \frac{C_1}{C_2} = \frac{4}{6} = \frac{2}{3}$$

13. (C)



$$\frac{1}{C_{eq}} = \frac{1}{36} + \frac{1}{(6+4+8)}$$

$$C_{eq} = 12\,\mu F$$

Charge flown through battery $= 12 \times 40 = 480\,\mu C$

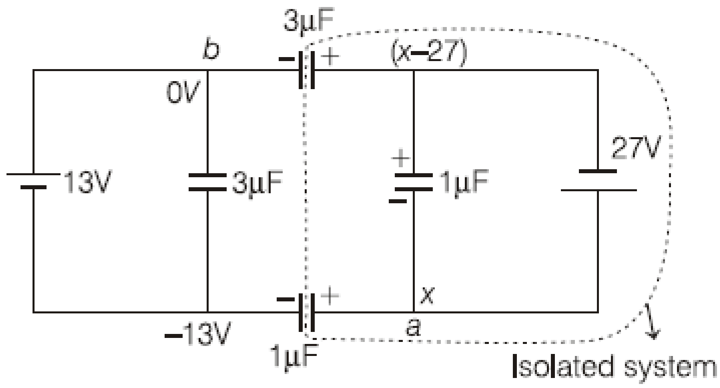
Charge on $8\,\mu F$ capacitor $= \frac{8}{18} \times 480 = 213.33\,\mu C$

$$\simeq 214\,\mu C$$

Potential difference across $8\mu F$ capacitor

$$= \frac{q}{C} = \frac{214}{8} \simeq 27V$$

14. (C)



Let $V_b = 0$ and $V_a = x$

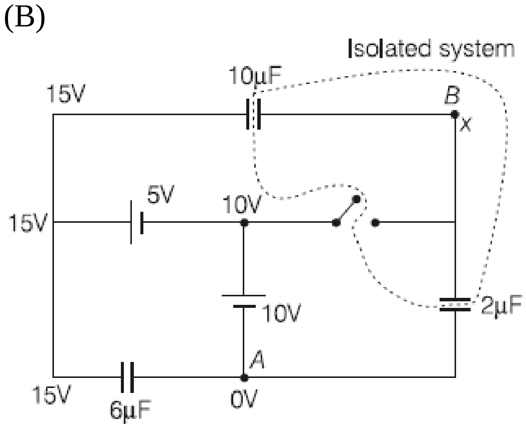
Applying charge conservation for the isolated system,

$\Rightarrow \quad 3[(x-27)-0]+1[x-(-13)]=0$

$\Rightarrow \quad x = 17V$

$V_a - V_b = 17 - 0 = 17\ V$

15. (B)

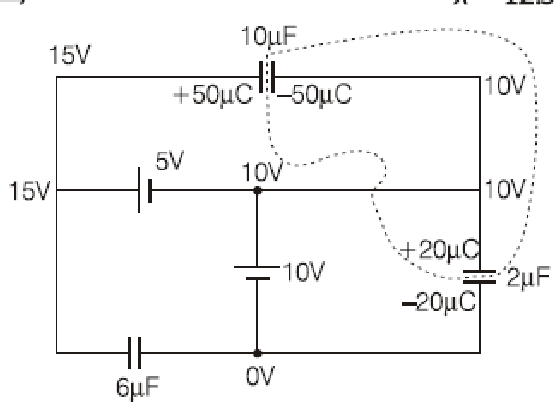


Let $V_A = 0$ and $V_B = x$

Total charge on the isolated system = 0

$\Rightarrow \quad 10(x-15)+2(x-0)=0$

$\Rightarrow \quad x = 12.5\ V$

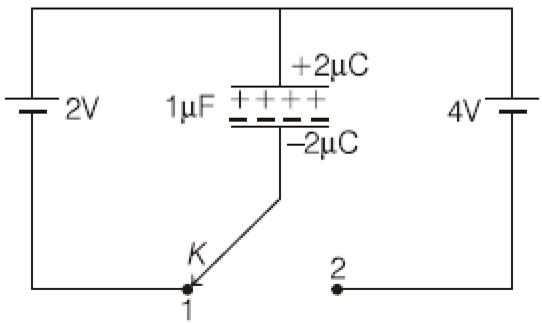


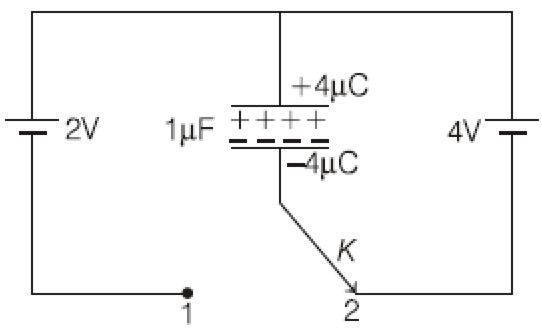
Change in charge of the isolated system

$$= (-50+20)-0$$
$$= -30\,\mu C$$

Charge flown through switch $= 30\mu$ C

16. (A)





Charge flown through 4V cell $= 4-2 = 2\,\mu C$

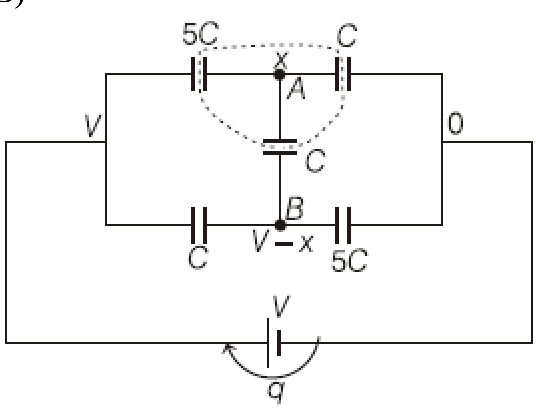
Work done by 4V cell $= 4\times 2 = 8\,\mu J$

$$\Delta U = \frac{(4\times 10^{-6})^2}{2\times 1\times 10^{-6}} - \frac{(2\times 10^{-6})^2}{2\times 1\times 10^{-6}} = 6\,\mu J$$

Heat produced $= W_{cell} - \Delta U$

$$= 8-6 = 2\,\mu J$$

17. (B)



Applying KCL at junction A,

$5C(x-V)+C[x-(V-x)]+C(x-0)=0$

$x=\frac{3V}{4}$

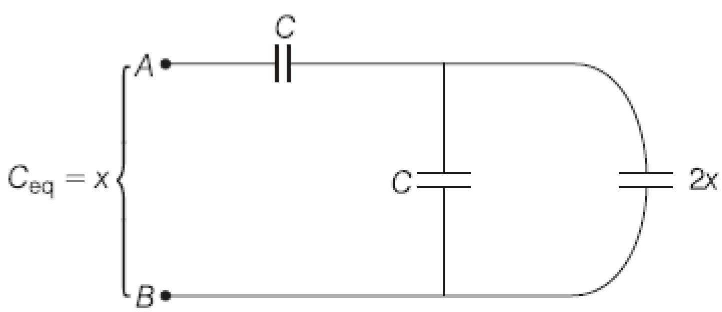
$q=$ Charge flown the battery $=5C\left(V-\frac{3V}{4}\right)+C\left(\frac{3V}{4}\right)$

$=2CV$

$C_{eq}=\frac{q}{V}=\frac{2CV}{V}=2C$

18. (A)

Let equivalent capacitance between A and B be x.



$$\frac{1}{x}=\frac{1}{C}+\frac{1}{(2x+C)}$$

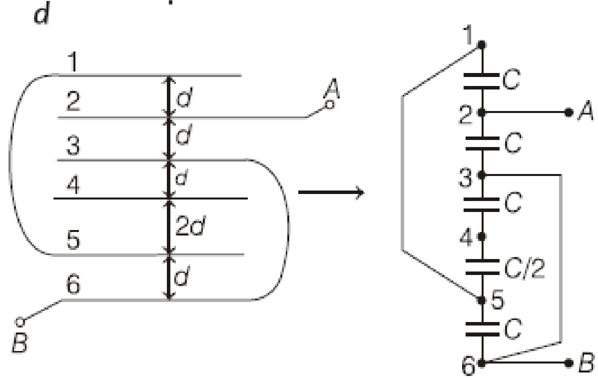
$$\Rightarrow \quad \frac{1}{x}=\frac{2x+2C}{C(2x+C)}$$

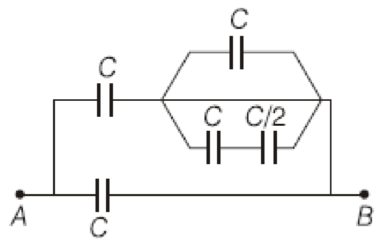
$$\Rightarrow \quad 2x^2+2Cx=2Cx+C^2$$

$$\Rightarrow \quad x=\frac{C}{\sqrt{2}}$$

19. (B)

Let $\frac{A\varepsilon_0}{d}=C=7\,\mu F$





$$C_{eq} = \frac{11C}{7} = \frac{11}{7}(7\,\mu F) = 11\,\mu F$$

20. (D)

Charge will remain same as the capacitor is isolated.

$$E = \frac{E_0}{K} \text{ and } V = \left(\frac{E_0}{K}\right)d = \frac{V_0}{K}$$

As dielectric constant becomes three times,

$$E = \frac{E_0}{3} \text{ and } V = \frac{V_0}{3}$$

21. (A)

$$C' = \frac{A\varepsilon_0}{(d-t)+\frac{t}{K}} \quad \Rightarrow \quad \frac{7}{6}\frac{A\varepsilon_0}{d} = \frac{A\varepsilon_0}{\frac{d}{3}+\frac{2d}{3K}}$$

$$\Rightarrow \quad \frac{7}{6} = \frac{3K}{K+2} \quad \Rightarrow \quad K = \frac{14}{11}$$

22. (C)

$$V = Ed \Rightarrow 25 = E(5\,\text{mm})$$

$$\Rightarrow \quad E = 5\text{ V/mm}$$

$$V' = E(2\,\text{mm}) + \left(\frac{E}{K}\right)(3\,\text{mm})$$

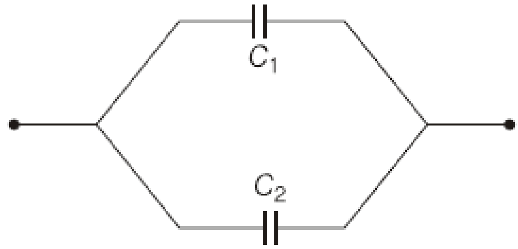
$$= 5\times 2 + \frac{5}{10}\times 3 = 11.5\text{ V}$$

23. (A)

$$C_2 = \frac{C_1}{2} \quad \Rightarrow \quad \frac{A\varepsilon_0}{\frac{3}{4}+\frac{5}{\varepsilon_r}} = \frac{1}{2}\left(\frac{A\varepsilon_0}{\frac{3}{4}}\right)$$

$$\Rightarrow \quad \varepsilon_r = \frac{20}{3}$$

24. (C)



$$C_1 = K\frac{A}{2}\frac{\varepsilon_0}{d}, \quad C_2 = \left(\frac{A}{2}\right)\frac{\varepsilon_0}{d}$$

$$C = C_1 + C_2$$

$$= K\frac{A}{2}\frac{\varepsilon_0}{d} + \frac{A}{2}\frac{\varepsilon_0}{d}$$

$$= (K+1)\frac{A\varepsilon_0}{2d}$$

25. (D)

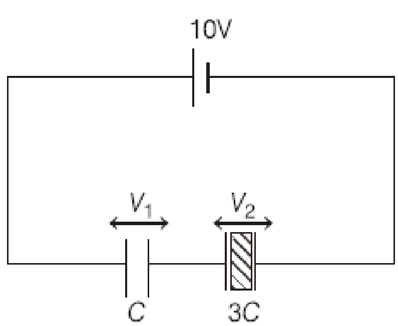
$$C = C_1 + C_2$$

$$= \frac{6\left[\pi\left(\frac{r}{2}\right)^2\right]\varepsilon_0}{d} + \frac{1\left[\pi r^2 - \pi\left(\frac{r}{2}\right)^2\right]\varepsilon_0}{d}$$

$$= \frac{9}{4}\frac{(\pi r^2)\varepsilon_0}{d}$$

$$= \frac{9C}{4}$$

26. (C)



$$V_1 + V_2 = 10 \qquad \text{...(i)}$$

and $$\frac{V_1}{V_2} = \frac{3}{1} \qquad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$V_1 = 7.5\text{ V}$$

and $$V_2 = 2.5\text{ V}$$

27. (C)

Let the final common potential drop be V.

$$q_1 + q_2 = q_1' + q_2'$$

$\Rightarrow \quad 1 \times 100 - 2 \times 20 = (1 \times 4)\,V + 2V$

$\Rightarrow \quad V = 10\ \text{V}$

$$U_1 = \frac{(1 \times 100 \times 10^{-6})^2}{2 \times 4 \times 10^{-6}} + \frac{1}{2} \times 2 \times 10^{-6} \times (20)^2$$

$$= 1.65 \times 10^{-3}\ \text{J}$$

$$U_2 = \frac{1}{2} \times (4 \times 10^{-6})\,(10)^2 + \frac{1}{2} \times (2 \times 10^{-6})\,(10)^2$$

$$= 0.3 \times 10^{-3}\ \text{J}$$

Heat dissipated $= U_1 - U_2$

$$= 1.65 \times 10^{-3} - 0.3 \times 10^{-3}$$

$$= 1.35 \times 10^{-3}\ \text{J}$$

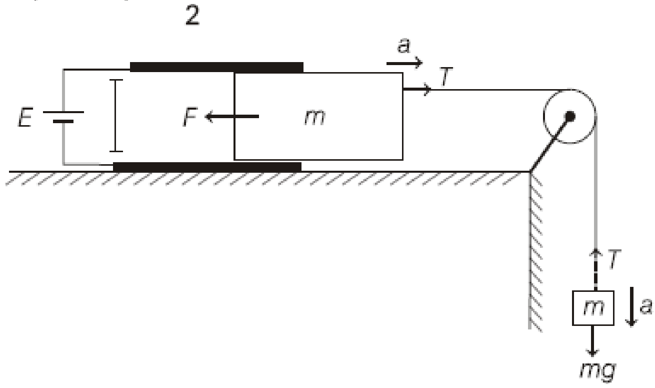
28. (C)

Force on dielectric slab due to capacitor

$$= \frac{1}{2}\frac{b\varepsilon_0 V^2}{d}(K-1)$$

$$\Rightarrow \quad F = \frac{1}{2}\frac{b\varepsilon_0}{d}\left(\sqrt{\frac{mgd}{b(K-1)\varepsilon_0}}\right)^2 (K-1)$$

$$\Rightarrow \quad F = \frac{mg}{2}$$



$$mg - T = ma \qquad \text{...(i)}$$

$$T - F = ma \qquad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$a = \frac{mg - F}{2m} = \frac{mg - \frac{mg}{2}}{2m} = \frac{g}{4}$$

29. (46)

Equivalent capacitance across battery $= C_{eq}$

$$\frac{1}{C_{eq}} = \frac{1}{12} + \frac{1}{10} + \frac{1}{5} \quad \Rightarrow \quad C_{eq} = \frac{60}{23}\,\mu\text{F}$$

Charge flown through battery $= C_{eq}E = \left(\frac{60}{23}\right)E$

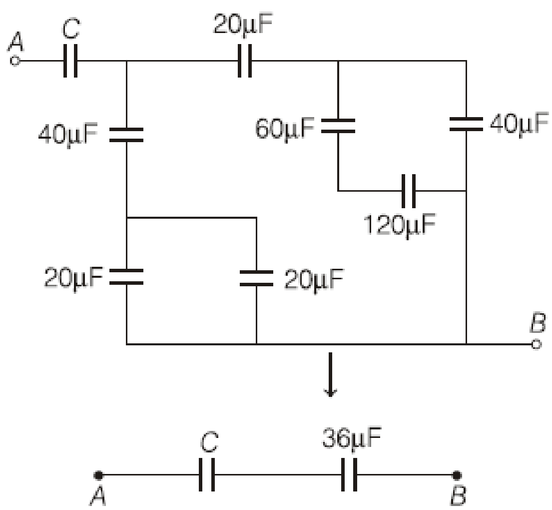
Charge on $12\,\mu\text{F}$ capacitor $= \frac{4}{4+6} \times \frac{60}{23}E = \frac{24E}{23}$

Potential difference across $12\,\mu\text{F}$ capacitor

$$= \frac{q}{C} = \frac{24E/23}{12} = \frac{2E}{23}$$

$$\frac{2E}{23} = 4 \quad \Rightarrow \quad E = 46\text{V}$$

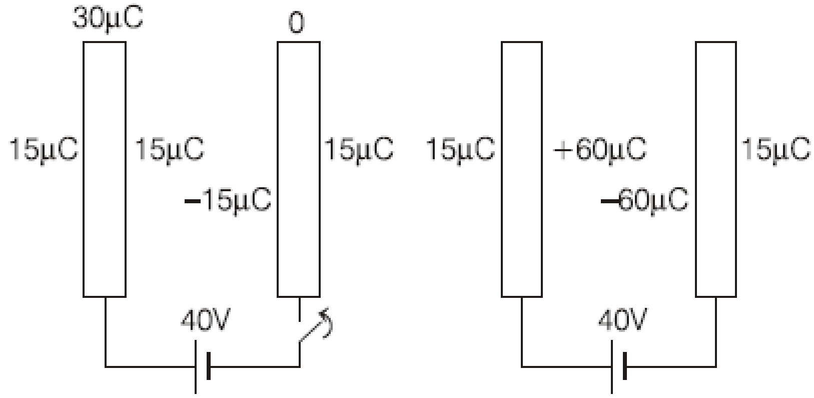
30. (18)



$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{36} \quad \Rightarrow \quad \frac{1}{12} = \frac{1}{C} + \frac{1}{36}$$

$\Rightarrow \qquad C = 18\,\mu\text{F}$

31. (1800)



Charge flown through battery $= 60 - 15 = 45\,\mu C$

Work done by battery $= (45\,\mu C)(40\,V) = 1800\,\mu J$

32. (42)

$q_1 + q_2 = q_1' + q_2'$

$\Rightarrow \quad 8 \times 125 + 0 = (8 + C_0)20$

$\Rightarrow \quad C_0 = 42\,\mu F$

33. (24)

Charge on capacitor $q = CV$

$= 5 \times 120 \times 10^{-12} \times 50$

$= 30 \times 10^{-9}\,C = 30\,nC$

Induced charges on mica $= q\left(1 - \frac{1}{K}\right) = (30\,nC)\left(1 - \frac{1}{5}\right)$

$= 24\,nC$

34. (3)

For $x < 2$; $E_0 = -\frac{dV}{dx} = \frac{10-4}{2-0} = 3\,V/mm$

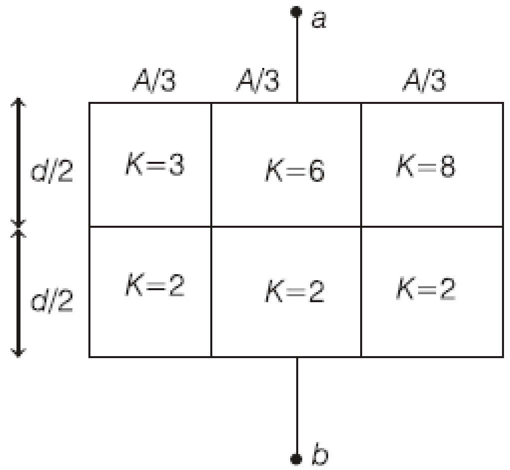
For $2 < x < 3$; $E = -\frac{dV}{dx} = \frac{4-3}{3-2} = 1\,V/mm$

$E = \frac{E_0}{K}$

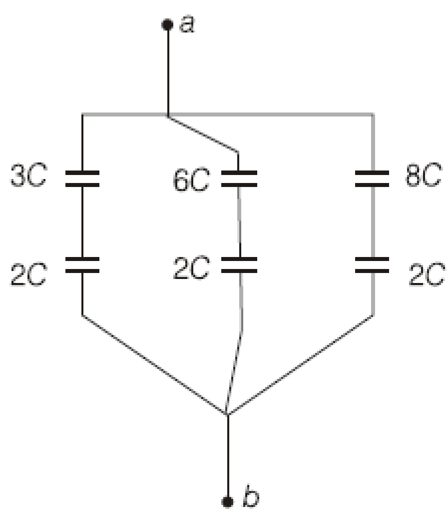
$\Rightarrow \quad 1\,V/mm = \frac{3\,V/mm}{K}$

$\Rightarrow \quad K = 3$

35. (43)



Let $$\frac{\frac{A}{3}\varepsilon_0}{\frac{d}{2}} = C \Rightarrow C = \frac{2A\varepsilon_0}{3d}$$



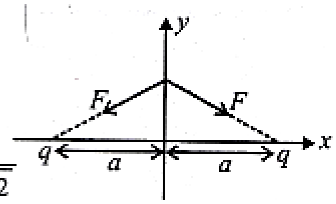
$$C_{eq} = \frac{(3C)(2C)}{3C+2C} + \frac{(6C)(2C)}{6C+2C} + \frac{(8C)(2C)}{8C+2C}$$

$$= \frac{43C}{10} = \frac{43}{10} \times \frac{2A\varepsilon_0}{3d}$$

$$= \frac{43}{15}\frac{A\varepsilon_0}{d}$$

1. (A)

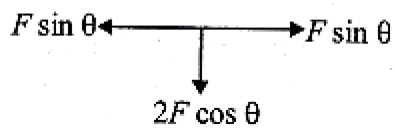
$$F_{net} = 2F\cos\theta$$



$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}}$$

$(\because y << a)$



$\Rightarrow \frac{kq^2 y}{a^3}$ So, $F \propto y$

2. (B)

$F = qE = mg$ $(q = 6e = 6\times 1.6\times 10^{-19})$

$$\text{Density (d)} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi r^3} \quad \text{or } r^3 = \frac{m}{\frac{4}{3}\pi d}$$

Putting the value of d and m $m\left(=\frac{qE}{g}\right)$ and solving we get r = 7.8×10^{-7} m

3. (C)

(c) $q = 1\mu C = 1\times10^{-6} C, r = 4\,cm = 4\times10^{-2}\,m$

There will be no potential due to induced charges.

Potential $V = \frac{kq}{r} = \frac{9\times10^9\times10^{-6}}{4\times10^{-2}} = 2.25\times10^5\,V.$

Induced electric field $E = -\frac{kq}{r^2}$

$[\because \vec{E}_i + \vec{E}_{1\mu c} = 0 \Rightarrow \vec{E}_i = -\vec{E}_{1\mu c}]$

$$= \frac{9\times10^9\times1\times10^{-6}}{16\times10^{-4}} = -5.625\times10^6\,V/m$$

4. (C)

The work done in moving a charge along an equipotential surface is always zero.
The direction of electric field is perpendicular to the equipotential surface or lines.

5. (C)

(c) As, $C = \frac{Q}{V} = \frac{It}{V}$

$$\Rightarrow \frac{V}{t} = \frac{I}{C} = \frac{2}{1\times10^{-6}} = 2\times10^6\,V/s$$

6. (C)

(c) Potential difference between any two points in electric field is given by,

$$dV = -\vec{E}\cdot\vec{dx} \Rightarrow \int_{V_O}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_O = -[10x^3]_0^2 = -80\,J/C$$

7. (A)

(a) Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by, $E = \frac{\sigma}{K\varepsilon_0}$

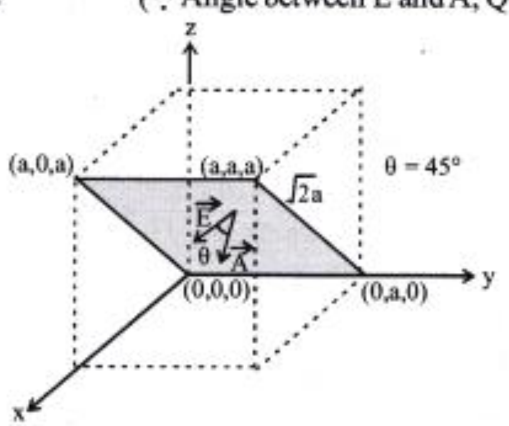
Then, charge density, $\sigma = K\varepsilon_0 E$
$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4 \approx 6 \times 10^{-7}\ C/m^2$

8. (C)

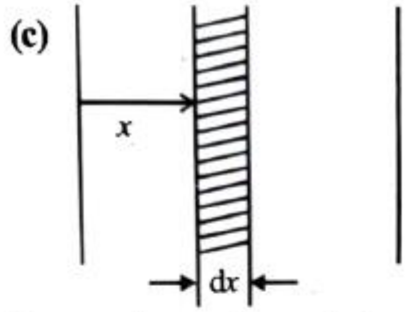
(c) Given $\vec{E} = E_0\,\hat{x}$

i.e, electric field $\vec{E}$ acts along $+x$ direction and is a constant. Therefore the electric flux through the shaded portion whose area $\vec{A} = a \times \sqrt{2}\,a = \sqrt{2}\,a^2$

$\phi = \vec{E}.\vec{A} = EA\cos\theta = E_0(\sqrt{2}\,a^2)\cos\ 45° = E_0(\sqrt{2}\,a^2)\times\frac{1}{\sqrt{2}}$
$= E_0 a^2$ ($\because$ Angle between E and A, Q = 45°)



9. (C)



Let us take an elemental capacitor of width 'dx'. Then,

$$C_{el} = \frac{(K_0 + \lambda x) A \in_0 d}{dx \ d} = \frac{(K_0 + \lambda x) C_0 d}{dx}, C_0 = \frac{A \in_0}{d}$$

$$\int \frac{1}{c_{el}} = \int_0^d \frac{dx}{(K + \lambda x) C_0 d} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{\lambda C_0 d} [\ln(K + \lambda x)]_0^d$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{\lambda C_0 d} \left[\ln\left(\frac{K + \lambda d}{K}\right) \right] \Rightarrow C_{eq} = \frac{\lambda C_0 d}{\ln\left(1 + \frac{\lambda d}{K}\right)}$$

10. (C)

Field line originate perpendicular from positive charge and terminate perpendicular from at negative charge. Further this system can be treated as an electric dipole.

11. (A)

(a) $\frac{KQ}{R} = V_0$

Now, $V = \frac{KQ}{r}$, $r \geq R$

$V = \frac{KQ}{2R^3}(3R^2 - r^2)$, $r < R$

when $r = R_1$, $V = \frac{3V_0}{2}$

$\therefore \frac{3V_0}{2} = \frac{3}{2}\frac{KQ}{R} - \frac{KQR_1^2}{R^3} \Rightarrow \frac{3V_0}{2} = \frac{3}{2}V_0 - \frac{KQ}{R^3} \cdot R_1^2$

$\Rightarrow R_1 = 0$

Similarly, $R_2 = \frac{R}{\sqrt{2}}, R_3 = \frac{4R}{3}, R_4 = 4R$

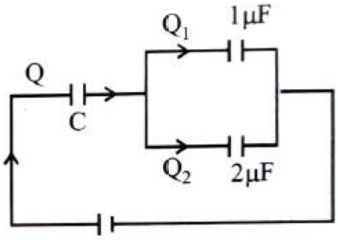
So, $R_4 - R_3 = \frac{8R}{3} > \frac{R}{\sqrt{2}}$

i.e. $R_2 < R_4 - R_3$

So, option (a) is correct.

12. (D)

(d)

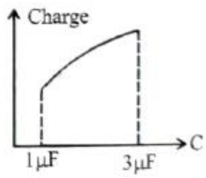


From figure, $Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q$

$Q = E\left(\frac{C \times 3}{C+3}\right)$

Therefore graph d correctly dipicts.

$\therefore Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$



13. (D)

Given: Length of wire $L = 20$ cm

charge $Q = 10^3 \varepsilon_0$

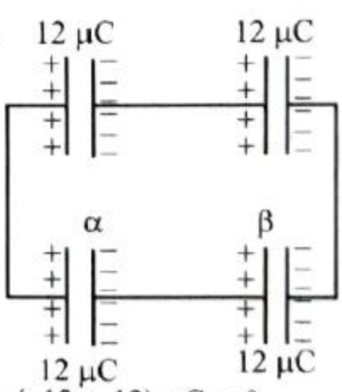
We know, electric field at the centre of the semicircular arc

$$E = \frac{2K\lambda}{r} \Rightarrow E = \frac{2K\left(\frac{2Q}{\pi r}\right)}{r}\left[As\lambda = \frac{2Q}{\pi r}\right]$$

$$= \frac{4KQ}{\pi r^2} = \frac{4KQ\pi^2}{\pi L^2} = \frac{4\pi KQ}{L^2} = 25 \times 10^3 N/C\hat{i}$$

14. (A)

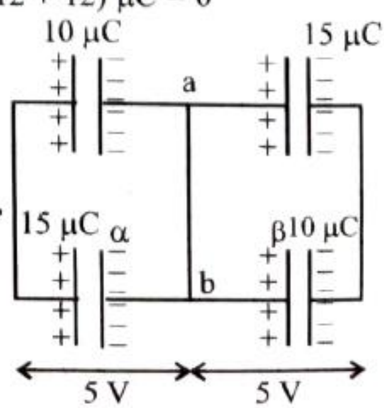
(a) Initially,



$(Q_\alpha + Q_\beta)_i = (-12 + 12)\ \mu C = 0$

Finally,

Finally,



$(Q_\alpha + Q_\beta)_f = -15 + 10 = -5\ \mu C$

So, 5 μC charge from b → a

so that not charge on plate α and β is zero.

15. (C)

Inside the cavity net charge is zero.

$\therefore\ Q_1 = 0$ and $\sigma_1 = 0$

There is no effect of point charges $+Q$, $-Q$ and induced charge on inner surface on the outer surface.

$\therefore\ Q_2 = 0$ and $\sigma_2 = 0$

16. (C)

(c) Applying Gauss's law

$$\oint_S \vec{E}\cdot\overrightarrow{ds} = \frac{Q}{\epsilon_0};\ Q = \int_a^r \frac{A}{r} 4\pi r^2 dr = 2\pi A[r^2 - a^2]$$

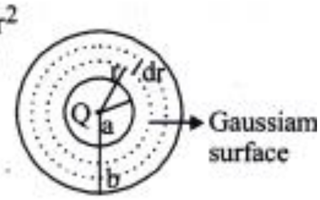
$$\therefore E \times 4\pi r^2 = \frac{Q + 2\pi A r^2 - 2\pi A a^2}{\epsilon_0}$$

$$E \times 4\pi r^2 = (Q - 2\pi A a^2) + 2\pi A r^2$$

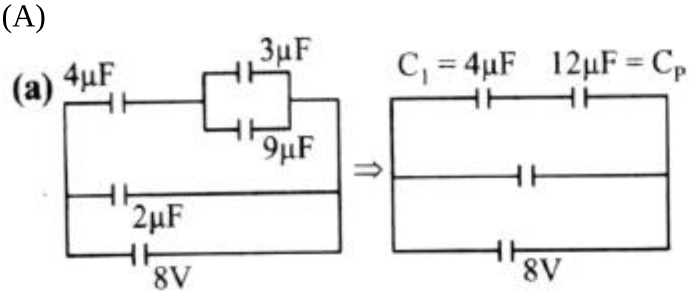
For E to be independent of 'r'

$$Q - 2\pi A a^2 = 0$$

$$\Rightarrow A = \frac{Q}{2\pi a^2}$$



17. (A)



By voltage division rule,

$$\text{Charge on } C_1 \text{ is } q_1 = \left[\left(\frac{12}{4+12}\right)\times 8\right]\times 4 = 24\mu C$$

$$\text{The voltage across } C_P \text{ is } V_P = \frac{4}{4+12}\times 8 = 2V$$

$\therefore$ Voltage across 9μF is also 2V

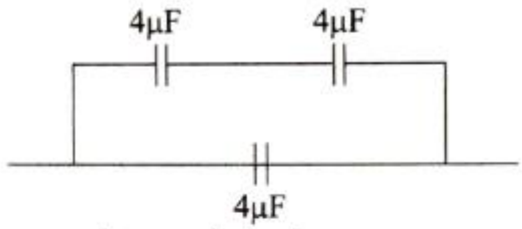
$\therefore$ Charge on 9μF capacitor $= 9 \times 2 = 18\mu C$

$\therefore$ Total charge on 4 μF and 9μF = 42μC

$$\therefore E = \frac{KQ}{r^2} = 9\times 10^9 \times \frac{42\times 10^{-6}}{30\times 30} = 420\ NC^{-1}$$

18. (D)

(d) To get effective capacitance of 6 μF two capacitors of 4 μF each connected in sereies and one of 4 μF capacitor in parallel with them.



Two capacitances in series

$\therefore \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

1 capacitor in parallel

$\therefore C_{eq}=C_3+C=4+2=6\,\mu F$

19. (A)

(a) $T = PE\sin\theta$ Torque experienced by the dipole in an electric field, $\vec{T}=\vec{P}\times\vec{E}$

$\vec{p}=p\cos\theta\,\hat{i}+p\sin\theta\,\hat{j}$; $\vec{E}_1=E\vec{i}$

$\vec{T}_1=\vec{p}\times\vec{E}_1=(p\cos\theta\,\hat{i}+p\sin\theta\,\hat{j})\times E(\hat{i})$

$\tau\hat{k}=pE\sin\theta(-\hat{k})$ ...(i)

$\vec{E}_2=\sqrt{3}E_1\hat{j}$; $\vec{T}_2=p\cos\theta\hat{i}+p\sin\theta\hat{j})\times\sqrt{3}E_1\hat{j}$

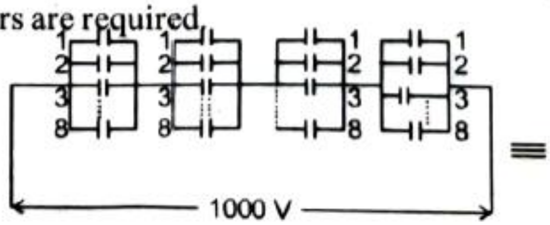
$\tau\hat{k}=\sqrt{3}\,pE_1\cos\theta\hat{k}$ ...(ii)

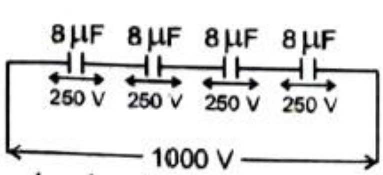
From eqns. (i) and (ii)

$pE\sin\theta=\sqrt{3}\,pE\cos\theta$; $\tan\theta=\sqrt{3}$ $\therefore$ $\theta=60°$

20. (B)

(b) To get a capacitance of 2μF arrangement of capacitors of capacitance 1μF as shown in figure 8 capacitors of 1μF in parallel with four such branches in series i.e., 32 such capacitors are required.





$\frac{1}{C_{eq}}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $\therefore$ $C_{eq}=2\,\mu F$

21. (C)

(c) The net flux linked with closed surfaces S_1, S_2, S_3 & S_4 are

For surface S_1, $\phi_1 = \frac{1}{\varepsilon_0}(2q)$

For surface S_2, $\phi_2 = \frac{1}{\varepsilon_0}(q+q+q-q) = \frac{1}{\varepsilon_0}2q$

For surface S_3, $\phi_3 = \frac{1}{\varepsilon_0}(q+q) = \frac{1}{\varepsilon_0}(2q)$

For surface S_4, $\phi_4 = \frac{1}{\varepsilon_0}(8q-2q-4q) = \frac{1}{\varepsilon_0}(2q)$

Hence, $\phi_1 = \phi_2 = \phi_3 = \phi_4$ i.e. net electric flux is same for all surfaces.

Keep in mind, the electric field due to a charge outside (S_3 and S_4), the Gaussian surface contributes zero net flux through the surface, because as many lines due to that charge enter the surface as leave it.

22. (C)

(c) Potential gradient is given by,

$\Delta V = E.d \Rightarrow 0.8 = Ed\,(\max)$

$\Delta V = Ed\cos\theta = 0.8 \times \cos 60 = 0.4$

Hence, maximum potential at a point on the sphere = 589.4 V

23. (D)

(d) Energy of sphere $= \frac{Q^2}{2C}$

$\Rightarrow 4.5 = \frac{16 \times 10^{-12}}{2C} \Rightarrow C = \frac{16 \times 10^{-12}}{9} = 4\pi\varepsilon_0 R$

(capacity of spherical conductor)

$R = \frac{16 \times 10^{-12}}{9} \times \frac{1}{4\pi\varepsilon_0} \qquad \because \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

$= 9 \times 10^9 \times \frac{16}{9} \times 10^{-12} = 16\text{ mm}$

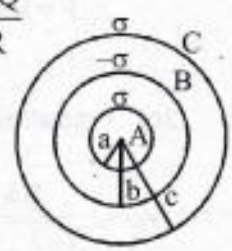
24. (B)

(b) Potential outside the shell, $V_{outside} = \frac{KQ}{r}$

where r is distance of point from the centre of shell

Potential inside the shell, $V_{inside} = \frac{KQ}{R}$

where 'R" is radius of the shell



$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

$$V_B = \frac{1}{4\pi \epsilon_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

25. (A)

(a) $Q_i = Q_{Plate}\left(1 - \frac{1}{K}\right)$

$$= KCV\left(1 - \frac{1}{K}\right) = CV(K-1) = 90\,pF \times 2 \times \left(\frac{5}{3} - 1\right) = 1.2\,nC$$

26. (A)

Equilibrium position will shift to point where resultant force = 0

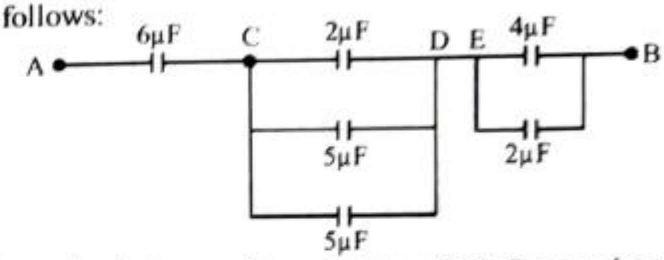
$$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$$

Total energy $= \frac{1}{2}m\omega^2 A^2 + \frac{1}{2}kx_{eq}^2$

Total energy $= \frac{1}{2}m\omega^2 A^2 + \frac{1}{2}\frac{q^2E^2}{k}$

27. (D)

(d) The simplified circuit of the circuit given in question as follows:



The equivalent capacitance between C & D capacitors of 2μF, 5μF and 5 μF are in parallel.

$\therefore$ $C_{CD} = 2 + 5 + 5 = 12\ \mu F$ ($\because$ In parallel grouping $C_{eq} = C_1 + C_2 + ... + C_n$)

Similarly equivalent capacitance between E & B $C_{EB} = 4 + 2 = 6\mu F$

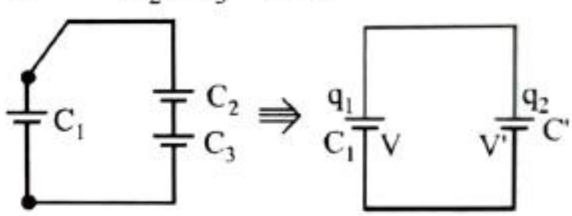
Now equivalent capacitance between A & B

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$$

$$\Rightarrow C_{eq} = \frac{12}{5} = 2.4\ \mu F$$ ($\because$ In series grouping, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$)

28. (D)

(d) $C' = \frac{C_2 C_3}{C_2 + C_3} = \frac{3 \times 6}{3 + 6} = 2\mu F$



As $V = V' \Rightarrow \frac{q_1}{C_1} = \frac{q_2}{C'} \Rightarrow \frac{q_1}{1} = \frac{q_2}{2} \Rightarrow q_1 = \frac{q_2}{2}$

and, $q_1 + q_2 = (60 \times 1)$

$\Rightarrow q_1 + 2q_1 = 60 \Rightarrow 3q_1 = 60 \Rightarrow q_1 = 20\mu C$

and, $q_2 = 60 - 20 = 40\ \mu C$.

29. (B)

(b) By Gauss law,

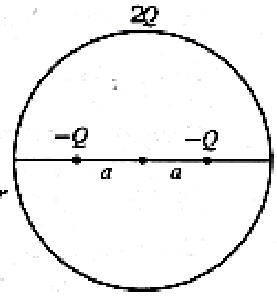
$$\oint \vec{E} . d\vec{A} = \frac{Q_{in}}{\epsilon_o} = \frac{Q + q - q}{\epsilon_o} = \frac{Q}{\epsilon_o}$$

or $E \times 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, r \geq R$

So, electric field outside is same as point charge inside the shell.

30. (A)

(a) $\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$



or $E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int S(4\pi r^2)dr$

$\Rightarrow E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r (kr)(4\pi r^2)dr$

or $E \times 4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\left(\frac{r^4}{4}\right)$

$\therefore \quad E = \frac{k}{4\varepsilon_0}r^2$ ...(i)

Also $2Q = \int_0^R (kr)\,(4\pi r^2)dr = 4\pi k\left|\frac{r^4}{4}\right|_0^R$

$Q = \frac{\pi k R^4}{2}$ ....(ii)

From above equations,

$E = \frac{Qr^2}{2\pi\varepsilon_0 R^4}$. For $r = a$, $E = \frac{Qa^2}{2\pi \epsilon_o R^4}$ ....(iii)

According to given condition

$QE = \frac{1}{4\pi \epsilon_o}\frac{Q^2}{(2a)^2} = Q\cdot\frac{Qa^2}{2\pi \epsilon_o R^4} = \frac{Q^2}{4\pi \epsilon_o \cdot 4a^2}$

$\Rightarrow R = a8^{1/4} \Rightarrow a = 8R^{-1/4}$

(b) $\tau = -PE\sin\theta$

$I\alpha = -PE\,(\theta) \Rightarrow \alpha = \frac{PE}{I}(-\theta)$

On comparing with

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$

31. (B)

(b) $\tau = -PE\sin\theta$

$I\alpha = -PE\,(\theta) \Rightarrow \alpha = \frac{PE}{I}(-\theta)$

On comparing with

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$

32. (B)

(b) Potential energy of a dipole is given by

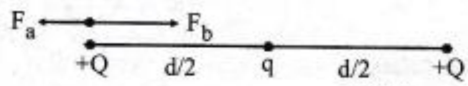
$U = -\vec{P}.\vec{E} \ = -\text{PE}\cos\theta$

[Where θ = angle between dipole and perpendicular to the field]

$= -(10^{-29})(10^{3})\cos 45°$

$= -0.707 \times 10^{-26}\ \text{J} = -7 \times 10^{-27}\text{J}$

33. (A)



Force due to charge + Q, $F_a = \dfrac{KQQ}{d^2}$

Force due to charge q, $F_b = \dfrac{KQq}{\left(\dfrac{d}{2}\right)^2}$

For equilibrium, $\vec{F}_a + \vec{F}_b = 0$

$\Rightarrow \dfrac{kQQ}{d^2} + \dfrac{kQq}{(d/2)^2} = 0 \qquad \therefore q = -\dfrac{Q}{4}$

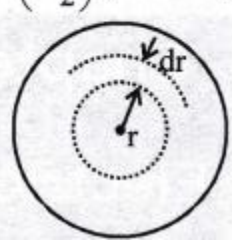
34. (B)

$Q = \int \rho dv = \int_0^R \dfrac{A}{r^2} e^{-2r/a}\left(4\pi r^2 dr\right)$

$= 4\pi A \int_0^R e^{-2r/a} dr = 4\pi A \left(\dfrac{e^{-2r/a}}{-\dfrac{2}{a}}\right)_0^R = 4\pi A\left(-\dfrac{a}{2}\right)\left(e^{-2R/a} - 1\right)$

$Q = 2\pi a A(1 - e^{-2R/a})$

$R = \dfrac{a}{2}\log\left(\dfrac{1}{1 - \dfrac{Q}{2\pi a A}}\right)$



35. (D)

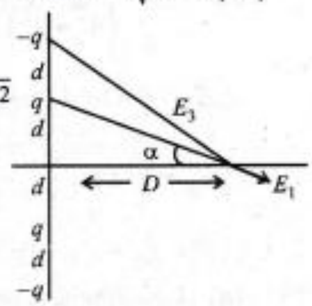
$\vec{E} = (\vec{E}_1 + \vec{E}_2) + (\vec{E}_3 + \vec{E}_4)$ or $E = 2E\cos\alpha - 2E\cos\beta$

$= \dfrac{2kq}{(D^2 + d^2)} \times \dfrac{D}{\sqrt{D^2 + d^2}} - \dfrac{2kq}{(D^2 + (2d)^2} \times \dfrac{D}{\sqrt{D^2 + (2d)^2}}$

$= \dfrac{2kqD}{(D^2 + d^2)^{3/2}} - \dfrac{2kqD}{[D^2 + (2d)^2]^{3/2}}$

For d << D

$E \propto \dfrac{D}{D^3} \propto \dfrac{1}{D^2}$

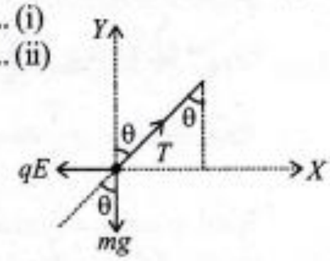


36. (D)

At equilibrium resultant force on bob must be zero, so

$T\cos\theta = mg$ ..... (i)

$T\sin\theta = qE$ ..... (ii)



Solving (i) and (ii) we get

$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2}$$

[Here, $q = 5\times10^{-6}$ C, E = 2000 v/m, m = 2×10^{-3} kg]

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$$

37. (B)

Electric field on the axis of a ring of radius R at a distance h from the centre,

$$E = \frac{kQh}{\left(h^2+R^2\right)^{3/2}}$$

Condition: for maximum electric field $\frac{dE}{dh} = 0$

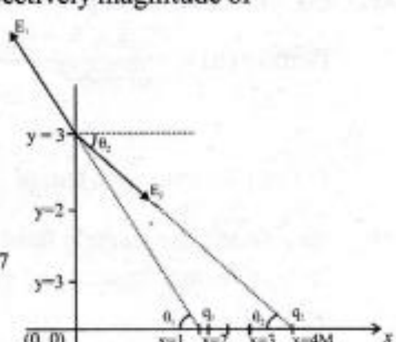
$$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{\left(R^2+h^2\right)^{3/2}}\right] = 0$$

By using the concept of maxima and minima we get,

$$h = \frac{R}{\sqrt{2}}$$

38. (A)

Let $\vec{E}_1$ and $\vec{E}_2$ are the vaues of electric field due to charge, q_1 and q_2 respectively magnitude of



$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\sqrt{10}\times10^{-6}}{\left(1^2+3^2\right)}$$

$$= \left(9\times10^9\right)\times\sqrt{10}\times10^{-7}$$

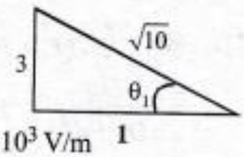
$$= 9\sqrt{10}\times10^2$$

$$\therefore \vec{E}_1 = 9\sqrt{10}\times10^2\left[\cos\theta_1\left(-\vec{i}\right)+\sin\theta_1\vec{j}\right]$$

$$\Rightarrow E_1 = 9\times\sqrt{10}\times10^2\left[\frac{1}{\sqrt{10}}\left(-\hat{i}\right)+\frac{3}{\sqrt{10}}\hat{j}\right]$$

$$\Rightarrow E_1 = 9\times10^2\left[-\hat{i}+3\hat{j}\right] = \left[-9\hat{i}+27\hat{j}\right]10^2$$

Similarly, $E_2 = \frac{1}{4\pi \epsilon_0}\frac{q_2}{r^2}$



$E_2 = \frac{9\times10^9\times(25)\times10^{-6}}{\left(4^2+3^2\right)}$ $E_2 = 9\times10^3$ V/m

$\therefore \vec{E}_2 = 9\times10^3\left(\cos\theta_2\hat{i} - \sin\theta_2\hat{j}\right)$ $\because \tan\theta_2 = \frac{3}{4}$

$\therefore \vec{E}_2 = 9\times10^3\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right) = \left(72\hat{i} - 54\hat{j}\right)\times10^2$

$\therefore \vec{E} = \vec{E}_1 + \vec{E}_2 = \left(63\hat{i} - 27\hat{j}\right)\times10^2\, V/m$

39. (B)

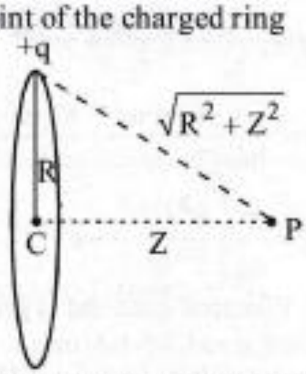
(b) The electric potential at the bisector is zero and electric field is antiparallel to the dipole moment.

$$\therefore V = 0 \text{ and } \vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-\vec{P}}{d^3}\right)$$

40. (C)

(c) Potential at any point of the charged ring

$V_p = \frac{Kq}{\sqrt{R^2+Z^2}}$



R = 3a, Z = 4a

$\ell = \sqrt{R^2+Z^2} = 5a$

The minimum velocity (v_0) should just sufficient to reach the point charge at the center, therefore

$$\frac{1}{2}mv_0^2 = q\,[V_C - V_P] = q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

$$\Rightarrow \quad v_0^2 = \frac{4Kq^2}{15ma} = \frac{4}{15}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{ma}$$

$$\Rightarrow v_0 = \sqrt{\frac{2}{m}}\left(\frac{2q^2}{15\times4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$$

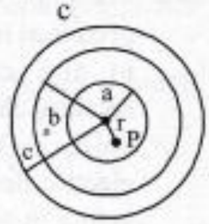
41. (B)

Electric potential is constant inside a charged spherical shell and outside it varsely with distance.

42. (D)

(d) Potential at point P, $V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$

Since surface charge densities are equal to one another i.e., $\sigma_a = \sigma_b = \sigma_c$



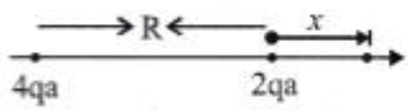
$\therefore Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$

$\therefore Q_a = \left[\frac{a^2}{a^2+b^2+c^2}\right]Q$, $Q_b = \left[\frac{b^2}{a^2+b^2+c^2}\right]Q$

$Q_c = \left[\frac{c^2}{a^2+b^2+c^2}\right]Q \quad \therefore V = \frac{Q}{4\pi\epsilon_0}\left[\frac{(a+b+c)}{a^2+b^2+c^2}\right]$

43. (D)

(d) Let at a distance 'x' from point B, both the dipoles produce same potential



$\therefore \frac{4qa}{(R+x)} = \frac{2qa}{(x^2)}$

$\Rightarrow \sqrt{2}x = R + x \Rightarrow x = \frac{R}{\sqrt{2}-1}$

Therefore distance from A at which both of them produce the same potential

$= \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}$

44. (C)

(c) Using conservation of energy

$U_i = U_F + \frac{1}{2}mv^2$; $\frac{kq_1q_2}{r_1} = \frac{kq_1q_2}{r_2} + \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}mv^2 = kq_1q_2\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \Rightarrow v^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= \frac{2 \times 9 \times 10^9 \times 10^{-12}}{4 \times 10^{-6} \times 10^{-3}}\left[1 - \frac{1}{9}\right] = 4 \times 10^{+6}$

$v = 2 \times 10^3$ m/s

45. (D)

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D + \frac{d}{2}\right)} + \frac{(-q)Q}{\left(D - \frac{d}{2}\right)}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right], \text{Ignoring } \frac{d^2}{4}$$

46. (B)

(b) Using, $[K+U]_i = [K+U]_f$

or $0 + Vq = mv^2 + v'q$ or $mv^2 = (V - V')q$

$$= -q\int_{r_0}^{r} Edr = q\int_{r_0}^{r} \frac{\lambda}{2\pi \in_0 r} dr = \frac{\lambda q}{2\pi \in_0} \ln\left(\frac{r}{r_0}\right)$$

$$\Rightarrow v \propto \sqrt{l\text{n}\frac{r}{r_0}}$$

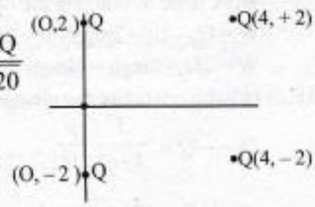
47. (B)

(b) Potential at origin

$$v = \frac{KQ}{2} + \frac{KQ}{2} + \frac{KQ}{\sqrt{20}} + \frac{KQ}{\sqrt{20}}$$

and potential at $\infty = 0$

$$= KQ\left(1 + \frac{1}{\sqrt{5}}\right)$$



$\therefore$ Work required to put a fifth charge Q at origin W = VQ

$$= \frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right)$$

48. (A)

(a) For parallel combination

$q = 10(C_1 + C_2)$

$q_1 = 500\ \mu C$

$500 = 10(C_1 + C_2)$

$C_1 + C_2 = 50\mu F$ ....(i)

For Series Combination–

$$q_2 = 10\frac{C_1C_2}{(C_1 + C_2)}$$

$80 = 10\dfrac{C_1C_2}{50}$ From equation ....(ii)

$C_1C_2 = 400$ ....(iii)

From equation (i) and (ii)

$C_1 = 10\mu F$ $\quad C_2 = 40\mu F$

49. (B)

(b) $U = U_f - U_i = \dfrac{q}{2}\left(\dfrac{1}{C_f} - \dfrac{1}{C_i}\right)$

$$= \frac{(5\times 10)^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right) \times 10^6 = 3.75 \times 10^{-6} J$$

50. (B)

(b) Capacitance of a capacitor with a dielectric of dielectric constant k is given by

$$C = \frac{k \in_0 A}{d}$$

$$\because\ E = \frac{V}{d} \qquad \therefore\ C = \frac{k \in_0 AE}{V}$$

$$15 \times 10^{-12} = \frac{k \times 8.86 \times 10^{-12} \times 10^{-4} \times 10^{6}}{500}$$

$$k = 8.5$$

51. (B)

(b) Energy stored in the system initially

$$U_i = \frac{1}{2} CE^2$$

$$U_f = \frac{1}{2}\frac{Q^2}{C_{eq}} = \frac{(CE)^2}{2 \times 4C} = \frac{1}{2}\frac{CE^2}{4}$$

[As $Q = CE$, and $C_{eq} = 4C$]

$$\Delta U = \frac{1}{2} CE^2 \times \frac{3}{4} = \frac{3}{8} CE^2 = \frac{3}{8}\frac{Q^2}{C}$$

52. (C)

(c) $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

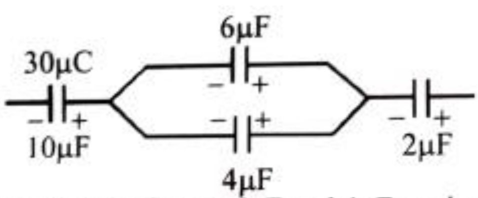
$\therefore Q = \varepsilon_0 . E . A$

$= 8.85 \times 10^{-12} \times 100 \times 1$

$= 8.85 \times 10^{-10} C$

53. (D)

(d)



As given in the figure, 6μF and 4μF are in parallel. Now using charge conservation.

Charge on 6μF capacitor $= \frac{6}{6+4} \times 30 = 18\mu C$

Since charge is asked on right plate therefore is $+18\mu C$

54. (B)

(b) $W = -\Delta U$

$= U_i - U_f$

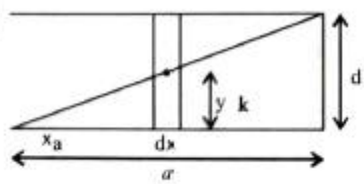
$$= U_i - \frac{U_i}{K} = U_i\left(1 - \frac{1}{K}\right) = \frac{1}{2} CV^2 \left(1 - \frac{1}{K}\right) = 508 PJ$$

55. (B)

(b) From figure, $\frac{y}{x} = \frac{d}{a} \Rightarrow y = \frac{d}{a}x$

$dy = \frac{d}{a}(dx) \Rightarrow \frac{1}{dc} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$

$\frac{1}{dc} = \frac{1}{(dx)a\varepsilon_0}\left(\frac{y}{k} + d - y\right)$



$\int dc = \int \frac{\varepsilon_0 adx}{\frac{y}{k} + d - y}$

$\Rightarrow c = \varepsilon_0 a.\frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)}$

$= \frac{\varepsilon_0 a^2}{\left(\frac{1}{k} - 1\right)d}\left[\ell n\left(d + y\left(\frac{1}{k} - 1\right)\right)\right]_0^d$

$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right)$

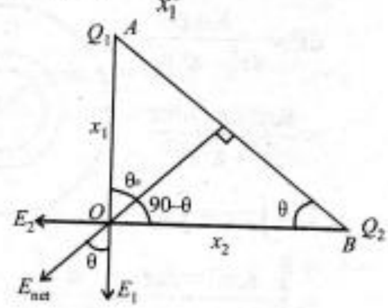
$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k \in_0 a^2 \ell nk}{(k-1)d}$

56. (C)

Electric field due charge Q_2, $E_2 = \frac{kQ_2}{x_2^2}$

Electric field due charge Q_1, $E_1 = \frac{kQ_1}{x_1^2}$

From figure,



$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2}$

$\Rightarrow \frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$

$\Rightarrow \frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{x_2}{x_1}$

or, $\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$.

57. (C)

For spherical shell

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad \text{(if } r \geq R\text{)}$$
$$= 0 \qquad \text{(if } r < R\text{)}$$

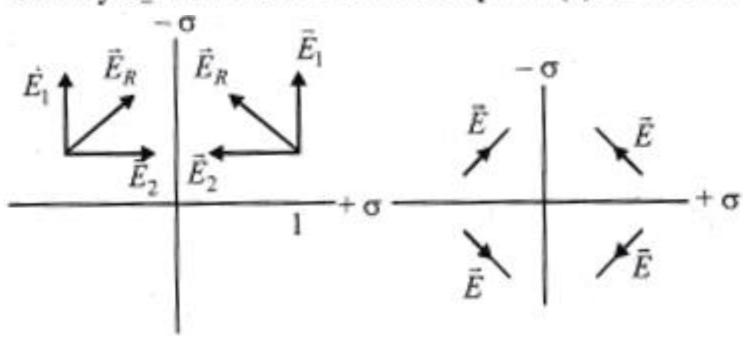
Force on charge in electried field, $F = qE$

$\therefore F = 0 \qquad$ (For $r < R$)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \qquad \text{(For } r > R\text{)}$$

58. (C)

The electric field produced due to uniformly charged infinite plane is uniform. So option (b) and (d) are wrong. And +ve charge density σ_+ is bigger in magnitude so its field along Y direction will be bigger than field of –ve charge density σ_- in X direction. Hence option (c) is correct.



59. (C)

Given, Electric field, $E = E_0(1 - x^2)$

$\therefore$ Force, $F = qE = qE_0(1 - x^2)$

Also, $F = ma = mv\frac{dv}{dx} \qquad \left(\because a = v\frac{dv}{dx}\right)$

$\therefore mv\frac{dv}{dx} = qE_0(1 - x^2) \Rightarrow v\,dv = \frac{qE_0(1 - x^2)dx}{m}$

Integrating both sides we get,

$\Rightarrow \int_0^v v\,dv = \int_0^x \frac{qE_0(1 - x^2)dx}{m} \Rightarrow \frac{v^2}{2} = \frac{qE_0}{m}\left(x - \frac{9x^3}{3}\right) = 0$

$\Rightarrow x = \sqrt{\frac{3}{a}}$

60. (B)

$F_x = 0,\ a_x = 0,\ (v)_x = \text{constant}$

Time taken to reach at $'P' = \frac{d}{v_0} = t_0$ (let) ...(i)

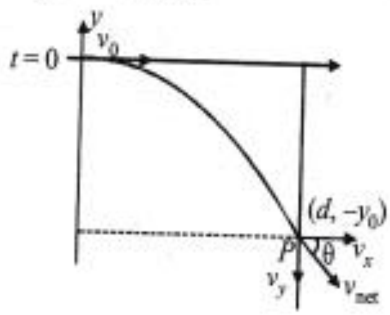
(Along $-y$), $y_0 = 0 + \frac{1}{2} \cdot \frac{qE}{m} \cdot t_0^2$ ...(ii)

$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m \cdot v_0},\ \left(t = \frac{d}{v_0}\right)$

$\tan\theta = \frac{qEd}{m \cdot v_0^2}$, Slope $= \frac{-qEd}{mv_0^2}$

No electric field $\Rightarrow F_{\text{net}} = 0,\ \vec{v} =$ const.

$y = mx + c, \left\{ \begin{matrix} m = \frac{qEd}{mv_0^2} \\ (d, -y_0) \end{matrix} \right\}$



$-y_0 = \frac{-qEd}{mv_0^2}, d + c$

$\Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$

$y = \frac{-qEd}{mv_0^2} x - y_0 + \frac{qEd^2}{mv_0^2} \quad \Rightarrow y_0 = \frac{1}{2} \cdot \frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2} \frac{qEd^2}{mv_0^2}$

$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2} \frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$

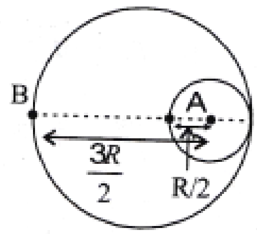
$y = \frac{-qEd}{mv_0^2} + \frac{1}{2} \frac{qEd^2}{mv_0^2} \Rightarrow y = \frac{\text{qEd}}{\text{mv}_0^2}\left(\frac{d}{2} - x\right)$

61. (B)

Electric field at A $\left(R' = \frac{R}{2}\right)$

$$E_A.ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \vec{E}_A = \frac{\rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\varepsilon_0 \cdot 4\pi\left(\frac{R}{2}\right)^2}$$



$$\Rightarrow \vec{E}_A = \frac{\sigma(R/2)}{3\varepsilon_0} = \left(\frac{\sigma R}{6\varepsilon_0}\right)$$

Electric fields at 'B'

$$\vec{E}_B = \frac{k \times \rho \times \frac{4}{3}\pi R^3}{R^2} - \frac{k \times \rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$\Rightarrow \vec{E}_B = \frac{\sigma R}{3\varepsilon_0} - \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(\sigma)}{\left(\frac{3R}{2}\right)^2}\frac{4\pi}{3}\left(\frac{R}{2}\right)^3$$

$$\Rightarrow \vec{E}_B = \frac{\sigma R}{3\varepsilon_0} - \frac{\sigma R}{54\varepsilon_0} \Rightarrow E_B = \frac{17}{54}\left(\frac{\sigma R}{\varepsilon_0}\right)$$

$$\left|\frac{E_A}{E_B}\right| = \frac{1 \times 54}{6 \times 17} = \left(\frac{9}{17}\right) = \frac{9}{17} \times \frac{2}{2} = \frac{18}{34}$$

62. (C)

Since $\vec{r} \cdot \vec{p} = 0$

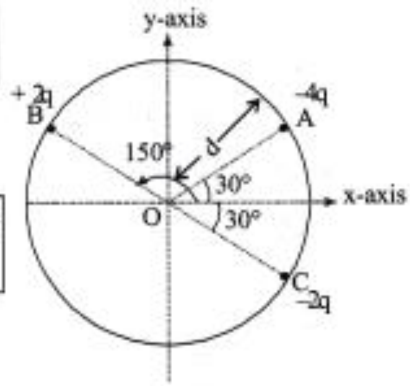
$\vec{E}$ must be antiparallel to $\vec{p}$

$\therefore\ \hat{E}$ is parallel to $\left(\hat{i} + 3\hat{j} - 2\hat{k}\right)$

63. (A)

Electric field due to charge +2q at centre O

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{d^2}\left[\frac{\sqrt{3}\hat{i} - \hat{j}}{2}\right]$$



Electric field due to charge $-2q$ at centre O

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{d^2}\left[\frac{\sqrt{3}\hat{i} - \hat{j}}{2}\right]$$

Electric field due to charge $-4q$ at centre O

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0} \times \frac{4q}{d^2}\left[\frac{\sqrt{3}\hat{i} + \hat{j}}{2}\right]$$

∴ Net electric field at point O

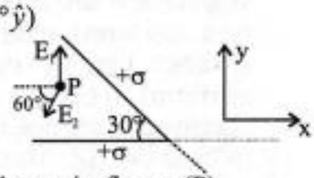
$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\sqrt{3}\,q}{\pi\varepsilon_0 d^2}\hat{i}$$

64. (D)

From figure, $\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{y}$ and

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}(-\cos 60°\hat{x} - \sin 60°\hat{y})$$

$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$



Electric field in the region shown in figure (P)

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\left[-\frac{1}{2}\hat{x} + \left(1 - \frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

$$\text{or, } \vec{E}_P = \frac{\sigma}{2\varepsilon_0}\left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$$

65. (B)

(b) Let v be the speed of dipole.
Using energy conservation

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow 0 - \frac{2k \cdot p_1}{r^3} p_2 \cos(180°) = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 0$$

$\left(\because \text{ Potential energy of interaction between dipole} \right.$

$$\left. -\vec{P_1} \cdot \vec{E_2} = -\vec{P_2} \cdot \vec{E_1} = \frac{-2p_1 p_2 \cos\theta}{4\pi \epsilon_0 r^3} \right)$$

$$\Rightarrow mv^2 = \frac{2kp_1 p_2}{r^3} \Rightarrow v = \sqrt{\frac{2kp_1 p_2}{mr^3}}$$

When $p_1 = p_2 = p$ and $r = a$

$$v = \frac{p}{a}\sqrt{\frac{1}{2\pi \epsilon_0 ma}}$$

66. (B)

(b) By Gauss law

$$\oint \vec{E} \cdot \vec{da} = \frac{q_{in}}{\epsilon_o}$$

If E = cons. and $\vec{E} \parallel_r \vec{A}$, then $|E|\,|A| = \frac{q_{in}}{\epsilon_o}$

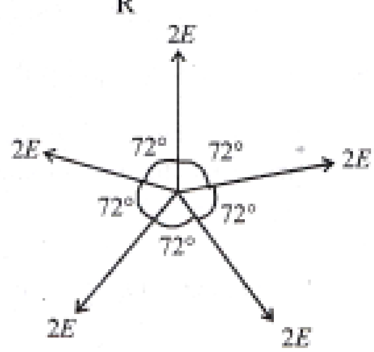
$\Rightarrow |E| = \frac{q_{in}}{\epsilon_o A} \cdot$ And $\vec{E}$ is always $\perp$r to equipotential surface so, gaussian surface is equipotential.

67. (C)

(c) Potential at the centre, $V_C = \frac{KQ_{net}}{R}$

$\because Q_{net} = 0 \quad \therefore V_C = 0$

Let E be electric field produced by each charge at the centre, then resultant electric field will be $E_C = 0$, since equal electric field vectors are acting at equal angle so their resultant is equal to zero.



68. (A)

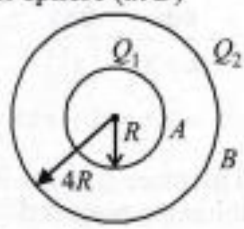
(a) We have given two metallic hollow spheres of radii R and $4R$ having charges Q_1 and Q_2 respectively.

Potential on the surface of inner sphere (at A)

$$V_A = \frac{kQ_1}{R} + \frac{kQ_2}{4R}$$

Potential on the surface of outer sphere (at B)

$$V_B = \frac{kQ_1}{4R} + \frac{kQ_2}{4R}$$



$$\left(\text{Here, } k = \frac{1}{4\pi\varepsilon_0}\right)$$

Potential difference,

$$\Delta V = V_A - V_B = \frac{3}{4} \cdot \frac{kQ_1}{R} = \frac{3}{16\pi \epsilon_0} \cdot \frac{Q_1}{R}$$

69. (D)

(d) By using energy conservation,

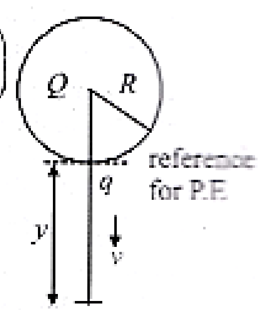
$$\Delta KE + (\Delta PE)_{\text{Electro}} + (\Delta PE)_{\text{gravitational}} = 0$$

$$\frac{1}{2}mV^2 + \left(k\frac{Qq}{R+y} - k\frac{Qq}{R}\right) + (-mgy) = 0$$

$$\Rightarrow \frac{1}{2}mV^2 = mgy + kQq\left(\frac{1}{R} - \frac{1}{R+y}\right)$$

$$\Rightarrow V^2 = 2gy + \frac{2kQq}{m}\frac{y}{R(R+y)}$$

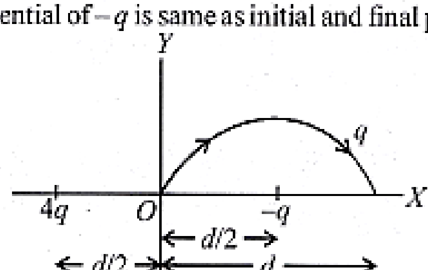
$$\text{or, } V^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$$



70. (D)

(d) Change in potential energy, $\Delta u = q(V_f - V_i)$

Potential of $-q$ is same as initial and final point of the path.

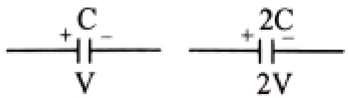


$$\Delta u = q\left(\frac{k4q}{3d/2} - \frac{k4q}{d/2}\right) = -\frac{4q^2}{3\pi\varepsilon_0 d}$$

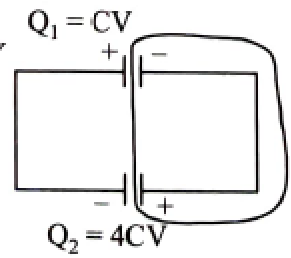
–ve sign shows the energy of the charge is decreasing.

71. (B)

(b) When capacitors C and $2C$ capacitance are charged to V and 2V respectively.



$Q_1 = CV \quad Q_2 = 2C \times 2V = 4CV$



When connected in parallel

By conservation of charge

$4CV - CV = (C + 2C)V_{\text{common}}$

$\Rightarrow V_{\text{common}} = \dfrac{3CV}{3C} = V$

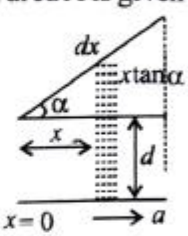
Therefore final energy of this configuration,

$U_f = \left(\dfrac{1}{2}CV^2 + \dfrac{1}{2} \times 2CV^2\right) = \dfrac{3}{2}CV^2$

72. (A)

(a) Consider an infinitesimal strip of capacitor of thickness dx at a distance x as shown.

Capacitance of parallel plate capacitor of area A is given by $C = \dfrac{\varepsilon_0 A}{t}$



[Here t = seperation between plates]

So, capacitance of thickness dx will be

$\therefore dC = \dfrac{\varepsilon_0 a dx}{d + x\tan\alpha}$

Total capacitance of system can be obtained by integrating with limits x = 0 to x = a

$$\therefore C_{eq} = \int dC = a\varepsilon_0 \int_{x=0}^{x=a} \frac{dx}{x\tan\alpha + d}$$

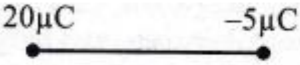
$$= a\varepsilon_0 \int_{x=0}^{x=a} \frac{dx}{d\left(1 + \frac{x}{d}\tan\alpha\right)}$$

[By Binomial expansion]

$$\Rightarrow C_{eq} = \frac{a\varepsilon_0}{d}\int_0^a \left(1 - \frac{x\tan\alpha}{d}\right)dx = \frac{a\varepsilon_0}{d}\left(x - \frac{x^2\tan\alpha}{2d}\right)_0^a$$

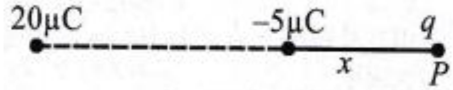
$$\Rightarrow C_{eq} = \frac{a^2\varepsilon_0}{d} = \left(1 - \frac{a\tan\alpha}{2d}\right) = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

73. (B)



Let, charge q be placed at P.

At point P forces due to 20 μC & $-5\mu C$ should be in opposite direction



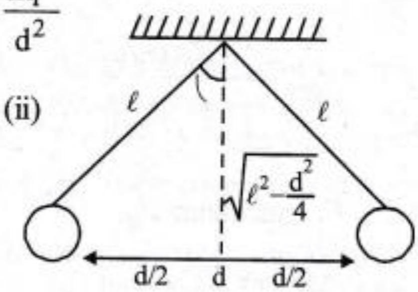
For net force $\vec{F} = 0$ & from coulomb's law force

$$F = \frac{k\,q_1\,q_2}{r^2} \Rightarrow k\frac{20q}{(5+x)^2} = \frac{k5q}{x^2} \Rightarrow x = 5\,\text{cm}$$

74. (B)

$$T\cos\theta = m\,g \qquad \text{.......(i)}$$

Force due to charges $= \dfrac{kq^2}{d^2}$

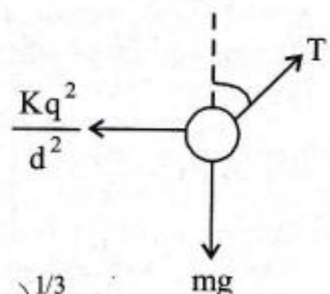


$$T\sin\theta = \frac{kq^2}{d^2} \qquad \text{.......(ii)}$$

From (i) and (ii) we get

$$\tan\theta = \frac{\frac{kq^2}{d^2}}{m\,g}$$

as $\tan\theta \approx \sin\theta \approx \dfrac{d}{2\ell}$



$$\frac{kq^2}{m\,g\,d^2} = \frac{d}{2\ell}$$

$$\Rightarrow d^3 = \frac{2kq^2\ell}{m\,g}$$

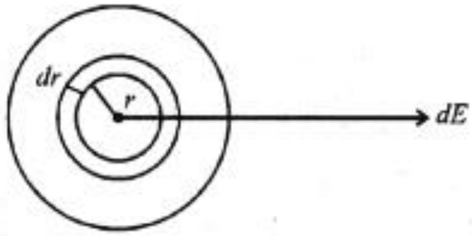
$$\Rightarrow \quad d = \left(\frac{2kq^2\ell}{m\,g}\right)^{1/3} = \left(\frac{q^2\ell}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

75. (A)

Consider a small ring of radius r and thickness dr on disc.

$dq = \sigma 2r\pi\, dr$

$$dE = \frac{Kdqz}{(r^2+z^2)^{3/2}}$$



$$= \frac{K\sigma(2\pi r)drz}{(r^2+z^2)^{3/2}}$$

$$E = \int dE = \int dE$$

$$= \int_0^R \frac{K\sigma 2\pi r drz}{\left(z^2+r^2\right)^{3/2}} = \frac{\sigma}{2\epsilon_0}\left[1-\frac{z}{\sqrt{R^2+z^2}}\right]$$

76. (B)

Charge per unit length of the rod,

$$\lambda = \left(\frac{-Q}{R\theta}\right) = \left(\frac{-Q}{R\frac{2\pi}{3}}\right) = \frac{-3Q}{2\pi R}$$

$$E = \int dE\cos\theta \Rightarrow E = \int_{-\pi/3}^{\pi/3} \frac{K\times(+\theta)}{\frac{2\pi}{3}R}\times\frac{Rd\theta}{R^2}\cos\theta$$

$$\Rightarrow E = \frac{3}{2\pi}\frac{K\theta}{R^2}[\sin\theta]_{-\pi/3}^{\pi/3} = \frac{3}{2\pi}\frac{K\theta}{R}\times\frac{2\sqrt{3}}{2}$$

$$\Rightarrow E = \frac{3\sqrt{3}Q}{8\pi^2\epsilon_0 R^2}\left(+\hat{i}\right)$$

77. (B)

The electrostatic field will balance the weight of oil drop,

$$m_{oil}\times g_{oil} = qE$$

$$\Rightarrow \frac{4}{3}\pi r^3\times\rho\times g = neE \quad (n = \text{no. of excess electrons})$$

$$\Rightarrow n = \frac{\frac{4}{3}\pi r^3\rho g}{eE} = \frac{\frac{4}{3}\times\pi\times(2\times10^{-3})^3\times(3\times10^3)\times 9.81}{1.6\times10^{-19}\times3.55\times10^5}$$

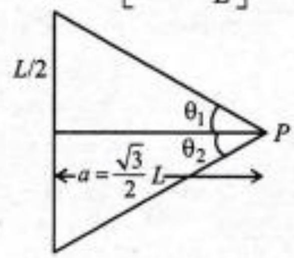
$$= 173.65\times10^8;\ 1.73\times10^{10}$$

78. (D)

Electric field at point P

$$E = \frac{k\lambda}{a}(\sin\theta_1 + \sin\theta_2)$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{L} \times \frac{1}{\left(\frac{\sqrt{3}L}{2}\right)} \times 2\sin\theta \qquad \left[\because \lambda = \frac{Q}{L}\right]$$



From figure,

$$\tan\theta = \frac{\frac{L}{2}}{\frac{\sqrt{3}L}{2}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{2Q}{\sqrt{3}L^2} \times \left(\frac{2\times 1}{2}\right) \qquad \left[\because \sin 30^\circ = \frac{1}{2}\right]$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 L \times \sqrt{3}\frac{L}{2}} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

79. (B)

(b) At bottom surface, electric field is zero as $y = 0$

$\therefore$ Electric flux, $\phi_1 = 0$; At top surface, $y = 0.5$

$\therefore$ Electric flux, $\phi_2 = EA = (150y^2)(0.5)^2$

$= 150 \times (0.5)^2 \times (0.5)^2$

And flux through all other surface is zero because $\vec{E} \perp \vec{A}$ for each of them

$= \frac{150}{4}(0.5)^2 = \frac{150}{16} \quad \therefore \phi_{total} = \phi_1 + \phi_2 = \frac{150}{16}$

Using Gauss's law $\phi = \frac{Q_{in}}{\epsilon_0} \Rightarrow \frac{150}{16} = \frac{Q_{in}}{\epsilon_0}$

$\Rightarrow Q_{in} = \frac{150}{16} \times 8.85 \times 10^{-12} = 8.3 \times 10^{-11}\,C$

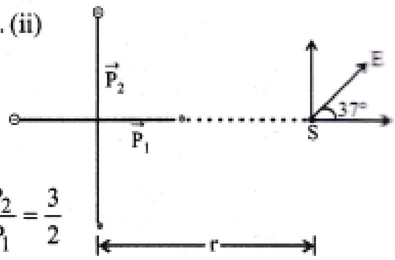
80. (C)

(c) Electric field due to $\vec{P}_1$ at axis point S

$E_{axis} = \frac{2kP_1}{r^3}$ ... (i)

Electric field due to $\vec{P}_2$ at perpendicular bisector at point S.

$E_{equator} = \frac{kP_2}{r^3}$ ... (ii)



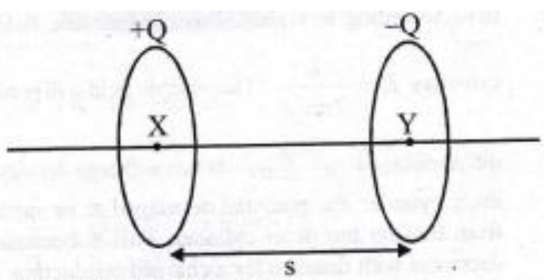
$\tan 37° = \frac{E_{equator}}{E_{axis}}$

$\Rightarrow \frac{3}{4} = \frac{\frac{KP_2}{r^3}}{K\frac{2P_1}{r^3}} \Rightarrow \frac{P_2}{P_1} = \frac{3}{2}$

$\Rightarrow \frac{P_1}{P_2} = \frac{2}{3}$

81. (D)

(d)



Potential at the centre of ring X

$V_X = \frac{Q}{4\pi \epsilon_0 a} - \frac{Q}{4\pi \epsilon_0 \sqrt{a^2+s^2}}$

Potential at the centre of ring Y

$V_Y = \frac{-Q}{4\pi \epsilon_0 a} + \frac{Q}{4\pi \epsilon_0 \sqrt{a^2+s^2}}$

$V_X - V_Y = \frac{2Q}{4\pi \epsilon_0 a} - \frac{2Q}{4\pi \epsilon_0 \sqrt{a^2+s^2}}$

$= \frac{Q}{2\pi \epsilon_0}\left(\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right)$

82. (C)

(c) $V = \dfrac{qd}{A \in_0}$

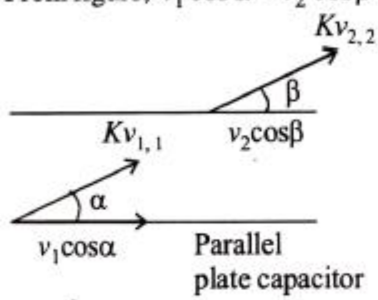
$i = \dfrac{V}{R} = \dfrac{qd}{A \in_0 \rho \dfrac{d}{A}}$ $\left(\because R = \rho \dfrac{d}{A}\right)$

$i_{max} = \dfrac{q_{max}}{\rho K \in_0} \Rightarrow i_{max} = \dfrac{2 \times 10^{-12} \times 40}{200 \times 50 \times 8.85 \times 10^{-12}}$

$= 0.9\ mA$

83. (C)

(c) From figure, $v_1 \cos\alpha = v_2 \cos\beta$

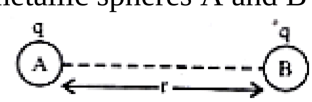


$\therefore \dfrac{K_1}{K_2} = \dfrac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \dfrac{v_1^2}{v_2^2} = \dfrac{\cos^2\beta}{\cos^2\alpha}$

84. (B)

Consider two metallic spheres A and B both have charge q.

$F = \dfrac{kq^2}{r^2}$

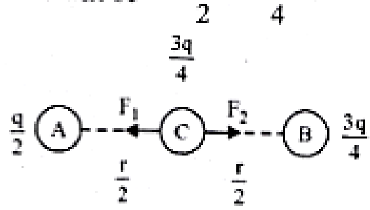


When sphere C is placed in contact with sphere A, charge on sphere A and sphere C will be = $\dfrac{q}{2}$

Now sphere C is placed in contact with sphere B, charge on sphere B and sphere C will be = $\dfrac{q + \frac{q}{2}}{2} = \dfrac{3q}{4}$

Now,

Then $F_1 = \dfrac{\frac{kq}{2} \cdot \frac{3q}{4}}{\frac{r^2}{4}}$



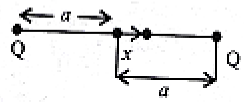
$F_2 = \dfrac{\frac{k3q}{4} \frac{3q}{4}}{\frac{r^2}{4}}$

The force experienced by sphere C,

$F' = F_2 - F_1 = \dfrac{\left(k\frac{3q}{4} - k\frac{q}{2}\right)}{\frac{r^2}{4}} \cdot \dfrac{3q}{4} = \dfrac{3kq^2}{4r^2} = \dfrac{3F}{4}$

85. (A)

Force on charge 'Q'



$$K = \frac{KQq_0}{(a-x)^2} - \frac{KQq_0}{(a+x)^2}$$

$$= KQq_0\left(\frac{4ax}{(a^2-x^2)^2}\right) \text{ as } a << x$$

So, $F = \frac{KQq_0 \times 4ax}{a^4} = \frac{4Kxq_0Q}{a^3}$

As, $F = m\omega^2 x$

So, $m\omega^2 = \frac{4KQq_0}{a^3} \Rightarrow \omega = \sqrt{\frac{4KQq_0}{ma^3}} \Rightarrow T = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0Q}}$

86. (B)

Let distance between the two divided charges be r.

From Coulomb's law, force between two charge,

$$F = \frac{Kq(4-q)}{r^2}$$

For F to be maximum,

$$\frac{dF}{dq} = \frac{K}{r^2}[4-2q] = 0 \Rightarrow q = 2$$

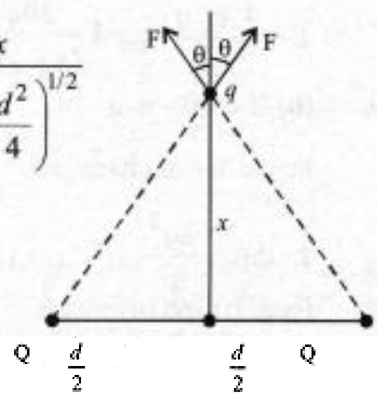
87. (D)

We have, from the given figure

$$F_{net} = 2F\cos\theta$$

$$F_{net} = 2\frac{KQq}{x^2+\frac{d^2}{4}} \cdot \frac{x}{\left(x^2+\frac{d^2}{4}\right)^{1/2}}$$

$$F_{net} = \frac{2KQqx}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}$$



For maximum F_{net}

$$\frac{dF_{net}}{dx} = 0$$

$$\Rightarrow x \times -\frac{3}{2}\left(x^2+\frac{d^2}{4}\right)^{-5/2}.2x + \left(x^2+\frac{d^2}{4}\right)^{-3/2} = 0$$

$$\Rightarrow \left(x^2+\frac{d^2}{4}\right)^{-5/2}\left[-3x^2+x^2+\frac{d^2}{4}\right] = 0$$

$$\Rightarrow 2x^2 = \frac{d^2}{4} \Rightarrow x^2 = \frac{d^2}{8} \Rightarrow x = \frac{d}{2\sqrt{2}}$$

88. (D)

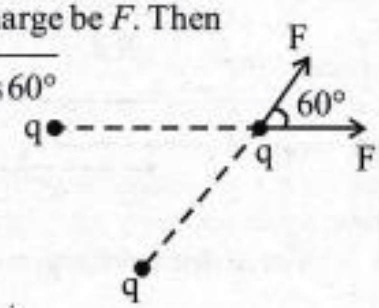
Let force between two charge be F. Then

$F_{net} = \sqrt{F^2 + F^2 + 2.F.F\cos 60^\circ}$

$= \sqrt{3}F$

so, required ratio

$= \frac{F_{net}}{F} = \frac{\sqrt{3}F}{F} = \sqrt{3}$



89. (B)

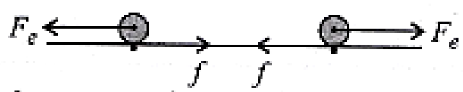
From law of conservation of charge $q_i = q_f$

$\Rightarrow 64q = Q \Rightarrow \frac{Q}{q} = 64$ and, $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$

$\Rightarrow R = 4r \Rightarrow \frac{r}{R} = \frac{1}{4}$

So, $\frac{\sigma_{bigger}}{\sigma_{smaller}} = \frac{\frac{Q}{4\pi R^2}}{\frac{q}{4\pi r^2}} = \frac{Q}{q} \times \left(\frac{r}{R}\right)^2 = 64 \times \frac{1}{16} = \frac{4}{1}$

90. (A)

(a) 

For charge to stay in equilibrium $F_e = f$

$\frac{kq^2}{L^2} = \mu mg$

$L = \sqrt{\frac{kq^2}{\mu mg}} = \sqrt{\frac{k}{\mu mg}}.q = \sqrt{\frac{9 \times 10^9}{0.25 \times 10^{-2} \times 10}} \times 2 \times 10^{-7}$

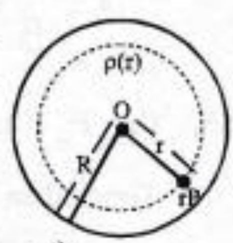
$= 12$ cm

91. (A)

(a) Here, $E.4\pi r^2 = \frac{\int_0^r \rho_0 \left(\frac{3}{4} - \frac{r}{R}\right) 4\pi r^2 dr}{\varepsilon_0}$

$\Rightarrow \quad E.4\pi r^2 = \frac{\rho_0 4\pi}{\varepsilon_0}\left(\frac{3}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right)$

$\Rightarrow \quad Er^2 = \frac{\rho_0 r^3}{4\varepsilon_0}\left(1 - \frac{r}{R}\right) \quad \Rightarrow E = \frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)$



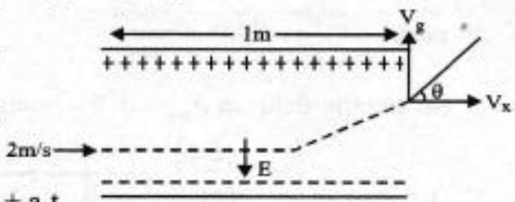
92. (B)

(b) $a_y = \frac{F_y}{m} = \frac{e(E)}{m} = \frac{e\left(\frac{8m}{e}\right)}{m} = 8\ m/s^2$

$s_x = u_x t$

$\Rightarrow 1 = 2 \times t$

$\Rightarrow t = \frac{1}{2} sec$



and, $v_y = u_y + a_y t$

$\Rightarrow v_y = 0 + 8 \times \frac{1}{2} \Rightarrow v_y = 4\ m/s$

$\tan\theta = \frac{V_y}{V_x} = \frac{4}{2} = 2$

$\Rightarrow \theta = \tan^{-1}(2)$

93. (D)

Here $a = \frac{qE}{m} = \frac{40\times10^{-6}\times10^{5}}{100\times10^{-6}} = 40000\ m/s^2$

As charge particle is moving opposite to direction of $\vec{E}$ So $\vec{E}$ will declerate the charge and charge will come to rest.

Now, $v^2 = u^2 + 2as$

$0 = 200^2 + 2\times(-40\times10^3)\times s$

$80\times10^3 s = 4\times10^4$

$\therefore s = \frac{4\times10^4}{80\times10^3} = \frac{40}{80} = 0.5\ m.$

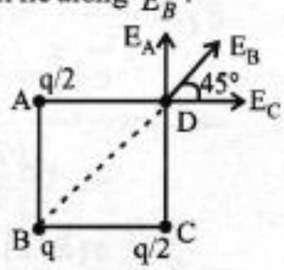
94. (A)

(a) Here, sum of $\vec{E}_C$ and $\vec{E}_A$ will lie along $\vec{E}_B$.

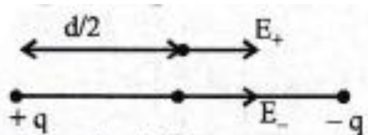
So, $\left|\vec{E}_{net}\right| = \left|\vec{E}_B\right| + \left|\vec{E}_C + \vec{E}_B\right|$

$= \frac{Kq}{(\sqrt{2}a)^2} + \left[\sqrt{2}\frac{kq}{2a^2}\right]$

$= \frac{kq}{a^2}\left[\frac{1}{2} + \frac{1}{\sqrt{2}}\right]$



95. (B)



So, $E_{net} = E_+ + E_-$

$= 2\frac{kq}{\left(\frac{d}{2}\right)^2} = \frac{8kq}{d^2} \Rightarrow 6.4 \times 10^4 = 8 \times 9 \times 10^9 \times \frac{8\times10^{-6}}{d^2}$

$\Rightarrow d^2 = \frac{72\times8\times10^{+3}}{6.4\times10^4} \Rightarrow d^2 = 9 \Rightarrow d = 3\text{ m}$

96. (B)

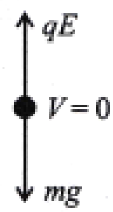
As water droplet is at rest

So, $\overline{F}_{\text{net}} = 0$

$\Rightarrow mg = qE \Rightarrow q = \frac{mg}{E}$

$\Rightarrow q = \frac{0.1\times10^{-3}\times9.8}{4.9\times10^5}$

$\Rightarrow q = 2\times10^{-9}\,C$



97. (None)

(None) Total flux through complete spherical surface is $\frac{q}{\epsilon_0}$.

So flux, through curved surface will be $\frac{q}{2\epsilon_0}$.

The flux through flat surface will be zero.

98. (C)

Electric field due to infinite sheet is given by $E = \frac{\bar{0}}{2\epsilon_0}$,

clearly $|\vec{E}|$ is independent of distance

So, $E_1 = E_2 = \frac{\bar{0}}{2\epsilon_0}$

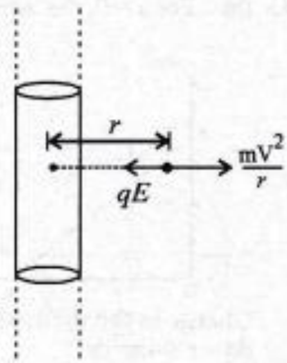
99. (A)

(a) As charge particle is moving in circular path. So,

$$qE = \frac{mv^2}{r}$$

$$\Rightarrow mv^2 = qEr$$

$$\Rightarrow \frac{mv^2}{2} = \frac{1}{2}qEr \quad \text{...(i)}$$



Now, by Gauss's law

$$E \times 2\pi rl = \frac{q_{in}}{\epsilon_0}$$

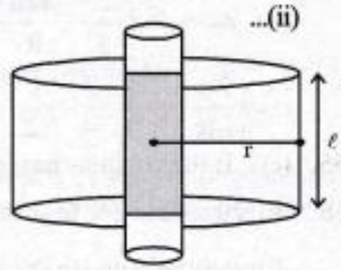
$$E \times 2\pi rl = \frac{\rho \times \pi R^2 l}{\epsilon_0}$$

$$E = \frac{\rho R^2}{2\epsilon_0 r} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{1}{2}mv^2 = \frac{1}{2}q \times \frac{\rho R^2}{2\epsilon_0 r} \times r$$

$$\Rightarrow K.E. = \frac{\rho q R^2}{4\epsilon_0}$$



100. (B)

(b) Since, $V_A = V_B$

$$\Rightarrow \frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}.$$

Here, $K = \frac{1}{4\pi\epsilon_o} \Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B} = \frac{1}{2}$

$$E_A = \frac{kQ_A}{R_A^2}; E_B = \frac{kQ_B}{R_B^2} \Rightarrow \frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{R_B^2}{R_A^2} = \frac{R_B}{R_A} = \frac{2}{1}$$

101. (D)

(d) We have

$V = 3x^2$ Volt

$$E_x = \frac{-\partial V}{\partial x} = -6x \Rightarrow E_{x/x=1} = -6$$

$$E_y = -\frac{\partial V}{\partial y} = 0$$

$$E_z = -\frac{\partial V}{\partial z} = 0$$

So, $|\vec{E}| = \sqrt{-6^2 + 0^2 + 0^2} = 6$

In vector form, $\vec{E} = -6\hat{i}\,V/m$

102. (A)

(a) Equivalent capacitance in series,

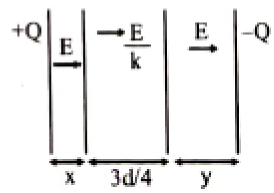
$$C_{eq} = \frac{C(KC)}{C+KC} = \frac{KC}{K+1}$$

$$24 = \frac{K40}{K+1} \qquad \left[\because C_{eq} = 24\mu F\right]$$

$$24(K+1) = 40K \therefore K = 1.5$$

103. (A)

(a) We have, $x + y + \frac{3d}{4} = d$



$$\Rightarrow x + y = \frac{d}{4}$$

$$\Delta V = Ex + \frac{E}{k} \times \frac{3d}{4} + Ey$$

$$\Rightarrow \Delta V = \frac{3Ed}{4k} + E(x+y) \Rightarrow \Delta V = E\left[\frac{3d}{4k} + \frac{d}{4}\right]$$

$$\Delta V = \frac{\sigma}{\epsilon_0}\left[\frac{3d + dk}{4k}\right] = \frac{Qd}{A\epsilon_0}\left[\frac{3+k}{4k}\right]$$

$$\frac{Q}{\Delta V} = C = \frac{A\epsilon_0}{d}\left[\frac{4k}{3+k}\right] = \frac{4kC_0}{k+3} \qquad \left[\because C_0 = \frac{A\epsilon_0}{d}\right]$$

104. (A)

(a) Given that all capacitors are connected in parallel so, the equivalent capacitance will be $C_{eq} = C_1 + C_2 + C_3 + C_4$
$= 1 + 2 + 4 + 3 = 10\,\mu F$
Voltage of battery $V = 20$ V
We have, $Q = CV = 10\,\mu F \times 20 = 200\,\mu C$

105. (A)

(a) As $Q = CV$
$\Rightarrow Q \propto V$. so, graph will be straight line between V and Q.
Now, $Q_{max} = CV_{max}$

$$\Rightarrow V_{max} = \frac{Q_{max}}{C} = \frac{5}{2\times10^{-6}} = 2.5 \times 10^6\,V$$

So, most suitable option is (A).

106. (D)

(d) We have

$$U = \frac{Q^2}{2C} = \frac{Q_0^2 e - 2t/\tau}{2C} = U_0 e^{-2t/\tau}, U = e^{-2t/\tau}$$

as, $U_{t=t_1} = \frac{U_0}{2}$

$$U_0 e^{-2t_1/\tau} = \frac{U_0}{2}$$

$$e^{-t_1/\tau} = \frac{1}{2} \Rightarrow t_1 = \frac{\tau}{2} ln2 \text{ and, } Q_{t=t_2} = \frac{Q_0}{8}$$

$$Q_0 e^{-t_2/\tau} = \frac{Q_0}{8} \Rightarrow e^{-t_2/\tau} = \frac{1}{8} \Rightarrow t_2 = \tau\, ln8 \Rightarrow t_2 = 3\tau\, ln2$$

So, $\frac{t_1}{t_2} = \frac{\frac{\tau}{2} ln2}{3\tau ln2} = \frac{1}{6}$

107. (A)

(a) $\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20} = \frac{6+4+3}{60} = \frac{60}{13} \mu F$

So, $Q = C_{eq} V = \frac{60}{13} \times 13 \mu C = 60 \mu C$

In series, charge across each capacitor is same and is equal to net charge.

So, $Q_{15\mu F} = 60 \mu C$

108. (C)

(c) We have, $C_i = \frac{A\epsilon_0}{d} = 4\mu F$

$$C_f = \frac{A\epsilon_0}{d - t + \frac{t}{k}} = \frac{A\epsilon_0}{d - \frac{d}{2} + \frac{d}{2\times 3}} = \frac{A\epsilon_0}{d\left(1 - \frac{1}{2} + \frac{1}{6}\right)}$$

$$= \frac{4\mu F}{\frac{2}{3}} = 6\mu F$$

109. (D)

(d) $q = CV$

$$\Rightarrow q = \frac{kA\epsilon_0}{d}.Ed \Rightarrow k = \frac{q}{A\epsilon_0 E}$$

$$= \frac{7 \times 10^{-6}}{30\pi \times 10^{-4} \times \frac{1}{4\pi \times 9 \times 10^9} \times 3.6 \times 10^7} = \frac{7}{3} = 2.33$$

110. (A)

(a) $U = \frac{q^2}{2c}$. As C = constant, $U \propto q^2$

So, $\frac{U_2}{U_1} = \left(\frac{q_2}{q_1}\right)^2 \Rightarrow q_1^2 = \frac{U_1}{U_2} q_2^2$

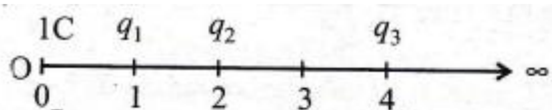
$\Rightarrow q_1^2 = \frac{U}{1.44U} \times (q_1+2)^2 . \left(\frac{q_1}{q+2}\right)^2 = \frac{1}{1.44}$

$\Rightarrow \frac{q_1}{q_1+2} = \frac{1}{1.2}$

$\Rightarrow 1.2q_1 = q_1 + 2 \Rightarrow 0.2\,q_1 = 2$

$\Rightarrow q_1 = 10\,C$

111. (12)



$F_{\text{total}} = \left[\frac{kq_1q_2}{r_1^2} + \frac{kq_1q_3}{r_2^2} + \frac{kq_1q_4}{r_3^2} + \ldots\right]$

$\Rightarrow \quad F = (k)(10^{-6})\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \ldots\right]$

$= \frac{(k)10^{-6}}{1-\frac{1}{4}} = \frac{(9\times10^9)\times4\times10^{-6}N}{3} = 12\times10^3 N$

112. (36)

When two spheres charges $q'_1 = 2.1$ nC and $q'_2 = -0.1$ nC are brought into contact and then separated by a distance $r = 0.5$ m then,

$q_1' = q_2' = \frac{Q_1+Q_2}{2} = 1$ nC

Electrostatic force between the two charged sphere,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1' q_2'}{r^2} = 9\times10^9 \times \frac{10^{-9}\times10^{-9}}{(0.5)^2} = 36\times10^{-9}\,N$$

$\therefore\ x = 36$

113. (640)

(640) Flux, $\phi = \vec{E}\cdot\vec{A} = \left[\frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}\right]\cdot\vec{A}$

$= \frac{E_0}{5}(2\hat{i}+3\hat{j})\cdot(0.4\hat{i}) \quad = \frac{4000}{5}(2\times0.4) = 640\ \text{Nm}^2\text{C}^{-1}$

114. (226)

(226) $\phi_{Total} = \frac{q}{\varepsilon_0}$,

From symmetry electric flux

$$\phi_{Sq} = \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{12\times10^{-6}}{6\times8.85\times10^{-12}} = 225.98\times10^3$$

$$= 226\times10^3\ \frac{Nm^2}{C}$$

115. (128)

(128) Potential, $V = \frac{Kq}{r}$

If charge on each drop $= q$ and radius $= r$

$$\therefore V_0 = \frac{Kq}{r} = 2\text{ V}$$

$$\frac{4}{3}\pi R^3 = 512\times\frac{4}{3}\pi r^3$$

$$\Rightarrow R = (512\times r^3)^{1/3} = 8r$$

Potential of bigger drop,

$$V = \frac{K(512)q}{8r} = \frac{512}{8}\frac{Kq}{r} = \frac{512}{8}\times2\text{ V} = 128\text{ V}.$$

116. (16)

(16) When battery is connected,

$$Q_1 = C_1V = 2\times10 = 20\ \mu C$$

When battery is removed and the capacitor is connected

$$Q_1 = C_1V + C_2V \Rightarrow 20 = (2+8)V \qquad \therefore V = 2V$$

Therefore charge in $C_2 = C_2V = 16\mu C$

117. (6)

(6) Maximum torque,

$|\tau|_{max} = PE$ or, $\tau_1 = P_1E_1$ and $\tau_2 = P_2E_2$

$$\therefore \frac{\tau_1}{\tau_2} = \frac{P_1E_1}{P_2E_2} = \frac{1.2\times10^{-30}\times5\times10^4}{2.4\times10^{-30}\times15\times10^4} = \frac{1}{6} = \frac{1}{x}$$

$\therefore$ x=6

118. (45)

(45) Here, $\phi_e = \int \vec{E}.dA \Rightarrow \frac{\phi_e}{A} = E$

$$\Rightarrow \frac{\phi_e}{A} = \frac{\rho R}{3\epsilon_o} = \frac{2\times10^0\times6}{3\times8.85\times10^{-12}} = 45\times10^{10}\ NC^{-1}$$

119. (60)

(60) Capacitances of the capacitors,

$$C_1 = \frac{\epsilon_0 \epsilon_{r_1} A}{t_1},\ C_2 = \frac{\epsilon_0 \epsilon_{r_2} A}{t_2}$$

Let V be the voltage of conducting foil. As the capacitors are connected in series, so charge on the capacitors should be same

$Q_1 = Q_2$

$\Rightarrow\ C_1(100 - V) = C_2 V\quad (\because\ Q = CV)$

$$\Rightarrow\ \frac{\epsilon_0 \epsilon_{r_1} A}{t_1}(100 - V) = \frac{\epsilon_0 \epsilon_{r_2} AV}{t_2}$$

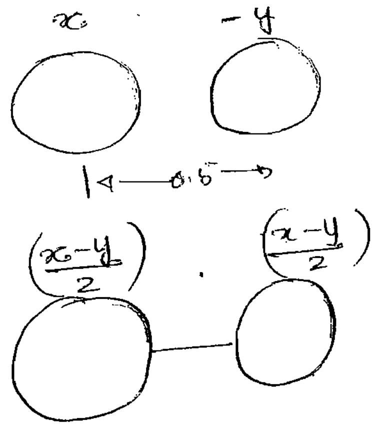
$$\Rightarrow\ \frac{3\times(100 - V)}{0.5\times10^{-3}} = \frac{4\times V}{1\times10^{-3}}$$

$\Rightarrow\ 600 - 6V = 4V \Rightarrow V = 60\ V$

Electrostatics.

Ex – I

1. (B)



$$\frac{kxy}{(0.5)^2} = 0.108.$$

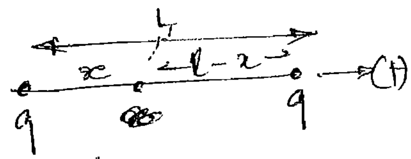
$$\frac{k(x-y)^2}{4(0.5)^2} = 0.036.$$

Eliminating k.

$$3x^2 + 3y^2 = 10xy.$$

Eliminate.

2. (d)

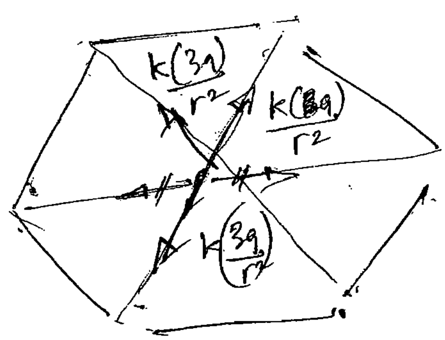


$$E = \frac{kq}{x^2} - \frac{kq}{(l-x)^2}$$

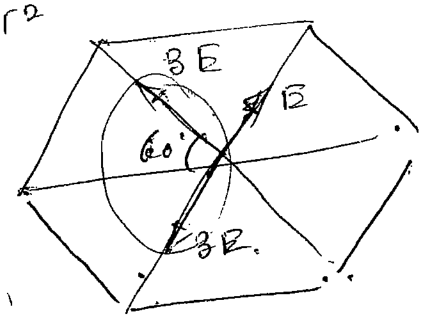
$$= \frac{kq\,l\,(l-2x)}{[x(l-x)]^2}.$$

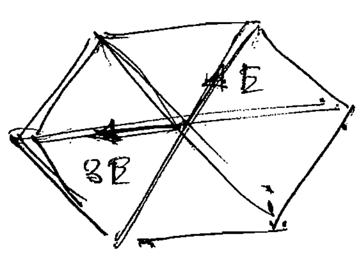
check the sign scheme of $l(l-2x)$ & note that E is undefined for ~~x = l~~ $x = 0, l$.

3. (D)



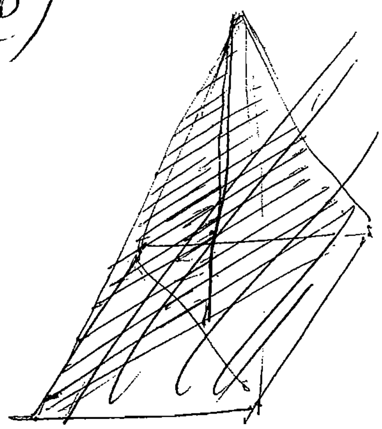
$$\frac{kq}{r^2} = E$$

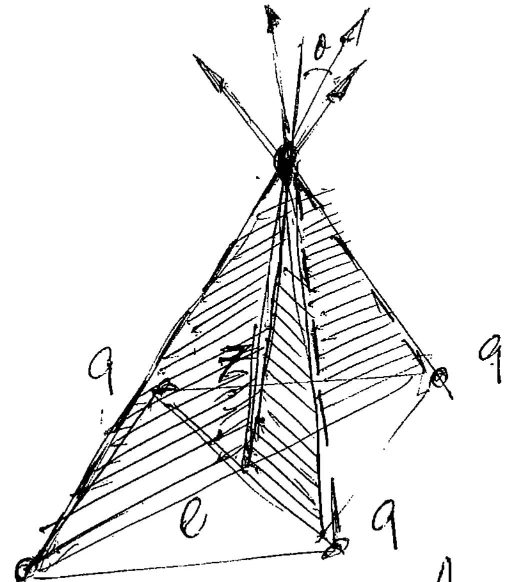




Resultant isn't along any of the given lines.

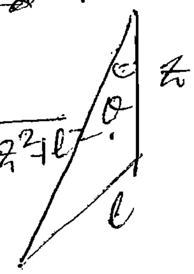
4 4.(b)





$$E = 4E_0 \cos\theta$$

$$= 4\frac{kq}{(z^2+l^2)}\frac{z}{\sqrt{z^2+l^2}}$$



$$= \frac{4kqz}{(z^2+l^2)^{3/2}}$$

$$\frac{dE}{dz} = \frac{4kq}{(z^2+l^2)^3}\left[(z^2+l^2)^{3/2}\cdot 1 - \frac{3}{2}(z^2+l^2)^{1/2}\cdot 2z\cdot z\right]$$

$$= \frac{4kq}{(z^2+l^2)^3}\left[\sqrt{z^2+l^2}\,(z^2+l^2-3z^2)\right]$$

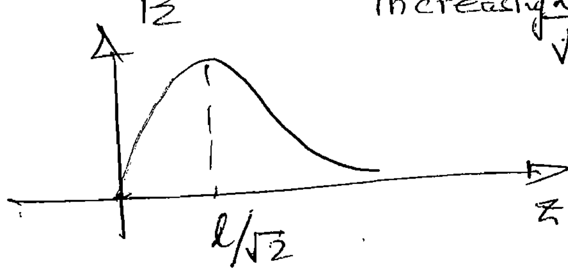
$$= \frac{4kq}{(z^2+l^2)^{5/2}}(l^2-2z^2)$$

sign scheme of $\frac{dE}{dz}$.

+ve | −ve

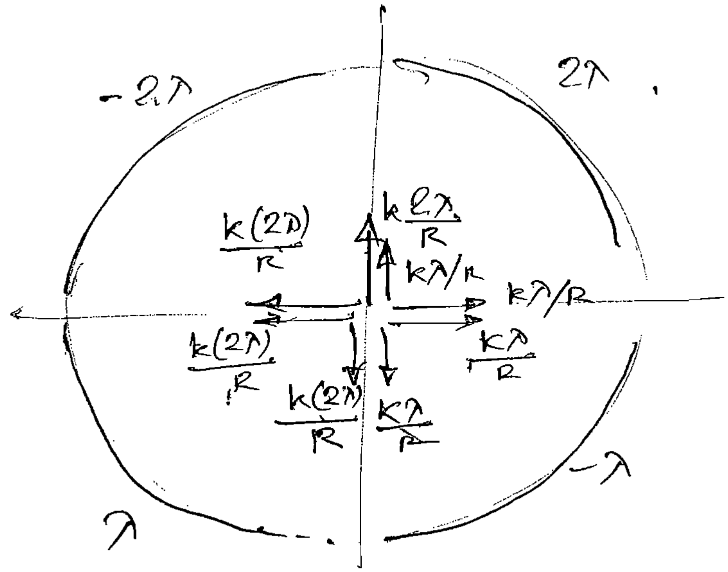
increasing $\frac{l}{\sqrt{2}}$ decreasing

E



5. (a)

(5)



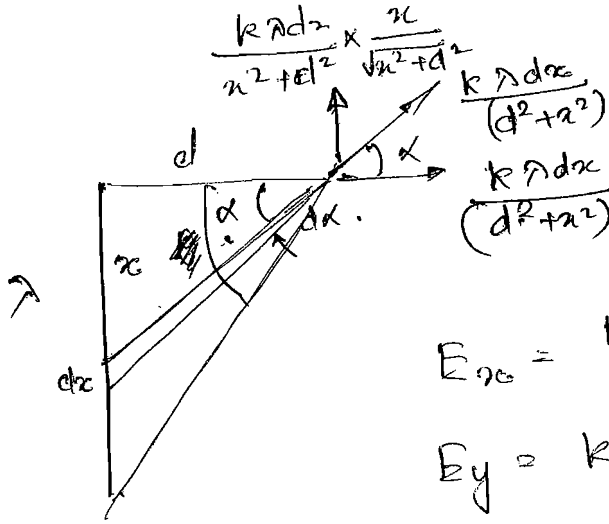
$\frac{k(2\lambda)}{R}$

$= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{R}$

$= \frac{\lambda}{2\pi\epsilon_0 R}$.

6. (a)

(6)



$\frac{k\lambda dx}{(d^2+x^2)} \cos\alpha = \frac{k\lambda dx}{(d^2+x^2)} \frac{d}{\sqrt{d^2+x^2}}$.

$E_x = k\lambda d \int \frac{1}{(d^2+x^2)^{3/2}} dx$.

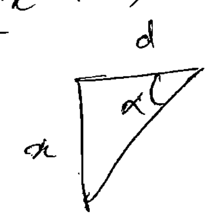
$E_y = k\lambda \int \frac{x}{(x^2+d^2)^{3/2}} dx$.

$\int \frac{1}{(d^2+x^2)^{3/2}} dx$

$= \int \frac{1}{d^3 \sec^3\alpha} d\sec^2\alpha\, d\alpha$.

$= \frac{1}{d^2} \int_0^{60^\circ} \cos\alpha\, d\alpha$.

$= \frac{\sqrt{3}}{2d^2}$ $\quad E_x = \frac{k\lambda\sqrt{3}}{2d}$.



$\tan\alpha = \frac{x}{d}$

$x = d\tan\alpha$.

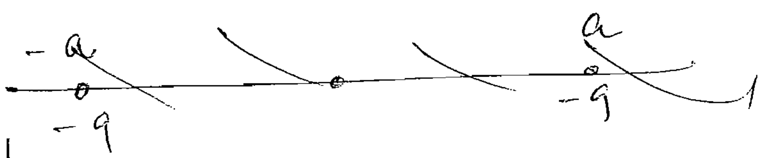
$dx = d\sec^2\alpha\, d\alpha$.

$\int \frac{x}{(x^2+d^2)^{3/2}} dx = \frac{1}{2} \int \frac{d(x^2+d^2)}{(x^2+d^2)^{3/2}} = \frac{1}{2} \int z^{-3/2} dz$

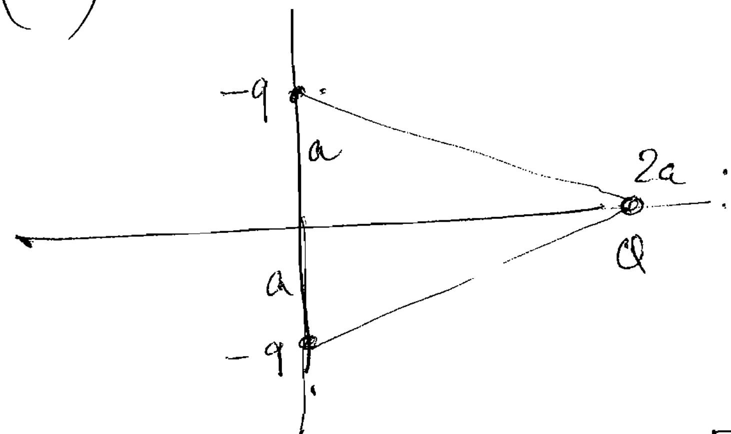
$= -\left[\frac{1}{\sqrt{z}}\right] = -\left[\frac{1}{\sqrt{x^2+d^2}}\right]^{d\sqrt{3}}$.

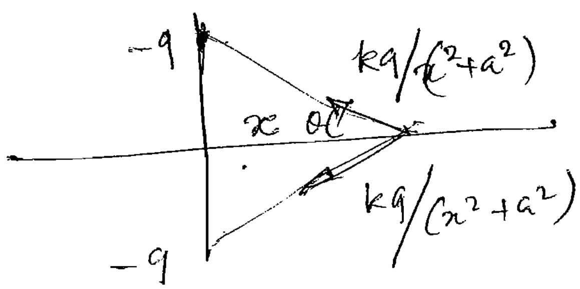
$= -\left[\frac{1}{2d} - \frac{1}{d}\right] = \frac{1}{2d}$.

$E_y = k\lambda$ $\quad E_y \quad k\lambda/2d \quad = 1$



7. (d)



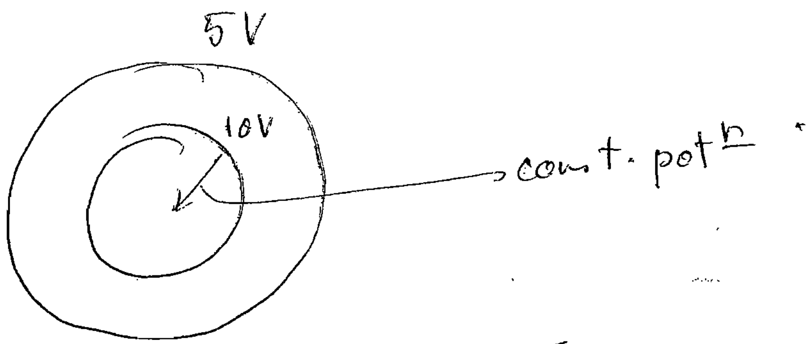


$$F = \left[2\,\frac{kq}{(x^2+a^2)} \times \frac{x}{\sqrt{x^2+a^2}} \right] Q$$

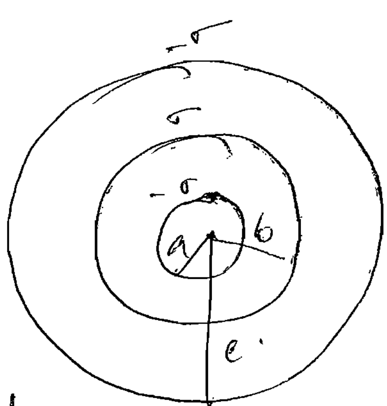
$$F = \frac{2kqQx}{(x^2+a^2)^{3/2}}$$

$F \not\propto x$ except for small oscillation.

8. (a)



9 (c)

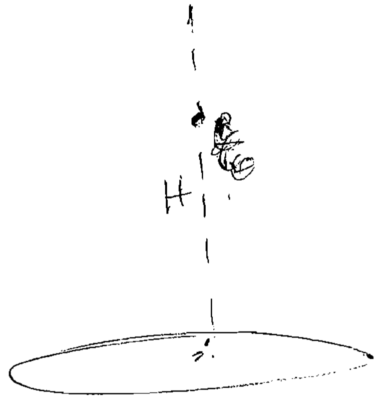


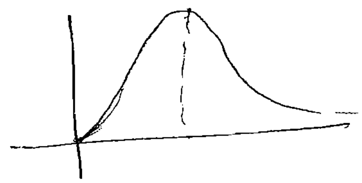
$$\frac{\sigma}{\epsilon_0}(-c) + \frac{\sigma}{\epsilon_0} b + \frac{\sigma}{\epsilon_0}(-a)$$

$\sigma \, [\, b - a - c \,]$

10. (b)

(10)

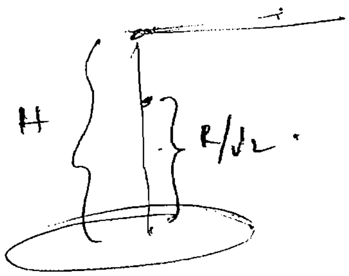




For ring E is max. at $\frac{R}{\sqrt{2}}$

$$\frac{KQHq}{(R^2+H^2)^{3/2}} = mg$$

If $H > \frac{R}{\sqrt{2}}$

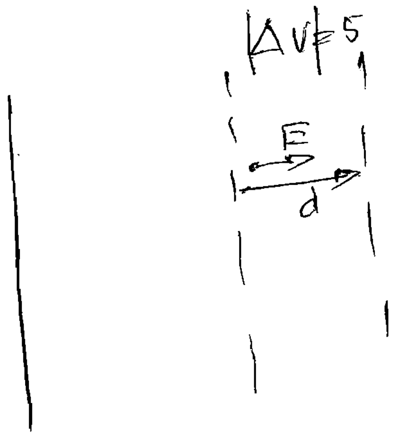


↑ Taken up, E↓, mg is same, accelerat down

↓ " down, E↑ mg is same, accelerat up.

Stable.

(11) (b)



$$|\Delta V| = Ed = \frac{\sigma}{2\epsilon_0} d$$

$$5 = \frac{10^{-7}}{2 \cdot 8.85 \times 10^{-12}} d$$

$$d = 8.85 \times 10^{-4}$$

$$d = 8.85 \times 10^{-3}\ m$$

$$= 0.885 \times 10^{-3}$$

$$d \approx 0.88\ mm$$

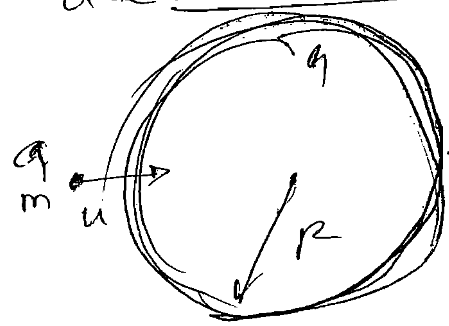
(12) (b)

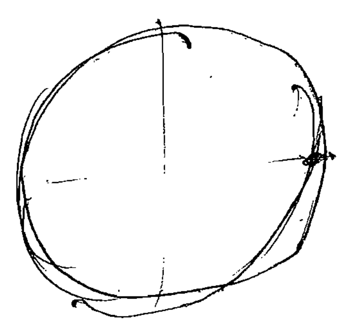
$$\frac{1}{2}mu^2 + \frac{Kq^2}{R} = \left(\frac{3}{2}\frac{Kq}{R}\right) q$$

$$\frac{mu^2}{2} = \frac{Kq^2}{2R}$$

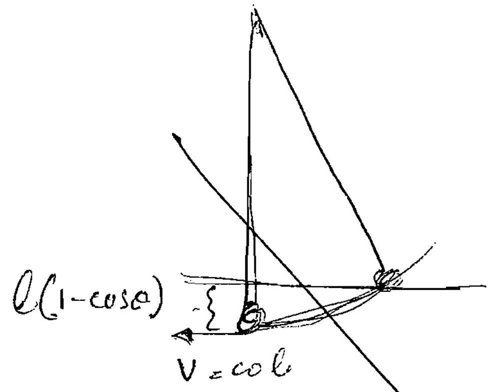
$$u = q\sqrt{\frac{k}{mR}}$$

$$u = \frac{q}{\sqrt{4\pi\epsilon_0 mR}}$$





13.

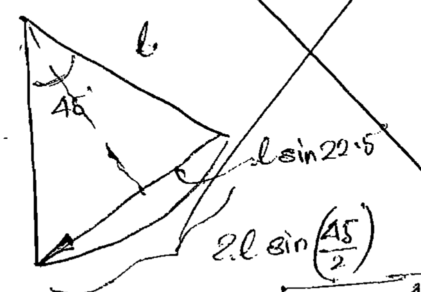


$\Delta E = W_E$.

$\frac{1}{2} m \omega^2 l^2 - 0 - mgl(1-\cos\theta)$

$= q \frac{mg}{q} 2l \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$.

$\omega^2 l^2 = 2gl\left(1 - \frac{1}{\sqrt{2}}\right)$

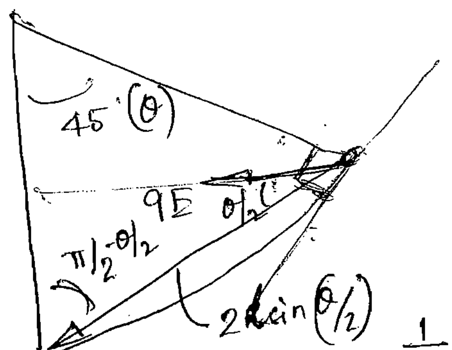


$2l \sin\left(\frac{45}{2}\right)$

$2l \sqrt{\frac{1-\cos 45}{2}}$

$2l \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

13. (B)



$W_E = qE\, 2l\sin(\theta/2)\cos(\theta/2)$

$= qE l \sin\theta$

$= q \cdot \frac{mg}{q} l \sin\theta$

$\Delta E = W_E$

$\frac{1}{2} m\omega^2 l^2 - mgl(1-\cos\theta)$

$= mgl\sin\theta$

$\omega^2 l^2 = 2gl(1-\cos\theta) + 2gl\sin\theta$

$\omega^2 = \frac{2g}{l}(1 + \sin\theta - \cos\theta)$

$\theta = 45^\circ$

$\omega = \sqrt{\frac{2g}{l}}$.

14. (a)

(14)

$$\frac{1}{2}mv^2 + \frac{kQq}{4a} = \frac{kQq}{a}$$

$$\frac{1}{2}mv^2 = \frac{3kQq}{4a}$$

$$\frac{1}{2}mv^2 = 3Vq$$

$$v = \sqrt{\frac{6Vq}{m}}$$

$$\frac{kQ}{4a} = V$$

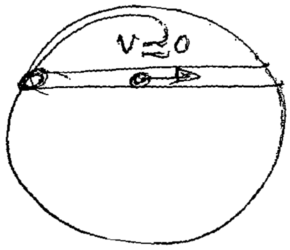
15. (a)

(15)

$$\frac{kQq}{R} + \frac{1}{2}mu^2 = \frac{11}{8}\frac{kQq}{R} \Rightarrow \frac{1}{2}mu^2 = \frac{3kQq}{8R}$$

$$u^2 = \frac{3kQq}{4mR} = \frac{3\rho \cdot \frac{4}{3}\pi R^3 \cdot q}{4 \cdot 4\pi\epsilon_0 mR}$$

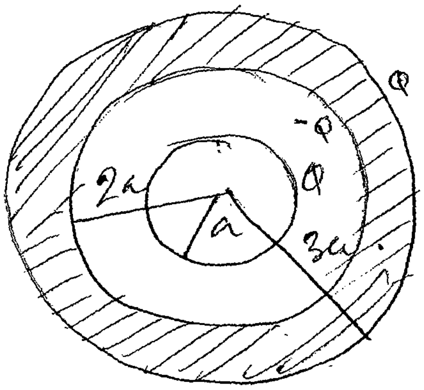
$$u = \sqrt{\frac{\rho q R^2}{4\epsilon_0 m}}$$



$\because q = 1$

$$u = \sqrt{\frac{\rho R^2}{4\epsilon_0 m}}$$

(16) 16. (a)

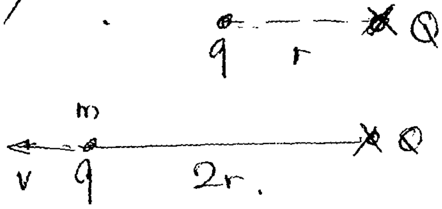


$$U = \frac{kQ^2}{2a} - \frac{kQ^2}{4a} + \frac{kQ^2}{3a}$$

$$= \frac{5kQ^2}{12a}$$

17. (b)

(17)



$$\frac{1}{2}mv^2 + \frac{kQq}{2r} = \frac{kQq}{r}$$

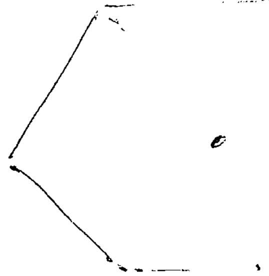
$$\frac{mv^2}{2} = \frac{kQq}{2r}$$

$$v = \sqrt{\frac{kQq}{mr}}$$

Impulse = mv

$$\sqrt{mkQq} = \sqrt{Qqm}$$

18. (b)

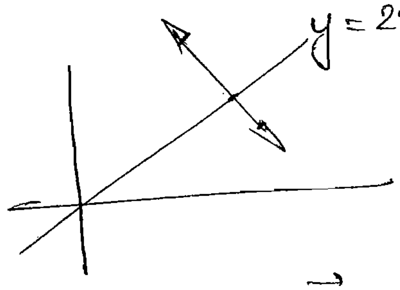


$E = \frac{kq}{r^2}$

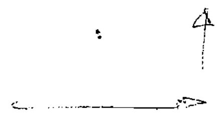
$V = (n-1)\frac{kq}{r}$

$\frac{V}{E} = r(n-1)$

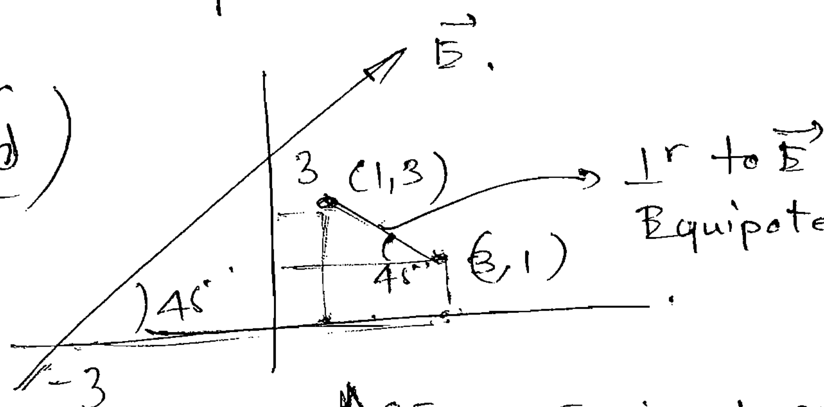
19. (d)



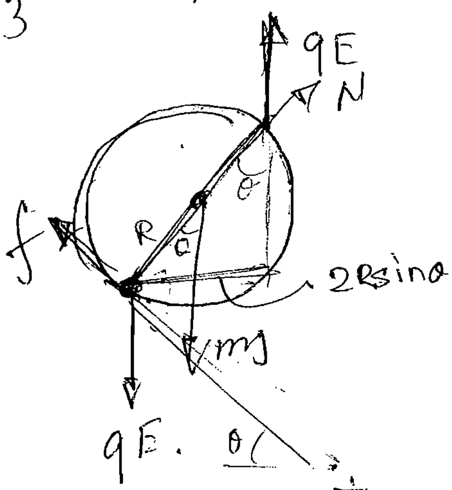
E ⊥ equipotential line. only (d)



20. (d)



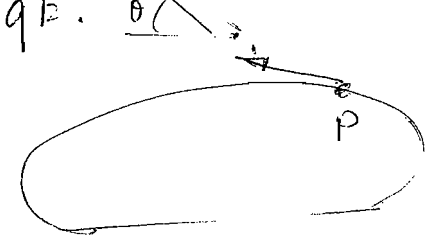
21. (b)



Taking torque abt bottommost pt. $qE\,2R\sin\theta = mgR\sin\theta$

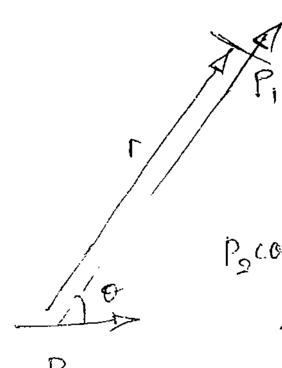
$E = \frac{mg}{2q}$.

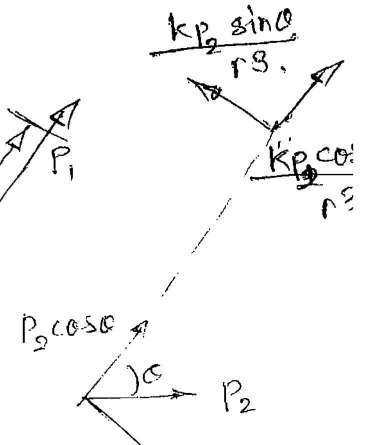
22. (b)

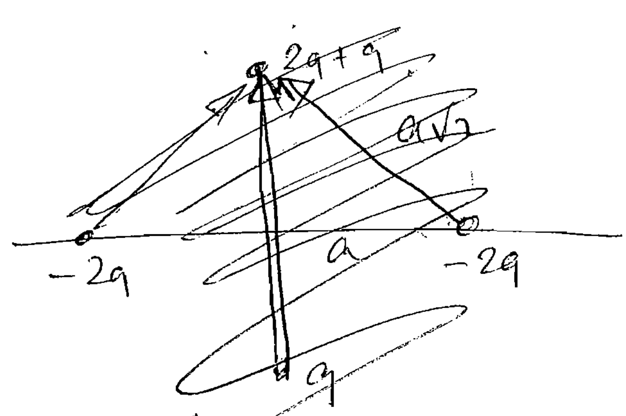


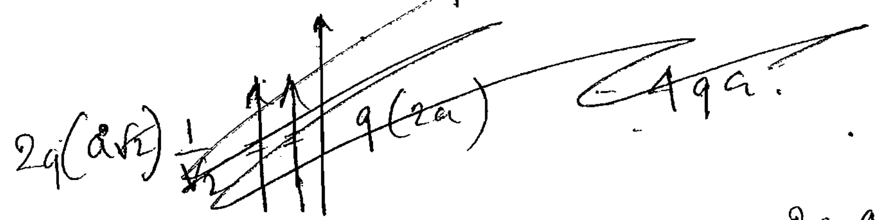
23. (b) $U = -\vec{p}_1 \cdot \vec{E}_2$

$= -p_1\hat{e}_1 \cdot (E_r\hat{e}_r + E_\theta\hat{e}_\theta)$



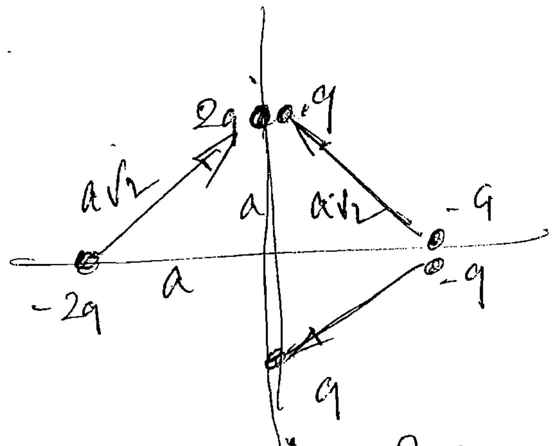


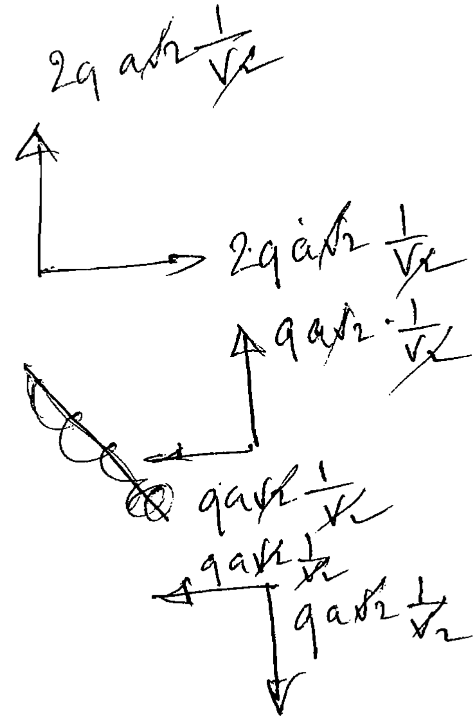


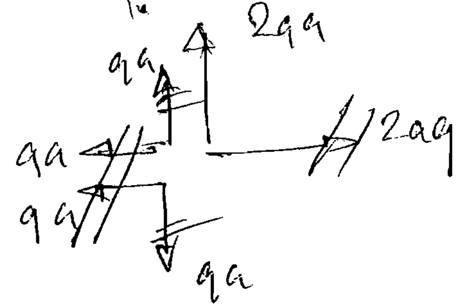


24. (a)

(24)







25. A (b)

(25) A

B (B) (d) Pot^n only due to $-q+Q$.
rest cancelled

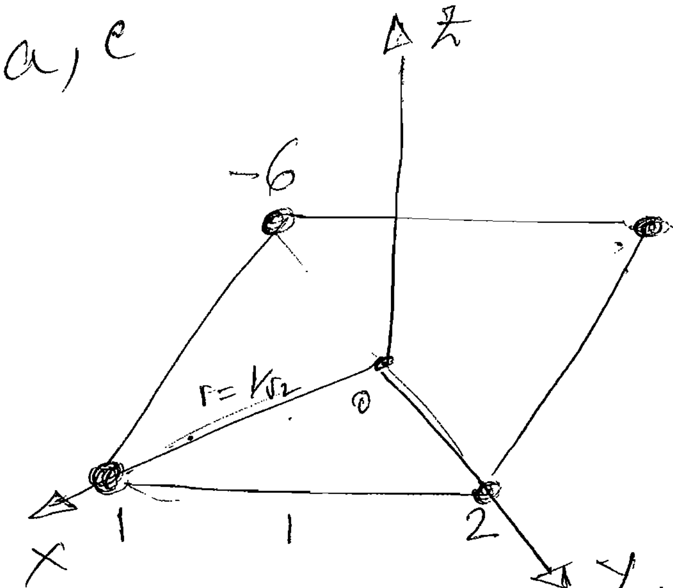
$\frac{KQ}{r} + \frac{K(-q)}{r}$

$K\frac{(Q-q)}{}$

~~26.(a)~~ Apply superposition. Exercise #2
Electrostatics

27. a, c

(1)



$V_o = \frac{k(1)}{r} + \frac{k(2)}{r} + \frac{k(3)}{r} + \frac{k(-6)}{r}$

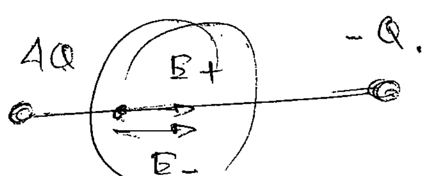
$= 0$

Dist. of any pt. on z axis from each charge is same.

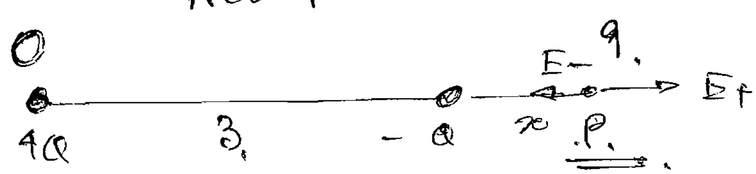
& $V_z = 0$

28.

(2)



Add up. ∴ (a)



$E = \frac{k4Q}{(3+x)^2} - \frac{kQ}{x^2}$

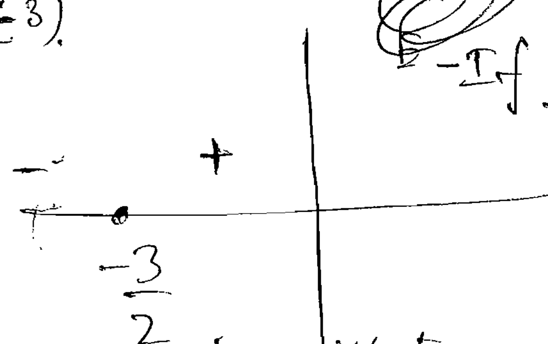
$= \frac{kQ(2x+3)(x-3)}{x^2(3+x)^2}$

~~If shifted~~

~~E+~~

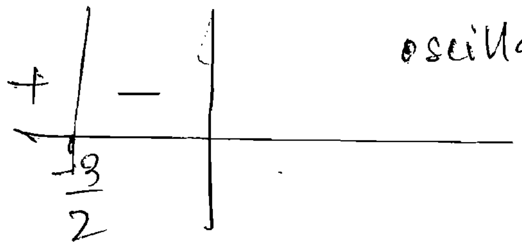
If $q > 0$

$F = qE$



... oscillate.

If $q<0$



oscillates.

(3) (a, c) check signs of E as above.

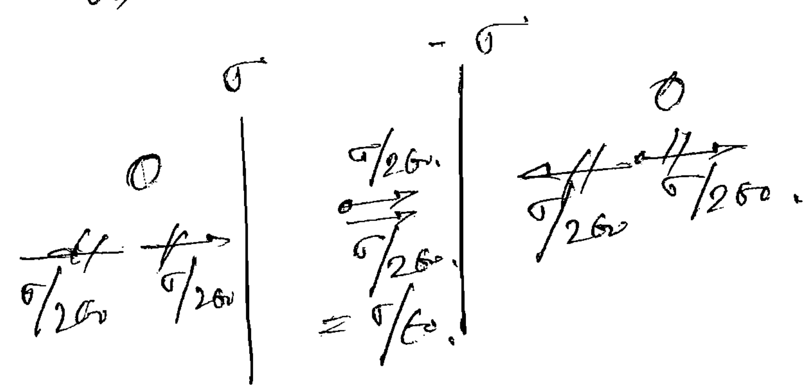
(4) (b), (c) check configs.

U is min

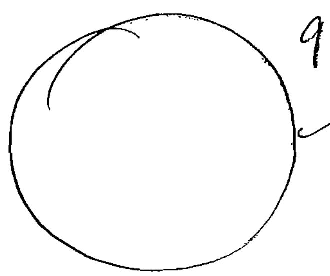
$\Rightarrow \frac{dU}{dx} = 0$

$E = -\frac{dU}{dx} = 0 \Rightarrow E = 0.$

(5) (a, c)



32. (d)



→ Equipotential.

$E_{inside} = 0$, irrespective of E_{ext}

Distribution of charge uniform / non uniform based on $E_{int} = 0$

33 (b)(a)

(6)

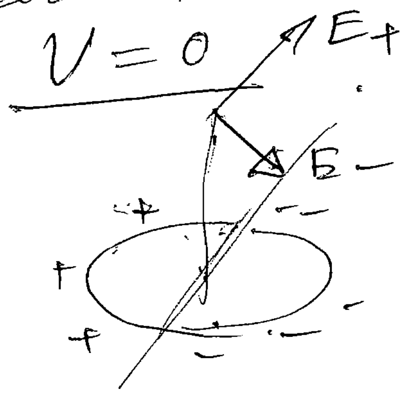
$E = \frac{kQx}{(x^2+R^2)^{3/2}}$

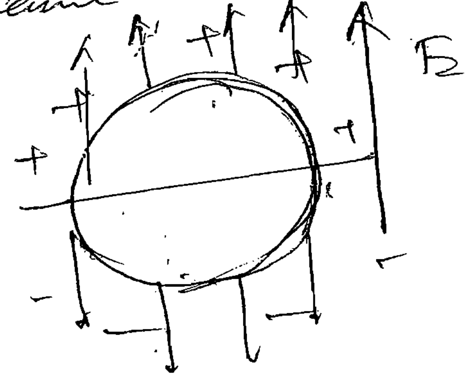
$V = \frac{kq}{\sqrt{x^2+R^2}}$ decreasing.

34 (a)

Pot is scalar. Each 1/2 is at same dist

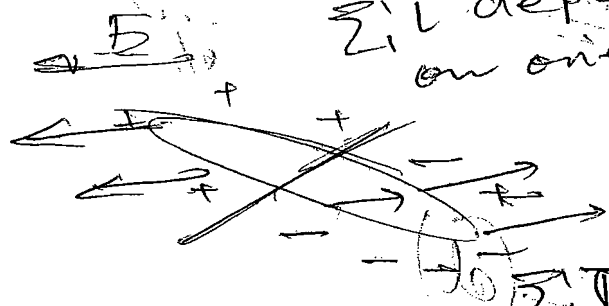
V = 0





$\sum_i F = 0$

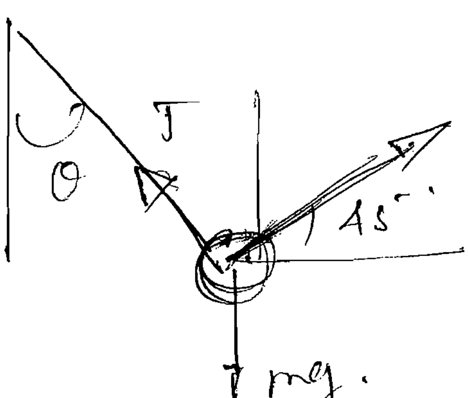
$\sum_i \tau$ depends on orientation.



$\sum \tau \neq 0$

35 (a, b)

(7)



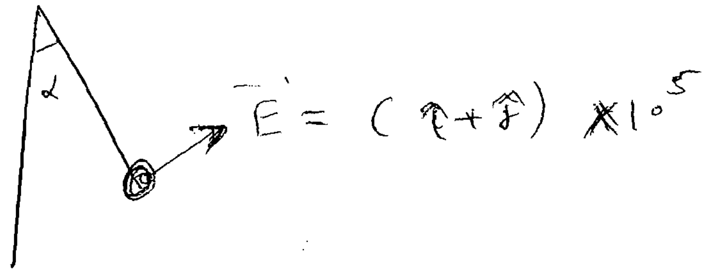
$T\sin\theta + \frac{qE}{\sqrt{2}} = mg$

$\frac{qE}{\sqrt{2}} = T\cos\theta$

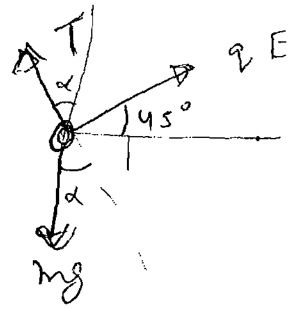
$\therefore T(\sin\theta + \cos\theta) = mg$

θ = 30°, 60°

(35)



(1)



Applying lami's theorem,

$$\frac{T}{\sin(135^\circ)} = \frac{mg}{\sin(45^\circ+\alpha)}$$

$$\Rightarrow \frac{2mg}{(1+\sqrt{3})\cdot\frac{1}{\sqrt{2}}} = \frac{mg}{\frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha}$$

$$\Rightarrow \therefore \cos\alpha + \sin\alpha = \frac{(\sqrt{3}+1)}{2}$$

$$=$$

$$\therefore \alpha = 30^\circ \text{ or } 60^\circ.$$

(36) (8)

$\vec{P} = (2\hat{i}+3\hat{j})\ \mu Cm.$

$\vec{E} = (3\hat{i}+2\hat{k}) \times 10^5\ NC^{-1}.$

$$\vec{\tau} = \vec{P}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} i(6) - \hat{j}(4) \\ +\hat{k}(-9) \end{pmatrix} \times 10^{-1}$$

(b) Potential Energy

$= -\vec{P}\cdot\vec{E}$

$= -(2\hat{i}+3\hat{j})\cdot(3\hat{i}+2\hat{k}) \times 10^{-6} \times 10^{5}$

$= -(6) \times 10^{-1}$

$= -0.6\ J$

(c) Pot Energy $= |\vec{P}||\vec{E}| \cos\theta$

Max value $= |\vec{P}|\,|\vec{E}|$

$= \sqrt{2^2+3^2}\ \sqrt{3^2+2^2} \times 10^{-6} \times 10^{5}$

$= 13 \times 10^{-1} = 1.3\ J$.

9 (37)

Potential decreases in the direction of electric field. $\therefore V_A < V_B$. (d)

More dense the electric field lines, more the electric field at that point.

$E_A > E_B$ (b)

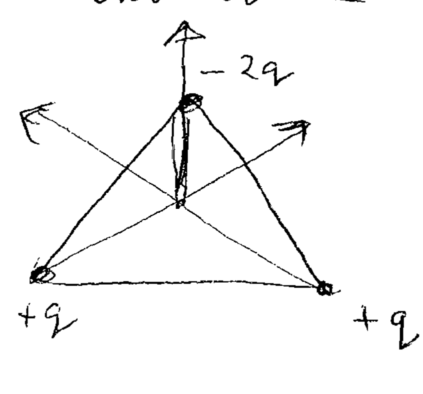
10 (38)

Pot at centre

$= k\left[\frac{-2q}{r} + \frac{q}{r} + \frac{q}{r}\right] = 0$

(where r is the distance of any one of the vertices from the centroid).

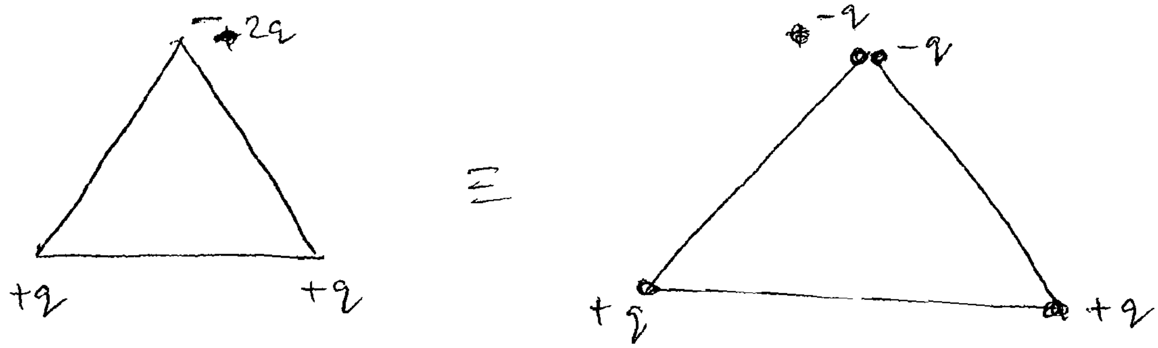
For Electric field, we have to add individual electric fields vectors.

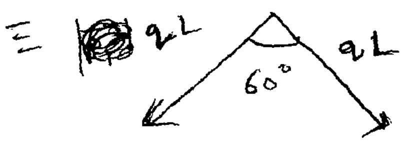


We see that all electric fields will add up. So, sum cannot be zero.

(8) Dipole moment.

(2)





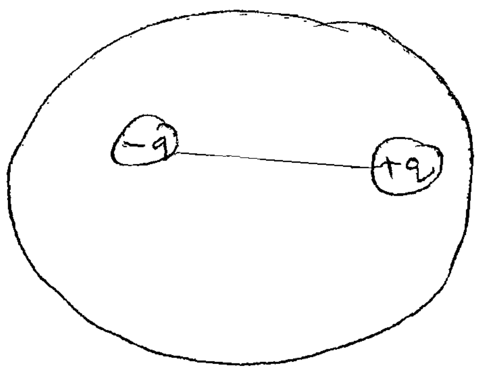
Magnitude of resultant

$$= \sqrt{(qL)^2 + (qL)^2 + 2(qL)(qL)\cos 60^\circ}$$

$$= qL\sqrt{1+1+2\frac{1}{2}} = \sqrt{3}qL.$$

options a, d.

(12) (29)

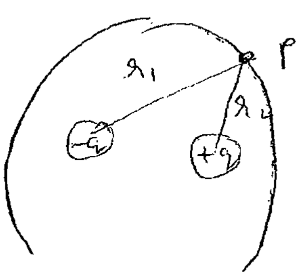


net charge enclosed = 0

∴ Net flux = 0.

Electric field at any point will not be zero.

Say, Electric potential will be zero at point P on surface.

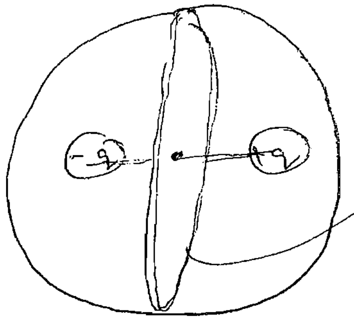


Then,

$$k\left[\frac{-q}{r_1} + \frac{q}{r_2}\right] = 0.$$

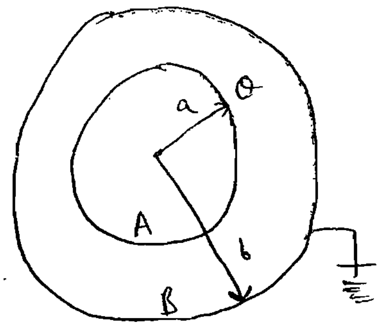
$$\therefore r_1 = r_2.$$

whose centre lies midway b/w $-q$ and q



circle on which potential $= 0$.

12



Shell B is earthed.

$\therefore$ potential at B will be zero.

$$\therefore \frac{kQ}{b} + \frac{kq}{b} = 0$$

$$\therefore \boxed{q = -Q}$$

where q is the charge on the outer shell.

Electric field at $a \le r \le b$

$$= |\vec{E}| = k\frac{Q}{r^2} \quad \text{(by gauss law)}$$

Option (a)

Potential at r, $a \le r \le b$ will be

Potential due inner shell + Potential due to outer

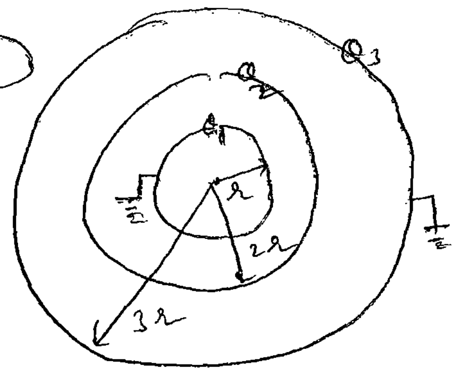
$$= \frac{kQ}{r} + \frac{k(-Q)}{b} = kQ\left(\frac{1}{r} - \frac{1}{b}\right).$$

Option (d)

Potential difference b/w A and B

is $\text{Pot}_A - \text{Pot}_B$ ($\text{Pot}_B \to 0$)

(41)



(13)

~~$V_A = 0$~~

$V_1 = 0$ and $V_3 = 0$.

$V_1 = 0$

$\therefore \; K\left[\frac{Q_1}{r} + \frac{Q_2}{2r} + \frac{Q_3}{3r}\right] = 0$ — (1)

Also $V_3 = 0$

$\therefore \; k\left[\frac{Q_1}{3r} + \frac{Q_2}{3r} + \frac{Q_3}{3r}\right] = 0$ — (2)

From (1) and (2), we have,

$Q_1 + \frac{Q_2}{2} + \frac{Q_3}{3} = 0$

and $Q_1 + Q_2 + Q_3 = 0$. (option (a) is right.)

$\therefore \; \frac{Q_2}{2} + \frac{Q_3}{3} = Q_2 + Q_3$.

$\Rightarrow \; -\frac{2Q_3}{3} = +\frac{Q_2}{2}$.

$\therefore \; \boxed{\frac{Q_3}{Q_2} = -\frac{3}{4}}$ → (option (d) is wrong).

Also $Q_1 + Q_2 + \left(-\frac{3}{4}Q_2\right) = 0$

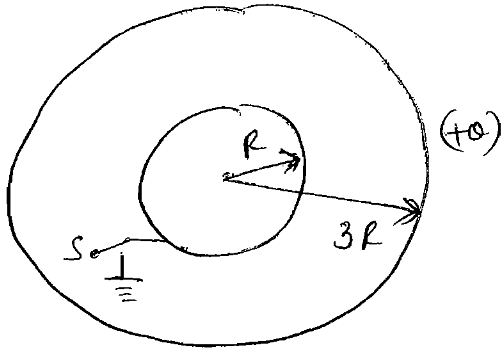
$\therefore \; Q_1 + \frac{Q_2}{4} = 0$

$\therefore \; \boxed{\frac{Q_1}{1} = -\frac{Q_2}{4}}$ → option (b) is right

and $\frac{Q_3}{Q_1} = \frac{-3/4 \, Q_2}{-Q_2/4} = 3$.

$\therefore$ option (c) is also right

(14) (42)



When switch is open,

$V_{inner} = V_{due\ inner\ charge} + V_{due\ outer}$

$= 0 + \frac{Q}{3R}$

$V_{outer} = V_{due\ inner} + V_{due\ outer}$

$= 0 + \frac{Q}{3R}$ (equal) (a) ✓

Switch closed,

$V_{inner} = 0$ ✓ (connected to earth) (b) ✓

Let q charge attained by inner (switch closed).

then $V_{in} = 0$

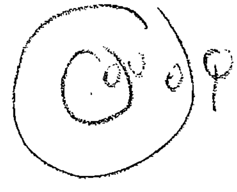
$\therefore \frac{Kq}{R} + \frac{KQ}{3R} = 0$

$\therefore q = -\frac{Q}{3}$. (c) ✓

Capacitance initial :→

$C_{in} = 4\pi\epsilon_0 3R$

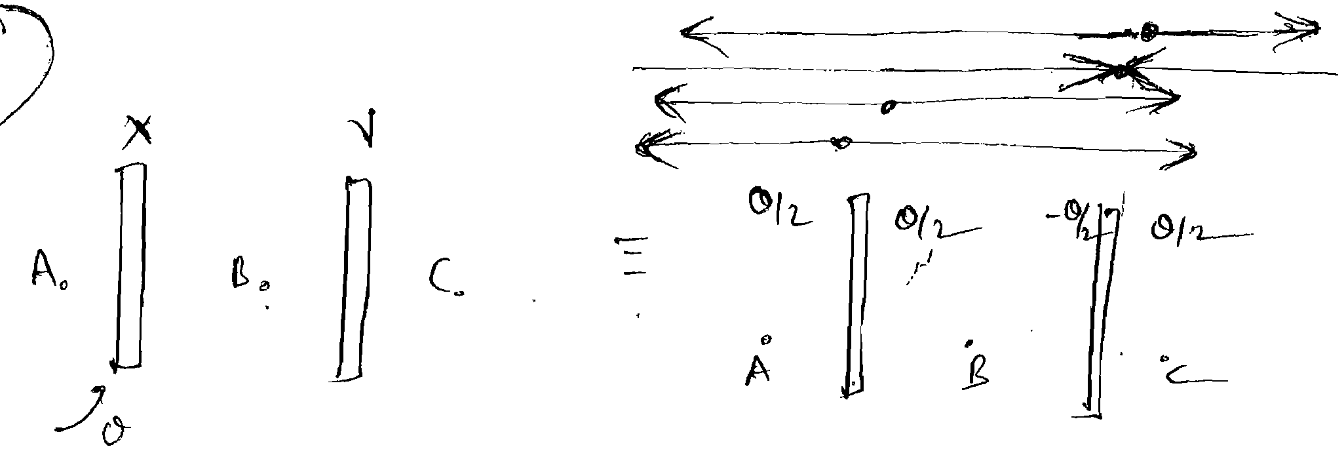
$C_f = \frac{4\pi\epsilon_0 R \cdot 3R}{3R - R}$ + $4\pi\epsilon_0 3R$



$6\pi\epsilon_0 R + 12\pi\epsilon_0 R$

$= 18\pi\epsilon_0 R$

(15) (43)



Then, $E_A = \frac{Q/2}{2\epsilon_0 A} + \frac{Q/2}{2\epsilon_0 A} = \frac{Q}{2\epsilon_0 A}$

$E_B = \frac{Q/2}{2\epsilon_0 A} + \frac{Q/2}{2\epsilon_0 A} = \frac{Q}{2\epsilon_0 A}$

$E_C = \frac{Q/2}{2\epsilon_0 A} + \frac{Q/2}{2\epsilon_0 A} = \frac{Q}{2\epsilon_0 A}$.

(Fields due to individual surfaces are represented by arrows).

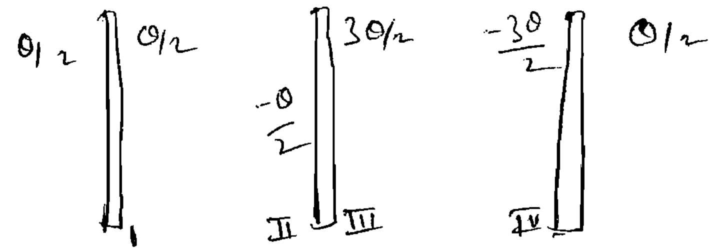
(16) (44)

a, c, d

Charge on outer surfaces →

$\frac{Q + Q - Q}{2} = \frac{Q}{2}$.

∴ Charge distribution :→



(a) ✓

(b) ✓

(c) ✓

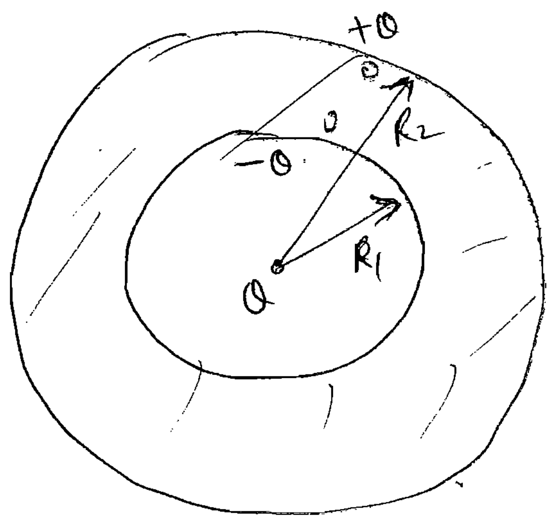
$$V_A - V_C = (V_A - V_B) + (V_B - V_C)$$

$$= \left(\frac{Q/2}{\epsilon_0 A}\right) d + \left(\frac{3Q/2}{\epsilon_0 A}\right) d$$

$$= \frac{2Qd}{\epsilon_0 A} \neq V_C - V_B$$

Hence wrong option is (d).

~~45~~ ~~107~~ 29



(Figure shows the charge distribution

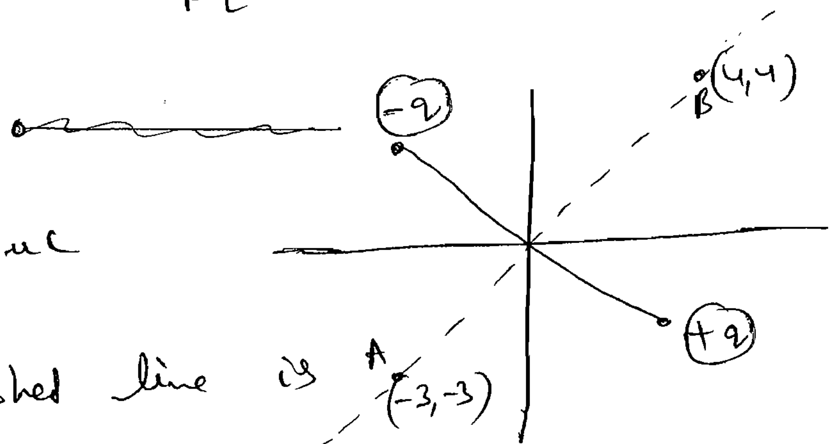
Now, $V_{\text{inner surface}} = V_{\text{due point charge } Q}$

$+ V_{\text{due inner surface charge}}$

$$= \frac{KQ}{R_1} - \frac{KQ}{R_1} + \frac{KQ}{R_2}$$

$$= \frac{KQ}{R_2} \quad \text{(none of these)}$$

~~48~~ (30) $q = 2\mu C$

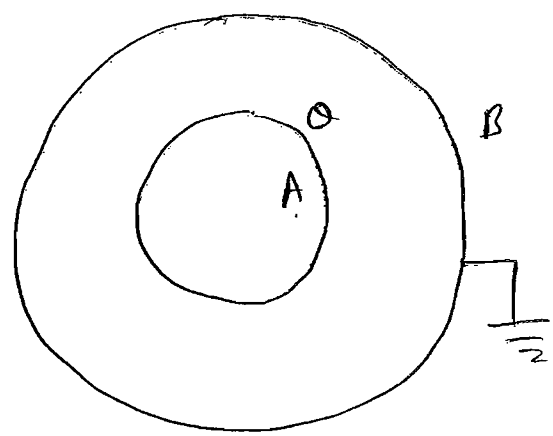


The dashed line is the equipotential surface. each point having the same potential on the dashed line.

So, No work done for displacement from A to B.

~~49~~ (31) B earthed

$\therefore V_B = 0$



But $V_B = K\left[\frac{Q}{r_B} + \frac{q}{r_B}\right] = 0$

(where q is the charge which comes on B).

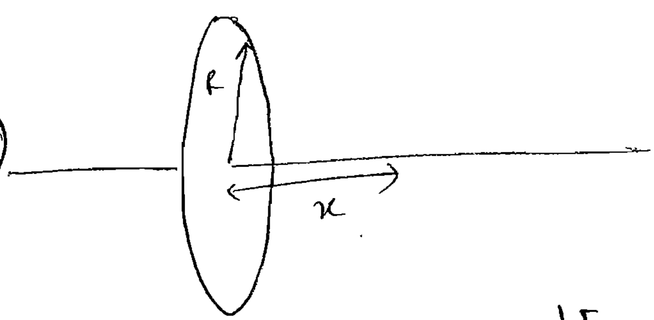
$\therefore Q + q = 0 \quad \therefore \boxed{q = -Q}$

field inside A = 0 (Gauss law)

field ~~inside~~ b/w A and B ≠ 0 (Gauss law charge enclosed ≠ 0).

(c) ✓

(32)



We have

$$E_x = \frac{K\,Q\,x}{(a^2+x^2)^{3/2}}$$

For E_x to be max,

$$\frac{dE_x}{dx} = 0.$$

$$\therefore\ x = \frac{a}{\sqrt{2}}.$$

and $(E_x)_{max} = \left(E_{a/\sqrt{2}}\right) = K \cdot \frac{2q}{3\sqrt{3}\,R^2}$.

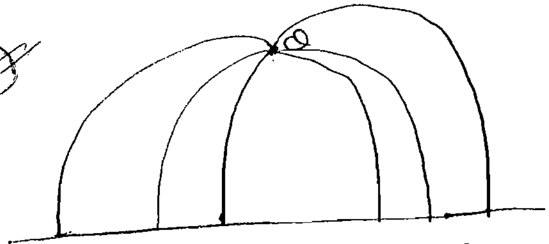
(~~Do~~ Read theory).

(33) Potential energy at A is same as that at B and C.

∴ Work done in taking the charge from P to any of A or B or C is same.

$W_A = W_B = W_C$.

(34)



Lines of force will be

(i) perpendicular to infinite metal plate

Also option (D) is wrong because (6)

there cannot be any 'kink' in lines of force as they would mean 2 directions at a particular point.

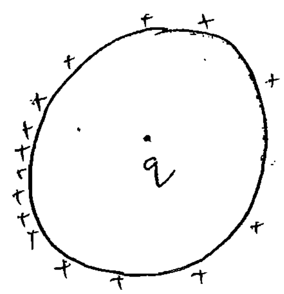
~~Hence~~.

In option (B), fields are going into the charge Q. So, incorrect.

Option (A) is right.

~~53~~

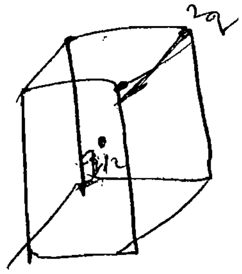
(35) The charge on the shell will distribute till that time and in that way such that the final



electric field at any point inside the conductor becomes zero due to charges present on the shell as well as outside the shell. Thus, q is shielded from any electric field and force on q is zero

~~54~~

(36)



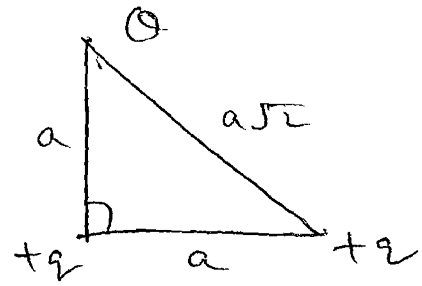
flux through cube

$= \frac{q_{enclosed}}{\epsilon_0}$

$= \frac{q/12}{\epsilon_0}$

Option (a)

(55) (37)



Net E. energy

$$= k\left[\frac{q^2}{a} + \frac{Qq}{a} + \frac{Qq}{a\sqrt{2}}\right]$$

$$= 0.$$

$$\therefore \frac{q}{1} + \frac{Q}{1} + \frac{Q}{\sqrt{2}} = 0$$

$$\therefore q + Q\left(\frac{1}{1} + \frac{1}{\sqrt{2}}\right) = 0$$

$$\therefore \frac{-q}{1+\frac{1}{\sqrt{2}}} = Q$$

$$\Rightarrow \boxed{\frac{-\sqrt{2}\,q}{\sqrt{2}+2} = Q} \quad \therefore (6)$$

(58) (38)

2.24 mg

$= 2.24 \times 10^{-3}$ g

Atomic mass of Fe = 56 amu.

6×10^{23} atoms $\rightarrow$ 56 g

$\therefore$ 1 g $\rightarrow \frac{6 \times 10^{23}}{56}$ atoms.

2.24×10^{-3} g $\rightarrow \frac{6 \times 10^{23}}{56} \times 2.24 \times 10^{-3}$

$= 6 \times 10^{23} \times 10^{-5} \times 4$ atoms.

Each atom has 26 electrons.

$\therefore$ Total electrons removed

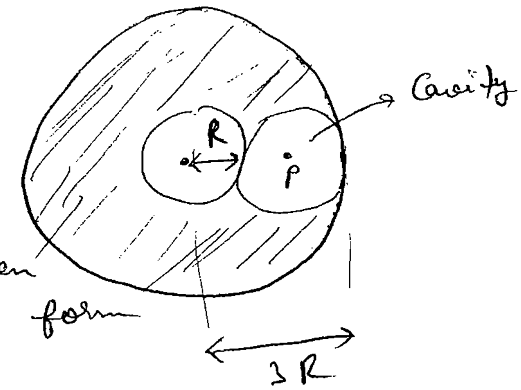
$= \frac{.02}{100} \times 26 \times 6 \times 10^{23} \times 10^{-5} \times 4$

Total charge $= 1.6 \times 10^{-19} \times \frac{.02}{100} \times 26 \times 6 \times 10^{23} \times 10^{-5} \times 4$

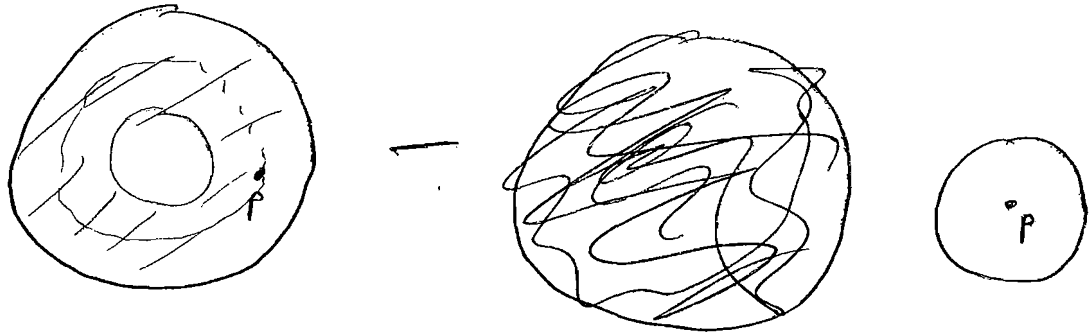
(57) (39) To find :

$E_{\text{cavity centre}} \equiv E_{cc}$

$\vec{E}_{C.C} = \vec{E}_{\text{no cavity}} - \vec{E}_{\text{solid sphere}}$ which has been removed to form cavity



$\equiv$



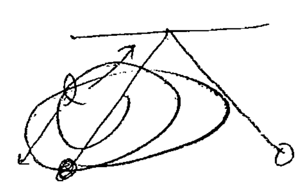
$$= \frac{q_{\text{enclosed}}}{\varepsilon_0 \cdot 4\pi (2R)^2} - 0$$

$$= \frac{\left[\frac{4}{3}\pi (2R)^3 - \frac{4}{3}\pi R^3\right]\sigma}{\varepsilon_0 4\pi (2R)^2}$$

$$= \frac{1}{3} \frac{7R^3\sigma}{4\varepsilon_0 R^2} = \frac{7\sigma R}{12\varepsilon_0}$$ (d)

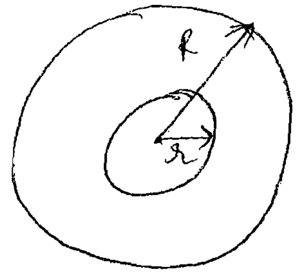
(58) (40) The electric field at any point is due to all the charges inside or outside the gaussian surface

(59)



(60)

41



q on inner surface
$= 4\pi r^2 \sigma$

q on outer surface
$= 4\pi R^2 \sigma$

Electric potential at centre.

$$K\left[\frac{4\pi R^2\sigma}{R} + \frac{4\pi r^2\sigma}{r}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot 4\pi\sigma\,(R+r)$$

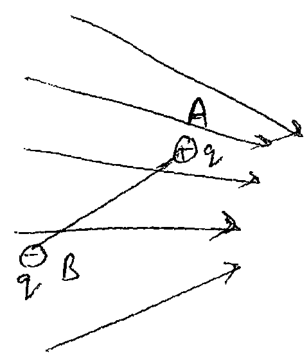
$$= \frac{\sigma}{\varepsilon_0}(R+r)$$

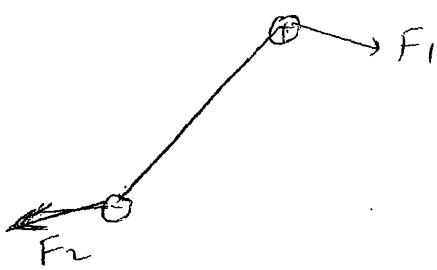
(61)

42

Field at A is more than field at B.

So, $F_1 > F_2$

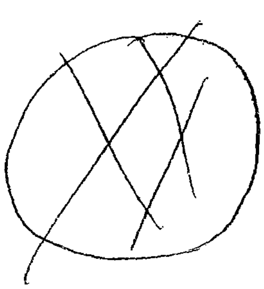


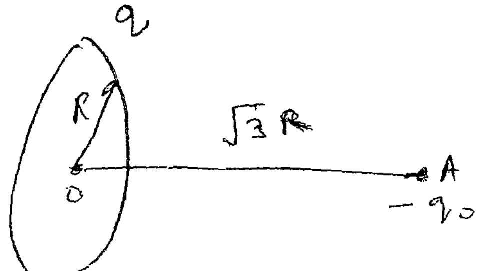


Net force towards right.
Net torque clockwise (into the plane).

(62)

43





$$P.E. \text{ at } A = \frac{-kq\,q_0}{\sqrt{r^2+(\sqrt{3}r)^2}} = \frac{-kq\,q_0}{2r}$$

~~$\Delta KE = \frac{1}{2}mv^2$~~

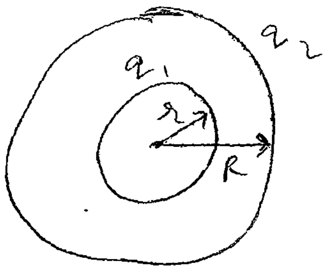
$$\Delta KE + \Delta PE = 0$$

$$\therefore \left(\frac{1}{2}mv^2 - 0\right) + \left(-\frac{kqq_0}{r} - \frac{-kqq_0}{2r}\right) = 0$$

~~$\frac{1}{2}mv^2 = \frac{kqq_0}{2r}$~~

$$KE = \frac{kqq_0}{2r}$$

~~63~~ 44



$$q_1 + q_2 = Q$$

$$\frac{q_1}{q_2} = \frac{4\pi r^2 \sigma}{4\pi R^2 \sigma}$$

$$\frac{q_1}{q_2} = \frac{r^2}{R^2}$$

$$q_1 + \frac{q_1 R^2}{r^2} = Q$$

$$q_1\left(1 + \frac{R^2}{r^2}\right) = Q$$

$$q_1 = \frac{Qr^2}{R^2 + r^2}, \quad q_2 = \frac{QR^2}{r^2 + R^2}$$

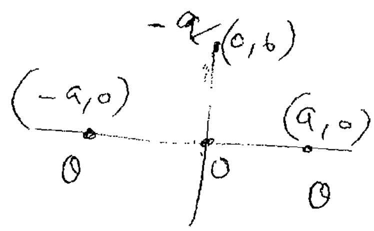
Potential at centre

$$= \frac{kq_1}{r} + \frac{kq_2}{R}$$

$$= k\left[\frac{Qr}{R^2 + r^2} + \frac{QR}{R^2 + r^2}\right]$$

$$= \frac{kQ}{R^2 + r^2}(r + R)$$

(64) (17) Ex#2



Distance b/w -q and Q initially = $\sqrt{a^2+b^2}$.

Initial P.E. = $\dfrac{-kQq}{\sqrt{a^2+b^2}} \times 2$

P.E. when -q reaches O

$= \dfrac{-kQq}{a} \times 2$

$\Delta KE + \Delta PE = 0$

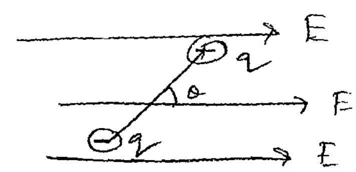
$\therefore \left(\frac{1}{2} m v_0^2 - 0\right) + \left(\left(-\frac{kQq}{a} \times 2\right) - \left(-\frac{kQq\,2}{\sqrt{a^2+b^2}}\right)\right) = 0$

$\Rightarrow \frac{1}{2} m v_0^2 = 2kQq\left[\frac{1}{a} - \frac{1}{\sqrt{a^2+b^2}}\right]$

$v_0 = \sqrt{\dfrac{4kQq}{m}\left[\dfrac{1}{a} - \dfrac{1}{\sqrt{a^2+b^2}}\right]}$

(65) (18) Ex#2

given θ is small
(slightly rotated)



$|\vec{\tau}| = |\vec{P} \times \vec{E}|$

$= (qL) \sin\theta \, E$

$= qLE \sin\theta$.

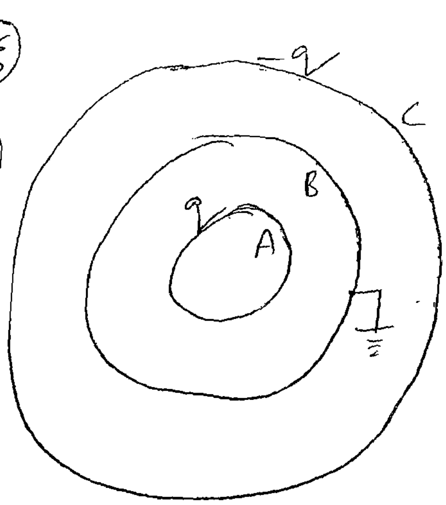
If θ is small, $\sin\theta \approx \theta$

$\therefore |\vec{\tau}| = qLE\theta$.

$\therefore$ S.H.M. + oscillatory

(19)

Ex#2



Let the charge on B be Q

Then,

$V_B = 0$

$\therefore \frac{Kq}{b} + \frac{KQ}{b} - \frac{Kq}{c} = 0$

$\Rightarrow K\left[\frac{q}{b} + \frac{Q}{b} - \frac{q}{c}\right] = 0$

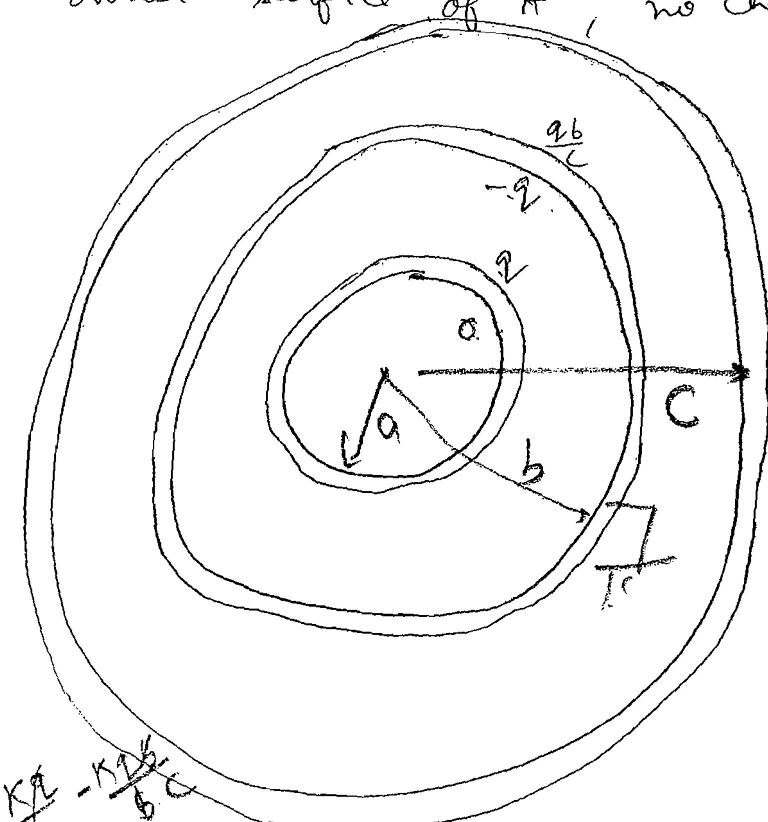
$\Rightarrow \frac{Q}{b} = q\left[\frac{1}{c} - \frac{1}{b}\right]$

$\frac{Q}{b} = q\left[\frac{b-c}{cb}\right]$

$Q = \frac{q(b-c)}{c}$

$\boxed{Q = q\frac{b}{c} - q}$

Inner surface of A, no charge (If we consider the guassian surface in the material of A, the total inside charge must be 0)



Outer surface of A → q.

Inner surface of B → −q

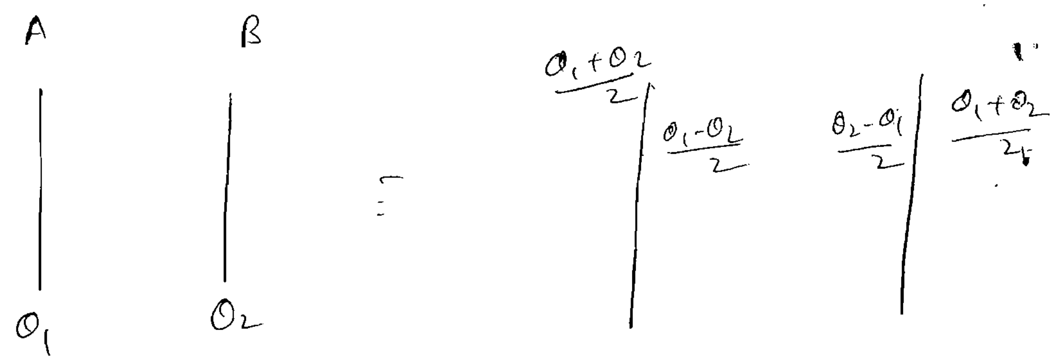
Outer surface of B → $\left(\frac{qb}{c} - q\right) - (-q)$

$= \frac{qb}{c}$.

(67)

(20)

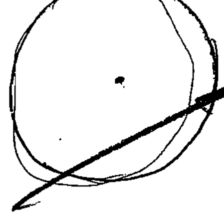
Ex#2



Hence, all options correct

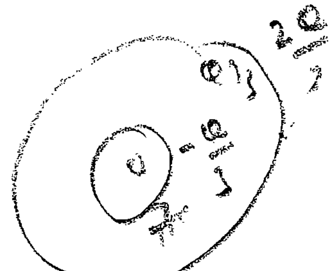
(68)

~~balloon is non-conducting.~~



remove

ROUGH



$\frac{\sigma}{2\epsilon_0} d$ $\frac{12}{8.85\times10}$ $\frac{7}{10}$ 5

$5 = \frac{\sigma}{2\epsilon_0} \cdot d$

$d = \frac{10^{10}}{6}$ $= \frac{8.85\times10^{-12}\times10}{10^{-7}}$ 8.85×10

$q = \frac{4\pi R^2 \sigma}{2} + 4\pi R^2 \sigma$

$= 6\pi R \sigma$

$= 18$

Exercise #1 (Electrostatics)

(26) $\vec{E}$ inside cavity $\propto \vec{l}$
$\vec{F} \propto \vec{l}$.

(27) Potential must be equal at every point.

(28) only potential will be zero at all points on its axis

(45) $\vec{E}_x = \frac{(16-4)\hat{i}}{4} = 3\hat{i}$ $\quad\quad \vec{E} = 3\hat{i} - 4\hat{j}$

$\vec{E}_y = -\frac{(12-4)}{2}\hat{j} = -4\hat{j}$

(46) $r = \sqrt{(8-2)^2 + (-5-3)^2} = 10$

$E = \frac{9\times10^{9} \times 50\times10^{-6}}{(10)2} = 4500\,\frac{V}{m}$.

(47) charge within the surface

$= \frac{(\sigma.\pi r^2)}{\epsilon_0} = \frac{\sigma.\pi(R^2-x^2)}{\epsilon_0}$

(48) $\vec{E} = \vec{E}_1 + \vec{E}_2$

$= \frac{\lambda}{2\pi\epsilon_0 r}\hat{i} + \frac{\sigma R}{\epsilon_0 r}\hat{i}$

$\vec{E} = 0$

$\boxed{\lambda = -2\pi\sigma R}$

(49) $\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial z}{\partial z}\hat{k}\right)$

$= -2K\left[2x\hat{i} - y\hat{j} + z\hat{k}\right)$

$\vec{E}(1,1,1) = 2K(2\hat{i} - \hat{j} + \hat{k})$

$|\vec{E}| = 2K\sqrt{6}$,

(50) $r = \sqrt{1+1+4} = \sqrt{6} < 5$

inside point $\Rightarrow \boxed{E = 0}$

Exercise # 2 : Electrostatics

(21) $\vec{E_A}$ is along $\vec{OA}$ & $\vec{E_B}$ is along $\vec{OB}$

$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{OB} = \hat{i} + \hat{j} - \hat{k}$

$\vec{OA} \cdot \vec{OB} = 0$
$\Rightarrow \vec{OA} \perp \vec{OB}$
$\Rightarrow \vec{E_A} \perp \vec{E_B}$

$|\vec{E}| \propto \frac{1}{r^2}$

$|\vec{OC}| = 2|\vec{OB}| \Rightarrow |\vec{E_B}| = 4|\vec{E_C}|$

(22) $AC = 5\,m$

$V = \frac{Kq}{AC} = 1.8\,kV$

$V_B = (V_B)_{\text{due to } q} + (V_B)_i$

$1.8 \times 10^3 = \frac{Kq}{AB} + (V_B)_i = 2.25 \times 10^3 + (V_B)_i$

(23) $\frac{Kq_A}{R} + \frac{Kq_B}{2R} = 2V$ & $\frac{Kq_A}{2R} + \frac{Kq_B}{2R} = \frac{3V}{2}$

$\Rightarrow \boxed{q_A/q_B = \frac{1}{2}}$

$V_B = 0 \Rightarrow q'_B = -q'_A = -q_A$

After ~~changing~~ earthing

$$V_A - V_B = Kq_A\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{Kq_A}{2R}$$

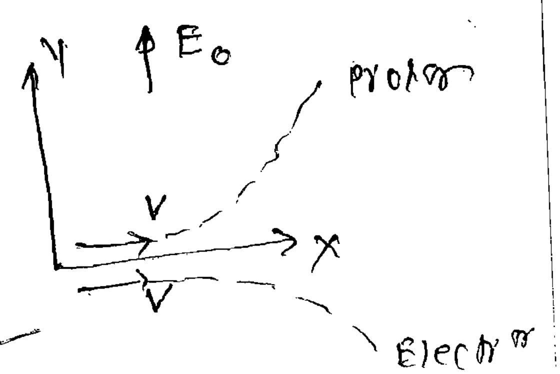
$$= V/2$$

$V_B = 0 \Rightarrow \boxed{V_A = V/2}$

(24) $\bar{E}$ must be continuous & defined

(25) use property of conductor & symmetry.

Exercise #3 Electrostatics



Passage-I

- Trajectory will be parabola.

$$Y = \frac{1}{2}\left(\frac{E_0 q}{m}\right)\left(\frac{x}{v}\right)^2$$

$$Y = \frac{1}{2}\cdot\left(\frac{E_0 q}{mv^2}\right)x^2 = \frac{E_0 q}{4K}\cdot x^2$$

if $K_1 = K_2$

Passage #2

- $F = F_{repulsion} = \dfrac{2K\varphi\cdot\varphi}{d^2} = \dfrac{2K\varphi^2}{d^2}$.

- $\frac{1}{2}(2m)v_0^2 = \frac{1}{2}\cdot 2m v^2 + \dfrac{2K\varphi^2}{d}$

$$v = \sqrt{v_0^2 - \frac{2K\varphi^2}{md}}$$

Impulse $= mv - mv_0$

- $\dfrac{2K\varphi^2}{d_{min}} = \frac{1}{2}\cdot 2m v_0^2 \Rightarrow d_{min} = \dfrac{2K\varphi^2}{m v_0^2}$

- $F_{2m} = \dfrac{K\cdot 2\varphi^2}{d_{min}^2} \Rightarrow a_{2m} = \dfrac{F_{2m}}{2m}$

- use energy conservation & linear momentum conservation

$$2mv_0 = (2m+m)V \quad \text{(I)}$$

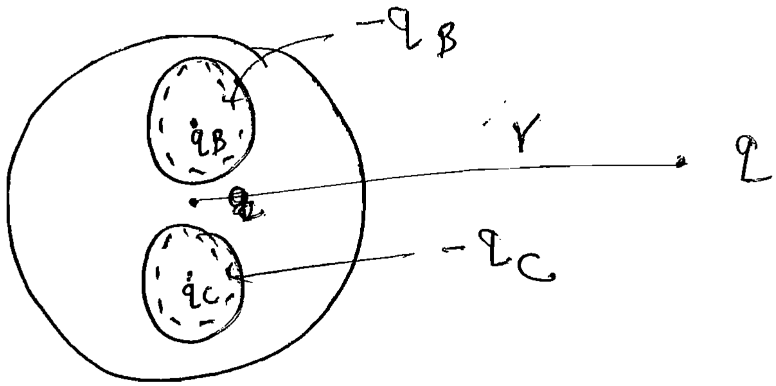
$$\frac{1}{2}\cdot 2mv_0^2 = \frac{1}{2}(2m+m)V^2 + \frac{K\cdot 2\varphi\cdot\varphi}{d_{min}} \quad \text{(II)}$$

solve for d_{min}.

Matching type problem

1 → use property of $\vec{E}$ in shell

2 →

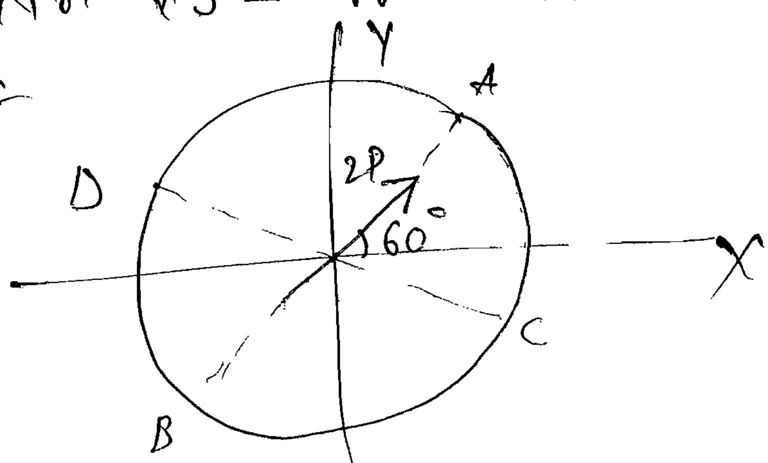


$\therefore F_q = \frac{K \cdot q (q_B + q_C)}{r^2}$

. use the concept of contribution of induced charge on $\vec{E}$ calculation i.e
$\vec{E}$ at any outside point of conducting shell due to all inside charges is zero and vice versa.

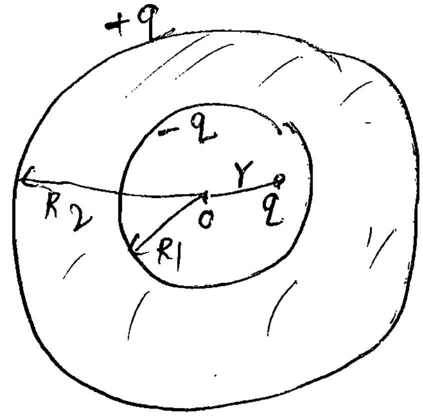
3 → Resultant dipole moment has magnitude $\sqrt{(\sqrt{3}p)^2 + p^2} = 2p$ at an angle $\theta = \tan^{-1}\sqrt{3} = 60^\circ$ with +ve x-axis

use property of dipole.

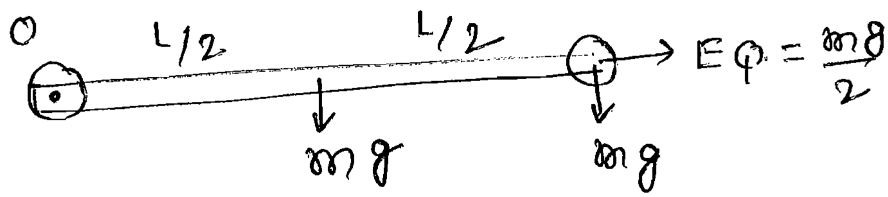


Passage #3

- $V_0 = \frac{Kq}{r} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$
- charge distribution on inner surface unknown.
- $V_{conductor} = \frac{Kq}{R_2}$.



Passage #4



- $\tau_0 = I_0 \alpha$

$$\alpha = \frac{\tau_0}{I_0} = \frac{mg \cdot \frac{L}{2} + mgL}{\frac{mL^2}{3} + mL^2} = \frac{9g}{8L}$$

$$a_t = \alpha r = \alpha L = 9g/8$$

- use energy conservation.

$$\frac{1}{2} I \omega^2 = mg\frac{L}{2} + mgL - \left(\frac{mg}{2}\right)L$$

$$\omega = \sqrt{\frac{3g}{2L}} \Rightarrow v = \omega L = \sqrt{\frac{3gL}{2}}$$

- $a = \sqrt{a_t^2 + a_c^2}$

(4) use property of conductor

(5) use property of conductor & charge induced on surfaces of conductor.

Exercise # 4 (Electrostatics)

(20)

$$U_{in} = \frac{q^2}{8\pi\epsilon_0 R_1} + \frac{q q_0}{4\pi\epsilon_0 R_1}$$

$$U_f = \frac{q^2}{8\pi\epsilon_0 R_2} + \frac{q q_0}{4\pi\epsilon_0 R_2}$$

$$W = -\Delta U = -(U_f - U_{in})$$

$$= \frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

(21) (a) $V_R = K\left[\frac{q}{R} - \frac{2q}{2R} + \frac{3q}{3R}\right] = \frac{Kq}{R}$

$V_{3R} = K\left[\frac{q}{3R} - \frac{2q}{3R} + \frac{3q}{3R}\right] = K \cdot \frac{2q}{3R}$

(b) using Gauss's Theorem

$\epsilon_0 \oint \vec{E} \cdot \vec{ds} = q_{enclosed} = -q$

$E\left[4\pi \left(\frac{5R}{2}\right)^2\right] = -q/\epsilon_0$

$E = -\frac{q^2}{4\pi\epsilon_0} \cdot \frac{1}{25R^2}$

(c) u = Energy density $= \frac{1}{2}\epsilon_0 E^2$

$U = \frac{K^2}{2}\epsilon_0 \left[\int_R^{2R} \frac{q^2}{r^4}(4\pi r^2)\,dr + \int_{2R}^{3R} (4\pi r^2)\,dV + \int_{3R}^{4R} \left(\frac{4q^2}{r^4}\right)(4\pi r^2)\,dV\right]$

$= \frac{Kq^2}{R}$

(22) (i) $\frac{1}{2}K x_{max}^2 = (qE)\,x_{max}$

$x_{max} = \frac{2qE}{K}$

(ii) In eqn. position $F_R = 0$

$Kx_0 = qE$

$x_0 = qE/K = \frac{1}{2}x_{max}$

(iii) $F = K(x + x_0) - qE = -Kx \Rightarrow$ SHM

(23) since $\frac{Kq^2}{L^2} > mg \Rightarrow$ string will not slack.

$$\frac{1}{2} m u_{min}^2 = mg(2L)$$

$$u_{min} = \sqrt{4gL} = 6 \text{ m/s}$$

(24) $V(x, y) = 0$

$$\frac{Kq}{[(x-b)^2+y^2]^{1/2}} + \frac{K(-cq/b)}{[(x-\frac{c^2}{b})^2+y^2]^{1/2}} = 0$$

solving: $\boxed{x^2+y^2 = c^2}$ circle

(25) $V_0 = (V_+) + (V_-)$

$$V_+ = \frac{3}{2} \cdot \frac{Q}{4\pi\epsilon_0 R} = \frac{3}{2} \cdot \frac{\rho \cdot (\frac{4}{3}\pi R^3)}{4\pi\epsilon_0 R} = \frac{\rho R^2}{2\epsilon_0}$$

$$V_- = \frac{-Q}{4\pi\epsilon_0 R/2} = \frac{-\rho \cdot \frac{4}{3}\pi (R/2)^3}{4\pi\epsilon_0 (R/2)} = -\frac{\rho R^2}{4\epsilon_0}$$

$$\boxed{V_0 = \rho R^2 / 4\epsilon_0}$$

$$V_{0_1} = (V_+) - (V_-)$$

$$V_+ = \frac{Q}{8\pi\epsilon_0 R^3}(3R^2 - r^2) = \frac{\rho \cdot (4/3\,\pi R^3)}{8\pi\epsilon_0 R^3}[3R^2 - (R/2)^2] = \frac{11\rho R^2}{24\epsilon_0}$$

$$V_- = -\frac{3}{2} \frac{Q}{4\pi\epsilon_0 (R/2)} = -\frac{3}{2} \cdot \frac{\rho \cdot \frac{4}{3}\pi (R/2)^3}{4\pi\epsilon_0 (R/2)} = -\frac{\rho R^2}{8\epsilon_0}$$

PHYSICS SOLUTIONS

Electrostatics Booklet **Exercise #2 (Subjective)**

1. By application of Gauss' Law, the net electrostatic flux associated with a closed surface and the net electrostatic charge enclosed within it are related by the expression $\oint \vec{E}.d\vec{S} = \frac{q_{enc}}{\varepsilon_0}$

Therefore when the position of a charged particle is moved from the geometric center to some asymmetric point within the same spherical Gaussian surface, the LHS of the above equation is unchanged, therefore the RHS which represents the total flux through it remains **UNCHANGED**.

2. From the definition of Electric Potential at a given point, $V = \sum\left(\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}\right)$, where q_i, r_i are the charges and respective distances of charges from the given point in the system, the presence of a proton (positively charged) additionally in a system will INCREASE the Electric Potential at every point in the vicinity of it.

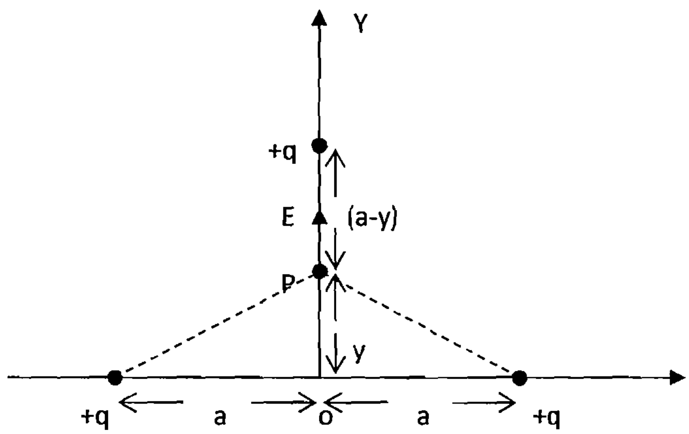
When a proton (of charge +e) is released inside a uniform Electric Field $\vec{E}$, the workdone by the constant electrostatic force $\vec{F} = e\vec{E}$ acting on it is given by $W = \int e\vec{E}.d\vec{r} = eEs$ where s is the displacement of the proton and can be shown to be equal to $W = \frac{1}{2}mv^2$ where v and m are instantaneous speed and mass of the proton, by application of Work-Energy theorem. Therefore, the workdone by the Electrostatic Force (internal conservative force considering the proton and the uniform Elec field to be a 'system') is positive for any given interval.

Therefore, by definition, the change in Electrostatic Potential Energy $\Delta U = -W_{\text{int}\,C} = -\int e\vec{E}.d\vec{r} = -\frac{1}{2}mv^2$ Or simply stated the Electrostatic Potential Energy DECREASES.

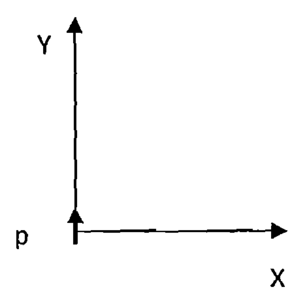
3. The electric field at a point on the 'Axial' line for an electric dipole at a distance of 'r' from the center of the dipole is given by the expression $E \approx \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ when r is very large compared to the geometrical size of the dipole. Hence when the distance is doubled, the Electric Field will reduce by a factor of (1/8), therefore the force acting on the particle at the new position $F' = \frac{F}{8}$

4. (a) Assuming the geometrical size of an electric dipole to be 'small', when placed in a non-uniform Electric filed $\vec{E}$, the net force acting on it can be shown to be $\vec{F} = \vec{\nabla}E.\vec{p}$, where $\vec{\nabla}E = \frac{\delta E}{\delta x}\hat{i} + \frac{\delta E}{\delta x}\hat{j} + \frac{\delta E}{\delta z}\hat{k}$ is the 'gradient' of the Electric field $\vec{E}$ and $\vec{p}$ is the electric dipole moment . Now the above equation can be simplified

to $F = \frac{dE}{dx} \times p$ for a one-dimensional situation where $\vec{E}$ and $\vec{p}$ are both along the same direction (X-axis). This is the case in the given problem. Let us consider the dipole to be along the Y-axis and it's bottom to be the origin. From symmetry, the electric field at any point on the y-axis due to the three charged particles q_1, q_2 and q_3 will also be along the Y-axis. To calculate the same consider the following diagram



The electric Potential at the point P (0,y) is given by $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{\sqrt{a^2+y^2}} + \frac{q}{(a-y)}\right]$ Therefore the Electric Filed at P, $E = -\frac{dV}{dy} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2qy}{\left(a^2+y^2\right)^{3/2}} - \frac{q}{(a-y)^2}\right]$ (along the Y-axis) as shown. Now, for a small Electric dipole placed at 'O'



The net Force acting on it $F = \frac{dE}{dy} \times p = \frac{1}{4\pi\varepsilon_0}\left[2q\frac{\left(a^2+y^2\right)^{3/2} - 3y^2\left(a^2+y^2\right)^{1/2}}{\left(a^2+y^2\right)^3} - q\frac{2}{(a-y)^3}\right] \times p$,

also since the 'position' of the dipole is y=0, substituting in the above equation,

$$F == \frac{1}{4\pi\varepsilon_0}\left[2q\frac{\left(a^2\right)^{3/2}}{\left(a^2\right)^3} - q\frac{2}{(a)^3}\right] \times p = 0$$

(b) Electrostatic Potential Energyof the system $U = U_{q_1,q_2} + U_{q_1,q_3} + U_{q_2,q_3} + U_{p,(q_1+q_2+q_3)}$ i.e the total energy being the sum of the Potential energies for all 'pairs' of components in the given system.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a\sqrt{2}} + \frac{q^2}{2a} + \frac{q^2}{a\sqrt{2}}\right] + \frac{1}{4\pi\varepsilon_0}\left[0 + \frac{pq}{a^2} + 0\right]$$

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a}\left(\sqrt{2}+2\right) + \frac{pq}{a^2}\right]$$

5. Workdone to remove each of them to infinity is $W = \Delta U = 0 - U$ where U is the Potential Energy of the system in the given configuration whereas when removed to infinity the PE will be 0.

$$U = \sum\left[\frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[\left(\frac{-1}{1}+\frac{-1}{1}+\frac{-1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-1}{1}+\frac{-1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-1}{1}+\frac{-1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+..\right.$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[\left(\frac{-3}{1}+\frac{3}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-2}{1}+\frac{3}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-2}{1}+\frac{2}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-1}{1}+\frac{2}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-2}{1}+\frac{1}{\sqrt{2}}\right)+\left(\frac{-1}{1}+\frac{1}{\sqrt{2}}\right)+\left(\frac{-1}{1}\right)\right.$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[\left(-1\times 12\right)+\left(\frac{1}{\sqrt{2}}\times 12\right)+\left(\frac{-1}{\sqrt{3}}\times 4\right)\right]$$

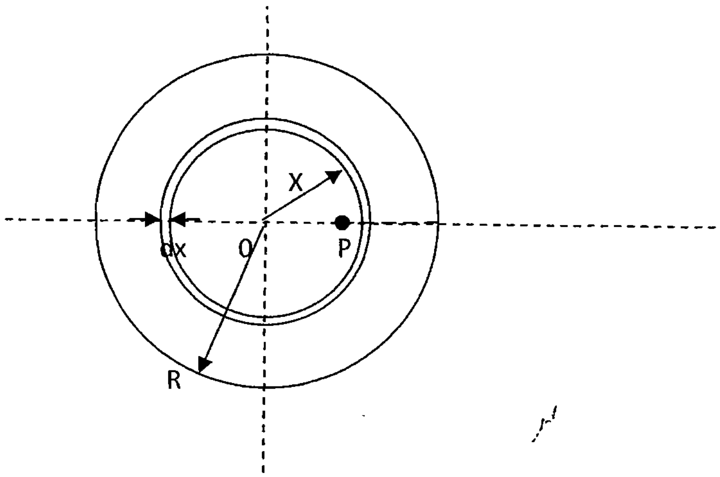
Note: The above expression can be most simply deduced from the fact that the given cube has 12 sides, 12 face diagonals and 4 body diagonals (geometrically) and any side chosen has opposite sigh charges at both ends, any face diagonal chosen has same sign charges and any body diagonal opposite charges (from the charge distribution)

Therefore, $U = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{a}\left[3\times\left(\frac{1}{\sqrt{2}}-1\right)-\left(\frac{1}{\sqrt{3}}\right)\right]$, $W = -U = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{a}\left[3\times\left(1-\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{3}}\right)\right]$

6. If the particle is projected with a speed v, the condition for it to penetrate to a point P at $r = (R/2) = 20cms$ from the center is given by $\frac{1}{2}mv^2 \geq +q(V_P - V_A)$ where V_A and V_P are the electric potentials at the initial point of projection A (r>R) and at P. (For minimum value of v take limiting condition in the inequality)

$V_A \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (since A is a point 'external' to the dielectric shell)

Whereas for the Point P which is 'inside' the shell, potential can be calculated by integrating over differential spherical shells of radii 'x' and thickness 'dx' ($0 < x < R$)



The distance OP= (R/2). Now, for a shell with x<(R/2), the potential

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R/2)}, \text{ where } dq = \rho \times 4\pi x^2 dx \text{ and } \rho = \frac{3Q}{4\pi R^3} \text{ or } dq = \frac{3Q}{R^3}x^2 dx$$

Whereas for a shell with (R/2) <x<R, the potential

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x} \text{ where. } dq = \frac{3Q}{R^3}x^2 dx$$

Therefore $V_P = \frac{1}{4\pi\varepsilon_0}\int_{x=0}^{x=R/2}\frac{dq}{(R/2)} + \frac{1}{4\pi\varepsilon_0}\int_{x=R/2}^{x=R}\frac{dq}{x}$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{6Q}{R^4}\int_{x=0}^{x=R/2} x^2 dx + \frac{1}{4\pi\varepsilon_0}\frac{3Q}{R^3}\int_{x=R/2}^{x=R} x dx$$

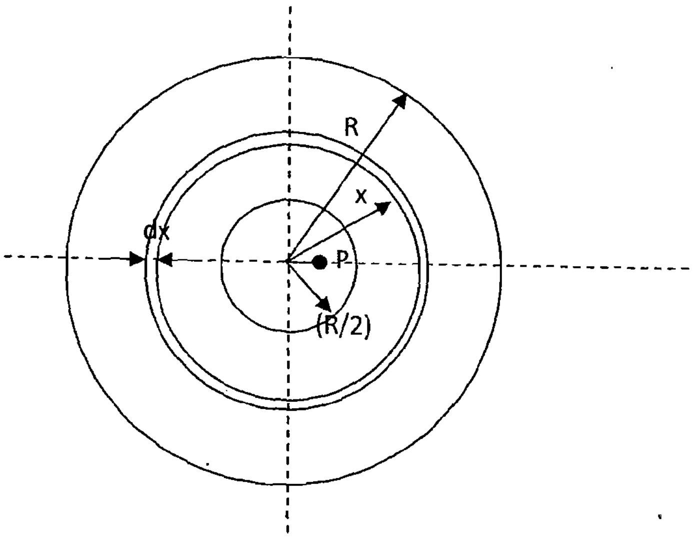
$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[\frac{6}{3}\left(\frac{1}{2^3} - 0\right) + \frac{3}{2}\left(1 - \frac{1}{2^2}\right)\right]$$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[\frac{11}{8}\right]$$

Therefore from the equation $\frac{1}{2}mv^2 \geq +q(V_P - V_A)$

$$v = \sqrt{\frac{1}{2\pi\varepsilon_0}\frac{Qq}{m}\left[\frac{11}{8R} - \frac{1}{r}\right]}$$

7.



The **Electric Field** at the point P will be **ZERO** (By application of Gauss's Law and spherical symmetry)

To calculate the Potential at the point 'P' as shown, consider differential spherical shells of radii 'x' and thickness 'dx' ($(R/2) < x < R$), potential due to such a shell at P,

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x}$$, where $dq = \rho \times 4\pi x^2 dx$ and $\rho = \dfrac{3Q}{4\pi\left(R^3 - \dfrac{R^3}{8}\right)}$ or $dq = \dfrac{24Q}{7R^3}x^2 dx$

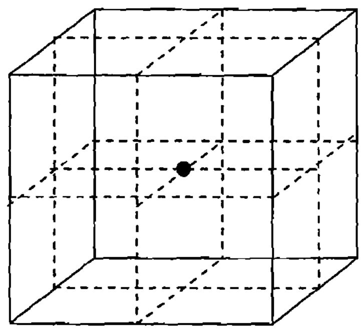
$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\int_{x=R/2}^{x=R}\frac{dq}{x}$$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{24Q}{7R^3}\int_{x=R/2}^{x=R} x\,dx$$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{24Q}{7R}\left[\frac{1}{2}\left(1 - \frac{1}{2^2}\right)\right] = \frac{1}{8\pi\varepsilon_0}\cdot\frac{24Q}{7R}\cdot\frac{1}{2}\cdot\frac{2}{4}$$

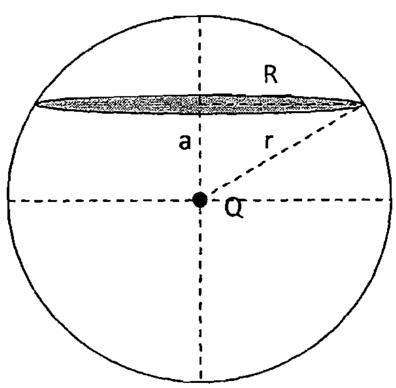
$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{9Q}{7R}\right)$$

8. The flux here can be calculated by application of superposition and symmetry in Gauss's Law. Consider a cubical Gaussian surface of dimension ($2L \times 2L \times 2L$), with the particle +q at the geometrical center as shown



Now, the total flux through the entire cubical Gaussian surface is $\oint \vec{E}.d\vec{S} = \frac{q_{enc}}{\varepsilon_0}$ with the flux through each of the sides of dimensions ($2L \times 2L$) being equal (by symmetry) and therefore equivalent to $\frac{q_{enc}}{6\varepsilon_0}$, now further subdividing each side into four symmetrical squares of dimensions ($L \times L$), the flux through each again being equal (by symmetry) and therefore equivalent to $\frac{1}{4} \times \frac{q_{enc}}{6\varepsilon_0} = \frac{q}{24\varepsilon_0}$

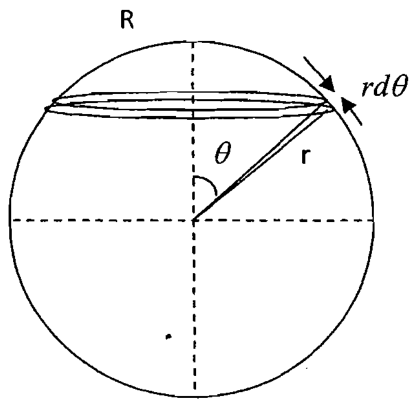
9.



The total flux associated with a spherical Gaussian surface of radius $r = \sqrt{R^2 + a^2}$ as shown in the figure is $\oint \vec{E}.d\vec{S} = \frac{Q}{\varepsilon_0}$ Now, this flux is distributed symmetrically in 3D, therefore the fraction of it passing through the shaded disc shown will be proportionate to the solid angle subtended by the disc at the center (where the charge Q is located). Therefore if this is (1/4th) of the total flux, the solid angle subtended by the portion of the spherical surface 'sliced' out by the disc must be π (total solid angle for a closed surface being 4 π steradians). Hence the area of the portion of the spherical surface

$$S = \pi r^2 \text{ (solid angle } = \frac{S}{r^2})$$

Therefore from the figure below



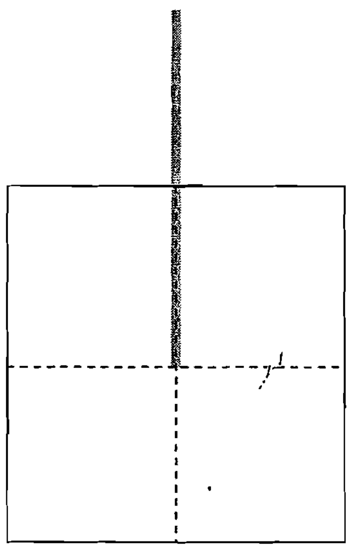
Taking an elemental ring of radius $rSin\theta$ and thickness $rd\theta$, the area of the sliced portion,

$$S = \int_{\theta=0}^{\theta=Cos^{-1}(a/r)} 2\pi r^2 Sin\theta d\theta = \pi r^2$$

$$\Rightarrow \left[1 - \frac{a}{r}\right] = \frac{1}{2} \Rightarrow r = 2a \Rightarrow \sqrt{a^2 + R^2} = 2a$$

$$\Rightarrow R = a\sqrt{3}$$

10. For the given situation, in order to minimize the flux, the portion of the rod enclosed within the cubical surface has to be minimized which will be for the config shown below giving Flux $\oint \vec{E}.d\vec{S} = \frac{q_{enc}}{\varepsilon_0} = \frac{Q}{2\varepsilon_0}$



11. The Electric Field at E at a any point on the axis of the disc shaped hole which is at a distance 'x' from the sheet can be calculated by application of superposition, $E = E_{InfiniteSheet} - E_{Disc}$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right) = \frac{\sigma}{2\varepsilon_0}\frac{x}{\sqrt{R^2 + x^2}}$$

Therefore the potential difference between the two points P ($x = R\sqrt{3}$) and O (x=0) can be calculated from the relation $\Delta V = -\int \vec{E}.d\vec{r}$

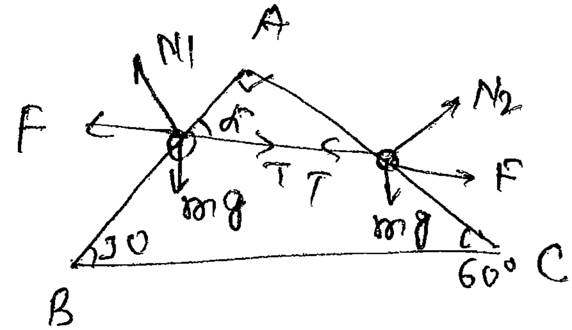
$$\Delta V = -\int_{x=R\sqrt{3}}^{x=0} \frac{\sigma}{2\varepsilon_0}\frac{x}{\sqrt{R^2 + x^2}}dx = \left| -\frac{\sigma}{2\varepsilon_0}\sqrt{R^2 + x^2}\right|_{x=R\sqrt{3}}^{x=R} = \frac{\sigma R}{2\varepsilon_0}.$$

Now, by application of Work Energy to the electron released at P,

$$\frac{1}{2}mv^2 = e\Delta V = \frac{e\sigma R}{2\varepsilon_0}$$

$$\Rightarrow v = \sqrt{\frac{e\sigma R}{m\varepsilon_0}}$$

(12) (a)



(i) $T\cos\alpha = F\cos\alpha + mg\sin 30°$

(ii) $F\sin\alpha + N_1 = mg\cos 30° + T\sin\alpha$

(iii) $T\sin\alpha = F\sin\alpha + mg\cos 30°$

(iv) $N_2 + F\cos\alpha = T\cos\alpha + mg\cos 60°$

solving $\Rightarrow$ $\boxed{\alpha = 60°}$

$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{l^2}$

$\Rightarrow T = mg + \frac{q_1 q_2}{4\pi\epsilon_0 l^2}$

$N_1 = \sqrt{3}\, mg$, $N_2 = mg$

(b) $T = 0$

$\boxed{F = -mg}$

$q_1 q_2 = (4\pi\epsilon_0 l^2)\, mg$.

12. use concept of equilibrium by drawing FBD.

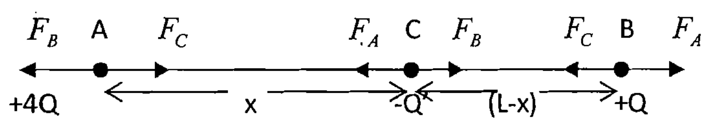
13. Consider a section of the rod of differential length 'dy' at a distance 'y' above the surface of the ring. The force experienced by this differential element would be $dF = dqE$, where $dq = \lambda dy$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Qy}{\left(R^2+y^2\right)^{3/2}}$ is the Electric Field due to the charged ring at the location of the diff' element.

$$\Rightarrow F = \int_{y=0}^{y\to\infty} \frac{1}{4\pi\varepsilon_0}\frac{\lambda Q y\,dy}{\left(R^2+y^2\right)^{3/2}}$$

$$\Rightarrow F = \frac{1}{4\pi\varepsilon_0}\lambda Q\left[\frac{-1}{\sqrt{R^2+y^2}}\right]_{y=0}^{y\to\infty}$$

$$\Rightarrow F = \frac{\lambda Q}{4\pi\varepsilon_0 R}$$

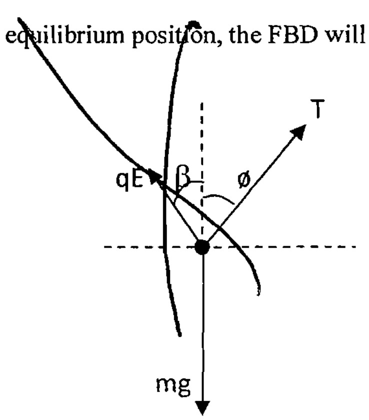
14. Since all the three charges will have to be in equilibrium, the third charge has to be a negative one placed somewhere between the other two positive charges as shown in the figure below



Balancing forces,

$$\frac{4Q^2}{L^2} = \frac{4QQ'}{x^2} \text{ and } \frac{4QQ'}{x^2} = \frac{QQ'}{(L-x)^2} \text{ solving these, } -Q' = -\frac{4}{9}Q \text{ and } x = \frac{2}{3}L \text{ (distance from +4Q)}$$

15. At the equilibrium position, the FBD will be as follows



Therefore at equilibrium $\vec{T} = m\vec{g} + q\vec{E}$ (T: Tension in the string, E is Electric Field)

$$\Rightarrow |\vec{T}| = \sqrt{(mg)^2 + (qE)^2 - 2(mg)(qE)\text{Cos}\beta} = mg'$$ where the time period can now be expressed as

$$t = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{ml}{\sqrt{(mg)^2 + (qE)^2 - 2(mg)(qE)\text{Cos}\beta}}}$$

and the angle ø is given by $\phi = \text{Tan}^{-1}\left(\frac{qE\text{Sin}\beta}{mg - qE\text{Cos}\beta}\right)$

15

16. The force acting on the particle is given by

$\vec{F} = -q\vec{E}$ where E is the electric field due to the positively charged ring given by $E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{3/2}}$

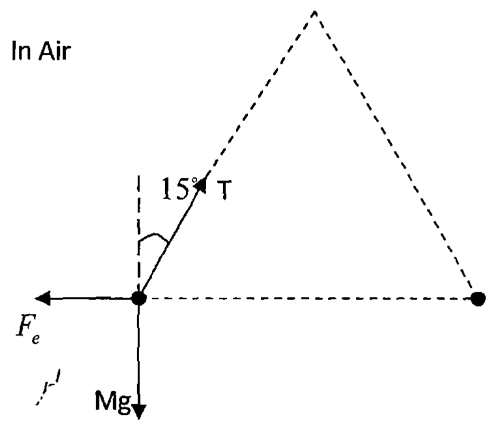
where x is the distance from the center of the ring and the direction of the field is directly away from the center along the axis.

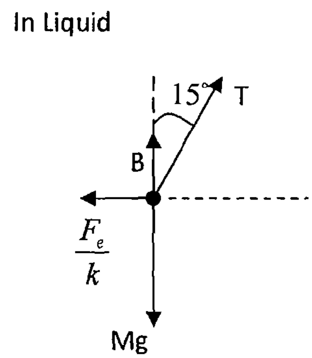
Since the particle is negatively charged, it experiences a force $F = \frac{1}{4\pi\varepsilon_0}\frac{qQx}{(R^2 + x^2)^{3/2}} \approx \frac{1}{4\pi\varepsilon_0}\frac{qQx}{R^3}$ for x<< R

directed towards the origin (center of the ring). Hence if released from such a position, it will experience an acceleration $a = -\frac{d^2x}{dt^2} \approx \frac{1}{4\pi\varepsilon_0}\frac{qQx}{mR^3}$ (for x<<R). Comparing with the standard equation of SHM

$\frac{d^2x}{dt^2} = -\omega^2 x$, time period $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{qQ}}$

17. The FBDs in 'air' and inside the liquid are as follows





From these it is evident that if the angle remains the same 15°, B= Buoyant Force = Wt of liquid displaced, F_e: Electrostatic force and k is the dielectric constant for the medium,

$\frac{F_e}{Mg} = \frac{(F_e / k)}{(Mg - B)}$ and also for a completely submerged object $B = \frac{Mg}{(\rho_m / \rho_l)}$ where ρ_m and ρ_l are densities of the material of the object and the liquid respectively.

$$k = \frac{(Mg)}{(Mg - B)} = \frac{1}{1 - (\rho_l / \rho_m)} = 2$$

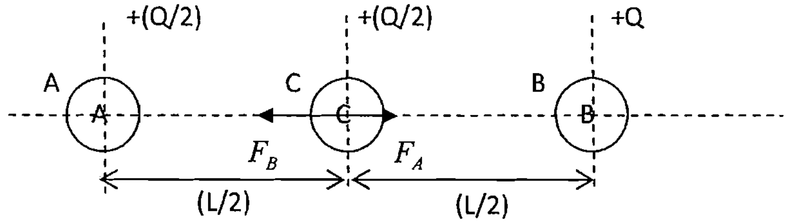
18. As is evident from the question, the second particle might be either attracted or repulsed by the first due to electrostatic forces such that the maximum value of this Electric Force is equal to the limiting value of static friction $F_e \le \mu mg \Rightarrow \frac{1}{4\pi\varepsilon_0} \frac{q \times 1.0 \times 10^{-6}}{(0.1)^2} \le 0.1 \times 80 \times 10^{-3} \times 10$

$\Rightarrow |q| \le 8.7 \times 10^{-8} C$. Therefore the charge q can range between $-(8.7 \times 10^{-8})C$ and $+(8.7 \times 10^{-8})C$

19. Let the charge on spheres A and B be '+Q' each and the distance between them is 'L'

Therefore the electrostatic repulsion $F = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{L^2} = 2 \times 10^{-5} N$

Now, when a third sphere C is made to touch C and placed at the mid-point, the charge on A gets 'shared' equally between them (they have identical capacitance)



Therefore, the net force on C,

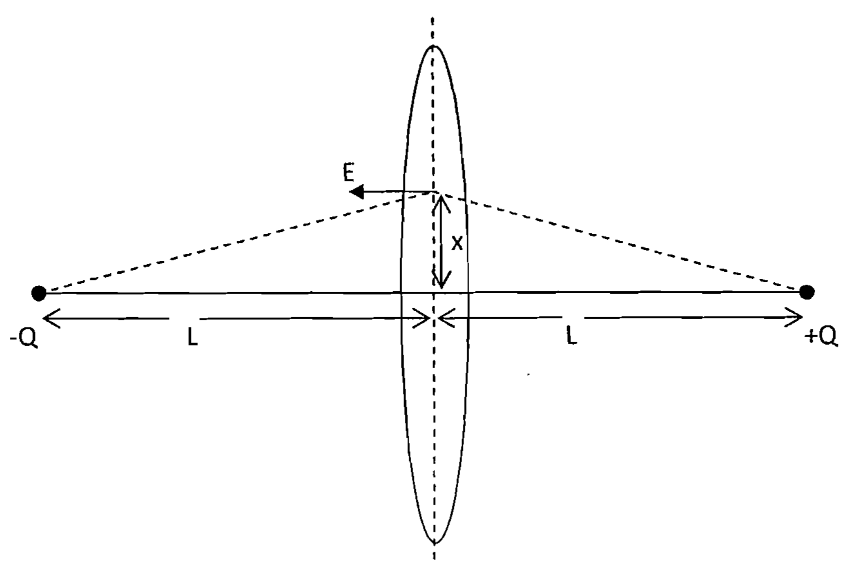
$$F = F_B - F_A = \frac{1}{4\pi\varepsilon_0}\frac{Q\times(Q/2)}{(L/2)^2} - \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)^2}{(L/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{L^2} = 2\times10^{-5}N$$

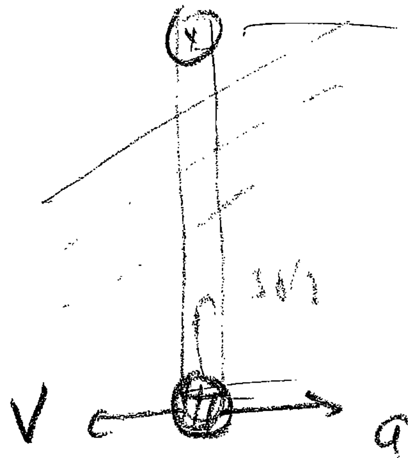
20. The electric field at any point on the disc at a distance of 'r' from the center can be calculated from the formula (E at an equatorial point for an electric dipole)

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q\times 2L}{(r^2+L^2)^{3/2}}$ with direction as shown in the figure (normal to the plane of the disc). Therefore the electric flux through a differential 'ring' sliced out on the disc of radius 'r' and thickness 'dr' would be

$d\phi_E = E\times dS = \frac{1}{4\pi\varepsilon_0}\frac{2QL}{(r^2+L^2)^{3/2}}\times 2\pi r dr = \frac{QL}{\varepsilon_0}\frac{rdr}{(r^2+L^2)^{3/2}}$, therefore the total flux through the

disc $\phi_E = \frac{QL}{\varepsilon_0}\int_{r=0}^{r=R}\frac{rdr}{(r^2+L^2)^{3/2}} = \frac{QL}{\varepsilon_0}\left[\frac{-1}{\sqrt{(r^2+L^2)}}\right]_{r=0}^{r=R} = \frac{Q}{\varepsilon_0}\left(1-\frac{L}{\sqrt{R^2+L^2}}\right)$





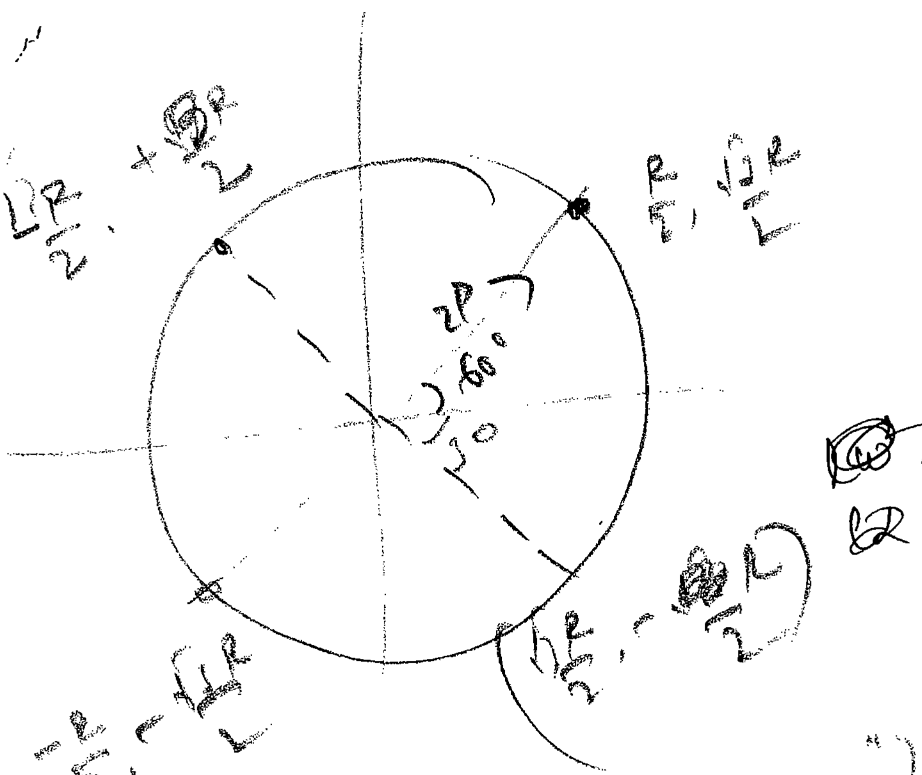
$v^2 = \frac{3g}{2}$

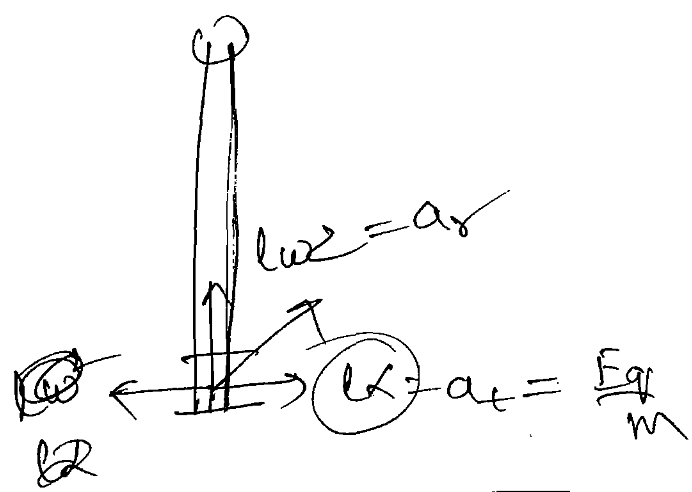
$a_t = \frac{Eq}{m} = g/2$

$\sqrt{\left(\frac{3g}{2}\right)^2 + \left(\frac{g}{2}\right)^2}$

$= \sqrt{\frac{9+1}{4}}$

$\frac{g}{2}\sqrt{5}$





$\sqrt{\left(\frac{g}{2}\right)^2 + (l\omega^2)^2}$

$\bar{p} = 2p\left(\frac{\sqrt{3}}{2}\ \frac{1}{2}\right)$

$= 2p(\cos 30° i + \sin 30° j)$

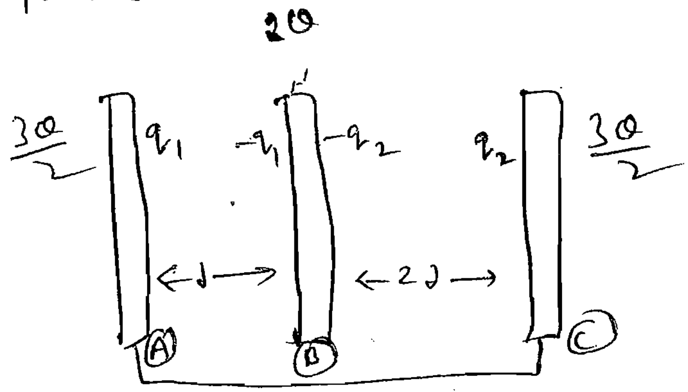
capictance Ex#1

Capacitance Ex-1

Q. 20.

Final

(1)



$$\frac{3Q}{2} + \frac{3Q}{2} + q_1 + q_2 = Q.$$

$$\boxed{q_1 + q_2 = -2Q}$$

$$V_A = V_C$$

$$\therefore V_A - V_B = V_C - V_B$$

$$\Rightarrow \left(\frac{q_1}{\varepsilon_0 A}\right) d = \left(\frac{q_2}{\varepsilon_0 A}\right) 2d$$

$$\boxed{q_1 = 2q_2}$$

$$q_1 + \frac{q_1}{2} = -2Q$$

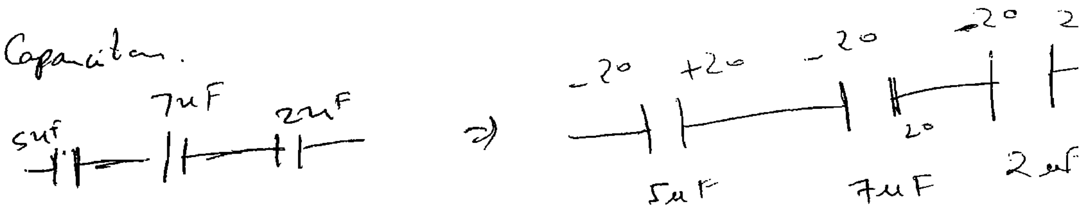
Plate ...

Charge flown

$= Q - \frac{Q}{6} = \frac{5Q}{6}$

(2) 2

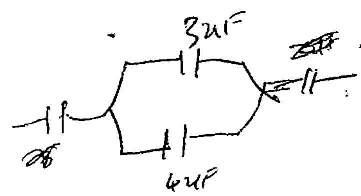
Eq. Capacitor.



Charge on 7uF = 20uC.

$\therefore V = \frac{Q}{C} = \frac{20uC}{7uF}$ (V across 7uF)

Now



V is same,

$\therefore$ Charge on 3uF

$Q = CV$

$= 3uF \times \frac{20uC}{7uF}$

$= \frac{60}{7}$ uC

$= 8.57$ uC.

On right plate,

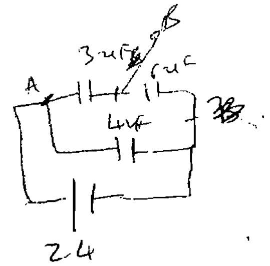
$Q = +8.57$ uC.

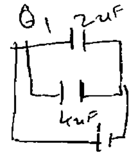
Ans → (A)

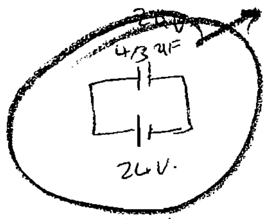
(3) 5uF is short-circuited, ∴

(D)

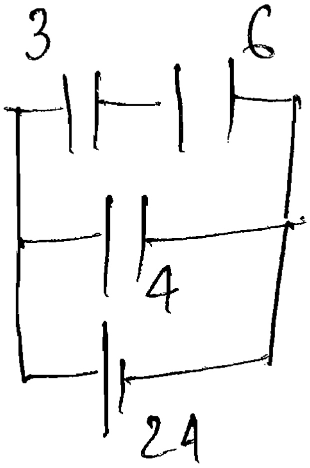
4. Eq. Circuit =







(4)



6

$V_6 = 24 \times \frac{3}{9} = 8$

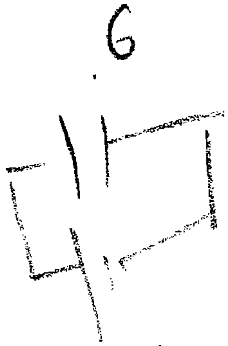
$U = \frac{1}{2} C V^2$

$= 32$

→ ~~Total Q = CV~~

~~= 24 × 4/3 uF~~

~~= 32uC.~~



$Q_1 = CV$

$= 2uF \times 24V$

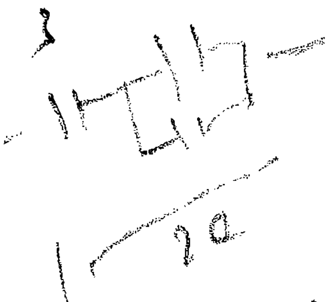
$= 48uC.$

P.D across A.B $= \frac{48uC}{3uF}$

$= 16V.$

∴ P.D across 1uF capacitor $= 8V.$

16 ... = 8



$\frac{1}{2} \cdot 1 \cdot 64$

$Q = CV$

$= 1uF \times 8.$

$= 8uC.$

∴ Energy $= \frac{Q^2}{2C}$

$= \frac{64}{2 \times (1)}$

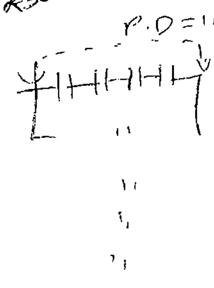
$= 32uJ$ ✓

(c)

5. Each capacitor — 8 µF, 250V.

(5)

P.D = 1000V.



1000V will be produced by 4 in series (250 each)

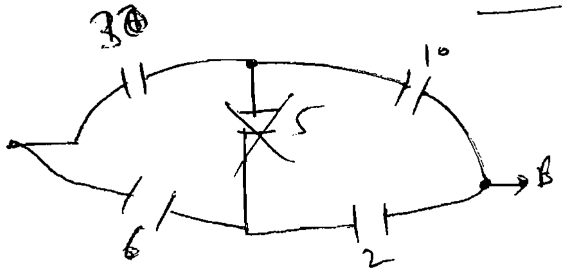
But then, series → 2µF ∴

So, 8 in parallel rows.

∴ 8 × 4 = 32.

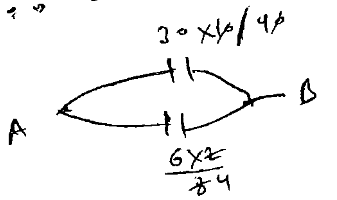
∴ 8 × 4 = 32.

(6)



Balanced wheat stone.

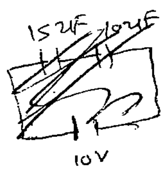
∴ 5 can be neglected

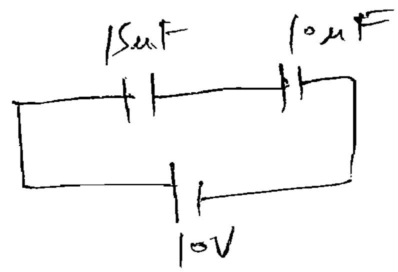


= 9 µF.

(7). (D) System is not changing in any way.

(8) Eq. Capacitance





$= \frac{15 \times 10}{15+10}$ µF

(B) = 6 µF

(9) Total Q = CV

= ~~6µF × 10~~ = 6 × 10 µC = 60 µC divided in 5 µF and 10 µF

= ~~60 µC~~

~~[illegible]~~

Q = CV

V same in 5µF and 10µ

So charge divided in ratio of capacitance

∴ 1 : 2

$\frac{60}{15} = 4V \rightarrow$ P.D.

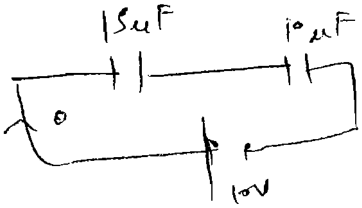
∴ Q = CV

= 5µF × 4

= ~~20µC~~ 20 µC.

10 (10) P.D across 5uF is 4V.

∴ across 6uF is 6V.



(A)

Charge flows same

$Q = CV$ So Potential divides

inverse of capacitance

15 uF → 4 V

10 uF → 6 V → same in 6 uF

---

(11) P-D across AB is 4V, BC is 6V.

(11)

∴ Energy of 10uF $= \frac{1}{2}CV^2$

$= 5u \times (4)^2$

$= 80$

Energy of 5uF $= 40$

Energy of 4uF $= \frac{1}{2}(4)(6)^2$

$= 72$

Energy of 6uF $= \frac{1}{2}(6)(6)^2$

$= 108.$

(B) ∴ Max. Energy is of 6 uF Capacitor.

12. $W = \frac{Q^2}{2C}$

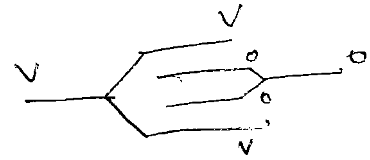
$= \frac{64 \times 10^{-36}}{2 \times 100 \times 10^{-6}}$

$= 32 \times 10^{-32}$

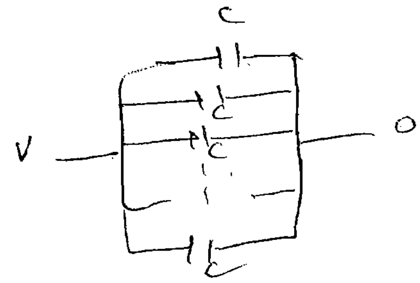
(b)

(12) It is effectively two Capacitor of Capacitance $\frac{\varepsilon_0 A}{d}$ in parallel arrangement.

∴ Net Capacitance $= \frac{2\varepsilon_0 A}{d}$



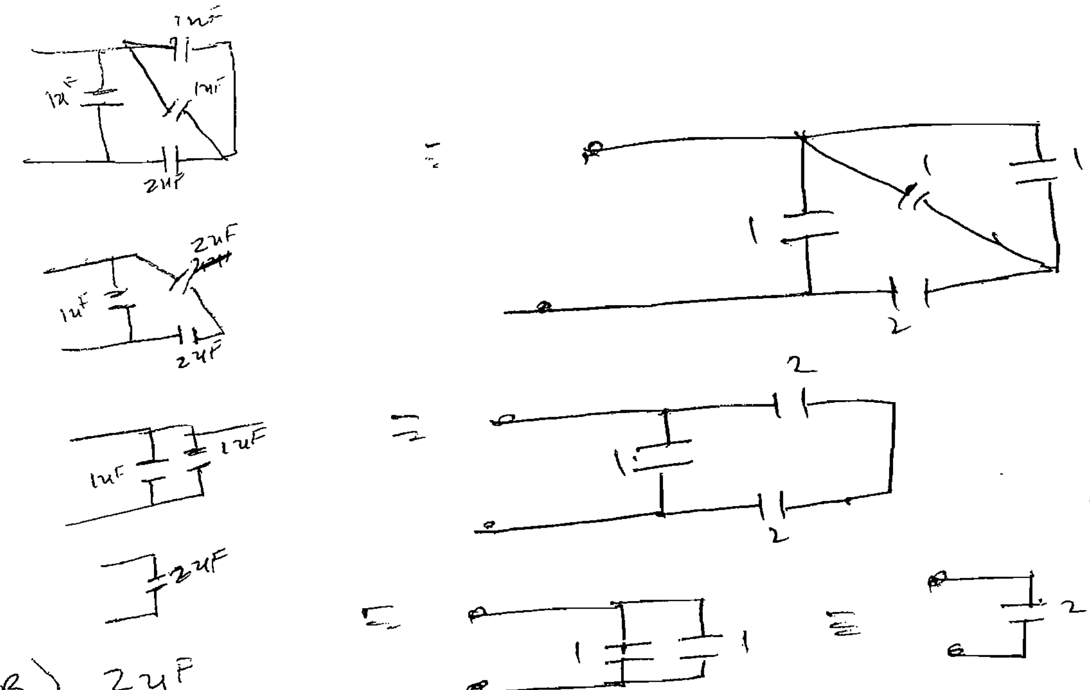
(13) Energy in each Capacitor $= \frac{1}{2}CV^2$



$\therefore$ Total $= \frac{1}{2}nCV^2$

(D)

(14)



(B) 2uF

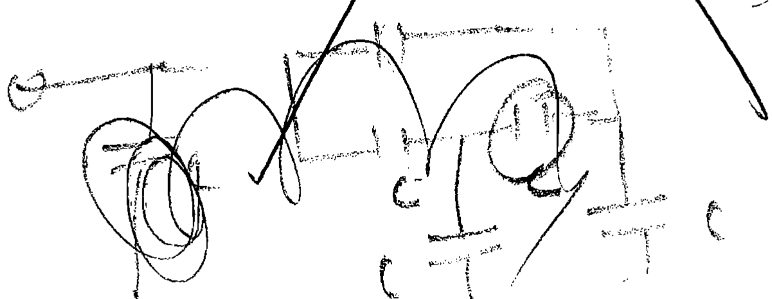
(15) Effectively, There are four capacitors in series, and the resultant is ~~parallel with the other~~

$\therefore$ net $= \frac{C}{4}$.

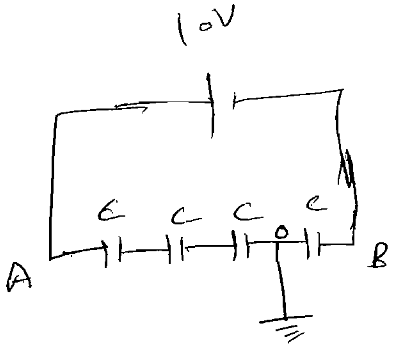
$\therefore$ P.D across each C is $-2.5V$.

$\therefore$ at B its $-2.5V$.

(B) at A ; P.D $= -2.5V + 10V$
$= 7.5V$.



(15)



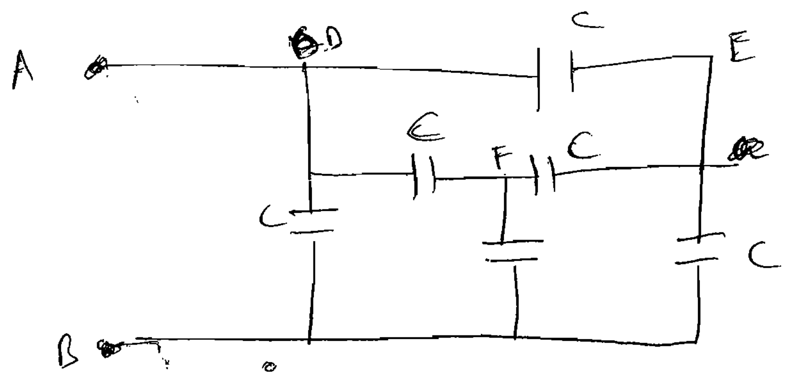
Potential will be equally divided among 4 capacitors in series such that sum = 0.

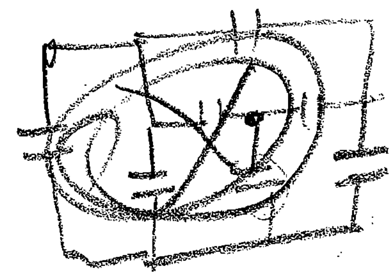
$\therefore$ B → −2.5 V

A → 7.5 V

(because potential across each capacitor = 2.5 V)

(16)





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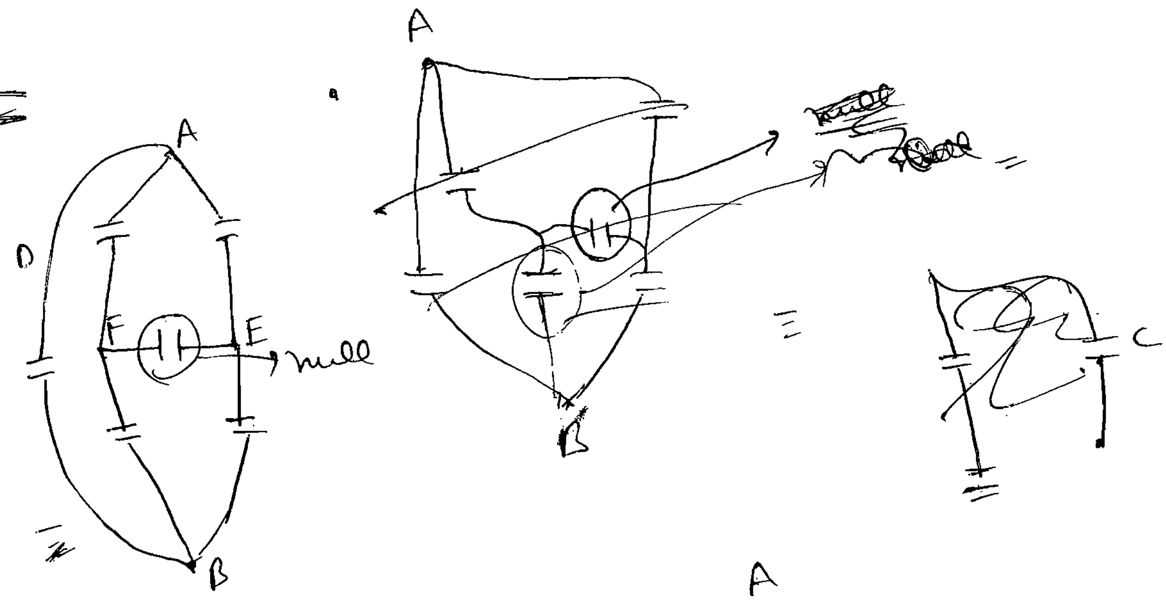

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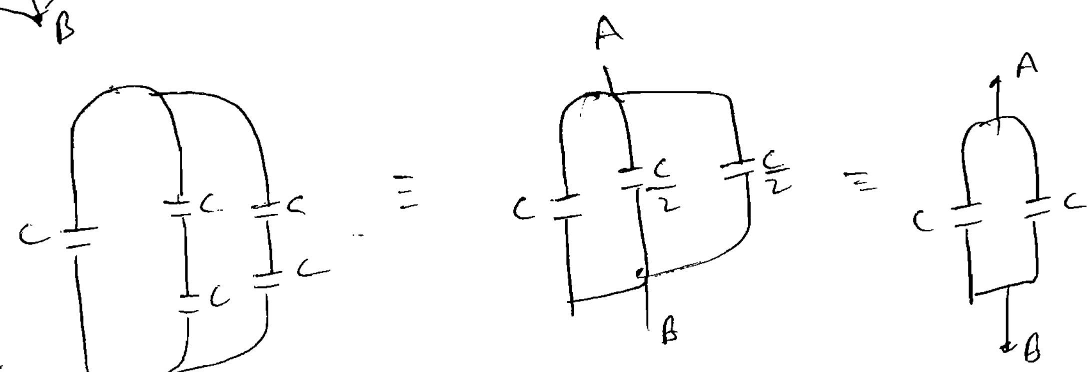

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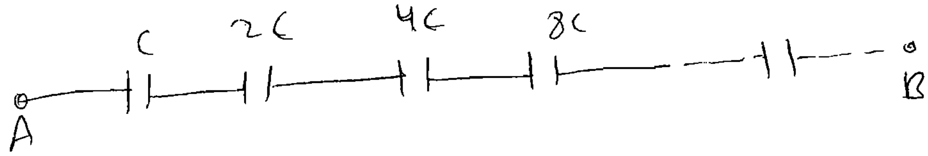
17



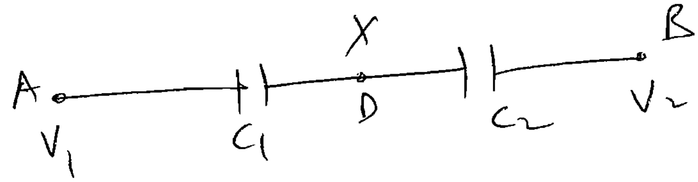
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{4C} + \frac{1}{8C} \text{ ---- } \infty$$

$$= \frac{1}{C}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \text{ --- } \infty\right]$$

$$\frac{1}{C_{eq}} = \frac{1}{C}[2]$$

$$C_{eq} = \frac{C}{2}$$

18



Let $V_D = X$

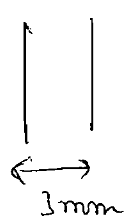
Then $(X - V_1)C_1 + (X - V_2)C_2 = 0$ (K.J.L.)

$\Rightarrow XC_1 - V_1C_1 + XC_2 - V_2C_2 = 0$

$\Rightarrow X(C_1 + C_2) = V_1C_1 + V_2C_2$

$\therefore X = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

19

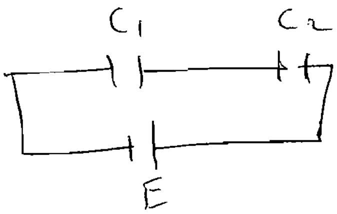


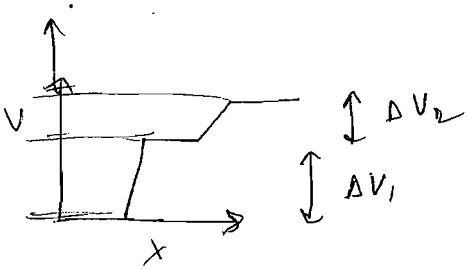
$\Delta V = 3 \times 10^5$ V.

$e\Delta V = 3 \times 10^5$ eV

(21)

(20)





q will be same (series)

$$\left(\frac{dV}{dx}\right)_{C_1} > \left(\frac{dV}{dx}\right)_{C_2}$$

$\Delta V_1 > \Delta V_2$

$\therefore \; E_{C_1} > E_{C_2}$.

$C_1 = \frac{q}{\Delta V_1}$

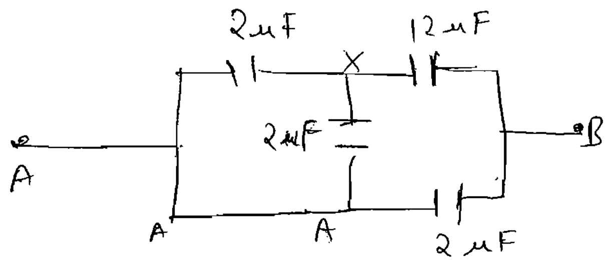
$C_2 = \frac{q}{\Delta V_2}$

$\Delta V_1 > \Delta V_2$

$\therefore \; C_1 < C_2$

(22)

(21)



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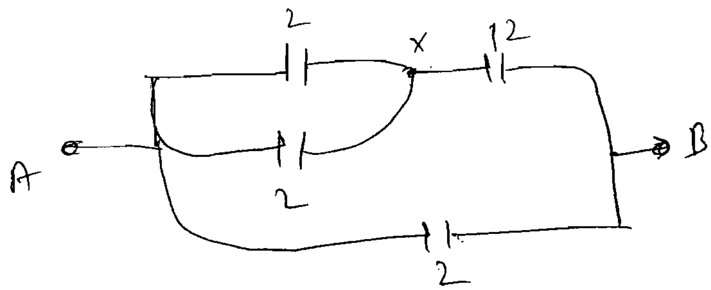

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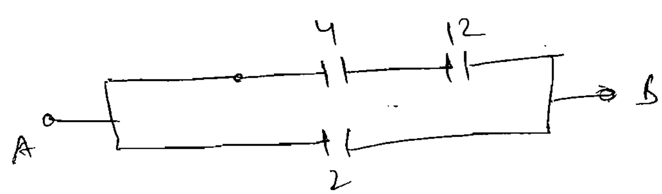
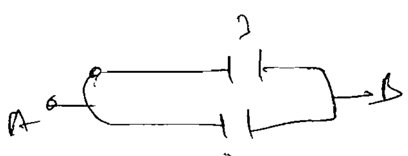
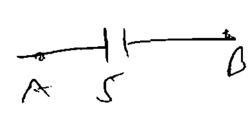


$\frac{4 \times 12}{16} = 3$

$=$

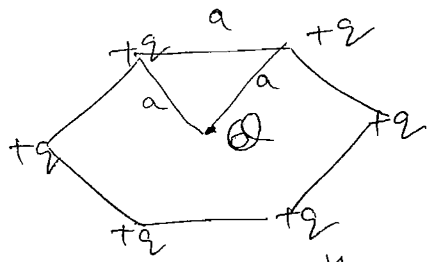


$\equiv$



(23)

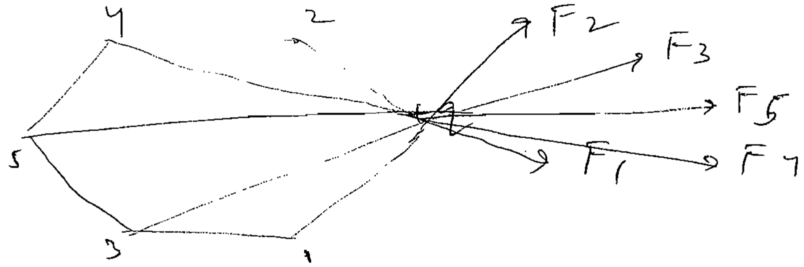
(22)



Let charge at centre be Q.

Then, on any charge,

Net force due to all q is

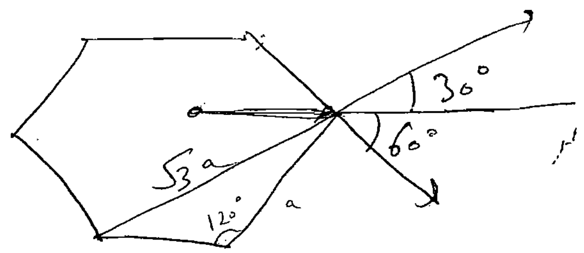


=

$2F_1 \cos 60^\circ$

$+ 2F_2 \cos 30^\circ$

$+ F_5$



$a^2 + a^2 + 2aa \cos 60$?

$\sqrt{3}a$

$$= 2 \times \frac{kq^2}{a^2} \cdot \frac{1}{2} + 2 \times \frac{kq^2}{(\sqrt{3}a)^2} \cdot \frac{\sqrt{3}}{2} + \frac{kq^2}{(2a)^2}$$

$$= \frac{kq^2}{a^2}\left[1 + \frac{1}{\sqrt{3}} + \frac{1}{4}\right].$$

This should be $-$ve of $\frac{kQq}{a^2}$.

$$\therefore \frac{kQq}{a^2} = -\frac{kq^2}{a^2}\left[1 + .58 + .25\right]$$

$$= -\frac{kq^2}{a^2}\left[1.83\right]$$

$\therefore$ $\boxed{Q = -1.83q}$

(24)

(23)

$Q = CV$

$= 3 \times 300 \times 10^{-6}$ $\quad$ $= 5 \times 500 \times 10^{-6}$

$= 9 \times 10^{-4}$ $\quad$ $= 25 \times 10^{-4}$.

$E_1 = \frac{1}{2} C_1 V_1^2$ $\quad$ $E_2 = \frac{1}{2} C_2 V_2^2$

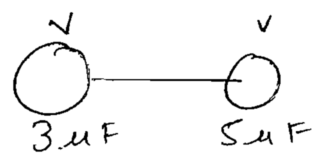
$= \frac{1}{2} (3 \times 10^{-6}) . (300)^2$ $\quad$ $= \frac{1}{2} \times (5 \times 10^{-6}) \ (500)^2$

$= \frac{1}{2} \times 27 \times 10^{-2}$ $\quad$ $= \frac{1}{2} \times 125 \times 10^{-2}$.

Initial total $= \frac{1}{2} \times [27 + 125] \times 10^{-2}$

$= 76 \times 10^{-2}$. J

Final.



Potential will be same. (Let V)

Then $(3+5).V \times 10^{-6} = (25+9) \times 10^{-4}$ Also total charge is same.

$\therefore \ 8 V \times 10^{-6} = 34 \times 10^{-4}$

$\therefore \ V = \frac{34}{8} \times 100 = \frac{1700}{4} = 425$ volt

$U_f = \frac{1}{2} (C_1 + C_2) V^2$

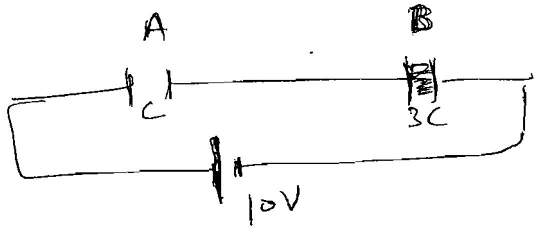
$= \frac{1}{2} (8 \times 10^{-6}) (425)^2$

$= 72.25 \times 10^{-2}$ J

Loss = 3.75×10^{-2} J

(25)

(24)



Charge flow is same.

$$\Delta V_1 = \frac{q}{C_1} = \frac{q}{C}$$

$$\Delta V_2 = \frac{q}{C_2} = \frac{q}{2C}.$$

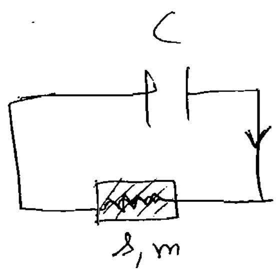
$$\therefore \frac{\Delta V_1}{\Delta V_2} = \frac{3}{1}$$

Also $\Delta V_1 + \Delta V_2 = 10.$

$\therefore \Delta V_1 = 7.5$, $\Delta V_2 = 2.5$

(26)

(25)



~~$q = CV[1 - e^{-t/RC}]$~~

~~$q = CVe^{-t/RC}$~~

$$i = \frac{CV}{RC} e^{-t/RC}.$$

Heat produced $= ms\Delta T$.

But heat produced $= \int_0^\infty i^2 R\,dt$

$$= \int_0^\infty i_0^2 R\,dt$$

$$= \int_0^\infty \frac{V^2}{R^2} e^{-2t/RC} \cdot R \cdot dt.$$

$$= \frac{V^2}{R} \frac{e^{-2t/RC}}{-2/RC}$$

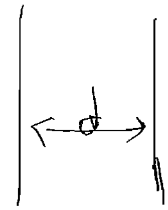
$$= \left[-CV^2 e^{-2t/RC}\right]_0^\infty$$

$$= CV^2 [1] = \frac{1}{2} CV^2.$$

$$ms\Delta T = \frac{1}{2} CV^2$$

$$\boxed{\sqrt{\frac{2ms\Delta T}{C}} = V}$$

(27)



$V = V$

$q = CV$

where $C = \frac{\varepsilon_0 A}{d}$

$q = \frac{\varepsilon_0 A V}{d}$

Battery disconnected, $q \rightarrow$ constant.

$\therefore$ New $C \rightarrow CK$

$q \rightarrow q$

$V \rightarrow V/K$

$V_{final} = \frac{V_0}{K}$

$E_{final} = \frac{V_0}{Kd}$

$W_{all} = \Delta KE$

$W_{ext} + W_{intc} = \Delta KE$

$W_{ext} = - W_{intc}$

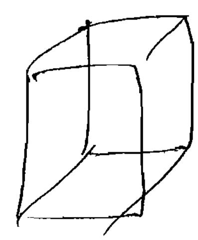
$= \Delta PE$

Work done on system $=$ P.E final $-$ P.E. initial

$= \frac{1}{2}(Ck)\left(\frac{V}{k}\right)^2 - \frac{1}{2}CV^2$

$= \frac{1}{2}CV^2\left(\frac{1}{k} - 1\right)$

(28) Total flux $= q/\varepsilon_0$

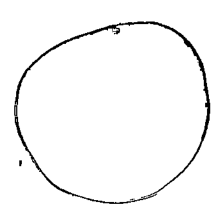


flux throgh One wall $= \frac{q}{6\varepsilon_0}$

(29) Uniformly charged. Non-conducting

By shell theorem,

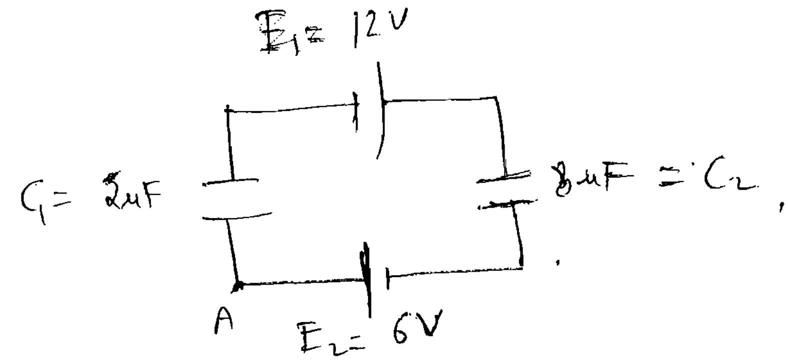
$E = 0$ (inside), ✗



Outside electric field $\rightarrow \frac{kQ}{r^2}$ (by gauss law)

... potential constant ...

(30)

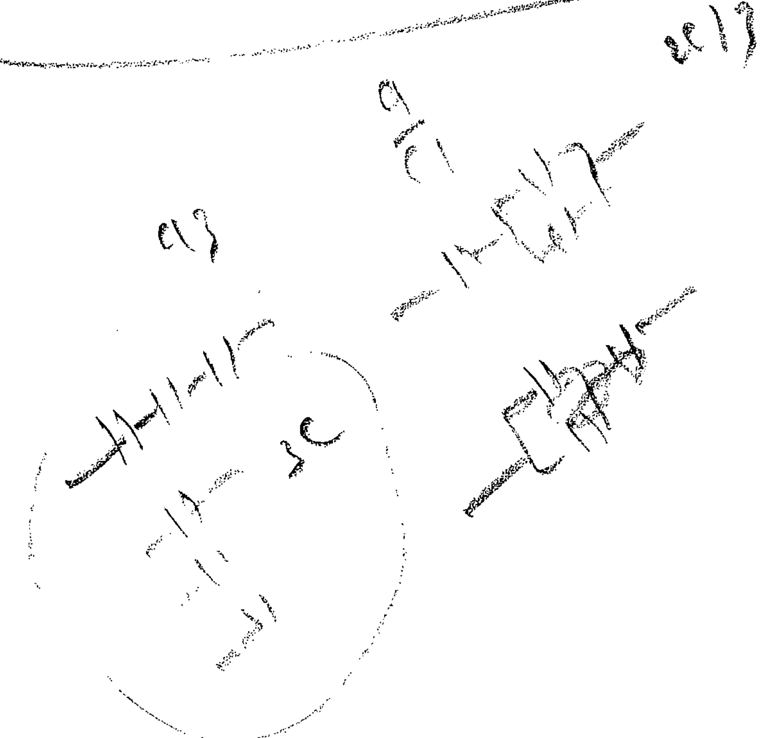


Charge flowing through C_1 and C_2 will be same

$V_{C_1} = \frac{q}{C_1}$, $V_{C_2} = \frac{q}{C_2}$.

$C_1 < C_2$

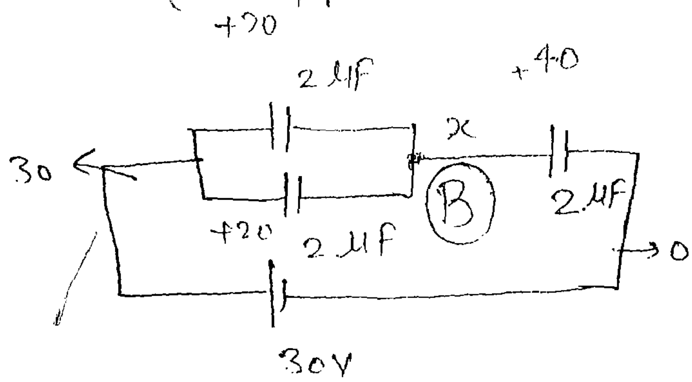
$\therefore V_{C_1} > V_{C_2}$.



Acharya 24/3/12 Exercise-I (subjective)

1.

(1)



$(x-0)2 + (x-30)4 = 0$

$3x = 60$

$x = 20V$

So initial charge on ① & ② ⇒ 20 μC

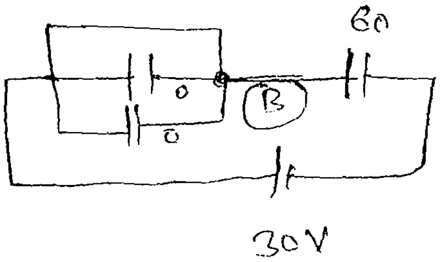
③ ⇒ 40 μC

(a) when S is closed,

final charge ① & ② ⇒ 0

③ ⇒ 60 μC

So charge flown = 20 μC



(b) initial Energy $= 2\left[\frac{1}{2} \times 2 \times (10)^2\right] + \frac{1}{2} \times 2 \times (20)^2$

$= 200 + 400 = 600$ μJ

final Energy $= \frac{1}{2} \times 2 \times (30)^2 = 900$ μJ

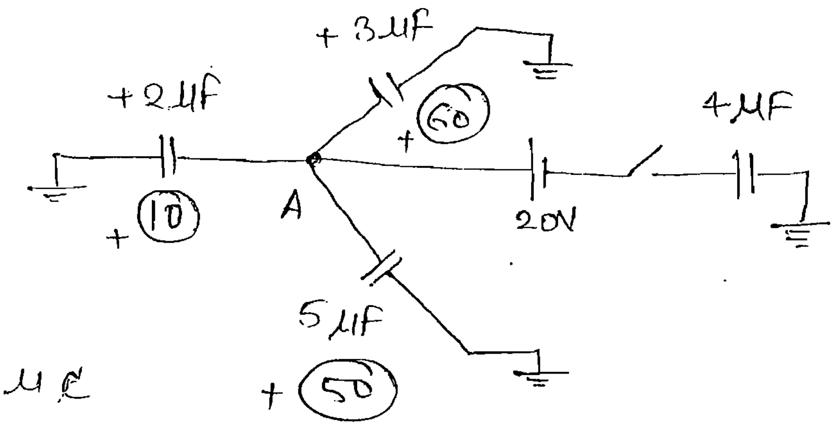
(c) Work done by battery $= 30 \times 20 = 600$ μJ

Energy loss $= 600 + 600 - 900 = 300$ μJ

(d) Charge flown through S = 60

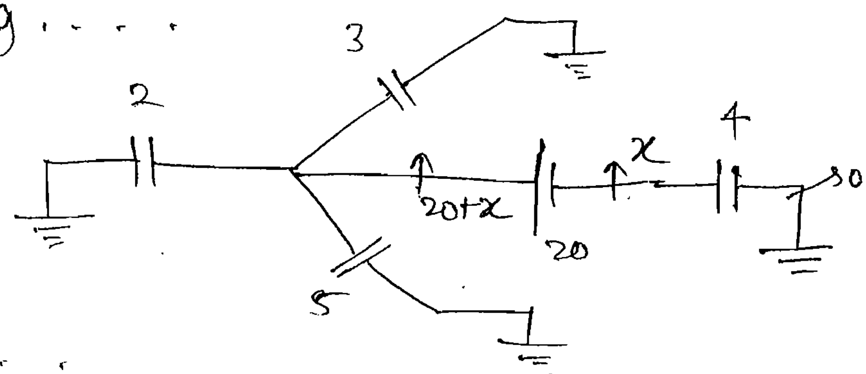
initial charge at 'B' = 0 μC
final charge at 'B' = 60 μC

2.



initial charge at junction 'A' = 120 μC

after Connecting....



(a) applying Nodal...

$(20+x)\,10 + (x-0)\,4 = 120$

$5(20+x) + 2x = 60$

$7x + 100 = 60$

$x = -\frac{40}{7}$ ✓

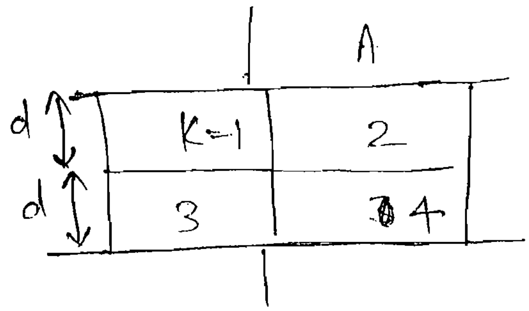
So potential of junction 'A' $= 20 - \frac{40}{7} = \frac{100}{7}$ V

(b) charges ~~of~~ on each Capacitors.

200 300 500 −160

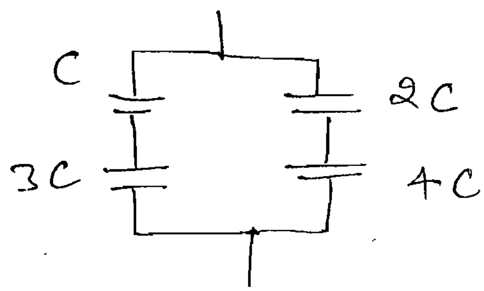
3.

(3)

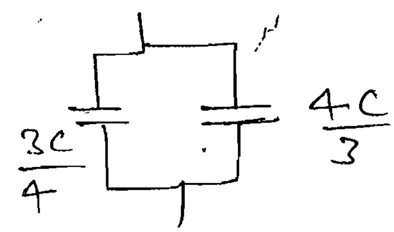


$$C_1 = \frac{K\epsilon_0 A}{d} = \frac{\epsilon_0 A}{2d} = C$$

$$C_2 = \frac{2\epsilon_0 A}{2d} = 2C \qquad C_3 = 3C, \; C_4 = 4C$$

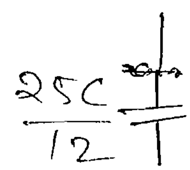


$\Rightarrow$



$$\frac{3C^2}{4C} = \frac{3C}{4}$$

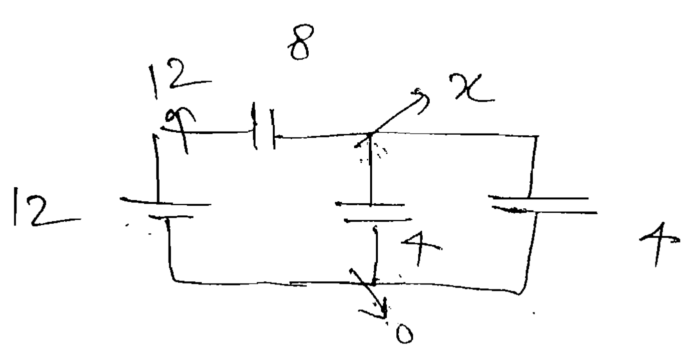
$$\frac{8C^2}{6C} = \frac{4C}{3}$$



$$\frac{3C}{4} + \frac{4C}{3} = \frac{25C}{12}$$

$$C_{eff} = \frac{25C}{12} = \frac{25}{12} \times \frac{\epsilon_0 A}{2d} = \frac{25\epsilon_0 A}{24 d}$$

(4.)



$$(x-12)8 + (x-0)4 + (x-0)4 = 0$$

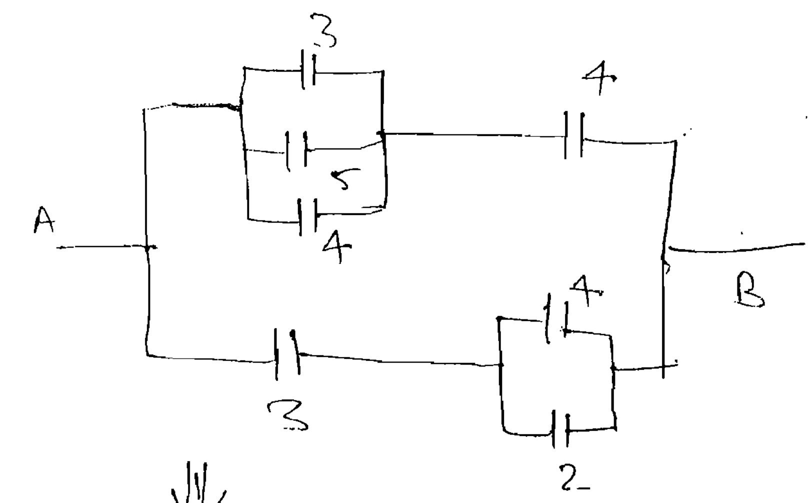
$$4x = 24$$

$$x = 6\text{ V}$$

Charge on '8' = 48 μC

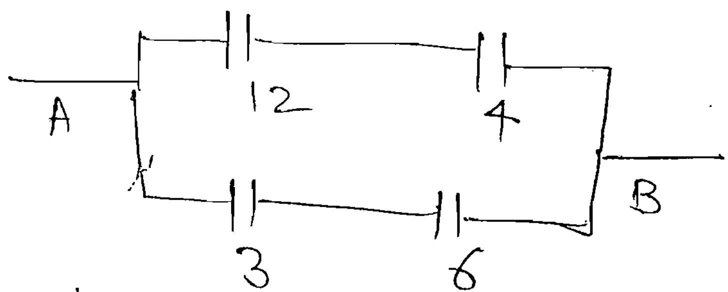
" " '4' = 24 μC

(5.)



$3+5+4 = 12$

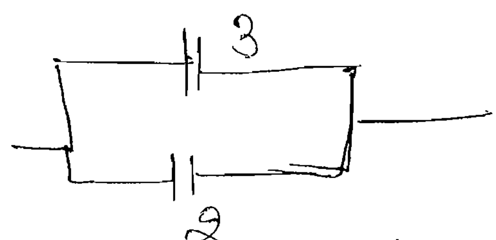
⇓



$\frac{12 \times 4}{16} = 3$

$\frac{3 \times 6}{3+6} = 2$

⇓



⇓

$5\,\mu f = C_{eff}$.

potential drop ~~accor~~ across '3' = $\frac{120}{5} = 24V$

" " '12' = 24V (all in parallel)

'12' & '4' are in series so both should have same charge ⇒

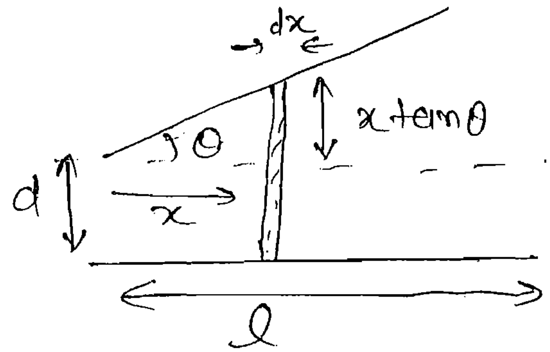
$12 \times 24 = 4 \times V$

P.D across AB = 24 + 72 = 96V

P.D across '3' = $\frac{6}{3+6} \times 96 =$

$= \frac{2}{3} \times 96 = 64V$

(6)



Capacitence of small plate is "dc" = $\frac{\epsilon_0 A}{d}$

$\Rightarrow \quad dc = \frac{\epsilon_0 \, l \, dx}{d + x\tan\theta}$

as all plates are in $||^{rl}$

$$C = \int dc = \int_0^l \frac{\epsilon_0 \, l \, dx}{d + x\tan\theta}$$

$$= \frac{\epsilon_0 l}{\tan\theta} \ln\left(\frac{d + l\tan\theta}{d}\right)$$

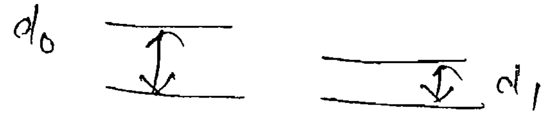
as θ is very small $\tan\theta \simeq \theta$

$$C = \frac{\epsilon_0 l}{\theta}\left(\frac{l}{d}\theta - \frac{l^2\theta^2}{2d^2}\right)$$

$$C = \frac{\epsilon_0 l^2}{d}\left(1 - \frac{l\theta}{2d}\right)$$ [ln(1+x)

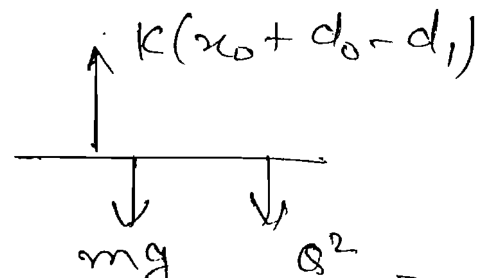
1. Suppose initial extension is $= x_0$

(a) therefore $Kx_0 = mg$



distance decrease b/w plates $= d_0 - d_1$

on upper plate



$\frac{Q^2}{2A\epsilon_0} = \frac{\epsilon_0 A V^2}{2 d_1^2}$

LOM for New equilibrium

$$mg + \frac{\epsilon_0 A V^2}{2 d_1^2} = K(x_0 + d_0 - d_1)$$

$$K = \frac{\epsilon_0 A V^2}{2 d_1^2 (d_0 - d_1)}$$

(b) $$V^2 = \frac{2K d_1^2 (d_0 - d_1)}{\epsilon_0 A}$$

$y = d_1^2 (d_0 - d_1)$

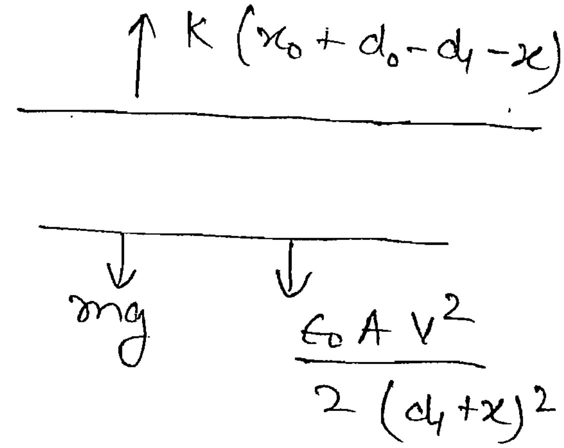
for max. value of $V \Rightarrow$ $2d_1 d_0 - 3d_1^2 = 0$

$d_1 = \frac{2d_0}{3}$

$$V^2 = \frac{2K}{\epsilon_0 A} \frac{4d_0^2}{9} \times \frac{d_0}{3}$$

$$V = \sqrt{\frac{K}{\epsilon_0 A}} \left(\frac{2}{3} d_0\right)^{3/2}$$

(c) Suppose ~~block~~ plate is displaced by a distance 'x', therefore -



$F_{net} = ma$

$$K(x_0 + d_0 - d_1 - x) - mg - \frac{\epsilon_0 A V^2}{2(d_1 + x)^2} = ma$$

$$\frac{\epsilon_0 A V^2}{2 d_1^2} \;\; \text{~~K(d0 - d1)~~} - Kx - \frac{\epsilon_0 A V^2}{2(d_1 + x)^2} = ma$$

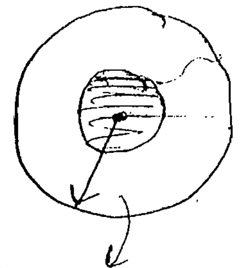
$$-Kx + \frac{\epsilon_0 A V^2}{2}\left[\frac{1}{d_1^2} - \frac{1}{(d_1 + x)^2}\right] = ma$$

$$-Kx + \frac{\epsilon_0 A V^2}{2}\left[\frac{x^2 + 2 d_1 x}{d_1^2 (d_1 + x)^2}\right] = ma \quad (x^2 \to 0)$$

$$ma = -\frac{[K d_1^3 - \epsilon_0 A V^2]}{d_1^3}\, x \qquad \left| \; d_1 + x \approx d_1 \right.$$

$$\omega = \left[\frac{K d_1^3 - \epsilon_0 A V^2}{m d_1^3}\right]^{1/2}$$

8.



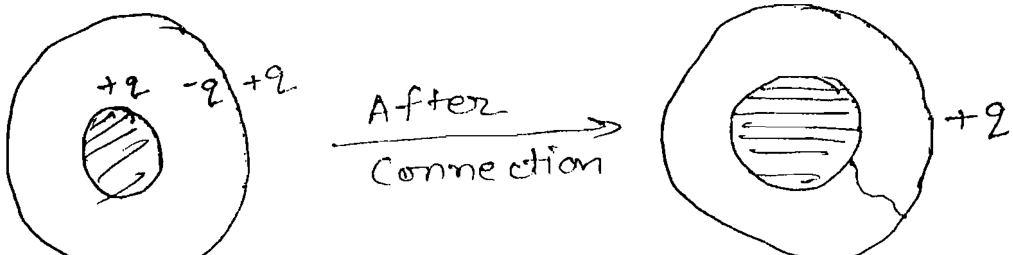
$r = 10$ cm

$q = 20\,\mu C$ 20 cm

$$\text{Initial Energy} = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2}\frac{(20\,\mu C)^2}{\frac{K \times .1 \times .2}{.1}} = \frac{(20)^2}{K \times .4}$$

when wire is connected, all charge will transfer to outer surface.



So we can say, inner capacitor will discharge if we connect both sphere.

$$\text{initial energy} = \frac{(20 \times 10^{-6})^2 \times 9 \times 10^9}{.4}$$

$$= 9\ J$$

$$\text{final energy} = 0\ J$$

$$\text{loss} = 9 - 0 = 9\ J$$

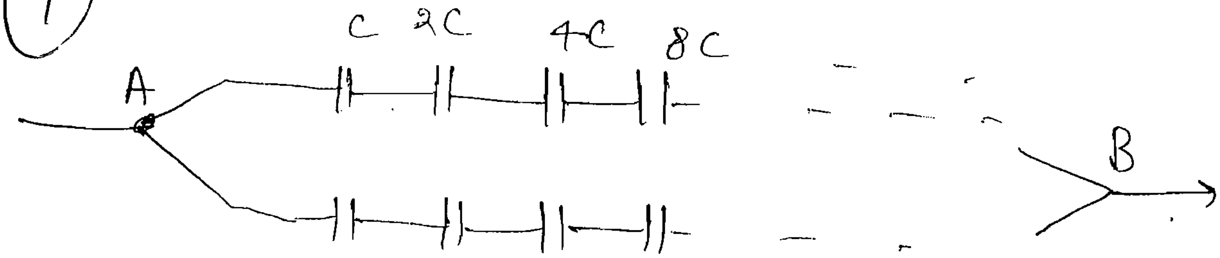
Ans

$$\frac{Q^2}{2C}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$C = \epsilon_0 \frac{4\pi R_1 R_2}{R_2 - R_1}$$

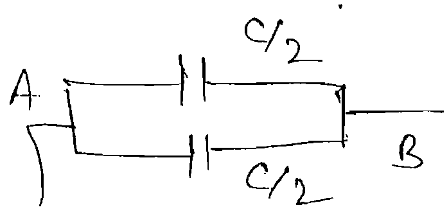
9. (9)



we can remove all middle capacitence (C) as ~~both ar~~ p.D acorross them = 0

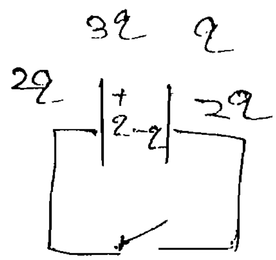
$$\frac{1}{C_{eff}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{4C} - - -$$

$$\frac{1}{C_{eff}} = \frac{2}{C} \Rightarrow C_{eff} = C/2$$



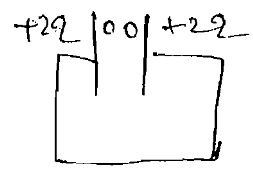
$$\Rightarrow \frac{C}{2} + \frac{C}{2} = C$$

(10.)



$$C = \frac{\epsilon_0 A}{d}$$

⇓



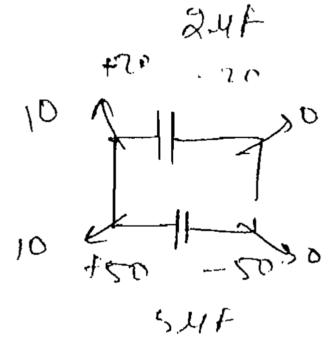
Capacitor would be in discharged condition.

initial Energy $= \frac{1}{2}\frac{Q^2}{C} = \frac{q^2 d}{2\epsilon_0 A}$

final energy $= 0$

Heat $= \frac{1}{2}\frac{q^2 d}{}$

11.



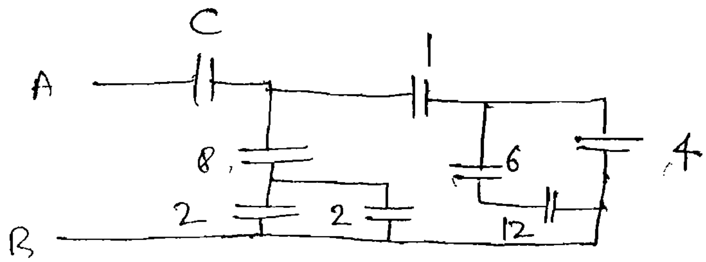
$\Delta V = \frac{20}{2} = 10\ V$

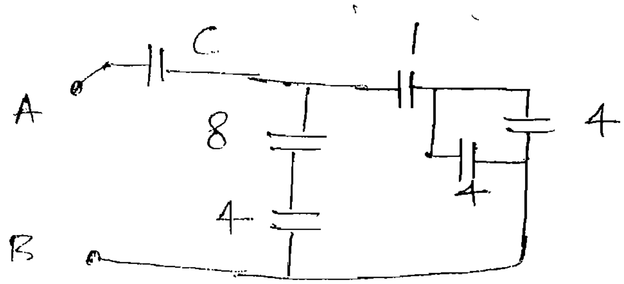
$\Delta V = \frac{50}{5} = 10$

if we ~~join~~ put switch in on condition, then no charge will flow as they are at same potential.

$Q = 0$

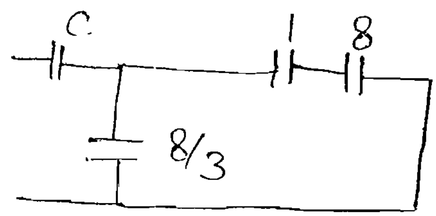
12.





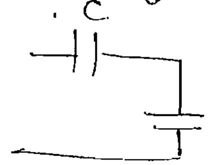
$\frac{6 \times 12}{18} = 4$

$2 + 2 = 4$



$\frac{8 \times 4}{12} = \frac{8}{3}$

⇓



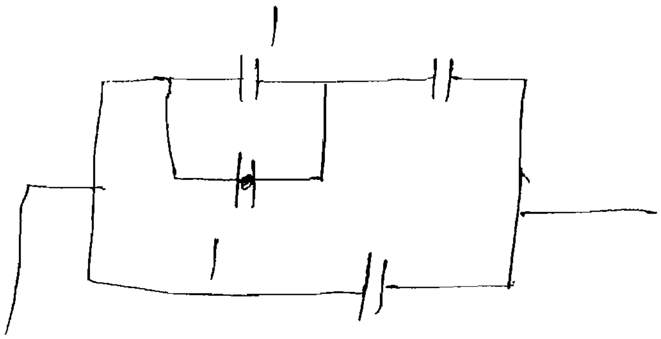
$\frac{32}{3}$

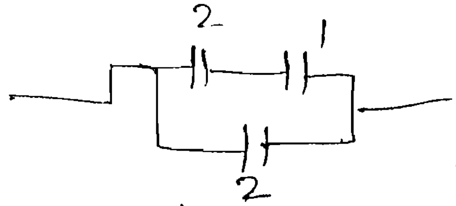
$$1 = \frac{C \times 32/3}{C + 32/3}$$

??

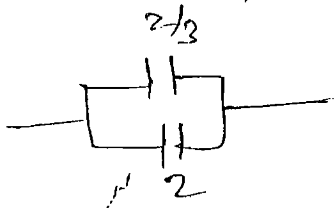
13.

(13)



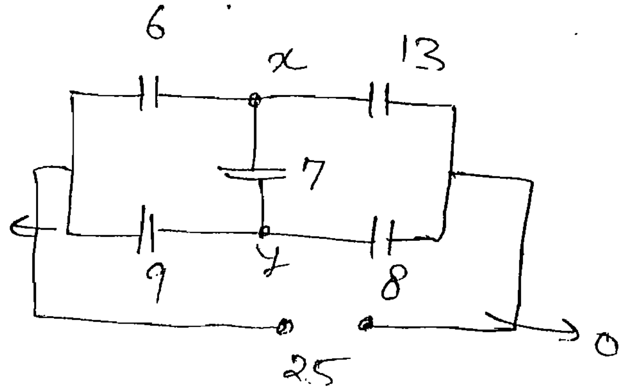


$\Rightarrow \frac{2\times 1}{3} = \frac{2}{3}$



$C_{eff} = 2 + \frac{2}{3} = \frac{8}{3}\ \mu F$

(14)



$(x-25)6 + (x-0)13 + (x-y)7 = 0$

$(y-25)9 + (y-0)8 + (y-x)7 = 0$

$26x - 7y = 150$

$24y - 7x = 225$

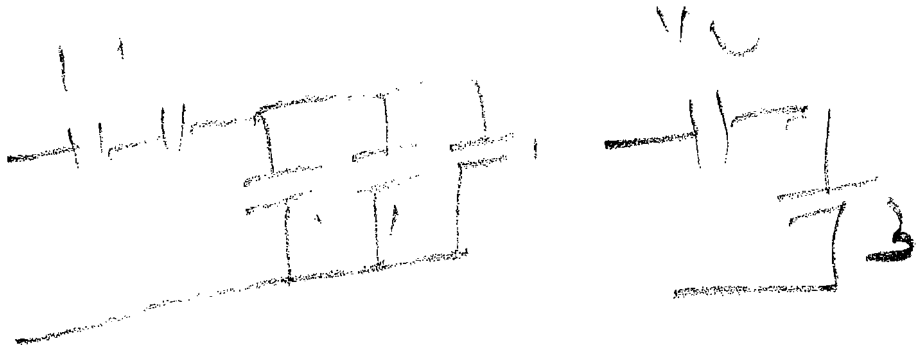
Solving both eq.

$x = 9V$

$y = 12V$

Voltage across ...

(b)

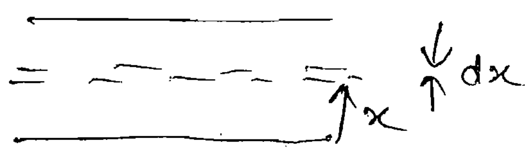


$\frac{1}{c} = \frac{1}{5} + 1 = \frac{7}{5}$

(15)

$k = k_1 \left[1 + \sin \frac{\pi}{d} x \right]$

$dc =$



$dc = \frac{k \epsilon_0 A}{dx}$

all would be in series

$$\frac{1}{C_{eff}} = \int \frac{1}{dc} = \int_0^d \frac{dx}{k \epsilon_0 A}$$

$$= \int_0^d \frac{dx}{k_1 \left(1 + \sin \frac{\pi}{d} x\right) \epsilon_0 A}$$

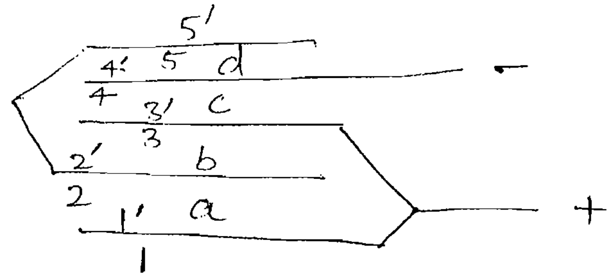
$$\frac{1}{C_{eff}} = \frac{1}{k_1 \epsilon_0 A \frac{\pi}{d}} \int \left(\frac{1 - \sin \frac{\pi}{d} x}{1 - \sin^2 \frac{\pi}{d} x} \right) dx$$

$$= \frac{1}{k_1 \epsilon_0 A \frac{\pi}{d}} \left[\tan \frac{\pi}{d} x + \frac{1}{\cos \frac{\pi}{d} x} \right]_0^d$$

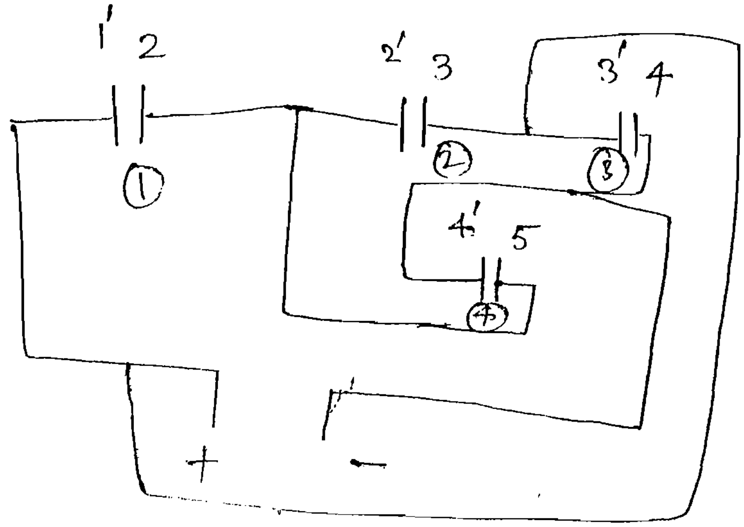
$\frac{1}{\ } = \frac{2d}{\ }$ ... $k_1 \epsilon_0 A \pi$

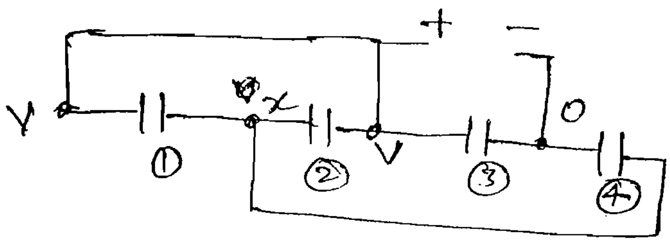
~~16~~ 17

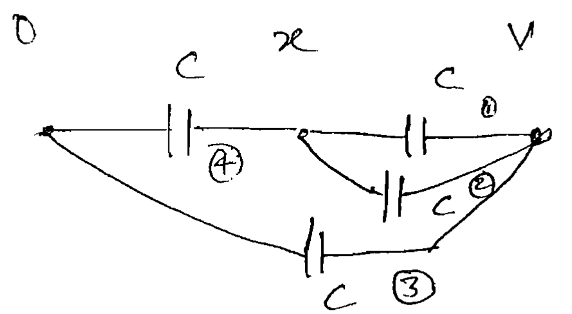
(16)

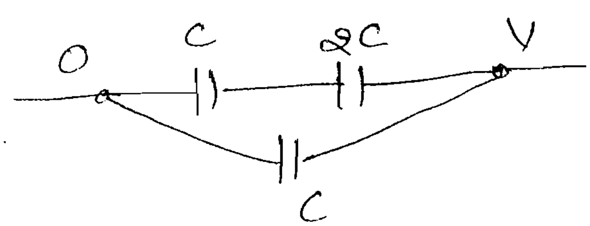


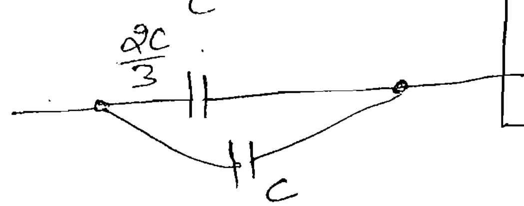
we can form 4 capacitor











$(x-0)C + 2(x-V)C = 0$

$3x = 2V$

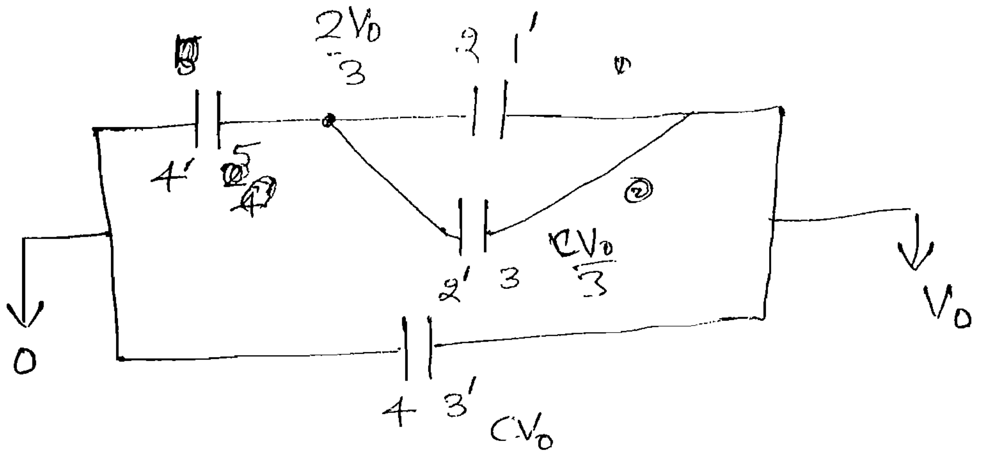
$x = \frac{2V}{3}$

$Q_1 = \frac{2CV}{3}$

$Q_2 = \frac{CV}{3} = Q_3$

$Q_4 = CV$

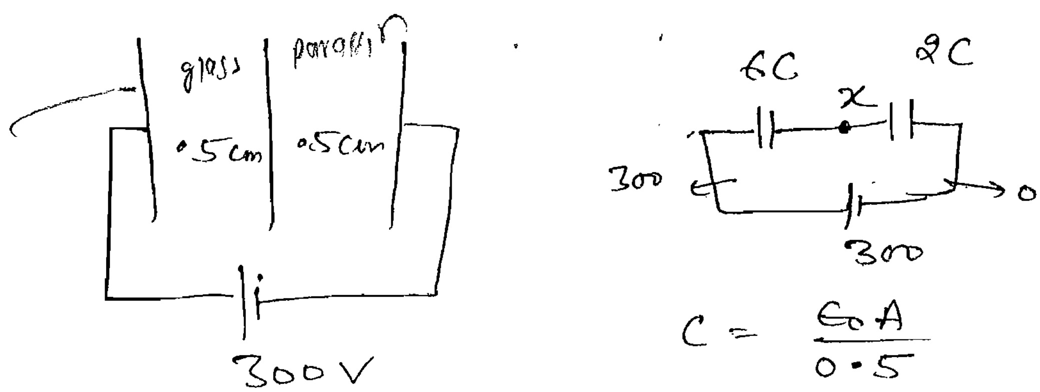
$C_{eff} = C + \frac{2C}{3} = \frac{5C}{3} = \frac{5}{3}\frac{\varepsilon_0 A}{d}$



charge on plate 5 = $\frac{2V_0}{3} \times C = \frac{2CV_0}{3}$

charge on plate 3 = $CV_0 + \frac{CV_0}{3} = \frac{4CV_0}{3}$

(17)



$C = \frac{\epsilon_0 A}{0.5}$

$(x-300)\,6C + (x-0)\,2C = 0$

$3x + x = 900$

$x = \frac{900}{4} = 225V$

(i) $E_1 = \frac{V_1}{d} = \frac{75}{.5\,cm} = 150 \times 10^2 = 1.5 \times 10^4$ V/m

$E_2 = \frac{V_2}{d} = \frac{225}{.5} = 4.5 \times 10^4$ V/m

(ii) drop ⟹ 75V

then ⟹ 225 V

(iii)

$Q_1 = C_1 V_1$

$Q_1 = 6C\ 75$

$= 6 \frac{\epsilon_0 A}{d} \times 75$

$\sigma_1 = \frac{Q_1}{A} = \frac{450\ \epsilon_0}{.5\,cm} =$

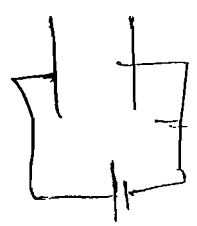
$= 900 \times 10^2 \times 8.85 \times 10^{-12}$

$= 7.965 \times 10^{-7}\ C/m^2$

$\approx 8 \times 10^{-7}\ C/m^2$

~~18.~~ ~~19.~~ (18)

$q = CV_0$



$C = \frac{\epsilon_0 A}{d}$

now battery is revomed.

then charge remain Conserved.

CV_0 |K| CV_0 new Capacitance $= KC$

initial Energy $= \frac{1}{2} C V_0^2$

final Energy $= \frac{1}{2} \frac{Q^2}{C} = \frac{C^2 V_0^2}{2KC}$

$= \frac{C V_0^2}{2K}$

$W_0 = \Delta K \to 0$

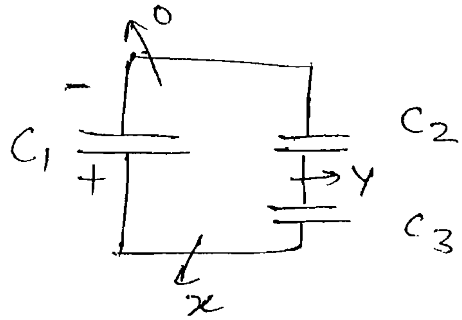
$W_{ext} + W_{cap.} = 0$

$\frac{1}{2} C V_0^2 \left(1 - \frac{1}{K}\right)$ ans

~~19.~~
20

(19)

initial charge on $C_1 \Rightarrow C_1 V$



$(x-0)C_1 + (x-y)C_3 = C_1 V$

$(y-0)C_2 + (y-x)C_3 = 0$

$(C_1+C_3)x - C_3 y = C_1 V$

$(C_2+C_3)y - C_3 x = 0$

$$\frac{(C_1+C_3)(C_2+C_3)y}{C_3} - C_3 y = C_1 V$$

$$y = \frac{C_1 C_3 V}{C_1 C_2 + (C_1+C_2)C_3}$$

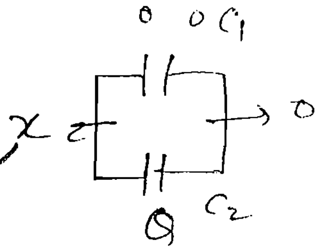
Charge on 'C_2' $= (y-0)C_2$

$$= \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$q_1 = x C_1$

$= \frac{(C_2+C_3)y}{C_3} \cdot C_1 = \frac{(C_2+C_3)C_1}{C_3}$

20 (20)



$(x-0)\,C_1 + (x-0)\,C_2 = Q$

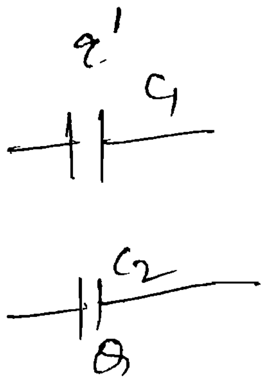
$x = \dfrac{Q}{C_1+C_2}$

charge on ~~plate~~ capacitor ① is

$C_1 x = \dfrac{C_1 Q}{C_1+C_2} = q$ — ①

charge transfer will stop between C_1 & C_2 when both will be at same potential.

$\Rightarrow \quad \dfrac{q'}{C_1} = \dfrac{Q}{C_2}$



$q_1' = \dfrac{C_1}{C_2} Q =$

$q_1' = \dfrac{Q\,q}{Q-q}$

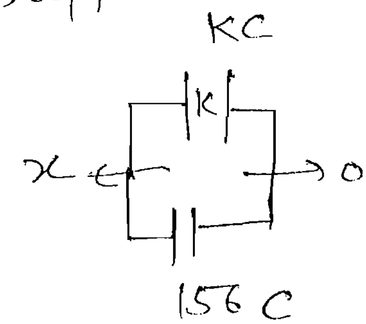
from eq. ①

$\dfrac{C_1}{C_1+C_2} = \dfrac{q}{Q}$

$\dfrac{C_1}{C_2} = \dfrac{q}{Q-q}$

21. (21)

Suppose capacitance $= C$



$K = \alpha V$

$(x-0)\,KC + (x-0)\,C = 156\,C$

$(K+1)\,x = 156$

$K = \alpha V = 1 \times x$

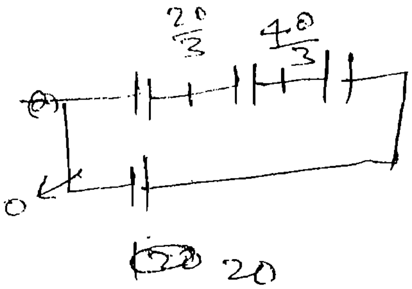
$(1+x)\,x = 156$

$x^2 + x = 156$

22.
~~21.~~
23

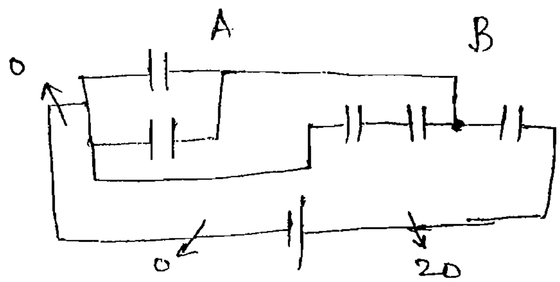
(22)

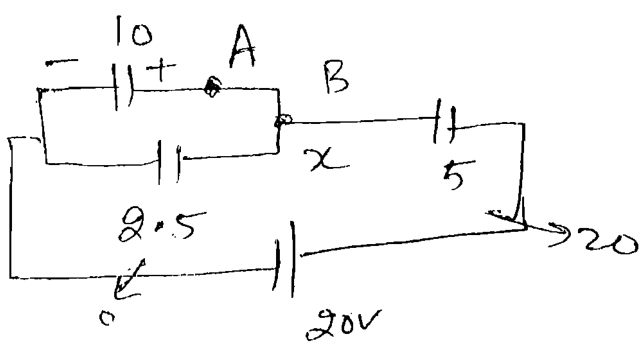
when A& switch is not closed ⇒



initial charge $= \frac{100}{3}\mu C$
on each capacitor

charge at junction 'B' = 0





$$(x-0)\,12.5\;2.5 + (x-20)5 = 0$$

$$3.5x = 20$$

$$x = \frac{40}{7}$$

So charge on capacitor '10' $= \frac{400}{7}$

charge will flow from B → A

$400\,\mu C$

(30) No effect of closing S

Heat dissipated $= 0$

(31) use series, parallel rule for dividing applied potential & ensure voltage less than break down.

(32) $Q = C_1V - C_2V = (C_1 - C_2)V$

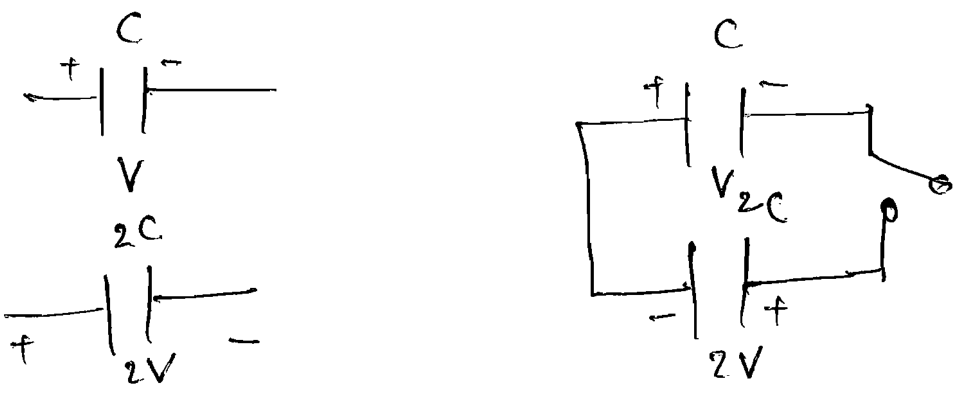
$V_2 = V_1 = \frac{Q}{C_1 + C_2} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)V = \frac{100}{3}$

$E_{in} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$

$E_f = \frac{1}{2}(C_1 + C_2)V_1^2$

$\frac{E_f}{E_{in}} = \left(\frac{V_1}{V}\right)^2 = 1/9.$

(33)



Final common potential $V_0 = \frac{4CV - CV}{3C} = V$

$U_f = \frac{1}{2}CV^2 + \frac{1}{2} \cdot 2C \cdot V^2 = \frac{3}{2}CV^2$

(34)

Capacitance

Exercise #1.

(26) Final charge on (+)ve plate = $CV + Q = Q_1$

Final net charge on (-)ve plate = $-CV = Q_2$

Final charge on cap. = $\frac{Q_1 - Q_2}{2} = \frac{2CV + Q}{2}$

$$p.d = \frac{2CV + Q}{2C} = V + \frac{Q}{2C}.$$

(27) $d_0 = 3\ mm$

$$C_0 = \frac{(A\epsilon_0)\,4}{d_0} = \frac{4\epsilon_0 A}{3}$$

$$C_1 = \frac{A\epsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} = \frac{A\epsilon_0}{\frac{3}{4} + \frac{5}{K}}$$

$$\boxed{C_1 = \frac{1}{2}C_0} \Rightarrow \boxed{K = \frac{20}{3}}$$

(28) capacitance of Left section = $\frac{2\epsilon_0 A}{d} = 2C_0$ (say)

capacitance of right section = $\frac{6\epsilon_0 A}{2d} = 3C_0$

since charges will be same

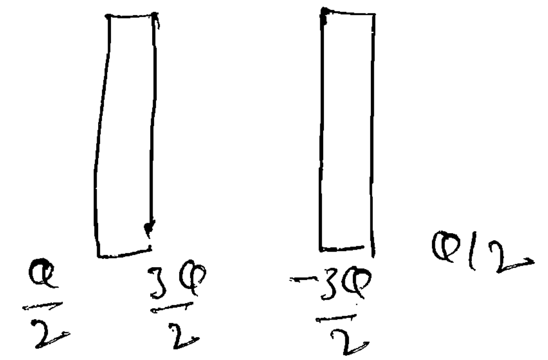
$$\Rightarrow V \propto \frac{1}{C} \Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1} = 3/2$$

(29) case I: $C_{eq} = \frac{KC}{K+1}$, $Q_1 = Q_2 = \frac{KCE}{K+1}$

case II, $Q_1' = Q_2' = \frac{CE}{2}$

$$\Rightarrow \frac{Q_1'}{Q_1} = \frac{Q_2'}{\ } = \frac{K+1}{\ }$$

(34)



$$U_{in} = q^2/2C = \frac{(3Q/2)^2}{2\left(\frac{A\epsilon_0}{d}\right)} = \frac{9Q^2 d}{8A\epsilon_0}.$$

Exercise # 2

(1) $\phi_{cube} = \frac{\Sigma Q}{\epsilon_0} = Q/\epsilon_0$

$\phi_{one\ face} = \frac{1}{6} \cdot \phi_{cube} = \frac{Q}{6\epsilon_0}$

(2) use std. expression of $\vec{E}$ inside & outside

(3) Both C_1 & C_2 are in series

$\Rightarrow Q_1 = Q_2$

$\Rightarrow V_1 = \frac{Q_1}{C_1}$, $V_2 = \frac{Q_2}{C_2} = \frac{Q_1}{C_2}$

(4) slab pulled out $\Rightarrow C_B \downarrow$ $\Rightarrow$ Net cap. $\downarrow$

$\Rightarrow q = CV$ $\Rightarrow$ $q_A \downarrow$

$\Rightarrow$ (+)ve charge move from a to b

but A & B are in series.

(5) initial $q_0 = C_0 V_0$

1st contact : $q_1 = \left(\frac{C_0}{C+C_0}\right) q_0$ (sharing of charge)

2nd contact : $q_2 = \left(\frac{C_0}{C+C_0}\right) q_1 = \left(\frac{C_0}{C+C_0}\right)^2 q_0$

nth contact : $q_n = \left(\frac{C_0}{C+C_0}\right)^n q_0$

$$\boxed{V = \frac{q_n}{C_0}} \Rightarrow C = C_0\left[\left(\frac{V_0}{V}\right)^{1/n} - 1\right]$$

(6) Initial

$C_0 = A\epsilon_0/d$

$Q_0 = C_0 V$

$V_0 = V$

$E_0 = V/d$

$U_0 = \frac{1}{2} C_0 V^2$

Final

$C = \frac{AK\epsilon_0}{d} = KC_0$

$Q = KC_0 \cdot V$

$V = V$

$E = V/d = E_0$

$U = \frac{1}{2}CV^2 = \frac{1}{2}KC_0V^2$

$$\Rightarrow \boxed{\begin{matrix} Q > Q_0, & U > U_0 \\ E = E_0, & V = V_0 \end{matrix}}$$

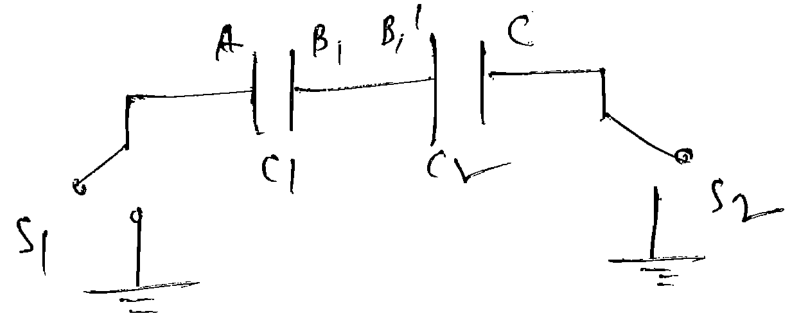
(7) $C_{net} = C + \frac{C}{2} + \frac{C}{4} + \cdots = 2C = 4F$

$U = \frac{1}{2} C_{net} \cdot V^2 = 200J$

charge on each row is same

since cap. in 1st row is max. hence

(8)

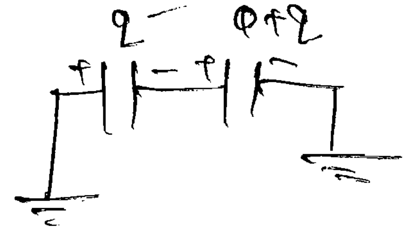


$C_1 = \frac{A\epsilon_0}{2d}$

$C_2 = \frac{A\epsilon_0}{d}$

$Q_{B_1} + Q_{B_1'} = Q$

- If S_1 is closed, charge on $C_1 = 0$
  $\Rightarrow Q_{B_1} = 0 \Rightarrow Q_{B_1'} = Q$
  $\Rightarrow -Q$ will flow thru S_1.
- If S_2 is closed $-Q$ will flow thru S_2.
- If S_1 & S_2 both closed



$$-\frac{q}{C_1} + \left(\frac{q+Q}{C_2}\right) = 0$$

$$\Rightarrow \boxed{q = -Q/3}$$

$-Q/3$ will flow thru S_1 $\Rightarrow$ $\frac{2Q}{3}$ will flow thru S_2.

(9) use std. expression taking charge on cap. to remain const.

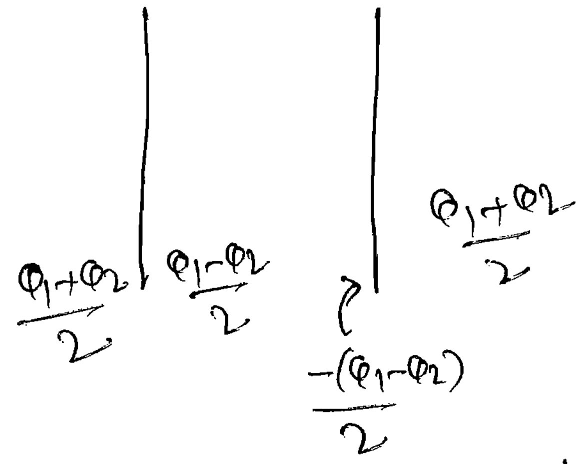
(10) $V_f = \frac{V_0 C_0}{C_0 + KC_0} = \frac{30 C_0}{C_0 + V_f C_0}$ $\Rightarrow \boxed{V_f = 5 \text{ volt}}$

$\boxed{\text{charge} = C \cdot V_f}$

$Q_A = (5C_0)5 = 25C_0$

$Q_B = (C_0)5 = 5C_0$

(11)



Heat produced $= \frac{1}{2}\frac{Q^2}{C_0} = \frac{1}{2C_0}\left(\frac{Q_1-Q_2}{2}\right)^2$

(12) use concept of sharing of charge.

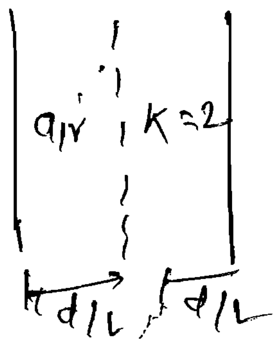
Exercise #3

Passage #1

use std. series, parallel arrangement.

Passage #2

$C_1 = \frac{\epsilon_0 A}{(d/2)} = \frac{2\epsilon_0 A}{d}$.



$C_2 = 2\cdot\left(\frac{\epsilon_0 A}{d/2}\right)$

$= \frac{4\epsilon_0 A}{d}$,

$\boxed{C_2 = 2C_1}$

$C_{in} = \frac{\epsilon_0 A}{d}$

$C_f = \frac{\epsilon_0 A}{\frac{d}{2} + \frac{d}{4}} = \frac{4\epsilon_0 A}{3d}$

$V = V_1 + V_2 = 1200$

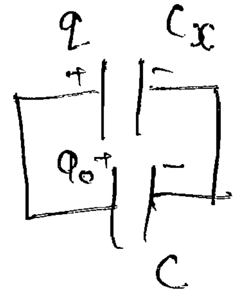
$V_2 = \left(\frac{C_1}{C_1+C_2}\right) V = \frac{C_1}{3C_1} V = V/3 = 400$ volt

$U_{air} = \frac{1}{2}\frac{Q^2}{C_1}$

$U_{di} = \frac{Q^2}{2C_2}$

$\frac{U_{air}}{U_{di}} = \frac{C_2}{C_1} = 2$

Passage #3



- $C_x = \frac{A\epsilon_0}{x}$, $C = \frac{A\epsilon_0}{d}$

charge on C_x, $q = Q_x = \frac{C_x \cdot Q_0}{C + C_x}$

$$F_x = \frac{Q_x^2}{2A\epsilon_0} = \frac{C_x^2 Q_0^2}{(C+C_x)^2 \, 2\epsilon_0 A}$$

$$W = \int_d^{2d} F_x \, dx = \frac{Q_0^2 d}{12 A\epsilon_0}$$

- $F_{ext} = F_x = \frac{Q_0^2}{2\left(\frac{x}{d}+1\right)^2 \cdot A\epsilon_0}$

- $V = \frac{Q_x}{C_x} = \frac{C_x \cdot Q_0}{(C+C_x)\, C_x} = \frac{Q_0}{C+C_x} = \frac{Q_0 x d}{A\epsilon_0 (x+d)}$

- $U_{in} = \frac{1}{2C}\left(\frac{Q_0}{2}\right)^2 = \frac{Q_0^2}{8C}$

Final charge distribution : $\frac{Q-q}{C/2} = \frac{q}{C}$

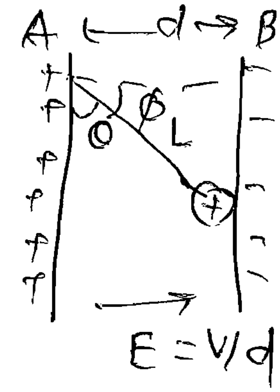
$\boxed{q = Q/3}$

Final energy stored in cap. 2

$$U = \frac{1}{2}(Q/3)^2 \cdot \frac{1}{C} = \frac{Q^2}{18C}$$

$$\Delta U = U_0 - U = \frac{5Q^2}{72C}$$

Passage # 4



- consider the ball in final position

$\sin\theta = \frac{d}{L} = 1/2 \Rightarrow \theta = 30^0$

$\boxed{\phi = 60^0}$

- When the ball again touches the plate A, then redistribution of charge takes place betn plate A & ball

$Q_1 : q_1 = C_0 : C_1$

$q_1 = \left(\frac{C_1}{C_1+C_0}\right) Q_0 = \left(\frac{C_1}{C_0+C_1}\right) C_0 V_0$

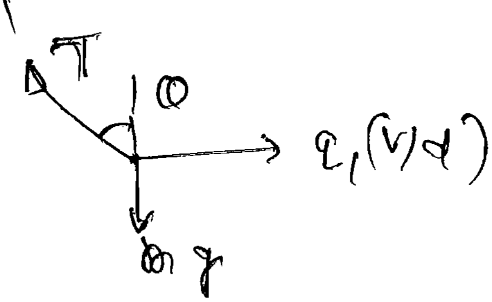
new potential ; $V = \frac{q_1}{C_1} = \frac{C_0 V_0}{C_0+C_1}$

- charge carried by ball in final position

$q_1 = C_1 V = \frac{C_1 C_0 V_0}{C_0 + C_1}$

- $Q_1 = C_0 V = \frac{C_0^2 V_0}{C_0+C_1} \Rightarrow Q_1 : q_1 = C_0 : C_1$

$q_1 = C_1 V = \frac{C_1 C_0 V_0}{C_0+C_1}$



- $mg\tan\theta = \frac{q_1 V}{d} = \frac{C_1 V^2}{d}$

$\Rightarrow \quad = \frac{C_1}{d}\left(\frac{C_0 V_0}{C_0+C_1}\right)^2$

$V_0 = \left(\frac{C_0+C_1}{C_0}\right)\sqrt{\frac{mgd\tan\theta}{C_1}}$

Passage #5

Finally both capacitor have charge CE each. The net charge crossing the cell is $= 2CE$

The work done by cell $= W = (2CE)E$

$= 2CE^2$

Energy stored ~~in~~ ~~cap~~ $= U = \frac{1}{2}CE^2 + \frac{1}{2}CE^2$

Heat prod. $= W - U$

$= CE^2$

Matching

1 →

	Q	C	V
(a)	C_0V_0	C_0	V_0
	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$V_0\left(\frac{K+1}{2K}\right)$
(b)	0	$\frac{C_0}{2d}$	0
	$V_0 \cdot \frac{C_0}{2d}$	$\frac{C_0}{2d}$	V_0
	$V_0 \frac{C_0}{2d}$	$\frac{C_0}{2d}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$V_0\left(\frac{K+1}{2K}\right)$
(c)	0	$\frac{KA\epsilon_0}{d}$	0
	KC_0V_0	KC_0	V_0
	KC_0V_0	KC_0	V_0
	KC_0V_0	$\frac{KC_0}{K+1}$	$(K+1)V_0$
(d)	C_0V_0	C_0	V_0
	KC_0V_0	KC_0	"
	$KC_0V_0/K+1$	$\frac{KC_0}{K+1}$	"

(2) $Q_{in} = CV_{in} = 2\,\mu C$

$Q_f = CV_f = 4\,\mu C$

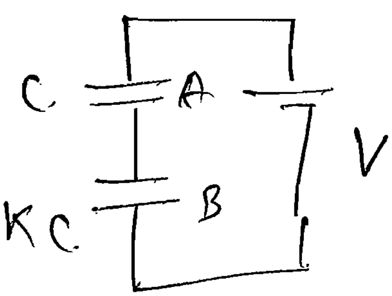
Net Q crossing 4V battery $= Q_f - Q_{in} = 2\,\mu C$

$|W| = (Q_f - Q_{in})\,4 = 8\,\mu J$

$\Delta U = \frac{1}{2} C (V_f^2 - V_{in}^2) = 6\,\mu J$

(3) As slab is inserted, $C_B \uparrow$, $C_{Total} \downarrow$

$\Rightarrow Q \downarrow$.



$Q = \frac{KC}{K+1} \cdot V$

$V_A = Q/C = \frac{KV}{K+1} > \frac{V}{2}$

$V_B = \frac{Q}{KC} = \frac{V}{1+K} < V/2$

(4) Battery is always connected $\Rightarrow V_C =$ const.

$C \uparrow \Rightarrow Q_C \uparrow$

Battery disconnected $\Rightarrow Q_C =$ const.

$C \uparrow \Rightarrow V_C \downarrow$

(5) $C_1 = 3\,\mu F$, $V_1 = 12\,V \Rightarrow Q_1 = 36\,\mu C$

$Q_{Total} = 36\,\mu C$, $C_{Total} = 9\,\mu F$

$V \ldots = 36/ \ldots$

Subjective Ex # 4 Capacitor

(23) • $Q_A = Q_B = Q \Rightarrow V_A = V_B = \frac{Q}{C}$

$E = V/d$

emf of battery $= \mathcal{E} = 2V = \frac{2Q}{C}$ — (I)

• Now $C_A = KC$; $C_B = C$

$V_A' + V_B' = \mathcal{E} \Rightarrow V_A' = \frac{Q'}{C_A} = \frac{Q'}{KC}$

$V_B' = \frac{Q'}{C_B} = Q'/C$

$\Rightarrow \mathcal{E} = \frac{Q'}{C}\left(1 + \frac{1}{K}\right)$ — (II)

$$\boxed{Q' = \left(\frac{2K}{1+K}\right) Q} \Rightarrow K > 1 \Rightarrow \boxed{Q' > Q}$$

$V_A' = \frac{2V}{K+1}$, $V_B' = \frac{2K}{K+1} \cdot V$

E. field & p.d. in cap. A ↓ by factor $\left(\frac{2}{K+1}\right)$ while in B ↑ by $\frac{2K}{K+1}$

$\Delta Q = Q' - Q = \left(\frac{K-1}{K+1}\right)\frac{C\mathcal{E}}{2}$

$U_{in} = 2 \cdot \frac{1}{2} CV^2 = CV^2$

$U_f = \frac{1}{2} C_A V_A'^2 + \frac{1}{2} C_B V_B'^2 = \left(\frac{4K}{1+K}\right)\frac{1}{2}(CV^2)$

$= \left(\frac{2K}{K+1}\right) U_{in}$

(24) S_1 : closed

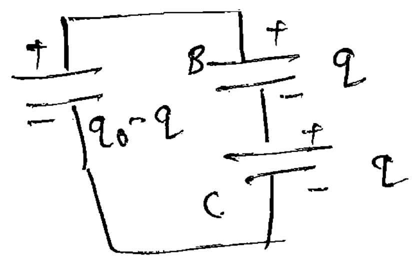
$$Q_A = C_1 V = 360\ \mu C$$

Energy supplied by battery $= Q_A \cdot \mathcal{E} = 0.0648\ J$

$$U_A = \frac{1}{2} Q_A \cdot \mathcal{E} = 0.0324\ J$$

S_1 : opened , S_2 : closed

$$q_0 = Q_A = 360\ \mu C$$



$$\frac{q}{C_2} + \frac{q}{C_3} - \left(\frac{q_0 - q}{C_1}\right) = 0$$

$$\boxed{q = 180\ \mu C}$$

$$U_{system} = U_A + U_B + U_C = \frac{(q_0 - q)^2}{2C_1} + \frac{q^2}{2C_2} + \frac{q^2}{2C_3}$$

$$= 0.0162\ J$$

$E_B = q/\epsilon_0 A$

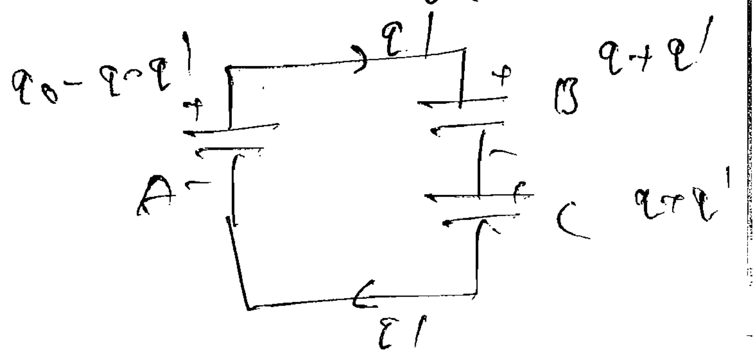
S_2 : open   dielectric slab of A → removed $C_1' = \frac{C_1}{K}$
$= 1\ \mu F$

Another slab $K = 2$ is inserted in B

$$C_2' = KC_2 = 6\ \mu F$$

Now S_2 is closed ⇒ redistribution of charge

$$\frac{q+q'}{C_2'} + \frac{q+q'}{C_3'} - \frac{q_0 - q - q'}{C_1'} = 0$$



$$\boxed{q' = 90\ \mu C}$$

Final charge on B = $Q + Q' = 220\ \mu C$

$$E' = \frac{Q+Q'}{\epsilon_0 K A}$$

$$\frac{E'}{E} = 0.75$$

$$\text{Energy lost} = \frac{(-Q')^2}{2C_1'} + \frac{(Q')^2}{2C_2} + \frac{(Q')^2}{2C_3'}$$

$$= 5.4\ mJ$$

(25) use Kirchoff's junction & loop rule.

(26) use concept of inverse symmetry.

(27) use Kirchoff's junction & loop rule.

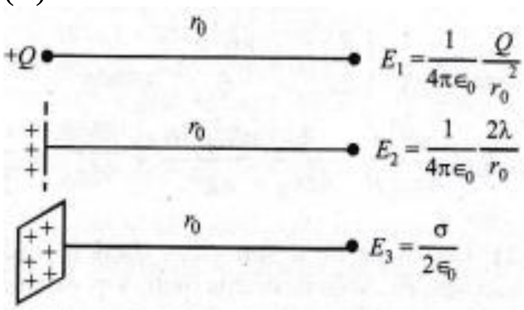
Only One Option Correct

1. (C)

$E_1 = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{R^2}$; $E_2 = \frac{1}{4\pi \epsilon_0} \cdot \frac{2Q}{R^2}$; $E_3 = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q/2}{R^2}$

Clearly $E_2 > E_1 > E_3$

2. (C)



$E_1(r_0) = E_2(r_0) = E_3(r_0)$ (Given)

(a) $E_1(r_0) = E_3(r_0) \therefore \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2} = \frac{\sigma}{2\epsilon_0} \Rightarrow Q = 2\pi\sigma r_0^2$

(b) $E_2(r_0) = E_3(r_0)$

$\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0} = \frac{\sigma}{2\epsilon_0} \Rightarrow r_0 = \frac{\lambda}{\sigma\pi}$

(c) $E_1(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{4Q}{r_0^2}$

$= \frac{1}{4\pi\epsilon_0} \times \frac{4 \times 2\lambda r_0}{r_0^2} = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$

$\because \quad E_1(r_0) = E_2(r_0) \quad \therefore \quad Q = 2\lambda r_0$

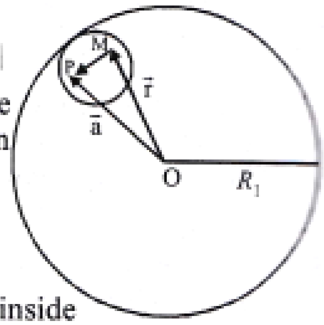
And $2E_2(r_0/2) = 2\left[\frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0}\right] = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$

(d) $E_2(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0/2} = \frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0} = \frac{\lambda}{\pi\epsilon_0 r_0}$

$4E_3(r_0/2) = \frac{4\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0} = \frac{2}{\epsilon_0} \times \frac{\lambda}{\pi r_0}$

3. (D)

Let us consider a point M inside the cavity where electric field has to be calculated. Assume the cavity to contain similar charge distribution of positive and negative charge as the rest of sphere. Electric field at M due to uniformly distributed charge of the whole sphere of radius R_1



$$\vec{E}_1 = \frac{\rho}{3\varepsilon}\vec{r}$$

Electric field at M due to the negative charge distribution in the cavity

$$\vec{E}_2 = \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

∴ The total electric field at M inside the cavity,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\left(\vec{a} - \vec{r}\right)\left[\because \ \vec{r} + \overrightarrow{MP} = \vec{a}\right]$$

$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{a}$$

Hence inside the cavity $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$.

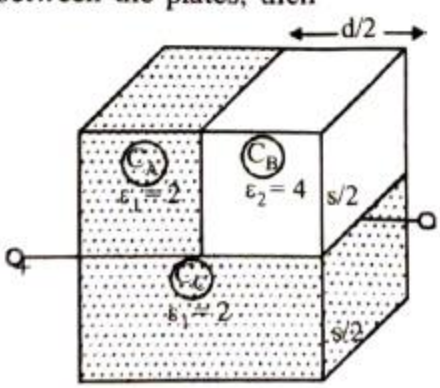
4. (D)

(d) As we know, the capacitance of a parallel, plate capacitor

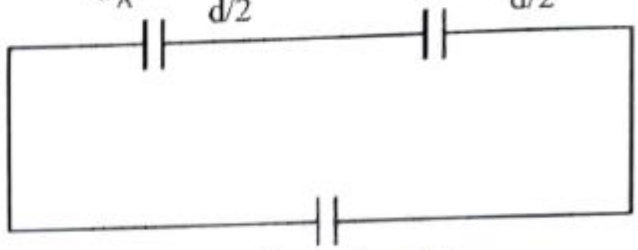
$$C = \frac{\varepsilon_0 A}{d}$$

Initialy, capacitance, $C_1 = \dfrac{\varepsilon_0 s}{d}$

When two dielectrics of permittivity $\varepsilon_1 = 2$ and $\varepsilon_2 = 4$ are introduced between the plates, then



$C_A = \frac{2\,\varepsilon_0\, s/2}{d/2}$ $\qquad C_B = \frac{4\,\varepsilon_0\, s/2}{d/2}$



$$C_C = \frac{2\,\varepsilon_0\, s/2}{d} = \frac{\varepsilon_0 s}{d}$$

$$C_2 = \frac{C_A \times C_B}{C_A + C_B} + C_C = \frac{\frac{2\varepsilon_0 s}{d} \times \frac{4\,\varepsilon_0 s}{d}}{\frac{6\varepsilon_0 s}{d}} + \frac{\varepsilon_0 s}{d}$$

$$= \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d}$$

$$\therefore \quad C_2 = \frac{7}{3}\frac{\varepsilon_0 s}{d} = \frac{7}{3}C_1 = \frac{C_2}{C_1} = \frac{7}{3} \qquad \left[\because C_1 = \frac{\varepsilon_0 s}{d}\right]$$

One or More than One Option Correct

1. (A, D)

   Clearly from figure electric field lines are originating from Q_1 and terminating on Q_2.

   $\therefore$ Q_1 is positive and Q_2 is negative.

   Number of electric field lines originating from Q_1 is more then terminating at Q_2.

   $\therefore$ $|Q_1| > |Q_2|$.

   Since at a finite distance to the right of Q_2, the electric field is zero. The electric field created by Q_2 at a particular point will cancel out the electric field created by Q_1. But at a finite distance to the left of Q_1 the electric field is non-zero.

2. (A, C, D)

   Due to symmetry, the net electric flux passing through $x = +\frac{a}{2}, x = -\frac{a}{2}, z = +\frac{a}{2}$ is same

   According to Gauss's theorem, net electric flux net electric flux crossing through any closed surface

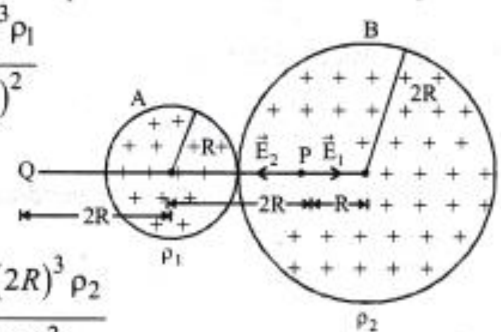
   $\phi = \frac{q_{in}}{\varepsilon_0}$

   $= \frac{-q + 3q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$.

3. (B, D)

Electric field E_1 due to smaller sphere 'A' at Q

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2}$$



Electric field E_2 due to bigger sphere 'B' at Q

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2}{(5R)^2}$$

$E_1 + E_2 = 0$

$$\frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2} + \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho 2}{(5R)^2} = 0$$

$$\therefore \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

Electric field E_1 due to smaller sphere 'A' at P

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\rho_1 \times \frac{4}{3}\pi R^3}{(2R)^2} = \frac{1}{4\pi\epsilon_0}\times\frac{\rho_1\pi R}{3} = \frac{\rho_1 R}{4\epsilon_0 \times 3}$$

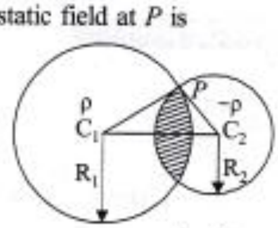
Electric field E_2 due to bigger sphere 'B' at P

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2 R}{(2R)^3} = \frac{\rho_2 R}{3\varepsilon_0}$$

$$E_1 = E_2 \qquad \therefore \frac{\rho_1 R}{4\epsilon_0 \times 3} = \frac{\rho_2 R}{3\epsilon_0} \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = 4$$

4. (C, D)

(c, d) Electrostatic field at P is



$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1P} + \frac{(-\rho)}{3\varepsilon_0}\overrightarrow{C_2P}$$

$$= \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1P} - \overrightarrow{C_2P}\right) = \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1P} + \overrightarrow{PC_2}\right) = \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1C_2}\right)$$

5. (B, D)

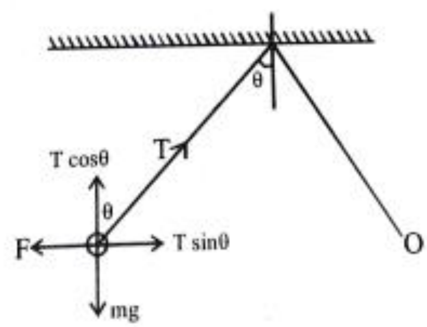
(b, d) When switch S_1 is pressed, the capacitor C_1 gets charged such that its upper plate acquires a positive charge $+ 2\ CV_0$ and lower plate $- 2\ CV_0$.

When switch S_2 is pressed and S_1 is release. As $C_1 = C_2$ the charge gets distributed equal. The upper plates of C_1 and C_2 now take charge $+ CV_0$ each and lower plate $- CV_0$ each.

When S_2 is released and S_3 is pressed, the charge on the upper plate of C_1 is CV_0 and charge on upper plate of C_2 is $- CV_0$.

6. [A, C]

The net electric force on any sphere is lesser but by Coulomb law the force due to one sphere to another remain the same.



In equilibrium

$T \cos\theta = mg$

and $T \sin \theta = F$

$\therefore \tan\theta = \dfrac{F}{mg}$ ...(i)

As force between two charge bodies according to coulomb's law does not depend upon the medium, hence force between them remain same.

Hence after immersed in dielectric medium

As given no change in angle θ.

$T' \sin\theta = \dfrac{F}{\varepsilon_r}$ and $T' \cos\theta = mg\left(1 - \dfrac{\rho_l}{\rho_s}\right)$

$\therefore \tan\theta = \dfrac{F}{mg(1 - \frac{\rho_l}{\rho_s})} \dfrac{1}{\varepsilon_r}$ ...(ii)

Now equating eqn. (i) & (ii)

$1 - \dfrac{\rho_l}{\rho_s} = \dfrac{1}{\varepsilon_r} \Rightarrow 1 - \dfrac{800}{\rho_s} = \dfrac{1}{21}$

$\therefore \rho_s = 840\ kg/m^3$

Hence option (c) is correct

And tension, $T' = \dfrac{T}{\varepsilon_r} = \dfrac{T}{21}$

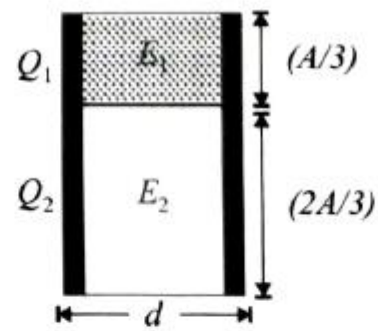
Hence option (d) is incorrect.

7. (A, D)

(a, d) The given capacitor is equivalent to two capacitors in parallel with capacitances

$$C_1 = \frac{K\varepsilon_0(A/3)}{d} = \frac{K\varepsilon_0 A}{3d}$$

$$C_2 = \frac{\varepsilon_0(2A/3)}{d} = \frac{2\varepsilon_0 A}{3d}$$



A = area of each plate and

d = distance between the plates

$\therefore C = C_1 + C_2$

$$= \frac{K\varepsilon_0 A}{3d} + \frac{2\varepsilon_0 A}{3d} = \frac{\varepsilon_0 A}{3d}(K+2)$$

$$\therefore \frac{C}{C_1} = \frac{\frac{\varepsilon_0 A(K+2)}{3d}}{\frac{\varepsilon_0 AK}{3d}} = \frac{K+2}{K}$$

Let V be potential difference between the plates

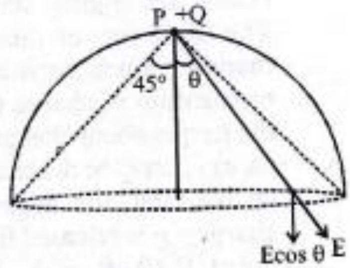
$$E_1 = \frac{V}{d} \text{ and } E_2 = \frac{V}{d} \qquad \therefore \quad \frac{E_1}{E_2} = 1$$

$$Q_1 = C_1 V = \frac{K\varepsilon_0 A}{3d} V \text{ and } Q_2 = C_2 V = \frac{2\varepsilon_0 A}{3d} V$$

$$\therefore \frac{Q_1}{Q_2} = \frac{K}{2}$$

8. (A, D)

(a, d) The circumference of the flat surface is an equipotential $V = \frac{KQ}{\sqrt{2}R}$ because the circumference is equidistant from +Q. The component of electric field per-pendicular to the flat surface is $E\cos\theta$.



Here E as well as θ changes for different point on the flat surface. The total flux through the curved and flat surface should be less than $\frac{Q}{\varepsilon_0}$.

The solid angle subtended by the flat surface at P

$= 2\pi(1-\cos\theta) = 2\pi(1-\cos 45°) = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$

$\therefore$ Flux passing through curved surface

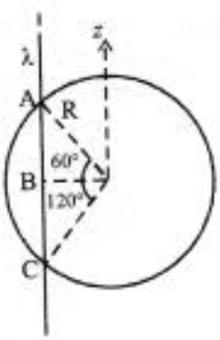
$$= -\frac{Q'}{\varepsilon_0}\frac{2\pi\left(1-\frac{1}{\sqrt{2}}\right)}{4\pi} = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right).$$

9. (A, B)

(a, b) According to Gauss's Law, Electric flux, ϕ

$= \frac{1}{\epsilon_0}q_{in} = \frac{1}{\epsilon_0}[\lambda \times 2R\sin 60°]$

$= \frac{\sqrt{3}\lambda R}{\epsilon_0}$

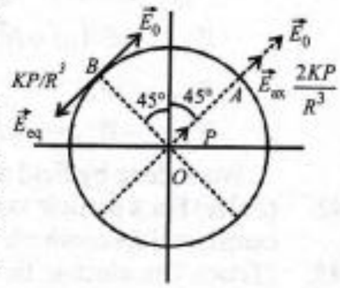


AB = R sin 60° or AC = 2R sin 60°

Also, electric field is perpendicular to the wire therefore its z-com ponent will be zero.

10. (B, D)

Given dipole moment of electric dipole



$\vec{P} = \frac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$ and circle is equipotential. Also electric field is normal to such a line that is the direction of electric field is either radial or the magnitude of electric field should be zero at points on the circle.

Now considering point A, the electric field due to dipole $\frac{2Kp}{R^3}$ (directed from O to A) as point A lies on the axial line of electric dipole. The external electric field E_0 should also be in the direction of O to A.

Now considering point B which is a point on the equitorial line of the electric dipole. The electric field here due to dipole is $\frac{Kp}{R^3}$ in a direction opposite to the dipole. The external electric field should cancel out this field. i.e., $E_B = 0$

Further $\vec{E}_0 = \frac{-K\vec{p}}{R^3}$ ...(i)

The electric field at A

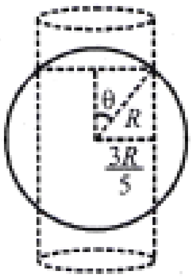
$$\vec{E}_A = \frac{2K\vec{p}}{R^3} + \vec{E}_0 = -2\vec{E}_0 + \vec{E}_0 = -\vec{E}_0$$

Again from eq. (i) $E_0 = \frac{1}{4\pi\epsilon_0}\frac{p_0}{\sqrt{2}(R^3)} \times \sqrt{2}$

$$\therefore \quad R^3 = \frac{p_0}{4\pi\epsilon_0 E_0} \quad \therefore \quad R = \left[\frac{p_0}{4\pi\epsilon_0 E_0}\right]^{1/3}$$

11. (A, B, C)

(a, b, c) (a) For $h > 2R$ and $r = \frac{3R}{5}$



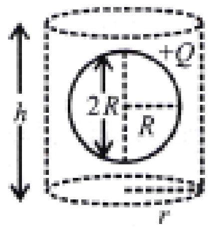
$$\sin\theta = \frac{3R/5}{R} = \frac{3}{5} = 37^\circ$$

$$q_{in} = Q[1 - \cos 37^\circ] = Q\left[1 - \frac{4}{5}\right] = \frac{Q}{5}$$

From Gauss's theorem $Q = \frac{q_{in}}{\varepsilon_0}$

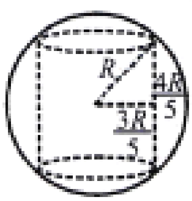
$$\therefore \quad \phi = \frac{Q}{5\,\epsilon_0}$$

(b) For $h > 2R$ and $r > R$



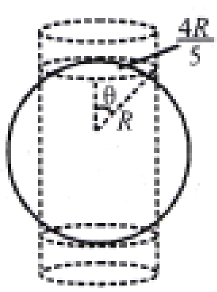
$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

(c) For $h < \frac{8}{5}R$ and $r = \frac{3}{5}R$



$$\phi = \frac{q_{in}}{\epsilon_0} = 0$$

(d) For $h > 2R$ and $r > \frac{4}{5}R$



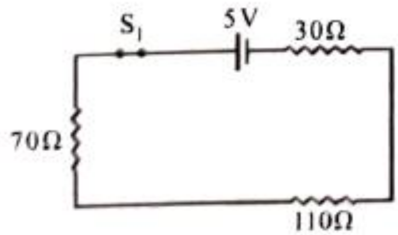
$$\sin\theta = \frac{4R/5}{R} = \frac{4}{5} = 0.8 \Rightarrow \theta = 53^\circ$$

$$q_{in} = Q[1 - \cos\theta] = Q\left[1 - \frac{3}{5}\right] = \frac{2Q}{5}$$

$$\therefore \quad \phi = \frac{2Q}{5\,\epsilon_0}$$

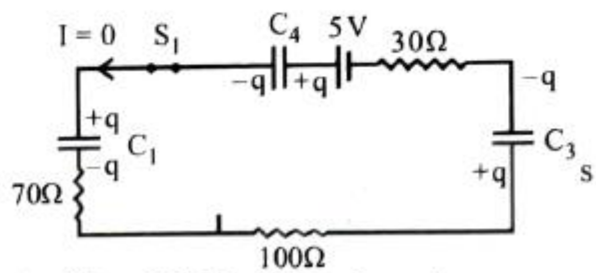
12. (C, D)

(c, d) +S_1 closed and S_2 open then at $t = 0$, charge on each capacitor is zero.



$$\therefore I = \frac{V}{R} = \frac{5}{70+100+30} = 0.025\text{ A} = 25\text{ mA}.$$

When switch S_1 is closed for a long time all the capacitors are fully charged. As the capacitors are in series these carry equal charge q. Current in the circuit is now zero as circuit is in steady state.



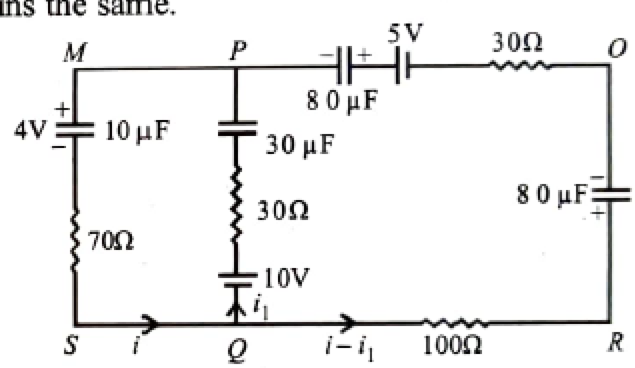
Applying Krichhoff's voltage law

$$5 - \frac{q}{80} - \frac{q}{10} - \frac{q}{80} = 0 \qquad \therefore \text{ q} = 40\ \mu\text{c}$$

Potential difference across C_1

$$V = \frac{q}{C_1} = \frac{40 \times 10^{-6}}{10 \times 10^{-6}} = 4\text{V}$$

Now just after closing the switch S_2 charge on each capacitor remains the same.



$V_P - 4 - 70 \times 25 \times 10^{-3} = V_Q$

$\therefore \quad V_P - V_Q = 4 + 1.75 = 5.75$ V

In loop MPQS

$+10 - 30\,i_1 - 4 - 70\,i = 0$

$70\,i + 30\,i_1 = 6$ ..... (i)

In loop QROPQ,

$$+10 - 30\,i_1 + \frac{40}{80} - 5 + (i - i_1) \times 130 + \frac{40}{80} = 0$$

$130\,i - 160\,i_1 = -6$ ..... (ii)

Solving (i) & (ii), we get $i = 0.05$ A

$\therefore \quad i_1 = 0.077$ A

13. (B, C)

Given: electric field, $\vec{E}_y = -400\sqrt{3}\ NC^{-1}$

Initial speed of charged particle, $u = 2\sqrt{10} \times 10^6$ m/s

Range, R = 5 m

F = ma and F = qE $\therefore a_y = \frac{q}{m}E_y = -400\sqrt{3} \times 10^{10}$

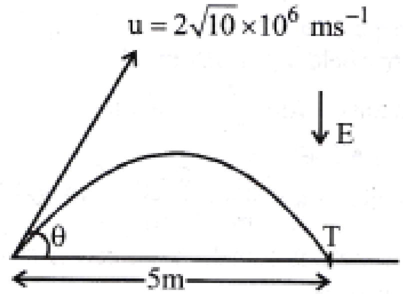
$$\left[\because \frac{q}{m} = 10^{10\,Ckg^{-1}} \text{ (given)}\right]$$

$$\text{Range}, R = \frac{u^2 \sin 2\theta}{a_y} \Rightarrow 5 = \frac{40 \times 10^{12} \sin 2\theta}{400\sqrt{3} \times 10^{10}}$$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$\Rightarrow 2\theta = 60^\circ$ or, $120 \Rightarrow \theta = 30^\circ$ or, 60°

Particle hits the target, if $\theta = 30^\circ$ or $\theta = 60^\circ$



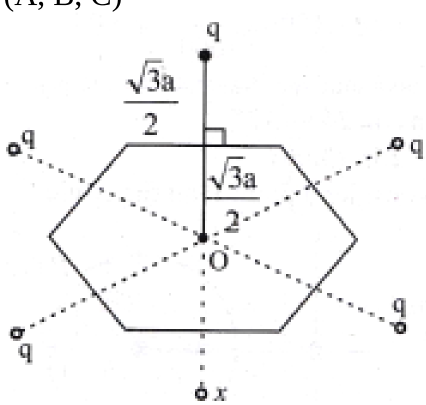
$$\text{Time of flight } T_1 = \frac{2u \sin\theta}{a_y} = \frac{2 \times 2\sqrt{10} \times 10^6 \times \frac{1}{2}}{400\sqrt{3} \times 10^{10}}$$

$$= \sqrt{\frac{5}{6}} \mu s \text{ (for } \theta = 30^\circ)$$

$$\text{Time of flight } T_2 = \frac{2u \sin\theta}{a_y} = \frac{2 \times 2\sqrt{10} \times 10^6 \times \frac{\sqrt{3}}{2}}{400\sqrt{3} \times 10^{10}}$$

$$= \sqrt{\frac{5}{6}} \mu s \text{ (for } \theta = 60^\circ)$$

14. (A, B, C)



(a) When x = q, the situation is symmetric
So, electric field at O is zero.
$\Rightarrow$ (a) is correct.

(b) When x = –q, then $E_0 = \frac{1}{4\pi \epsilon_0}\frac{q}{(\sqrt{3}a)^2} \times 2$

$\Rightarrow E_0 = \frac{q}{6\pi \epsilon_0 a^2}$

$\Rightarrow$ (b) is correct.

(c) When x = 2q

$V_0 = 5 \times \frac{Kq}{\sqrt{3}a} + \frac{K(2q)}{\sqrt{3}a} \Rightarrow V_0 = \frac{7q}{4\sqrt{3}\pi \epsilon_0 a}$

$\Rightarrow$ (c) is correct.

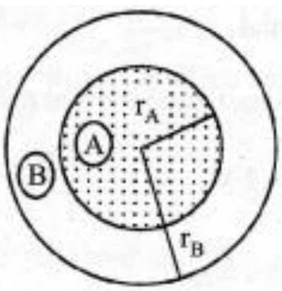
(d) When x = –3q

$V_0 = 5 \times \frac{Kq}{\sqrt{3}a} + \frac{K(-3q)}{\sqrt{3}a} = \frac{1}{4\pi \epsilon_0}\frac{2q}{\sqrt{3}a} = \frac{q}{2\sqrt{3}\pi \epsilon_0 a}$

$\Rightarrow$ (d) is incorrect.

15. (B)

We have



$q_A = \int_0^1 kr(4\pi r^2 dr) = \frac{4\pi k}{4}\left[R^4\right]_0^1 = \pi k$

$q_B = \int_1^r \frac{2k}{r} \times 4\pi r^2 dr = 8\pi k\left(\frac{r^2 - 1}{2}\right)$

$= 4\pi kr^2 - 4\pi k$

So, $q_{net} = q_A + q_B = 4\pi kr^2 - 3\pi k$

$= \pi k(4r^2 - 3)$

(A) $E_{net} = 0 \Rightarrow q_{net} = 0 \Rightarrow 4r^2 - 3 = 0 \Rightarrow r = \frac{\sqrt{3}}{2}$

So (a) is incorrect

(B) $V = \frac{1}{4\pi \epsilon_0}\frac{q_{net}}{r} = \frac{1}{4\pi \epsilon_0}\frac{\pi k(4r^2 - 3)}{r}$

$\Rightarrow V = \frac{k}{4 \epsilon_0}\left(4r - \frac{3}{r}\right) \Rightarrow V = \frac{k}{4 \epsilon_0}\left(4 \times \frac{3}{2} - \frac{3}{3} \times 2\right)$

$\Rightarrow V = \frac{k}{\epsilon_0}$. So (b) is correct.

(C) If $r_B = 2$ i.e $r = 2$.
then, $q_{net} = \pi k(16-3)$
$= 13\pi k$. So (c) is incorrect.

(D) $E = \frac{k\, q_{net}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\pi k(4r^2-3)}{r^2} = \frac{k}{4\epsilon_0}\left[4-\frac{3}{r^2}\right]$

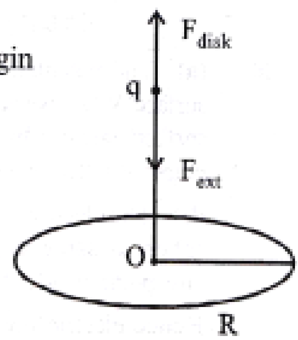
$E = \frac{k}{4\epsilon_0}\left[4-\frac{3}{25}\times 4\right] = \frac{k}{4\epsilon_0}\left[\frac{88}{25}\right] = \frac{22k}{25\epsilon_0}$

16. (A, C, D)

(a, c, d) Particle will reach the origin only if

$\Delta K \geq 0$

$\Rightarrow w_{el} + w_{ext} > 0$



$\Rightarrow \frac{\sigma q}{2\epsilon_0}\left[\left(\sqrt{R^2+0^2}-0\right)+\left(\sqrt{R^2+Z_0^2}-Z_0\right)\right]+CZ_0 \geq 0$

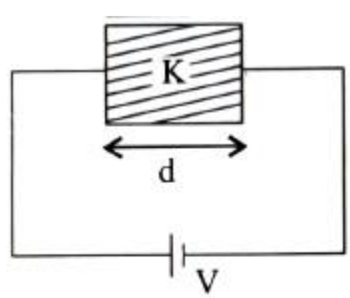
$\Rightarrow \frac{\sigma q}{2\epsilon_0 C}\left(R-\sqrt{R^2+Z_0^2}+Z_0\right)+\frac{CZ_0}{C} \geq 0$

$\Rightarrow \frac{1}{\beta}\left(R+Z_0-\sqrt{R^2+Z_0^2}\right)+Z_0 \geq 0$

Substitute Z_0 and β, and check the condition.
If $\Delta K > 0$, particle will reach origin if $\Delta K > 0$ otherwise if will not reach origin.

17. (B)

(b) Case a:

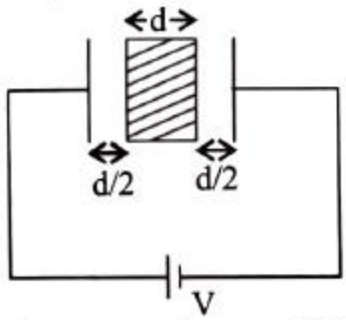


$C_a = \frac{KA\epsilon_0}{d}$

$q_a = C_a V = \frac{KA\epsilon_0}{d}V$

$E_a = \frac{q_a}{AK\epsilon_0} = \frac{KA\epsilon_0 V}{d.AK\epsilon_0} = \frac{V}{d}$

Case b:



$$C_b = \frac{A \in_0}{2d + \frac{d}{k} - d} = \frac{A \in_0}{d + \frac{d}{k}} = \frac{KA \in_0}{d(K+1)}$$

$$q_b = \frac{\in_0 AKV}{d(K+1)}$$

$$(E_b)_{dielectric} = \frac{(E_{air})_b}{K} = \frac{q_b}{KA \in_0} = \frac{\in_0 AKV}{d(K+1)KA \in_0}$$

$$= \frac{V}{d(K+1)}$$

So, capacitance decreases by factor of $\frac{1}{K+1}$ and same is true for electric field. So (a) is incorrect and (b) is correct work done in process, $U_f - U_i$

$$= \frac{1}{2}(C_f - C_i)V^2$$

$$= \frac{1}{2}V^2 \cdot \frac{KA \in_0}{d}\left(\frac{1}{K+1} - 1\right) = \frac{1}{2}V^2 \frac{KA \in_0}{d}\left(\frac{-K}{K+1}\right)$$

$$= \frac{1}{2}\frac{\in_0 AV^2}{d}\left(\frac{-K^2}{K+1}\right).$$

So option (d) is incorrect.

As voltage supply remains constant, so there will be no change in voltage across capacitor. So option (c) is incorrect.

Comprehensions Type

1. (A)

(a) In process 1, work done by battery, $w = q \times V = CV_o \times V_o = CV_o^2$

Energy stored in the battery $E_C = \frac{1}{2}CV_0^2$

Heat dissipated across resistance,

$E_D = W - E_C = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$

$\therefore \quad E_C = E_D$

2. (C)

(c) Let V_i and V_f be the initial and final voltage in each step of process 2. Then

Energy dissipated = $W_{battery} - \Delta U$

$= C(V_f - V_i)V_f - \frac{1}{2}C(V_f - V_i)^2$

$= \frac{1}{2}C(V_f - V_i)^2$

$\therefore$ Total energy dissipated across the resistance,

$$E_D = \frac{1}{2}C\left[\left(\frac{V_o}{3} - 0\right)^2 + \left(\frac{2V_o}{3} - \frac{V_o}{3}\right)^2 + \left(V_o - \frac{2V_o}{3}\right)^2\right]$$

or, $E_d = \frac{1}{6}CV_o^2$

3. (C)

(c) After colliding the top plate, the ball will gain negative charge and get repelled by the top plate and bounce back to the bottom plate. But ball do not execute simple harmonic motion as force on it $\propto x$

4. (D)

(d) Average current, $I_{av} \propto \frac{Q}{t}$ ...(i)

Here $Q \propto V_0$ ...(ii)

From $S = ut + \frac{1}{2}at^2$

$$h = \frac{1}{2}\frac{QE}{m}t^2 = \frac{1}{2}\left(\frac{Q \times 2V_0}{mh}\right) \times t^2 \quad \left(\because \; a = \frac{F}{m} = \frac{qE}{m}\right)$$

$\therefore \quad t \propto \frac{1}{V_0}$ – (iii) $\quad [\because Q \propto V_0]$

From eq. (i), (ii) and (iii)

$$I_{av} \propto \frac{V_0}{1/V_0} = I_{av} \propto V_0^2$$

Matrix-Match Type

1. [A]

If Q_1, Q_2, Q_3 and Q_4 are all positive, then the force will be along +*y*-direction as component of force along *x*-axis cancel out each other.

If Q_1, Q_2 are positive and Q_3, Q_4 are negative the force will act along +*x*-direction as components of forces along *y*-axis cancel out each other.

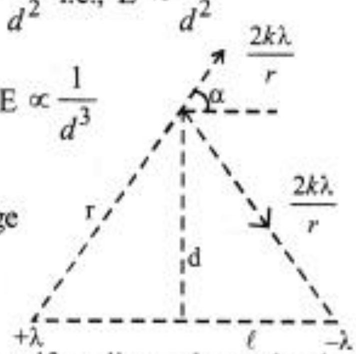
If Q_1, Q_4 are positive and Q_2, Q_3 are negative then attractive force will dominate repulsive force and the force will be along –*y* direction.

If Q_1, Q_3 positive and Q_2, Q_4 negative components of forces along *y*-axis cancel out each other. So net force on charge *q* along *x*-axis.

2. (B)

For a point charge $E = \frac{kQ}{d^2}$ i.e., $E \propto \frac{1}{d^2}$

and for a dipole $E = \frac{kp}{d^3}$ i.e., $E \propto \frac{1}{d^3}$



For an infinite long line charge

$E = \frac{2k\lambda}{d}$ i.e., $E \propto \frac{1}{d}$

For two infinite wires carrying uniform linear charge density.

$$E = \frac{2k\lambda}{r}\cos\alpha = \frac{2k\lambda}{\sqrt{d^2+\ell^2}} \times \frac{\ell}{\sqrt{d^2+\ell^2}} = \frac{2k\lambda\ell}{d^2+\ell^2}$$

or, $E \propto \frac{1}{d^2} \because 2l << d$

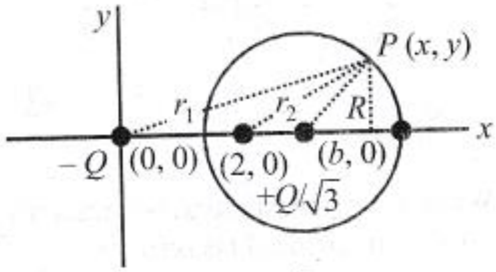
For infinite plane charge $E = \frac{\sigma}{2\epsilon_0}$ i.e., E is in dependent of d

Stem based Questions

1. (1.73)

2. (3)

Let us Consider a point P on the circle



$$V_P = 0 = \frac{k(-Q)}{r_1} + \frac{kQ/\sqrt{3}}{r_2} \Rightarrow \frac{kQ}{r_1} = \frac{kQ/\sqrt{3}}{r_2}$$

$$\Rightarrow \frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\sqrt{3}\sqrt{(x-2)^2+y^2}}$$

$\Rightarrow 3(x-2)^2 + 3y^2 = x^2 + y^2$

$\Rightarrow 3(x^2+4-4x) - x^2 + 2y^2 = 0$

$\Rightarrow 2x^2 + 12 - 12x + 2y^2 = 0$

$\Rightarrow x^2 + 6 - 6x + y^2 = 0$

$\Rightarrow (x-3)^2 + y^2 = (\sqrt{3})^2$

or $(x-b)^2 + y^2 = (\sqrt{3})^2 = R^2$

$\therefore R = \sqrt{3} = 1.73$ and $b = 3$

Integer / Numerical Answer Type

1. (6)

(6) The magnitude of the electric field at the point P which is at a distance $2R$ from the axis of the cylinder

$E = E_{total} - E_{cavity}$

$$= \frac{\gamma}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

$$Q_{sphere} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho = \frac{\pi R^3\rho}{6};\ \lambda_{cylinder} = \pi R^2\rho$$

$$\therefore E = \frac{\pi R^2\rho}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0}.\frac{\pi R^3\rho/6}{4R^2} = \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{16\times6\times\varepsilon_0}$$

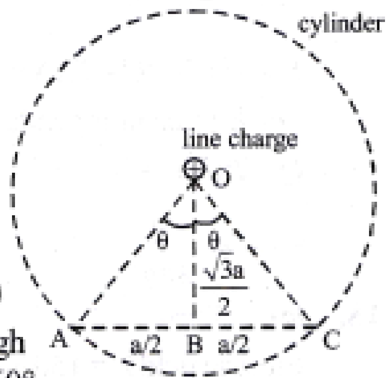
$\therefore k = 6$

2. (6)

(6) From figure $\tan\theta = \frac{BC}{OB} = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta = 30^\circ$

Electric flux through the complete cylinder by Gauss's theorem



ϕ cylinder = $\frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$

(Where L = length of cylinder)

$\therefore$ Electric flux passing through cylindrical surface i.e., for 60° angle = $\frac{\lambda L}{6\varepsilon_0}$ Hence, n = 6

3. (2)

(2) $\because$ F = ma $\therefore$ qE = $m\frac{dv}{dt}$

$\Rightarrow dv = \frac{qEdt}{m} = \frac{q\sin 1000t\,\hat{i}}{m}dt$

($\because$ E = $\sin 10^3 t\,\hat{i}$ given)

$\therefore \int_0^v dv = \frac{q}{m}\int_0^{\pi/\omega} \sin 1000t\,dt \left[\text{max. speed is at } \frac{T}{2} = \frac{2\pi}{\omega \times 2}\right]$

$\therefore \quad V = -\frac{q}{m}\left[\frac{\cos 1000t}{1000}\right]_0^{\pi/\omega} = -\frac{1}{10^{-3}} \times \frac{[\cos 1000t]_0^{\pi/\omega}}{1000}$

$\left(\begin{array}{l}\because m = 10^{-3}\,kg, q = 1C, E_0 = 1NC^{-1} \\ and\ w = 10^3 \text{ rads}^{-1} \text{ given}\end{array}\right)$

$\therefore \ V = -\left[\cos 1000 \times \frac{\pi}{1000} - \cos 0\right] = -[-1-1] = 2\ \text{ms}^{-1}$

Hence maximum speed attained by the particle.

4. (1.5)

(1.50) When switch S_1 is closed and S_2 is opened, capacitor C_3 becomes fully charged.

$\therefore$ Charge on capacitor C_3, $q_3 = C_3V = 1 \times 8\ \mu C = 8\ \mu C$

When switch S_2 is closed and S_1 is opened,

When all the capacitors reach equilibrium charge on C_3 is found to be 5 μC therefore charges on C_1 and C_2 are 3 μC each

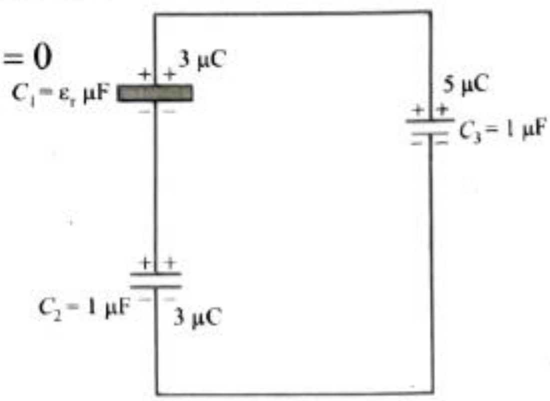
Applying Kirchhoff loop rule

$$\frac{CV_0 - q}{C} - \frac{q}{\varepsilon_r C} - \frac{q}{C} = 0$$

$$\Rightarrow \frac{5}{1} - \frac{3}{\varepsilon_r} - \frac{3}{1} = 0$$

$$\therefore 5 = 3\left[1 + \frac{1}{\varepsilon_r}\right]$$

$$\therefore \frac{1}{\varepsilon_r} = \frac{5}{3} - 1 = \frac{2}{3} \qquad \therefore \varepsilon_r = 1.5$$



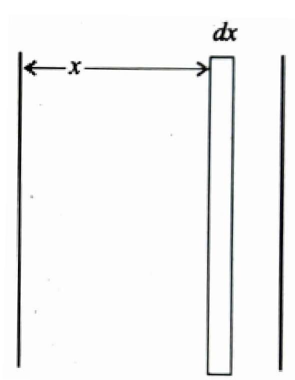
5. (1)

Let the region between the plates is filled with N dielectric layers.

m = number of electric layers within x

d = distance between e plates.

Here, $\frac{x}{m} = \frac{d}{N}$



$$dc = \frac{k\left(1 + \frac{m}{N} \in_0 A\right)}{dx}$$

$$\therefore \frac{1}{dc} = \frac{dx}{k\left(1 + \frac{m}{N}\right) \in_0 A}$$

$$\frac{1}{C} = \int dc = \int_0^d \frac{dx}{k\left(1 + \frac{m}{N}\right) \in_0 A}$$

$$= \int_0^d \frac{dx}{k\left(1 + \frac{x}{d}\right) \in_0 A} = \frac{d}{k \in_0 A} \int_0^d \frac{dx}{d + x}$$

or, $\frac{1}{C} = \frac{d}{k \in_0 A} \ln_\alpha \Rightarrow C = \frac{k \in_0 A}{d \ln^\alpha}$

Comparing this equivalent capacitance

$C=\frac{k\in_0 A}{d\ln_\alpha}$ with $\alpha\left(\frac{k\in_0 A}{d\ln_\alpha}\right)$ we get, $\alpha=1.00$

6. (6.40)

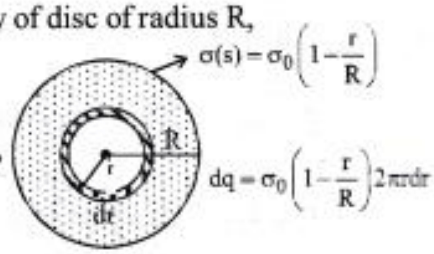
(6.40) Let us consider a ring element of radius r and thickness dr

Surface charge density of disc of radius R,

$\sigma(r)=\sigma_0\left(1-\frac{r}{R}\right)$

Charge of disc element,

$dq=\sigma_0\left(1-\frac{r}{R}\right)\,2\pi r dr$



Now from Gauss's theorem, Electric flux, through a large spherical surface that encloses the charged disc completely.

$$\phi_0=\frac{\int dq}{\varepsilon_0}=\frac{\int_0^R \sigma_0\left(1-\frac{r}{R}\right)2\pi r dr}{\varepsilon_0}$$

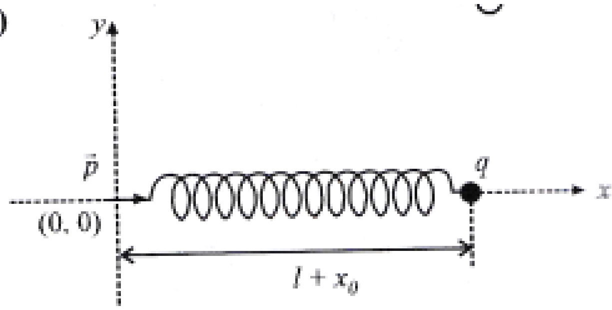
Electric flux through another spherical surface of radius R/4

$$\phi=\frac{\int dq}{\varepsilon_0}=\frac{\int_0^{R/4} \sigma_0\left(1-\frac{r}{R}\right)2\pi r dr}{\varepsilon_0}$$

$$\therefore \frac{\phi_0}{\phi}=\frac{\sigma_0 2\pi\int_0^R\left(r-\frac{r^2}{R}\right)dr}{\sigma_0 2\pi\int_0^{R/4}\left(r-\frac{r^2}{R}\right)dr}=\frac{\frac{R^2}{2}-\frac{R^2}{3}}{\frac{R^2}{32}-\frac{R^2}{3\times 64}}=\frac{32}{5}=6.40$$

7. (3.14)

(3.14)



Original frequency, $f = \frac{2}{\delta}\sqrt{\frac{K}{m}}$

If dipole appears at equilibrium,

$$\frac{2KP}{(\ell + x_0)^3} \cdot q = Kx_0 \quad \text{...(i)}$$

When displaced towards right by length x_0

$$f_{net} = \frac{2KP}{(\ell + x_0 + x)^3} \cdot q - K(x_0 + x)$$

$$ma = \frac{2KPq}{(\ell + x_0)^3}\left[1 + \frac{x}{\ell - x_0}\right]^3 - K(x_0 + x)$$

$$= \frac{2KPq}{(\ell + x_0)^3}\left[1 - \frac{3x}{\ell + x_0}\right]^3 - K(x_0 + x)$$

$$= \frac{6KPqx}{(\ell + x_0)^4} Kx = \frac{3x}{\ell + x_0} \; Kx_0 - Kx = -Kx\left[\frac{3x_0}{\ell + x_0} + 1\right]$$

As 'ℓ' is negative, $ma = -Kx4 \Rightarrow a = \frac{-4K}{m}x$

New frequency, $f' = \frac{1}{2\pi}\sqrt{\frac{4K}{m}} = 2f = \frac{1}{\pi}\sqrt{\frac{K}{m}}$

$\therefore \delta = \pi = 3.14$

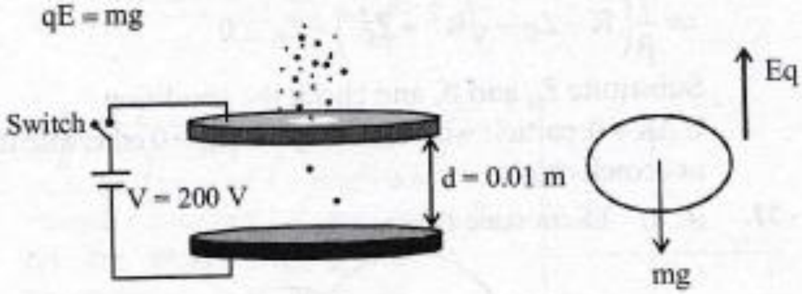
8. (6)

(6) Let n no. of electrons present in the oil drop

Electric field, $E = \frac{V}{d} = \frac{200}{0.01} = 2 \times 10^4 V/m$

When terminal velocity is achieved

$qE = mg$



As $q = ne$ and $m = \frac{4}{3}\pi R^3 \rho$

$\therefore n \times 1.6 \times 10^{-19} \times 2 \times 10^4 = \frac{4\pi}{3}(8 \times 10^{-7})^3 \times 900 \times 10$

$\Rightarrow n = \frac{4}{3}\pi \times \frac{(8 \times 10^{-7}) \times 900 \times 10}{2 \times 10^4 \times 1.6 \times 10^{-19}}$ $\therefore n = 6$

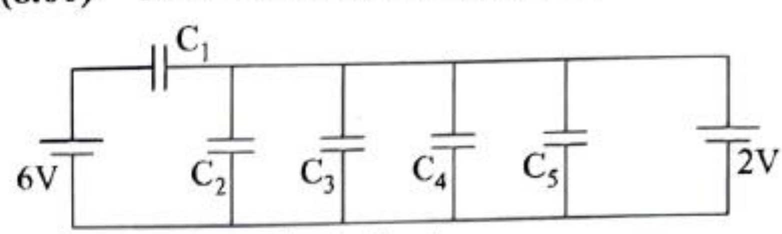
9. (3)

$\phi_{conc} = \frac{q}{2\epsilon_0} = \frac{3q}{6\epsilon_0}$

So, n = 3

10. (8.00)

(8.00) The circuit can be redrawn as



So, charge stored in C_3 is given as

$Q_3 = C_3 \times 2V = 4\mu F \times 2V = 8\mu C$