Ex 1(A). Determinant & Matrices.

1.(A) $n \neq 3k$

$$\begin{vmatrix} 1 & w^n & w^{2n} \\ w^{2n} & 1 & w^n \\ w^n & w^{2n} & 1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$.

$$\begin{vmatrix} 1+w^{2n}+w^n & w^n+1+w^{2n} & w^{2n}+w^n+1 \\ w^{2n} & 1 & w^n \\ w^n & w^{2n} & 1 \end{vmatrix} = 0.$$

2(A) $$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$$

$= (1+x)[(1+y)(1+z)-1] - 1(1+z-1) + 1(1-1-y)$

$= (1+x)(y+z+yz) - z - y$

$= (1+x)(y+z+yz) - z - y$

$= y+z+yz + xy + xz + xyz - z - y$

$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right).$

3.(C)

$$\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0.$$

$C_2 \to C_2 + C_1$

$$\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 + {}^{10}C_4 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 + {}^{11}C_6 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_8 + {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} {}^{10}C_4 & {}^{11}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{12}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{13}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0 \quad (\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_r)$$

$$\Rightarrow \quad m = 5.$$

4.(D) $a_1 = a,\ a_2 = ar,\ \ldots\ a_n = ar^{n-1}$

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$C_2 \to C_2 - C_1 \quad , \quad C_3 \to C_3 - C_1$

$$= \begin{vmatrix} \log a_n & \log\left(\frac{a_{n+1}}{a_n}\right) & \log\left(\frac{a_{n+2}}{a_n}\right) \\ \log a_{n+3} & \log\left(\frac{a_{n+4}}{a_{n+3}}\right) & \log\left(\frac{a_{n+5}}{a_{n+3}}\right) \\ \log a_{n+6} & \log\left(\frac{a_{n+7}}{a_{n+6}}\right) & \log\left(\frac{a_{n+8}}{a_{n+6}}\right) \end{vmatrix}$$

$$= \begin{vmatrix} \log a_n & \log r. & 2\log r. \\ \log a_{n+3} & \log r & 2\log r \\ \log a_{n+6} & \log r & 2\log r \end{vmatrix} = 0.$$

5.(A) $$\begin{vmatrix} 1 & 1 & 1 \\ (2^x+2^{-x})^2 & (3^x+3^{-x})^2 & (5^x+5^{-x})^2 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix} = 0.$$

6.(C) $x51 = 100x + 50 + 1$, $y41 = 100y + 40 + 1$

& $z31 = 100z + 30 + 1$

$$\Delta = \begin{vmatrix} 5 & 4 & 3 \\ 100x+50+1 & 100y+40+1 & 100z+30+1 \\ x & y & z \end{vmatrix}$$

$R_2 \to R_2 - 10R_1 - 100R_3$

$$= \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$

$C_2 \to C_2 - C_1$ $\quad$ $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 5 & -1 & -2 \\ 1 & 0 & 0 \\ x & y-x & z-x \end{vmatrix}$$

$= -(-z + x + 2y - 2x) = \cancel{z + 2y = 3x}.$

$= x - 2y + z.$

$= 0.$

7 (A) $a \neq b \neq c$

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$

$$-(x-a)\left[-(x+b)(x-c)\right] + (x-b)\left[(x+a)(x+c)\right] = 0$$

$$\Rightarrow -(x-a)\left[-(x+b)(x-c)\right] + (x-b)(x+a)(x+c) = 0.$$

$$\Rightarrow (x-a)(x+b)(x-c) + (x-b)(x+a)(x+c)$$

$$\Rightarrow x = 0.$$

B (C)
$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0.$$

$$C_1 \to C_1 + C_2 + C_3.$$

$$\Rightarrow \begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ \sin x + 2\cos x & \cos x & \sin x \end{vmatrix} = 0.$$

$$\Rightarrow (\sin x + 2\cos x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0.$$

$$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$$

$$\Rightarrow (\sin x + 2\cos x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$$\Rightarrow (\sin x + 2\cos x)(\sin x - \cos x)^2 = 0.$$

$$\Rightarrow x = \frac{\pi}{4}.$$

9 (B) $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & 2-\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$

put $\lambda = 0$

$$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -4 \\ -3 & 4 & 0 \end{vmatrix}$$

$$= 1(-12) + 3(4+6) = 18.$$

10. (B) $1 \times 2 \times 3 \times 4 \times 5 = 5!$

11. (B) $-\begin{vmatrix} 1 & 3 & 1 \\ 8 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$$= -\left[-2 - 3\times 8 + 16\right] = 10.$$

12. (B) standard property.

13. (D) $\Delta = 11$

$\Delta_c = 121 \Rightarrow \Delta^2 = 14641$

14. (B) $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$

$$= \left(\log_3 512 \times \log_4 9 - \log_4 3 \times \log_3 8\right)\left(\log_2 3 \log_3 4 - \log_3 4 \log_8 3\right)$$

$$= \left(\frac{\log 2^9}{\log 3} \times \frac{\log 3^2}{\log 2^2} - \frac{\log 3}{\log 2^2} \times \frac{\log 2^3}{\log 3}\right)\left(\frac{\log 3}{\log 2} \cdot \frac{\log 2^2}{\log 3} - \frac{\log 2^2}{\log 3} \cdot \frac{\log 3}{\log 2^3}\right)$$

$$= \left(\frac{9 \times 2}{2} - \frac{3}{2}\right)\left(2 - \frac{2}{3}\right) = 10.$$

15 (D) $\sum_{p=1}^{5} D_p = \begin{vmatrix} \Sigma p & 15 & 8 \\ \Sigma p^2 & 35 & 9 \\ \Sigma p^3 & 25 & 10 \end{vmatrix}$

$= \begin{vmatrix} 15 & 15 & 8 \\ 55 & 35 & 9 \\ 225 & 25 & 10 \end{vmatrix}$

16 (A) $\sum_{n=1}^{N} U_n = \begin{vmatrix} \Sigma n & 1 & 5 \\ \Sigma n^2 & 2N+1 & 2N+1 \\ \Sigma n^3 & 3N^2 & 3N \end{vmatrix}$

$= \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left(\frac{N(N+1)}{2}\right)^2 & 3N^2 & 3N \end{vmatrix}$

$= \frac{N(N+1)}{2} \begin{vmatrix} 1 & 1 & 5 \\ \frac{2N+1}{3} & 2N+1 & 2N+1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N \end{vmatrix}$

$= \frac{N^2(N+1)(2N+1)}{2} \begin{vmatrix} 1 & 1 & 5 \\ \frac{1}{3} & 1 & 1 \\ \frac{N+1}{2} & 3N & 3 \end{vmatrix}$

$= \frac{N^2(N+1)(2N+1)}{2} \left[(3-3N) - \left(1 - \frac{N+1}{2}\right) + 5\left(N - \frac{N+1}{2}\right) \right]$

$= 0.$

17. (B) $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$

$$\frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2$$

18 (D) $y = \sin mx.$

$$\Delta = \begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -m^3\cos mx & m^4\sin mx & m^5\cos mx \\ -m^6\sin mx & -m^7\cos mx & m^8\sin mx \end{vmatrix}$$

$$= m^3 \times m^6 \begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -\cos mx & m\sin mx & -m^2\cos mx \\ -\sin mx & -m\cos mx & m^2\sin mx \end{vmatrix}$$

$$= 0.$$

19. (D)

$$\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$$

$C_3 \to C_3 + C_2 - C_1$

$$\Delta(x) = \begin{vmatrix} 1 & \cos x & 0 \\ 1+\sin x & \cos x & 0 \\ \sin x & \sin x & 1 \end{vmatrix} = \cos x - \cos x - \sin x\cos x$$

$$= -\frac{\sin 2x}{2}.$$

$$\therefore \int_0^{\pi/2} -\frac{\sin 2x}{2}\, dx = -1/2.$$

20. $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$\Rightarrow$ a, b, c are in H.P.

21. (A) $\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$.

$1 - a(-a^2) = 0 \Rightarrow a = -1$.

22. (A) $\begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & k+3 \\ 2k+1 & 0 & 1 \end{vmatrix} = 0$.

$\Rightarrow +(k+3)\,2(2k+1) = 0$

$\Rightarrow \quad k = -3$.

23. (C) $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 3$.

$\cancel{\Delta_3 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & & \\ n & & \end{vmatrix}}$ $\quad \Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & n \end{vmatrix} = 0$

$\Rightarrow \quad n = 10$.

24 (D) $\begin{vmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0.$

$2(-2\lambda-1)+(\lambda-1)-(1+2)=0.$

$\Rightarrow -4\lambda-2+\lambda-1-3=0 \Rightarrow -3\lambda=6 \Rightarrow \lambda=-2$

25 (D) $\Delta = \begin{vmatrix} 0 & i-100 & i-500 \\ 100-i & 0 & 1000-i \\ 500-i & i-1000 & 0 \end{vmatrix}$

$\Delta^T = -\Delta.$

$\Rightarrow |\Delta| = 0.$

26 (C) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

standard result.

27. (B) Given $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix} = 0$

$\Rightarrow (x-a-x+b)(x-b-x+c)(x-c-x+a)\left[x-a+x-b+x-c\right]=0$

$\Rightarrow x = \dfrac{a+b+c}{3}.$

28. (D). $x^3+px+q=0 \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$

$\alpha+\beta+\gamma=0.$

$$\therefore \Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$$

$C_1 \to C_1+C_2+C_3$

$$\Delta = \begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = 0.$$

29. put $x=0$

$$P_6 = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} = 1.$$

$$\Delta'(0) = P_5 = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$P_5 = 1-2+1-2(1-4)+1$

$= 7.$

$(P_5, P_6): (7, 1)$

(30) $A = [a_{ij}]$, $a_{ij} = 0$ for $i \neq j$ and

$a_{ij} = k$ (constant) for $i = j$

$$\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$ scalar matrix.

(b) &.

(31) $|A| \neq 0$

then A^{-1} exist.

$AB = 0$ (Given)

$A^{-1}(AB) = A^{-1}(0)$

$(A^{-1}A)B = A^{-1}0$

$IB = 0$

$B = 0$

Then B is null matrix

& (a) &.

Q32

$$A = \begin{bmatrix} 2 & 5 & -7 \\ 0 & 3 & 11 \\ 0 & 0 & 9 \end{bmatrix}$$

upper triangular matrix. . A (c)

Q33

minimum number of zeroes in upper triangular matrix in $n \times n$ matrix

$$= (n-1) + (n-2) + (n-3) + \text{——} + 2 + 1$$

$$= \frac{(n-1)}{2}\left[(n-1)+1\right] = \frac{n(n-1)}{2}. \quad \text{Ans (a)}.$$

Q34

$$A = [a_{ij}]$$

For scalar matrix value of leading diagonal elements $a_{11} = a_{22} = a_{33} = \text{——}$

then trace of $A = \sum_i a_{ii}$ (d)

Q35

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \text{ then}$$

$$A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} \quad \text{A (c)}.$$

Q36

$$A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}, \text{ then } \alpha = a^2+b^2, \ \beta = 2ab.$$

Q37 $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $n \in N$

$A^2 = \begin{bmatrix} i^2 & 0 \\ 0 & -1 \end{bmatrix}$, $A^4 = \begin{bmatrix} i^2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} i^2 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{4n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for $n \in N$. & (A)

Q38 $A+B = \begin{bmatrix} a+1 & 0 \\ 2+b & -2 \end{bmatrix}$, $(A+B)^2 = $ ~~$\begin{bmatrix} a^2+1+2a & 0 \\ 2a-b+ab & 4 \end{bmatrix}$~~

$(A+B)^2 = \begin{bmatrix} a^2+1+2a & 0 \\ 2a-b+ab & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B^2 = \begin{bmatrix} a^2+b & a-1 \\ ab-b & b+1 \end{bmatrix}$

$A^2+B^2 = \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$

$(A+B)^2 = (A^2+B^2)$ from this

$a-1 = 0$, $b = 4$

$a = 1$, $b = 4$ & (b).

(39) 1×1 nodder & (b).

(40) use product of matrix & (c) &.

Q41 $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

The characteristic Eq of A is

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 2 & 1 & -\lambda \end{vmatrix} = 0$$

$\cancel{\lambda^3 - 3\lambda^2 + 1 = 0}$

~~the~~ $\cancel{A^3 - 3A^2 + I = 0}$

$\lambda^3 - 3\lambda^2 - 1 = 0$

then $A^3 - 3A^2 - I = 0$ A(b)

Q42 $(AB)^T = B^T A^T$ using Property of Transpose.

A (b)

Q43 Let $A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}_{3\times 2}$ & Let $B = \begin{bmatrix} m & n \\ o & p \\ q & r \end{bmatrix}_{3\times 2}$

$C = \begin{bmatrix} a & c & m \\ \alpha & \beta & \gamma \end{bmatrix}_{2\times 3}$

For addition & ~~subtraction~~ ~~odder~~ order of matrix are same. so $A' + B$ is not possible.

A (A)

Q44 Ans (b) $\boxed{A^T = -A}$ is for skew symmetric matrix.

Q45 For non-singular matrix

$$|A| \neq 0.$$

$$|A| = \begin{vmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix} = \lambda + 2 \neq 0$$

$\lambda \neq -2$ Ans (a)

Q46 For orthogonal Matrix

$$A A^T = I = A^T A.$$

Ans $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$

For this $AA^T = I$ so Matrix A is orthogonal.

Q47 $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$, $A^T = \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$

For symmetric Matrix

$$A^T = A$$

$$\begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix} = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$$

$2x-3 = x+2$

$x = 5$ Ans (b).

Q 48 $(A-B)^2 = (A-B)(A-B)$
$= A^2 - AB - BA + B^2$. (D) &

Q 49 $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$|A| = 1$ so $|A| \neq 0$ so A is inventible

Ans (d).

Q 50 $A^2 = 2A - I$

$A^3 = A^2 \cdot A$
$= (2A - I)A$
$= 2A^2 - A = 2(2A - I) - A = 3A - 2I$

$A^4 = A^3 \cdot A$
$= (2A^2 - A) \cdot A$
$= 2A^3 - A^2 = 2(3A - 2I) - (2A - I)$
$= 6A - 4I - 2A + I$
$= 4A - 3I$.

Then $A^n = nA - (n-1)I$ & (A) &

(S1) $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$ & (a)

Q.s 52 $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$

$|adj A| = |A|^{n-1}$ when n is order of matrix.

$|adj A| = |A|^{2-1} = |A| = 16-6 = 10.$

Ans- b.

Q-53 $|A - Ix| = 0$

$(x-3)(x+2) = 0$

$x^2 - x - 6 = 0$

$A^2 - A - 6 = 0$

$adj A (A^2 - A - 6) = adj A \cdot O$

$adj A \cdot A^2 - adj A \cdot A - 6 adj A = 0$

$4A - 4I - 6 adj A = 0$

$adj A = \frac{4A - 4I}{6}$

Then

$|adj A - Ix| = 0$

$\left|\frac{4A-4I}{6} - xI\right| = 0.$

Then Eigenvalue of (adj A) is

$4/3, -2$ & (3.)

Q54 $A^{-1} = \frac{1}{K} adj(A)$

we know that $A^{-1} = \frac{1}{|A|} adj(A)$

then $K = |A|$

$K = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 11$ & (d)

Q55 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$adj A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(adj A)$

$= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Ans-(b)

$$0 = |\kappa I - A|$$

$$0 = (5+\kappa)(8-\kappa)$$

[illegible]

[illegible]

48. (D). $(A-B)^2 = (A-B)(A-B)$

$= A^2 - AB - BA + B^2$

50. (A). $A^2 = 2A - I$.

$A^3 = 2A^2 - A$.

$= 2(2A-I) - A$

$= 3A - 2I$.

Similarly $A^4 = 3A^2 - 2A$.

$= 3(2A-I) - 2A$.

$= 4A - 3I$.

$\therefore \quad A^n = nA - (n-1)I$.

Ex 1(c). Determinant & Matrices.

1.(A) $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$A^2 = I$.

$A^{-1}A^2 = A^{-1}$

$\Rightarrow A = A^{-1}$

$\therefore$ Inverse of $\frac{A}{2} = 2A^{-1} = 2A$.

2.(A) $A^T = A$.

$B^T = B$.

$(ABA)^T = A^T B^T A^T$

$= ABA \Rightarrow$ symmetric.

3.(B) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

$|A| = 1$.

$A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = A^T$.

4.(C). order $A = 3\times 3$, $B = 3\times 2$, $C = \cancel{1\times 3}\ 3\times 1$

order of $AB = 3\times 2$ order of $(AB)^T = 2\times 3$

$\Rightarrow$ Exactly three are defined.

5.(D) $A = \begin{bmatrix} 3 & 4 \\ 1 & -6 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 5 \\ 6 & 1 \end{bmatrix}$

$$A + 2X = B \Rightarrow X = \frac{1}{2}(B - A).$$

$$X = \frac{1}{2}\begin{bmatrix} -5 & 1 \\ 5 & 7 \end{bmatrix}.$$

6.(C). (A) $\text{adj} A = |A| A^{-1}$ correct

(B) $\det(A^{-1}) = |\det(A)|^{-1}$ correct

(C) $(A+B)^{-1} = B^{-1} + A^{-1}$ wrong.

(D) $(AB)^{-1} = B^{-1}A^{-1}$ correct

7. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix}$$

$$\therefore A^2 - (a+d)A + k = 0. \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} + k = 0.$$

$$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix} - (a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix} + k = 0.$$

$$\Rightarrow \begin{bmatrix} bc-ad & bd-db \\ 0 & bc-ad \end{bmatrix} + k = 0.$$

$$\Rightarrow k = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}.$$

8. (D) Standard result.

9. (B) Standard result.

10. (A). For orthogonal matrix

$AA^T = I$.

(A) $A = \begin{bmatrix} 6/7 & 2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ 3/7 & -6/7 & 2/7 \end{bmatrix}$, $A^T = \begin{bmatrix} 6/7 & 2/7 & 3/7 \\ 2/7 & 3/7 & -6/7 \\ -3/7 & 6/7 & 2/7 \end{bmatrix}$

$AA^T = \begin{bmatrix} \frac{36}{49}+\frac{4}{49}+\frac{9}{49} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$

11. (C) For idempotent $A^2 = A$.

12. (C) $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow$ nilpotent.

13. (B) Standard result.

14. (B). order of $A = 3 \times 4$.

order of $A' = 4 \times 3$.

$\because$ $A'B$ & BA' are defined

therefore order of $B = \cancel{4 \times 3}$ 3×4.

order of $B' = 4 \times 3$.

order of $B'A = 4 \times 4$.

15. (c) $AB = \begin{bmatrix} \cos^2\alpha\cos^2\beta + \sin\alpha\cos\alpha\cdot\sin\beta\cos\beta & \cos^2\alpha\sin\beta\cos\beta + \sin^2\beta\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha\cos^2\beta + \sin^2\alpha\sin\beta\cos\beta & \sin\alpha\cos\alpha\sin\beta\cos\beta + \sin^2\alpha\sin^2\beta \end{bmatrix}$

$$AB = \begin{bmatrix} \cos\alpha\cos\beta(\cos(\alpha-\beta)). & \cos\alpha\sin\beta(\cos(\alpha-\beta)) \\ \sin\alpha\cos\beta\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{bmatrix} = 0$$

$\alpha-\beta =$ odd multiple of $\frac{\pi}{2}$.

16. (c). $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

$AA^T = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix} = I$.

$\Rightarrow$ orthogonal.

17. (c). $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$ $\quad xyz = 60$ & $8x+4y+3z = 20$.

$A \text{ adj} A = |A| I$.

$= (xyz + 24 + 4 - (4y + 8x + 3z))I$.

$= 68I$

18. (c) $A^2 + A + 2I = 0$.

$A^{-1} = -\frac{1}{2}(A+I)$.

A is symmetric $\Rightarrow$ wrong.

19. (C) $A+2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ — ①

$2A-B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ — ②.

Eqⁿ ② × 2 + eqⁿ ①.

$5A = \begin{bmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$

& $B = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$.

$\therefore \ tr(A) - tr(B) = 1-(-1) = 2$

20 (B) $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} = 0.$

$\Rightarrow (1-\lambda)(2-\lambda) - 6 = 0$

$\Rightarrow \lambda^2 - 3\lambda - 4 = 0.$

21. (A) $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$

$AA^{-1} = I.$

$\Rightarrow \begin{bmatrix} 1 & 0 & c+1 \\ 0 & 1 & 2c+2 \\ -4a+4 & 3a-3 & 2+ac \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$c = -1$
$a = 1$

22.(C). standard result.

23.(B) $B = \sum_{r=1}^{n} (2r-1)(A_{2r-1})^{2r-1}$

$B = A_1 + 3(A_3)^3 + \cdots$

$B^T = -(A_1 + 3(A_3)^3 + \cdots)$

$\Rightarrow$ skew symmetric

24.(D). $|A| = (\lambda-1)[-\lambda-7-3\lambda+6] - \lambda[2\lambda+14-3\lambda-9]$
$+ (\lambda+1)[2\lambda-4+\lambda+3]$

$+ (\lambda+1)(3\lambda-1).$

$= -(\lambda-1)(4\lambda+1) + \lambda(\lambda-5) + (\lambda+1)(3\lambda-1)$

$= -4\lambda^2+3\lambda+1 + \lambda^2 - 5\lambda + 3\lambda^2 + 2\lambda - 1$

$= 0.$

$\Rightarrow \lambda \in R.$

25. (C) $|A^{-1}| = k^n |B^{-1}|.$

26 (D). $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$B = A^{-1}$

~~10AB = 10I~~. $AB = I.$

$\Rightarrow \frac{2}{10} - \frac{\alpha}{10} + \frac{3}{10} = 0 \Rightarrow +\frac{\alpha}{10} = \frac{5}{10}$

$\alpha = 5.$

27. (C) $A^2 = 2A - I$.

$A^3 = 2A^2 - A$

$= 2(2A - I) - A$

$= 4A - 2I - A = 3A - 2I$.

Similarly $A^4 = 3A^2 - 2A$

$= 3(2A - I) - 2A$

$A^4 = 4A - 3I$.

$\therefore \quad A^n = nA - (n-1)I$.

28. (A) $|A| = 1(0-10) - 2(2) + 3(4)$.

$= -10 - 4 + 12 = -2$

$|A| \neq 0$.

unique solⁿ.

29 (B) $|A| = 2, \quad |B| = 3, \quad |C| = 5$.

$\det(A^2BC^{-1}) = \frac{4 \times 3}{5} = \frac{12}{5}$.

30. (A) $BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$.

$\therefore \ \mathrm{tr}A + \mathrm{tr}\left(\frac{A}{2}\right) + \mathrm{tr}\left(\frac{A}{4}\right) + \cdots \infty$.

$= 3 + \frac{3}{2} + \frac{3}{4} + \cdots \infty$.

$= 3\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots \infty\right)$

$= 3 \times \frac{1}{1 - \frac{1}{2}} = 6$.

31. (A) Given $PP^T = I$.

& $A^5 = A$.

$$Q = PAP^T$$

$$Q^2 = PA(P^TP)AP^T.$$

$$= PA^2P^T.$$

$$\therefore \quad Q^{2005} = PA^{2005}P^T$$

$$\therefore \quad X = (P^TP)A^{2005}(P^TP) = A^{2005} = A.$$

32. (D) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} a-b \\ c-d \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow a-b = -1 \text{ \& } c-d = 2.$$

$$A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A\begin{bmatrix} a-b \\ c-d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a-b \\ c-d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} a(a-b)+b(c-b) \\ c(a-b)+d(c-b) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}} \qquad \Rightarrow -a+2b = 1$$

$$\& \quad -c+2d = 0.$$

$$\cancel{a(a-b)+b(c-d) = 1.} \qquad \therefore \quad a = -1, \; b = 0.$$

$$\cancel{c(a-b)+d(c-d) = 0.} \qquad c = 4, \; d = 2.$$

$$\therefore \quad a+b+c+d = 5.$$

33 (D) $AA^T = I$.

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix} \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2\beta^2 - \gamma^2 = 0$ — (1)

$4\beta^2 + \gamma^2 = 1$ — (2)

$6\beta^2 = 1 \Rightarrow \beta = \pm \frac{1}{\sqrt{6}}$ & $\gamma = \pm \frac{1}{\sqrt{3}}$

$\therefore \alpha = \pm \frac{1}{\sqrt{2}}$

34. (A) Standard result

35. (B) $A = dig(2, -1, 3)$

$B = dig(-1, 3, 2)$.

$A^2B = dig(-4, 3, 18)$

36. (C) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$.

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} - 1 = 0$.

$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$.

37. (B) $AB = \begin{bmatrix} 3a_1 & 3a_2 & 3a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{bmatrix} = 3B.$

38. $A = \begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$

$|A| \neq 0$

$$\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix} \neq 0$$

$C_1 \to C_1 + C_2 + C_3$.

$$(3x+\lambda) \begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix} \neq 0.$$

$R_2 \to R_2 - R_1 \quad , \quad R_3 \to R_3 - R_1$

$$(3x+\lambda) \begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} \neq 0$$

$\Rightarrow (3x+\lambda)(\lambda^2) \neq 0$

$\Rightarrow 3x+\lambda \neq 0 \quad \& \quad \lambda \neq 0.$

39. (B). Standard Result.

40 (A). $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$PP^T = I$. , $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

$Q = PAP^T$

$Q^2 = PA(P^T P)AP^T = PA^2P^T.$

$Q^{2005} = PA^{2005}P^T.$

$X = (P^T P)A^{2005}(P^T P.) = A^{2005}.$

$= \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

41. (A). $\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$

$= a^2[\sin(n+2)x\cos(n+1)x - \cos(n+2)x\sin(n+1)x]$

$- a[\sin(n+2)x\cos nx - \cos(n+2)x\sin nx]$

$+ 1[\sin(n+1)x\cos nx - \cos(n+1)x\sin nx]$

$= a^2\sin x - a\sin 2x + \sin x.$

independent of n.

42. (D).
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0.$$

$$(1+a)[(1+b)(1+c)-1]-1(1+c-1)+1(1-1-b)=0.$$

$$(1+a)(1+b)(1+c)-1-a-c-b=0.$$

$$\Rightarrow (1+a)(1+b)(1+c)+abc-1-a-b-c=0$$

$$\Rightarrow 1+a+b+c+ab+bc+ca+abc$$

$$\Rightarrow abc\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right]=0.$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=-1$$

43.
$$D=\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$$

$$=1-\cos^2(\beta-\gamma)-\cos(\beta-\alpha)[\cos(\alpha-\beta)-\cos(\alpha-\gamma)\cos(\gamma-\beta)]$$
$$+\cos(\gamma-\alpha)[\cos(\alpha-\beta)\cos(\beta-\gamma)-\cos(\alpha-\gamma)]$$

$$=1-\cos^2(\beta-\gamma)-\cos^2(\alpha-\beta)+\cos(\alpha-\beta)\cos(\beta-\gamma)\cos(\gamma-\alpha)-\cos^2(\alpha-\gamma).$$
$$+\cos(\alpha-\beta)\cos(\beta-\gamma)\cos(\gamma-\alpha)$$

$$=1-\cos^2(\alpha-\beta)-\cos^2(\beta-\gamma)-\cos^2(\gamma-\alpha).$$

$$=\sin^2(\alpha-\beta)-\cos^2(\beta-\gamma)-\cos^2(\gamma-\alpha)$$

$$=-[\cos^2(\beta-\gamma)-\sin^2(\alpha-\beta)]-\cos^2(\gamma-\alpha)$$

$$=-[\cos(\alpha-\gamma)\cos(2\beta-\alpha-\gamma)]-\cos^2(\gamma-\alpha)$$

$$=-\cos(\gamma-\alpha)[\cos(2\beta-\alpha-\gamma)+\cos(\gamma-\alpha)]$$

$$=-2\cos(\gamma-\alpha)\cos(\beta-\alpha)\cos(\beta-\gamma).$$

44. (c) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0.$

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} - 1 = 0$

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1.$

45. (c). $\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & a+1 & a+2 \\ 1 & 1 & 1 \end{vmatrix} = 0.$

$C_2 \to C_2 - C_1, \quad C_3 \to C_3 - C_1$

$\begin{vmatrix} a^3 & (a+1)^3 - a^3 & (a+2)^3 - a^3 \\ a & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 0.$

$2[(a+1)^3 - a^3] - [(a+2)^3 - a^3] = 0$

$\Rightarrow a = -1.$