<div style="text-align:center">**INCHAPTER EXERCISE**</div>

Q.1 $y = [x^2 + 2\sqrt{x}]x = x^3 + 2(x)^{1/2+1} = x^3 + 2(x)^{3/2}$

$\Rightarrow \dfrac{dy}{dx} = 3x^2 + 2\left[\dfrac{3}{2}(x)^{3/2-1}\right] = 3x^2 + 3\sqrt{x}$

Q.2 $y = x^2 \cos x, \quad u = x^2 \Rightarrow \dfrac{du}{dx} = 2x$

$\quad \overset{\uparrow}{u} \quad \overset{\uparrow}{v}$

$\qquad\qquad v = \cos x \Rightarrow \dfrac{dv}{dx} = -\sin x$

$\Rightarrow \dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx} = \cos x(2x) + x^2(-\sin x)$

$= 2x\cos x - x^2\sin x$

Q.3 $y = (4x^2 - 7x + 5)\sec x$

$\qquad\quad \overset{\uparrow}{u} \qquad\quad \overset{\uparrow}{v} \qquad 4(2x)$

$\qquad\qquad\qquad\qquad\qquad \downarrow$

$u = 4x^2 - 7x + 5 \Rightarrow \dfrac{du}{dx} = 8x - 7$

$v = \sec x \Rightarrow \dfrac{dv}{dx} = \sec x \, \tan x$

$\dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx} = \sec x(8x - 7) + (4x^2 - 7x + 5)(\sec x \tan x)$

Q.4 $y = x^4(5\sin x - 3\cos x)$

$\quad \overset{\downarrow}{u} \quad \overset{\downarrow}{v}$

$u = x^4 \Rightarrow \dfrac{du}{dx} = 4x^3$

$v = 5\sin x - 3\cos x \Rightarrow \dfrac{dv}{dx} = 5[\cos x] - 3(-\sin x)$

$= 5\cos x + 3\sin x$

$\Rightarrow \dfrac{dy}{dx} = (5\sin x - 3\cos x)4x^3 - x^4(5\cos x + 3\sin x)$

Q.5 $y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$u = \sqrt{x} + 1 = (x)^{1/2} + 1$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1} + 0 = \frac{1}{2}(x)^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$v = \sqrt{x} - 1 = (x)^{1/2} - 1$$

$$\frac{dv}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1} - O = \frac{1}{2}(x)^{-1/2} = \frac{2}{2\sqrt{x}}$$

$$\begin{matrix} v & du/dx & u \\ \downarrow & \downarrow & \downarrow \end{matrix}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(\sqrt{x}-1\right)\dfrac{1}{2\sqrt{x}} - \left(\sqrt{x}+1\right)\left(\dfrac{1}{2\sqrt{x}}\right)}{\left(\sqrt{x}-1\right)^2}$$

$$= \frac{\sqrt{x}-1-\sqrt{x}+1}{2\sqrt{x}\left(\sqrt{x}-1\right)^2} = \frac{-2}{2\sqrt{x}\left(\sqrt{x}-1\right)^2}$$

$$= \frac{-1}{\sqrt{x}\left(\sqrt{x}-1\right)^2}$$

Q.6 $\quad y = \dfrac{x \ln x}{e^x} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}, \ v = e^x \Rightarrow \dfrac{dv}{dx} = e^x$

$$P = x \Rightarrow \frac{dp}{dx} = 1$$

$$u = (x)\,(\ln x), \quad \begin{matrix} \uparrow & \uparrow \\ P & Q \end{matrix} \quad Q = Lnx \Rightarrow \frac{dq}{dx} = \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = P\frac{dQ}{dx} + Q\frac{dp}{dx}$$

$$= (x)\left(\frac{1}{x}\right) + (\ln x)(1) = 1 + Lnx$$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} = \frac{(e^x)(1+Lnx) - (Lnx)(e)^x}{(e^x)^2}$$

$$= \frac{e^x[1 + Lnx - xLnx]}{e^x} = \frac{1 + Lnx - xLnx}{e^x}$$

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INCHAPTER EXERCISE

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Q.1 (i) $\quad y = \dfrac{x^3}{3} - x$

Step – 1 Find $\dfrac{dy}{dx}, \dfrac{dy}{dx} = \dfrac{1}{3}[3x^2] - 1 = x^2 - 1$

Step – 2 Equate $\dfrac{dy}{dx} = 0$

$$\Rightarrow x^2 - 1 = 0$$

i.e. $x^2 = 1 \Rightarrow x = 1$ or $x = -1$

Step -3 Find $\dfrac{d^2y}{dx^2} \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{d(x^2-1)}{dx}$

$\dfrac{d^2y}{dx^2} = 2(x) - 0 = 2x$

Step-4 Substitute x = 1and x = -1 in $\dfrac{d^2y}{dx^2}$ and compare it with zero

For x = 1in $\dfrac{d^2y}{dx^2} = (2x) = 2(1) = 2 > 0$ x = 1

$\Rightarrow x = 1 = \min ima$

And $[y_{min}] = \dfrac{[x=1]^3}{3} - (x=1)$

$\qquad$ Local

$\left(\because y = \dfrac{x^3}{3} - x \right)$

$\Rightarrow y_{min} = \dfrac{1}{3} - 1 = \dfrac{-2}{3}$

For x = -1 in $\dfrac{d^2y}{dx^2} (= 2x) = 2(-1) = -2 < 0$

$\Rightarrow x = -1 = \max ima$

And $[Y_{max}]_{local} = \dfrac{(x=-1)^3}{2} - (x=-1)$

$= \dfrac{-1}{3} + 1 = 2/3$

Q.1 **(ii)** $\quad y = 4x^3 - 21x^2 + 18x + 5$

$S^{-1} \Rightarrow 4(3x^2) - 21(2x) + 18(1) + 0$

$\qquad = 12x^2 - 42x + 18$

$S - 2 = \dfrac{dy}{dx} = 0$

$\Rightarrow 12x^2 - 42x + 18 = 0$

$\Rightarrow 6(2x^2 - 7x + 3) = 0$

Or $(2x^2 - 7x + 3) = 0$

$\Rightarrow 2x^2 - 6x - x + 3 = 0$

$2x(x-3) - 1(x-3) = 0$

$\Rightarrow (x-3)(2x-1) = 0$

$\Rightarrow x = 3$ or x = ½

$S - 3 \quad \dfrac{d^2y}{dx^2} = 12(2x) - 42(1) + 0$

$\qquad = 24x - 42$

$S - 4 \quad x = 3$ in $\dfrac{d^2y}{dx^2} = 24(3) - 42 = 30 > 0$

$\Rightarrow x = 3$ is minima.

$$x = \frac{1}{2} \text{ in } \frac{d^2y}{dx^2} = 24\left(\frac{1}{2}\right) - 42 = -30 > 0$$

$$\Rightarrow x = \frac{1}{2} = \text{maxima}$$

Q.1 **(iii)** $y = \sin x$

$S - 1 \quad \dfrac{dy}{dx} = \cos x$

$S - 2 \quad \dfrac{dy}{dx} = \cos x = 0$

$$\Rightarrow x = \frac{\pi}{2} \text{ or } x = \frac{3\pi}{2}$$

$S - 3 \quad \dfrac{d^2y}{dx^2} = -\sin x$

$S - 4 \quad x = \dfrac{\pi}{2} \text{ in } \dfrac{d^2y}{dx^2} = -\sin\left(\dfrac{\pi}{2}\right) = -1 < 0$

$\Rightarrow x = \pi/2 = \text{maxima}$

$$x = \frac{3\pi}{2} \text{ in } \frac{d^2y}{dx^2} = -\sin\left(\frac{3\pi}{2}\right) = -(-1) = 1 > 0$$

$$\Rightarrow x = \frac{3\pi}{2} = \text{minima}$$

Q.2 **(i)** $y = x^3 - 3x + 10$

$S - 1 \quad \dfrac{dy}{dx} = 3x^2 - 3(1) - 0 = 3x^2 - 3$

$S - 2 \quad \dfrac{dy}{dx} = 3x^2 - 3 = 0$

$\qquad \Rightarrow \cancel{3}x^2 = \cancel{3}$

$\qquad x^2 = 1$

$\qquad \Rightarrow x = 1 \text{ or } x = -1$

$S - 3 \quad \dfrac{d^2y}{dx^2} = 3(2x) - 0 = 6x$

$S - 4 * x = 1 \text{ in } \dfrac{d^2y}{dx^2}(= 6x) = 6(1) = 6 > 0$

$\qquad \Rightarrow x = 1 = \text{minima}$

$y_{min} = (1)^3 - 3(1) + (10) = 8$

$\left(\because y = x^3 - 3x + 10 \right)$

- $x = -1 \text{ in } \dfrac{d^2y}{dx^2} = 6x = 6(-1) = -6 < 0$

$\Rightarrow x = -1 = \text{maxima}$

$\Rightarrow y_{max} = (-1)^3 - 3(-1) + 10$

$y_{max} = -1 + 3 + 10 = 12$

Q.2 **(ii)** $y = \dfrac{x^2}{2} + \dfrac{1}{x}$

$x^{-1} - 1(x) = \dfrac{-1}{x^2}$

S -1 2 $\dfrac{dy}{dx} = \dfrac{1}{2}(2x) - \dfrac{1}{x^2} = x - \dfrac{1}{x^2}$

S -2 $\dfrac{dy}{dx} = x - \dfrac{1}{x^2} = 0 \Rightarrow x = \dfrac{1}{x^2}$

$\Rightarrow x^3 = 1 \Rightarrow x = 1$ only real value s of x

We only get 1 value of x hence either we'll get maxima or we'll get minima.

S -3 $\dfrac{d^2y}{dx^2} = 1 - \dfrac{(-2)}{x^3} = 1 + \dfrac{2}{x^3}$

S -4 $x = 1$ in $\dfrac{d^2y}{dx^2} = 1 + \dfrac{2}{(x=1)^3} = 3 > 0$

$\Rightarrow x = 1 = \min ima \Rightarrow y_{min} \dfrac{(1)^2}{2} + \dfrac{1}{1} = \dfrac{3}{2}$

$y = \dfrac{x^2}{2} + \dfrac{1}{x}$

INCHAPTER EXERCISE

Q.1 $\int \left(x^5 + \dfrac{2}{x^2} - \dfrac{1}{x} - \dfrac{4}{x} - \dfrac{4}{\sqrt{x}} + 10 \right) dx$

$= \int x5dx + \int \dfrac{2}{x^2} dx - \int \dfrac{1}{x} dx - \int \dfrac{4}{\sqrt{x}} dx + 10\int dx$

$= \dfrac{x^6}{6} \dfrac{-2}{x} = Lnx - 8\sqrt{x} + 10x + c$

Q.2 $\int \left(7e^x + 4\sin x - \dfrac{9}{x^3} + e \right) dx$

$7\int e^x + 4\int \sin x - 9\int \dfrac{dx}{x^3} + e\int dx$

$7e^x - 4\cos x + \dfrac{9}{2x^2} + ex + c$

INCHAPTER EXERCISE

Q.1 $\int x\sin(1 + x^2)dx$

Let $P = 1 + x^2$

$\Rightarrow \dfrac{dp}{dx} = 0 + 2x \Rightarrow \dfrac{dp}{dx} = 2x$

Or $\dfrac{dp}{2} = xdx$

$$\Rightarrow \int x\sin(1+x^2)dx = \int \sin(1+x^2)xdx = \int \sin p\left(\frac{dp}{2}\right)$$

$$= \frac{1}{2}(-\cos p) + c$$

$$= -\frac{\cos(1+x^2)}{2} + c$$

Q.2 $\quad \int \dfrac{2x}{1+x^2}dx$

Let $1+x^2 \Rightarrow \dfrac{dp}{dx} = 2x \Rightarrow dp = 2xdx$

$$\Rightarrow \int \frac{2xdx}{1+x^2} = \int \frac{dP}{P} = LnP + C$$

Ans. $\ln(1+x^2) + C$

INCHAPTER EXERCISE

Q.1 $\quad \displaystyle\int_{1}^{5}(3+2t)dt = \int_{1}^{5}3dt + \int_{1}^{5}2tdt$

$$= 3[t]^5 + 2\left(\frac{t^2}{2}\right)_{1}^{5}$$

$$= 3(5-1) + [(5)^2 - (1)^2]$$
$$= 12 + 24 = 36$$

Q.2 $\quad \displaystyle\int_{\pi/2}^{\pi/2}(2\sin x - \cos x)dx$

$$\int_{\pi/4}^{\pi/2}2\sin x\, dx - \int_{\pi/4}^{\pi/2}\cos x\, dx$$

$$= 2[-\cos x]_{\pi/4}^{\pi/2} - [\sin x]_{\pi/4}^{\pi/2}$$

$$= 2\left[\left(-\cos\frac{\pi}{2}\right) - \left(-\cos\frac{\pi}{4}\right)\right] - \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right]$$

$$= 2\left[0 + \frac{1}{\sqrt{2}}\right] - \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}} - 1$$

INCHAPTER EXERCISE

Q.1 $S = (2t + 4t^2)m$

$v = \dfrac{ds}{dt} = 2 + 8t \rightarrow$ at $t = 0 \Rightarrow v = 2$

at $t = 2 \Rightarrow v = 18$

at $t = 10 \Rightarrow v = 82$

$a = \dfrac{dv}{dt} = 8 = $ constant.

Hence same value at all times

Q.2 $v = 3t^2$

$* \int ds = \int v dt = \int 3t^2 dt = 3\left(\dfrac{t^3}{3}\right)^t = [t^3]$

$[s]_0^s = [t^3]_0^t \Rightarrow (s - 0) = (t^3 - 0)$

$*$ At $t = 0 \Rightarrow s = 0$ m

$*$ At $t = 2 \Rightarrow s = 8m$

$*$ At $t = 10 \Rightarrow S = 1000m$

$* \quad a = \dfrac{dv}{dt} = 6t \leftarrow$ variable acceleration

$*$ acceleration at $t = 0 \Rightarrow a = 0m/s^2$

$*$ acceleration at $t = 2 \Rightarrow a = 12$ m/s^2

$*$ acceleration at $t = 10 \Rightarrow a = 60m/s^2$

Q3.   If $v = u + at$

As $S = \displaystyle\int_0^t v dt$

$= \displaystyle\int_0^t (u + at)dt \quad a = $ constant

$= u\displaystyle\int_0^t dt + a\int_0^t t\, dt$

$= u[t]_0^t + a\left(\dfrac{t^2}{2}\right)_0^t$

$= ut + \dfrac{1}{2}at^2$

Q.4 $a = \dfrac{dV}{ds}\dfrac{ds}{dt} = v$

$a = v\dfrac{dv}{ds}$

$\Rightarrow ads = vdv$

$$\Rightarrow \int ads = \int vdv$$

$$a(s)_0^s = \left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow a(S-0) = \frac{v^2 - u^2}{2}$$

$$\Rightarrow v^2 - u^2 = 2as \Rightarrow v^2 = u^2 + 2as$$

EXERCISE # 1

1. $y = (x^2 - 3x + 3)(x^2 + 2x - 1)$

$$\underset{u}{\uparrow} \qquad \underset{v}{\uparrow}$$

$$u = x^2 - 3x + 3 \Rightarrow \frac{du}{dx} = 2x - 3(1) + 0 = 2x - 3$$

$$v = x^2 + 2x - 1 \Rightarrow \frac{dv}{dx} = 2x + 2(1) - 0 = 2x + 2$$

$$\frac{dy}{dx} = v\frac{dy}{dx} + u\frac{dv}{dx} = (x^2 + 2x - 1)(2x - 1) + (x^2 - 3x + 3)(2x + 2)$$

$$= 2x^3 + 4x^2 - 2x - 3x^2 - 6x + 3 + 2x^3 - 6x^2 + 6x + 2x^2 - 6x + 6$$

$$= 4x^3 - 3x^2 - 8x + 9$$

2. $\left.y = \frac{x+1}{x-1} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}\right\} \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$$u = x + 1 \Rightarrow \frac{du}{dx} = (1)$$

$$v = x - 1 \Rightarrow \frac{dv}{dx} = 1$$

3. $y = \dfrac{x}{x^2 + 1} \begin{matrix} \leftarrow 4 \\ \leftarrow v \end{matrix}$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$u = x \Rightarrow \frac{du}{dx} = 1$$

$$v = x^2 + 1 \Rightarrow \frac{dv}{dx} = 2x + 0 = 2x$$

Substitute and solve

4. $y = \dfrac{ax + b \leftarrow u}{cx + d \leftarrow v}$, a, b, c and d are constants

$$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$u = ax + b \Rightarrow \dfrac{du}{dx} = a(1) = a$$

$$v = cx + d \Rightarrow \dfrac{dv}{dx} = C(1) = C$$

Substitute values and solve

5. $z = \dfrac{x^2 + 1}{3(x^2 - 1)} + (x^2 - 1)(1 - x)$

             $\downarrow$ $\downarrow$

             P   Q

$$\dfrac{dz}{dx} = \dfrac{dP}{dx} + \dfrac{dQ}{dx}$$

$$P = \dfrac{x^2 + 1 \rightarrow u}{3(x^2 - 1) \rightarrow v}, \quad \dfrac{dP}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$u = x^2 + 1 \Rightarrow \dfrac{du}{dx} = 2x$$

$$v = 3(x^2 - 1) \Rightarrow \dfrac{dv}{dx} = 3(2x - 0) = 6x$$

$$\Rightarrow \dfrac{dp}{dx} = \dfrac{3(x^2 - 1)(2x) - (x^2 + 1)6x}{[3(x^2 - 1)^2]^2} = \dfrac{6x(x^2 - 1 - x^2 - 1)}{3(x^2 - 1)^2}$$

$$\dfrac{dP}{dx} = \dfrac{-12x}{9(x^2 - 1)^2} = \dfrac{-4x}{3(x^2 - 1)^2}$$

$$Q = (x^2 - 1)(1 - x), \quad \dfrac{dQ}{dx} = b\dfrac{dQ}{dx} + a\dfrac{db}{dx}$$

         $\uparrow$ $\uparrow$

         a   b

$$\left. \begin{array}{l} a = x^2 - 1 \Rightarrow \dfrac{dq}{dx} = 2x \\[2mm] b = 1 - x \Rightarrow \dfrac{db}{dx} = -1 \end{array} \right\} \dfrac{dQ}{dx} = (1 - x)(2x) + (x^2 - 1)(-1)$$

$$= 2x - 2x^2 - x^2 + 1$$

$$\dfrac{dQ}{dx} = -3x^2 + 2x + 1$$

$$\Rightarrow \dfrac{dz}{dx} = \dfrac{dp}{dx} + \dfrac{dQ}{dx} = \dfrac{-4x}{3(3x^2 - 1)^2} - 3x^2 + 2x + 1$$

6. $y = \dfrac{1-x^3}{1+x^3}$ $\leftarrow u \rightarrow \dfrac{du}{dx} = -3x^2$

$\leftarrow v \rightarrow \dfrac{du}{dx} = 3x^2$

$$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

Substitute to get the answer

7. $y = \dfrac{2}{x^3 - 1}$ $\leftarrow u \Rightarrow \dfrac{du}{dx} = 0$

$\leftarrow v \Rightarrow \dfrac{du}{dx} = 3x^2$

$$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

8. $y = \dfrac{x^2 - x + 1}{a^3 - 3}$,

Here a is constant hence $a^3 - 3 = $ constant

$$\Rightarrow y = \left[\dfrac{1}{a^3 - 3}\right](x^2 - x + 1)$$

$$\dfrac{dy}{dx} = \left[\dfrac{1}{a^3 - 3}\right]\dfrac{d(x^2 - x + 1)}{dx} = \left(\dfrac{1}{a^3 - 3}\right)(2x - 1)$$

9. $y = \dfrac{1-x^3}{\sqrt{\pi}} = \dfrac{1}{\sqrt{\pi}}[1 - x^3]$ constant

$$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\pi}}\dfrac{d(1 - x^3)}{dx}$$

$$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\pi}}(0 - 3x^2) = \dfrac{-3x^2}{\sqrt{\pi}}$$

10. $y = \sin x + \cos x$

$$\dfrac{dy}{dx} = \cos x + (-\sin x) = \cos x - \sin x$$

11. $y = \dfrac{2}{1 - \cos x}$ $\leftarrow u \Rightarrow \dfrac{du}{dx} = 1$

$\leftarrow v \Rightarrow \dfrac{dv}{dx} = 0 - (\sin x) = \sin x$

$$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$ Solve further

12. $y = \dfrac{\tan x}{x} \begin{matrix} \to u \Rightarrow \dfrac{du}{dx} = \sec^2 x \\ \to v \Rightarrow \dfrac{dv}{dx} = 1 \end{matrix}$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ Solve further

13. $y = [(x)(\sin x)] + [\cos x]$
$\qquad\quad\;\; \downarrow \qquad\;\; \downarrow$
$\qquad\quad\;\; P \qquad\;\; Q$

$\dfrac{dy}{dx} = \left[P\dfrac{dQ}{dx} + Q\dfrac{dP}{dx} \right] + \dfrac{d(\cos x)}{dx}$

$= [x(\cos x) + \sin x(1)] + [-\sin x]$

$= x\cos x + \sin x - \sin x$

$= x\cos x$

14. $y = \dfrac{\sin x}{x} + \dfrac{x}{\sin x} \Rightarrow \dfrac{dy}{dx} = \dfrac{dP}{dx} + \dfrac{qQ}{dx} + \dfrac{dQ}{dx}$
$\qquad\quad \downarrow \qquad \downarrow$
$\qquad\quad P \qquad Q$

$P = \dfrac{\sin x}{x} \begin{matrix} \to a \\ \to b \end{matrix} \Rightarrow \dfrac{da}{dx} = \cos x$

$\Rightarrow \dfrac{d_b}{d_x} = 1$

$\dfrac{dP}{dx} = \dfrac{b\dfrac{dq}{dx} - a\dfrac{db}{dx}}{b^2} \Leftarrow \text{division rule}$

$\dfrac{dP}{dx} = \dfrac{(x)\cos x - \sin x(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$Q = \dfrac{x}{\sin x} \to u \Rightarrow \dfrac{du}{dx} = 1$

$v \Rightarrow \dfrac{dv}{dx} = \cos x$

$\dfrac{dQ}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} = \dfrac{\sin x(1) - (x)\cos x}{(\sin x)^2}$

$\dfrac{dQ}{dx} = \dfrac{\sin x - x\cos x}{(\sin x)^2}$

$\dfrac{dy}{dx} = \dfrac{dP}{dx} + \dfrac{dQ}{dx}$

$= \dfrac{x\cos x - \sin x}{x^2} + \dfrac{\sin x - x\cos x}{(\sin x)^2}$

$\dfrac{dy}{dx} = x\cos x - \sin x\left[\dfrac{1}{x^2} - \dfrac{1}{(\sin x)^2} \right]$

15. $y = \dfrac{\sin x}{1 + \cos x}$

$\quad \to u \Rightarrow \dfrac{du}{dx} = \cos x$

$\quad \to v \Rightarrow \dfrac{dv}{dx} = 0 + (-\sin x) = -\sin x$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} = \dfrac{(1 + \cos x)(\cos x) - (\sin x)(-\sin x)}{(1 + \cos x)^2}$

$\quad = \dfrac{1 + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} = \dfrac{(1 + \cos x)}{(1 + \cos x)^2} = \dfrac{1}{1 + \cos x}$

16. $y = \dfrac{x}{\underbrace{\sin x + \cos x}_{=v}}$

$\quad \to u \Rightarrow \dfrac{du}{dx} = 1$

$\quad \Rightarrow \dfrac{dv}{dx} = \cos x - \sin x$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

17. $y = \dfrac{[(x)(\sin x)] \leftarrow u}{[1 + \tan x] \leftarrow v}$

$u = [\underset{\uparrow}{x}][\underset{\uparrow}{\sin(x)}], \dfrac{dp}{dx} = 1, \dfrac{dQ}{dx} = \cos x$

$\dfrac{du}{dx} = Q\dfrac{dP}{dx} + P\dfrac{dQ}{dx} = \sin x(1) + (x)\cos x$

$\dfrac{du}{dx} = \sin x + x\cos x$

$v = 1 + \tan x \Rightarrow \dfrac{dv}{dx} = 0 + \sec^2 x = \sec^2 x$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{(1 + \tan x)(\sin x)(x\cos x) - (x\sin x)\sec^2 x}{(1 + \tan x)^2}$

18. $y = \cos^2 x = \underset{\underset{u}{\uparrow}}{\cos x} \ \underset{\underset{v}{\uparrow}}{\cos x}$

$\dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx} = \cos x(-\sin x) + \cos x(-\sin x)$

$\qquad\qquad = -2\sin x \cos x = -\sin 2x$

19. $y = \dfrac{1}{4}\tan^4 x$

$\quad$ Let $P = \tan x \Rightarrow y = \dfrac{1}{4}(P)^4$

$$\frac{dP}{dx} = \sec^2 x, \quad \frac{dy}{dP} = \frac{1}{4}(4P^3)$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = (P)^3(\sec^2 x)$$

$$= (\tan x)^3 (\sec^2 x)$$

20. $y = \cos x - \dfrac{1}{3}\cos^3 x$

$$\frac{dy}{dx} = \frac{d(\cos x)}{dx} - \frac{1}{3}\frac{d(\cos^3 x)}{dx}$$

Let $Z = \cos^3 x., \dfrac{dZ}{dx} = \dfrac{d(\cos^3 x)}{dx} = ?$

Let $P = \cos x \Rightarrow z = p^3$

$$\frac{dP}{dx} = -\sin x, \quad \frac{dZ}{dP} = 3P^2$$

$$\Rightarrow \frac{dZ}{dx} = \frac{dZ}{dP} \times \frac{dP}{dx} = -3P^2 \times \sin x$$

$$\frac{dZ}{dx} = -3(\cos x)^2 \sin x$$

$$\Rightarrow \frac{dy}{dx} = -\sin x - \frac{1}{3}[-3(\cos x)2\sin x)$$

$$= -\sin x[\underbrace{1 - \cos^2 x}_{\sin^2 x}]$$

$$= -\sin^3 x$$

21. $y = 3\underset{\underset{P}{\downarrow}}{\sin^2 x} - \underset{\underset{Q}{\downarrow}}{\sin^3 x}, \quad \dfrac{dy}{dx} = \dfrac{3dP}{dx} - \dfrac{dQ}{dx}$

$P = \sin^2 x$

Let $z = \sin x \Rightarrow p = z^2$

$$\Rightarrow \frac{dz}{dx} = \cos x, \quad \frac{dp}{dz} = 2z$$

$$\frac{dP}{dx} = \frac{dP}{dz} \times \frac{dz}{dx} = 2z_x \cos x$$

$$= 2z\cos x = 2(\sin x)\cos x = \sin 2x$$

$Q = \sin^3 x$

Let $y = \sin x, Q = y^3$

$$\Rightarrow \frac{dy}{dx} = \cos x, \quad \frac{dQ}{dy} = 3y^2$$

$$\frac{dQ}{dx} = \frac{dQ}{dy} \times \frac{dy}{dx} = (3y^2) \times \cos x = 3y^2 \cos x = 3(\sin x)^2 \cos x$$

$$\Rightarrow \frac{dy}{dx} = 3\frac{dp}{dx} - \frac{dQ}{dx} = 3(\sin 2x) - 3\sin^2 x \cos x$$

$$= 6\sin x\cos x - 3\sin^2 x\cos x$$

22. $y = \dfrac{1}{3}\underset{\underset{a}{\downarrow}}{\tan^3 x} - \underset{\underset{b}{\downarrow}}{\tan x} + \underset{\underset{c}{\downarrow}}{x}$

$\dfrac{dy}{dx} = \dfrac{1}{3}\dfrac{da}{dx} - \underset{\underset{\sec^2 x}{\downarrow}}{\dfrac{db}{dx}} + \dfrac{dc}{dx}$

$a = \tan^3 x$

Let $P = \tan x \Rightarrow a \Rightarrow p^3$

$\dfrac{dP}{dx} = \sec^2 x, \dfrac{da}{dP} = 3P^2$

$\Rightarrow \dfrac{da}{dx} = \dfrac{da}{dP} \times \dfrac{dP}{dx} = 3P^2 \sec^2 x$

$= 3(\tan x)(\sec^2 x)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}[\tan^2 x \sec^2 x] - \sec^2 x + 1$

$= \tan^2 x \sec^2 x + [\sec^2 x - \tan^2 x] - \sec^2 x$

$= \tan^2 x(\sec^2 x - 1)$

$= (\tan^2 x)(\tan^2 x) = \tan^4 x$

23. $y = \underset{u}{\underbrace{(\sec^2 x)}} - \underset{v}{\underbrace{(\tan x)}} \Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} - \dfrac{dv}{dx} \Rightarrow \dfrac{dv}{dx} = \sec^2 x$

$u = \underset{\underset{a}{\downarrow}}{(x)}\underset{\underset{b}{\downarrow}}{(\sec^2 x)}, a = x \Rightarrow \dfrac{da}{dx} = 1$

$b = \sec^2 x$

Let $P = \sec x \Rightarrow b = p^2$

$\dfrac{dP}{dx} = \sec x \tan x, \dfrac{db}{dP} = 2P$

$\dfrac{db}{dx} = \dfrac{dp}{dP} \times \dfrac{dP}{dx} = 2P \times \sec x \tan x$

$\qquad\qquad = 2\sec x \sec x \tan x$

$\dfrac{db}{dx} = 2\sec^2 x \tan x$

$\dfrac{du}{dx} = b\dfrac{da}{dx} + a\dfrac{db}{dx} = \sec^2 x(1) + (x)2\sec^2 x \tan x$

$\dfrac{dy}{dx} = \dfrac{du}{dx} - \dfrac{dv}{dx} = (\sec^2 x + 2x \sec^2 x \tan x) - \sec^2 x$

$= 2x \sec^2 x \tan x$

24. $y = \underset{\underset{u}{\downarrow}}{x}\underset{\underset{v}{\downarrow}}{\ln x}$

$\dfrac{du}{dx} = 1, \dfrac{dv}{dx} = \dfrac{1}{x}$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

25. $y = Ln^2x = (Lnx)^2$

Let $P = Lnx \Rightarrow y = p^2$

$$\frac{dP}{dx} = \frac{1}{x}, \ \frac{dy}{dP} = 2P$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = (2p)\left(\frac{1}{x}\right) = \frac{2Lnx}{x}$$

26. $y = Ln(x^2)$

Let $p = x^2 \Rightarrow y = Ln\,P$

$$\frac{dP}{dx} = 2x, \frac{dy}{dP} = \frac{1}{P}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = \frac{1}{P} \times 2x = \frac{2x}{x^2} = \frac{2}{x}$$

27. $y = \sqrt{Lnx} = (Lnx)^{1/2}$

Let $P = Lnx \Rightarrow y = p^{1/2}$

$$\frac{dP}{dx} = \frac{1}{x}, \frac{dy}{dP} = \frac{1}{2}(P)^{\frac{1}{2}-1} = \frac{(P)^{-1/2}}{z} = \frac{1}{2\sqrt{P}}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = \frac{1}{2\sqrt{P}} \times \frac{1}{x} = \frac{1}{2\sqrt{Lnx}} \times \frac{1}{x}$$

28. $\int \sqrt{x}\,dx = \int (x)^{1/2}dx$

$$= \frac{(x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2(x)^{3/2}}{3} + C$$

29. $\int \sqrt[m]{x^n}\,dx = \int [x]^{\frac{n}{m}}dx$

$$\frac{[x]^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C = \frac{(x)^{\frac{n+m}{m}}}{\frac{n+m}{m}} + C$$

$$= \frac{m}{n+m}(x)^{\frac{n+m}{m}} + C$$

30. $\int \frac{dx}{x^2} = \int x^{-2}dx$

$$= \frac{(x)^{-2+1}}{-2+1} + C = \frac{(x)^{-1}}{-1} + C$$

$$= \frac{-1}{x} + C$$

31. $\int \frac{dx}{2\sqrt{x}} = \frac{1}{2} \int (x)^{-1/2} dx$

$$= \frac{1}{2} \frac{(x)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} = \frac{1}{2} \frac{(x)^{1/2}}{\frac{1}{2}} + C$$

$$= \sqrt{x} + C$$

32. $\int (1 - 2u) du = \int du - 2 \int u \, du$

$$= \int u^0 du - 2 \frac{(u)^{1+1}}{1+1}$$

$$= \frac{u^{0+1}}{0+1} - 2 \frac{u^2}{2} + C$$

$$= u - u^2 + C$$

EXERCISE # II

1. **(B)**

$$y = \frac{1}{\sqrt{x}} = (x)^{-1/2}$$

$$\frac{dy}{dx} = \frac{-1}{2}(x)^{\frac{-1}{2}-1} = \frac{-1}{2}(x)^{-3/2}$$

2. **(A)**

$$y = \frac{1}{(ax+b)^2} = (ax+b)^{-2} \ (0,6 = \text{constant})$$

$$P = (ax+b) \Rightarrow y = (P)^{-2}$$

$$\frac{dP}{dx} = a(1) + 0 = a \quad \frac{dy}{dP} = -2(P)^{-2-1} = -2P - 3 = \frac{-2}{P^3}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = (a)\left(\frac{-2}{P^3}\right) = \frac{-2a}{(ax+b)^3}$$

3. **(C)**

$$y = x^3 + \frac{1}{x^3} + 8 = x^3 + (x)^{-3} + 8$$

$$\frac{dy}{dx} = 3x^2 + -3(x)^{-4} + 0 = 3x^2 - 3x^{-4}$$

4. **(D)**

$y = \sin x^3$

Let $P = x^3 \Rightarrow y = \sin P$

$\dfrac{dP}{dx} = 3x^2 \Rightarrow \dfrac{dy}{dP} = \cos P$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dP} \times \dfrac{dp}{dx} = (\cos P)(3x^2) = (3x^2)\cos(x^3)$

5. **(D)**

$y = \sqrt{4x^3 - 5} = [4x^3 - 5]^{1/2}$

Let $P = 4x^3 - 5 \Rightarrow y = [P]^{1/2}$

$\dfrac{dp}{dx} = 4(3x^2) - 0, \dfrac{dy}{dP} = \dfrac{1}{2}(P)^{\frac{1}{2}-1}$

$= 12x^2 \qquad = \dfrac{1}{2}(P)^{-1/2} = \dfrac{1}{2\sqrt{P}}$

$\dfrac{dy}{dx} = \dfrac{dy}{P} \times \dfrac{dP}{dx}$

$= \dfrac{1}{2\sqrt{P}} \times 12x^2 = \dfrac{6x^2}{\sqrt{4x^3 - 5}}$

6. **(D)**

$y = \sin(Lnx)$

$P = Lnx \Rightarrow y = \sin P$

$\Rightarrow \dfrac{dP}{dx} = \dfrac{1}{x}, \dfrac{dy}{dP} = \cos(p)$

$\dfrac{dy}{dx} = \dfrac{dy}{dP} \times \dfrac{dP}{dx} = \cos(P) \times \dfrac{1}{x} = \dfrac{\cos(Lnx)}{x}$

7. **(B)**

$y = \sqrt{2x^2 + 1} = (2x^2 + 1)^{1/2}$

Let $P = (2x^2 + 1) \Rightarrow y = (P)^{1/2}$

$\dfrac{dP}{dx} = 2(2x) + 0 = 4x, \dfrac{dy}{dP} = \dfrac{1}{2}(P)^{1/2-1} = \dfrac{1}{2\sqrt{P}}$

$\dfrac{dy}{dx} = \dfrac{dy}{dP} \times \dfrac{dP}{dx} = \dfrac{1}{2\sqrt{p}} \times {}^2 4x = \dfrac{2x}{\sqrt{2x^2 + 1}}$

8. **(A)**

$y = (e)^{\sqrt{2x}}$

Let $P = \sqrt{2x} = \left(\sqrt{2}\right)(x)^{1/2} \Rightarrow y = e^p \Rightarrow \dfrac{dy}{dP} = e^P$

$\dfrac{dP}{dx} = \sqrt{2}\,\dfrac{1(x)}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}(x)^{-1/2} = \dfrac{1}{\sqrt{2}\sqrt{x}}$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = (e)^P \times \frac{1}{\sqrt{2x}} = \frac{(e)^{\sqrt{2x}}}{\sqrt{2x}}$$

9. **(D)**

$$y = x^4 - \sin x + 3\cos x$$

$$\frac{dy}{dx} = 4x^3 - 2(\cos x) + 3(-\sin x) = 4x^3 - 2\cos x - 3\sin x$$

10. **(A)**

$$y = \underbrace{(x^2)(\sin x)}_{u}\ \underbrace{(Lnx)}_{v}$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$= Lnx[2x\sin x + x^2\cos x] + (x^2\sin x)\left(\frac{1}{x}\right)$$

$$v = Lnx \Rightarrow \frac{dv}{dx} = \frac{1}{x}$$

$$u = (\underset{\underset{P}{\uparrow}}{x^2})(\underset{\underset{Q}{\uparrow}}{\sin x})$$

$$\frac{du}{dx} = Q\frac{dP}{dx} + P\frac{dQ}{dx}$$

$$= (\sin x)(2x) + x^2(\cos x)$$

$$\frac{du}{dx} = 2\sin x + x^2\cos x$$

11. **(A)**

$$y = \frac{x^2 + 1 \leftarrow u}{x + 1 \leftarrow v} \Rightarrow \frac{du}{dx} = 2x$$

$$\Rightarrow \frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

Solve further

12. **(D)**

$$v = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{dr} = \left(\frac{4}{3}\pi\right)(3r^2) = 4\pi r^2$$

13. **(D)**

$$xy = c^2$$

$$\Rightarrow y = \frac{c^2}{x} \Rightarrow y = c^2(x)^{-1}$$

$$\frac{dy}{dx} = c^2(-1)(x)^{-1-1} = \frac{-C^2}{x^2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}(xy)$$

$$= \frac{-y}{x}$$

14. **(B)**

$$y = \frac{1}{\sqrt{2x+1}} = (2x+1)^{-1/2}$$

Let $P = 2x + 1$, $\Rightarrow y = (P)^{-1/2}$

$$\frac{dP}{dx} = 2, \quad \frac{dy}{dP} = -\frac{1}{2}(P)^{-1/2-1} = -\frac{1}{2\left(P^{3/2}\right)}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = -\frac{1}{2\left(P^{3/2}\right)} \times 2 = -\frac{1}{P^{3/2}} = -\frac{1}{\left(2x+1\right)^{3/2}}$$

15. **(B)**

$$x = at^2, y = 2at$$

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

16. **(A)**

$$\int \sqrt[5]{x}\ dx = \int (x)^{1/5} x$$

$$= \frac{[x]^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C = \frac{5}{6}(x)^{6/5} + c$$

17. **(C)**

$$\int \frac{1}{(ax+b)^2}\,dx = \int (ax+b)^{-2}dx$$

We'll use integration by substitution
Let $P = ax + b$

$$\Rightarrow \frac{dP}{dx} = a \Rightarrow dx = \frac{dP}{a}$$

$$\Rightarrow \int (ax+b)^{-2}dx = \int (p)^{-2}\frac{dP}{a}$$

$$= \left(\frac{1}{a}\right)\int P^{-2}dP = \frac{1}{a}\left[\frac{P}{-2+1}\right]^{-2+1} + C$$

$$= \left(\frac{1}{a}\right)\left(\frac{P^{-1}}{-1}\right) + C = \frac{-1}{a} \times \frac{1}{P} + C$$

$$= \frac{1}{a(ax+b)} + C$$

18.   **(A)**

$$\int \sin x \cos x \, dx$$

Let $P = \sin x \Rightarrow \dfrac{dP}{dx} = \cos x \Rightarrow dP = \cos x \, dx$

$$\Rightarrow \int \sin x \cos x \, dx = \int P \, dP = \frac{P^2}{2} + C'$$

But options are in terms of cos2x, sin2x
So lets convert $\sin^2 x$ into cos2x

$$\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\frac{\left[\dfrac{1 - \cos 2x}{2}\right]}{2} + c' = \frac{1 - \cos 2x}{4} + c'$$

$$= \frac{-\cos 2x}{4} + \left[c' - \frac{1}{4}\right] \text{ new const} = c$$

$$= -\frac{\cos 2x}{4} + c$$

19.   **(B)**

$$\int \frac{x}{x^2 + a^2} \, dx$$

Let $P = x^2 + a^2$

$$\frac{dP}{dx} = 2x + 0 \Rightarrow \frac{dP}{2} = x \, dx$$

$$\Rightarrow \int \frac{x}{x^2 + a^2} \, dx = \int \frac{1}{p} \frac{dP}{2} = \frac{1}{2} \int \frac{dP}{P}$$

$$\Rightarrow \frac{1}{2} \mathrm{Ln}(p) + C = \frac{1}{2} \mathrm{Ln}(x^2 + a^2) + C$$

20.   **(C)**

$$\int\limits_{-\pi/2}^{\pi/2} \cos x \, dx = [\sin x]_{-\pi/2}^{\pi/2}$$

$$= \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= [1 - (-1)] = 2$$

21.   **(A)**

$$\int\limits_{0}^{\pi/2} \sqrt{1 + \cos x} \, dx$$

Half angle formula

$$\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$$

$$\int_{0}^{\pi/2} \sqrt{1 + 2\cos 2\left(\frac{x}{2}\right) - 1} = \int_{0}^{\pi/2} \sqrt{2\cos^2\left(\frac{x}{2}\right)}$$

$$\int_{0}^{\pi/2} \sqrt{2} = \cos\left(\frac{x}{2}\right) dx$$

$$\int_{0}^{\pi/2} \sqrt{2} = \cos\left(\frac{x}{2}\right) dx$$

$$\Rightarrow \text{Let } P = \frac{x}{2} \Rightarrow \frac{dP}{dx} = \frac{1}{2}$$

$$\Rightarrow dx = 2dP$$

$$\Rightarrow \sqrt{2} \int_{0}^{\pi/2} \cos\left(\frac{x}{2}\right) dx = \sqrt{2} \int_{0}^{\pi/2} \cos(P)(2dP)$$

$$= 2\sqrt{z} \int_{0}^{\pi/2} \cos P \, dP$$

$$= 2\sqrt{z} \int_{0}^{\pi/2} \cos P \, dP$$

$$= 2\sqrt{2}[\sin P]_{0}^{\pi/2} = 2\sqrt{2}\left[\sin\left(\frac{x}{2}\right)\right]_{0}^{\pi/2}$$

$$= 2\sqrt{2}\left[\sin\left(\frac{\pi/2}{2}\right) - \sin\left(\frac{0}{2}\right)\right]$$

$$= 2\sqrt{2}\left[\sin\left(\frac{\pi}{4}\right) - 0\right] = 2\sqrt{z}\left(\frac{1}{\sqrt{2}} - 0\right)$$

$$= 2$$

22. **(D)**

$$\int (1-x)\sqrt{x}\,dx$$

$$\int \sqrt{x}\,dx - \int x\sqrt{x}\,dx$$

$$\int (x)^{1/2}dx - \int \underbrace{(x)'(x)^{1/2}}_{x^{3/2}}\,dx$$

$$= \frac{[x]^{1/2+1}}{\frac{1}{2}+1} - \frac{(x)^{3/2+1}}{\frac{3}{2}+1} + C$$

$$= \frac{2}{3}(x)^{3/2} - \frac{2}{5}(x)^{5/2} + C$$

23. **(B)**

$$\int \frac{1}{\sin^2 x \cos 2x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x\, dx + \int \cos ec^2 x\, dx$$

$$= \tan x + (-\cot x) + C$$

$$= \tan x - \cot x + C$$

24. **(A)**

$$\int \frac{1}{1+e^{-x}} dx = \int \frac{1}{1+\dfrac{1}{e^x}} dx$$

$$= \int \frac{1}{\dfrac{e^x+1}{e^x}} dx = \int \frac{e^x dx}{e^x+1}$$

Let $p = e^x + 1$

$$\frac{dP}{dx} = e^x \Rightarrow dP = e^x dx$$

$$= \int \frac{e^x dx}{e^x+1} = \int \frac{dP}{P}$$

$$= Ln(P) + C = Ln(1+e^x) + c$$

25. **(A)**

$$\int \frac{\cos ec^2 x}{1+\cot x} dx$$

$$P = 1+\cot x \Rightarrow \frac{dP}{dx} = 0 + [-\cos ec^2 x]$$

$$\Rightarrow dP = -\cos ec^2 x\, dx \Rightarrow \cos ec^2 x\, dx = -dp$$

$$\Rightarrow \int \frac{\cos ec^2 x\, dx}{1+\cot x} = \int \frac{-dP}{P} = -1\int \frac{dP}{P}$$

$$= -Ln(P) + C = -Ln(1+\cot x) + C$$

26. **(B)**

$$\int \frac{Lnx}{x} dx$$

Let $P = Lnx \Rightarrow \dfrac{dP}{dx} = \dfrac{1}{x}$

$$\Rightarrow dP = \left[\frac{dx}{x} \right]$$

$$\int \frac{Lnx}{d} dx = \int P\, dP = \left[\frac{P^2}{2} \right] + C$$

$$= \frac{(\text{Ln} x)^2}{2} + C$$

27. **(A)**

$$\int_0^{\pi/2} [\sin x + \cos x]dx$$

$$\int_0^{\pi/2} \sin x \, dx + \int_0^{\pi/2} \cos x \, dx$$

$$[-\cos x]_0^{\pi/2} + [\sin x]_0^{\pi/2}$$

$$\left[\left(-\cos \frac{\pi}{2}\right) - (-\cos 0)\right] + \left[\left(\frac{\pi}{2}\right) - \sin 0\right]$$

$$= 0 + 1 + (1) - 0$$
$$= 2$$

28. **(A)**

$$\int_0^{\infty} e^{-x} \, dx$$

$$P = -x \Rightarrow \frac{dP}{dx} = -1 \Rightarrow dx = -dP$$

$$\int_0^{\infty} e^{-x} dx = \int_0^{\infty} e^{P}(-dP) = \int_0^{\infty} e^{p} dP$$

$$= -\left[e^{P}\right]_0^{\infty} = -[e^{-x}]_0^{\infty}$$

$$= -[e^{-\infty} - e^{-0}]$$

$$= -\left[\frac{1}{\underset{\substack{\downarrow \\ =\infty}}{e^{\infty}}} - \frac{1}{\underset{\substack{\downarrow \\ -1}}{e^{0}}} \right]$$

$$= -\left[\frac{1}{\infty} - \frac{1}{1}\right] = 1$$

1. $\mathbf{0.320\,\pi\,cm^2\,/\,s}$

$$\frac{dr}{dt} = 0.05 cm/s$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = (\pi)(2r) = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = (2\pi r)(0.05)$$

$$= 0.1\pi r = 0.01 \times 3.2\pi$$

$$= 0.32\pi$$

2. $\mathbf{900\,cm^3\,/s}$

$$\frac{dl}{dt} = 3 cm/s$$

$$v = \ell^3 \Rightarrow \frac{dV}{d\ell} = 3\ell^2$$

$$\frac{dv}{dt} = \frac{dv}{d\ell}\frac{d\ell}{dt} = 3\ell^2 \times 3$$

$$= 9\ell^2 = 9(10)^2$$

$$= 900\ cm^3$$

3. $\dfrac{1}{\boldsymbol{\pi}}\,\mathbf{cm\,/\,s}$

$$\frac{dv}{dt} = 300 cm^3/s$$

$$\frac{d_r}{dt} = ?$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr}\left[\frac{4}{3}\pi\right](3r^2) = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$300 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{900}{4\pi(r=15)^2} = \frac{900 \quad 60 \quad 4}{4\pi \times 15 \times 15}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{\pi} cm/s$$

4. $\dfrac{8}{3}$ **cm/s**
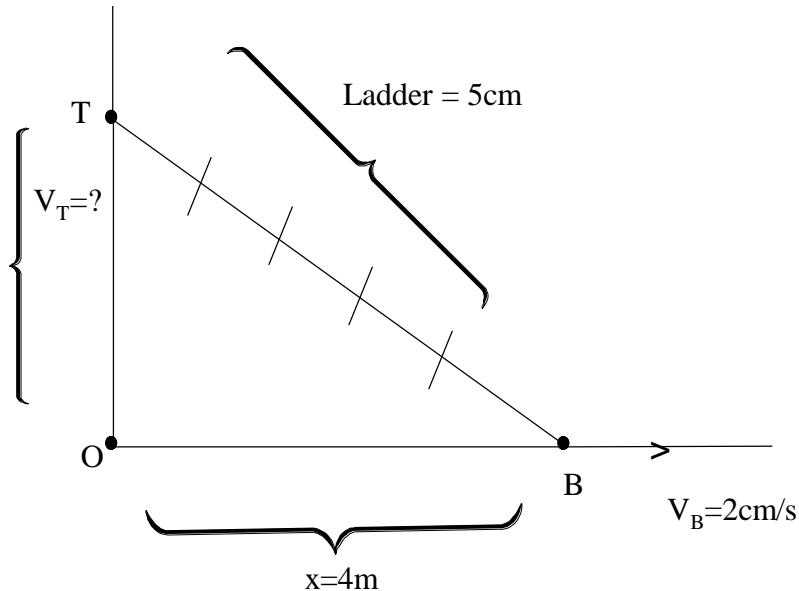
$x^2 + y^2 = 5^2$

$y^2 = 25 - x^2 \Rightarrow y = \sqrt{25 - x^2}$

Let $P = 25 - x^2 \Rightarrow y = \sqrt{p} = (P)^{1/2}$

$\Rightarrow \dfrac{dP}{dx} = 0 - 2x, \dfrac{dy}{dP} = \dfrac{1}{2}(P)^{-1/2}$

$\qquad = -2x$

$\dfrac{dy}{dx} = \left(\dfrac{dy}{dP}\right)\left(\dfrac{dP}{dx}\right) = \dfrac{1}{2}(P)^{-1/2}(-2x)$

$\dfrac{dy}{dx} = -\dfrac{x}{(P)^{1/2}} = \dfrac{-x}{\sqrt{25 - x^2}}$

$dy = -\dfrac{x}{\sqrt{25 - x^2}}dx$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{-x}{\sqrt{25 - x^2}}\dfrac{dx}{dt}$

$V_T = \dfrac{dy}{dt} \quad$ * Rate of change of y

$\dfrac{dx}{dt} = V_B$ * Rate of change of x

$\Rightarrow V_T = \dfrac{-x}{\sqrt{25 - x^2}}V_B$

$\qquad = \dfrac{-(4)}{\sqrt{25 - (4)^2}}(2)$

$V_T = \dfrac{-8}{\sqrt{9}} = \dfrac{-8}{3} \qquad$ -ve indicates that V_T is –ve and it should be as y is $\downarrow$ sing

5. $\dfrac{1}{48\pi}\,\text{cm/s}$

$$\dfrac{dV}{dt} = 12\,\text{cm}^3/s$$

$$V_{cone} = \dfrac{1}{3}\pi R^2 h, \quad h_{cone} = \dfrac{R}{6} \Rightarrow R = 6h$$

$$V = \dfrac{1}{3}\pi (R = 6h)^2 (h) = \dfrac{1}{3}\pi 36h^3 = 12\pi h^3$$

$$\dfrac{dv}{dh} = 12\pi(3h^2) = 36\pi h^2$$

$$dv = 36\pi h^2 dh$$

$$\Rightarrow \dfrac{dv}{dt} = 36\pi h^2 \dfrac{dh}{dt}$$

$$12 = 36\pi(h = 4)^2 \dfrac{dh}{dt}$$

$$\dfrac{dh}{dt} = \dfrac{1}{48\pi}$$

6. $\dfrac{5}{2}\,\text{km/h}$
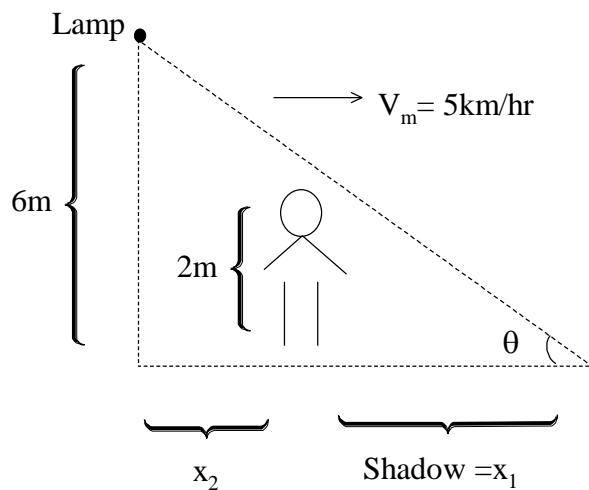
$$\tan\theta = \dfrac{6}{x_1 + x_2} = \dfrac{2}{x_1} \Rightarrow 3x_1 = x_1 + x_2$$

$$\Rightarrow 2x_1 = x_2$$

$$\dfrac{d(x_2)}{d(x_1)} = 2$$

$$\Rightarrow dx_2 = 2dx_1 \Rightarrow \dfrac{dx_2}{dt} = 2\dfrac{dx_1}{dt}$$

$$\dfrac{dx_2}{dt} = V_m = 5 = 2\left[\dfrac{dx_1}{dt} = V_{shadow}\right]$$

$$\dfrac{5}{2} = V_{shadow}$$

7. $\dfrac{15}{2}, \dfrac{15}{2}$

$x + y = 15$

$y = 15 - x$

Let, $z = x^2 + y^2 = x^2 + (15 - x)^2$

$z = x^2 + 225 + x2 - 30x$

$z = 2x^2 - 30x + 225$

Minimize z.

$S - 1 \ \dfrac{dZ}{dx} = 2(2x) - 30(1) + 0$

$\qquad\quad = 4x - 30$

$S - 2 \ \dfrac{dz}{dx} = 4x - 30 = 0 \Rightarrow x = 7.5$

$\qquad\qquad\qquad\qquad \Rightarrow y = 15 - x$

$\qquad\qquad\qquad\qquad\quad = 7.5$

8. **3 cm**

Box $\Rightarrow$ Base = square of length $18 - 2\ell$

$\qquad\qquad$ ht = 1

Volume (V) $= (18 - 2\ell)^2 \ (\ell)$

$\qquad\qquad = [324 + 4\ell^2 - 72\ell](\ell)$

$V = 4\ell^3 - 72\ell^2 + 324\ell$

$S - 1 \ \dfrac{dV}{d\ell} = 4(3\ell^2) - 72(2\ell) + 324$

$12\ell^2 - 144\ell + 324$

$S\text{ -2 } \dfrac{dv}{d\ell} = 12\ell^2 - 144\ell + 324 = 0$

$12[\ell^2 - 12\ell + 27] = 0$

$\Rightarrow \ell^2 - 12\ell + 27 = 0$

$\ell^2 - 9\ell - 3\ell + 27 = 0$

$\ell(\ell - 9) - 3(\ell - 9)$

$(\ell - 9)(\ell - 3) = 0$

$\Rightarrow \ell = 3 \text{ or } \ell = 9$

$S - 3 \ \dfrac{d^2v}{d\ell^2} = 24(\ell) - 144 = 24\ell - 144$

$S - 4 \text{ for } \ell = 3 \ \dfrac{d^2v}{dQ^2} = 24(3) - 144 = -72 < 0$

$\Rightarrow \ell = 3 \text{ maxima}$

(ii) $\quad$ For $\ell = g \ \dfrac{d^2v}{d\ell^2} = 24(9) - 144 = 216 - 144 = 72 > 0$

$\Rightarrow \ell = 9 = \text{minima}$

9.
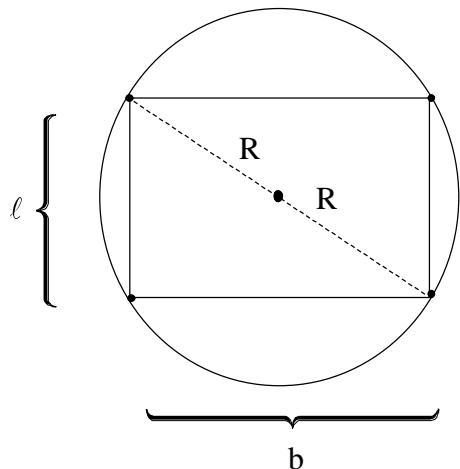
$\ell^2 + b^2 = (2R)^2$

$\ell^2 + b^2 = 4R^2$

$\ell = \sqrt{4R^2 - b^2}$

$Area = \ell \times b$

$A = \sqrt{4R^2 - b^2}\, b$

$S-1 \quad \dfrac{dA}{dB} = b\, \dfrac{d\sqrt{4R^2 - b^2}}{db} + \sqrt{4R^2 - b^2}\, \dfrac{d(b)}{db}$

$\dfrac{dA}{db} = b\left[\dfrac{-b}{\sqrt{4R^2 - b^2}}\right] + \sqrt{4R^2 - b^2}\,(1)$

$S-2 \quad \dfrac{dA}{dB} = 0$

$\dfrac{dA}{dB} = \dfrac{-b^2}{\sqrt{4R^2 - b^2}} + \sqrt{4R^2 - b^2} = 0$

$\sqrt{4R^2 - b^2} = \dfrac{b^2}{\sqrt{4R^2 - b^2}}$

$\sqrt{4R^2 - b^2}\,\sqrt{4R^2 - b^2} = b^2$

$4R^2 - b^2 = b^2$

$\Rightarrow 4R^2 = 2b^2 \text{ or } b = R\sqrt{2}$

As $\ell = \sqrt{4R^2 - b^2} \Rightarrow \ell\sqrt{4R^2 - \left(\sqrt{2}R\right)^2}$

$\ell = \sqrt{4R^2 - 2R^2}$

$\ell = R\sqrt{2}$

$\Rightarrow$ Area is maximum $b = \ell = R\sqrt{2}$

10. 44.1 m

$V = 29.4 - 9.8t$

$V = \dfrac{ds}{dt} = 29.4 - 9.8t = 0 \Rightarrow 29.4 = 9.8t \Rightarrow t = 3$

$$\frac{dS}{dt} = 29.4 - 9.8t$$

$$\int dS = 29.4 \int dt - 9.8 \int t\,dt$$

$$= 29.4 \left[\frac{t^0 + 1}{0+1} \right]_0^3 - 9.8 \left[\frac{t^2}{2} \right]_0^3$$

$$= 29.4[3-0] = 4.9[(3^2) - (0)^2]$$

$$= 88.2 - 4.9 \times 9$$

$$= 88.2 - 44.1$$

$$= 44.1 \text{ m}$$

EXERCISE # IV

1. $S = \frac{1}{4}t^4 - 4t^3 + 16t^2$

(i) S = 0 according to question

$$\Rightarrow \frac{t^4}{4} - 4t^3 + 16t^2 = 0$$

$$t^2 \left(\frac{t^2}{4} - 4t + 16 \right) = 0$$

$$\Rightarrow t^2 = 0 \text{ or } \frac{t^2}{4} - 4t + 16 = 0$$

$$\Rightarrow t = 0 \; ; \; \frac{t^2}{4} - 2t - 2t + 16 = 0$$

$$t \left(\frac{t}{4} - 2 \right) - 8 \left(\frac{t}{4} - 2 \right) = 0$$

$$\Rightarrow \left(\frac{t}{4} - 2 \right)(t - 8) = 0$$

$$\Rightarrow \frac{t}{4} - 2 = 0 \Rightarrow \frac{t}{4} = 2 \text{ or } t = 8 \text{ or } t - 8 = 0$$

$$\Rightarrow t = 8$$

(ii) $v = \frac{ds}{dt}, S = \frac{1}{4}t^4 - 4t^3 + 16t^2$

$$\Rightarrow \frac{ds}{dt} = \frac{1}{4}(4t^3) - 4(3t^2) + 16(2t)$$

$$v = t^3 - 12t^2 + 32t$$

$$v = 0 \Rightarrow t^3 - 12t^2 + 32t = 0$$

$$t[t^2 - 12t + 32] = 0$$

$$\Rightarrow t = 0 \text{ or } t^2 - 12t + 32 = 0$$

$$t^2 - 8t - 4t + 32 = 0$$

$$t(t - 8) - 4(t - 8) = 0$$

$$(t - 8)(t - 4) = 0$$

$$\Rightarrow t = 8 \text{ or } t = 4$$

2. $m = 3kg$, $s = (1 + t + t^2)cm$

$$v = \frac{ds}{dt} = 0 + 1 + 2t \, \frac{cm}{s}$$

$$V = 2t + 1 \, \frac{cm}{s}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(3)\underset{\underset{(t=5)}{\downarrow}}{(2t+1)^2}$$

$$= \frac{3}{2}(kg)(2\times 5 + 1 = 11)^2 \left(\frac{cm}{S}\right)^2$$

$$= \frac{3}{2}121kg \, \frac{cm^2}{s^2}$$

$$= \frac{363}{2} \frac{kgm^2}{S^2} \times \frac{1}{104} \quad \because 1cm^2 \, \frac{1}{10^4} m^2$$

$$= 0.01815 \frac{kgm^2}{s^2}$$

3. $S = t^3 - 4t^2 - 3t$, Find acc^n when $v = 0$

$$V = \frac{ds}{dt} = 3t^2 - 4(2t) - 3$$

$$= 3t^2 - 8t - 3 = 0$$

$$3t^2 - 9t + t - 3 = 0$$

$$3t[t-3] + 1[t-3] = 0$$

$$(t-3((3t+1)=0$$

$$t = 3, \, t = -\frac{1}{3} \times \text{not possible}$$

$$t = 3$$

$$a = \frac{dv}{dt} = 3(2t) - (8) = 6t - 8$$

$$= 6t - 8 = 6(3) - 8$$

$$= 18 - 8 = 10m/s^2$$

4. $t = \sqrt{x} + 3 \Rightarrow \sqrt{x} = t - 3$ squaring both sides

$$x = (t-3)^2$$

$$x = (t^2 + 9 - 6t)$$

$$v = \frac{dx}{dt} = 2t - 6$$

$$v = 2t - 6 = 0 \Rightarrow t = 3$$

$$X = (3)^2 + 9 - 1(t = 3)$$

$$= 9 + 9 - 18 = 0$$

5. $V = 3t^2 + 2t + 1$

$$\frac{ds}{dt} = 3t^2 + 2t + 1$$

$$ds = 3t^2dt + 2tdt + dt$$

$$\int ds = 3\int t^2 dt + 2\int t\,dt + \int t^0 dt$$

$$= 3\left[\frac{t^3}{3}\right]_0^{10} + 2\left[\frac{t^2}{2}\right]_0^{10} + \left[\frac{t^{0+1}}{0+1}\right]_0^{10}$$

$$= [(10)^3 - 0^3] + [(10)^2 - 0^2] + (10)' - (0)]$$

$$= 1000 + 100 + 10$$

$$= 1110$$

6. $V = 9t^2 - 8t$

$$\frac{ds}{dt} = 9t^2 - 8t \Rightarrow ds = 9\int t^2 dt - 8\int t\,dt$$

$$S = \cancel{9}^3\left[\frac{t^3}{\cancel{3}}\right] - \cancel{8}^4\left[\frac{t^2}{\cancel{2}}\right]$$

$$S = [3t^3]_3^4 - [4t^2]_3^4 \quad 4^{th} \text{ second means 3 to 4}$$

$$= 3[(4)^3 - (3)^3] - 4[(4)^2 - (3)^2]$$

$$= 3[64 - 27] - 4[16 - 9]$$

$$= 3(37) - 4(7)$$

$$= 111 - 28 = 83$$

7. $V = 6t^2 + 4$

$$\frac{ds}{dt} = 6t^2 + 4 \Rightarrow ds = 6t^2 dt + 4dt$$

$$\int ds = 6\int t^2 dt + 4\int t^0 dt$$

$$[S]_0^5 = 2\cancel{6}\left[\frac{t^3}{3}\right]_0^5 + 4\left[\frac{t^{0+1}}{0+1}\right]_0^5$$

$$S - 0 = 2(5^3 - 0) + 4(5 - 0)$$

$$= 2(125) + 20$$

$$S = 270$$

8. $S = s_0 + v_0 t + \dfrac{1}{2}gt^2$

$$\frac{ds}{dt} = 0 + v_0(1) + \frac{9}{2}(2t)$$

$$= v_0 + gt$$

9. $x = t^4 - 12t^2 - 40$

(i)   at $t = 2 \Rightarrow x = (2)^4 - 12(2)^2 - 40$

$$= 16 - 48 - 40$$

$$= -72$$

(ii) $v = \dfrac{dx}{dt} = 4t^3 - 12(2t) - 0$

$$V = 4t^3 - 24t$$

At $t = 2$, $v = 4(2)^3 - 24(2)$

$$= 32 - 48 = -16$$

(iii) $\quad a = \dfrac{dv}{dt} = 4(3t^2) - 24$

$\Rightarrow a = t - 24$

At $t = 2 \Rightarrow a = 12(2)^2 - 24$

$A = 24$