Main Booklet Solution

Binomial Theorem

EXERCISE - 1 [A]

1. (b)

$S=\sum_{m=0}^{100} {}^{100}C_m\left(x-3\right)^{100-m}\,2^m$

$\Rightarrow\ S=\left(x-3+2\right)^{100}=\left(x-1\right)^{100}$

Now, coefficient of x^{53} in S, $\left(x-1\right)^{100}$ is $\left(-1\right)^{53}$

${}^{100}C_{53}$

2. (b)

Let $E=\left(\sqrt{5}+1\right)^5-\left(\sqrt{5}-1\right)^5$

$\Rightarrow\ E=2\left\{{}^5C_1\left(\sqrt{5}\right)^4+{}^5C_3\left(\sqrt{5}\right)^2+{}^5C_3\left(\sqrt{5}\right)^0\right\}$

$\Rightarrow\ E=2\left\{125+50+1\right\}=352$

3. (a)

Since, coefficient of $\left(3r\right)^{\text{th}}$ term in $\left(1+x\right)^{2n}$ equals coefficients of $\left(r+2\right)^{\text{th}}$ term in $\left(1+x\right)^{2n}$.

$\therefore\ {}^{2n}C_{3r-1}={}^{2n}C_{r+1}$

$\Rightarrow 3r-1+r+1=2n$

$\therefore\ r=\frac{n}{2}$

4. (c)

$t_{r+1}={}^9C_r\left(\frac{4}{3}x^2\right)^{9-r}\left(\frac{-3}{2x}\right)^r$

$\Rightarrow\ t_{r+1}={}^9C_r\left(\frac{4}{3}x^2\right)^{9-r}\left(\frac{-3}{2x}\right)^r$

$\Rightarrow\ t_{r+1}={}^9C_r\,\frac{2^{18-3r}}{3^{9-2r}}\left(-1\right)^r\,x^{18-3r}$

For term independent of x, we get

$18-3r=0$

$r=6$

5. (b)

$$\frac{n(n-1)}{2!}x^2 = \frac{-1}{8}x^2$$

$$\Rightarrow\ n(n-1) = -\frac{1}{4}$$

$$\Rightarrow\ 4n^2 - 4n + 1 = 0$$

$$\therefore\ n = \frac{1}{2}$$

6. (d)

$$t_{n+1} = {}^{n}C_{n}\left(2^{\frac{1}{3}}\right)^{n-n}\left(\frac{-1}{\sqrt{2}}\right)^{n}$$

$$\Rightarrow \left(\frac{1}{3^{3^{\frac{2}{3}}}}\right)^{\log_3 8} = (-1)^n\, 2^{\left(\frac{-n}{2}\right)}$$

$$\Rightarrow\ 3^{\frac{-5}{3}\log_3 8} = (-1)^n\, 2^{\left(\frac{-n}{2}\right)}$$

$$\Rightarrow\ \frac{1}{32} = \frac{(-1)^n}{2^{\frac{n}{2}}}$$

$$\therefore\ n = 10$$

Now, $t_5 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^{10-4}\dfrac{(-1)^4}{\left(2^{\frac{1}{2}}\right)^4} \Rightarrow t_5 = {}^{10}C_4$

7. (a)

$$t_5 + t_6 = 0 \Rightarrow {}^{n}C_4\,(a)^{n-4}\,b^4 - {}^{n}C_5\,(a)^{n-5}\,b^5 = 0$$

$$\Rightarrow\ {}^{n}C_4\, a = {}^{n}C_5\, b$$

$$\Rightarrow\ \frac{a}{b} = \frac{{}^{n}C_5}{{}^{n}C_4} = \frac{n-4}{5}$$

8. (a)

General term, $t_{r+1} = {}^{4n-2}C_r\,(i)^r\,x^r$

$r = 2, 6, ..., 4n-2$

$\therefore\ n$ terms

9. (a)

Coefficient of t^{32} in $\left(1+t^{12}\right)\left(1+t^{24}\right)\left(1+t^2\right)^{12}$

$\Rightarrow$ Coefficient of t^{32} in $\left(1+r^{12}+t^{24}+t^{36}\right)\left(\sum_{r=0}^{12} {}^{12}C_r\, t^{2r}\right)$

$\Rightarrow$ Coefficient of t^{32} is ${}^{12}C_{10}+{}^{12}C_4=561$

10. (c)

$E=\left(1-2x^3+3x^5\right)\left(\sum {}^8C_r x^{-r}\right)$

Coefficient of x in e is $(-2)\,{}^8C_2+3\,{}^8C_4=154$

11. (a)

General term, $t_{r+1}={}^{15}C_r\,2^{\frac{1}{2}(15-r)}3^{\frac{1}{2}r}$

Now, $r\in w$ and $0\le r\le 15$

Also, $15-r$ is EVEN and r is EVEN

$\Rightarrow$ r is ODD and r is EVEN

$\Rightarrow\ \therefore\ e\in\phi$

Hence, no radical term exist in the given expansion $\Rightarrow$ all terms are irrational.

$\therefore$ Number of irrational terms = 16

12. (a)

General form, $t_{r+1}={}^{55}C_r\,x^{\frac{1}{5}(55-r)}y^{\frac{1}{10}r}$

$r\in w; 0\le r\le 55$

$\therefore$ $55-r$ is a multiple of 5 and r is a multiple of 10

$\Rightarrow$ $r=0,10,20,30,40,50$

$\therefore$ 6 terms are rational.

13. (a)

$\Rightarrow\ 2\,{}^nC_5={}^nC_4+{}^nC_6$

$\Rightarrow\ \dfrac{{}^nC_4}{{}^nC_5}+\dfrac{{}^nC_6}{{}^nC_5}=2$

$\Rightarrow\ \dfrac{5}{n-4}+\dfrac{n-5}{6}=2$

$\Rightarrow\ n^2-21n+108=0$

$\Rightarrow\ (n-9)(n-12)=0$

$\therefore\ n=9,12$

14. (a)

${}^5C_2\,x^3x^{2\log_{101}x}=10^6 \Rightarrow x^{3+2\log_{10}x}=10^5$

$\therefore\ 3\log_{10}x+2\left(\log_{10}x\right)^2-5=0$

$\Rightarrow\ 2\left(\log_{10}x\right)^2+5\log_{10}x-2\left(\log_{10}x\right)-5=0$

$\Rightarrow\ \log_{10}x=\dfrac{-5}{2},1$

$\therefore\ x = 10^{\frac{-5}{2}}, 10$

15. (d)

$${}^{8}C_{5}\frac{1}{x^{8}}x^{10}\left(\log_{10}x\right)^{5} = 5600 \Rightarrow x^{2}\left(\log_{10}x\right)^{5} = 100$$

$\therefore\ x = 10$

16. (a)

$$t_{3} = {}^{n}C_{2}\,2^{x(n-2)}\left(\frac{1}{4^{x}}\right);\ t_{2} = {}^{n}C_{1}\,2^{x(n-1)}\left(\frac{1}{4^{x}}\right)^{1}$$

$$\frac{t_{3}}{t_{2}} = 7 \Rightarrow \frac{{}^{n}C_{2}}{{}^{n}C_{1}}\ \frac{2^{x(n-2)}}{2^{x(n-1)}}\ \frac{\left(\frac{1}{4^{x}}\right)^{2}}{\left(\frac{1}{4^{x}}\right)^{1}} = 7$$

$$\Rightarrow \frac{n-1}{2}\frac{1}{2^{x}}\left(\frac{1}{4}\right)^{x} = 7$$

Also, ${}^{n}C_{1} + {}^{n}C_{2} = 36 \Rightarrow n^{2} + 2n - 72 = 0$

$\Rightarrow n = 8$

$\therefore\ 2^{3x+1} = 2^{0}$

$\Rightarrow x = \frac{-1}{3}$

17. (d)

$$\left(1+x\right)^{131}\left(1-x+x^{2}\right)^{130} = \left(1+x\right)\left(1+x^{3}\right)^{130}$$

Now each term in the expansion of $\left(1+x^{3}\right)^{130}$ will be of type x^{3r}.

Hence in $\left(1+x\right)\left(1+x^{3}\right)^{130}$ terms will be of type x^{3x} and x^{3r+1}.

But no number of type $3r$ and $3r+1$ can be 65.

Hence coefficient of $x^{65} = 0$.

18. (c)

We can write it as

$$\left(1+x\right)\left(\left(1+x\right)\left(1-x+x^{2}\right)\right)^{100}$$

$$\left(1+x\right)\left(1+x^{3}\right)^{100}$$

$$\left(1+x\right)\left({}^{100}C_{0} + {}^{100}C_{1}\,x^{3} + \dots{}^{100}C_{100}x^{300}\right)$$

Clearly, This multiplication we can’t get power of form $3r+2$.

19. (d)

$$\left(1+0.0001\right)^{10000} = \left(1+\frac{1}{10^{4}}\right)^{10^{4}} = 1 + {}^{10^{4}}C_{1}\frac{1}{10^{4}} + {}^{10^{4}}C_{2}\frac{1}{10^{8}} + {}^{10^{4}}C_{3}\frac{1}{10^{12}} + \dots + {}^{10^{4}}C_{10^{4}}\frac{1}{10^{10^{4}}}$$

$$\left(1+0.0001\right)^{10000}=1+1+\frac{9999}{10^4\times 2!}+\frac{9999\times 9998}{10^8\times 3!}+\ldots+\frac{1}{10^{10^4}}$$

Clearly except the first two terms all the rest are non integers

So, the positive integer just greater than $\left(1+0.0001\right)^{10000}$ is 3.

20. (b)

Coefficient of x^{-1} in $(1+x)^n\left(1+\frac{1}{x}\right)^n$

= Coefficient of x^{-1} in $\frac{(1+x)^{2n}}{x^n}$

= Coefficient of x^{n-1} in $(1+x)^{2n}$

= ${}^{2n}C_{n-1}$

$=\frac{(2n)!}{(n-1)!(n+1)!}$

21. (c)

$101^{100}-1$

$\Rightarrow\ \left(1+100\right)^{100}-1$

$\Rightarrow\ {}^{100}C_0+{}^{100}C_1 100+{}^{100}C_2 100^2+.......+{}^{100}C_{100}\left(100\right)^{100}-1$

$\Rightarrow\ \left(100\right)^2\left[1+{}^{100}C_2+.....\right]$

22. (c)

$${}^9C_4\left(2a\right)^5\left(\frac{a^2}{5}\right)^4;-{}^9C_5\left(2a\right)^4\left(\frac{a^2}{4}\right)^5$$

23. (a)

$t_3<t_4>t_5\ \Rightarrow\ {}^{10}C_2\left(2\right)^8\left(\frac{3}{8}|x|\right)^2<{}^{10}C_3\left(2\right)^7\left(\frac{3}{8}|x|\right)^3>{}^{10}C_4\left(2\right)^6\left(\frac{3}{8}|x|\right)^4$

$\Rightarrow\ {}^{10}C_2\ 2\left(\frac{3}{8}|x|\right)^{-1}<{}^{10}C_3>{}^{10}C_4\ \frac{1}{2}\left(\frac{3}{8}|x|\right)$

$\Rightarrow\ \frac{{}^{10}C_2}{{}^{10}C_3}2\cdot\frac{8}{6}<|x|$

$\Rightarrow\ \frac{3}{10-25}\cdot 2\cdot\frac{8}{3}<|x|$

$\therefore\ |x|>2$

$\frac{{}^{10}C_3}{{}^{10}C_4}2\cdot\frac{8}{3}>|x|$

$\frac{4}{10-3}\cdot 2\cdot\frac{8}{3}>|x|$

$|x|<\frac{64}{21}$

$\Rightarrow\ \ 2<|x|<\frac{64}{21}$

24. (b)

Let t_{r+1} is the greater term, then $t_{r+1} > t_r$

$\Rightarrow\ {}^nC_r\left(2x\right)_{n-r}\ 7^r > {}^nC_{r-1}\left(2x\right)^{n-r+1}\ 7^{r-1}$

$\Rightarrow\ \dfrac{{}^nC_r}{{}^nC_{r-1}}\ 7 > 2x$

$\Rightarrow\ \dfrac{n-r+1}{r}7 > 6$

$\Rightarrow\ 7\left(n+1\right) > 13r$

$\Rightarrow r < \dfrac{77}{13}$

$\therefore$ 6th terms the greatest terms.

25. (b)

$\left(1+2\right)^n = 6561$

$\Rightarrow\ n = 8$

$\therefore$ Greatest term is 5th term.

26. (d)

Let $P = 5^{99} = 5\times 5^{98} = 5(25)^{49} = 5(26-1)^{49}$

$= 5[{}^{49}C_0(26)^{49} - {}^{49}C_1(26)^{48} + {}^{49}C_2(26)^{47} - \ldots.. + {}^{49}C_{48}(26) - {}^{49}C_{49}.1]$

$= 5\times 26k - 5$ when k is an integer

$\therefore \dfrac{P}{13} = 10k - \dfrac{5}{13} = 10k - 1 + \dfrac{8}{13}$

Hence, the remainder is 8

27. (b)

Now, $\dfrac{3^{2003}}{28} = \dfrac{3^2\times 3^{2001}}{28} = \dfrac{9}{28}(3^3)^{667} = \dfrac{9}{28}(28-1)^{667}$

$= \dfrac{9}{28}\{(28)^{667} - {}^{667}C_1(28)^{666} + {}^{667}C_2(28)^{665} - \ldots.. + {}^{667}C_{666}(28) - 1\}$

$= 9k - \dfrac{9}{28}$ where k is an integer

$(9k-1) + \dfrac{19}{98}$

Or $\left\{\dfrac{3^{2003}}{28}\right\} = \left\{(9k-1) + \dfrac{19}{28}\right\} = \dfrac{19}{28}$

28. (c)

Write $\left(23\right)^{14} = \left(20+3\right)^{14}$ and see last two digit.

29. (b)

Write $\left(3\right)^{100} = \left(9\right)^{50} = \left(10-1\right)^{50}$ and expand.

30. (b)

Write $3^{37}=3\cdot\left(3^4\right)^9=3\cdot(80-1)^9$ and expand.

31. (a)

$$1+(1+x)+\left(1+x^2\right)+.....+(1+x)^n=\frac{(1+x)^{n+1}-1}{x}$$

Coefficient of x^k is ${}^{n+1}C_{k+1}$

32. (d)

$$S=\sum_{r=0}^{n}(2r+1)\,{}^{n}C_r$$

$$S=2\sum_{r=0}^{n}{}^{n^{n-1}}C_{r+1}+\sum_{r=0}^{n}{}^{n}C_r=2\cdot 2^{n-1}+2^n$$

$$=(n+1)2^n$$

33. (c)

Coefficient of x^0 in $(1+x)^n\left(1+\frac{1}{x}\right)^n={}^{2n}C_n$

Coefficient of x^0 in $(1+x)^n\left(1-\frac{1}{x}\right)^n=0$

$$\therefore\ C_1^2+C_3^2+....+C_n^2=\frac{{}^{2n}C_n}{2}=\frac{(2n)!}{(n!)^2\,2}$$

34. (a)

Use ${}^{n}C_r={}^{n}C_{n-r}$

${}^{n-1}C_{n-1}+{}^{n}C_{n-1}+{}^{n+1}C_{n-1}+{}^{n+2}C_{n-1}+.....+{}^{n+r-1}C_{n-r}$

$={}^{n+r}C_n$

35. (a)

Clearly it is van denounced 2

Rewrite the caprenian as ${}^{n}C_n\cdot{}^{n}C_r+{}^{n}C_{n-1}\,{}^{n}C_{r+1}+......$ By vandermont 2 ${}^{2n}C_{n+r}$

36. (a)

$(n_1-n_2)(n_1-n_2-1)=30$ ...(1)

$(n_1+n_2)(n_1+n_2-1)=90$ ...(2)

$\therefore\ n_1=8$ and $n_2=2$

37. (d)

$${}^{n-1}C_r=\left(k^2-3\right)\frac{n}{r+1}\,{}^{n-1}C_r$$

$$k^2 - 3 = \frac{r+1}{n}$$

I: $k^2 - 3 > 0$

II: $k^2 - 3 < 1$

38. (c)

$${}^nC_r \cdot r! = 5040\left({}^{n-1}C_{r-1} + {}^{n-1}C_r\right) \Rightarrow {}^nC_r \cdot r! = 5040 \cdot {}^nC_r$$

$$\Rightarrow r = 7$$

39. (a)

$${}^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{r=1}^{5} {}^{47-r}C_{40-r}$$

$$\Rightarrow {}^{35}C_8 + \left({}^{41}C_7 + {}^{42}C_7 + {}^{43}C_7 + ... + {}^{35}C_7\right) + \left({}^{46}C_7 + {}^{45}C_7 + {}^{44}C_7 + ... + {}^{42}C_7\right)$$

By ${}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^nC_r$

$${}^{35}C_8 + {}^{35}C_7 + {}^{36}C_7 + ... + {}^{41}C_7 = {}^{42}C_8 \ \& \ {}^{42}C_8 + {}^{42}C_7 + {}^{43}C_7 + ... + {}^{46}C_7 = {}^{47}C_8$$

40. (b)

$$\sum_{r=0}^{49} {}^{50}C_r \cdot {}^{50}C_{r+1} = \sum_{r=0}^{49} {}^{50}C_{50-r} \cdot {}^{50}C_{r+1}$$

$$= {}^{100}C_{51}$$

41. (b)

Put the values to get answer.

42. (c)

$$(1+x)^{21} + (1+x)^{22} + ... + (1-x)^{20}$$

$$= (1+x)^{21}\left[\frac{(1+x)^{10} - 1}{(1+x) - 1}\right] = \frac{1}{x}\left[(1+x)^{31} - (1+x)^{21}\right]$$

$\Rightarrow$Coefficient of x^5 in the given expression

$=$Coefficient of x^5 in $\left\{\frac{1}{x}\left[(1+x)^{31} - (1+x)^{21}\right]\right\}$

= Coefficient of x^6 in [(1 + x)^{31} – (1 + x)^{21}]

$= {}^{31}C_6 - {}^{21}C_6$

43. (b)

$$\frac{1}{n!}\left[\frac{n!}{1!(n-1)!} + \frac{n!}{3!(n-3)!} + ...\right] = \frac{1}{n!}\left({}^nC_1 + {}^nC_3 + ...\right)$$

$$= \frac{1}{n!} 2^{n-1}$$

44. (a)

We know that

$$(1-1)^{20} = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} - {}^{20}C_{11} + {}^{20}C_{12} - .... + {}^{20}C_{20} = 0$$

$$2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9) + {}^{20}C_{10} = 0 \quad \left[\because {}^{20}C_{20} = {}^{20}C_0,\ {}^{20}C_{19} = {}^{20}C_1\ etc\right]$$

$$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10}$$

$$= -\frac{1}{2}\,{}^{20}C_{10} + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$$

45. (a)

$$\because (1+x)^m + (1+x)^{m+1} + ... + (1+x)^n$$

$$= \frac{(1+x)^m\{(1+x)^{n-m+1} - 1\}}{(1+x)-1} = \frac{(1+x)^{n+1} - (1+x)^m}{x}$$

$\therefore$ Coefficient of x^m in

$(1+x)^m + (1+x)^{m+1} + ... + (1+x)^n$

Or coefficient of x^m in $\dfrac{(1+x)^{n+1} - (1+x)^m}{x}$

Or coefficient of x^{m+1} in $(1+x)^{n+1} - (1+x)^m$

$$= {}^{n+1}C_{m+1} - 0 = {}^{n+1}C_{m+1}$$

46. (c)

We have $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + ... + {}^{20}C_{20}x^{20}$

On dividing by x, we get

$$\frac{(1+x)^{20}}{x} = \frac{{}^{20}C_0}{x} + {}^{20}C_1 + {}^{20}C_2x + {}^{20}C_3x^2 + ... + {}^{20}C_{20}x^{19}$$

On differentiating w.r.t. x, we get

$$\frac{20(1+x)^{19}.x - (1+x)^{20}}{x^2} = \frac{-{}^{20}C_0}{x^2} + 0 + {}^{20}C_2 + 2.{}^{20}C_3x + ... + 19.{}^{20}C_{20}x^{18}$$

On putting x = 1, we get

$$20(2)^{19} - (2)^{20} = -\frac{1}{1} + {}^{20}C_2 + 2.{}^{20}C_3 + ... + 19.{}^{20}C_{20}$$

$$\therefore {}^{20}C_2 + 2.{}^{20}C_3 + ... + 19.{}^{20}C_{20} = 1 + 9.2^{20}$$

47. (b)

We have $T_{r+1} = {}^{29}C_r 3^{29-r}(7x)^r = ({}^{29}C_r \times 3^{29-r} \times 7^r)x'$

Coefficient of (r + 1)th term is ${}^{29}C_r \times 3^{29-r} \times 7^r$

And coefficient of rth term is ${}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$

From given condition,

$${}^{29}C_r \times 3^{29-r} \times 7^r = {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$$

$$\frac{{}^{29}C_r}{{}^{29}C_{r-1}} = \frac{3}{7} \Rightarrow \frac{30-r}{r} = \frac{3}{7}\ or\ r = 21$$

48. (b)

Let term be $\frac{30!}{n_1!\,n_2!\,n_3!}\left(x^3\right)^{n_2}\left(-x^6\right)^{n_3}$

$n_1+n_2+n_3=30$

$3n_2+6n_3=28$

LHS is multiple of 3 but RHS is not cell 10

49. (a)

Number of ways to distribute 8 identical objects in 3 distinct groups $={}^{8+2}C_2=\frac{10\cdot 9}{2!}=45$.

50. (b)

Number of ways to distribute n identical objects in 5 distinct groups $={}^{n+4}C_4$

51. (a)

$(1-ax)^{-1}(1-bx)^{-1}(1-cx)^{-1}$

$=(1+ax+....)(1+bx+....)(1+cx+....)$

Hence coefficient of $x=(a+b+c)$.

52. (a)

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{3}{2}x\right)^3=\frac{27}{128}x^3$$

53. (c)

$(abcd)^{10}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$

So instead find cell of $\frac{1}{a^2}\frac{1}{b^6}\cdot\frac{1}{c}\frac{1}{d}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^1$

$\frac{10!}{2!6!}=2520$

54. (d)

Let term be $\frac{5!}{n_1!\,n_2!\,n_3!}\left(x^2\right)^{n_1}(-x)^{n_2}(-2)^{n_3}$

Now $n_1+n_2+n_3=5$ and $2n_1+n_2=5$

Put $n_1=0 \Rightarrow n_2=5$ & $n_3=0$

Coefficient is –1

If $n_1=1 \Rightarrow n_2=3, n_3=1$

Coefficient is $\frac{5!}{3!}\times 2=40$

If $n_1=2 \Rightarrow n_2=1,\ n_3=2$

Coefficient is $-\frac{5!}{2!2!}\times 4 \Rightarrow -120$

Add all coefficient –81

55. (d)

Let term be $\frac{20!}{n_1!\,n_2!\,n_3!}(1)^{n_1}(-x)^{n_2}(y)^{n_3}$

$n_1+n_2+n_3=20$

$n_2=2$

$n_3=3$

$n_1=15$

Coefficient $\frac{20!}{15!\,3!\,2!}$

56. (d)

$(1+3\text{x}+2\text{x}^2)^6=[1+x(3+2x)]^6$

$=1+{}^6C_1x(3+2x)+{}^6C_2x^2(3+2x)^2+{}^6C_3x^3(3+2x)^3+{}^6C_4x^4(3+2x)^4+{}^6C_5x^5(3+2x)^5$

$+{}^6C_6x^6(3+2x)^6$

We get x^{11} only from ${}^6C_6x^6(3+2x)^6$. Hence, coefficient of x^{11} is ${}^6C_5\times 3\times 2^5=576$

EXERCISE - 1 [B]

1. (a)

$$\frac{n^2+n-14}{2}=\frac{n(n+1)}{2}-7$$

$$\Rightarrow\ x^{\frac{n^2+n-14}{2}}=\frac{x\cdot x^2\cdot x^3\cdot......x^n}{x^7}$$

Hence, we need to find those terms which are product of all the x^r terms in each bracket except those in which the sum of powers is 7.

Such terms are $(x^7),(x\cdot x^6),(x^2\cdot x^5),(x^3x^4),(x\cdot x^2\cdot x^4),(x\cdot x^3\cdot x^3)$

Hence required coefficient is $-(7)+(1\cdot 6)+(2\cdot 5)+(3\cdot 4)-(1\cdot 2\cdot 4)=13$

2. (a)

Conceptual

3. (b)

${}^{\min}C_m = A \,\&\, B={}^{m+n}C_n$

Clearly $A=B$

4. (a)

General $\tan^{100}(r(i))^r$

$A=1,3,5,7,........,99\Rightarrow 50$ terms

5. (c)

Expand

6. (c)

Let $E=\left(\sqrt{5}+1\right)^5-\left(\sqrt{5}-1\right)^5$

$$\Rightarrow\ E=2\left\{{}^5C_1\left(\sqrt{5}\right)^4+{}^5C_3\left(\sqrt{5}\right)^2+{}^5C_3\left(\sqrt{5}\right)^0\right\}$$

$$\Rightarrow\ E=2\{125+50+1\}=352$$

7. (b)

Expand

8. (b)

General term, $t_{r+1}={}^{15}C_r\,2^{\frac{1}{2}(15-r)}3^{\frac{1}{2}r}$

Now, $r\in w$ and $0\le r\le 15$

Also, $15-r$ is EVEN and r is EVEN

$\Rightarrow$ r is ODD and r is EVEN

$\Rightarrow\ \therefore\ e\in\phi$

Hence, no radical term exist in the given expansion $\Rightarrow$ all terms are irrational.

$\therefore$ Number of irrational terms = 16

9. (c)

$$\left(x^3 - \frac{1}{x^2}\right)^n$$

General terms $T_{r+1} = \frac{n!}{r!(n-r)!}(-1)^{n-r}\ x^{5r-2n}$

If $5r - 2n = 5,$ then 5r = 2n + 5 or $r = \frac{2n}{5} + 1$

If $5r - 2n = 10,$ then $5r = 2n + 10$ or $r = \frac{2n}{5} + 2$

x^5 *and* x^{10} terms occurs if n = 5k

Given that sum of x^5 *and* x^{10} is zero

$$\Rightarrow \frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0$$

Or $\frac{1}{3k-1} - \frac{1}{2k+2} = 0$

Or k = 3 $\Rightarrow$ n = 15

10. (b)

$(1-x)(1-x)^n$

$= (1-x)[1 + n(-x) + ... + {}^nC_{n-1}(-x)^{n-1} + {}^nC_n(-x)^n]$

Therefore, coefficient of x^n is

${}^nC_n(-1)_n - {}^nC_{n-1}(-1)^{n-1} = (-1)^n + (-1)^n n$

$= (-1)^n(1+n)$

11. (b)

Put x = i

$$(1+i)^5 = (a_0 - a_2 + a_4) + i(a_1 - a_3 + a_5)$$

$\Rightarrow |1+i|5 = |(a_0 - a_2 + a_4) + i(a_1 - a_3 + a_5)]$

$\Rightarrow (a_0 - a_2 + a_4)^2 + (a_1 - a_3 + a_5)^2 = 2^5 = 32$

12. (c)

For $\left(ax^2 + \left(\frac{1}{bx}\right)\right)^{11}, T_{r+1} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r a^{11-r}\frac{1}{b^r}x^{22-3r}$

For x^7 22- 3r = 7

Or $3r = 15$

Or r = 5

$\Rightarrow T_6 = {}^{11}C_5 a^6 \frac{1}{b^5}x^7$

$\Rightarrow$Coefficient of x^7 is ${}^{11}C_5\,\frac{a^6}{b^5}$

Similarly, coefficient of x^{-7} in $\left(ax-\left(\frac{1}{bx^2}\right)\right)^{11}$ is ${}^{11}C_6\,\frac{a^5}{b^6}$

Given that ${}^{11}C_5\,\frac{a^6}{b^5}={}^{11}C_6\,\frac{a^5}{b_6}$

$\Rightarrow a=\frac{1}{b}$

Or $ab=1$

13. (b)

$t_{r+1}={}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r={}^{10}C_r x^{5-5n2}(-k)^r$

For this to be independent of x, r must be 2, so that ${}^{10}C_2k^2=405\ or\ k\pm 3$

14. (b)

$T_5={}^nC_4a^{n-1}(-2b)^4$

And $T_6={}^nC_5a^{n-5}(-2b)^5$

As $T_5+T_6=0$, we get

${}^nC_4 2^4\,a^{n-4}b^4={}^nC_5 2^5 a^{n-5}b^5$

Or $\frac{a^{n-4}b^4}{a^{n-5}b^5}=\frac{n!2^5}{5!(n-5)!}.\frac{4!(n-4)!}{n!2^4}$

Or $\frac{a}{b}=\frac{2(n-4)}{5}$

15. (a)

We have $S=\frac{1(2^{2000}-1)}{2-1}=2^{2000}-1=(2^2)^{1000}-1$

$=(5-1)^{1000}-1$

$=(5^{1000}-{}^{1000}C_1.5^{999}+{}^{1000}C_2.5^{998}.....+{}^{1000}C_{998}.5^2-{}^{1000}C_{999}.5+1)-1$

$=5(5^{999}-{}^{1000}C_1.5^{998}+{}^{1000}C_2.5^{997}-....-{}^{1000}C_{999})$

$\therefore$ Remainder is 0

16. (c)

$6^{83}+8^{83}=(7-1)^{83}+(7+1)^{83}$

$=2(7^{83}+{}^{83}C_2.7^{81}+{}^{83}C_4.7^{79}+.....+{}^{83}C_{80}7^3+{}^{83}C_{82}7)$

$=2\{49m+{}^{83}C_{82}.7\}$

Where m is an integer

$=98m+2.\,{}^{83}C_1.7=98m+2.83.7$

$=98m+2(77+6).7=49(2m+22)+84$

$=49(2m+22)+49+35$

$= 49(2m+23)+35 = 49n+35$

Where n is an integer

Hence, the remainder is 35

17. (c)

Middle term of $(1+\alpha x)^4$ is T_3

Its coefficient is ${}^4C_2(\alpha)^2 = 6\alpha^2$

Middle term of $(1-\alpha x)^6$ is T_4

Its coefficient is ${}^6C_3(-\alpha)^3 = -20\alpha^3$

According to question

$6\alpha^2 = -20\alpha^3$

Or $3\alpha^2 + 10\alpha^3 = 0$

Or $\alpha^2(3+10\alpha) = 0$

Or $\alpha = -\frac{3}{10}$

18. (d)

$3^{400} = 81100 = (1+80)^{100}$

$= {}^{100}C_0 + {}^{100}C_1 80 + ... + {}^{100}C_{100} 80^{100}$

Thus, the last two digits are 01.

19. (a)

$\frac{2^{4n}}{15} = \frac{(15+1)^n}{15}$

$= \frac{({}^nC_0 15^n + {}^n C_1 15^{n-1} + ... + {}^n C_{n-1} 15 + {}^n C_n)}{15}$

= Integer $+\frac{1}{15}$

Hence, the fractional part of $\frac{2^{4n}}{15}$ *is* $\frac{1}{15}$

20. (c)

Use concept of middle term.

21. (c)

$$(103)^{86}-(86)^{103}$$

$$\Rightarrow (1+102)^{86}-(1+85)^{103}$$

$$\Rightarrow \left[{}^{86}C_0(102)^0+{}^{86}C_1\,102+{}^{86}C_2\,102^2+\dots\right]$$

$$-\left[{}^{103}C_0\,85^0+{}^{103}C_1\,85+{}^{103}C_2\,85^2+\dots\right]$$

$$\Rightarrow \left[1+{}^{86}C_1\,102+{}^{86}C_2\,102^2+\dots\right]-\left[1+{}^{103}C_1\,85+{}^{103}C_2(85)^2+\dots\right]$$

$$\Rightarrow \left[{}^{86}C_1\,102+{}^{86}C_2\,102^2+\dots\right]-\left[{}^{103}C_1\,85+{}^{103}C_2\,85^2+\dots\right]$$

$$\Rightarrow 17\left\{\left({}^{86}C_1\,6+{}^{86}C_2\,6(102)+\dots\right)-\left({}^{103}C_1\,5+{}^{103}C_2\,5(85)+\dots\right)\right\}$$

Thus, $\boxed{\text{it is divisible by 17.}}$

Hence, $\boxed{\text{C is correct option.}}$

22. (c)

The correct option is **C**

$\frac{8}{31}$

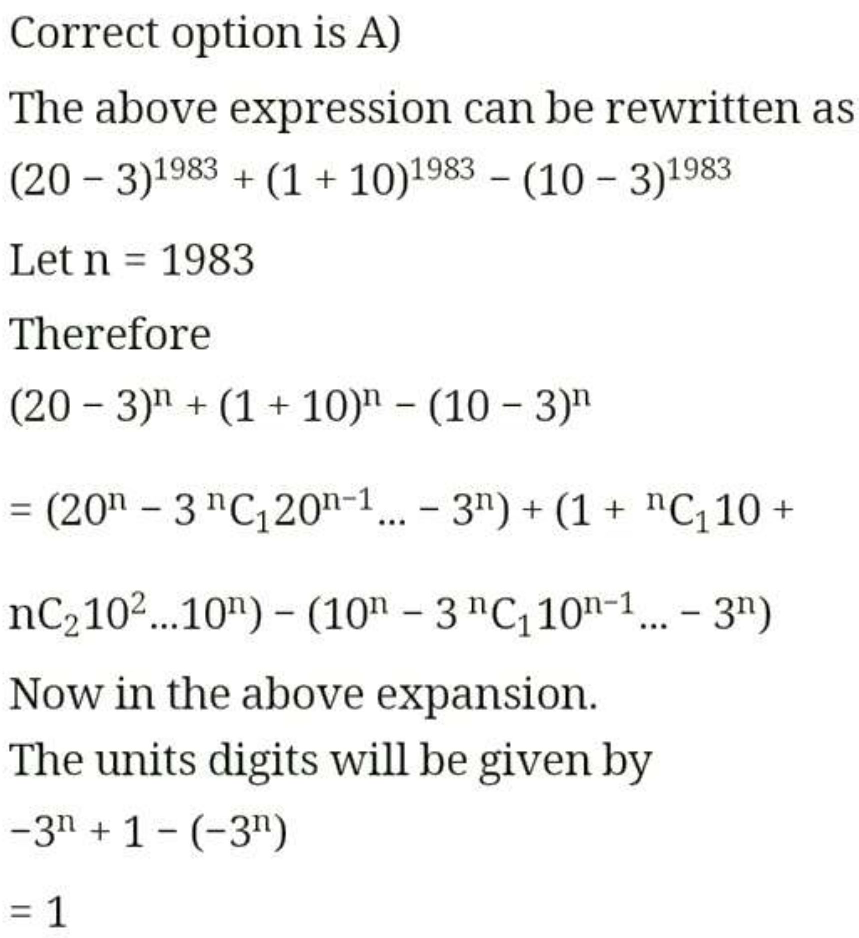
$2^{78} + 2^3.2^{75}= 8\ 25^{15} = 8(1+31)^{15}= 8$

${}^{15}C_0 + {}^{15}C_1 31+\dots\dots\dots\dots\dots\dots\dots+{}^{15}C_{15}(31)^{15}$

$2^{78} = 8$ + an integer multiple of 31

$\frac{2^{78}}{31}=\frac{8}{31}$ + an integer

23. (a)



Correct option is A)

The above expression can be rewritten as

$(20-3)^{1983}+(1+10)^{1983}-(10-3)^{1983}$

Let n = 1983

Therefore

$(20-3)^n+(1+10)^n-(10-3)^n$

$= (20^n - 3\,{}^nC_1 20^{n-1} \ldots - 3^n) + (1 + {}^nC_1 10 +$

$nC_2 10^2 \ldots 10^n) - (10^n - 3\,{}^nC_1 10^{n-1} \ldots - 3^n)$

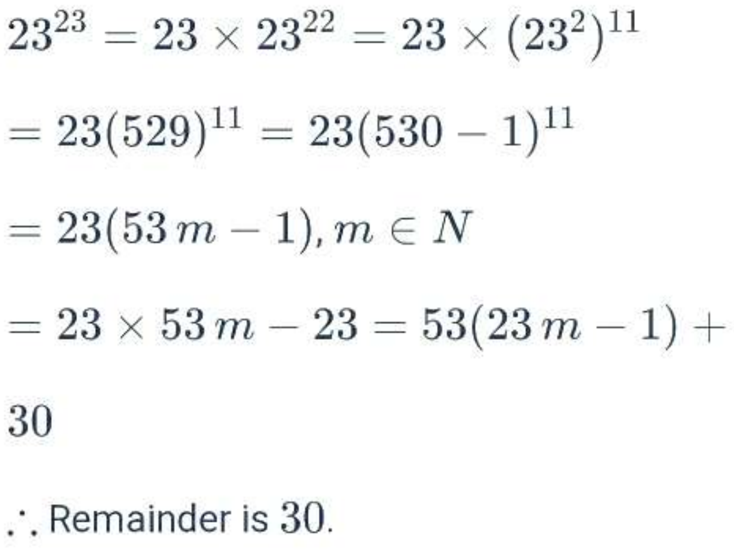
Now in the above expansion.

The units digits will be given by

$-3^n + 1 - (-3^n)$

$= 1$

24. (b)



$23^{23} = 23 \times 23^{22} = 23 \times (23^2)^{11}$

$= 23(529)^{11} = 23(530-1)^{11}$

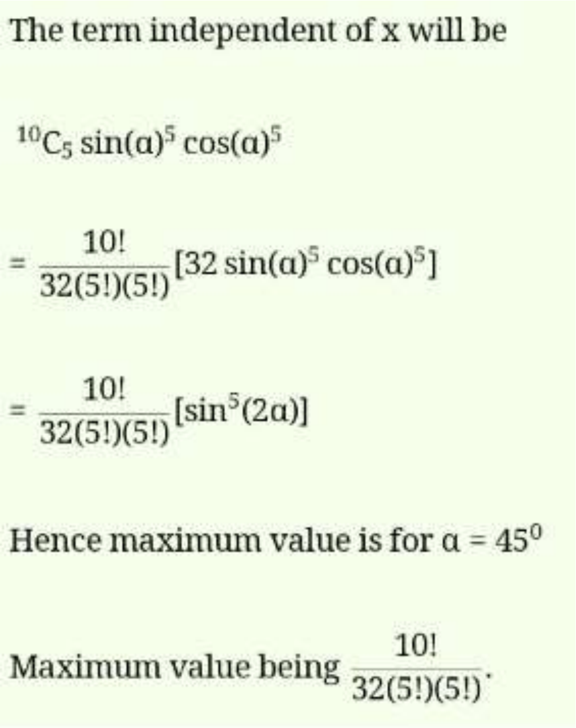
$= 23(53\,m - 1), m \in N$

$= 23 \times 53\,m - 23 = 53(23\,m - 1) +$

30

$\therefore$ Remainder is 30.

25. (c)



The term independent of x will be

$^{10}C_5 \sin(\alpha)^5 \cos(\alpha)^5$

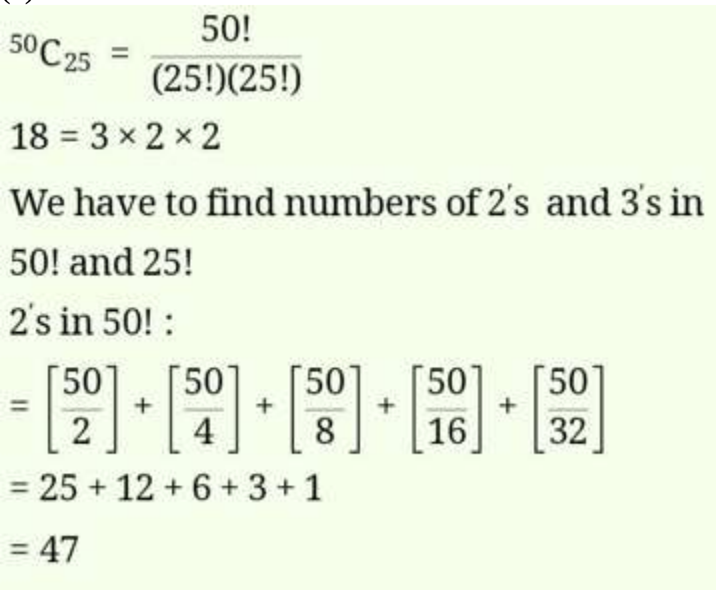
$$= \frac{10!}{32(5!)(5!)}[32 \sin(\alpha)^5 \cos(\alpha)^5]$$

$$= \frac{10!}{32(5!)(5!)}[\sin^5(2\alpha)]$$

Hence maximum value is for $\alpha = 45^0$

Maximum value being $\frac{10!}{32(5!)(5!)}$.

26. (c)



$$^{50}C_{25} = \frac{50!}{(25!)(25!)}$$

$18 = 3 \times 2 \times 2$

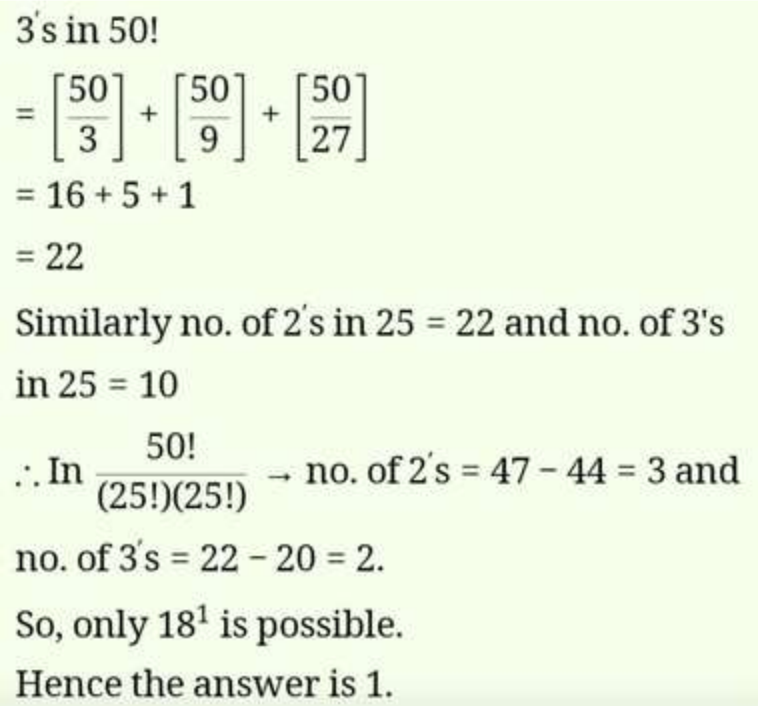
We have to find numbers of 2's and 3's in 50! and 25!

2's in 50! :

$$= \left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right]$$

$= 25 + 12 + 6 + 3 + 1$

$= 47$



$3'$s in 50!

$$= \left[\frac{50}{3}\right] + \left[\frac{50}{9}\right] + \left[\frac{50}{27}\right]$$

= 16 + 5 + 1

= 22

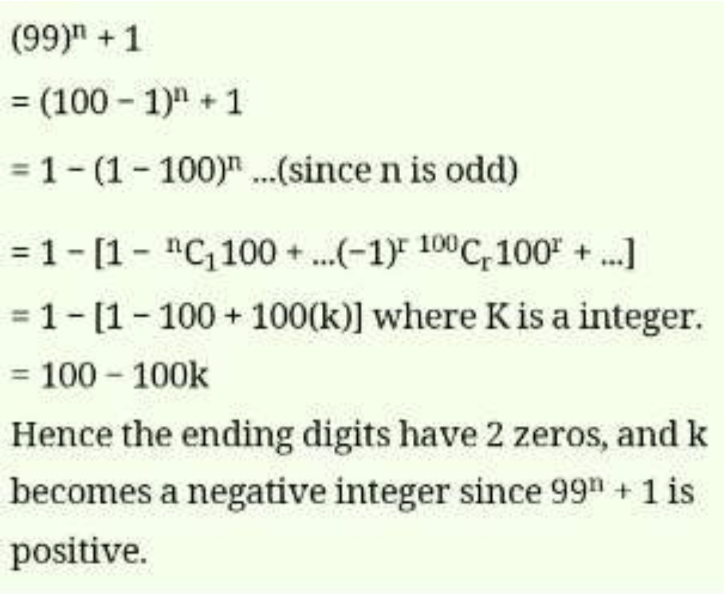
Similarly no. of $2'$s in 25 = 22 and no. of 3's in 25 = 10

$\therefore$ In $\frac{50!}{(25!)(25!)} \rightarrow$ no. of $2'$s = 47 − 44 = 3 and no. of $3'$s = 22 − 20 = 2.

So, only 18^1 is possible.

Hence the answer is 1.

27. (c)



$(99)^n + 1$

$= (100 - 1)^n + 1$

$= 1 - (1 - 100)^n$ ...(since n is odd)

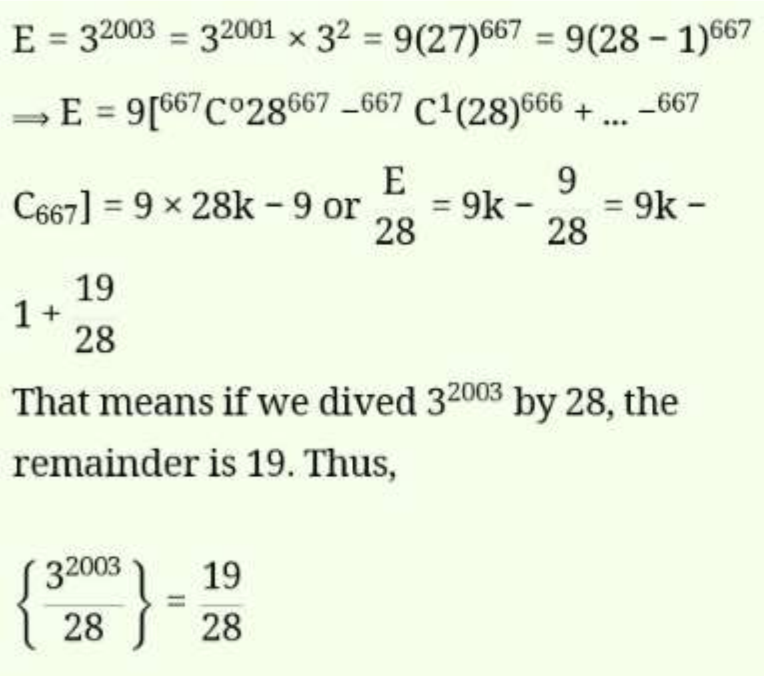
$= 1 - [1 - {}^nC_1 100 + ...(-1)^r\, {}^{100}C_r 100^r + ...]$

$= 1 - [1 - 100 + 100(k)]$ where K is a integer.

$= 100 - 100k$

Hence the ending digits have 2 zeros, and k becomes a negative integer since $99^n + 1$ is positive.

28. (c)



$E = 3^{2003} = 3^{2001} \times 3^2 = 9(27)^{667} = 9(28 - 1)^{667}$

$\Rightarrow E = 9[{}^{667}C^{o}28^{667} - {}^{667}C^{1}(28)^{666} + \ldots - {}^{667}$

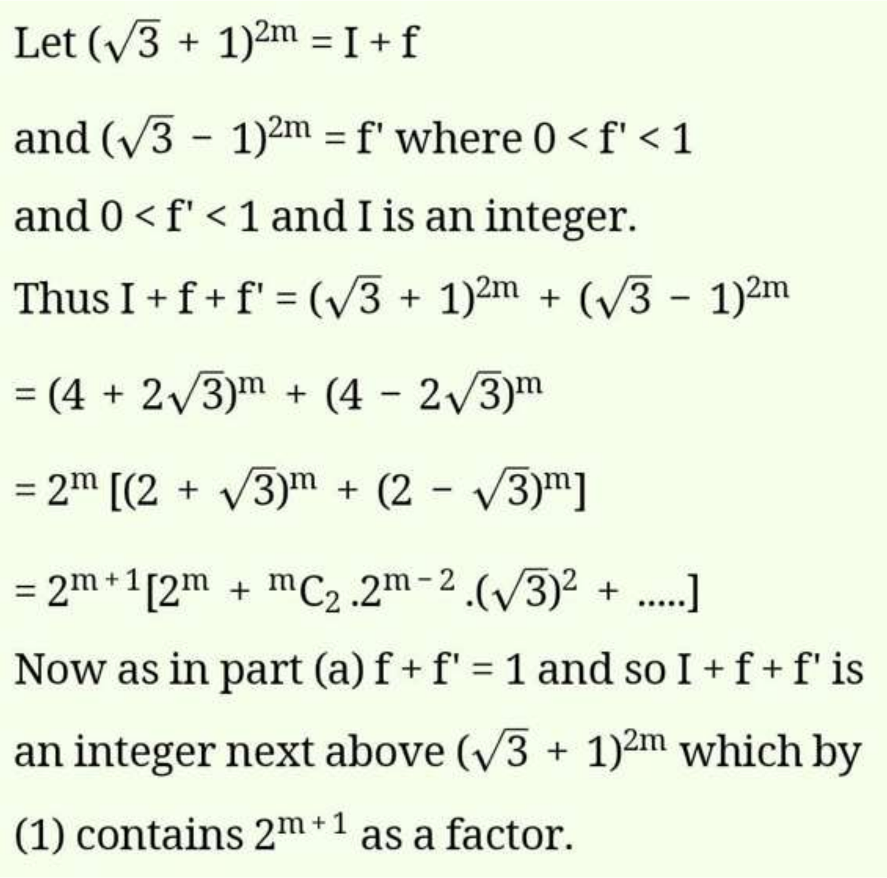
$C_{667}] = 9 \times 28k - 9$ or $\frac{E}{28} = 9k - \frac{9}{28} = 9k -$

$1 + \frac{19}{28}$

That means if we dived 3^{2003} by 28, the remainder is 19. Thus,

$$\left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$$

29. (a)



Let $(\sqrt{3} + 1)^{2m} = I + f$

and $(\sqrt{3} - 1)^{2m} = f'$ where $0 < f' < 1$

and $0 < f' < 1$ and I is an integer.

Thus $I + f + f' = (\sqrt{3} + 1)^{2m} + (\sqrt{3} - 1)^{2m}$

$= (4 + 2\sqrt{3})^{m} + (4 - 2\sqrt{3})^{m}$

$= 2^{m}[(2 + \sqrt{3})^{m} + (2 - \sqrt{3})^{m}]$

$= 2^{m+1}[2^{m} + {}^{m}C_{2}.2^{m-2}.(\sqrt{3})^{2} + \ldots..]$

Now as in part (a) $f + f' = 1$ and so $I + f + f'$ is an integer next above $(\sqrt{3} + 1)^{2m}$ which by (1) contains 2^{m+1} as a factor.

30. (a)

Write $(101)^{100} - 1 = (1+100)^{100} - 1$, then expand.

31. (a)

We have, $\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}=\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^{n+1}C_{r+1}}$

$$=\sum_{r=0}^{n-1}\frac{{}^nC_r}{\frac{n+1}{r+1}\,{}^nC_r}\sum_{r=0}^{n-1}\frac{r+1}{n+1}$$

$$=\frac{1}{n+1}[1+2+....+n]=\frac{n(n+1)}{2(n+1)}=\frac{n}{2}$$

32. (b)

We have ${}^{39}C_{3r-1}+{}^{39}C_{3r}={}^{39}C_{r^2}+{}^{39}C_{r^2-1}$

$\Rightarrow {}^{40}C_{3r}={}^{40}C_{r^2}$

$\Rightarrow 3r=r^2 \ or \ 40-3r=r^2$

$\Rightarrow r=0,\,3 \ or \ r^2+3r-40=0$

$\Rightarrow (r+8)(r-5)=0 \Rightarrow r=0,3,5,-8$

But r = 0, -8 do not satisfy the given equation $\therefore r=3,5$

33. (d)

$$\sum_{r=0}^{20} r(20-r)({}^{20}C_r)^2$$

$$\sum_{r=0}^{20} r\times{}^{20}C_r\,(20-r)\times{}^{20}C_{20-r}=\sum_{r=0}^{20}20.\ {}^{19}C_{r-1}\times20\times\ {}^{19}C_{19-r}$$

$$=400\sum_{r=0}^{20}{}^{19}C_{r-1}\times{}^{19}C_{19-r}$$

$$=400\sum_{r=0}^{20}{}^{19}C_{r-1}\times{}^{19}C_{19-r}$$

$$=400\times \ coefficient \ of \ x^{18} \ in \ (1+x)^{19}(1+x)^{19}$$

$$=400\times{}^{38}C_{18}=400\times{}^{38}C_{20}$$

34. (c)

As we know that ${}^nC_0-{}^nC_1^2+{}^nC_2^2-{}^nC_3^2+.....+(-1)^n\ {}^nC_n^2=0$ (if n is odd) and in the question n = 15 (odd). Hence sum of given series is 0.

35. (a)

$$\sum_{r=0}^{40} r\,{}^{40}C_r\,{}^{30}C_r$$

$$=40\sum_{r=0}^{40}{}^{39}C_{r-1}\ {}^{30}C_r$$

$$=40\sum_{r=0}^{40}{}^{39}C_{r-1}\ {}^{30}C_{30-r}$$

$$=40^{39+30}C_{r-1+30-r}$$

$$=40\ {}^{69}C_{29}$$

36. (a)

$$\frac{r\times 2r}{(r+2)!}=\frac{(r+2-2)2^r}{(r+2)!}$$

$$=\frac{2^r}{(r+1)!}-\frac{2^{r+1}}{(r+2)!}$$

$$=-\left(\frac{2^{r+1}}{(r+2)!}-\frac{2^r}{(r+1)!}\right)$$

= (V(r) – V(r – 1))

$$\Rightarrow\sum_{r=1}^{15}\frac{r\times 2^r}{(r+2)!}=-(V(15)-V(0))$$

$$=-\left(\frac{2^{16}}{17!}-\frac{2}{2!}\right)$$

$$=1-\frac{2^{16}}{(17)!}$$

37. (b)

Put $x=1$

$$a_0+a_1+a_2+a_3..........a_{12}=0$$

Put $x=-1$

$$1-a_1+a_2-a_3.........+a_{12}=2^6$$

Add $2\left(1+a_2+a_4+.....a_12\right)=2^5$

$$a_2+a_4+.......a_{12}=2^5$$

38. (c)

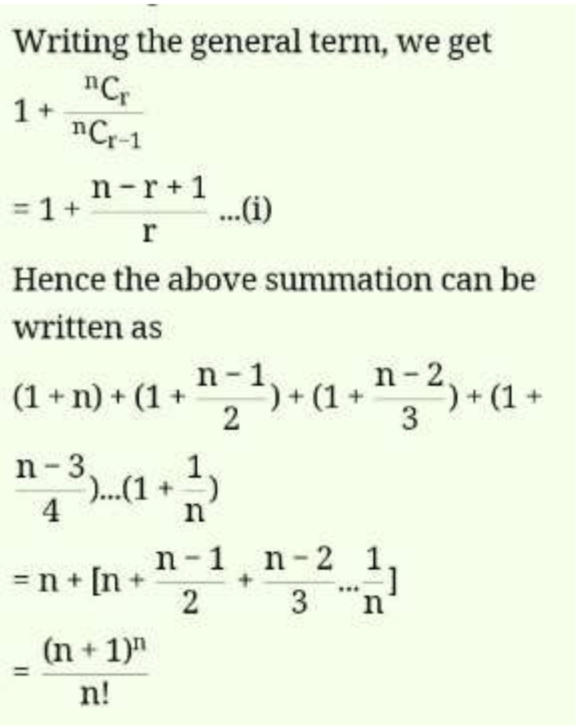
$$\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3}k.{}^{k}C_r$$

$$\sum_{k=1}^{\infty}\frac{1}{3^k}\left(2^k\right)$$

Which is a G.P.

$$\frac{\frac{2}{3}}{1-\frac{2}{3}}=2$$

39. (c)



Writing the general term, we get

$1 + \frac{{}^nC_r}{{}^nC_{r-1}}$

$= 1 + \frac{n - r + 1}{r}$ ...(i)

Hence the above summation can be written as

$(1 + n) + (1 + \frac{n-1}{2}) + (1 + \frac{n-2}{3}) + (1 + \frac{n-3}{4})...(1 + \frac{1}{n})$

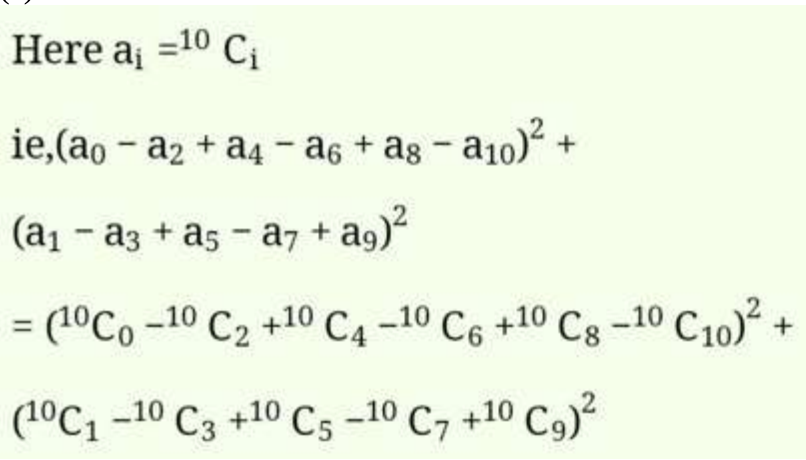
$= n + [n + \frac{n-1}{2} + \frac{n-2}{3} ... \frac{1}{n}]$

$= \frac{(n+1)^n}{n!}$

40. (a)

Do yourself.

41. (c)



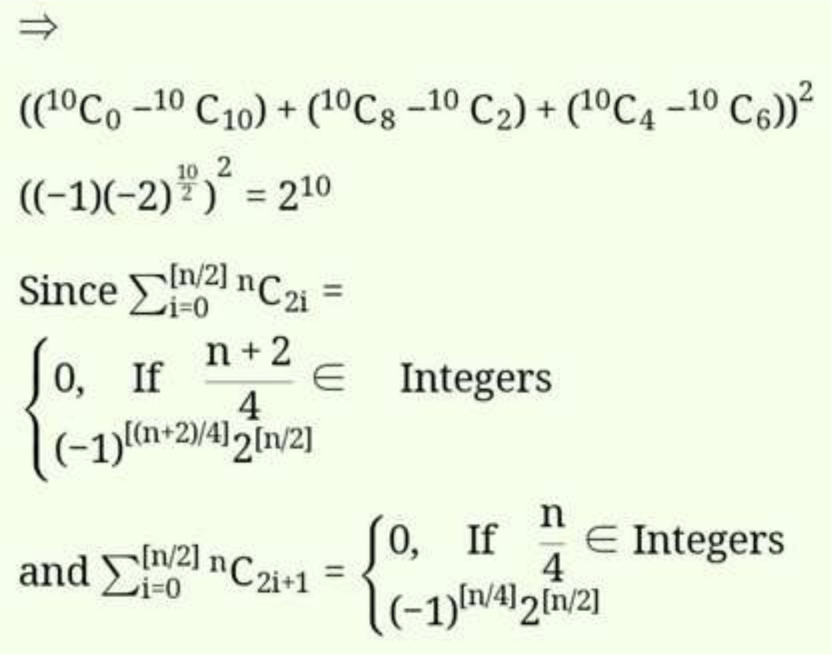
Here $a_i = {}^{10}C_i$

ie,$(a_0 - a_2 + a_4 - a_6 + a_8 - a_{10})^2 +$

$(a_1 - a_3 + a_5 - a_7 + a_9)^2$

$= ({}^{10}C_0 - {}^{10}C_2 + {}^{10}C_4 - {}^{10}C_6 + {}^{10}C_8 - {}^{10}C_{10})^2 +$

$({}^{10}C_1 - {}^{10}C_3 + {}^{10}C_5 - {}^{10}C_7 + {}^{10}C_9)^2$



⇒

$$((^{10}C_0 - ^{10}C_{10}) + (^{10}C_8 - ^{10}C_2) + (^{10}C_4 - ^{10}C_6))^2$$

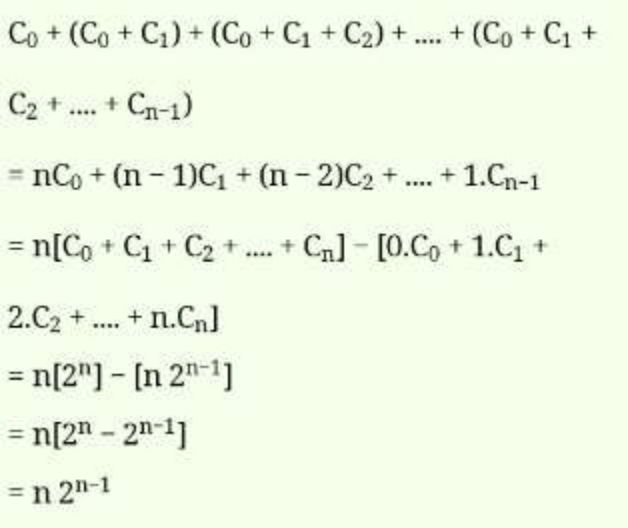
$$((-1)(-2)^{\frac{10}{2}})^2 = 2^{10}$$

Since $\sum_{i=0}^{[n/2]} {}^nC_{2i} =$

$$\begin{cases} 0, & \text{If } \dfrac{n+2}{4} \in \text{Integers} \\ (-1)^{[(n+2)/4]}2^{[n/2]} \end{cases}$$

and $\sum_{i=0}^{[n/2]} {}^nC_{2i+1} = \begin{cases} 0, & \text{If } \dfrac{n}{4} \in \text{Integers} \\ (-1)^{[n/4]}2^{[n/2]} \end{cases}$

42. (a)



$C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + ... + (C_0 + C_1 +$

$C_2 + ... + C_{n-1})$

$= nC_0 + (n-1)C_1 + (n-2)C_2 + ... + 1.C_{n-1}$

$= n[C_0 + C_1 + C_2 + ... + C_n] - [0.C_0 + 1.C_1 +$

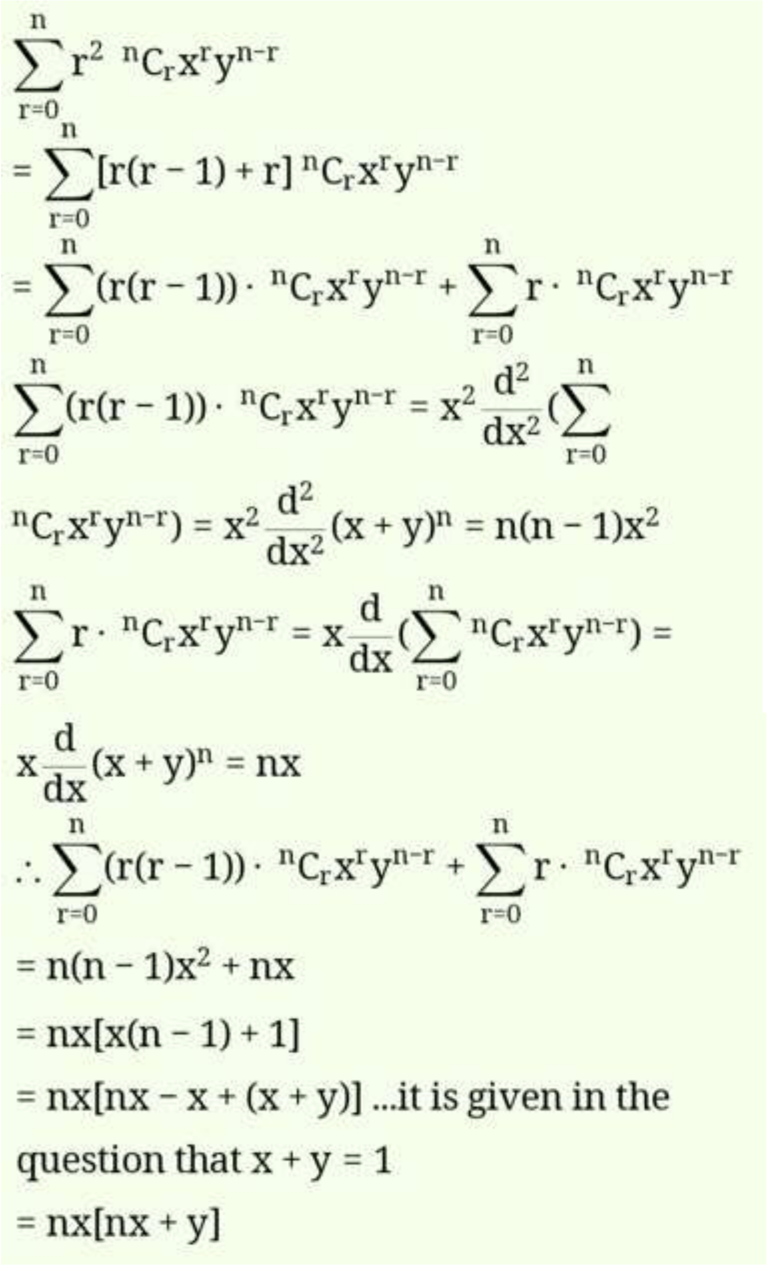
$2.C_2 + ... + n.C_n]$

$= n[2^n] - [n\,2^{n-1}]$

$= n[2^n - 2^{n-1}]$

$= n\,2^{n-1}$

43. (c)



$$\sum_{r=0}^{n} r^2 \ {}^nC_r x^r y^{n-r}$$

$$= \sum_{r=0}^{n} [r(r-1) + r] \ {}^nC_r x^r y^{n-r}$$

$$= \sum_{r=0}^{n} (r(r-1)) \cdot \ {}^nC_r x^r y^{n-r} + \sum_{r=0}^{n} r \cdot \ {}^nC_r x^r y^{n-r}$$

$$\sum_{r=0}^{n} (r(r-1)) \cdot \ {}^nC_r x^r y^{n-r} = x^2 \frac{d^2}{dx^2} (\sum_{r=0}^{n}$$

$${}^nC_r x^r y^{n-r}) = x^2 \frac{d^2}{dx^2} (x+y)^n = n(n-1)x^2$$

$$\sum_{r=0}^{n} r \cdot \ {}^nC_r x^r y^{n-r} = x \frac{d}{dx} (\sum_{r=0}^{n} {}^nC_r x^r y^{n-r}) =$$

$$x \frac{d}{dx} (x+y)^n = nx$$

$$\therefore \sum_{r=0}^{n} (r(r-1)) \cdot \ {}^nC_r x^r y^{n-r} + \sum_{r=0}^{n} r \cdot \ {}^nC_r x^r y^{n-r}$$

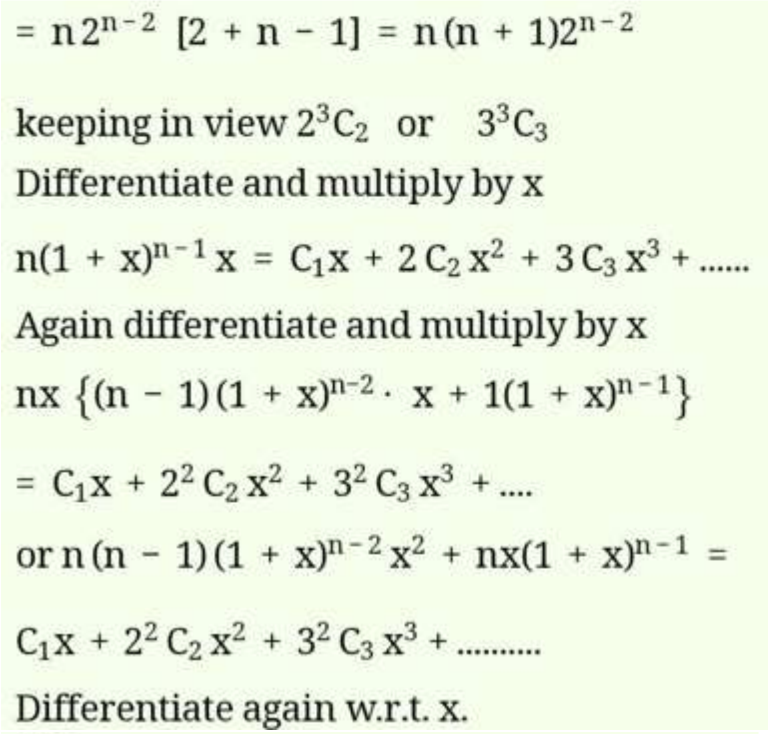
$$= n(n-1)x^2 + nx$$

$$= nx[x(n-1) + 1]$$

$= nx[nx - x + (x+y)]$ ...it is given in the question that $x + y = 1$

$$= nx[nx + y]$$

44. (a)



$= n2^{n-2}\ [2 + n - 1] = n(n + 1)2^{n-2}$

keeping in view 2^3C_2 or 3^3C_3

Differentiate and multiply by x

$n(1 + x)^{n-1}x = C_1x + 2C_2x^2 + 3C_3x^3 + ...$

Again differentiate and multiply by x

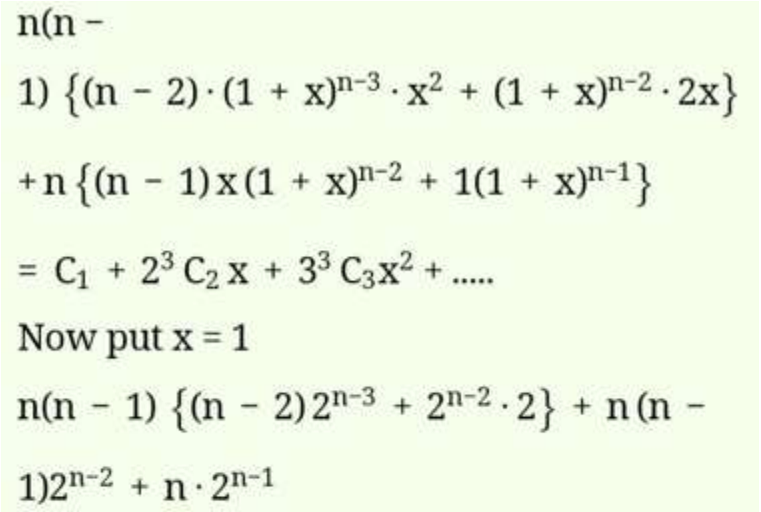
$nx\left\{(n - 1)(1 + x)^{n-2}\cdot x + 1(1 + x)^{n-1}\right\}$

$= C_1x + 2^2C_2x^2 + 3^2C_3x^3 + ...$

or $n(n - 1)(1 + x)^{n-2}x^2 + nx(1 + x)^{n-1} =$

$C_1x + 2^2C_2x^2 + 3^2C_3x^3 + ...$

Differentiate again w.r.t. x.



$n(n -$

$1)\left\{(n - 2)\cdot(1 + x)^{n-3}\cdot x^2 + (1 + x)^{n-2}\cdot 2x\right\}$

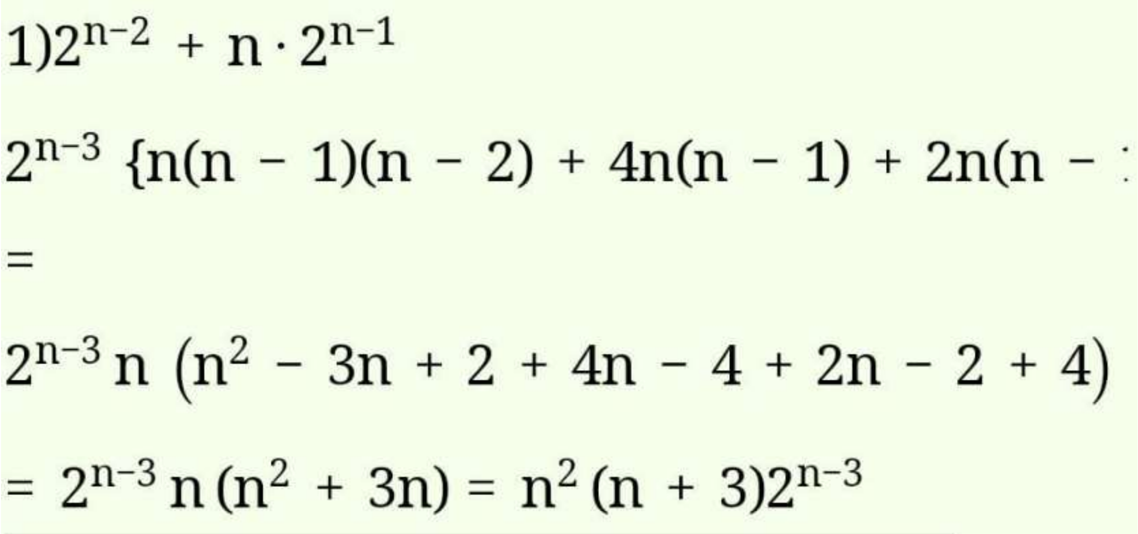
$+ n\left\{(n - 1)x(1 + x)^{n-2} + 1(1 + x)^{n-1}\right\}$

$= C_1 + 2^3C_2x + 3^3C_3x^2 + ...$

Now put x = 1

$n(n - 1)\left\{(n - 2)2^{n-3} + 2^{n-2}\cdot 2\right\} + n(n -$

$1)2^{n-2} + n\cdot 2^{n-1}$



$1)2^{n-2} + n\cdot 2^{n-1}$

$2^{n-3}\ \{n(n - 1)(n - 2) + 4n(n - 1) + 2n(n -$

$=$

$2^{n-3}n\ \left(n^2 - 3n + 2 + 4n - 4 + 2n - 2 + 4\right)$

$= 2^{n-3}n(n^2 + 3n) = n^2(n + 3)2^{n-3}$

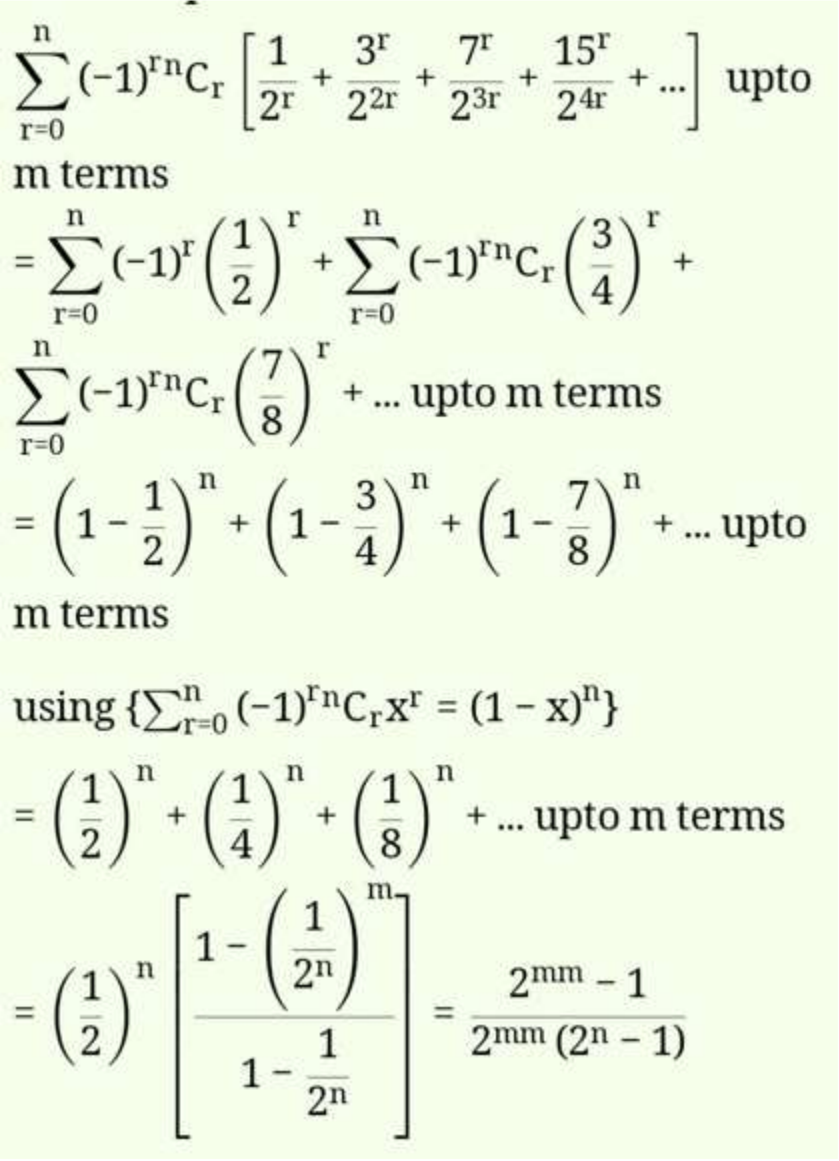
45. (b)

$$\sum_{r=0}^{n} r^3\left(\frac{n-r+1}{r}\right)^2=\sum_{r=0}^{n} r\left(n-r+1\right)^2$$

$$=\sum_{r=0}^{n}\left(n+1\right)^3 r-\sum_{r=0}^{n}2\left(n+1\right)r^2+\sum_{r=0}^{n}r^3$$

$$=\frac{1}{12}n\left(n+1\right)^2\left(n+2\right)$$

46. (a)



$$\sum_{r=0}^{n}(-1)^{r\,n}C_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\ldots\right] \text{ upto m terms}$$

$$=\sum_{r=0}^{n}(-1)^r\left(\frac{1}{2}\right)^r+\sum_{r=0}^{n}(-1)^{r\,n}C_r\left(\frac{3}{4}\right)^r+\sum_{r=0}^{n}(-1)^{r\,n}C_r\left(\frac{7}{8}\right)^r+\ldots \text{ upto m terms}$$

$$=\left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\left(1-\frac{7}{8}\right)^n+\ldots \text{ upto m terms}$$

using $\{\sum_{r=0}^{n}(-1)^{r\,n}C_r x^r=(1-x)^n\}$

$$=\left(\frac{1}{2}\right)^n+\left(\frac{1}{4}\right)^n+\left(\frac{1}{8}\right)^n+\ldots \text{ upto m terms}$$

$$=\left(\frac{1}{2}\right)^n\left[\frac{1-\left(\frac{1}{2^n}\right)^m}{1-\frac{1}{2^n}}\right]=\frac{2^{mm}-1}{2^{mm}(2^n-1)}$$

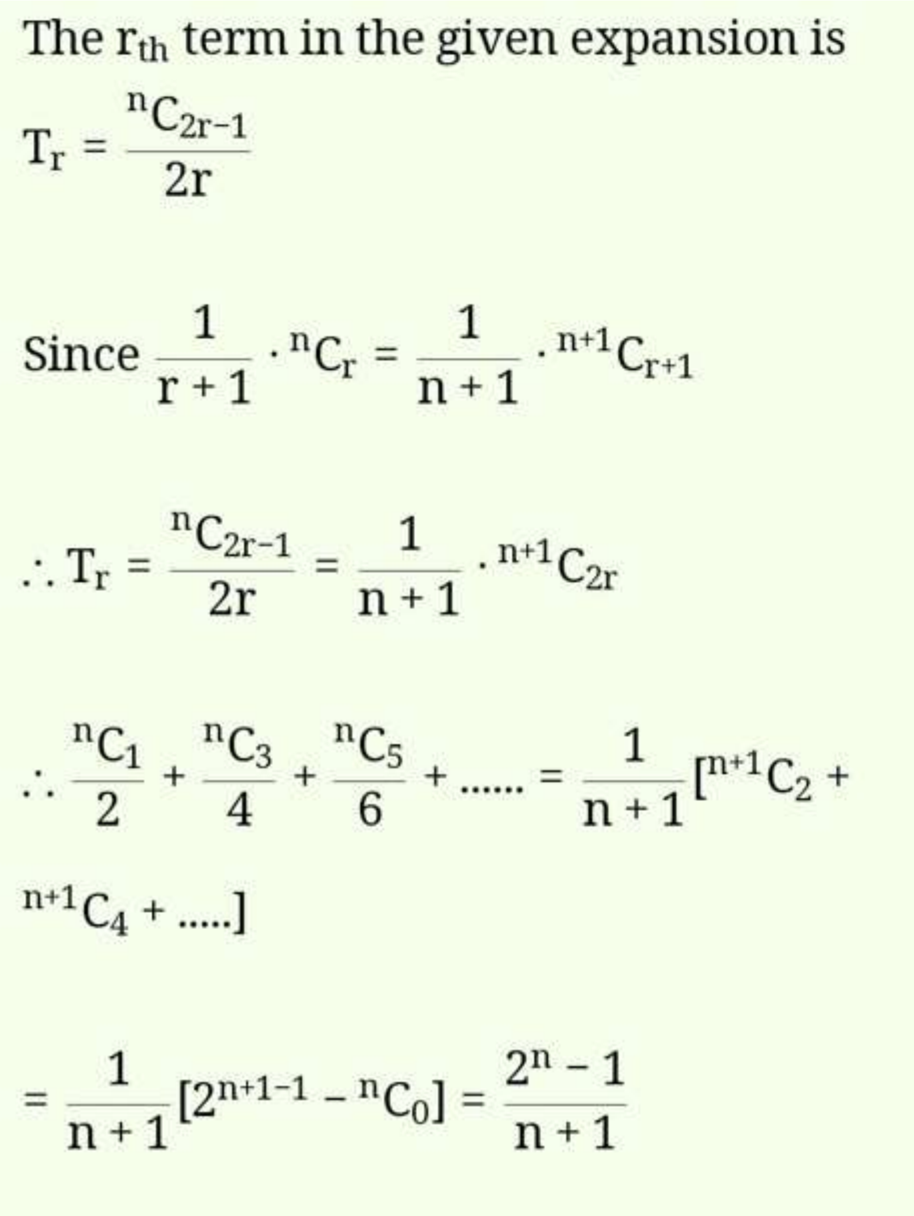
47. (b)

$$S=\sum_{i=0}^{r}{}^{n_1}C_{r-i}\;{}^{n_2}C_i$$

Coefficient of x^r in $\left(1+x\right)^{n_1}\left(1+x\right)^{n_2}=\left(1+x\right)^{n_1+n_2}$

$$={}^{n_1+n_2}C_r$$

48. (c)



The r_{th} term in the given expansion is

$$T_r = \frac{{}^nC_{2r-1}}{2r}$$

Since $\frac{1}{r+1} \cdot {}^nC_r = \frac{1}{n+1} \cdot {}^{n+1}C_{r+1}$

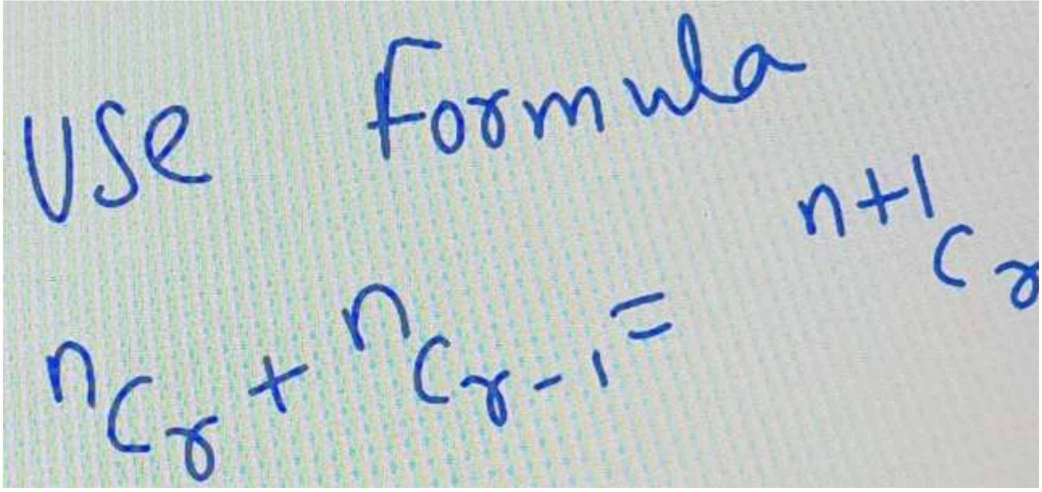
$$\therefore T_r = \frac{{}^nC_{2r-1}}{2r} = \frac{1}{n+1} \cdot {}^{n+1}C_{2r}$$

$$\therefore \frac{{}^nC_1}{2} + \frac{{}^nC_3}{4} + \frac{{}^nC_5}{6} + \ldots\ldots = \frac{1}{n+1}[{}^{n+1}C_2 + {}^{n+1}C_4 + \ldots..]$$

$$= \frac{1}{n+1}[2^{n+1-1} - {}^nC_0] = \frac{2^n - 1}{n+1}$$

49. (c)

50. (d)



Use formula

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

51. (d)

$\left(1+2x+3x^2\right)^{10} = a_0 + a_1x + ...a_{20}a^{20}$ ………….(1)

Diff $10\left(1+2x+3x^2\right)^9\left(2+6m\right) = a_1 + 2a_2x + ...$

Put $x = 0$

$20 = a_1$ In equation (1)

Put $x = 1$

$a_0 + a_1 + a_2 + ...a_{20} = 6^{10}$

Clearly coefficient of x^{20} is 3^{10}

Use PNC to find coefficient of x^2

$3.{}^{10}C_1 + 4.\frac{10}{2}$

$30 + 180 = 210$

52. (c)

$\Rightarrow 1 + x + x^2 + x^3 = [1\,(1+x) + x^2\,(1+x)]$

then $(1 + x + x^2 + x^3)^5 = [\,(1+x)(1+x^2)\,]^5 = (1+x)^5\,(1+x^2)^5$

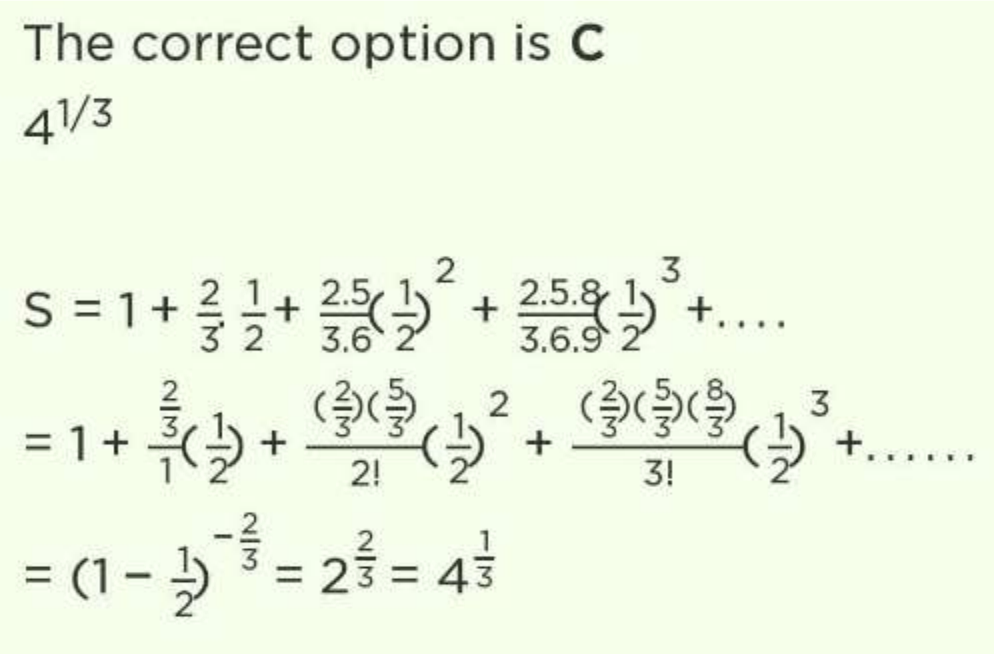
$=(1+x)^5 \times [\,{}^5C_0\,1^5 + {}^5C_1\,1^4\,x^2 + {}^5C_2\,1^3x^4 + {}^5C_3\,1^2\,x^6 + {}^5C_4\,1^1\,x^8 + {}^5C_5\,1^0\,x^{10}\,]$

a_{10} = *Coefficient of* $x^{10} = ({}^5C_0\,1^5 \times {}^5C_5\,1^0) + ({}^5C_2\,1^3 \times {}^5C_4\,1^1) + ({}^5C_4\,1^1 \times {}^5C_3\,1^2)$

$= 1 + 50 + 50$

So, $a_{10} = 101$

53. (b)



The correct option is **C**

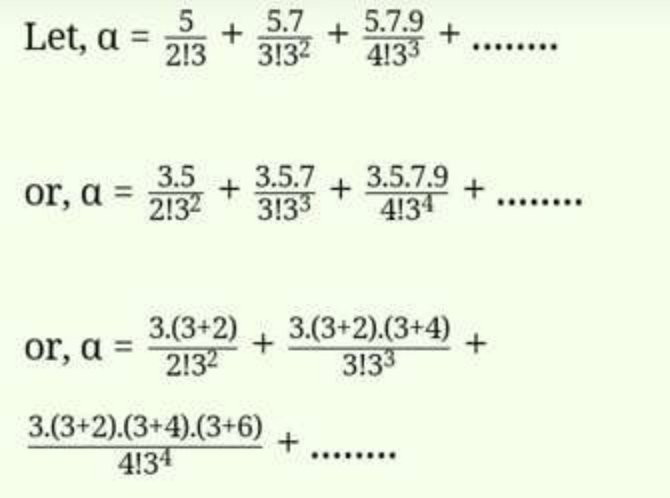
$4^{1/3}$

$$S = 1 + \frac{2}{3}.\frac{1}{2} + \frac{2.5}{3.6}\left(\frac{1}{2}\right)^2 + \frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3 + \ldots.$$

$$= 1 + \frac{\frac{2}{3}}{1}\left(\frac{1}{2}\right) + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{2!}\left(\frac{1}{2}\right)^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)}{3!}\left(\frac{1}{2}\right)^3 + \ldots\ldots$$

$$= \left(1 - \frac{1}{2}\right)^{-\frac{2}{3}} = 2^{\frac{2}{3}} = 4^{\frac{1}{3}}$$

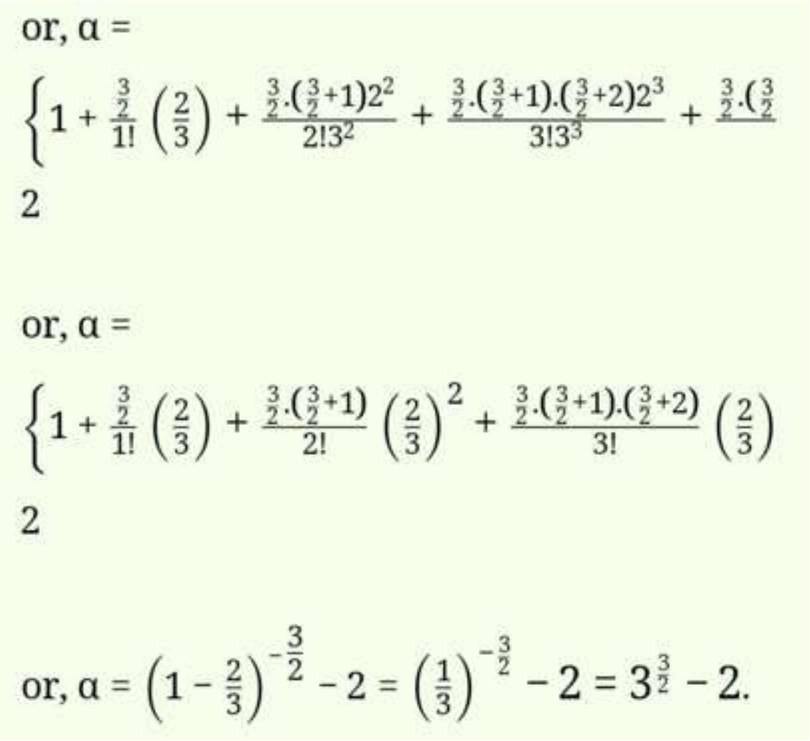
54. (b)



Let, $\alpha = \frac{5}{2!3} + \frac{5.7}{3!3^2} + \frac{5.7.9}{4!3^3} + \ldots\ldots..$

or, $\alpha = \frac{3.5}{2!3^2} + \frac{3.5.7}{3!3^3} + \frac{3.5.7.9}{4!3^4} + \ldots\ldots..$

or, $\alpha = \frac{3.(3+2)}{2!3^2} + \frac{3.(3+2).(3+4)}{3!3^3} + \frac{3.(3+2).(3+4).(3+6)}{4!3^4} + \ldots\ldots..$



or, α =

$$\Bigg\{1 + \frac{\frac{3}{2}}{1!}\left(\frac{2}{3}\right) + \frac{\frac{3}{2}.(\frac{3}{2}+1)2^2}{2!3^2} + \frac{\frac{3}{2}.(\frac{3}{2}+1).(\frac{3}{2}+2)2^3}{3!3^3} + \frac{\frac{3}{2}.(\frac{3}{2}}{}$$

2

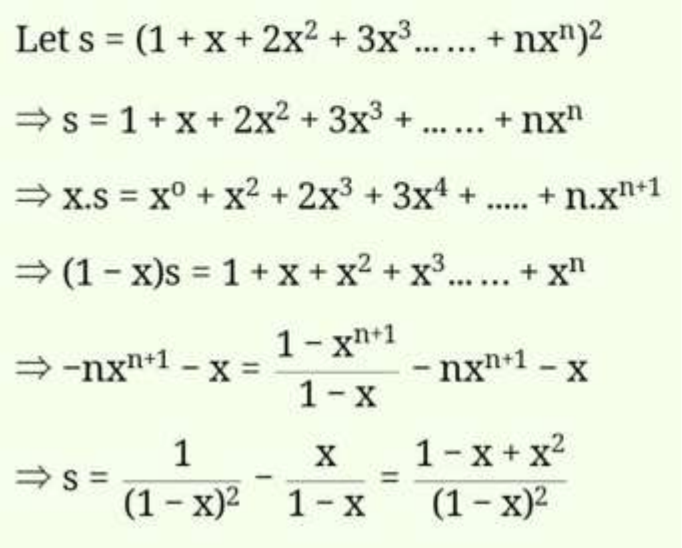
or, α =

$$\Bigg\{1 + \frac{\frac{3}{2}}{1!}\left(\frac{2}{3}\right) + \frac{\frac{3}{2}.(\frac{3}{2}+1)}{2!}\left(\frac{2}{3}\right)^2 + \frac{\frac{3}{2}.(\frac{3}{2}+1).(\frac{3}{2}+2)}{3!}\left(\frac{2}{3}\right)$$

2

$$\text{or, } \alpha = \left(1 - \frac{2}{3}\right)^{-\frac{3}{2}} - 2 = \left(\frac{1}{3}\right)^{-\frac{3}{2}} - 2 = 3^{\frac{3}{2}} - 2.$$

55. (a)



Let $s = (1 + x + 2x^2 + 3x^3 \ldots\ldots + nx^n)^2$

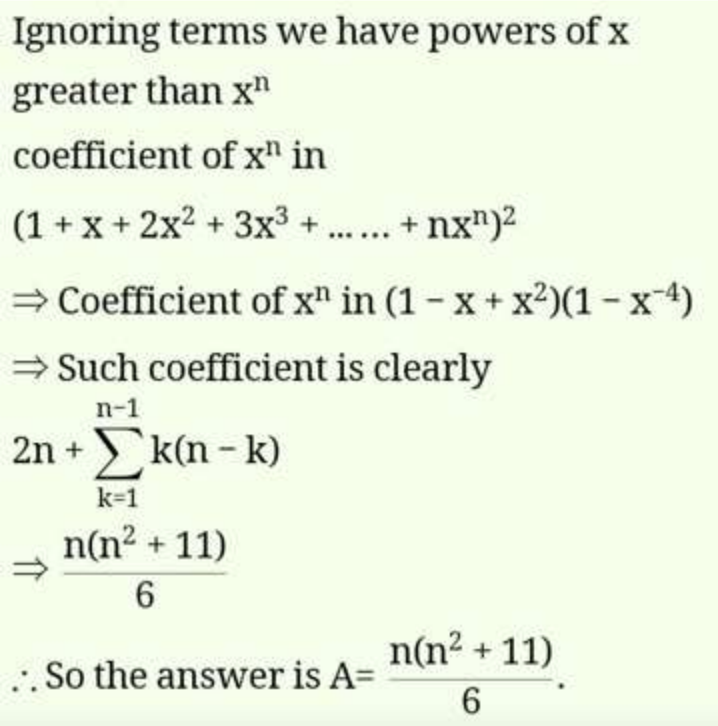
$\Rightarrow s = 1 + x + 2x^2 + 3x^3 + \ldots\ldots + nx^n$

$\Rightarrow x.s = x^o + x^2 + 2x^3 + 3x^4 + \ldots.. + n.x^{n+1}$

$\Rightarrow (1 - x)s = 1 + x + x^2 + x^3 \ldots\ldots + x^n$

$\Rightarrow -nx^{n+1} - x = \frac{1 - x^{n+1}}{1 - x} - nx^{n+1} - x$

$\Rightarrow s = \frac{1}{(1-x)^2} - \frac{x}{1 - x} = \frac{1 - x + x^2}{(1 - x)^2}$



Ignoring terms we have powers of x greater than x^n

coefficient of x^n in

$(1 + x + 2x^2 + 3x^3 + \dots\dots + nx^n)^2$

$\Rightarrow$ Coefficient of x^n in $(1 - x + x^2)(1 - x^{-4})$

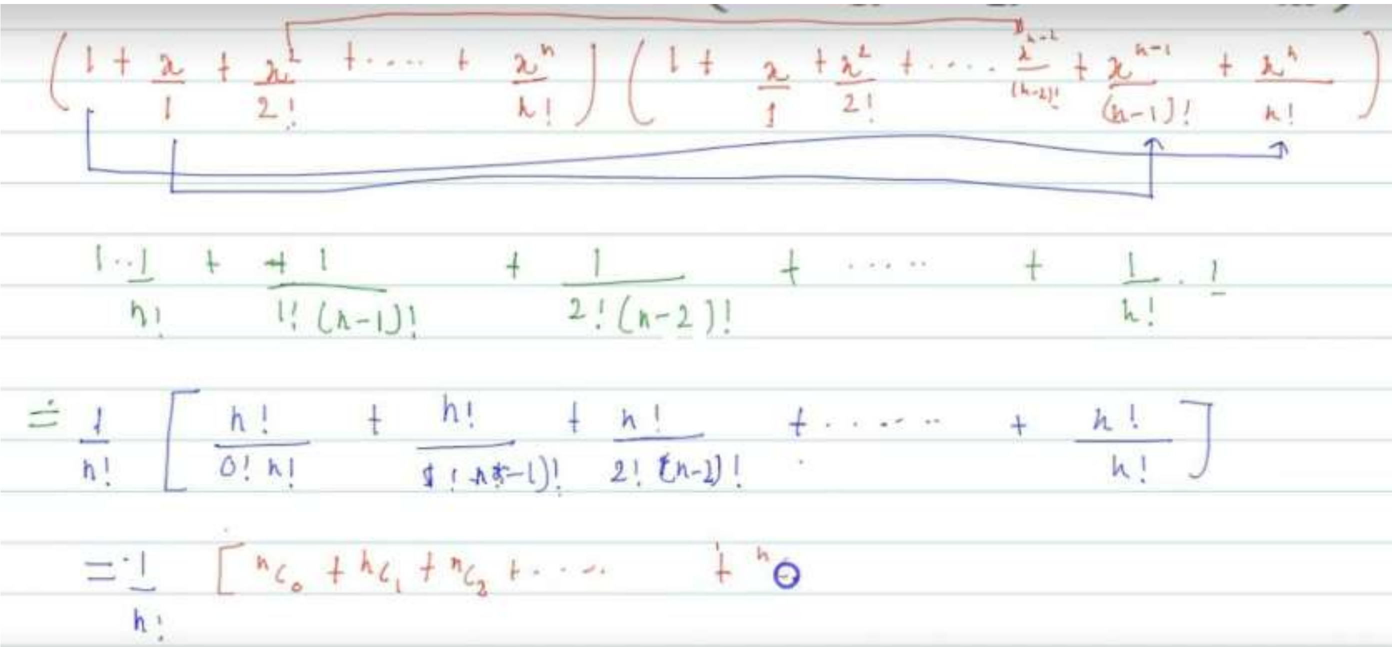
$\Rightarrow$ Such coefficient is clearly

$$2n + \sum_{k=1}^{n-1} k(n - k)$$

$$\Rightarrow \frac{n(n^2 + 11)}{6}$$

$\therefore$ So the answer is A= $\frac{n(n^2 + 11)}{6}$.

56. (b)



$$\left(1+\frac{x}{1}+\frac{x^2}{2!}+\dots+\frac{x^n}{n!}\right)\left(1+\frac{x}{1}+\frac{x^2}{2!}+\dots\frac{x^{n-2}}{(n-2)!}+\frac{x^{n-1}}{(n-1)!}+\frac{x^n}{n!}\right)$$

$$1\cdot\frac{1}{n!}+\frac{1}{1!(n-1)!}+\frac{1}{2!(n-2)!}+\dots+\frac{1}{n!}\cdot 1$$

$$=\frac{1}{n!}\left[\frac{n!}{0!\,n!}+\frac{n!}{1!(n-1)!}+\frac{n!}{2!(n-2)!}+\dots+\frac{n!}{n!}\right]$$

$$=\frac{1}{n!}\left[{}^nC_0+{}^nC_1+{}^nC_2+\dots+{}^n\Theta\right.$$

57. (c)
A.G.P.

58. (b)

$$(abc)^{12}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^{12}$$

$\Rightarrow$Find coefficient of $\frac{1}{a^4}\frac{1}{b^2}\frac{1}{c^6}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^{12}$

Then $\frac{12!}{4!2!6!}$

59. (d)

$$\left(1+2x+3x^2+...\right)^{-3/2}=\left[\left(1-x\right)^{-2}\right]^{-3/2}$$

$$=(1-x)^3=1-3x+3x^2-x^3$$

Therefore, coefficient of x^5 is 0.

60. (d)

$$(1+x+x^2+x^3+....)^2=\left(\left(1-x\right)^{-1}\right)^2=\left(1-x\right)^{-2}$$

$$=1+2x+3x^2+.....$$

Therefore, coefficient of x^n is n + 1

61. (d)

$$\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$$

$$=\frac{\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)-\left(1+\frac{3}{2}x+3\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

$$=\frac{-3}{8}x^2(1-x)^{-1/2}$$

$$=-\frac{3}{8}x^2\left(1+\frac{x}{2}\right)$$

$$=-\frac{3}{8}x^2$$

62. (d)

$$\frac{1}{(1-ax)(1-bx)}=a_0+a_1x+a_2x^2+......+a_nx^n+...$$

But $(1-ax)^{-1}(1-bx)^{-1}=(1+ax+a^2x^2+....)(1+bx+b^2x^2+....)$

$\Rightarrow$Coefficient of x^n is $b^n+ab^{n-1}+a^2b^{n-2}+....+a^{n-1}b+a^k=\frac{b^{n+1}-a^{n-1}}{b-a}$

$$\Rightarrow a_n=\frac{b^{n+1}-a^{n+1}}{b-a}$$

EXERCISE - 1 [C]

1. (15)

$$\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}=\sum_{i=0}^{m}{}^{10}C_i\,{}^{20}C_{m-i}$$

$$={}^{10}C_0\cdot{}^{20}C_m+{}^{10}C_1\cdot{}^{20}C_{m-1}+{}^{10}C_2\cdot{}^{20}C_{m-2}+\ldots+{}^{10}C_m\cdot{}^{20}C_0$$

= Coefficient of x^m in the expansion of product $(1+x)^{10}(1+x)^{20}$

= Coefficient of x^m in the expansion of $(1+x)^{30}={}^{30}C_m$

To get maximum value of the given sum, ${}^{30}C_m$ should be maximum.

Which is so, when $m=\frac{30}{2}=15$

2. (141)

$$\because\ \left(x+\frac{1}{x}+1\right)^6=\sum_{r=0}^{6}{}^{6}C_r\left(x+\frac{1}{x}\right)^r$$ for constant term r must be even integer.

$$\therefore\ a_0={}^6C_0+{}^6C_2\times{}^2C_1+{}^6C_4\times{}^6C_2+{}^6C_6\times{}^6C_3$$

$$=1+30+90+20=141$$

3. (210)

Since, last term in the expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$

$$=\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}\Rightarrow{}^nC_n\cdot\left(-\frac{1}{\sqrt{2}}\right)^n=\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$$

$$\Rightarrow\ (-1)^n\cdot\left(\frac{1}{2}\right)^{n/2}=\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}=\left(3^{-5/3}\right)^{\log_2 2^3}$$

$$=3^{-\frac{5}{3}\times 3\times\log_3 2}=3^{-5\log_3 2}=3^{\log_3 2^{-5}}=2^{-5}=\left(\frac{1}{2}\right)^5$$

$$\Rightarrow\ (-1)^n\cdot\left(\frac{1}{2}\right)^{n/2}=\left(\frac{1}{2}\right)^5$$

$$\therefore\ n=10$$

Now, 5th term beginning $={}^{10}C_4\left(\sqrt[3]{2}\right)^6\left(-\frac{1}{\sqrt{2}}\right)^4$

$$={}^{10}C_4\cdot 2^2\cdot\frac{1}{2^2}={}^{10}C_4={}^{10}C_6$$

4. (5)

Here, $f(x)=\sum_{r=1}^{n}\left\{r^2\left({}^nC_r-{}^nC_{r-1}\right)+(2r+1)\,{}^nC_r\right\}$

$$=\sum_{r=1}^{n}\left(r^2+2r+1\right){}^nC_r-r^2\cdot{}^nC_{r-1}$$

$$=\sum_{r=1}^{n}\left((r+1)^2\cdot{}^nC_r-{}^nC_r-r^2\cdot{}^nC_{r-1}\right)$$

$$=(n+1)^2\cdot{}^nC_n-1^2\cdot{}^nC_0$$

$$=(n+1)^2-1=\left(n^2+2n\right)$$

$\therefore\quad f(30)=(30)^2+2(30)=960$

$$=30\times32=30(2)^5=30(2)^\lambda$$

Hence, $\lambda=5$

5. (2)

$\because\ 9^{100}=(2\cdot4+1)^{100}=4n+1$ [say] [Where n is positive integer]

$\therefore\ 2^{9^{100}}=2^{4n+1}=2^{4n}\cdot2=(16)^n\cdot2$

The digit at unit's place in $(16)^n\cdot2=2$

$\therefore$ The digit at unit's place in $(16)^n\cdot2=2$

6. (5)

Here, $a_r={}^nC_r$

$\therefore\ b_r=1+\dfrac{a_r}{a_{r-1}}=1+\dfrac{{}^nC_r}{{}^nC_{r-1}}$

$$=1+\frac{n-r+1}{r}=\frac{(n+1)}{r}$$

$\Rightarrow\ \prod_{r=1}^{n}b_r=\prod_{r=1}^{n}\dfrac{(n+1)}{r}$

$$=\frac{(n+1)}{1}\cdot\frac{(n+1)}{2}\cdot\frac{(n+1)}{3}\dots\frac{(n+1)}{n}=\frac{(n+1)^n}{n!}$$

$$=\frac{(101)^{100}}{100!}\quad\text{[Given]}$$

$\therefore\ n=100\Rightarrow\ \dfrac{n}{20}=5$

7. (8)

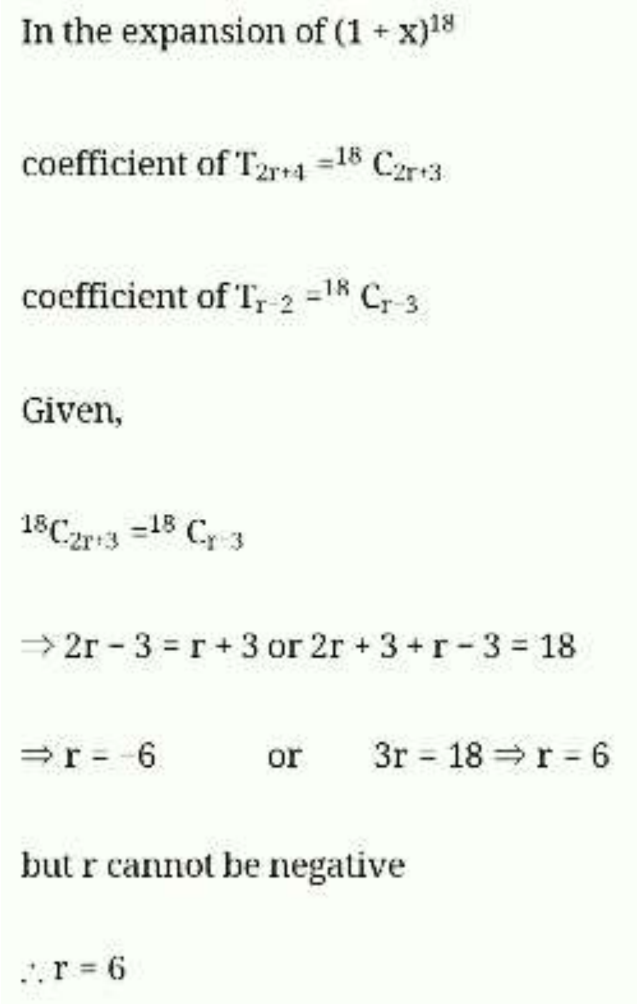
We have, $2^{2006}=2^2\left(2^3\right)^{668}$

$$=4(1+7)^{668}=4(1+7k)=4+28k$$

$\therefore\ 2^{2006}+2006=4+28k+7\times(286)+4$

Hence, remainder is 8.

8. (6)



In the expansion of $(1 + x)^{18}$

coefficient of $T_{2r+4} = {}^{18}C_{2r+3}$

coefficient of $T_{r-2} = {}^{18}C_{r-3}$

Given,

${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

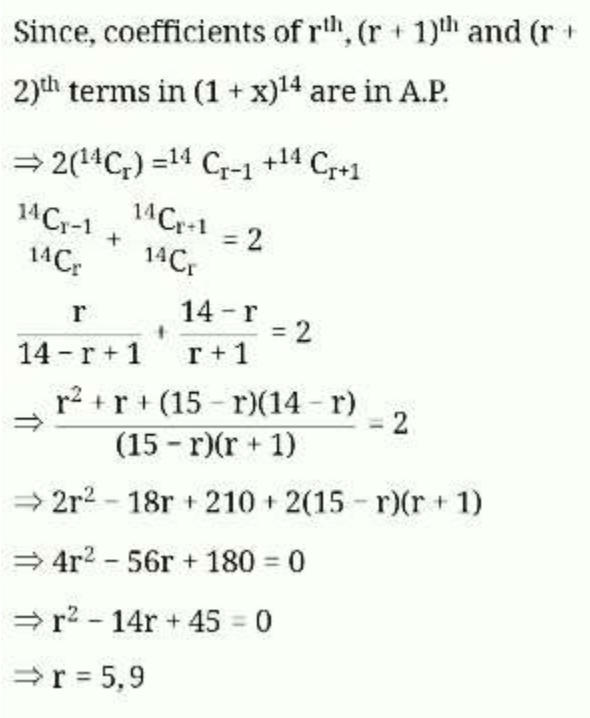
$\Rightarrow 2r - 3 = r + 3$ or $2r + 3 + r - 3 = 18$

$\Rightarrow r = -6$ or $3r = 18 \Rightarrow r = 6$

but r cannot be negative

$\therefore r = 6$

9. (9)



Since, coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in $(1 + x)^{14}$ are in A.P.

$\Rightarrow 2({}^{14}C_r) = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

$$\frac{{}^{14}C_{r-1}}{{}^{14}C_r} + \frac{{}^{14}C_{r+1}}{{}^{14}C_r} = 2$$

$$\frac{r}{14 - r + 1} + \frac{14 - r}{r + 1} = 2$$

$$\Rightarrow \frac{r^2 + r + (15 - r)(14 - r)}{(15 - r)(r + 1)} = 2$$

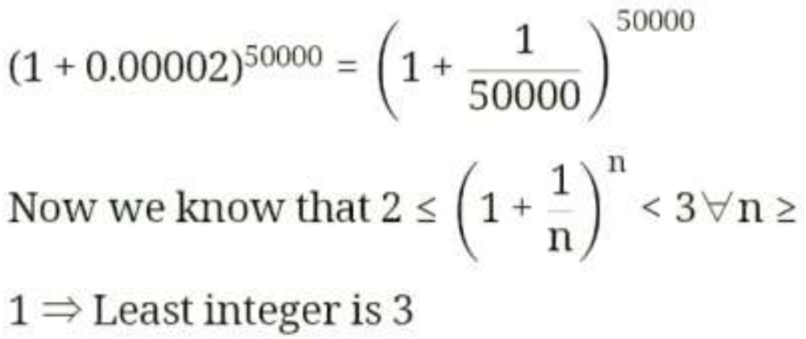
$\Rightarrow 2r^2 - 18r + 210 + 2(15 - r)(r + 1)$

$\Rightarrow 4r^2 - 56r + 180 = 0$

$\Rightarrow r^2 - 14r + 45 = 0$

$\Rightarrow r = 5, 9$

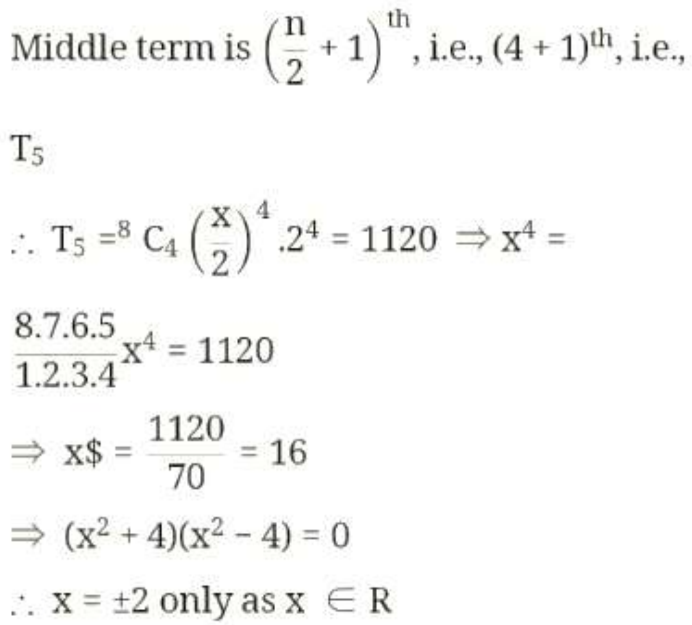
10. (3)



$$(1 + 0.00002)^{50000} = \left(1 + \frac{1}{50000}\right)^{50000}$$

Now we know that $2 \le \left(1 + \frac{1}{n}\right)^{n} < 3 \forall n \ge$ $1 \Rightarrow$ Least integer is 3

11. (0)



Middle term is $\left(\frac{n}{2} + 1\right)^{th}$, i.e., $(4 + 1)^{th}$, i.e.,

T_5

$\therefore\ T_5 = {}^8C_4 \left(\frac{x}{2}\right)^4 .2^4 = 1120 \Rightarrow x^4 =$

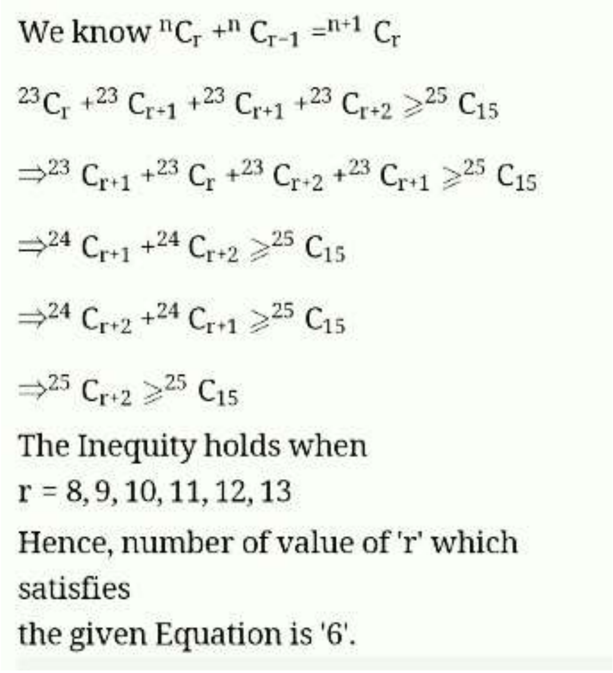
$\frac{8.7.6.5}{1.2.3.4} x^4 = 1120$

$\Rightarrow\ x\$ = \frac{1120}{70} = 16$

$\Rightarrow\ (x^2 + 4)(x^2 - 4) = 0$

$\therefore\ x = \pm 2$ only as $x \in R$

12. (5)



We know ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

${}^{23}C_r + {}^{23}C_{r+1} + {}^{23}C_{r+1} + {}^{23}C_{r+2} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{23}C_{r+1} + {}^{23}C_r + {}^{23}C_{r+2} + {}^{23}C_{r+1} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{24}C_{r+1} + {}^{24}C_{r+2} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{24}C_{r+2} + {}^{24}C_{r+1} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{25}C_{r+2} \geqslant {}^{25}C_{15}$

The Inequity holds when
r = 8, 9, 10, 11, 12, 13
Hence, number of value of 'r' which
satisfies
the given Equation is '6'.

13. (4)

$$\left(5^{\frac{1}{5}\log\sqrt{4^{x}+44}} + \frac{1}{5^{\log 5\sqrt{2^{x-1}+7}}}\right)$$

$$= \left((\sqrt{4^{x} + 444})^{2/5} + \left(\frac{1}{\sqrt[3]{2^{x-1} + 7}}\right)\right)^{8}$$

$$= \left((4^{x} + 444)^{2/5} + \left(\frac{1}{(2^{x-1} + 7)^{1/3}}\right)\right)^{8}$$

Now $T_4 = T_{3+1} = {}^{8}C_3((4^{x} +$

$44)^{1/5})^{8-3}\frac{1}{((2^{x-1} + 7)^{1/3})^{3}}$

Given $336 = {}^{8}C_3\left(\frac{4^{x} + 44}{2^{x-1} + 7}\right)$

Let $2^{x} = y$

$\Rightarrow\ 336 = {}^{8}C_3\left(\frac{y^2 + 44}{(y/2) + 7}\right)$

$\Rightarrow\ 366 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1}\left(\frac{2(y^2 + 44)}{y + 14}\right)$

$\Rightarrow\ y^2 - 3y + 2 = 0\ \Rightarrow y = 0, 2$

14. (4)

We have b = coefficient of x^3 in

$((1 + x + 2x^2 + 3x^3) + 4x^4)^4$

= coefficient of x^3 in $[{}^4C_0(1 + x + 2x^2 + 3x^3)^4(4x^4)^0 + {}^4C_1(1 + x + 2x^2 + 3x^3)^3(4x^4)^1 + \ldots]$

= coefficient of x^3 in $(1 + x + 2x^2 + 3x^3)^4$ =

Hence, 4a/b = 4.

15. (1)

$$\sum_{k=0}^{4}\left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)$$

$$=\sum_{k=0}^{4}\left(\frac{4!}{(4-k)!k!}3^{4-k}.\,x^4.\,\frac{1}{4!}\right)$$

$$=\sum_{k=0}^{4}\frac{.^4\,C_k.3^{4-k}.\,x^4}{04!}$$

$$=\frac{(3+x)^4}{4!}$$

According to the question ,

$$\frac{(3+x)^4}{4!}=\frac{32}{3}$$

or $(3+x)^4=256$

or $x+3=256$

or $x=1$

1. (c)

We are given that $7^{2022} + 3^{2022}$

$=(49)^{1011}+(9)^{1011}=(50-1)^{1011}+(10-1)^{1011}$

$=5\lambda-1+5K-1=5m-2$

$\Rightarrow$ Remainder = 5 – 2 = 3

2. (c)

$(202)^{2023}=(7K-2)^{2023}$

$= {}^{2023}C_0(7\lambda)^{2023}-\ldots{}^{2023}C_{2023}2^{2023}=7\mu-2^{2023}$

$\therefore\ -2^{2023}=-2\times2^{2022}$

$=-2\times\left(2^3\right)^{674}=-2(1+7\gamma)^{674}=-(7\beta+2)$

$\Rightarrow$ remainder = –2 or +5

3. (a)

Given expression is

$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+\ldots+x^{500}$

Take $(5+x)^{500}$ common, then it forms G.P.

$$\left[(5+x)^{500}\left(\left(\frac{x}{5+x}\right)^{501}-1\right)\right]\left(\frac{x}{5+x}\right)-1$$

$$=\frac{(5+x)^{501}-x^{501}}{5}$$

$\Rightarrow$ Coefficient x^{101} in given expression is calculate by

$\left(\frac{1}{5}(x+5)^{501}\right)$ only $=\frac{{}^{501}C_{101}5^{400}}{5}={}^{501}C_{101}5^{399}$

4. (d)

$(1+x)^{20}=C_0+C_1x+C_2x^2+\ldots.+C_{20}x^{20}$

Diff. w.r.t. x

$20(1+x)^{19}=C_1+2C_2x+\ldots+20C_{20}x^{19}$

Multiple by x both side

$20x(1+x)^{19}=C_1x+2C_2x^2+\ldots+20C_{20}x^{20}$

Diff. w.r.t. x

$20(1+x)^{19}+20.x.19(1+x)^{18}$

$=C_1+4C_2x+\ldots.+20^2C_{20}x^{19}$

Let $x=1$

$\Rightarrow\ 20\times19.2^{18}+20.2=C_1+4C_2+\ldots.+20^2C_{20}$

$= 420 \times 2^{18}$.

5. (d)

$$\frac{3^{200}}{8} = \frac{1}{8}\left(9^{100}\right)$$

$$= \frac{1}{8}(1+8)^{100} = \frac{1}{8}\left[1 + n \cdot 8 + \frac{n(n+1)}{2} \cdot 8^2 + \ldots\right] = \frac{1}{8} + \text{Integer}$$

$$\therefore \quad \left\{\frac{3^{200}}{8}\right\} = \left\{\frac{1}{8} + \text{integer}\right\} = \frac{1}{8}$$

6. (a)

$$\left(x^2 + \frac{1}{x^3}\right)^n$$

General term $T_{r+1} = {}^nC_r \left(x^2\right)^{n-r} \left(\frac{1}{x^3}\right)^r = {}^nC_r \cdot x^{2n-5r}$

To Find coefficient of x, $2n - 5r = 1$

Given ${}^nC_r = {}^nC_{23} \Rightarrow r = 23$ or $n - r = 23$

$\therefore$ $n = 58$ or $n = 38$, Minimum value is $n = 38$

7. (d)

$$\left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1} + \sqrt{5x^2-1}}\right]^8$$

After rationalize the polynomial we get

$$\left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1} - \sqrt{5x^3-1}}{\sqrt{5x^3+1}\sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1} - \sqrt{5x^3-1}}{\sqrt{5x^3+1}\sqrt{5x^3-1}}\right]^8$$

$$= \left[\frac{\sqrt{5x^3+1} + \sqrt{5x^3-1}}{\left(5x^3+1\right) - \left(5x^3-1\right)}\right]^8 + \left[\frac{\sqrt{5x^3+1} + \sqrt{5x^3-1}}{\left(5x^3+1\right) - \left(5x^3-1\right)}\right]^8$$

$$= \frac{1}{2^8}\left[\left(\sqrt{5x^3+1} + \sqrt{5x^3-1}\right)^8 + \left(\sqrt{5x^3+1} - \sqrt{5x^3-1}\right)^8\right]$$

$$= \frac{1}{2^8}\left[{}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(\sqrt{5x^3-1}\right)^2 + {}^8C_4\left(\sqrt{5x^3+1}\right)^4\left(\sqrt{5x^3-1}\right)^4\right.$$
$$\left. + {}^8C_6\left(\sqrt{5x^3+1}\right)^2\left(\sqrt{5x^3-1}\right)^6 + {}^8C_8\left(\sqrt{5x^3-1}\right)^8\right]$$

$$= \frac{1}{2^8}\left[{}^8C_0\left(5x^3+1\right)^4 + {}^8C_2\left(5x^3+1\right)^3\left(5x^3-1\right) + {}^8C_4\left(5x^3+1\right)^2\left(5x^3-1\right)^2\right.$$
$$\left. + {}^8C_6\left(5x^3+1\right)\left(5x^3-1\right)^3 + 8C_8\left(5x^3-1\right)^4\right]$$

So, the degree of polynomial is 12,

Now, coefficient of x^{12}

$$= \left[{}^{8}C_0\, 5^4 + {}^{8}C_2\, 5^4 + {}^{8}C_4\, 5^4 + {}^{8}C_6\, 5^4 + {}^{8}C_8\, 5^4 \right]$$

$$= 5^4 \times \frac{2^8}{2} = 5^4 \times 2^4 \times \frac{2^4}{2} = 10^4 \times 2^3 = 8(10)^4$$

8. (c)

$$\frac{{}^{n}C_r}{1} = \frac{{}^{n}C_{r+1}}{7} = \frac{{}^{n}C_{r+2}}{42}$$

By solving we get $r = 6$

So, it is 7th term.

9. (c)

General term of $\left(ax^{1/8} + bx^{-1/12}\right)^{10}$ is

$$= {}^{10}C_r \left(ax^{1/8}\right)^{10-r} \left(bx^{-1/12}\right)^r$$

On solving $r = 6$

$$\Rightarrow \left(a^4 b^6\right)^{\frac{1}{4}} \ge \frac{}{2\left(\frac{1}{a^2} + \frac{1}{b^2}\right)} \Rightarrow a^4 b^6 \ge \left(\frac{1}{2}\right)^4$$

$$\Rightarrow {}^{10}C_6 \left(\frac{1}{2}\right)^4 = \frac{210}{16} \Rightarrow \frac{105}{8}$$

10. (b)

General term :

$$T_{r+1} = {}^{60}C_r \cdot \left(3^{\frac{1}{4}}\right)^{60-r} \cdot \left(5^{\frac{1}{8}}\right)^r = {}^{60}C_r \cdot 3^{15\frac{r}{}} = \frac{r}{}$$

Term will be rational if r divisible by 8.

$\therefore\ r = 0, 8, 16, 24, 32, 40, 48, 56$

So, total number of irrational terms = n = 61 – 8 = 53

Hence, n – 1 = 52 is divisible by 26.

11. (c)

General term $= T_{r+1} = {}^{10}C_r \left(\sqrt{x}\right)^{10-r} \cdot \left(-\frac{k}{x^2}{}^r\right)$

$$= {}^{10}C_r (-k)^r \cdot x^{\frac{10-r}{2} - 2r} = {}^{10}C_r (-k)^r \cdot x^{\frac{10-5r}{2}}$$

Since, it is constant term, then

$$\frac{10-5r}{2} = 0 \Rightarrow r = 2;\therefore\ {}^{10}C_2 (-k)^2 = 405$$

$$\Rightarrow k^2 = \frac{405 \times 2}{10 \times 9} = \frac{81}{9} = 9$$

$$\therefore\ |k| = 3$$

12. (c)

Given expression can be written as

$$\left[\frac{\left(x^{1/3}\right)^{2}+1^{3}}{x^{2/3}-x^{1/3}+1}-\frac{\left(\sqrt{x}\right)^{2}-1^{2}}{\sqrt{x}\left(\sqrt{x}-1\right)}\right]^{10}$$

$$=\left(\left(x^{1/3}+1\right)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}=\left(x^{1/3}+1-1-\frac{1}{\sqrt{x}}^{10}\right)$$

$$=\left(x^{1/3}-x^{-1/2}\right)^{10}$$

General term $=T_{r+1}$

$$={}^{10}C_{r}\left(x^{1/3}\right)^{10-r}\left(-x^{-1/2}\right)^{r}={}^{10}C_{r}\,x^{\frac{10-r}{3}}\cdot(-1)^{r}\cdot x^{-\frac{r}{2}}$$

$$={}^{10}C_{r}\,(-1)^{r}\cdot x^{\frac{10-r}{3}-\frac{r}{2}}$$

Term will be independent of x when $\frac{10-r}{3}-\frac{r}{2}=0$

$\Rightarrow\ r=4$

So, required term $=T_{5}={}^{10}C_{4}=210$

13. (b)

Given expression is $\sum_{t=1}^{20}\left(r^{2}+1\right)(r!)$

$$\Rightarrow\ \sum_{x=1}^{20}\left((r+1)^{2}-2r\right)r!$$

$$\Rightarrow\ \sum_{x=1}^{20}\left((r+1)(r+1)!r.r!\right)-\sum_{r=1}^{20}r.r!$$

$$\Rightarrow\ \sum_{x=1}^{20}\left((r+1)(r+1)!-r.r!\right)-\sum_{r=1}^{20}\left((r+1)!-r!\right)$$

$$=\left(21.\lfloor\underline{21}-1\right)-\left(\lfloor\underline{21}-1\right)=20.21!=22!-2.21!$$

14. (a)

Given expansion is $\sum_{\substack{i,\,j=0\\ i\neq j}}^{n}{}^{n}C_{i}\ {}^{n}C_{j}$

$$=\sum_{i=0}^{n}{}^{n}C_{i}\cdot\sum_{j=0}^{n}{}^{n}C_{j}-\sum_{i=j=0}^{n}\left({}^{n}C_{i}\right)^{2}$$

$$=\left(2^{n}\right)\left(2^{n}\right)-{}^{2n}C_{n}=2^{2n}-{}^{2n}C_{n}$$

15. (d)

Constant term in $\left(3x^{3}-2x^{2}+\frac{5}{x^{5}}\right)^{10}$ to make one term without x

Let, $x^{50}\left(3x^8-2x^7+5\right)^{10}$

How general term of the expression is

$$\frac{10!}{p!q!r!}\left(3x^8\right)^p\left(-2x^7\right)^q\left(5\right)^r$$

Here $8p+7q=50$ and $p+q+r=10$

$\Rightarrow$ $p=1, q=6, r=3$ in only valid solution.

$\therefore$ $\frac{10!}{1!6!r!}3^1 2^6\cdot 5^3=2^K\cdot l\Rightarrow K=9$

16. (a)

Given expression is $\sum_{K=1}^{31}{}^{31}C_K\cdot{}^{31}C_{K-1}$

$={}^{31}C_1\cdot{}^{31}C_0+{}^{31}C_2\cdot{}^{31}C_1+....+{}^{31}C_{31}\cdot{}^{31}C_{30}$

Here, ${}^nL_r={}^nL_{n-r}$

$={}^{31}C_0\cdot{}^{31}C_{30}+{}^{31}C_1\cdot{}^{31}C_{29}+.....+{}^{31}C_{30}\cdot{}^{31}C_0={}^{62}C_{30}$

Similarly

$$\sum_{K=1}^{30}\left({}^{31}C_K\cdot{}^{31}C_{K-1}\right)={}^{60}C_{29}$$

$$=1\ {}^{62}C_{30}-{}^{60}C_{29}=\frac{62!}{30!32!}-\frac{60!}{29!31!}$$

$$=\frac{60!}{29!31!}\left\{\frac{62.61}{30.32}-1\right\}=\frac{60!}{30!31!}\left(\frac{2822}{32}\right)$$

Compare above equation with $\frac{\alpha(60!)}{(30!)(31!)}$

So, $\alpha=\frac{2822}{32}$

$\therefore$ $16\alpha=16\times\frac{2822}{32}=1411$

17. (b)

We are given that the expression is

$$\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11};\ x\neq 0$$

$\because$ General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

${}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r$ [$\because$ General term of $\left(x+y\right)^n$ is ${}^nC_r\left(x\right)^{n-r}\cdot y^r$]

Now general term of ${}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r x^{33-5r}$

$\therefore$ Term independent of x is

$1\times$ coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ $+-1\times$coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

For coefficient of x^0 $33-5r=0$ not possible

For coefficient of x^{-2} $33-5r=-2$

$\Rightarrow\ 35=5r\ \Rightarrow\ r=7$

For coefficient of x^{-3} $33-5r=-3$

$\Rightarrow\ 36=5r$ not possible

So term independent of x is

$(-1)\,{}^{11}C_7\left(\frac{5^4}{2}\right)\left(-\frac{1}{5}\right)^7=\frac{33}{200}$

18. (c)

$\sum_{k=0}^{20}{}^{20}C_k\cdot{}^{20}C_{20-k}$

$=\left({}^{20}C_0\right)^2+\left({}^{20}C_1\right)^2+\left({}^{20}C_2\right)^2+\ldots+\left({}^{20}C_{20}\right)^2$

$={}^{20}C_0\cdot{}^{20}C_{20}+{}^{20}C_1\cdot{}^{20}C_{19}\ldots{}^{20}C_{20}\cdot{}^{20}C_0$ $\left(\because\ {}^nC_r={}^nC_{n-r}\right)$

= coefficient of x^{20} in $(1+x)^{20}\cdot(1+x)^{20}$

= coefficient of x^{20} in $(1+x)^{40}={}^{40}C_{20}$

19. (a)

The given series, $\sum_{r=0}^{20}{}^{50-r}C_6$

$={}^{50}C_6+{}^{49}C_6+{}^{48}C_6+{}^{47}C_6+\ldots+{}^{32}C_6+{}^{31}C_6+{}^{30}C_6$

$=\left({}^{30}C_7+{}^{30}C_6\right)+{}^{31}C_6+{}^{32}C_6+\ldots+{}^{49}C_6+{}^{50}C_6-{}^{30}C_7$

$=\left({}^{31}C_7+{}^{31}C_6\right)+{}^{32}C_6+\ldots+{}^{49}C_6+{}^{50}C_6-{}^{30}C_7$

$=\left({}^{31}C_7+{}^{32}C_6\right)+\ldots+{}^{49}C_6+{}^{50}C_6-{}^{30}C_7$

………………………………………………

………………………………………………

………………………………………………

$={}^{51}C_7-{}^{30}C_7$

20. (b)

Given, ${}^{20}C_1+2^2\cdot{}^{20}C_2+3^2\cdot{}^{20}C_3+\ldots+20^2\cdot{}^{20}C_{20}$

$=A\left(2^{\beta}\right)$ Taking L.H.S., $=\sum_{r=1}^{20}r^2\cdot{}^{20}C_r=20\sum_{r=1}^{20}r\cdot{}^{19}C_{r-1}$

$=20\left[\sum_{r=1}^{20}(r-1)\,{}^{19}C_{r-1}+\sum_{r=1}^{20}{}^{19}C_{r-1}\right]$

$$=20\left[19\sum_{r=2}^{20}{}^{18}C_{r-2}+2^{19}\right]=20\left[19\cdot 2^{18}+2^{19}\right]=420\times 2^{18}$$

Now, compare it with R.H.S., A = 420 and $\beta=18$

21. (a)

Let $a=\left(\left(1+2x+3x^2\right)^6+\left(1-4x^2\right)^6\right)$

$\therefore$ Coefficient of x^2 in the expansion of the product $\left(2-x^2\right)\left(\left(1+2x+3x^2\right)^6+\left(1-4x^2\right)^6\right)$

= 2 (Coefficient of x^2 in a) – 1 (Constant of expansion)

In the expansion of $\left(\left(1+2x+3x^2\right)^6+\left(1-4x^2\right)^6\right)$.

Constant = 1 + 1 = 2

Coefficient of $x^2=$ [Coefficient of x^2 in $\left({}^6C_0\left(1+2x\right)^6\left(3x^2\right)^0\right)$]

+ [Coefficient of x^2 in $\left({}^6C_1\left(1+2x\right)^5\left(3x^2\right)^1\right)-\left[{}^6C_1\left(4x^2\right)\right]$

= 60 + 6 × 3 – 24 = 54

$\therefore$ The coefficient of x^2 in $\left(2-x^2\right)\left(\left(1+2x+3x^2\right)^6+\left(1-4x^2\right)^6\right)$

$=-2\times 54-1(2)=108-2=106$

22. (a)

We have $\left({}^{21}C_1+{}^{21}C_2....+{}^{21}C_{10}\right)-\left({}^{10}C_1+{}^{10}C_2....{}^{10}C_{10}\right)$

$=\frac{1}{2}\left[\left({}^{21}C_1+....+{}^{21}C_{10}\right)+\left({}^{21}C_{11}+....{}^{21}C_{20}\right)\right]-\left(2^{10}-1\right)$

$\left(\because\ {}^{10}C_1+{}^{10}C_2+....+{}^{10}C_{10}=2^{10}-1\right)$

$=\frac{1}{2}\left[2^{21}-2\right]-\left(2^{10}-1\right)=\left(2^{20}-1\right)-\left(2^{20}-1\right)=2^{20}-2^{10}$

23. (c)

We know that $\left(a+b\right)^n+\left(a-b\right)^n$

$=2\left[{}^nC_0\,a^n\,b^0+{}^nC_2\,a^{n-2}b^2+{}^nC_4a^{n-4}b^4...\right]$

$\left(1-2\sqrt{x}\right)^{50}+\left(1+2\sqrt{x}\right)^{50}$

$2\left[{}^{50}C_0+{}^{50}C_2\left(2\sqrt{x}\right)^2+{}^{50}C_4\left(2\sqrt{x}\right)^4...\right]$

$=2\left[{}^{50}C_0+{}^{50}C_2 2^2x+{}^{50}C_4 2^4x^2++...\right]$

Putting $x=1$, we get,

$${}^{50}C_0+{}^{50}C_2\,2^2+{}^{50}C_4 2^4....=\frac{3^{50}+1}{2}$$

24. (24)

Given binomial expression is

$\left(3+6x\right)^n = {}^nC_0 3^n + {}^nC_1 3^{n-1}\left(6x\right)^1 + ...$

General term is shown below.

$T_{r+1} {}^nC_r 3^{n-r}.\left(6x\right) r = {}^nC_r 3^{n-r}.6^r.x^r$

$= {}^nC_r 3^{n-r}.3^r.2^r.\left(\frac{3}{2}\right)^r = {}^nC_r 3^n.3^r \qquad \left[\text{for } x = \frac{3}{2}\right]$

T_9 is greatest of $x = \frac{3}{2}$

So, $T_9 > T_{10}$ and $T_9 > T_8$

Here, $\frac{T_9}{T_{10}} > 1$ and $\frac{T_9}{T_8} > 1 \Rightarrow \frac{{}^nC_8 3^n.3^8}{{}^nC_9 3^n.3^9} > 1$ and $\frac{{}^nC_8 3^n.3^8}{{}^nC_7 3^n.3^7} > 1$

So, $\frac{{}^nC_8}{{}^nC_7} > \frac{1}{3}$ and $\frac{n-7}{8} > \frac{1}{3} \Rightarrow \frac{29}{3} < n < 11 \Rightarrow n = 10 = n_0$

So, in $\left(3+6x\right)^n$ for $n = n_0 = 10$

Now, Take $\left(3+6x\right)^{10}$, here $T_{r+1} = {}^{10}C_r\ 310 - r\ 6^r\ x^r$

$T_7 = {}^{10}C_6 3^4.6^6.x^6 = 210.3^{10}.2^6 x^6$

$T_4 = {}^{10}C_6 3^7.6^3.x^3 = 120.3^{10}.2^3 x^3$

Ratio of coefficients of x^6 and coefficient of $x^3 = k \therefore k = \frac{210.3^{10}.2^6}{120.3^{10}.2^3} = \frac{7}{4} \times 2^3 = 14$

Therefore, $k = n_0 = 14 + 10 = 24$.

25. (23)

Given expression is $\left(1+x\right)^p\left(1-x\right)^q$ and coefficients of x and x^2 is –3 and –5 respectively.

$\left(1+x\right)^p\left(1-x\right)^q = \left(1 + {}^pC_1 x + {}^pC_2 x^2 ...\right)\left(1 - {}^qC_1 x + {}^qC_2 x^2 ...\right)$

According to questions,

$-{}^qC_1 + {}^pC_1 = -3 \Rightarrow p - q = -3 \qquad ...(\text{i})$

${}^qC_2 + {}^pC_2 - {}^pC_1 {}^qC_2 = -5$

$\frac{q\left(q-1\right)}{2} - pq + \frac{p\left(p-1\right)}{2} = -5$

$q^2 - q - 2pq + p^2 - p = -10$

$\left(p-q\right)^2 - \left(p+q\right) = -10$

From (i),

$\left(-3\right)^2 - \left(p+q\right) = -10$

$9 + 10 = \left(p+q\right) = 19$

Add (i) & (ii), p = 8, q = 11.

Coefficient of $x^3 = (1+x)^8 (1-x)^{11} = \left(1-x^2\right)^8 (1-x)^3$

$= 1\times(-1)+(-8)\times(-3) = -1+24 = 23$

26. (5)

$T_{r+1} = (-1)^r \cdot {}^{15}C_r . 2^{15-r} x^{\frac{15-2r}{5}} \Rightarrow m = {}^{15}C_{10} 2^5$

For coefficient of x^{-1}

$\frac{15-r}{5} - \frac{r}{5} = -1 \Rightarrow r = 10 \Rightarrow n = -1$

Given, $mn^2 = {}^{15}C_5 2^5$

27. (83)

Given binomial expansion is $\left(2x^3 + \frac{3}{x}\right)^{10}$.

$T_{r+1} = {}^{10}C_r \left(2x^3\right)^{10-r} \left(\frac{3}{x}\right)^r = 10C_r 2^{10-r} 3^r \ x^{30-4r}$

Put $r = 0, 1, 2, ... 7$

$= {}^{10}C_0 2^{10} 3^0 + {}^{10}C_1 2^9 3 + {}^{10}C_2 2^8 3^2 + ... + {}^{10}C_{10} 2^0 3^{10} - \left({}^{10}C_8 2^2 3^8 + {}^{10}C_9 \cdot 2 \cdot 3^9 + {}^{10}C_0 3^{10}\right)$

Use $(a+b)^n$ expansion,

$= (2+3)^{10} - \left(3\times5\times4\times3^9 + 2\times5\times2\times3^9 + 3\cdot3^9\right)$

$= (5)^{10} - 3^9 (60+20+3) = 5^{10} - 8^3 9^3$

Compare with given equation.

Then, $B = 83$.

28. (13)

$T_{r+1} = {}^{22}C_r \cdot \left(x^m\right)^{22-r} \cdot \left(\frac{1}{x^2}\right)^r$

$T_{r+1} = {}^{22}C_r \cdot x^{22m-mr-2r}$

$\because\ 22m - mr - 2r = 1 \Rightarrow r = \frac{22m-1}{m+2} \Rightarrow r = 22 - \frac{3\cdot3\cdot5}{m+2}$

So, possible value of m = 1, 3, 7, 13, 43 $(\because\ r \in W)$

But ${}^{22}C_r = 1540$

$\therefore$ Only possible value of m = 13.

29. (84)

$$\frac{T_5}{T_{n-3}} = \frac{{}^nC_4 \left(2^{1/4}\right)^{n-4} \left(3^{-1/4}\right)^4}{{}^nC_{n-4} \left(2^{1/4}\right)^4 (3-1/4)^{n-4}} = \frac{\sqrt[4]{6}}{1}$$

$$\Rightarrow\ (6)^{\frac{n-8}{4}} = 6^{1/4} \Rightarrow\ 6^{n-8} = 6$$

$$\Rightarrow n-8=1 \Rightarrow n=9$$

$$T_6 = {}^9C_5\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^5 = \frac{84}{\sqrt[4]{3}}$$

$$\therefore\ \alpha=84$$

30. (57)

Coefficient of middle term

${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P.

$$2(-6\beta) = {}^4C_2\frac{\beta^2}{6} - {}^6C_3\frac{\beta^3}{8}$$

$$\beta^2 - \frac{5}{2}\beta h3 = -12\beta$$

$$\beta = \frac{12}{5} \text{ or } \beta = -2$$

$$\therefore\ \beta = \frac{12}{5}$$

Common difference

$$d = \frac{72}{5} - \frac{144}{25} = -\frac{504}{25}$$

$$\therefore\ 50 - \frac{2d}{\beta^2} = 57$$

31. (924)

Sum of coefficient of $(x+y)^n = 2^n \Rightarrow 2^n = 4096$

$$\Rightarrow\ 2^n = 2^{12} \Rightarrow n=12$$

Greatest coefficient $= {}^{12}C_6 = \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}$

$$=11\times3\times4\times7=924$$

32. (1)

Middle term of $(1+x)^{20} = T_{11}$

$\therefore\ T_{11} = T_{10+1} = {}^{20}C_{10}x^{10}$

Coefficient $= {}^{20}C_{10}$

Middle terms of $(1+x)^{19} = T_{10}$ and T_{11}

$\therefore\ T_{10} = T_{9+1} = {}^{19}C_{10}x^{10}$ and $T_{11} = T_{10+1} = {}^{19}C_{10}x^{9}$

Sum of coefficient $= {}^{19}C_9 + {}^{19}C_{10} = {}^{20}C_{10}$

So, required ratio $= \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$.

33. (99)

From given expressional $1+\left(1+2^{49}\right)\left(2^{49}-1\right)=2^{98}$

$\Rightarrow\ m=1, n=98 \Rightarrow\ m+n=99$

34. (102)

$${}^{40}C_0 + {}^{41}C_1 + {}^{41}C_2 + ... + {}^{59}C_{19} + {}^{60}C_{20}$$

$$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + ... + {}^{60}C_{40}$$

35. (286)

Given expansion is

$$(1+x)^{10} = C_0 + C_1x + C_2x^2 + ... + C_{10}x^{10}$$

Differentiating

$$10(1+x)^9 = C_1 + 2C_2x + 3C_3x^2 + ... + 10C_{10}x^9$$

Replace x by x^2

$$10\left(1+x^2\right)^9 = C_1 + 2C_2x^2 + 3C_3x^4 + ... + 10C_{10}x^{18}$$

$$10x\left(1+x^2\right)^9 = C_1x + 2C_2x^3 + 3C_3x^5 + ... + 10C_{10}x^{19}$$

Differentiate w.r.t. x.

$$10\left(\left(1+x^2\right)^9.1 + x.9\left(1+x^2\right)^8 2x\right)$$

$$= C_1x + 2X_2.3x^3 + 3.5.C_3x^4 + ... + 10.19C_{10}x^{18}$$

Put $x=1$,

$$10\left(2^9 + 18.2^8\right)$$

$$= C_1 + 3.2.C_2 + 5.3.C_3 + ... + 19.10C_{10}$$

$$C_1 + 3.2.C_2 + ... + 19.10C_{10} = 10.2^9.10 = 100.^{2^9}$$

Take, $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_9}{11} + \frac{C_{10}}{11} = \frac{2^{11}-1}{11}$

$\uparrow \quad \uparrow$

10th term 11th term

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_9}{11} = \frac{2^{11}-2}{11}$$

Now, $100.2^9 = \frac{\alpha.2^{11}}{2^{\beta}-1}\left(\frac{2^{11}-2}{11}\right)$;

$$25.2^{11} = \frac{\alpha.2^{11}}{2^{\beta}-1}\cdot\left(\frac{2^{11}-2}{11}\right)$$

Compare the above equation,

$\alpha = 25\times 11 = 275$ & $\beta = 11 \Rightarrow \alpha+\beta = 275+11 = 286$

36. (315)

General term:

$$\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha}\left(2b\right)^{\beta}.\left(4ab\right)^{\gamma}=\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha+\gamma}b^{\beta+\gamma}.2^{\beta}.4^{\gamma}$$

$\Rightarrow \quad \alpha+\beta+\gamma=10 \qquad (i)$

$\alpha+\gamma=7 \qquad (ii)$

$\beta+\gamma=8 \qquad (iii)$

Solving equations (i), (ii) and (iii), we get

$\gamma=5, \alpha=2, \beta=3$

So, coefficients $=\frac{10!}{2!3!5!}2^{3}.2^{10}$

$$=\frac{10\times9\times8\times7\times6\times5!}{2\times3\times2\times5!}\times2^{13}=315\times2^{16}\Rightarrow k=315$$