SOLUTIONS

1. (B)

Looking, we see that the area of $[\Delta EBA]$ is 16 and the area of $[\Delta ABC]$ is 12.

Set the area of $[\Delta ADB]$ to be x.

We want to find $[\Delta ADE]-[\Delta CDB]$.

So, that would be $[\Delta EBA]-[\Delta ADB]=16-x$ and $[\Delta ABC]-[\Delta ADB]=12-x$

Therefore, $[\Delta ADE]-[\Delta DBC]=(16-x)-(12-x)=16-x-12+x=4$

2. (B)

Let the lengths of the vertical sides of the shaded trapezium be p and q.

Using similar triangles, $\frac{p}{3}=\frac{q}{3+5}=\frac{8}{3+5+8}$.

Hence $p=\frac{3}{2}$ and $q=4$.

Therefore the area of the trapezium is $\frac{1}{2}\times\left(\frac{3}{2}+4\right)\times 5$ that is $\frac{55}{4}\text{cm}^2$.

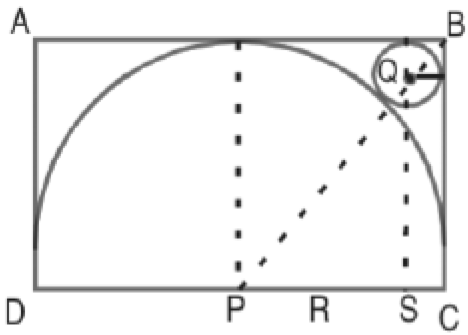
3. (C)

Let v be the length of the smallest piece.

Then the middle piece has length $\frac{3}{2}v$ and the largest had length $\frac{3}{2}\times\frac{3}{2}v=\frac{9}{4}v$.

So the total length is $\frac{9}{4}v+\frac{3}{2}v+v=\frac{19}{4}v$ and hence v = 20 m and the longest piece has length $\frac{9}{4}v$, that is 45 m.

4. (D)



Let radius of the semicircle be R and radius of the circle be r.
Let P be the centre of semicircle and Q be the centre of the circle.
Draw QS parallel to BC.
Now, $\Delta PQS \sim \Delta PBC$

$\therefore \frac{PQ}{PB}=\frac{QS}{BC}$

$\Rightarrow \frac{R+r}{\sqrt{2}R}=\frac{R-r}{R}$

$\Rightarrow R+r=\sqrt{2}R-\sqrt{2}r$

$\Rightarrow r(1+\sqrt{2})=R(\sqrt{2}-1)$

$$\Rightarrow\ r = R\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}\times\frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$$

$$\Rightarrow\ r = R(\sqrt{2}-1)^2$$

$$\text{Required Ratio } = \frac{\pi r^2}{\pi R^2}\times 2$$

$$= \frac{\pi R^2(\sqrt{2}-1)^4\times 2}{\pi R^2}$$

$$= 2(\sqrt{2}-1)^4 : 1$$

5. (D)

You can do this by the method of simulation. For eg.

Let the three numbers be 1, 3 & 5. So option (A) is $1^2\ 3^2\ 5^2 = 225$,

which is odd. (B) is $3(1^2+3^3)5^2 = 2100$, which is even. $5+3+5^4 = 633$,

which is odd. (D) is $5^2\frac{\left(1^4+3^4\right)}{2} = 1025$, which is not even and hence, the answer.

6. (A)

50 numbers are divisible by 2, 33 numbers are divisible by 3 and 20 numbers are divisible by 5.
3 numbers are divisible by all 2, 3 and 5.
16 numbers are divisible by both 2 and 3, therefore 13 numbers are divisible by 2 and 3 but not by 5.
10 numbers are divisible by both 2 and 5, therefore 7 numbers are divisible by 2 and 5 but not by 3.
6 numbers are divisible by both 3 and 5, therefore 3 numbers are divisible by 3 and 5 but not by 2.
Total number of numbers that are divisible by one or more among 2, 3 and 5
= 27 + 14 + 7 + 13 + 3 + 7 + 3 = 74
Hence, the required number = 100 – 74 = 26.

7. (B)

Number of 2 in the product of all integers form 1 to 100

$$= \frac{100}{2}+\frac{100}{4}+\frac{100}{8}+\frac{100}{16}+\frac{100}{32}+\frac{100}{64}$$

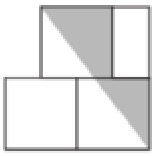
= 50 + 25 + 12 + 6 + 3 + 1 = 97 and

number of 5 in the product of all integers from 1 to 100 $= \frac{100}{5}+\frac{100}{25} = 20+4 = 24$

Hence, number of zeroes at the end = Lowest of the (number of 2, number of 5) = 24.

8. (C)

The centre of the top square is directly above the common edge of the lower two squares. Hence a rectangle half the size of the square, and so of area $\frac{1}{2}\text{cm}^2$, can be added to the diagram to form a right-angled triangle as shown. The area of the shaded region and the added rectangle is equal to $\left(\frac{1}{2}\times 2\times 1\frac{1}{2}\right)\text{cm}^2 = 1\frac{1}{2}\text{cm}^2$.



9. (D)

The number of terms of the series form the sum of first n natural numbers i.e. $\frac{n(n+1)}{2}$.

Thus the first 23 letters will account for the first $\frac{23\times 24}{2}=276$ terms of the series.

The 288$^{\text{th}}$ term will be the 24$^{\text{th}}$ letter which is x.

10. (B)

To maximise the value of the wealth, we must carry more of the one whose value per kilogram is more.

Value per kilogram of ruby $=\left(\frac{4}{0.3}\right)=\text{Rs.}13.33\text{ crore}$,

and value per rupee of each emerald

$=\left(\frac{5}{0.4}\right)=\text{Rs.}12.5\text{ crore}$.

It is obvious that we should carry entire 12 kg of ruby.

This would amount to $\left(\frac{12}{0.3}\right)=40\text{ rubies}$.

11. (B)

$(a+b+c)^2=4$

$a^2+b^2+c^2+2(ab+bc+ca)=4$

$\therefore\quad ab+bc+ca=\frac{3}{2}$

$\therefore\quad \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{ab+bc+ca}{abc}=\frac{1}{2}$

12. (C)

$a^3-a^3+a+2=0 \quad\Rightarrow\quad a=-2$

13. (B)

$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}+3-3$

$=\frac{a+b+c}{b+c}+\frac{b+c+a}{c+a}+\frac{a+b+c}{a+b}-3$

$=(a+b+c)\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right)-3$

$=3\left(\frac{10}{3}\right)-3=7$

14. (B)

$N=2^{64}$

$\therefore\ \lambda=64$

15. (C)

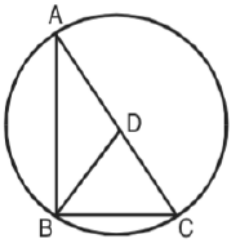
$x^3+y^3+(-1)^3=-3xy$

$\therefore\ x+y-1=0$ or $x=y=-1$

$\therefore\ |x+y|=2$ Ans.

16. (8)
89 is a larger number to be the sum of the digits of a ten digit number.
In fact, the largest possible digital sum is 10×9 or 90.
Since 89 is only 1 less than 90, the number in question must be composed of nine 9's and one 8.
In order that the number be divisible by 2, the last digit must be 8.

17. (3)



In a right-angled triangle, the length median to the hypotenuse is half the length of the hypotenuse. Hence, BD = $\frac{1}{2}$ AC = 3 cm. This relationship can be verified by knowing that the diameter of a circle subtends a right angle at the circumference e.g. in the above figure D is the centre of the circle with AC as diameter. Hence, $\angle ABC$ should be 90°. So BD should be the median to the hypotenuse. Thus, we can see that BD = AD = CD = Radius of this circle.

Hence, $BD=\frac{1}{2}$ diameter $=\frac{1}{2}AC=\frac{1}{2}\times$hypotenuse.