PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / HYDERABAD / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / PATNA / BARAMATI

IIT – JEE: 2026 **MAJOR TEST - 2** **DATE: 19/10/24**

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	C	**31.**	C	**61.**	C
2.	C	**32.**	B	**62.**	B
3.	B	**33.**	B	**63.**	B
4.	B	**34.**	B	**64.**	A
5.	D	**35.**	C	**65.**	A
6.	B	**36.**	C	**66.**	C
7.	A	**37.**	C	**67.**	B
8.	A	**38.**	B	**68.**	B
9.	A	**39.**	B	**69.**	B
10.	A	**40.**	A	**70.**	C
11.	A	**41.**	D	**71.**	A
12.	A	**42.**	A	**72.**	D
13.	C	**43.**	B	**73.**	A
14.	D	**44.**	D	**74.**	A
15.	C	**45.**	B	**75.**	B
16.	B	**46.**	C	**76.**	A
17.	B	**47.**	C	**77.**	A
18.	D	**48.**	B	**78.**	C
19.	A	**49.**	A	**79.**	D
20.	B	**50.**	C	**80.**	A
21.	10	**51.**	5	**81.**	2
22.	3	**52.**	3	**82.**	10
23.	10	**53.**	2	**83.**	0
24.	20	**54.**	3	**84.**	4
25.	2	**55.**	3	**85.**	1
26.	5	**56.**	7	**86.**	5
27.	2	**57.**	200	**87.**	4
28.	10	**58.**	9	**88.**	0
29.	2	**59.**	7	**89.**	5
30.	5	**60.**	8120	**90.**	4

PART (A) : PHYSICS

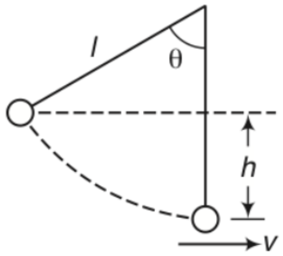
1. (C)

At $\theta = 180°, |\Delta \mathbf{P}| = 2mv =$ maximum

At $\theta = 360°, |\Delta \mathbf{P}| = 0 =$ minimum

2. (C)

$h = l - l\cos\theta = l(1-\cos\theta)$



$$v^2 = 2gh = 2gl(1-\cos\theta) = v_{max}^2$$

$$\therefore \quad K_{max} = \frac{1}{2}mv^2 = mgl(1-\cos\theta)$$

3. (B)

$\frac{v^2}{R} = a_t = a$ (Here, $a_t = a$ say)

or $\frac{(at)^2}{R} = a$

$$\therefore \quad t = \sqrt{\frac{R}{a}} = \sqrt{\frac{20}{5}} = 2\text{ s}$$

4. (B)

As, initial velocity $\mathbf{v} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ m/s

For projectile motion along horizontal direction,

$x = t$ .....(i)

Motion along vertical direction,

$$y = ut - \frac{1}{2}gt^2$$

where, u is a vertical component of velocity

$y = 2t - \frac{1}{2}10t^2$ .....(ii)

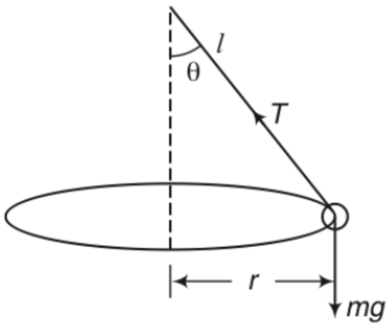
From Eqs. (i) and (ii), we get

$$y = 2x - 5x^2$$

5. (D)

$T\cos\theta = mg$ .....(i)

$T\sin\theta = mr\omega^2 = m(l\sin\theta)\omega^2$ .....(ii)



Solving these two equations we get,

$$\cos\theta = \frac{g}{l\omega^2} = \frac{g}{l(2\pi n)^2}$$

$$= \frac{10}{\left[2\pi\times\frac{2}{\pi}\right]^2} \qquad (l = 1\,\text{m})$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{5}{8}\right)$$

6. (B)

$$\frac{d_B}{d_C} = \frac{v_B t}{v_C t} = \frac{2.5}{2} = \frac{5}{4}$$

7. (A)

Decrease in gravitational potential energy of block = increases in spring potential energy

$$\therefore \quad mg\left(x_m \sin\theta\right) = \frac{1}{2}K\,x_m^2$$

$$\therefore \quad x_m = \frac{2mg\sin\theta}{K}$$

8. (A)

$$\mathbf{v} = \frac{d\,\mathbf{S}}{dt} = (4t)\,\hat{\mathbf{i}}$$

$$P = \mathbf{F}\cdot\mathbf{v} = 12t^2$$

$$\therefore \quad W = \int_0^2 Pdt = \int_0^2 (12)t^2\,dt = 4t^3\Big|_0^2$$

$$= 32 \text{ J}$$

9. (A)

Decrease in potential energy = Work done against friction

$$\therefore \quad mg\left(h+d\right) = F\cdot d$$

Here F = average resistance

$$\Rightarrow \quad F = mg\left(1+\frac{h}{d}\right)$$

10. (A)

$$F = kx \Rightarrow x = \frac{F}{k}$$

Now, $U = \frac{1}{2}kx^2$

$$= \frac{1}{2}k\left(\frac{F}{k}\right)^2$$

or $U \propto \frac{1}{k}$

k_B is double. Therefore, U_B will be half.

11. (A)

$$K_f - K_i = W = \int F dx$$

$$\therefore \quad K_f = K_i + \int_{20}^{30} (-0.1\,x)\,dx$$

$$= \frac{1}{2}\times 10 \times (10^2) - \left[0.1\frac{x^2}{2}\right]_{20}^{30}$$

= 475 J

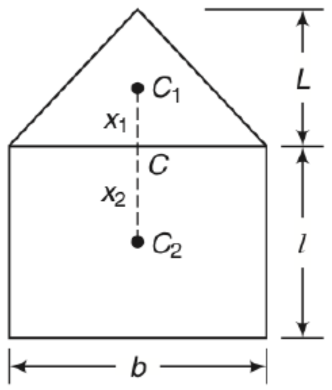
12. (A)

$$E_i = E_f$$

$$0 = m_2gh - m_1gh + \frac{1}{2}(m_1 + m_2)v^2$$

$$\therefore \quad v = \sqrt{2gh\left(\frac{m_1 - m_2}{m_1 + m_2}\right)}$$

13. (C)



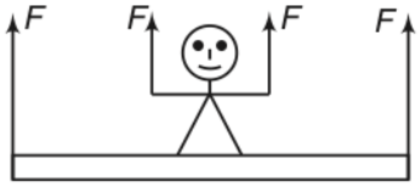
$$A_1x_1 = A_2x_2$$

$$\therefore \quad \frac{1}{2}(bL)\left(\frac{L}{3}\right) = (bl)\left(\frac{l}{2}\right)$$

$$\therefore \quad l = \frac{L}{\sqrt{3}}$$

14. (D)

Total upward force $= 4\left(\frac{mg}{2}\right) = 2\,mg$



Total downward force = $(m + m)\,g = 2\,mg$

$\therefore$ Net force = 0

15. (C)

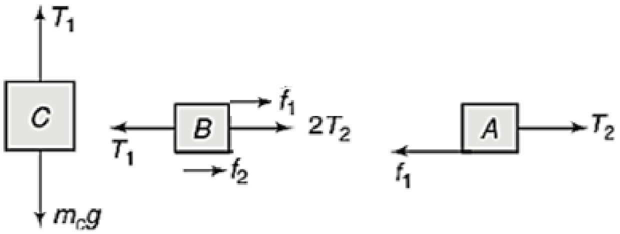
Maximum value of friction between A and B

$$(f_1)_{max} = \mu N_1 = \mu m_A g$$
$$= 0.3 \times 50 \times 10$$
$$= 150\text{ N}$$

Maximum value of friction between B and ground

$$(f_2)_{max} = \mu N_2 = \mu(m_A + m_B)g$$
$$= (0.3)(120)(10) = 360\text{ N}$$

Force diagram is as shown below



$$T_2 = (f_1)_{max} = 150\text{ N}$$
$$T_1 = 2T_2 + (f_1)_{max} + (f_2)_{max}$$
$$= 300 + 150 + 360 = 810\text{ N}$$
$$T_1 = m_c g$$
$$\therefore \quad m_c = \frac{T_1}{g} = \frac{810}{10} = 81\text{ kg}$$

16. (B)

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (8\hat{\mathbf{i}} - 4t\,\hat{\mathbf{j}})$$

At 1**s** $\quad \mathbf{F}_{net} = m\mathbf{a} = (1)(8\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = (8\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$

$$= \mathbf{W} + \mathbf{F}$$

Where, **F** = force on cube

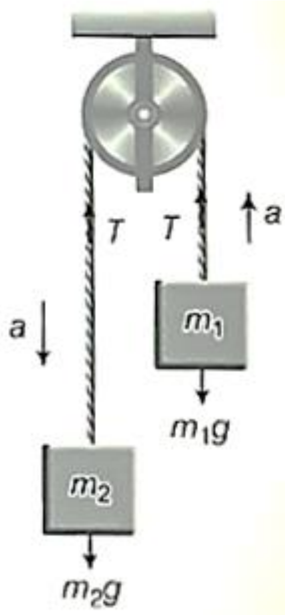
$$\therefore \quad \mathbf{F} = (8\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) - \mathbf{w}$$
$$= (8\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) - (-10\hat{\mathbf{j}})$$
$$= (8\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$$

or $\quad |\mathbf{F}| = \sqrt{(8)^2 + (6)^2}$

$$= 10\text{ N}$$

17. (B)

As the string is inextensible, both masses have the same acceleration a. also, the pulley is massless and frictionless, hence the tension at both ends of the string is the same. Suppose, the mass m_2 is greater than mass m_1, of the heavier mass m_2 is acceleration downward and the lighter mass m_1 is accelerating upwards.



Therefore, by Newton's 2nd law

$$T - m_1 g = m_1 a \quad \text{...(i)}$$

and $$m_2 g - T = m_2 a \quad \text{...(ii)}$$

After solving Eqs. (i) and (ii), we get

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} \cdot g = \frac{g}{8} \quad \text{[given]}$$

So, $$\frac{g}{8} = \frac{m_2(1 - m_1/m_2)}{m_2(1 + m_1/m_2)} \cdot g \quad \text{...(iii)}$$

Let $\frac{m_1}{m_2} = x$

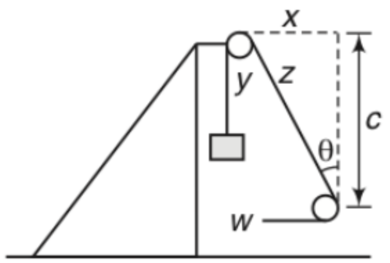
Thus, Eq. (iii) becomes

$$\frac{1-x}{1+x} = \frac{1}{8} \text{ or } x = \frac{7}{9} \text{ or } \frac{m_2}{m_1} = \frac{9}{7}$$

So, the ratio of the masses is 9:7.

18. (D)

$$z = \sqrt{x^2 + c^2}$$



Now, $w + y + z = l$

or $$w + y + \sqrt{x^2 + c^2} = l$$

$$\therefore \frac{dw}{dt} + \frac{dy}{dt} + \frac{x}{\sqrt{x^2 + c^2}} \cdot \frac{dx}{dt} = 0$$

or $$\left(-\frac{dw}{dt}\right) + \frac{x}{z}\left(-\frac{dx}{dt}\right) = \frac{dy}{dt} \quad \text{.....(i)}$$

$$-\frac{dw}{dt} = -\frac{dx}{dt} = v_2$$

$$\frac{dy}{dt} = v_1$$

and $\frac{x}{z} = \sin\theta$

Substituting these values in Eq. (i) we have

$v_2(1+\sin\theta)=v_1$

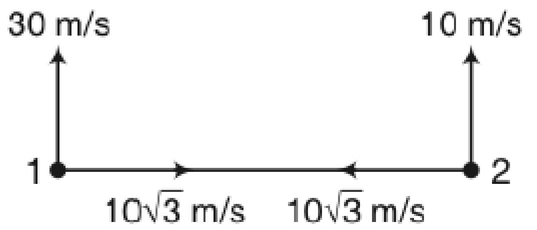
19. (A)

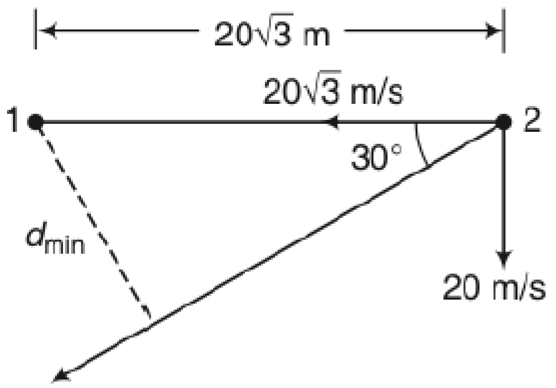
$R=\frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha+\beta)+\sin\beta\right]$

$u=10\text{ m/s},\ g=10\ \text{m/s}^2,\ \beta=30°$ and $\alpha=60°$

20. (B)

$a_1=a_2=g$ (downwards)





∴ $a_{12}=0$

∴ Relative motion between them is uniform.

Relative velocity v_{21}

$d_{min}=20\sqrt{3}\sin 30°$

$=10\sqrt{3}\text{ m}$

21. (10)

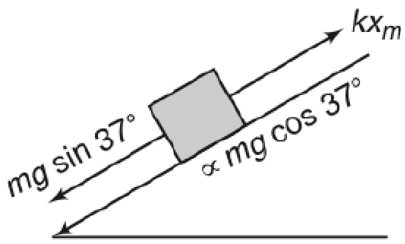
$a_t=8\text{ m/s}^2$

$a_r=\frac{v^2}{R}=6$

$a=\sqrt{a_t^2+a_r^2}=10$

22. (3)

Let X_m is maximum extension of spring. Then decreases in potential energy of M = increase in elastic potential energy of spring



$\therefore \quad \frac{1}{2}kX_m^2 = MgX_m$

or $\quad X_m = \frac{2Mg}{k}$

Now, $kX_m = mg\sin 37° + \mu mg\cos 37°$

or $\quad 2Mg = (mg)\left(\frac{3}{5}\right) + \left(\frac{3}{4}\right)mg\left(\frac{4}{5}\right)$

$\therefore \quad M = \frac{3}{5}m$

23. (10)
If speed of block of 1.0 kg is 1 m/s then speed of 4.0 kg block at this instant would be 2 m/s.
Applying,

$E_i - E_f =$ work done against friction

$\therefore \quad 0 - \left[\frac{1}{2}\times 1.0\times(1)^2 + \frac{1}{2}\times 4.0\times(2)^2 - 1\times 10\times 1\right] = \mu_k \times 4\times 10\times(2)$

Solving this equation we get,

$\mu_k = 0.12$

24. (20)
Maximum speed with which the boy can throw stone is

$u = \sqrt{2gh} = \sqrt{2\times 10\times 10} = 10\sqrt{2}$ m/s

Range is maximum when projectile is thrown at an angle of $45°$.

Thus, $R_{max} = \frac{u^2}{g} = \frac{(10\sqrt{2})^2}{10} = 20$ m

25. (2)

$x_{CM} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$

$= \frac{(ab)(0) - \left(\frac{ab}{4}\right)\left(\frac{a}{4}\right)}{ab - \frac{ab}{4}}$

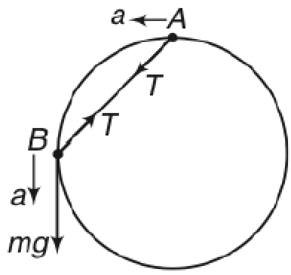
$= -\frac{a}{12}$

$y_{CM} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$

$$= \frac{(ab)(0) - \left(\frac{ab}{4}\right)\left(\frac{b}{4}\right)}{ab - \frac{ab}{4}}$$

$$= -\frac{b}{12}$$

26. (5)



For A $T\cos 45° = ma$

or $T = \sqrt{2}\, ma$ ...(i)

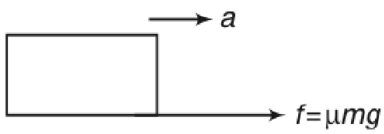
For B $mg - T\cos 45° = ma$

$\therefore \quad mg - ma = ma$ or $a = \frac{g}{2}$

Substituting in Eq. (i), we get $T = \frac{mg}{\sqrt{2}}$

27. (2)

$a = \frac{f}{m} = \propto g = 3\ \text{m/s}^2$



Relative motion will stop when velocity of block also becomes 6 m/s by the above acceleration.

$v = at$

$\therefore \quad t = \frac{v}{a} = \frac{6}{3} = 2\ \text{s}$

28. (10)

It is just like a projectile motion with g to be replaced by $g\sin 45°$.

After 2 s,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{\left(10\sin 45° - \frac{g}{\sqrt{2}\times 2}\right) + (10\cos 45°)^2}$$

= 10 m/s

29. (2)

Let v_x and v_y be the components of v_0 along x and y directions.

$(v_x)(2) = 2$

$\therefore \quad v_x = 1\,\text{m/s}$

$v_y(2) = 10$

or $\quad v_y = 5\,\text{m/s}$

$v_0 = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{26}\ \text{m/s}$

30. (5)

Time of descent $t = \sqrt{\dfrac{2H}{g}} = \sqrt{\dfrac{2\times 500}{10}}$

$= 10\ \text{s}$

Now $v_x = ay = 3y$

or $\quad \dfrac{dx}{dt} = 3\left(\dfrac{1}{2}gt^2\right) = 15t^2$

$\therefore \quad \int_0^x dx = 15\int_0^t t^2 dt \Rightarrow x = 15\left(\dfrac{t^3}{3}\right) = 5000$

PART (B) : CHEMISTRY

31. (C)

$Ag^{+} \rightarrow$ Pseudo inert gas configuration cation.

32. (B)

33. (B)

$$\begin{array}{ccccccccc} H_2C & = & CH & - & C & \equiv & C & - & H \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ sp^2 & & sp^2 & & sp & & sp & & \end{array}$$

34. (B)

$O_2 \quad \frac{BO}{2}$

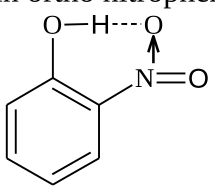
(a) O_2^{+} 2.5 (b) If the e^{-} is lost from ABMO bond energy $\uparrow$

O_2^{-} 1.5

(d) BO $\uparrow$ BS $\uparrow$ BL $\downarrow$

35. (C)

In ortho nitrophenol we observe intramolecular hydrogen bonding



36. (C)

For all irreversible processes

$w - P_{ext}(v_2 - v_1)$

PV^{n} = constant (is valid for reversible adiabatic)

37. (C)

Process AB and CD – isobaric

Work done $= -P\Delta V = -nR\Delta T$

$WAB = nR(100)$

$WCD = -nR(100)$

Process BC and DA $\Rightarrow$ isothermal

Work done $= nRT \ln \frac{P_1}{P_2}$

$W_{BC} = nRT \ln \frac{1}{2} = -400R \ln \frac{1}{2}$

$W_{\Delta A} = nRT\ln 2 = 300R\ln 2$

Total work done $= 100nR - 100nR + 400R\ln 2 - 300R\ln 2$

38. (B)

Given : Mol fraction of $H_2O = 0.85$

If total number of moles of H_2SO_4 & H_2O in H_2SO_4 solution = 1

Then no. of moles of $H_2SO = (1-0.85) = 0.15$

Molality $= \dfrac{\text{No.of moles of } H_2SO_4}{\text{Mass of } H_2O(\text{in gm})} \times 1000$

$= \dfrac{0.15}{0.85 \times 10} \times 1000 = 9.8$

39. (B)

(A) Correct order $\rightarrow Ca^{2+} > K^+ > Cl^- > S^{2-}$ (ionisation energy)

For isoelectronic species (I.E. $\propto Z_{eff}$)

(B) Correct order $\rightarrow C < N < F < O(2^{nd}\text{ I.E.})$

Second electron removal from oxygen requires more energy as it acquires stable $2s^2 2p^3$ configuration after removal of one electron.

(C) Correct order $\rightarrow B > Tl > ln > Ga > Al$ (Electronegativity)

In general EN increases in boron family from top to bottom due to increase in Z_{eff} on valence shell while boron has highest E.N. due to its vary small size.

(D) Correct order $\rightarrow Na^+ > Li^+ > Mg^{2+} > Al^{3+} > Be^{2+}$ (Ionic radius) Ionic radius depends on Z_{eff} and number of shells.

40. (A)

s orbital has one subshell which means it has only one magnetic quantum number possible. $1 = 0$.

$m = -1$ is not possible for *s*-orbital

41. (D)

42. (A)

$m = +2$

d subshell

$n = 3 \rightarrow 3$ waves

43. (B)

None of these have trigonal Bipyramidal shape.

$SO_2 \rightarrow$ Bent

$SF_4 \rightarrow$ See saw

$ClF_3 \rightarrow$ T shaped

$BrF_5 \rightarrow$ square Pyramidal

$XeF_4 \rightarrow$ square planar.

44. (D)
Some number of neutron excess.

45. (B)
For unielectronic species;
$1s < 2s = 2p < 3s = 3p = 3d < 4s$

46. (C)
$A \rightarrow A^{+} + e$ (Ionisation energy should be low)
$B + e^{+} \rightarrow B^{-}$ (electron affinity should be high)
& for Ionic bond, cation should be large & anion should be small.

47. (C)
In F and Ca electronegativity difference is maximum. So they will form electrovalent bond.

48. (B)
$CO_3^{2-} \rightarrow BO = \frac{4}{3}$
$CO_2 \rightarrow BO = 2$
$HCOO^{\ominus} \rightarrow BO = 1.5$
$CO \rightarrow BO = 3$

49. (A)
Moles of Na_2SO_4 = Molarity × Volume (in 1)
$= 0.2 \times 0.5 = 0.1$
Moles of SO_4^{2-} from $Na_2SO_4 = 0.1$
Volume of $Al_2(SO_4)_3$ solution = 100 ml
$\frac{w}{v}\% = 17.1$
$\therefore$ 100 ml solution contains $= 17.1$ gm$Al_2(SO_4)_3$
Moles of $Al_2(SO_4)_3 = \frac{17.1}{342} = \frac{1}{20}$
No. of moles of SO_4^{-2} from $Al_2(SO_4)_3 = 3 \times \frac{1}{20} = \frac{3}{20}$
Total initial volume of both solutions $= 500 + 100 = 600$ ml
After dilution, volume of solution becomes $= 600 \times 5 = 3000$ ml
Molarity of SO_4^{2-} in final solution $= \frac{\text{Total moles of } SO_4^{-2}}{\text{Total volume of solution (in ml)}} \times 1000$

50. (C)

51. (5)
$NO \rightarrow 2.5$
$O_2^{-} \rightarrow 1.5$

$N_2 \rightarrow 3.0$

$H_2^{\oplus} \rightarrow 0.5$

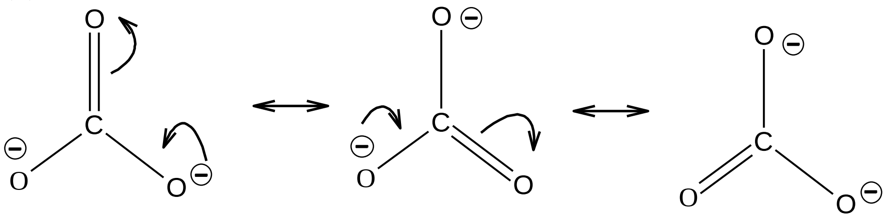
$CO \rightarrow 3.0$

$C_2^{2-} \rightarrow 3.0$

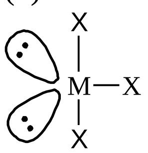
$O_2 \rightarrow 2.0$

$H_2^{-} \rightarrow 0.5$

52. (3)



53. (2)



54. (3)

Given that $\frac{P}{V} = \text{constant}$

$PV^{-1} = \text{constant}$

$\gamma = -1$

$$\frac{\Delta U}{w} = \frac{nC_v\Delta T}{\frac{nR\Delta T}{\gamma - 1}}$$

$$= -2\frac{C_v}{R}$$

$$= -2 \times \frac{3/2\,R}{R}$$

$$= -3$$

55. (3)

Let total number of moles = x

Moles of $N_2 = 0.6x$

Moles of $O_2 = 0.4x$

For the reaction :

	N_2	+ $2O_2 \rightarrow 2NO_2$
Moles	0.6x	0.4x
$\frac{\text{Moles}}{\text{S.Coeff}}$	$\frac{0.6x}{1}$	$\frac{0.4x}{2}$

O_2 is limiting reagent.

Number of moles of NO_2 produced $= \frac{2}{2} \times 0.4x$

Given : $0.4x = \frac{92}{46} \bar{A}x = 5$

Number of moles of $N_2 = 0.6 \times 5 = 3$

Ans is 3.

56. (7)

57. (200)

$w = -mRT \ln \frac{P_1}{P_2}$

$-4200 = \frac{-w}{40} \times 2 \times 300 \times \ln \frac{16}{4}$

$\therefore W = 200$ g

58. (9)

59. (7)

For $A \to B$

$600V_1^{\gamma-1} = 60V_2^{\gamma-1}$ $\left(\gamma = \frac{5}{3}\right)$

(Reversible adiabatic)

$\Rightarrow \;\; 600(V_1)^{2/3} = 60(V_2)^{2/3}$

$\Rightarrow \;\; 10 = \left(\frac{V_2}{V_1}\right)^{2/3}$

$\Rightarrow 10 = \left(\frac{V_2}{10}\right)^{2/3}$

$\Rightarrow V_2 = 10(10)^{3/2} = 10^{5/2}$

Now, $q_{net} = RT_2 \ln 10 = 60R \ln 10 = q_{AB} + q_{BC}$

$\because q_{AB} = 0$

$\Rightarrow q_{BC} = 60\ R\ \ln 10 = 60\ R \ln \frac{V_3}{V_2}$ $\quad$ [$\because B \to C$ is reversible isothermal]

$\Rightarrow 60R \ln 10 = 60\ R\ \ln\left(\frac{V_3}{10^{5/2}}\right)$

$\Rightarrow \log 10 = \log V_3 - \frac{5}{2}$

$\log V_3 = \frac{7}{2} \Rightarrow 2 \log V_3 = 7$

60. (8120)

$X \to Y$ is an isothermal process an ideal gas:

$\Delta H = 0$

$Y \rightarrow Z$ is an isochoric process

$\therefore\ w = 0$

$\Delta U = mC_{v,m}\left(T_2 - T_1\right)$

$= 5\times10\left(415-335\right)$

$= 4800\ J$

$\Delta H = \Delta U + \Delta\left(PV\right)$

$= \Delta U + nR\Delta T$

$= 4800+5\times8.3\times\left(415-335\right)$

$= 8120\ J$

PART (C) : MATHEMATICS

61. (C)

Since, $\sin\theta, \sqrt{2}(\sin\theta+1), 6\sin\theta+6$ are in GP.

$\therefore \quad 2(\sin\theta+1)^2 = \sin\theta(6\sin\theta+6)$

$\Rightarrow \quad 2\sin^2\theta+4\sin\theta+2 = 6\sin^2\theta+6\sin\theta$

$\Rightarrow \quad 4\sin^2\theta+2\sin\theta-2=0$

$\Rightarrow \quad 2\sin^2\theta+\sin\theta-1=0$

$(\sin\theta+1)(2\sin\theta-1)=0$

$\sin\theta \neq -1, \quad \therefore \sin\theta = \frac{1}{2}$

Then $\frac{1}{2}, \frac{3\sqrt{2}}{2}, 9, ...$ are in G.P.

Common ratio $= \dfrac{\frac{3\sqrt{2}}{2}}{\frac{1}{2}} = 3\sqrt{2}$

$\therefore$ Fifth term $= \frac{1}{2}(3\sqrt{2})^4 = \frac{324}{2} = 162$.

62. (B)

Since x, $|x+1|$, $|x-1|$ are in AP

$\therefore \quad 2|x+1| = x+|x-1|$

Case I : If $x<-1$

Then $-2(x+1) = x-(x-1)$

$\Rightarrow \quad -2x-2=1 \Rightarrow x=-\frac{3}{2}$

Then series $-\frac{3}{2}, \frac{1}{2}, \frac{5}{2}, ...$

$S_{20} = \frac{20}{2}\left[-3+(20-1)2\right]$

$=10(35)=350$

Case II : If $-1 \le x < 1$

Then $2(x+1) = x-(x-1)$

$\Rightarrow \quad 2x+2=1$

$\therefore \quad x=-\frac{1}{2}$

Then series $-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, ...$

$S_{20} = \frac{20}{2}\left[-1+(20-1)\cdot 1\right]$

$=10\times 18 = 180$.

Case III : If $x \geq 1$

Then $2(x+1) = x + (x-1)$

$\Rightarrow \quad 2 = -1$ (impossible)

Hence, sum is 180 or 350.

63. (B)

$$\because \; a_1 a_2 a_3 .... a_n = \prod_{r=1}^{n} a_r = \prod_{r=1}^{n} \frac{a_r b_r}{b_r}$$

$$= \prod_{r=1}^{n} \frac{b_{r-1}}{b_r}$$

$$= \frac{b_0}{b_1} \cdot \frac{b_1}{b_2} \cdot \frac{b_2}{b_3} \cdot .... \frac{b_{n-1}}{b_n} = \frac{b_0}{b_n}$$

$$= \frac{x-y}{b_n}$$

64. (A)

$$\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} = \sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^{n+1}C_{r+1}}$$

$$= \sum_{r=0}^{n-1} \frac{{}^nC_r}{\left(\frac{n+1}{r+1}\right) {}^nC_r}$$

$$= \sum_{r=0}^{n-1} \left(\frac{r+1}{n+1}\right)$$

$$= \frac{1+2+3+....+n}{(n+1)}$$

$$= \frac{n(n+1)}{2(n+1)} = \frac{n}{2}$$

65. (A)

$(1-2+3)^n = 128 \Rightarrow 2^n = 128$

$\therefore \quad n = 7$

Here, n is odd.

$\therefore$ Greatest coefficient $= {}^7C_3 = 35$

66. (C)

$3^{400} = (3^4)^{100}$

$= (81)^{100} = (1+80)^{100}$

$= 1 + {}^{100}C_1(80) + {}^{100}C_2(80)^2 + ... + {}^{100}C_{100}(80)^{100}$

$= 1 + 8000 +$ (last two digits in each term is 00)

$\therefore$ Last two digits = 01

67. (B)

We have,

$$|4\sin x - 1| < \sqrt{5}$$

$\Rightarrow$ $-\sqrt{5} < 4\sin x - 1 < \sqrt{5}$

$\Rightarrow$ $-\left(\frac{\sqrt{5}-1}{4}\right) < \sin x < \frac{(\sqrt{5}+1)}{4}$

$\Rightarrow$ $-\sin\frac{\pi}{10} < \sin x < \cos\frac{\pi}{5}$

$\Rightarrow$ $\sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$

$\Rightarrow$ $\sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{3\pi}{10}\right)$

$x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ $\quad \left\{\because\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right\}$

68. (B)

$\because$ $\cos(315\pi + x) = (-1)^{315}\cos x = -\cos x$

$\therefore$ $4\cos^3 x - 4\cos^2 x - \cos(315\pi + x) = 1$

or $4\cos^3 x - 4\cos^2 x + \cos x - 1 = 0$

$\Rightarrow$ $(4\cos^2 x + 1)(\cos x - 1) = 0$

$\cos x = 1,\ 4\cos^2 x + 1 \neq 0$

$\Rightarrow$ $\cos x = \cos 0$

$\therefore$ $x = 2n\pi,\ n \in I$

$\therefore$ $x = 2\pi, 4\pi, 6\pi, 8\pi, \ldots, 100\pi$ $\quad (\because\ 0 < x < 315)$

$(ie,\ 100\pi < 315 < 101\pi)$

$\therefore$ Required arithmetic mean

$$= \frac{2\pi + 4\pi + 6\pi + 8\pi + \ldots + 100\pi}{50}$$

$$= \frac{2\pi(1 + 2 + 3 + 4 + \ldots + 50)}{50}$$

$$= \frac{2\pi \cdot \frac{50}{2} \cdot 51}{50} = 51\pi$$

69. (B)

If $|f(x) + g(x)| = |f(x)| + |g(x)|$

$\Leftrightarrow$ $f(x)\cdot g(x) \geq 0$

$\therefore$ $\tan x \sec x \geq 0$

$\Rightarrow$ $\frac{\sin x}{\cos^2 x} \geq 0$

$\Rightarrow$ $\sin x \geq 0$

But $\cos x \neq 0$

$\therefore \quad x \in [0, \pi] \sim \left\{\frac{\pi}{2}\right\}$

or $\quad x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

70. (C)

$5^{\frac{1}{4}(\log_5^2 x)} \geq 5x^{\frac{1}{5}(\log_5 x)}$

Taking logarithm on base 5, then

$$\left(\frac{1}{4}\right)(\log_5^2 x) \geq 1 + \frac{1}{5}(\log_5 x)(\log_5 x)$$

$\Rightarrow \quad \frac{1}{20}\log_5^2 x \geq 1$

or $\quad (\log_5 x)^2 \geq 20$

or $\quad \log_5 x \geq 2\sqrt{5}$ and $\log_5 x \leq -2\sqrt{5}$

or $\quad x \geq 5^{2\sqrt{5}}$ and $x \leq 5^{-2\sqrt{5}}$

But $x > 0$

$\therefore \quad x \in (0, 5^{-2\sqrt{5}}] \cup [5^{2\sqrt{5}}, \infty)$

71. (A)

$\because \quad \tan\theta = \frac{1}{\sqrt{7}}$

$$\therefore \quad \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(1+\cot^2\theta) - (1+\tan^2\theta)}{(1+\cot^2\theta) + (1+\tan^2\theta)}$$

$$= \frac{(\cot^2\theta - \tan^2\theta)}{2 + \tan^2\theta + \cot^2\theta}$$

$$= \frac{7 - \frac{1}{7}}{2 + \frac{1}{7} + 7} = \frac{48}{14 + 1 + 49} = \frac{48}{64}$$

$$= \frac{3}{4}$$

72. (D)

$$\left(\frac{x^{\frac{1}{3}}}{3}+\frac{x^{\frac{-2}{3}}}{2}\right)^{18}$$

$$t_7={}^{18}C_6\left(\frac{x^{\frac{1}{3}}}{3}\right)^{12}\left(\frac{x^{-\frac{2}{3}}}{2}\right)^{6}={}^{18}C_6\,\frac{1}{(3)^{12}}\cdot\frac{1}{2^6}$$

$$t_{13}={}^{18}C_{12}\left(\frac{x^{\frac{1}{3}}}{3}\right)^{6}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{12}={}^{18}C_{12}\,\frac{1}{(3)^{6}}\cdot\frac{1}{2^{12}}\cdot x^{-6}$$

$$m={}^{18}C_6\cdot 3^{-12}\cdot 2^{-6};\ n={}^{18}C_{12}\cdot 2^{-12}\cdot 3^{-6}$$

$$\left(\frac{n}{m}\right)^{\frac{1}{3}}=\left(\frac{2^{-12}\cdot 3^{-6}}{3^{-12}\cdot 2^{-6}}\right)^{\frac{1}{3}}=\left(\frac{3}{2}\right)^{2}=\frac{9}{4}$$

73. (A)

$${}^{n-1}C_r=\left(k^2-8\right){}^{n}C_{r+1}$$

$$\underbrace{r+1\ge 0,\ r\ge 0}_{r\ge 0}$$

$$\frac{{}^{n-1}C_r}{{}^{n}C_{r+1}}=k^2-8$$

$$\frac{r+1}{n}=k^2-8$$

$$\Rightarrow\ k^2-8>0$$

$$\left(k-2\sqrt{2}\right)\left(k+2\sqrt{2}\right)>0$$

$$k\in\left(-\infty,-2\sqrt{2}\right)\cup\left(2\sqrt{2},\infty\right)\qquad\ldots(1)$$

$$\therefore\ n\ge r+1,\ \frac{r+1}{n}\le 1$$

$$\Rightarrow\ k^2-8\le 1$$

$$k^2-9\le 0$$

$$-3\le k\le 3\qquad\ldots\ldots(\text{ii})$$

From equation (I) and (II) we get

$$k\in\left[-3,-2\sqrt{2}\right)\cup\left(2\sqrt{2},3\right]$$

74. (A)

Sum of coefficients in the expansion of $\left(1-3x+10x^2\right)^n = A$ then $A=\left(1-3+10\right)^n = 8^n$ (put $x=1$)

and sum of coefficients in the expansion of $\left(1+x^2\right)^n = B$ then $B=\left(1+1\right)^n = 2^n$

$A = B^3$

75. (B)

Sum $= 8 = -\frac{b}{a}$

Product $= 12 = \frac{1}{a} \Rightarrow a = \frac{1}{12}$

$b = -\frac{2}{3}$

$2a + b = \frac{2}{12} - \frac{2}{3} = -\frac{1}{2}$

$6a + b = \frac{6}{12} - \frac{2}{3} = -\frac{1}{6}$

Sum = –8

P = 12

$x^2 + 8x + 12 = 0$

76. (A)

By newton's theorem

$a_{n+2} - \left(t^2 + 5t + 6\right)a_{n+1} + a_n = 0$

$\therefore\ a_{2025} + a_{2023} = \left(t^2 - 5t + 6\right)a_{2024}$

$\therefore\ \frac{a_{2025} + a_{2023}}{a_{2024}} = t^2 - 5t + 6$

$\because\ t^2 - 5t + 6 = \left(t - \frac{5}{2}\right)^2 - \frac{1}{4}$

$\therefore$ minimum value $= -\frac{1}{4}$

77. (A)

$(8)^{2x} - 16\cdot(8)^x + 48 = 0$

Put $8^x = t$

$t^2 - 16t + 48 = 0$

$\Rightarrow\ t = 4$ or $t = 12$

$\Rightarrow\ 8^x = 4 \qquad 8^x = 12$

$\Rightarrow\ x = \log_8 4 \qquad x = \log_8 12$

Sum of solution $= \log_8 4 + \log_8 12$

$= \log_8 48 = \log_8 (6.8)$

$=1+\log_8(6)$

78. (C)

$S=1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+.....$

$\frac{S}{6}=\frac{1}{6}+\frac{5}{6^2}+\frac{12}{6^3}+\frac{22}{6^4}+.....$

On subtraction

$\frac{5}{6}S=1+\frac{4}{6}+\frac{7}{6^2}+\frac{10}{6^3}+\frac{13}{6^4}+.....$

$\frac{5}{36}S=1+\frac{4}{6^2}+\frac{7}{6^3}+\frac{10}{6^4}+\frac{13}{6^5}+.....$

On subtraction

$\frac{25}{36}S=1+\frac{3}{6}+\frac{3}{6^2}+\frac{3}{6^3}+.....=\frac{8}{5}$

$S=\frac{288}{125}$

79. (D)

$\sin 12°+\sin 12°-\sin 72°$

$=\sin 12°-2\cos 42°\sin 30°$

$=\sin 12°-\sin 48°$

$=-2\cos 30°\sin 18°$

$=-2\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{5}-1}{4}$

$=\frac{\sqrt{3}}{4}(1-\sqrt{5})$

80. (A)

$\tan 15°=2-\sqrt{3}$

$\frac{1}{\tan 75°}=\cot 75°=2-\sqrt{3}$

$\frac{1}{\tan 105°}=\cot(105°)=-\cot 75°=\sqrt{3}-2$

$\tan 195°=\tan 15°=2-\sqrt{3}$

$\therefore\ 2(2-\sqrt{3})=2a\Rightarrow a=2-\sqrt{3}$

$\Rightarrow\ a+\frac{1}{a}=4$

81. (2)

$\therefore$ nth term $T_n = \dfrac{1+2+3+...+n}{1^3+2^3+3^3+....+n^3}$

$$= \dfrac{\dfrac{n(n+1)}{2}}{\left[\dfrac{n(n+1)}{2}\right]^2} = \dfrac{2}{n(n+1)}$$

$$= 2\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$$

Putting $n = 1, 2, 3, 4, ……, n$

$\therefore$ $T_1 + T_2 + T_3 + ... + T_n$

$$= 2\left(1 - \dfrac{1}{2} + \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{4} + ... + \dfrac{1}{n} - \dfrac{1}{n+1}\right)$$

$$= 2\left(1 - \dfrac{1}{n+1}\right)$$

$$S_n = 2\left(1 - \dfrac{1}{n+1}\right) \quad \Rightarrow \quad S_\infty = 2$$

82. (10)

Since, fourth term of

$$\left(\sqrt{x^{\left(\frac{1}{1+\log_{10} x}\right)}} + \sqrt[12]{x}\right)^6 = 200$$

$$\therefore \quad {}^6C_3\left(\sqrt{x^{\left(\frac{1}{1+\log_{10} x}\right)}}\right)^3 (\sqrt[12]{x})^3 = 200$$

$$\Rightarrow \quad \frac{6\cdot 5\cdot 4}{1\cdot 2\cdot 3}\cdot x^{\frac{3}{2}\left(\frac{1}{1+\log_{10} x}\right)}\cdot x^{1/4} = 200$$

$$\Rightarrow \quad x^{\frac{3}{2}\left(\frac{1}{1+\log_{10} x}\right)+\frac{1}{4}} = 10$$

$$\Rightarrow \quad \left[\frac{3}{2}\left(\frac{1}{1+\log_{10} x}\right)+\frac{1}{4}\right]\log_{10} x = 1$$

Let $\log_{10} x = t$

$$\therefore \quad \left\{\frac{3}{2(1+t)}+\frac{1}{4}\right\}t = 1$$

$$\Rightarrow \quad \left\{\frac{6+1+t}{4(1+t)}\right\}t = 1$$

$$\Rightarrow \quad t^2 + 7t = 4t + 4$$

$$t^2 + 3t - 4 = 0$$

$(t+4)(t-1)=0$

$t=-4,1$

$\log_{10} x=-4,1$

$\therefore \quad x=10^{-4}.10^{1}$

83. (0)

$1+\sin x\sin^2\frac{x}{2}=0$

$1+\sin\left(\frac{1-\cos x}{2}\right)=0$

$\Rightarrow \quad 2+\sin x-\sin x\cos x=0$

$\Rightarrow 4+2\sin x=\sin 2x$

LHS $\in[2, 6]$ but

RHS $\in[-1, 1]$

Hence, no solution

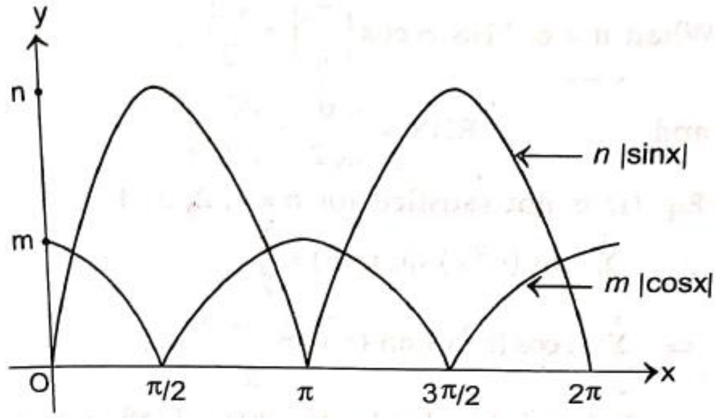
ie, Number of solutions = zero

84. (4)

Let $y=n|\sin x|=m|\cos x|$

The curve $y=n|\sin x|$ and $y=m|\cos x|$

Intersect at 4 points in $[0, 2\pi]$



85. (1)

$2^x\left(1+\frac{1}{2}+\frac{1}{4}\right)=7^x\left(1+\frac{1}{7}+\frac{1}{7^2}\right)$

$\Rightarrow \quad 2^x\left(\frac{7}{4}\right)=7^x\left(\frac{57}{49}\right)$

$\Rightarrow \quad \left(\frac{7}{2}\right)^{x-2}=\left(\frac{7}{57}\right)$

or $\quad (x-2)\log\left(\frac{7}{2}\right)=\log\left(\frac{7}{57}\right)$

$$\therefore \quad x = 2 + \frac{\log\left(\frac{7}{57}\right)}{\log\left(\frac{7}{2}\right)}.$$

86. (5)

$K = 2^9 + 2^8 \cdot 3 + 2^7 \cdot 3^2 + ... + 3^9$

$$= \frac{2^9\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\frac{3}{2} - 1} = 3^{10} - 2^{10}$$

Now, $3^{10} - 2^{10} = \left(3^5 - 2^5\right)\left(3^5 + 2^5\right)$

$= (211)(275)$

$= (35 \times 6 + 1)(45 \times 6 + 5)$

$= 6\lambda + 5$

Remainder is 5.

87. (4)

$$\sin 10°\left(\frac{1}{2}.2\sin 20°\sin 40°\right).\sin 10°\sin(60° - 10°)\sin(60° + 10°)$$

$$\sin 10°\frac{1}{2}(\cos 20° - \cos 60°)\cdot\frac{1}{4}\sin 30°$$

$$\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\sin 10°\left(\cos 20° - \frac{1}{2}\right)$$

$$= \frac{1}{32}(2\sin 10°\cos 20° - \sin 10°)$$

$$= \frac{1}{32}(\sin 30° - \sin 10° - \sin 10°)$$

$$= \frac{1}{32}\left(\frac{1}{2} - 2\sin 10°\right)$$

$$= \frac{1}{64}(1 - 4\sin 10°)$$

$$= \frac{1}{64} - \frac{1}{16}\sin 10°$$

Hence, $\alpha = \frac{1}{64} \Rightarrow \frac{1}{16\alpha} = 4$

88. (0)

$$(1-x)(1-x)^{2007}\left(1 + x + x^2\right)^{2007}$$

$$(1-x)\left(1-x^3\right)^{2007}$$

$$(1-x)\left({}^{2007}C_0 - {}^{2007}C_2\left(x^3\right) + ...\right)$$

General term

$(1-x)\left((-1)^{r\,2007}C_r x^{3r}\right)$

$(-1)^{r\,2007}C_r x^{3r}-(-1)r^{2007}C_r x^{3r+1}$

$3r=2012$

$r\neq\frac{2012}{3}$

$3r+1=2012$

$3r=2011$

$r\neq\frac{2011}{3}$

Hence there is no term containing x^{2012}.

So, coefficient of $x^{2012}=0$.

89. (5)

$T_{r+1}={}^{24}C_r\cdot 7^{\frac{24-r}{2}}\cdot 11^{\frac{r}{6}}$

$\therefore\quad r=$ multiple of 6

$\therefore\quad r=0, 6, 12, 18, 24$

Total 5 values of r.

90. (4)

$\frac{\alpha^{10}+\beta^{10}+\sqrt{2}\left(\alpha^9+\beta^9\right)}{2\left(\alpha^8+\beta^8\right)}$

$\frac{\alpha^8\left(\alpha^2+\sqrt{2}\alpha\right)+\beta^8\left(\beta^2+\sqrt{2}\beta\right)}{2\left(\alpha^8+\beta^8\right)}$

$\frac{8\alpha^8+8\beta^8}{2\left(\alpha^8+\beta^8\right)}=4$