SOLUTIONS

1. (A)

Since α, β all zeros then $a\alpha^2 + b\alpha + c = 0$

$$\alpha(a\alpha + b) + c = 0$$

$$a\alpha + b = \frac{-c}{\alpha}$$

And $$a\beta^2 + b\beta + c = 0$$

$$\beta(a\beta + b) = -c$$

$$\alpha/\beta + b = -c/\beta$$

$$\frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = \frac{\alpha}{-c} + \frac{\beta}{c} = \frac{-1}{c}(\alpha + \beta)$$

$$= \frac{-1}{c}\left(\frac{-b}{a}\right)$$

$$= \frac{+b}{ac}$$

$$\alpha + \beta = \frac{-b}{a}$$

2. (B)

We let x be the side-length of the four small unshaded squares and let y be the side-length of the three shaded squares.

It follows that the rectangle $PQRS$ has width $4x + y$ and height $3y$.

Also, the large unshaded square has width $4x$ and height $3y - 4$.

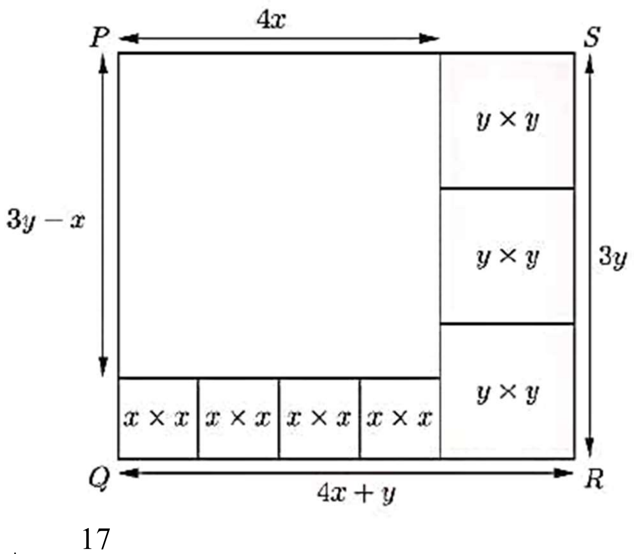
Because it is a square,

$$4x = 3y - x$$

and hence

$$5x = 3y.$$

Therefore $x = \frac{3}{5}y$. It follows that



$$4x + y = 4 \times \frac{3}{5}y + y = \frac{12}{5}y + y = \frac{17}{5}y.$$

Hence the area of the rectangle $PQRS$ is $\frac{17}{5}y \times 3y$.

The three shaded squares form a rectangle with width y and height $3y$. Therefore the area of the rectangle $PQRS$ that is shaded is $y \times 3y$.

It follows that the fraction of the area of the rectangle $PQRS$ that is shaded is

$$\frac{y \times 3y}{\frac{17}{5}y \times 3y} = \frac{y}{\frac{17}{5}y} = \frac{1}{\frac{17}{y}} = \frac{5}{17}.$$

3. (A)

Let $x = \sqrt{30+\sqrt{30+\sqrt{30+....}}}$

$x = \sqrt{30+x}$

$x^2 = 30 + x$

$x^2 - x - 30 = 0$

$x^2 - 6x + 5x - 30 = 0$

$(x-6)(x+5) = 0$

$x = 6 \;\&\; -5$

Since root cannot be negative x = 6

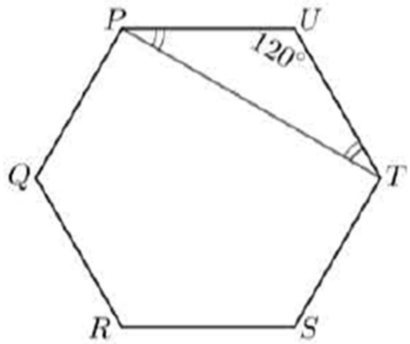
4. (A)

The interior angles of a regular hexagon are all 120º.

Therefore $\angle TUP = 120°$.

The angles of a triangle add to $180°$. Therefore, from the triangle *UTP*, we have

$\angle UTP + \angle UTP + 120° = 180°$ .....(1)



Because *PQRSTU* is a regular hexagon, *PU* = *TU*.

Therefore the triangle *PTU* is isosceles. Hence $\angle UPT = \angle UTP$.

It follows from (1) that

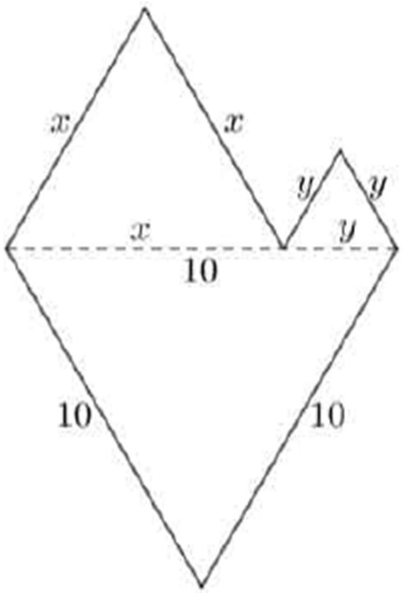
$$2 \times \angle UPT + 120° = 180°.$$

Therefore

$$\angle UPT = \frac{1}{2}(180° - 120°) = \frac{1}{2}(60°) = 30°.$$

5. (B)

Let the side-lengths, in cm, of the two smaller equilateral triangles be *x* and *y*, as shown in the diagram.



We see from the diagram that the sum of the side-lengths of the two smaller equilateral triangles is equal to the side-lengths of the largest equilateral triangle.

That is, $x + y = 10$.

Therefore the perimeter, in cm, of the hexagon is

$$10 + 10 + x + x + y + y = 20 + 2(x+y)$$
$$= 20 + 2 \times 10$$
$$= 20 + 20$$
$$= 40.$$

Note: Note that this answer does not depend on the values of *x* and *y* but only on the fact that $x + y = 10$.

6. (A)

Let no of girls = x

No of boys = y

$x + y = 10$ and $x = 4 + y$

$x - y = 4$

Adding both equation $2x = 14$

$x = 7$

And $y = 3$

$\Rightarrow$ No of girls $= x = 7$

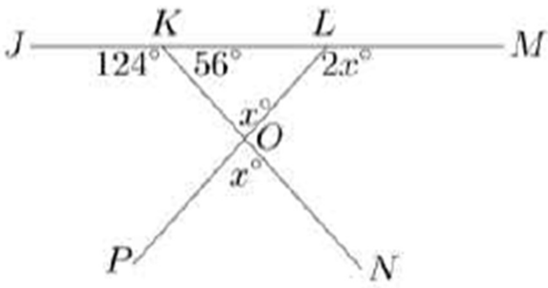
No of boys $= y = 3$

7. (B)

The four smaller equilateral triangles into which the large equilateral triangle is divided share sides. So their side lengths are all the same. Therefore these four equilateral triangles have the same area.
It follows that the area of each of these triangles is one-quarter of the area of the large triangle.
Similarly, the area of each of the three shaded triangles is one-quarter of the area of the central triangle.
Therefore three-quarter of the central triangle is shaded.
Hence three-quarters of one-quarter of the large triangle is shaded.
It follows that the fraction of the area of the large triangle that is shaded is given by

$$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}.$$

8. (D)



To avoid fractions, we let $\angle OLM = 2x°$.
Then, because $\angle OLM$ is twice $\angle PON$, we have $\angle OPN = x°$.
Because the angles $\angle PON$ and $\angle KOL$ are vertically opposite, $\angle KOL = \angle PON = x°$.
Because angles on a line have sum $180°$, it follows that $\angle OKL + \angle JKO = 180°$ and therefore $\angle OKL = 180° - 124° = 56°$.
By the External Angle Theorem, $OLM = \angle KOL + \angle OKL$. Therefor

$$2x = x + 56.$$

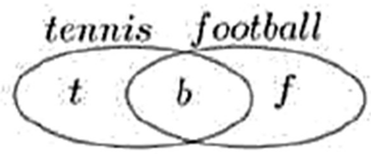
It follows that

$$x = 56.$$

Hence $2x = 2 \times 56 = 112$.
We conclude that $\angle OLM$ is $112°$.

9. (D)

We let t be the number of children that only play tennis, f be the number of children that only play football, and b be the number of children that play both tennis and football.
It follows that the total number of children who play football is $b + f$.



There are 42 children in the group. Therefore

$$t + b + f = 42 \qquad ...(1)$$

The number who play tennis is the same as the number who just play football. Therefore

$$t + b = f \qquad ...(2)$$

Twice as many play both tennis and football as play just tennis. Therefore

$$b = 2t \qquad ...(3)$$

Substituting from (2) in (1), we obtain

$$f + f = 42,$$

from which it follows that

$$f = 21 \qquad ...(4)$$

Substituting from (3) and (4) in (2), we obtain

$$t + 2t = 21,$$

That is,

$$3t = 21.$$

It follows that

$$t = 7.$$

Hence, by (3),

$$b = 14 \qquad ...(5)$$

Therefore, by (4) and (5),

$$b + f = 14 + 21$$
$$= 35$$

Therefore the number of children who play football is 35.

10. (A)

$2+\sqrt{3}$ is one root so secound root must be $2-\sqrt{3}$

$\therefore$ $x^2 - 4x + 1$ is a factor other factor must be $x^2 + 6x + 7$

$\therefore$ two other roots are $-3+\sqrt{2}, -3-\sqrt{2}$

11. (D)

Given:

In parallelogram *ABCD*, *AL* and *CM* are perpendicular to *CD* and *AD* respectively.

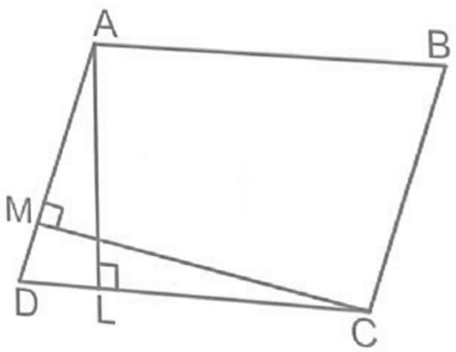
AL = 20 cm, *CD* = 18 cm and *CM* = 15 cm

Formula used:

Area of parallelogram = Base $\times$ Height

Perimeter of parallelogram $= 2 \times$ (Sum of parallel sides)

Calculation:



Area of *ABCD* with base *DC* = $AL \times DC = 20 \times 18$

$\Rightarrow$ $360\ cm^2$

Again, Area of *ABCD* with base *AD* = *CM* $\times$ *AD*

$= 15 \times AD$

$\Rightarrow$ $360\text{ cm}^2 = 15 \times AD$

$\Rightarrow$ $AD = 24\text{ cm}$

$\therefore$ $AD = BC = 24\text{ cm}, DC = AB = 18\text{ cm}$

Perimeter of $ABCD = 2 \times (24 + 18)$

$\Rightarrow$ $2 \times 42 = 84\text{ cm}$

$\therefore$ **The required result = 84 cm**

12. (C)

$(A+B)^2 = A^2 + B^2 + 2AB$

$(a-b+c)^2 + (b-c+a)^2 + 2(a-b+c)(b-c+a)$

$= \quad ((a-b+c)+(b-c+a))^2$

$= \quad (2a)^2 = 4a^2$

13. (B)

$x + y + 3 = 0$

Formula used:

$(a+b)^3 = a^3 + b^2 + 3ab(a+b)$

Calculation:

$x + y + 3 = 0$

$\Rightarrow x + y = -3$ .....(1)

$\Rightarrow (x+y)^3 = (-3)^3$ [Taking cube of both sides]

$\Rightarrow x^3 + y^3 + 3xy(x+y) = -27$

$\Rightarrow x^3 + y^3 + 3xy(-3) = -27$ [$\because x + y = -3$]

$\Rightarrow x^3 + y^3 - 9xy + 9 = -27 + 9$ [Adding 9 in both sides]

$\Rightarrow x^3 + y^3 - 9xy + 9 = -18$

$\therefore$ **The value of $x^3 + y^3 - 9xy + 9$ is (–18)**

14. (B)

Let, price of 1 pencil = x, price of 1 pen = y and price of one eraser = z

Then, $8x + 5y + 3z = 111$ ...(1)

$9x + 6y + 5z = 130$ ...(2)

$16x + 11y + 3z = 221$ ...(3)

Adding (1), (2) and (3), we get

$33x + 22y + 11z = 462$

$\Rightarrow 3x + 2y + z = 42$

$\Rightarrow 39x + 26y + 13z = 546$ (multiplying with 13)

15. (A)

$1+p+q+r+pq+qr+pr+pqr$

$=1+p+q+pq+r(1+p+q+pq)$

$=(1+r)(1+p+q+pq)$ $\quad=(1+r)(1+p)(1+q)$.

16. (5)

The four smallest primes are 2, 3, 5 and 7.
An integer n leaves a remainder of 1 when divided by each of these primes provided that $n - 1$ is a multiple of each of them.
So we need to find integers that are multiples of 2, 3, 5 and 7. These are precisely the integers that are multiples of their product $2\times3\times5\times7$.
Now $2\times3\times5\times7=210$. So the multiples of 2, 3, 5 and 7 are precisely the multiples of 210.
Therefore the two smallest positive integers that are multiples of 2, 3, 5 and 7 are 210 and 420.
It follows that the two smallest positive integers, greater than 1, that leave a remainder of 1 when divided by each of the four smallest primes are 211 and 421.
The difference between these numbers is 421 – 211, that is, 210.

17. (8)

The given:

Polynomial: $p(x)=x^3+3x^2-6x-a$

One zero of the polynomial: $x=2$

Concept Used:

If $x=2$ is a zero of the polynomial, then $p(2)=0$.

Calculation:

Since, $x=2$ is a zero,

$p(2)=0$:

$\Rightarrow\ 2^3+3(2)^2-6(2)-a=0$

$\Rightarrow\ 8+12-12-a=0$

$\Rightarrow\ 8-a=0$

$\Rightarrow\ a=8$

Now, $P(x)=x^3+3x^2-6x-8$

Since 2 is the zero of the polynomial, so $P(x)$ can be factorized as

$P(x)=x^3+3x^2-6x-8=(x-2)(x^2+5x+4)$

Now, (x^2+5x+4)

$\Rightarrow\ (x^2+4x+x+4)$

$\Rightarrow\ x(x+4)+1(x+4)$

$\Rightarrow\ (x+4)(x+1)$

So, other zeroes are –4 and –1

Thus, sum of the squares of the other zeros of the polynomial

$\Rightarrow\ (-4)^2+(-1)^2$

$\Rightarrow\ 16+1=17$

$\therefore$ **Sum of digit is 8.**