IIT – JEE: 2025 ADVANCED | **MAJOR TEST - 5** | **DATE: 04/08/24**

ANSWER KEY

PAPER – 1 (Code – 11)

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	D	**18.**	A	**35.**	C
2.	B	**19.**	A	**36.**	D
3.	B	**20.**	B	**37.**	A
4.	C	**21.**	B	**38.**	A
5.	A, D	**22.**	B, C	**39.**	A, B, C
6.	B or ABC	**23.**	A, B, C	**40.**	A, C
7.	A, C, D	**24.**	A, D	**41.**	A, B, C
8.	4	**25.**	3	**42.**	7
9.	30	**26.**	5	**43.**	3
10.	4	**27.**	2	**44.**	10
11.	2	**28.**	3	**45.**	4
12.	3	**29.**	0	**46.**	3
13.	8	**30.**	4	**47.**	7
14.	A	**31.**	B	**48.**	A
15.	C	**32.**	B	**49.**	A
16.	B	**33.**	A	**50.**	A
17.	A	**34.**	D	**51.**	A

PART (A) : PHYSICS

1. (D)

The angular velocity ω, which we are supposed to find is given by vector addition of the angular velocities ω_0 and ω_y. The vectors are shown in the figure.

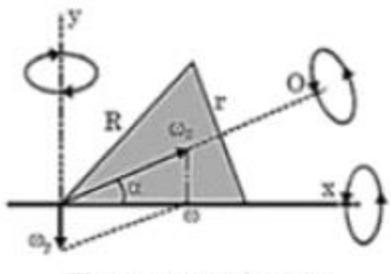


Figure : Analysis of the rotational motion.

Since the moment takes place on a rough surface, the cone cannot slip, so it moves circularly about its apex. We can immediately see the simple relation between ω_y and ω_0.

$$R\omega_y = r\omega_0$$

We introduce the angle α between ω and ω_0 : using the cosine law, we can write

$$\omega_y^2 = \omega^2 + \omega_0^2 - 2\omega\omega_0 \cos\alpha$$

The angle α is the angle at the apex and satisfies the relation

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{r^2}{R^2}} = \frac{\omega}{\omega_0}$$

Substituting into the cosine law (2), we obtain $\omega^2 = \omega_0^2 - \omega_y^2$;

That is the Pythagorean theorem, thus ω must line in the ground plane, as the picture hints. Using the relation (1), we can finally express the magnitude of the angular velocity

$$\omega = \omega_0\sqrt{1-\frac{r^2}{R^2}}$$

2. (B)

$$\alpha = \frac{mg\frac{\ell}{2}}{\frac{m\ell^2}{3}} = \frac{3g}{2\ell}$$

For $x \geq \frac{2\ell}{3}$

$a \geq g$

Wherever $a > g$ the beads will be left behind, losing contact with the plank.

3. (B)

For C_1 :

9 MSD = 10 VSD

1 VSD = 0.9 MSD = 0.9 mm

$\therefore\ LC = 1MSD - 1VSD = 1 - 0.9 = 0.9 = 0.1\text{ mm} = 0.01\text{cm}$

So, the reading of C_1 is given by,

Reading $= MSR + (LC \times VCR)$

Reading $= 2.8 + (0.01 \times 7) = 2.87\text{cm}$

For C_2 :

11 MSD = 10 VSD

1 VSD = 1.1 MSD = 1.1 mm

$\therefore LC = 1MSD - 1VSD = 1 - 1.1 = -0.1\text{mm} = -0.01\text{cm}$

So, the reading of C_2 is given by,

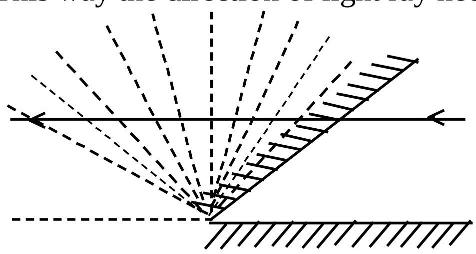
Reading $= MSR + (LC \times VCR)$

Reading $= 2.8 + [0.01 \times (10 - 7)] = 2.83\text{ cm}$

4. (C)

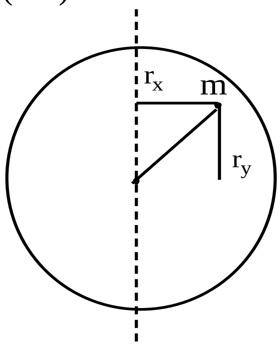
We can draw the mirror image of lower mirror w.r.t upper mirror & see the second reflection from the perspective of the top mirror.

This way the direction of light ray need not be changed & this process can be repeated.



$\therefore$ || intersections correspond to || reflections.

5. (AD)



On particle of mass ‘m’.

$\frac{GMmr_x}{R^3}$ force acts on left R^3 and $m\omega^2 r_x$ acts on right in the frame of Earth.

Disintegration happens when forces equate.

$\therefore m\omega_0^2 r_x = \frac{GMmr_x}{R^3}$

$\therefore \omega_0 = \sqrt{\frac{GM}{R^3}}$

Since no assumption about position of particle was taken and ω_0 is independent of position, disintegration happens simultaneously and particles move away from axis of rotation.

Horizontal force on particles is $m\omega^2 r_x$ for $\omega \le \omega_0$ & $\frac{GMmr_x}{R^3}$ for $\omega > \omega_0$.

6. (B or ABC)
NCERT Theory (Validity of Ray optics & Wave optics)

7. (ACD)
With node at O

$\upsilon = \sqrt{\frac{T}{\mu}}$

$\upsilon' = \sqrt{\frac{T}{4\mu}} - \frac{1}{2}\upsilon$

$\Rightarrow m\frac{1}{2l}\sqrt{\frac{T}{\mu}} = n\frac{1}{2(2l)}\sqrt{\frac{T}{4\mu}}$

$\Rightarrow m = \frac{n}{2}\times\frac{1}{2}$

$\Rightarrow \frac{m}{n} = \frac{1}{4}$

$\therefore m = 1, n = 4$

With antinode at O

$m\frac{1}{4l}\sqrt{\frac{T}{\mu}} = n\frac{1}{4(2l)}\sqrt{\frac{T}{4\mu}}$

$\Rightarrow \frac{m}{n} = \frac{1}{2}\times\frac{1}{2}$

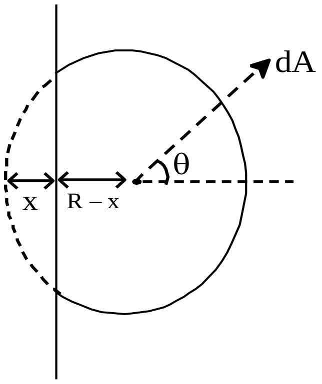
$\Rightarrow \frac{m}{n} = \frac{1}{4}$

$\therefore m = 1,\ f_{min} = \frac{1}{4}\sqrt{\frac{T}{\mu}} = \frac{\upsilon_0}{2}$

(B is wrong)
Also, when node at O.
Total nodes = 6
(C is correct)

8. (4)

$\Delta P = \frac{4\gamma}{R}$, This excess pressure provides force for SHM



Force $= \int \Delta P dA \cos\theta$

$= \Delta P \underbrace{\int dA \cos\theta}_{\text{projected area of area of flattened part}}$

Area of circle $= \pi\left(R^2 - (R - x)^2\right)$

$\approx 2\pi Rx$

$\therefore F = \frac{4\gamma}{R} \times 2\pi Rx$

$\therefore \omega = \sqrt{\frac{8\pi\gamma}{m}} = \sqrt{\frac{8\pi(2)}{\pi}} = 4$

9. (30)

Since the mass of the astronaut is much less than the mass of the spaceship. Tension of rope does not affect motion of spaceship much.

Let spaceship orbit at radius r.

$\Rightarrow \omega = \sqrt{\frac{GM}{r^3}}$ (M = Mass of Earth)

For motion of astronaut

$T + \frac{GMm}{(r+h)^2} = m\omega^2 (r+h)$

$\Rightarrow T = \frac{GMm(r+h)}{r^3} - \frac{GMm}{(r+h)^2}$

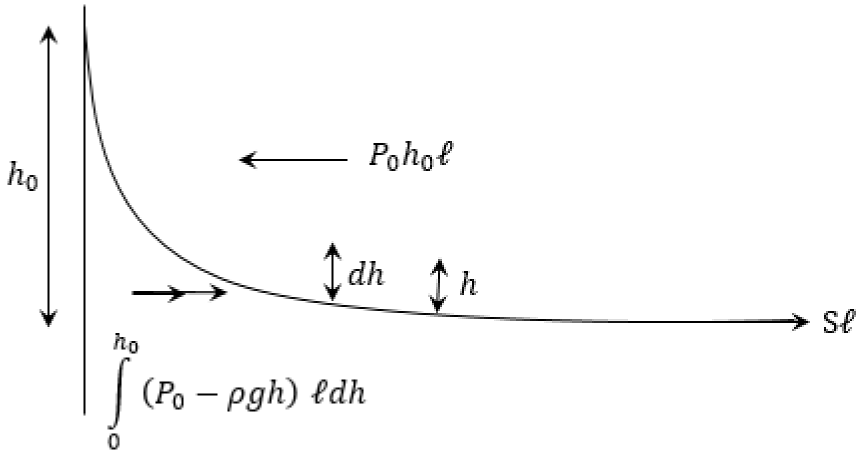
$\approx \frac{3GMmh}{r^3}$

$\approx \frac{3GMmh}{R^3}$ [R = Radius of earth]

$\approx \frac{3mgh}{R}$

≈ 29 or 30

10. (4)



Let 'ℓ' be length of the wall of fish tank.
Balancing horizontal forces on the curved water surface:

$$S\ell + \int_0^{h_0} (P_0 - \rho g h)\,\ell dh = P_0 h_0 \ell$$

$$\Rightarrow S = \frac{\rho g h_0^2}{2}$$

$$\Rightarrow h_0 = \sqrt{\frac{2S}{\rho g}}$$

$$= 3.74$$

11. (2)

Acceleration due to gravity $= \dfrac{GM}{R^2}$

$$\Delta\left(\frac{GM}{R^2}\right) \to 0 \text{ for small } \Delta$$

$$\therefore \frac{d\left(\frac{GM}{R^2}\right)}{dR} = 0 \text{ at } R = \text{ Radius of earth}$$

$$\Rightarrow R^2 \frac{dM}{dR} - 2MR = 0$$

$$dM = \rho_{crust}\ 4\pi R^2 dR \ \& \ M = \rho_{avg} \frac{4\pi}{3} R^3$$

Substitute these expressions.

$$\therefore \frac{\rho_{crust}}{\rho_{avg}} = \frac{2}{3}$$

12. (3)

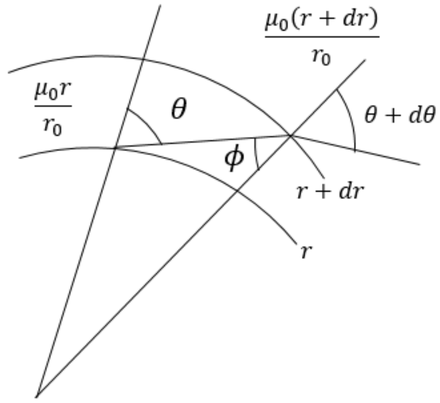
$$C = \frac{2\pi ab\varepsilon_0 (k+1)}{(b-a)}$$

$$R = \frac{1}{\alpha} \frac{(b-a)}{2\pi ab}$$

$$RC = \frac{2\pi ab\varepsilon_0 (k+1)}{(b-a)} \frac{(b-a)}{\sigma 2\pi ab} = \frac{3\varepsilon_0}{\sigma}$$

Both capacitance and resistances are in parallel to each other. In the part where there is no medium inserted, sigma is 0 and k is 1. And time constant is RC.

13. (8)



Sine law:

$$\frac{\mu_0 r}{r_0} \sin\phi = \frac{\mu_0 (r+dr)}{r_0} \sin(\theta + d\theta) \quad ....(1)$$

Snell law: $\frac{r+dr}{\sin\theta} = \frac{r}{\sin\phi} \quad ...(2)$

$$\therefore \frac{r+dr}{\sin\theta} = \frac{r}{\frac{(r+dr)}{r}\sin(\theta+d\theta)}$$

$$\Rightarrow r^2 \sin\theta = (r+dr)^2 \sin(\theta + d\theta)$$

$$\therefore r^2 \sin\theta = \text{const}$$

14. (A)

In Bohr's formulae, substituting 4 (reduced mass) in place of $m_{electron}$ results in following transformations

$r \rightarrow$ distance between nucleus & electron

$v \rightarrow$ relative velocity of nucleus & electron

But, new radius & velocity of electron is half of these values.

$$\therefore r_{new} = r_{old}, \quad V_{new} = \frac{1}{2} V_{old}$$

$$\left(\mu = \frac{m_1 m_2}{m_1 + m_2} \right)$$

15. (C)

16. (B)

When a fluid is in the pipe, the viscosity force generated by friction between the two laminae is given below.

$$F = -\eta A\left(\frac{d\upsilon}{dr}\right)$$

Where A is the lamina's curved surface with radius r and length L;

$$A = -2\pi rL$$

$\frac{d\upsilon}{dr}$ is the velocity gradient, which means that velocity changes as the radius changes.

Because this force opposes the fluid's velocity, the sign is negative.

Then put all the values F become

$$F = -2\pi\eta rL\left(\frac{d\upsilon}{dr}\right)$$

The fluid is flowing through the pipe as a result of the pressure difference.

$$P = P_1 - P_2$$

When expressed in terms of external force per unit area the force is given as;

$$F = P\pi r^2$$

When external force equal laminar frictional force, the condition is said to be in equilibrium. We get the magnitude by equating both forces.

$$P\pi r^2 = 2\pi\eta rL\frac{d\upsilon}{dr}$$

After simplifying, we get

$$\frac{d\upsilon}{dr} = \frac{Pr}{2\eta L}$$

Then,

$$d\upsilon = \left(\frac{Pr}{2\eta L}\right)dr$$

When we integrate both sides from r to R, we get

$$\int d\upsilon = \frac{P}{2\eta L}\int_r^R rdr$$

On solving, we obtain

$$\upsilon = \left[\frac{P}{4\eta L}\right]\left(R^2 - r^2\right)$$

$$dV = 2\pi rdr \times P\left(\frac{R^2 - r^2}{4\eta L}\right)$$

We obtain when we solve

$$dV = \frac{P\pi\left(r.R^2 - r^3\right)}{2\eta L}dt$$

Because we want to find the total volume of fluid in the pipe, we integrate both sides from 0 to R once more. Then we have

$$\int dV = \frac{P\pi}{2\eta L}\int_0^R\left[r \cdot R^2 - r^3\right]dr$$

We obtain when we solve

$$V = \frac{P\pi}{2\eta L}\left[\frac{\left(R^2 \cdot R^2\right)}{2} - \frac{R^4}{4}\right]$$

$$V=\frac{P\pi}{2\eta L}\left[\frac{R^4}{2}-\frac{R^4}{4}\right]$$

We obtain when we solve

$$V=\frac{P\pi R^4}{8\eta L}$$

17. (A)

When a fluid is in the pipe, the viscosity force generated by friction between the two laminae is given below.

$$F=-\eta A\left(\frac{d\upsilon}{dr}\right)$$

Where A is the lamina's curved surface with radius r and length L;

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$$P\pi r^2=2\pi\eta rL\frac{d\upsilon}{dr}$$

After simplifying, we get

$$\frac{d\upsilon}{dr}=\frac{Pr}{2\eta L}$$

Then, $d\upsilon=\left(\frac{Pr}{2\eta L}\right)dr$

When we integrate both sides from r to R, we get

$$\int d\upsilon=\frac{P}{2\eta L}\int_r^R rdr$$

On solving, we obtain

$$\upsilon=\left[\frac{P}{4\eta L}\right]\left(R^2-r^2\right)$$

$$dV=2\pi rdr\times P\left(\frac{R^2-r^2}{4\eta L}\right)$$

We obtain when we solve

$$dV=\frac{P\pi\left(r.R^2-r^3\right)}{2\eta L}dt$$

Because we want to find the total volume of fluid in the pipe, we integrate both sides from 0 to R once more. Then we have

$$\int dV = \frac{P\pi}{2\eta L}\int_0^R \left[r \cdot R^2 - r^3\right] dr$$

We obtain when we solve

$$V = \frac{P\pi}{2\eta L}\left[\frac{\left(R^2 \cdot R^2\right)}{2} - \frac{R^4}{4}\right]$$

$$V = \frac{P\pi}{2\eta L}\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

We obtain when we solve

$$V = \frac{P\pi R^4}{8\eta L}$$

PART (B) : CHEMISTRY

18. (A)
Since $2NO + O_2$ is an endothermic reaction, the equilibrium shifts towards the product on increasing the temperature.
Hence, option B is correct.
Note: Increasing temperature at constant volume does not affect the molarity of any species, hence no sudden spike is observed.

19. (A)
OH^- ligand is named as hydroxide, anionic complex is suffixed by –ate

20. (B)

21. (B)
Factual. Refer NCERT types of sols.

22. (B), (C)
$Z = 1 + A/V + B/V^2 + \ldots..$
This equation can be converted to $Z = 1 + A_1P + B_1P^2 + \ldots\ldots$
$Z = PV/RT$
$1/V = P/ZRT$
Substitute into Virial Equation
$1 + A_1P + B_1P^2 + \ldots = 1 + [b-(a/RT)](P/ZRT) + BP^2/Z^2R^2T^2 + \ldots.$
Cancel out 1 and P
$A_1 + B_1P + \ldots. = (b-a/RT)/ZRT + \ldots..$
Now take Limit $P \to 0$
$A_1 = (b - a/RT)/RT$
Now, $A_1 + B_1P + \ldots = A_1/Z + (b/RT)^2 (P/Z^2) + \ldots$
$A_1(1 - 1/Z) + B_1P = (b/RT)^2(P/Z^2) + \ldots.$
$A_1(Z - 1)/ZP + B_1 = (b/RTZ)^2$
Again apply limit:
$A_1^2 + B_1 = (b/RT)^2$
Now find B_1

23. (A), (B), (C)

24. (A), (D)
Factual. Refer Surface Chemistry NCERT

25. (3)

26. (5)
Pyrene is aromatic because of peripheral resonance. The pi bond in the middle ring is ignored because it is unstable. Cyclooctatetraene is non-aromatic due to non-planarity of compound, while disodium

cyclooctatetraene is aromatic.10-annulene is non-aromatic due to non-planarity induced by repulsion between hydrogen atoms. Benzyne is also aromatic due to false pi bond. Azulene is also aromatic.

27. (2)
Chromium has a half-filled 4d subshell
Elements from the p-block show valencies differing by 2
Zn is considered a post transition element.
Electronegativity is applied in real cases and is observed through compounds of the element only.

28. (3)
pKa of tartaric acid: 3.3.
molar mass of tartaric acid: 150, molar mass of potassium hydrogen tartarate: 188.
Apply Helmholtz - Hasselbalch equation to get answer.

29. (0)
$d(z^2)$ orbital has 2 angular nodes, but zero planar nodes.

30. (4)
Ag^+ at anode = sqrt($K_{sp\,(AgI)}$), Ag^+ at cathode: ($K_{sp\,(AgCl)}$)/[Cl^-]
Apply Nernst equation.

31. (B)
32. (B)
In Q31, SN2 reaction takes place. In Q32 also, SN2 reaction takes place, but since Iodide ion is a good leaving group and a good nucleophile, it again attacks the product. Hence, racemisation takes place due to repeated inversions taking place in the product.

33. (A)
3 isomers

34. (D)
11 isomers

PART (C) : MATHEMATICS

35. (C)

Possible coordinates of intersection are $(0,\pm5),(\pm3,\pm4),(\pm4,\pm3),(\pm5,0)$ ie, 12 points

Total pairs $= {}^{12}C_2$

Tangents $=12$

Lines passing through origin = 6

Vertical lines = 7

Vertical & through origin = 1

$\therefore \quad {}^{12}C_2+12-6-7+1$

= 66

36. (D)

$a_{n+1}=1+a_1a_2.....a_n$

$\Rightarrow \quad (a_{n+1}-1)=(a_1a_2a_3.....a_{n-1})a_n$

$\Rightarrow \quad (a_{n+1}-1)=(a_n-1)a_n$

$\Rightarrow \quad \dfrac{1}{a_{n+1}-1}=\dfrac{1}{a_n-1}-\dfrac{1}{a_n}$

$\Rightarrow \quad \dfrac{1}{a_n}=\dfrac{1}{a_n-1}-\dfrac{1}{a_{n+1}-1}$ for $n\ge 2$

$\therefore \quad$ sum $=\dfrac{1}{a_1}+\dfrac{1}{a_2-1}+0 \quad (\text{since } a_\infty=\infty)$

= 2

37. (A)

$xy+yz+zx=xyz$

$\Rightarrow \quad \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=1$

Let $a=\dfrac{1}{x}, b=\dfrac{1}{y}, c=\dfrac{1}{z}$

$\Rightarrow \quad a+b+c=1 \qquad \therefore a,b,c\in(0,1)$

Consider $f(x)=\ell n\left(x+\dfrac{1}{x}\right)$

$$f'(x)=\frac{1-\frac{1}{x^2}}{x+\frac{1}{x}}$$

$f'(x)$ increases in (0,1) since numerator increases & denominator decreases thus, $f(x)$ is concave up in (0,1)

Applying Jensen Inequality

$$\frac{\ln\left(a+\frac{1}{a}\right)+\ln\left(b+\frac{1}{b}\right)+\ln\left(c+\frac{1}{c}\right)}{3} \geq \ln\left(\frac{a+b+c}{3}+\frac{1}{\frac{a+b+c}{3}}\right)$$

$$\therefore \left(a+\frac{1}{a}\right)\left(b+\frac{1}{b}\right)\left(c+\frac{1}{c}\right) \geq \frac{1000}{27} \approx 37$$

38. (A)

$$f_n(\theta)=\sum_{r=1}^{n}\left(\frac{\sin(4r-1)\theta}{\cos(4r+1)\theta\cdot\cos(4r-3)\theta}\right)$$

$$=\frac{1}{2\sin 2\theta}\sum_{r=1}^{n}\frac{2\sin(4r-1)\theta\ \sin 2\theta}{\cos(4r+1)\theta\cdot\cos(4r-3)\theta}$$

$$=\frac{1}{2\sin 2\theta}\sum_{r=1}^{n}\frac{\cos(4r-3)\theta-\cos(4r+1)\theta}{\cos(4r+1)\theta\cdot\cos(4r-3)\theta}$$

$$=\frac{1}{2\sin 2\theta}\sum_{r=1}^{n}\left(\frac{1}{\cos(4r+1)\theta}-\frac{1}{\cos(4r-3)\theta}\right)$$

$$=\frac{1}{2\sin 2\theta}\left(\frac{1}{\cos(4n+1)\theta}-\frac{1}{\cos\theta}\right)$$

$$f_{11}\left(\frac{\pi}{4}\right)=-\sqrt{2}$$

39. (A), (B), (C)

Let $f(x)=(x-0)(x-x_1)(x-x_2).....(x-x_4)$

Given: $f'(x_1)=f'(x_2)=f'(x_3)=f'(x_4)=4$

$\therefore\ f'(x)-4=5(x-x_1)(x-x_1).....(x-x_4)$

Since $f'(x)$ is a 4 degree equation with leading coefficient 5, this equation is true.

Let $s_1=x_1+x_2+x_3+x_4$

$s_2=x_1x_2+x_1x_3+x_1x_4+x_2x_3+x_2x_4+x_3x_4$

and so on.

$f'(x)=5x^4-4s_1x^3+3s_2x^2-2s_3x+s_4$

$\therefore\quad 5x^4-4s_1x^3+3s_2x^2-2s_3x+s_4$

$=5x^4-5s_1x^3+5s_2x^2-5s_1x+5s_4+4$

Compare & equate the coefficients of the powers of x.

40. (A), (C)

Let there be ‘n’ distinct objects and (n – 2) like boxes to fill them in.

Let the objects be numbers 1 to n.

Case 1: nth object is kept singly in a box and the remaining (n – 1) objects kept in (n – 3) boxes.

$\therefore\ P(n-1, n-3)$ to be added.

Case 2: Objects numbered 1 to n – 1 kept in the n – 2 boxes and the nth object now kept in any one box.

$\therefore\ P(n-1, n-2)\times {}^{n-2}C_1$ added

Thus $P(n, n-2) = P(n-1, n-3) + {}^{n-2}C_1 P(n-1, n-2)$

41. (A), (B), (C)

Let $\cos^2\theta = a$ & $\sin^2\theta = b$

$a + b = 1$ & both a, b are irrational.

$f(n) = \left[\frac{n}{a}\right]$ & $g(n) = \left[\frac{n}{b}\right]$

$\therefore f(n)$ & $g(n)$ are quotients (without remainders) when n is divided by a & b respectively.

Since a & b are less than 1, as n varies, quotients will have to change. Thus, f & g are bijective.

Let's say α is a common element of A & B

$\therefore\ n_1 = \alpha a + r_1 \quad (r_1 \in (0, a))$

$n_2 = \alpha b + r_2 \quad (r_2 \in (0, b))$

$\therefore\ n_1 + n_2 = \alpha + r_1 + r_2 \ [\because a + b = 1]$

LHS is an integer but RHS is not because $r_1 + r_2 < a + b = 1$

$\therefore\ A \cap B = \phi$

Now let's day α is neither an elements of A nor of B.

$\therefore\ n_1 < \alpha a, (\alpha+1)a < n_1 + 1$

$n_2 < \alpha b, (\alpha+1)b < n_2 + 1$

$\Rightarrow\ n_1 + n_2 < 2\alpha, 2\alpha + 2 < n_1 + n_2 + 2$

This is a contradiction since two integers cannot be found in an interval of 2 openly bound by two other integers.

$\therefore\ A \cup B = N$

42. (7)

$\frac{2}{x}\int_x^{2x} f(y)dy = x + 2 \ \Rightarrow\ 2\int_x^{2x} f(y)dy = x^2 + 2x$

Differentiate

$\Rightarrow\ 2[2f(2x) - f(x)] = 2x + 2 \qquad \Rightarrow \qquad 2f(2x) - f(x) = x + 1 \qquad .....(i)$

$\Rightarrow\ 4f(4x) - 2f(2x) = 2(2x+1) = 4x + 2 \qquad .....(ii)$

$\Rightarrow\ 8f(8x) - 4f(4x) = 2(8x+2) = 16x + 4 \qquad .....(iii)$

Add (i) + (ii) and (iii) we get

$\Rightarrow\ 8f(8x) - f(x) = 21x + 7$

$\Rightarrow\ \int_0^1 (8f(8x) - f(x) - 21x)dx = \int_0^1 7dx = 7$

43. (3)

Let the number be $a_n a_{n+1} \ldots\ldots a_1$ (n digits)

Given: $57 \times a_{n-1} \ldots\ldots a_1 = a_n a_{n-1} \ldots\ldots a_1$

$\Rightarrow \quad 56 \times a_{n-1} \ldots\ldots a_1 = a_n \times 10^{n-1}$

$\Rightarrow \quad 7 \times 8 \times a_{n-1} \ldots a_1 = a_n \times 10^{n-1}$

Since LHS is divisible by 7, RHS is also divisible by 7 Thus, $a_n = 7$

$$\Rightarrow \quad a_{n-1} \ldots\ldots a_1 = \frac{10^{n-1}}{8} = 12500\ldots\ldots$$

$\therefore$ Sum of digits $= 7 + 1 + 2 + 5$
$= 15$

44. (10)

Number of ways in which $a + b + c = n$ is ${}^{n+2}C_2$ (since $a, b, c \geq 0$)

Thus, reducing summation to one variable

$$\sum_{n=0}^{\infty} \left(\frac{{}^{n+2}C_2}{n!} \right) = \frac{1}{2} \sum_{n=0}^{\infty} \frac{n^2 + 3n + 2}{n!}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \frac{n(n-1) + 4n + 2}{n!}$$

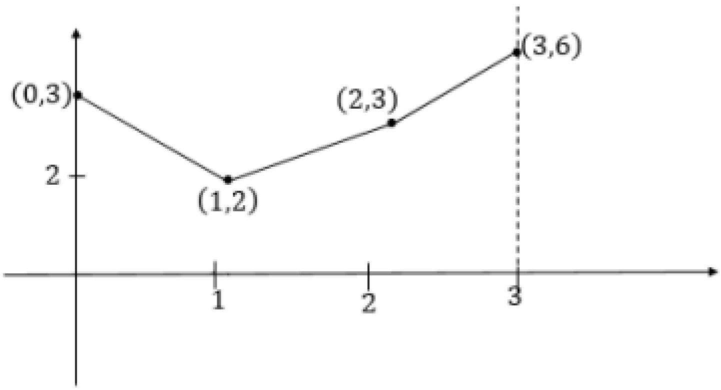
$$= \frac{1}{2} \left(\sum_{n=2}^{\infty} \frac{1}{(n-2)!} + 4 \sum_{n=1}^{\infty} \frac{1}{(n-1)!} + 2 \sum_{n=0}^{\infty} \frac{1}{n!} \right)$$

$$= \frac{7e}{2} \left(\text{since} \sum_{n=0}^{\infty} \frac{1}{n!} = e \right)$$

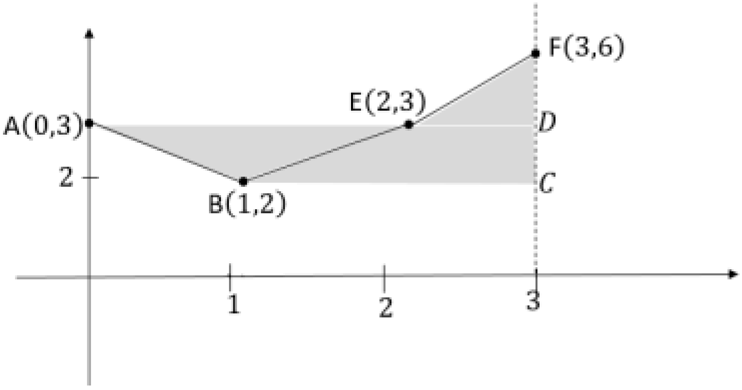
$$\approx 9.51$$

45. (4)

Graph of $y = |x-1| + |x-2| + x$ in [0,3]



Applying transformation of graph $[f(x) \to f(kx)]$ from $k \in [0, 1]$ stretches graph horizontally right word from given graph to infinity. Thus area



Area = Area of trapezium $ABCD + $ Area of ΔDEF

$= 4$

46. (3)

Let $b = \tan\beta$ & $a = \tan\alpha$

Thus, expression becomes : $\dfrac{\tan\beta - \tan\alpha}{\beta - \alpha}$ for $0 < \alpha < \beta \le \dfrac{\pi}{3}$

By LMVT, this equals $\sec^2(c)$ for some $c \in (\alpha, \beta)$.

Thus, expression reaches a maximum value just less than 4.

47. (7)

Let $\dfrac{\Pi \cdot 3^k}{3^{100}+1} = \theta$

Then,
$$1 + 2\cos(2\theta)$$
$$= 1 + 2\left(1 - 2\sin^2\theta\right)$$
$$= 3 - 4\sin^2\theta$$
$$= \frac{3\sin\theta - 45\sin^3\theta}{\sin\theta}$$
$$= \frac{\sin 3\theta}{\sin\theta}$$

$$\prod_{k=1}^{100}\left(1 + 2\cos\left(\frac{2\Pi \cdot 3^k}{3^{100}+1}\right)\right)$$

$$= \prod_{k=1}^{100}\frac{\sin\left(\dfrac{\Pi \cdot 3^{k+1}}{3^{100}+1}\right)}{\sin\left(\dfrac{\Pi \cdot 3^k}{3^{100}+1}\right)}$$

$$= \frac{\sin\left(\dfrac{\Pi \cdot 3^{101}}{3^{100}+1}\right)}{\sin\left(\dfrac{\Pi \cdot 3}{3^{100}+1}\right)}$$

$$=1\left(\sin ce \frac{\Pi\cdot 3^{101}}{3^{100}+1}+\frac{\Pi\cdot 3}{3^{100}+1}=3\Pi\right)$$

48. (A)

Consider $y=ke^{x}$ and $y=x$

Let $\left(\alpha, ke^{\alpha}\right)$ be a point on $y=ke^{x}$

If it lies on y = x also then $\alpha=ke^{\alpha}$

$\frac{dy}{dx}=ke^{x}$

$\therefore \left.\frac{dy}{dx}\right|_{x=\alpha}=ke^{\alpha}=\alpha=1$ {$\because$ y = x is tangent to $y=ke^{x}$ at one point}

$\therefore 1=ke$ i.e. $k=\frac{1}{e}$

49. (A)

Consider $y=ke^{x}$ and $y=x$

From above question $e^{x}=\frac{x}{k}$ if we decrease the value of k from $\frac{1}{e}$, then slope of $y=\frac{x}{k}$ increases

$\therefore y=e^{x}$ and $y=\frac{x}{k}$ intersect at two distinct points

$\therefore k\in\left(0,\frac{1}{e}\right)$

50. (A)

$$\int_{4}^{\alpha}\sin x=\int_{\alpha}^{\frac{3\pi}{4}}\sin x$$

$\Rightarrow \cos|_{0}^{\alpha}=-\cos x|_{\alpha}^{3\pi/4}$

$\Rightarrow -\cos\alpha-1(-1)=-\cos\frac{3\pi}{4}-(-\cos\alpha)$

$\Rightarrow 2\cos\alpha=1-\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\alpha=\frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)$

51. (A)

Let $g(x)=\int_{0}^{x}f(x)dx$

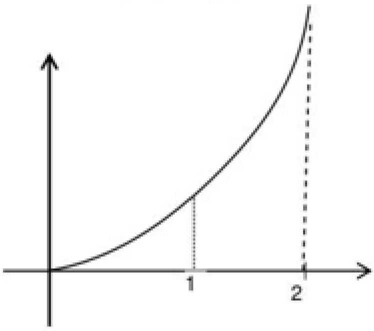
$g'(x)=f(x)$

$g''(x)=f'(x)>0$

$\therefore$ $g(x)$ is concave upwards with $g(0)=0$

given equation is $g(2)=2g(c) \Rightarrow g(c)=\frac{1}{2}g(2)$

Assume that graph of $g(x)$ looks like this



So c is always greater than 1 and is unique.

IIT – JEE: 2025 ADVANCED | **MAJOR TEST - 5** | **DATE: 04/08/24**

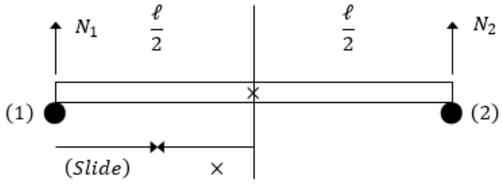
ANSWER KEY

PAPER – 2 (Code – 21)

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	B	**18.**	A	**35.**	D
2.	D	**19.**	D	**36.**	A
3.	D	**20.**	B	**37.**	A
4.	C	**21.**	D	**38.**	C
5.	AC or A, B, C, D	**22.**	A, B, C, D	**39.**	B, C
6.	A, B, C, D	**23.**	A, C, D	**40.**	C, D
7.	A, B, D	**24.**	A, B, C, D	**41.**	A, B
8.	5	**25.**	4	**42.**	1
9.	5	**26.**	22	**43.**	771
10.	3 or 9	**27.**	20	**44.**	0
11.	13	**28.**	6	**45.**	1
12.	90	**29.**	6	**46.**	3
13.	20	**30.**	11	**47.**	3
14.	9	**31.**	5.52	**48.**	4.5
15.	1900	**32.**	9.35	**49.**	27.5
16.	3	**33.**	0.25	**50.**	0.5
17.	3	**34.**	1.16 to 1.17	**51.**	4

PART (A) : PHYSICS

1. (B)



$N_1 + N_2 = mg$

$N_1 x = N_2 y$

$N_1 + \frac{N_1 x}{y} = mg$

$N_1 = \frac{mgy}{x+y}$

While (1) slides static friction at N_2 is sufficient to prevent sliding rod. When $\rho_1 = \rho_2$, slides will witch.

$N_1 x = N_2 \frac{\ell}{2}$ at switch friction $N_1 > N_2$

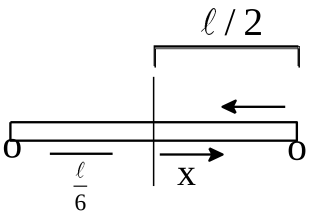
Then $\left(\frac{\mu}{3}\right) N_1 = \mu N_2 \rightarrow f_{N_2}$ is no longer able to counter $f_1 \Rightarrow$ m now (2) will slide

$N_1 = 3N_2$

$x(3N_2) = N_2 \frac{\ell}{2}; n = \frac{\ell}{6}; \frac{\ell}{2} \rightarrow \frac{\ell}{6}$

Friction $= \frac{\mu}{3} N_1 \Rightarrow \frac{\mu}{3}\left(\frac{mg\left(\frac{\ell}{2}\right)}{\frac{\ell}{2}+x}\right) = \frac{\mu mg\ell}{3(\ell+2x)}$

Work done $\Rightarrow \int_{\frac{\ell}{6}}^{\frac{\ell}{2}} \frac{\mu ms\ell}{3(\ell+2x)} dx$



$N_1 \times \frac{\ell}{6} = N_2 \times x \quad \Rightarrow x = \frac{\ell}{18}; x = \frac{y}{3}$

$\mu N_1 = \left(\frac{\mu}{3}\right) N_2$

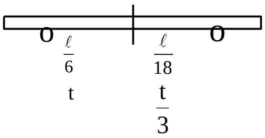
$N_2 = 3N_1$

$\frac{\ell}{2}$ to $\frac{\ell}{18}$

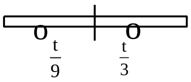
$$\text{Work n } = \frac{\mu}{3}\left(\frac{msy}{x+y}\right)$$

$$\Rightarrow \frac{\mu mg}{3}\left(\frac{\ell/6}{x+\ell/6}\right) = \frac{\mu mg}{3}\left(\frac{\ell}{6x+\ell}\right)$$

$$\int_{\ell/18}^{\ell/2} \frac{\mu mg\ell}{3(\ell+6x)}$$



$$\frac{\mu mg}{3}\left(\frac{t/3}{x+t/3}\right) = \int_{t}^{t/9} \frac{\mu mgtdx}{3(3x+t)}$$



$$\frac{\mu ms}{3}\left(\frac{t/9}{x+t/9}\right)$$

$$= \int_{\frac{t}{3}}^{\frac{t}{27}} \frac{\mu mtdx}{3(9x+t)} \quad \text{let } 3x = z$$

$$\int_{z}^{\frac{z}{9}} \frac{\mu mt}{3(3z+t)} \quad (3) \quad \frac{1}{3} \text{ factor each friction}$$

$$\text{Work done} \Rightarrow = \int_{\frac{\ell}{6}}^{\frac{\ell}{2}} \frac{\mu mg\ell\, dx}{3(\ell+2x)} + \int_{\frac{\ell}{6}}^{\frac{\ell}{2}} \frac{\mu mg\ell\, dx}{3(\ell+6x)} + \int_{\frac{t}{9}}^{t} \frac{\mu mgt\, dx}{3(t+3x)}\left(1+\frac{1}{3}+\frac{1}{9}+...\right)$$

$$= \frac{\mu mg}{6}\left(\ln\left(x+\frac{\ell}{2}\right)\right)_{\frac{\ell}{6}}^{\frac{\ell}{2}} + \frac{\mu mg\ell}{3\times 6}\ln\left(x+\frac{\ell}{6}\right)_{\frac{\ell}{18}}^{\frac{\ell}{2}} + \frac{\mu mg\ell}{3\times 6-3}\ln\left(t+\frac{t}{3}\right)\Big|_{t/9}^{t}\left(\frac{1}{1-\frac{1}{3}}\right)$$

$$= \frac{\mu mg\ell}{6}\ln\left(\frac{1}{\frac{4\ell}{6}}\right) + \frac{\mu mg\ell}{6\times 3}\ln\left(\frac{\frac{4\ell}{6}}{\frac{4\ell}{18}}\right) + \frac{\mu mg\ell}{18\times 3}\ln\left(\frac{\frac{4t}{3}}{\frac{4t}{9}}\right)\cdot\left(\frac{3}{2}\right)$$

$$= \frac{\mu mg\ell}{6}\ell n\left(\frac{3}{2}\right) + \frac{\mu mg\ell}{18}\ell(3) + \frac{\mu mg\ell}{18}\times\frac{1}{2}\ln(3)$$

$$= \frac{\mu mg\ell}{6}\left[\ln\left(\frac{3}{2}\times(3)^{1/3}\times(3)^{1/6}\right)\right] = \frac{\mu mg\ell}{6}\ln\left(\frac{3\sqrt{3}}{2}\right)$$

2. (D)

In the first case, $E = \frac{v}{d}$ will develop in the lower region. Let us assume lower plate to be at 0 potential as

$v_a - v_b$ will not depend on this

$v_b = \frac{v}{d} \times \frac{d}{4} = \frac{v}{4}$

$va = v$

$v_a - v_b = \frac{3v}{4}$

In the second scenario, $E = \frac{v}{\left(\frac{d}{2}\right)} = \frac{2v}{d}$

$v_a = v$ and $v_b = \frac{2v}{d} \times \frac{d}{4} = \frac{v}{2}$

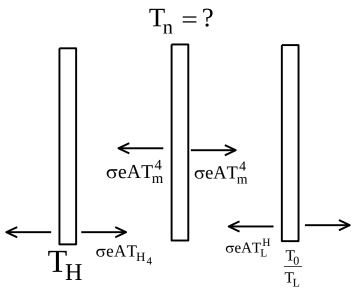
$v_a = v_b = \frac{v}{2}$

The ratio in both cases is $\frac{3}{2}$

D is the correct ans

3. (D)
Conceptual

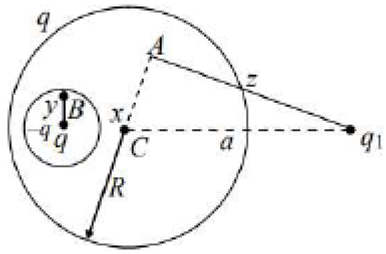
4. (C)



$\sigma eAT_H^4 + \sigma eAT_L^4 = 2\sigma eAT_m^4$

$T_m = \left[\frac{T_H^4 + T_L^4}{2}\right]^{1/4}$

5. (AC or ABCD)
The charge distribution is shown in figure.
Due to q_1, the charge induction is non-uniform but net induced charge due to q_1 is zero.
At point A field is zero and potential is equal to the potential at center.



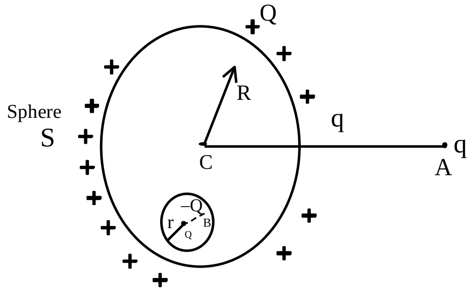
$$V_A = V_C = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a}$$

At point B, field is due to charge q only

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}$$

At point B, potential is due to charge q at center of cavity, $-q$ induced on inner surface of cavity in addition to the potential at the center.

$$\therefore V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a} + \frac{q}{R} + \frac{q}{y} - \frac{q}{r}\right].$$



$$V_B = V_{Q,\text{center}} + V_{(-Q),\text{ind}} + V_{+Q,\text{ind}} + V_{q_1A}$$

$$= \frac{kQ}{y} + \frac{k(-Q)}{r} + \left(V_{+Q,\text{ind}} + V_{q_1A}\right)$$

$$\left.\begin{aligned} V_{Q_{\text{ind}},}B - V_\infty^0 &= -\int_\infty^B \vec{E}_Q \cdot \overrightarrow{d\ell} \\ V_{q_1A} - V_\infty^0 &= -\int_\infty^B \overrightarrow{E_q}.\overrightarrow{d\ell} \end{aligned}\right\}$$

$$V_{Q_{\text{ind}},B} + V_{q_1A} = -\int_\infty^B \left(\overrightarrow{E_q} + \overrightarrow{E_Q}\right)\cdot\overrightarrow{d\ell}$$

$$= -\int_\infty^S \left(\overrightarrow{E_q} + \overrightarrow{E_Q}\right)\cdot\overrightarrow{d\ell} - \left.\int_S^B \left(\overrightarrow{E_Q} + \overrightarrow{E_q}\right)\cdot\overrightarrow{d\ell}\right\} \to 0$$

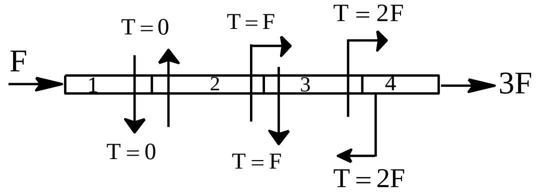
$$-\int_S^C \left(\overrightarrow{E_Q} + \overrightarrow{E_q}\right)\cdot\overrightarrow{d\ell} = -\int_\infty^C \left(\overrightarrow{E_Q} + \overrightarrow{E_q}\right).d\ell$$

$$= V_{Q,C} + V_{q,C}$$

$$\therefore \left(V_{+Q_{\text{ind}}+V_{q_1A}}\right)_B = \left(V_{Q_{\text{ind}}} + V_{2A}\right)_C = \frac{kq}{a} + \frac{kQ}{R}$$

$$\therefore V_B = \frac{kQ}{y} - \frac{kQ}{r} + \frac{kQ}{R} + \frac{kq}{qa}$$

6. (ABCD)



$a = \frac{4F}{M}$; net force on each piece

$\Rightarrow F_{net} = \frac{4F}{M} \times \frac{M}{4} = F$

$\Delta\ell = \frac{(F_1 + F_2)}{2Ay}$ for

(Conceptual)

$\Rightarrow \Delta\ell_1 = \frac{-F}{2Ay} \quad \Delta\ell_2 = \frac{+F}{2Ay} \quad \Delta\ell_3 = \frac{3F}{2Ay}$

$\Delta\ell_4 = \frac{5F}{2Ay} \quad \Delta_{net} = \frac{8F}{2AY}$

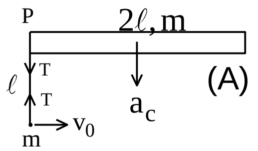
$\therefore \Delta\ell_1 = \frac{\Delta\ell_3}{3} = \frac{\Delta\ell}{8}$

$\Delta\ell_2 = \frac{\Delta\ell_4}{5} = \frac{\Delta\ell}{8}$

$U_1 < U_3 < U_4$ by $\Delta\ell_1 < \Delta\ell_3 > \Delta\ell_4$

$U_1 < \frac{U}{4} < U_4$

7. (ABD)



$a_c = \frac{7}{m}$

$a_p = \frac{T}{m} + \alpha \cdot \ell = \frac{47}{m}$

$T.\ell = \frac{m(2\ell)^2}{12} \cdot \alpha \Rightarrow \alpha = \frac{37}{m\ell}$

wrt P particle perform circular motion of radius $= \ell$

$\Rightarrow T + \left(\frac{4T}{m}\right)m = \frac{mv_0^2}{\ell}$

$T = \frac{mv_0^2}{5\ell}$

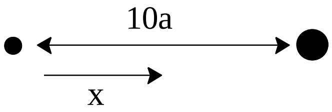
$\alpha = \frac{3v_0^2}{8\ell^2} + 8 \quad \omega = 0$

$$a_c = \frac{v_0^2}{5\ell}$$

8. (5)

Bigger star will pull the satellite m as soon as its force exceeds the force exerted by smaller star.

$\therefore\ F_{bigger} = F_{smaller}$



$$\frac{G(16M)m}{(10a-x)^2} = \frac{GMm}{x^2}$$

$\Rightarrow\ 16x^2 = (10a-x)^2$

$\Rightarrow\ 4x = 10a - x$

$\Rightarrow\ 5x = 10a$

$\Rightarrow\ x = 2a$

Now, change in potential energy = Kinetic energy of satellite

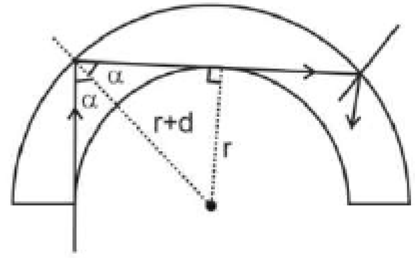
$$\therefore\ \left[\frac{-GMm}{x} - \frac{G(16M)m}{(10a-x)}\right] - \left[\frac{-GM(m)}{a} - \frac{G(16M)m}{9a}\right] = KE$$

$$KE = \frac{GMm}{a}\left[1 + \frac{16}{9} - \frac{1}{2} - 2\right]$$

$$= \frac{GMm}{a}\left[\frac{5}{18}\right]$$

$\therefore\ x = 5$

9. (5)



$$\sin\alpha = \frac{r}{r+d}$$

For TIR

$$\sin\alpha \ge \frac{1}{\mu}$$

$$\frac{10}{10+d} \ge \frac{1}{1.5}$$

$15 \ge 10 + d$

$d_{max} = 5cm$

10. (3 or 9)

$K = 1\times10^{-11}$ N-m/deg.

$P = 3mW = 3\times10^{-3}W$

$\ell = 6\ cm = 0.06\ m$

Now, $CK\theta = P\frac{\ell}{2}$

Where $C = 3\times10^{8}$ m/s

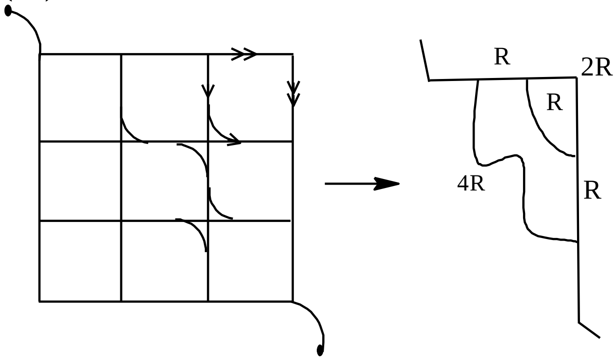
$\therefore\ 3\times10^{8}\times1\times10^{-11}\times\theta = 3\times10^{-3}\times0.06$

$\theta = 0.03$ degrees

$\theta = 3\times10^{-2}$ degrees

$\therefore\ \alpha = 3$

11. (13)



$\frac{12R}{7} + 2R$

$= \frac{26R}{3}$

$R_{resis} = \frac{13R}{7}$

12. (90)

13. (20)

$2\mu_0 t = \frac{\lambda}{2}$

$t = \frac{\lambda}{4\mu_0} = 20$

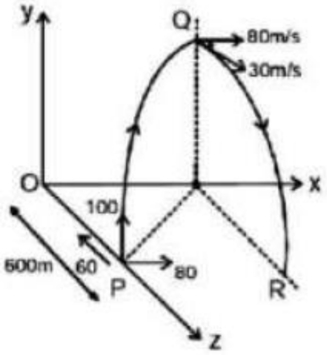
14. (9)

Since wall is smooth time of flight remains unchanged

$T = \frac{2u_y}{g} = \frac{2\times100}{10} = 20s$

The particle reaches the wall

at $t = \frac{600}{60}s = 10s$



At $t = 10s$, the particle is at its maximum height. Hence, $t = 10s$ and $\theta = 90°$
Let Q be the point of impact on the wall. So its X-co-ordinate $= 80 \times 10 = 800$ m

Y-co-ordinate $= \frac{u_y^2}{2g} = \frac{(100)^2}{2 \times 10} = 500\,m$

Z-co-ordinate = 0

$\vec{V}_Q = 80\hat{i} + e \times 60\hat{k} = (80\hat{i} + 30\hat{k})\,m/s$

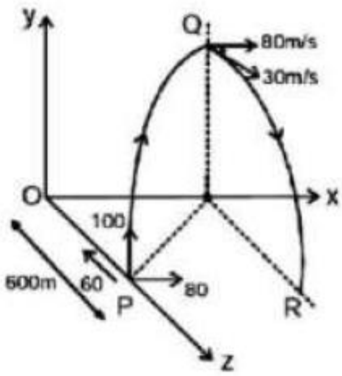
And time to reach the XZ-plane is 10s from Q
X-co-ordinate $= 80 \times 20 = 1600$ m
Y-co-ordinate = 0
Z-co-ordinate $= 30 \times 10 = 300\,m$

15. (1900)
Since wall is smooth time of flight remains unchanged

$T = \frac{2u_y}{g} = \frac{2 \times 100}{10} = 20s$

The particle reaches the wall

at $t = \frac{600}{60} s = 10s$



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Z-co-ordinate = 0

$\vec{V}_Q = 80\hat{i} + e \times 60\hat{k} = (80\hat{i} + 30\hat{k})\,m/s$

And time to reach the XZ-plane is 10s from Q
X-co-ordinate $= 80 \times 20 = 1600$ m

Y-co-ordinate = 0
Z-co-ordinate $= 30 \times 10 = 300\,\text{m}$

16. (3)

$$E_1 = E_0 \times \frac{x_1}{100}$$

$$= 10 \times \frac{30}{100} = 3$$

17. (3)

Current will flow in ABCD

$$i = \frac{E_1}{r + R}$$

$$V_{AD} = E_1 - \left(\frac{E_1}{r + R}\right) \cdot r$$

$$= \frac{E_1 R}{r + R}$$

$$= x_2 \times \frac{E_0}{\ell}$$

But $E_1 = x_1 \times \dfrac{E_0}{\ell}$

$$\frac{x_1 E_0}{\ell} = \frac{x_2 E_0}{\ell}\left(\frac{R}{r + R}\right)$$

$$\frac{x_2}{x_1} = \frac{r + R}{R} = 1 + \frac{r}{R}$$

$$r = R\left(\frac{x_2}{x_1} - 1\right) = 10 \times \left(\frac{5}{20}\right) = 3$$

PART (B) : CHEMISTRY

18. (A)
19. (D)
20. (B)
21. (D)
22. (ABCD)
23. (ACD)
24. (ABCD)
25. (4)
26. (22)
27. (20)
28. (6)
29. (6)
30. (11)
31. (5.52)
32. (9.35)
33. (0.25)
34. (1.16 to 1.17)

PART (C) : MATHEMATICS

35. (D)

Image of A in $y=x$ lies on $BC = A^1(3,1)$

Image of A in $y=-2x$ lies on $BC = A^{11}(-3,1)$

Equation BC = y = 1

Ex centre opposite to $A \to I_1(0,0)$

AD $\to$ Equation $y=3x$

$$\Rightarrow D=\left(\frac{1}{3},1\right)$$

$$\frac{AI_1}{I_1D}=\frac{3}{1} \Rightarrow \frac{AI}{ID}=\frac{3}{1}$$

$$\therefore I=\left(\frac{1}{2},\frac{3}{2}\right)$$

36. (A)

Since the parabolas are the same shape, the "rolling" parabola will always be the reflection of the stationary parabola over the tangent line at the point of contract Hence, the focus of rolling Parabola will be reflection of Focus of fixed Parabola about any arbitrary common tangent to both these Parabola. As reflection of focus about any tangent to Parabola lies on its Directrix and hence Focus of rolling Parabola lies in directrix of fixed Parabola. Hence, Focus of rolling Parabola must lie on Directrix of $y=x^2-x+1s$. Hence, locus is y = 1.

37. (A)

Let the focus be (h, k) then equation of the tangent at vertex is $y=-\frac{k}{h}(x-h)$

i.e. $kx+hy-hk=0$

its distance from $(h,k)=\left|\frac{hk}{\sqrt{h^2+k^2}}\right|=a$

$\therefore$ the locus is $a^2(x^2+y^2)=x^2y^2$

i.e. $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{a^2}$, $x>0, y>0$

38. (C)

We have,

$$y=\tan^{-1}\left(\frac{\log e-\log x^2}{\log e+\log x^2}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$=\tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$=\tan^{-1}1-\tan^{-1}(2\log x)+\tan^{-1}3+\tan^{-1}(2\log x)$$

$$=\tan^{-1}1+\tan^{-1}3$$

$\Rightarrow \dfrac{dy}{dx}=0 \Rightarrow \dfrac{d^2y}{dx^x}=0$

39. (B), (C)

If $f(a)=f(b)\rightarrow f(f(a))=f(f(b))$ then $-a=-b\rightarrow f$ in one-one.

Since $f(a)=-x$ for $a=f(x)$, is covers whole of R, $\Rightarrow f$ is onto.

If function continuous, then since it is bijective it must monotonous.

$\Rightarrow f(f(x))>f(f(y))\ \forall\ x>y$ (say)

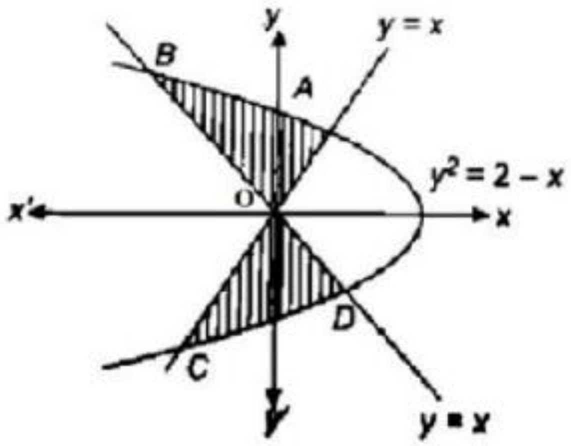
$\Rightarrow -x>-y\ \forall\ x>y$

$y<x\ \forall\ x>y$

Contradiction

$\Rightarrow$ f is cannot be continuous.

40. (C), (D)



Let A = (1, 1) & B = (–2, 2)

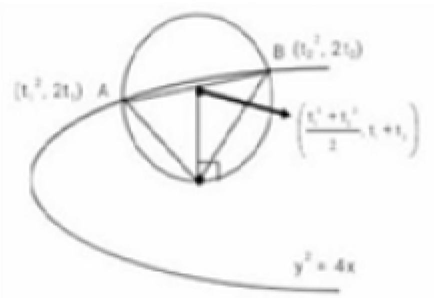
Maximum value of x^2+y^2 corresponding to B is 8

Minimum value of x^2+y^2 corresponding to O to 0

Integral points is R arc : (1, 1) (1, –1) (0, 0) (0, 1) (0, –1) (–1, 1) (–1, –1) (–2, 2) (–2, –2) $\Rightarrow$ 9 points

41. (A), (B)

Slope of $PV=\dfrac{2t-0}{t^2-0}=\dfrac{2}{t}$



Equaiton of QV is $y=-\dfrac{t}{2}(x)$

On solving with $y^2=4x$, $Q=\left(\dfrac{16}{t^2},\dfrac{-8}{t}\right)$

Area of ΔPVQ is $\frac{1}{2}PV.VQ = 20$

$\Rightarrow PV.VQ = 40$

B solving above equaiton $t = \pm 4, \pm 1$

42. (1)

$x = y$ is the line equidistant from (0, 3) & (3, 0)

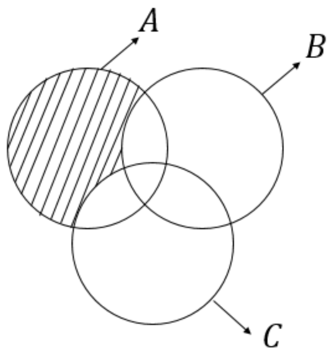
$\therefore P$ has to lie on $x = y$ as well as $3x + 4y + 1 = 0$

Solving x = y & 3x + 4y + 1 = 0, we get

$(a, b) = \left(-\frac{1}{7}, -\frac{1}{7}\right)$

$\therefore \frac{a}{b} = 1$

43. (771)



$A \equiv$ Divisible by 3

$B \equiv$ Divisible by 5

$C \equiv$ Divisible by 7

$\therefore$ Reqd $= n(A) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$n(A) \Rightarrow$ Divisible by = 3 = 333

$n(A \cap B) \Rightarrow$ Divisible by 15 = 66

$n(B \cap C) \Rightarrow$ Divisible by 21 = 47

$n(A \cap B \cap C) \Rightarrow$ Divisible by 105 = 9

Required $\Rightarrow$ 229

$\therefore m = 229; n = 1000$

44. (0)

This family of lines passes through a fixed point (0, 2) which is also the focus of the given parabola.

$\therefore$ The shortest intercept will be the latus rectum whose equation is $y = 2$

$\therefore m = 0$

45. (1)

46. (3)

Equation of the normal is

$ax\cos\theta + by\cot\theta = a^2 + b^2$

The normal at P meets the coordinates axes at $G\left(\frac{a^2+b^2}{a}\sec\theta, 0\right)$ and $g\left(0, \frac{a^2+b^2}{b}\tan\theta\right)$

$$\therefore\ PG^2 = \left(\frac{a^2+b^2}{a}\sec\theta - a\sec\theta\right)^2 + (b\tan\theta - 0)^2$$

$$PG^2 = \frac{b^2}{a^2}\left(b^2\sec^2\theta + a^2\tan^2\theta\right)$$

When $\tan\theta = 0$

$$PG = \frac{b^2}{a}$$

47. (3)

48. (4.5)

49. (27.5)

$$S_1 = \sum_{K=1}^{10} K(K-1)^{10}C_K = 90\sum_{K=1}^{10} {}^8C_{K-2} = 90\cdot 2^8 = 45\cdot 2^9$$

$$S_2 = \sum_{K=1}^{10} K\cdot {}^{10}C_K = 10\sum_{K=1}^{10} {}^9C_{K=1} = 10\cdot 2^9$$

$$S_3 = S_1 + S_2 = 55\cdot 2^9$$

50. (0.5)

51. (4)

$$x_n + y_n = 4(x_{n-1} + y_{n-1})$$

$$= 4^2(x_{n-2} + b_{n-2})$$

$$\ldots\ldots = 4^n(x_0 + y_0) = 6.4^n = x_n + y_n$$

$$y_n - x_n = 2y_{n-1}$$

Place $x_n = 6\cdot 4^n - y_n$

$$\Rightarrow 2y_n - 6.4^n = 2y_{n-1}$$

$$y_n - y_{n-1} = 3.4^n$$

$$\Rightarrow y_n - y_0 = 3\left(4 + 4^2 + 4^3 + \ldots. + 4^n\right)$$

$$y_n = 3\left[1 + 4 + 4^2 + \ldots\ldots.4^n\right] = 3\cdot\left(\frac{4^{n+1}-1}{4-1}\right) = 4^{n+1} - 1$$

$$y_n = 4^{n+1} - 1 \Rightarrow x_n = 2.4^n + 1$$