TIME: 1 Hr. **OLYMPIAD SELECTION TEST** **DATE: 04/08/24**

PHYSICS

ANSWER KEY

1. (D)
2. (C)
3. (A)
4. (A)
5. (A)
6. (C)
7. (AD)
8. (BCD)
9. (ABD)
10. (AB)
11. (C)
12. (B)
13. (B)
14. (D)
15. (7)
16. (3)
17. (3)
18. (3)
19. (1)

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PHYSICS **MARKS: 66**

SOLUTIONS

1. (D)

Ranges for complementary angles are same

$\therefore$ Required angle $\frac{\pi}{2}-\frac{5\pi}{36}=\frac{13\pi}{36}$

2. (C)

$v=(180-16x)^{1/2}$

As $a=\frac{dv}{dt}=\frac{dv}{dx}\cdot\frac{dx}{dt}$

$\therefore a=\frac{1}{2}(180-16x)^{-1/2}\times(-16)\left(\frac{dx}{dt}\right)$

$-8(180-16x)^{-1/2}\times(180-16x)^{1/2}=-8m/s^2$

3. (A)

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right]$

$=\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\right)\right]$

$=\frac{d}{dx}\left[\tan^{-1}\left[\frac{1-\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right]\right]=\frac{d}{dx}\left[\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]=-\frac{1}{2}$

4. (A)

Force is parallel to a line $y=\frac{3}{2}x+c$

The equation of given line can be written as

$y=-\frac{k}{3}x+\frac{3}{5}$.

Work done will be zero, when it is perpendicular to the displacement i.e., the above two lines are perpendicular or $m_1m_2=-1$

Or $\left(\frac{3}{2}\right)\left(-\frac{k}{3}\right)=-1$

Or $k=2$

5. (A)

$\vec{F} = k\left(v_y \hat{i} + v_x \hat{j}\right)$

$\therefore F_x = kv_y \hat{i}, F_y = kv_x j$

$\frac{mdv_x}{dt} = kv_y \Rightarrow \frac{dv_x}{dt} = \frac{k}{m} v_y$

Similarly, $\frac{dv_y}{dt} = \frac{k}{m} v_x$

$\frac{dy_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dx_y = \int v_x dv_x$

$v_y^2 = v_x^2 + C$

$v_y^2 - v_x^2 =$ constant

$\vec{v} \times \vec{a} = \left(v_x \hat{i} + v_y \hat{j}\right) \times \frac{k}{m}\left(v_y \hat{i} + v_x \hat{j}\right)$

$\vec{v} \times \vec{a} = \left(v_x \hat{i} + v_y \hat{j}\right) \times \frac{k}{m}\left(v_y \hat{i} + v_x \hat{j}\right)$

$= \left(v_x^2 k - v_y^k k\right)\frac{k}{m} = \left(v_x^2 - v_y^2\right)\frac{k}{m} k =$ constant

6. (C)

Maximum height of ball $= 5m$

So velocity of projection $\Rightarrow u = \sqrt{2gh} = 10 m/s$

Time interval between two balls (time of ascent)

$= \frac{u}{g} = 1 sec = \frac{1}{60 \min}$.

So number of all thrown per min $= 60$

7. (AD)

The mirror is in XY-plane. So, normal will be perpendicular to XY-plane (i.e.z-axis)

If angle of incidence is θ. Then $\cos\theta = \left(\hat{e}_1 \cdot n\right)$

Here, $\hat{e}_1 =$ unit vector along incidence ray

$$= \frac{\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} + k}{\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 + 1^2}} = \frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{\sqrt{2}} k$$

$n =$ unit vector along normal on incidence point $= k$

$\therefore \quad \cos\theta = \left|\hat{e}_1 \cdot n\right| = \frac{1}{\sqrt{2}} \Rightarrow 0 = 45^\circ$

8. (BCD)

The two particles will collide after time t if the distance moved along x-axis of A is equal to that of B and distance moved along y –axis of A is equal to that of B

So, $u_1 t = \frac{1}{2} a_2 t^2$ and $u_2 t = \frac{1}{2} a_1 t^2$

$\therefore \quad \frac{u_1}{u_2} = \frac{a_2}{a_1}$ or $a_1 u_1 = a_2 u_2$

For first particle, $x = u_1 t, y = \frac{1}{2}a_1 t^2$

Or $y = \frac{a_1}{2}\left(\frac{x}{u_1}\right)^2 \Rightarrow x^2 = \frac{2u_1^2}{a_1}y$

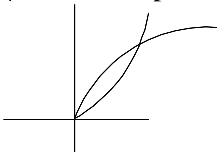
(upward opening parabola)

For the second particle,

$x = \frac{1}{2}a_2 t^2, y = u_2 t$

$\therefore \quad x = \frac{a_2}{2}\left(\frac{y}{u_2}\right)^2 \Rightarrow y^2 = \frac{2{u_2}^2}{a^2}x$

(Leftward opening parabola)



The point of intersection $x^2 = \frac{2u_1^2}{a_1}y$ and $y^2 = \frac{2u_2^2}{a_2}x$ are

(0,0) and $\left(\sqrt[3]{\frac{8u_1^4u_2^2}{a_2a_1^2}}, \frac{a_1}{2u_1^2}\left(\frac{8u_1^4u_2^2}{a_2a_1^2}\right)^{2/3}\right)$

So, their paths must intersect at some point. The particles will have same speed at some point after time t if, $\sqrt{u_1^2 + (a_1t)^2} = \sqrt{u_2^2 + (a_2t)^2}$

$u_1^2 - u_2^2 = (a_2^2 - a_1^2)t^2$

$t = \sqrt{\frac{u_1^2 - u_2^2}{a_2^2 - a_1^2}}$

T will be positive if $u_1 > u_2$ and $a_2 > a_1$

9. (ABD)

Let u_x and u_y be horizontal and vertical comments of velocity respectively at t $= 0$.Then , $v_y = u_y - gt$

Hence, $v_y - t$ graph is straight line

$x = v_x t$

Hence, graph is straight line passing through origin.

The relation between y and $y = u_y t - \frac{1}{2}gt^2$

Hence, y-t graph is parabolic.

$v_x =$ constant

Hence - t $v_x - t$ graph is straight line

10. (AB)

$\because$ Particle is in equilibrium.

$\sum F_x = 0$ and $\sum F_y = 0$

$\therefore$ The sum of x-component is

$\sum F_x = 10\cos 30^\circ + F_2 \sin 30^\circ - 15\sin 53^\circ = 0$

Or $5\sqrt{3}+\frac{F_2}{2}-12=0$

$\therefore F_2 = 24-10\sqrt{3} = 24-17 = 7N$

The sum of y-component of force is

$$\sum F_y = F_2\cos 30^\circ + 10\sin 30^\circ + 15\cos^\circ - F_1 = 0$$

Or $\frac{\sqrt{3}F_2}{2}+5+15\times\frac{3}{5}-F_1=0$

Or $\frac{\sqrt{3}}{2}\left(24-10\sqrt{3}\right)+5+9-F_1=0$

Or $12\sqrt{3}-15+14-F_1=0$

$\therefore$ $F_1 = 12\sqrt{3}-1 = 20.4-1 = 19.4N$

11. (C)

As $v = 12t - 3t^2 = 0$ at $t = 4\,\text{sec}$

That means the direction of motion reverses at $4\,\text{sec}$

(position of particle)

$x_3 = 27m\left(\text{at } t = 3s\right)$

$x_4 = 32m\left(\text{at } t = 4s\right)$

$x_5 = 25m\left(\text{at } t = 5s\right)$

Distance $= 32-27+32-25 = 12m$

12. (B)

$x_0 = 0, x_4 = 32m, x_6 = 0$

Total distance travelled $= 32m + 32m = 64m$

$$\text{Average speed} = \frac{\text{TotalDistanced travelled}}{\text{Total time}}$$

$$= \frac{64m}{6s} = \frac{32}{3}\,m/s$$

13. (B)

The position of the particle, $\vec{r} = \left(5t^2\hat{i} + 15t^2\hat{j}\right)m$

$\frac{d\vec{r}}{dt} = \left(10t\hat{i} + 30t\hat{j}\right)m/s$

Acceleration, $\frac{d^2\vec{r}}{dt^2} = \left(10\hat{i} + 30\hat{j}\right)m/s^2$

14. (D)

The position of the particle, $\vec{r} = \left(5t^2\hat{i} + 15t^2\hat{j}\right)m$

$\frac{d\vec{r}}{dt} = \left(10t\hat{i} + 30\hat{j}\right)m/s$

Acceleration, $\frac{d^2\vec{r}}{dt^2} = \left(10\hat{i} + 30\hat{j}\right)m/s^2$

$\vec{F_1} + \vec{F_2} + \vec{F_3} = m\vec{a}$

$$\Rightarrow 100\hat{i}+\left(F_2\cos30^\circ \hat{i}+F_2\sin30^\circ \hat{j}\right)+\left(F_1\cos150^\circ \hat{i}+F_1\sin150^\circ \hat{j}\right)=10\left(10\hat{i}+30\hat{j}\right)$$

$$100\hat{i}+F_2\left(\sqrt{\frac{3}{2}}\hat{i}+\frac{1}{2}\hat{j}\right)-F_1\left(\frac{\sqrt{3}}{2}\hat{i}-\frac{1}{2}\hat{j}\right)=100\hat{i}+300\hat{j}$$

$$\left(100+\frac{\sqrt{3}}{2}F_2-\frac{\sqrt{3}}{2}F_1\right)\hat{i}+\left(\frac{1}{2}F_2+\frac{1}{2}F_1\right)\hat{j}+100\hat{i}+300\hat{j}$$

$$\left(100+\frac{\sqrt{3}}{2}F_2-\frac{\sqrt{3}}{2}F_1\right)=100\Rightarrow F_2=F_1$$

$$\left(\frac{F_1}{2}+\frac{F_2}{2}\right)=300\Rightarrow F_1=F_2=300\text{N}$$

15. (7)

$$a_u=\mu g=2.5\text{m/s}^2$$

$$a_L=\frac{12.5}{4}\text{m/s}^2$$

$$a_{u/L}=a_u-a_L=\frac{10}{4}-\frac{-12.5}{4}=\frac{-2.5}{4}$$

$$s_{u/L=-1}=0+\frac{1}{2}a_{u/L}t^2\Rightarrow t=\frac{4}{\sqrt{5}}$$

16. (3)

$$\frac{s_1}{v_1}+\frac{s_2}{v_2}=\frac{s_1+s_2}{\sqrt{v_1v_2}}\ \text{(given)}$$

$$\Rightarrow \frac{\left(\frac{s_1}{s_2}\right)}{v_1}+\frac{1}{v_2}=\frac{\left(\frac{s_1}{s_2}\right)+1}{\sqrt{v_1v_2}}$$

Let $\frac{s_1}{s_2}=\lambda$

$$\Rightarrow \frac{\lambda}{v_1}+\frac{1}{v_2}=\frac{\lambda+1}{\sqrt{v_1v_2}}$$

$$\Rightarrow \lambda\left[\frac{1}{v_1}-\frac{1}{\sqrt{v_1v_2}}\right]=\frac{1}{\sqrt{v_1v_2}}-\frac{1}{v_2}$$

$$\Rightarrow \lambda\left[\frac{\sqrt{v_2}}{v_1\sqrt{v_2}}-\frac{\sqrt{v_1}}{v_1\sqrt{v_2}}\right]=\frac{\sqrt{v_2}}{v_2\sqrt{v_1}}-\frac{\sqrt{v_1}}{v_2\sqrt{v_1}}$$

$$\Rightarrow \frac{\lambda\left(\sqrt{v_2}-\sqrt{v_1}\right)}{v_1\sqrt{v_2}}=\frac{\sqrt{v_2}-\sqrt{v_1}}{v_2\sqrt{v_1}}$$

$$\Rightarrow \frac{\lambda}{v_1\sqrt{v_2}}=\frac{1}{v_2\sqrt{v_2}}\Rightarrow \lambda=\frac{v_1\sqrt{v_2}}{v_2\sqrt{v_1}}$$

$$\Rightarrow \lambda=\sqrt{\frac{v_1}{v_2}}$$

$$\Rightarrow \frac{s_1}{s_2}=\sqrt{\frac{v_1}{v_2}}$$

$\therefore \frac{s_1}{s_2} = \sqrt{\frac{90}{40}} = 1.5 \Rightarrow 2\left(\frac{s_1}{s_2}\right) = 3$

17. (3)

Relative displacement = relative velocity $\times$ time

$8 \times 3 = (10-2)t \Rightarrow 3\text{sec}$

18. (3)

We have, $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a} \cdot \vec{b}\right)^2 = 144$

$|\vec{a}|^2 |\vec{b}|^2 \sin^2\theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta = 144$

$|\vec{a}|^2 |\vec{b}|^2 = 144; |\vec{b}| = 3 \qquad \left[\because |\vec{a}| = 4\right]$

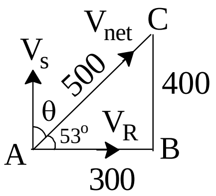
19. (1)

Time taken by Ram $\left(t_{Ram}\right) = \frac{700\text{m}}{7\text{m}} = 100\,\text{sec}$

To swim m across AC

$V_s \sin\theta = V_R \sin 53$

$4\sin\theta = 3 \times \frac{4}{5}$



$\sin\theta = \frac{3}{5} = \theta = 37^\circ$

$V_{net} = V_s \cos\theta + V_R \cos 53$

$= 4x\frac{4}{5} + 3\frac{3}{5} = \frac{25}{5} = 5\text{ m/s}$

$t_{shyam} = \frac{500}{5} = 100\,\text{sec}$

$\frac{t_{Ram}}{t_{shyam}} = \frac{100}{100} = 1$

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CHEMISTRY

ANSWER KEY

20. (A)
21. (C)
22. (B)
23. (B)
24. (A)
25. (B)
26. (C)
27. (ABC)
28. (ACD)
29. (B)
30. (C)
31. (A)
32. (B)
33. (C)
34. (125)
35. (28)
36. (5)
37. (200)
38. (2)

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CHEMISTRY

SOLUTIONS

20. (A)

$Ag_2CO_3 \rightarrow 2Ag + CO_2 + \frac{1}{2}O_2$

276 g Ag_2CO_3 gives $2 \times 108 = 216$ g residue

2.76 g Ag_2CO_3 gives 2.16 g residue (silver)

21. (C)

Let a g of Cu be oxidised to give CuO, ie., $\frac{(63.6+16)a}{63.6}$ g

Thus, final weight

$$= (3.18 - a) + \frac{(63.6+16)a}{63.6} = 3.92$$

$\therefore \quad a = 2.94g$

Thus, % of Cu left unoxidies

$\frac{(3.18 - 2.94)}{3.18} \times 100 = 7.55$

22. (B)

$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda} = \frac{3}{2}k_BT$ (From kinetic theory)

$\Rightarrow T = \frac{h^2}{3m\lambda}$

$$= \frac{(6.625 \times 10^{-34})^2}{3 \times 9 \times 10^{-31}(76.3 \times 10^{-9})^2 \times 1.38 \times 10^{-23}} = 2K$$

23. (B)

$M + HCl \rightarrow H_2$

$N_2 + 3H_2 \rightarrow 2NH_3$

1 mol 1 mol 2 mol

1 vol. 3 vol. 2 vol.

? 16.8 mL

3 mL H_2 combine with $= 1mL\ N_2$

16.8mL combine with $= 5.6mL\ N_2$

24. (A)
n,l and m are related to size, shape and orientation respectively .

25. (B)
Each period consist of series of element whose atom has same principal quantum number of outermost shell i.e in second period $n = 2$ this shell has 4 orbitals which have 8 electrons hence second period contain 8 element from atomic number 3 to 10.

26. (C)
Probability (P) between 0 and $2a_0 = \int_0^{2a_0} \Psi_{ls}^2 \, d\tau = 4\pi r^2 dr$.

$$\therefore P = \int_0^{2a_0} \left(\pi a_0^3\right)^{-1} e^{-2r/a0} 4\pi r^2 dr = \frac{4}{a_0^3} \int_0^{2a_0} e^{-\frac{2r}{a_0}} r^2 dr$$

$$= \frac{4}{a_0^3} a_0^3 \left[a_0^3 e^{-4} + a_0^3 e^{-4} + \frac{a_0^3}{4} \right] = -4\left(3e^{-4} + \frac{e^{-4}}{4} - \frac{1}{4} \right) = 0.7624$$

27. (ABC)
$N_2 + 3H_2 \rightarrow 2NH_3$

$\frac{140}{28}$ $\frac{40}{2}$ 0

5 mole 20 mole 0
0 5 mole 10 mole
$10 \times 17 = 170$ gram

$$\Rightarrow \% \text{ Yield} = 80 = \frac{\text{Actual Wt.}}{\text{expeted wt.}} \times 100$$

$$\frac{80}{100} = \frac{w}{30}$$

$\therefore$ $w = 24$ gram
$\Rightarrow$ N_2 (L.R) completely consumed in reaction.

$$\Rightarrow \frac{50}{100} = \frac{w}{170}$$

Expected wr × 100
$\therefore$ $w = 85$ gram .

28. (ACD)
Kinetic energy of photoelectrons is linearly related to frequency of incident radiation but has no relation with intensity of incident radiation.

29. (B)
For H atom, first Balmer line series is

$$E_3 - E_2 = \frac{-E_1(H)}{(3)^2} - \frac{E_1(H)}{(2)^2} = \frac{5E_1(H)}{36}$$

For H^+ ion (Z = 2)

$$E_6 - E_4 = -\frac{E_1(H) \times (2)^2}{6^2} - \frac{E_1(H) \times (2)^2}{4^2}$$

$$= -E_1(H) \times 2^2 \left| \frac{16 - 36}{16 \times 36} \right|$$

$$= \frac{4\times 20}{36\times 16} E_1(H) = \frac{5E_1(H)}{36}$$

30. (C)

31. (A)

32. (B)

33. (C)

34. (125)
Possible subshells are present in the 8th period: $8s\, 7d\, 6f\, 5g\, 8p$
Total electrons = 5 + 25 + 35 + 45 + 15 = 125

35. (28)
$$E = \frac{nhc}{\lambda}$$
$$10^{-17} = \frac{n\times 6.625\times 10^{-34}\times 3\times 10^{8}}{550\times 10^{-9}}$$
$n = 27.67 \quad \therefore\ 28$

36. (5)
Number of moles $= \dfrac{70\times 10^{-3}}{7}$
$= 1\times 10^{-2}$
Amount of energy required $= 1\times 10^{-2}\times 200\ \text{kJ} = 5\ \text{kJ}$

37. (200)
Let the volume of O_3 = x mL
Volume of O_2 present = (600 – x)mL
22400 mL of O_3 and O_7 at NTP will weight 48g and 32g respectively.
The weight of x mL of $O_3 = \dfrac{x\times 48}{22400}$ g
The weight of $(600-x)$ mL of $O_2 = \dfrac{(600-x)}{22400}\times 32$
The weight of ozonised O_2 (600 mL)
$$= \frac{48x}{22400} + \frac{(600-x)\times 32}{22400} = 1.0$$
$\therefore\ x = 200\ \text{mL}$

38. (2)
$$133.7 = \frac{4\times 91.2\times 3^2}{(3^2-4)\times z^2} - \frac{91.2\times 2^2}{(2^2-1)z^2}$$
On Simplification we get z = 2

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MATHEMATICS

ANSWER KEY

39. (C)
40. (B)
41. (D)
42. (A)
43. (C)
44. (A)
45. (AB)
46. (ABD)
47. (BC)
48. (BD)
49. (B)
50. (D)
51. (C)
52. (D)
53. (4)
54. (0)
55. (9)
56. (5)
57. (11)

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MATHEMATICS

SOLUTION

39. (C)

As $f(x)=x^3-3b^2x+2c^3$ is divisible by $x-a$ and $x-b$, therefore

$f(a)=0 \Rightarrow a^3-3b^2a+2c^3=0 \quad ...(i)$

and $f(b)=0 \Rightarrow b^3-3b^3+2c^3=0 \quad ...(ii)$

From (ii), $b=c$

From (i), $a^3-3ab^2+2b^3=0$ (Putting $b=c$)

$\Rightarrow (a-b)\left(a^2+ab-2b^2\right)=0$

$\Rightarrow a=b$ or $a^2+ab=2b^2$

Thus $a=b=c$ or $a^2+ab=2b^2$ and $b=c$

$a^2+ab=2b^2$ is satisfied by $a=-2b$. But $b=c$

$\therefore a^2+ab-2b^2$

and $b=c$ is equivalent to $a=-2b=-2c$

40. (B)

$\tan 3A=\tan\left(\left(A+2B\right)+2\left(A-B\right)\right)=1$

41. (D)

$2b=a+c$

$\Rightarrow 2\sin B=\sin A+\sin C$

$\Rightarrow 2\sin\frac{B}{2}=\cos\frac{A-C}{2}$

$\Rightarrow \cos\frac{A}{2}.\cos\frac{C}{2}=3\sin\frac{A}{2}.\sin\frac{C}{2}$

$\Rightarrow \tan\frac{A}{2}.\tan\frac{C}{2}=\frac{1}{3}$

$$\Rightarrow \frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}}=\frac{\cos\frac{B}{2}}{\cos\frac{A}{2}.\cos\frac{C}{2}}\cdot\frac{\sin\frac{B}{2}}{\cos\frac{B}{2}}$$

$$=\frac{\cos\frac{A+C}{2}}{\cos\frac{A}{2}.\cos\frac{C}{2}}$$

$$=1-\tan\frac{A}{2}.\tan\frac{C}{2}=1-\frac{1}{3}=\frac{2}{3}$$

42. (A)

$3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow (3\cos x - 1)(\cos x - 3) = 0$

But $\cos x \neq -3$

So $\cos x = \frac{1}{3}$

$\Rightarrow x = 2n\pi \pm \cos^{-1}\frac{1}{3}; n \in I$

43. (C)

Adding and subtracting the given relation,

we get $(m+n) = a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha + 3a\cos^2\alpha.\sin\alpha + a\sin^3\alpha$

$= a(\cos\alpha + \sin\alpha)^3$

and similarly $(m-n) = a(\cos\alpha - \sin\alpha)^3$

Thus, $(m+n)^{2/3} + (m-n)^{2/3}$

$= a^{2/3}\left\{(\cos\alpha + \sin\alpha)^2 + (\cos\alpha - \sin\alpha)^2\right\}$

$= a^{2/3}\left\{2\left(\cos^2\alpha + \sin^2\alpha\right)\right\} = 2a^{2/3}$

44. (A)

(1) $2\sin 3\theta\cos 2\theta - \sin 3\theta = 0$

$\Rightarrow \sin 3\theta(2\cos 2\theta - 1) = 0$

$\Rightarrow \sin 3\theta = 0 \Rightarrow \theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi \qquad 0 \le \theta \le \pi$

and $\cos 2\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

solutions $= 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$

(2) $\cos 5x + \cos x = 0$

$\cos x \neq 0 x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$2\cos 3x\cos 2x = 0 \; 0 < x < 2\pi$

$\Rightarrow \cos 3x = 0$

$\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

and $\cos 2x = 0$

$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Solutions: $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{3\pi}{2}, \frac{11\pi}{6}$

(3) $\tan\theta = \sqrt{3}\,\text{cosec}\,\theta = -\frac{2}{\sqrt{3}}$

$\theta = n\pi \pm \frac{\pi}{3}$

$\theta = (2n+1)\pi + \frac{\pi}{3}, 2n\pi - \frac{\pi}{3}$

But $\tan\theta$ positive in III quadrant

$\Rightarrow \theta = (2n+1)\pi + \frac{\pi}{3} = 2n\pi + \frac{4\pi}{3}$

(4) $\cos\theta = -\frac{1}{\sqrt{2}}, \tan\theta = 1$

$\Rightarrow \theta = (2n+1)\pi \pm \frac{\pi}{4}, \theta = n\pi \pm \frac{\pi}{4}$

But $\tan\theta + \text{ve}$ and $\cos\theta - \text{ve}$ in III quadrant

$\Rightarrow \theta = (2n+1)\pi + \frac{\pi}{4}$

45. (AB)

Any term of S looks like

$\cos(2\theta)\cdot \text{cosec}(3\theta)$ or $\frac{\cos(2\theta)}{\sin(3\theta)}$

We know $2\cos 2\theta + 1 = \frac{\sin 3\theta}{\sin\theta}$

$\therefore \frac{\cos 2\theta}{\sin 3\theta} = \frac{\sin 3\theta - \sin\theta}{2\sin\theta\sin 3\theta} = \frac{1}{2}\left(\frac{1}{\sin\theta} - \frac{1}{\sin 3\theta}\right)$

$S = \therefore \frac{\cos 2\theta}{\sin 3\theta} + \frac{\cos 6\theta}{\sin 9\theta} + \frac{\cos 18\theta}{\sin 27\theta}$

$= \frac{1}{2}\left(\frac{1}{\sin\theta} - \frac{1}{\sin 3\theta} + \frac{1}{\sin 3\theta} - \frac{1}{\sin 9\theta} + \frac{1}{\sin 9\theta} - \frac{1}{\sin 27\theta}\right)$

$= \frac{1}{2}\left(\frac{1}{\sin\frac{\pi}{28}} - \frac{1}{\sin\frac{27\pi}{28}}\right)$

$= 0$

46. (ABD)

$\alpha = 18°$

$\Rightarrow 2\alpha + 3\alpha = 90°$

$\Rightarrow 2\alpha.3^{r-1} = 90°.3^{r-1} - 3^r\alpha$

$\cos\left(2.3^{r-1}\alpha\right) = \cos\left(90°.3^{r-1} - 3^r\alpha\right)$

$r = \text{odd} \Rightarrow \cos\left(2.3^{r-1}\alpha\right) = \sin\left(3^r\alpha\right)$

$r = \text{even} \Rightarrow \cos\left(2.3^{r-1}\alpha\right) = -\sin\left(3^r\alpha\right)$

Now check the options.

47. (BC)

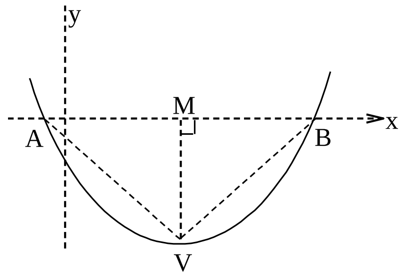
$2\cos^2 2x - 2\cos 2x\cos\left(\frac{2014\pi^2}{x}\right) = -2\sin^2 2x$

$\Rightarrow \cos 2x \cos\frac{2014\pi^2}{x} = 1$

$\cos 2x, \cos\left(\frac{2014\pi^2}{x}\right)$ should both be simultaneously 1 or -1.

Positive solutions : $\pi, 19\pi, 53\pi, 1007\pi$
Sum $= 1080\pi$

48. (BD)



$MA = MB \Rightarrow \Delta$ is right angled isosceles

$\Rightarrow MV = MA = MB = \left|-\frac{D}{4a}\right|$

$\Rightarrow$ difference of roots $= |\alpha - \beta| = \frac{\sqrt{D}}{a} = 2\left|\frac{D}{4a}\right|$

$\Rightarrow \frac{D}{a^2} = \frac{D^2}{4a^2} \Rightarrow D = 4$

Also $b^2 - 4ac = D = 4 \quad \{a > 0 \ \& \ c < 0\}$

$\Rightarrow 4ac = b^2 - 4 < 0 \ \Rightarrow \ b \in (-2, 2)$

& vertex $-\frac{b}{2a} > 0 \Rightarrow b < 0$

49. (B)

$x^3 - x^2(1 + \sin\theta + \cos\theta) + x(\sin\theta + \cos\theta + \sin\theta\cos\theta) - \sin\theta\cos\theta = 0$

$\Rightarrow$ Roots are $1, \sin\theta, \cos\theta$

$\left(\frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2 = \frac{1}{4}\left(1 + \sin^2\theta + \cos^2\theta\right) = \frac{1}{2}$

50. (D)

$x^3 - x^2(1 + \sin\theta + \cos\theta) + x(\sin\theta + \cos\theta + \sin\theta\cos\theta) - \sin\theta\cos\theta = 0$

$\Rightarrow$ Roots are, $\sin\theta, \cos\theta$

For at least 2 to be equal

$\sin\theta = 1$ or $\cos\theta = 1$ or $\sin\theta = \cos\theta$ or $\sin\theta = \cos\theta = 1$

$\sin\theta = 1 \Rightarrow \ \theta = \frac{\pi}{2}$

$\cos\theta = 1 \ \Rightarrow \ \theta = 0, 2\pi$

$\sin\theta = \cos\theta \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\sin\theta = \cos\theta = 1 \Rightarrow$ No solution.

51. (C)

For $x \geq 4$, the last digit of $1! + 2! + ... + x!$ is3

For $x < 4$, the given equation has only solutions

$x = 1, K \pm 1$ and $x = 3, K = \pm 3$

$\alpha = 1, \beta = 3$

$$\left(a+\sqrt{b}\right)^{x^2-15}+\left(a-\sqrt{b}\right)^{x^2-15}=2a$$
$$\therefore x^2-15=\pm1\Rightarrow x=\pm4,\pm\sqrt{14}$$
$$\alpha_1=-4,\alpha_2=4,\alpha_3=-\sqrt{14},\alpha_4=\sqrt{14}$$
$$\therefore\left|\alpha_1+\alpha_2+\alpha_3+\alpha_4-\alpha_1\alpha_2\alpha_3\alpha_4\right|=\left|0-16\times14\right|=224$$

52. (D)

$$a^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....}=a$$
$$\sqrt{x\sqrt{x\sqrt{x........\infty}}}=x \text{ and } \sqrt{49+20\sqrt{6}}=5+2\sqrt{6}$$
$$x^2-3>0 \text{ and } x>0$$
$$\Rightarrow x>\sqrt{3}$$
$$\left(5+2\sqrt{6}\right)^{\sqrt{a\sqrt{a\sqrt{a......\infty}}}}+\left(5-2\sqrt{6}\right)^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x....\infty}}}}=10$$
$$\Rightarrow\left(5+2\sqrt{6}\right)^{x^2-3}+\left(5-2\sqrt{6}\right)^{x^2-3}=10$$
$$\therefore x^2-3=1\Rightarrow x=2 \quad \left(\because x>\sqrt{3}\right)$$

53. (4)

$$\left(\sin x-1\right)^3+\left(\cos x-1\right)^3+\left(\sin x\right)^3$$
$$=\left(2\sin x+\cos x-2\right)^3$$
$$\Rightarrow\left(\sin x+\cos x-1\right)\left(\cos x+\cos x-1\right)\left(2\sin x-1\right)=0$$
$$\Rightarrow\sin x+\cos x=1 \;\&\; \sin x=\frac{1}{2}$$
$$\Rightarrow x=0,\frac{\pi}{6},\frac{\pi}{2},2\pi$$

54. (0)

Let $g(x)=p(x)(3+x)-2 \quad ...(1)$

Then $g(x)$ is 4 degree polynomial whose roots are 0,1,2,3

$$\Rightarrow g(x)=kx(x-1)(x-2)(x-3)$$
$$\Rightarrow g(-3)=-2\Rightarrow-2=k\times-3x-4x-5x-6\Rightarrow k=\frac{-1}{180}$$
$$\therefore g(x)=\frac{-x(x-1)(x-2)(x-3)}{180}=p(x)(3+x)-2\text{, put } x=6$$
$$\frac{-6\times5\times4\times3}{180}=p(6)9-2\Rightarrow p(6)=0$$

55. (9)

$$\sin B\left[3\sin A+5\cos A\right]+4\cos B=5\sqrt{2}$$
$$3\sin A+5\cos A\le\sqrt{34}$$
$$\&\; \sqrt{34}\sin B+4\cos B\le5\sqrt{2}$$
$$\therefore 3\sin A+5\cos A=\sqrt{34}$$
$$\&\; \sqrt{34}\sin B+4\cos B=5\sqrt{2}$$

$$\left(3\tan A-\sqrt{34}\sec A\right)^2-\left(\sqrt{34}\tan B-5\sqrt{2}\sec B\right)^2=9$$

56. (5)

$$1+\frac{(\sin x-\cos x)}{\sqrt{2}}=1+\cos 5x$$

$$\Rightarrow \cos 5x+\cos\left(x+\frac{\pi}{4}\right)=0$$

$$\Rightarrow 2\cos\left(3x+\frac{\pi}{8}\right)\cos\left(2x-\frac{\pi}{8}\right)=0$$

$$\Rightarrow 3x+\frac{\pi}{8}=(2n+1)\frac{\pi}{2}$$

or $2x-\frac{\pi}{8}=(2m+1)\frac{\pi}{2}, n, m\in I$

$$\Rightarrow x=\frac{n\pi}{3}+\frac{\pi}{8} \text{ or } x=\frac{m\pi}{2}+\frac{5\pi}{16}$$

In $[0,\pi]$, $x=\frac{\pi}{8},\frac{\pi}{3}+\frac{\pi}{8},\frac{2\pi}{3}+\frac{\pi}{8},\frac{5\pi}{16},\frac{\pi}{2}+\frac{5\pi}{16}$

$\Rightarrow$ Sum of all solutions $=\frac{5\pi}{2}$

57. (11)

We know,

$$-\sqrt{a^2+b^2}\le a\cos x+b\sin x\le\sqrt{a^2+b^2}$$

$\therefore$ $-\sqrt{3^2+4^2}\le 3\cos x+4\sin x\le\sqrt{3^2+4^2}$ $-5\le k+1\le 5-6\le k\le 4$

$\therefore$ Set of integers

$=-6,-5,-4,-3,-2,-1,0,1,2,3,4=$ Total 11 integers.