IIT – JEE: 2026 **MAJOR TEST - 1** **DATE: 27/07/24**

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	A	**31.**	D	**61.**	C
2.	B	**32.**	C	**62.**	B
3.	C	**33.**	A	**63.**	D
4.	D	**34.**	B	**64.**	C
5.	B	**35.**	B	**65.**	A or C
6.	B	**36.**	C	**66.**	C
7.	A	**37.**	C	**67.**	C
8.	B	**38.**	A	**68.**	C
9.	B	**39.**	C	**69.**	B
10.	B	**40.**	A	**70.**	B
11.	C	**41.**	D	**71.**	B
12.	A	**42.**	B	**72.**	B
13.	C or D	**43.**	B	**73.**	A
14.	A	**44.**	D	**74.**	A
15.	C	**45.**	A	**75.**	B
16.	C	**46.**	B	**76.**	D
17.	C	**47.**	D	**77.**	B
18.	D	**48.**	A	**78.**	C
19.	D	**49.**	C	**79.**	D
20.	B	**50.**	B	**80.**	D
21.	2	**51.**	4	**81.**	4
22.	5	**52.**	2	**82.**	9
23.	8	**53.**	3	**83.**	6
24.	5	**54.**	8	**84.**	0
25.	4	**55.**	4	**85.**	1
26.	6	**56.**	1	**86.**	2
27.	4	**57.**	6	**87.**	7
28.	1	**58.**	1	**88.**	1
29.	6	**59.**	9	**89.**	5
30.	5	**60.**	4	**90.**	4

PART (A) : PHYSICS

1. **(A)**

$\vec{v} = \vec{U} + \vec{a}\ t$

$= 4\hat{i} + 4\hat{j} + (=0.4\hat{i})t$

$= (4 - 0.4t)\hat{i} + 4\hat{j}$

Speed $= \sqrt{(4-0.4t)^2 + 16} = 5$

$(4-0.4t)^2 = 9$

$4 - 0.4t = \pm 3$

$t = 17.5\text{sec}, t = 2.5\text{sec}$

2. **(B)**

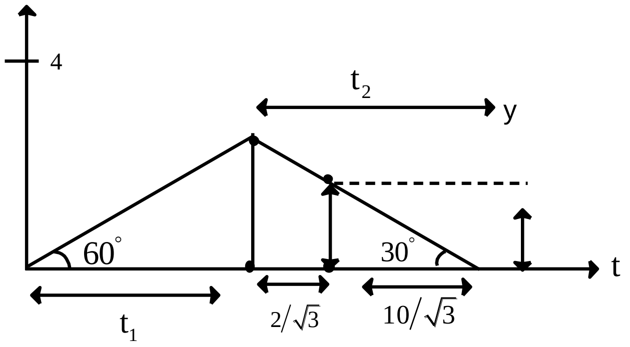
$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin}} \cos x$ [Using chain rule]

$= \frac{\cos x}{2\sqrt{\sin x}}$

3. **(C)**

Area under curve $= 10 + 20 + 15 + 10 = 55\text{m}$

4. **(D)**



$\tan 60^\circ = \frac{4}{t_1}$ $\Rightarrow t_1 = \frac{4}{\sqrt{3}}$

Remaining time $= 2\sqrt{3} - \frac{4}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{4}{t_2}$

$t_2 = 4\sqrt{3}\text{sec}$

$\tan 30^\circ = \frac{y}{10/\sqrt{3}}$

$y = 10/3\text{m/s}$

Average acceleration $= \frac{\frac{10}{3} - 0}{2\sqrt{3}} = \frac{5}{3\sqrt{3}} \text{m/s}^2$

5. **(B)**

$$t=\sqrt{\frac{2\times19.6}{9.8}}=2\sec$$

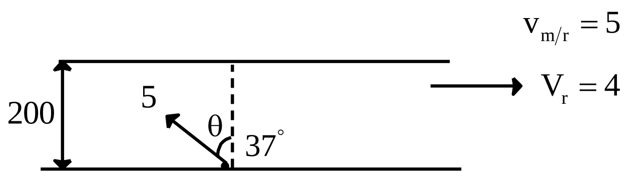
6. **(B)**

$$3u=u+gt\Rightarrow t=\frac{2u}{g}$$

7. **(A)**

$$\frac{H_1}{H_2}=\frac{\frac{U^2\sin^2\left(90^\circ\right)}{2g}}{\frac{U^2\sin^2\left(30^\circ\right)}{2g}}=\frac{1}{1/4}=4:1$$

8. **(B)**



$$5\cos37^\circ=4=\frac{0.200}{t}\Rightarrow t=\frac{0.1}{2}\text{hr}$$

$$v_x=4-5\sin37^\circ=1$$

Displacement in $x=1\times\frac{0.1}{2}$

$$=\frac{0.1}{2}\text{m}$$

time to go back $=\frac{0.1}{2}\times3\text{hr}$

$$=\frac{0.1\times60}{6}\text{min}$$

$$=1\text{min}$$

total time $=\frac{0.1}{2}\times60+1$

= 4 minutes

9. **(B)**

$$t=15\text{min}$$

$$=\frac{1}{4}\text{hours}$$

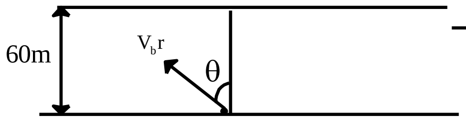
$$V_{br}\cos\theta\times\frac{1}{4}=1$$

$\cos\theta = \frac{4}{5}$

$v_{b/r}\sin\theta = v_r$

$5\times\frac{3}{5} = v_r = 3\,\text{km/hr}$

10. **(B)**



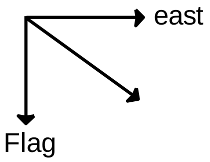
$V_r = 5$
$t = 5$

$v_{br}\cos\theta = 5$

$(v_{br}\sin\theta)\times\not{5} - 60$

$v_{br} = \sqrt{5^2 + 12^2 s} = 13\,\text{m/s}$

11. **(C)**



$20\,\text{km/hr}$

South
$20\,\text{km/hr}$

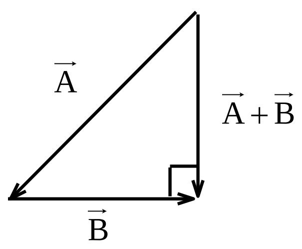
12. **(A)**

$y = x^2\sin x$

$\frac{dy}{dx} = \left(\frac{d}{dx}x^2\right)\sin x + x^2\frac{d}{x}(\sin x)$

$= 2x(\sin x) + x^2(\cos x)$

13. **(C or D)**



14. **(A)**

Slope at $t = 3$, $v = \frac{\Delta S}{\Delta t}$

$= \frac{0-(-8)}{4-2} = 4$

15. **(C)**

$\left|\vec{A}+\vec{B}\right|^2 = A^2 + B^2 + 2AB\cos\theta$

$\left|\vec{A}-\vec{B}\right|^2 = A^2 + B^2 - 2AB\cos\theta$

$A = 2B$

$A^2 + B^2 + 2AB\cos\theta = \dfrac{A^2 + B^2 - 2AB\cos\theta}{4}$

$3A^2 + 3B^2 = -10AB\cos\theta$

$15B^2 = -20B^2\cos\theta$

$\cos\theta = -\dfrac{3}{4}$

16. **(C)**

$\text{aug. acc} = \dfrac{\Delta V}{\Delta t}$

$\Delta V = \text{Area under } a-t \text{ graph } = \dfrac{10\times 40}{2} + 10\times 40 = 600$

$\Delta t = 20$

$\text{aug.acc.} = \dfrac{600}{20} = 30\,m/s^2$

17. **(C)**

Distance = displacement $\Rightarrow$ direction of motion should not change which is possible in constant velocity

18. **(D)**

$v_1 = \dfrac{s}{90}, v_2 = \dfrac{s}{60}$

Now, $t = \dfrac{s}{v_1 + v_2} = \dfrac{s}{\frac{s}{90}+\frac{s}{90}} = \dfrac{90\times 90}{90+60} = 36\,s$

19. **(D)**

$\int \cos\left(x^3\right).x^2 dx = \int\left(\cos t\right)\dfrac{dt}{3}$ put $x^3 = t$

$3x^2 dx = dt$

$= \dfrac{\sin t}{3} + c$

$= \dfrac{\sin\left(x^3\right)}{3} + c$

20. **(B)**

$\int_0^{\pi/2}\left(\cos 3t\right)dt$

$= \left[\dfrac{\sin 3t}{3}\right]_0^{\pi/2} = -\dfrac{1}{3}$

21. **(2)**

We know if a vector makes $\left(a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}\right)$ an angle α, β and γ with X, Y and Z respectively then

$$\cos\alpha=\frac{a}{\sqrt{a^2+b^2+c^2}},\ \cos\beta=\frac{b}{\sqrt{a^2+b^2+c^2}},\ \cos\gamma=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

So, $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=1-\cos^2\alpha+1-\cos^2\beta+1-\cos^2\gamma$

$$=3-\left(\cos^2\alpha+\cos^2\beta+\cos^2\gamma\right)$$

$$=3-1=2$$

22. **(5)**

$$\vec{\mathbf{P}}=7\hat{\mathbf{i}}+6\hat{\mathbf{j}}$$

$$\vec{\mathbf{P}}+\vec{\mathbf{Q}}=11\hat{\mathbf{i}}+9\hat{\mathbf{J}}$$

$\therefore$ $\vec{\mathbf{Q}}=4\hat{\mathbf{i}}+3\hat{\mathbf{j}}$

$$\left|\vec{\mathbf{Q}}\right|=5$$

23. **(8)**

Given $\vec{\mathbf{A}}\perp\vec{\mathbf{B}}$ (*i.e.*, component of $\vec{\mathbf{B}}$ along $\vec{\mathbf{A}}$ is 0)

$\therefore$ $\vec{\mathbf{A}}.\vec{\mathbf{B}}=8+24-4x=0$

$\Rightarrow$ $x=8$

24. **(5)**

$$\vec{\mathbf{A}}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}},\ \vec{\mathbf{B}}=\hat{\mathbf{i}}+4\hat{\mathbf{j}}$$

Area of parallelogram $=\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|=5$ units

25. **(4)**

Suppose the total distance be d,

Time taken for first $d/3$

$$m=\frac{d}{3\times 4}=\frac{d}{12}\text{sec}$$

Let body travels for next T sec then

$$\frac{T}{2}\times 2+\frac{T}{2}\times 6=\frac{2d}{3}\Rightarrow T=\frac{d}{6}$$

So average velocity $=\dfrac{d}{\dfrac{d}{12}+\dfrac{d}{6}}=4$ m/s

26. **(6)**

$$0=30t+\frac{1}{2}(-10)t^2\Rightarrow t=0,6$$

27. **(4)**

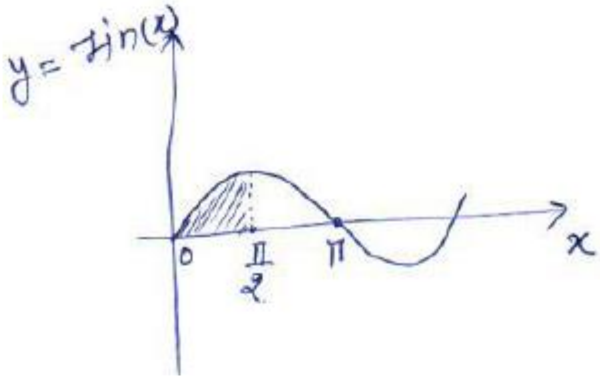
$$d=\int\left|\vec{\upsilon}\right|dt=\int_0^4\left|t-2\right|dt$$

$$=\int_0^2(2-t)\,dt+\int_2^4(t-2)\,dt=4\text{ metre}.$$

28. **(1)**

$$= \int_0^{\frac{\pi}{2}} \sin(x)\,dx$$

$$= -\left[\cos(x)\right]_0^{\frac{\pi}{2}} = 1$$



29. **(6)**

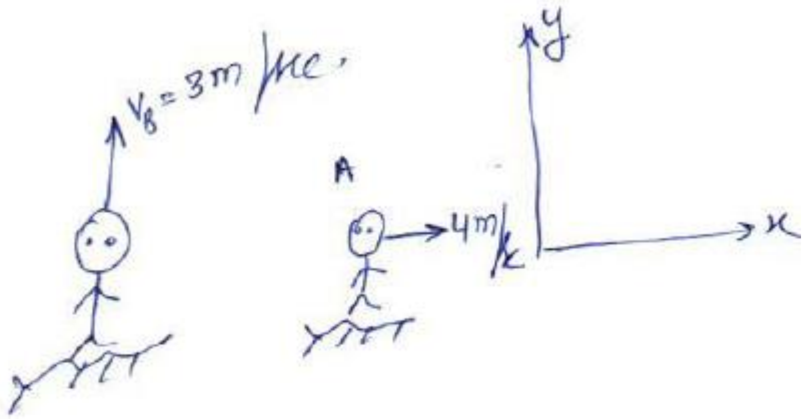
Area under a-t graph = change in velocity

$\int v dt = \Delta v = v_f - v_i$

$\frac{1}{2} \times \{2\}\{4\} = v - 2$

$4 + 2 = v = 6$ m/sec .

30. **(5)**



$\overrightarrow{V_A} = 4\hat{i}$

$\overrightarrow{V_B} = 3\hat{j}$

$\overrightarrow{V_{AB}} = \overrightarrow{V_A} - \overrightarrow{V_B} = 4\hat{i} - 3\hat{j}$

$\left|\overrightarrow{V_{AB}}\right| = 5$ m/sec .

PART (B) : CHEMISTRY

31. **(D)**

$$100\,amu=(100)\left(\frac{1g}{6.022\times10^{23}}\right)=1.66\times10^{-22}\,g$$

Mass of 7.0×10^{22} molecules $=\dfrac{7.0\times10^{22}}{6.022\times10^{23}}\times46\,g=5.35\,g$

Mass of $8.0\times10^{-1}\,mol=0.8\times46\,g=36.8\,g$

32. **(C)**

Volume of $100\,g$ solution is $V=\dfrac{m}{\rho}=\dfrac{100\,g}{1.14\,g\,cm^{-3}}=87.72\,cm^3$

Amount of sulphuric acid in 100 g solution is $n=\dfrac{m}{M}=\dfrac{20.0\,g}{98\,g\,mol^{-1}}=0.207\,mol$

Molarity of sulphuric acid is $M=\dfrac{n}{V}=\dfrac{0.207\,mol}{87.72\times10^{-3}\,dm^3}=2.32\,mol\,dm^{-3}$

33. **(A)**

Let V be the volume of $0.25\,M\,NaOH$ solution.

Total amount of NaOH after mixing the two solutions is

$$n=V\left(0.25\,mol\,L^{-1}\right)+\left(0.25\,L\right)\left(0.15\,mol\,L^{-1}\right)$$

Total volume of the solution $=V+0.25\,L$

Molalrity of the resultant solution, $n=\dfrac{V\left(0.25\,mol\,L^{-1}\right)+\left(0.25L\right)\left(0.15\,mol^{-1}\right)}{V+0.25\,L}$

Equating this 0.2 M, we get $\dfrac{V\left(0.25\,molL^{-1}\right)+\left(0.25L\right)\left(0.15\,mol\,L^{-1}\right)}{V+0.25\,L}=0.2\,mol\,L^{-1}$

34. **(B)**

$$m=N_Am_e=\left(6.022\times10^{23}\,mol^{-1}\right)\left(9.1\times10^{-31}kg\right)=5.48\times10^{-7}\,kg\equiv0.548\,mg$$

35. **(B)**

36. **(C)**

37. **(C)**

$$\Delta E=\frac{hc}{\lambda}=\frac{\left(6.626\times10^{-34}\,Js\right)\left(3\times10^{8}\,m\,s^{-1}\right)}{\left(97.26\times10^{-9}\,m\right)}=2.0438\times10^{-18}\,J$$

Also $\Delta E=2.18\times10^{-18}\,J\left(\dfrac{1}{n_2^2}-\dfrac{1}{1^2}\right)$

Hence $\frac{1}{n_2^2}=1-\left(\frac{\Delta E}{2.18\times10^{-18}\,J}\right)=1-\frac{2.0436\times10^{-18}}{2.18\times10^{-18}}=1.09374$

$n_2=\sqrt{1/(1-0.9374)}=4$

The transition $n_2=4\to n_1=3$ will emit the longest wavelength. Hence

$\Delta E=\left(2.18\times10^{-18}\,J\right)\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=1.06\times10^{-19}\,J$

$\lambda=\frac{hc}{\Delta E}=\frac{\left(6.626\times10^{-34}\,J\,s\right)\left(3\times10^{8}\,m\,s^{-1}\right)}{\left(1.06\times10^{-19}\,J\right)}=1.875\times10^{-6}\,m=1875\,nm$

38. **(A)**

First line in the Lyman series of hydrogen atom corresponds to

$\Delta E=R_H hc\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=\frac{3}{4}Rhc$

For He^+, the spectral line occurs at

$\Delta E=Z^2R_H\,hc\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)=(2)^2\,R_H\,hc\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\frac{3}{4}R_H\,hc$

39. **(C)**

Binding energy of electron per atom of the metal

$E_{bind}=\frac{193\times10^{3}\,mol^{-1}}{6.022\times10^{23}\,mol^{-1}}=3.20\times10^{-19}\,J$

$v=\frac{E_{bind}}{h}=\frac{3.20\times10^{-19}\,J}{6.626\times10^{-34}\,J\,s}=4.83\times10^{14}\,s^{-1}$

40. **(A)**

$KE=\frac{1}{2}m\upsilon^2=\frac{p^2}{2m}$

Thus $\frac{p^2}{2m}=eV$ or $p=\sqrt{2meV}$

Now, $\lambda=\frac{h}{p}=\frac{h}{\sqrt{2meV}}=\frac{6.626\times10^{-34}\,J\,s}{\left[(2)\left(9.1\times10^{-31}\,kg\right)\left(1.6\times10^{-19}\,C\right)(500\,V)\right]^{1/2}}$

$=5.49\times10^{-11}\,m=54.9\,pm$

41. **(D)**

$2Al+\frac{3}{2}O_2\to Al_2O_3$

n_{O_2} reacted $=\frac{1}{2}$

$\Rightarrow$ n_{Al} reacted $=\frac{1}{2}\times\frac{2}{2/3}=\frac{1}{2}\times\frac{4}{3}=\frac{2}{3}$

$\Rightarrow$ wt_{Al} reacted $=\frac{2}{3}\times27g=18\,g$

42. **(B)**

Moles of $Na_2CO_3 \cdot 10H_2O = 0.1$

No. of O atom in each molecule $= 13$

Mole of O atoms $= 0.1 \times 13 = 1.3$

$wt = 1.3 \times 16 = 20.8\,g$

43. **(B)**

$$X = 0.08 = \frac{n_{C_2H_5OH}}{n_{C_2H_5OH} + n_{H_2O}}$$

$\Rightarrow$ Let $n_{C_2H_5OH} + n_{H_2O} = 100$

$\Rightarrow n_{C_2H_5OH} = 8$

$\Rightarrow n_{H_2O} = 92 \Rightarrow wt_{H_2O} = 92 \times 18\,g = 1656\,g = 1.656\,kg$

$$\text{Molality} = \frac{8}{1.656} = 4.83 \text{ mol/kg}$$

44. **(D)**

Lyman line $\rightarrow 4$	(UV)
Balmer line $\rightarrow 3$	(visible)
Paschen line $\rightarrow 2$	(IR)
Brachett line $\rightarrow 1$	(IR)

No. of IR lines $= 3$

45. **(A)**

46. **(B)**

n = 3	3s	3p	3d
1 orbital	3 orbital	5 orbital	

$\Rightarrow$ degeneracy $= 1 + 3 + 5 = 9$

47. **(D)**

(A) Radical nodes for $4p = 4 - 1 - 1 = 2$

(B) Angular nodes for $3d_{xy} = 2$

(C) Total node $= n - 1 = 3 - 1 = 2$

48. **(A)**

$Cu^{2+} \rightarrow [Ar]4s^o 3d^9$

$n = 3,\ l = 2,\ m = -2$ or -1 or 0 or 1 or 2

$$s = -\frac{1}{2} \text{ or } +\frac{1}{2}$$

49. **(C)**

$$1=\frac{hc}{\lambda}-wF \Rightarrow \left(1=\frac{hc}{\lambda}-wF\right)\times 3$$

$$4=\frac{hc}{\lambda/3}-wF \Rightarrow 4=\frac{3hc}{\lambda}-wF$$

$$-1=-2wF$$

$$\Rightarrow wF=0.5eV$$

50. **(B)**

$$N_2+\frac{3}{2}O_2 \rightarrow N_2O_3$$

$$n_{N_2}\ \text{reacted}=\frac{7\times 0.8}{28}=0.2$$

$$\Rightarrow \quad n_{O_2}\ \text{reacted}=\frac{3}{2}\times 0.2=0.3$$

$$\Rightarrow \quad \text{no. of O atoms}=0.3\times 2\times 6.022\times 10^{23}$$

$$\equiv 3.6\times 10^{23}$$

51. **(4)**

100 g sample $\equiv$ 0.33 g iron

$\therefore$ 67200 g $\equiv$ 221.8 g iron

$\therefore$ Number of iron atoms per molecules of haemoglobin $=\frac{221.8}{56}=4$

52. **(2)**

meq. of $Na_2CO_3\cdot x\,H_2O$ in 20 mL $=19.8\times\frac{1}{10}$

$\therefore$ meq. of $Na_2CO_3\cdot x\,H_2O$ in 100 mL $=19.8\times\frac{1}{10}\times 5$

$\therefore$ $\frac{w}{E}\times 1000=19.8\times\frac{1}{10}\times 5$

or $\frac{0.7}{M/2}\times 1000=\frac{19.8}{2}$

$\therefore$ $M=141.41$

$\therefore$ $23\times 2+12+3\times 16+18x=141.41$

$\therefore$ $x=2$

53. **(3)**

Magnetic moment $=\sqrt{n(n+2)}=4.9,\ \therefore\ n=4$

Thus Mn ions has four unpaired electron (n).

$Mn^{+3}:1s^2,2s^2 2p^6,3s^2 3p^6 3d^4$

54. **(8)**

Here, $V_{solution} \approx V_{solvent}$

Since, in 1 L solution, 3.2 moles of solute are present.

So, 1 L solution $\approx$ 1 L solvent (d = 0.4 g/mL) $\approx$ 0.4 kg

$$\text{So, molality } (m) = \frac{\text{moles of solute}}{\text{mass of solvent (kg)}} = \frac{3.2}{0.4} = 8$$

55. **(4)**

$$M_{Na^+} = \frac{\text{mole of } Na^+}{\text{Mass of water in kg}} = \frac{92}{23} \times \frac{1}{1} = 4$$

56. **(1)**

millimole in conc. solution = millimole in dil solution

$$5 \times 1 = M \times 5$$

$$M_{H_2SO_4} = 1$$

57. **(6)**

$$n = 4; l = 0, 1, 2, 3; |m_l| = 1 \Rightarrow \pm 1; m_s = -\frac{1}{2}$$

For $l = 0$, $m_l = 0$

$$l = 1, m_l = -1, 0, +1$$

$$l = 2, m_l = -2, -1, 0, +1, +2$$

$$l = 3, m_l = -3, -2, -1, 0, +1, +2, +3$$

So, six electrons can have $|m_l| = 1$ and $m_s = -\frac{1}{2}$

58. **(1)**

Number of angular node = l = 1 (l = 1, for p–orbital)

59. **(9)**

No. of orbitals in 3rd shell = $n^2 = 3^2 = 9$

60. **(4)**

$$r_{(n+1)} - r_n = r_{n-1}$$

$\because \quad r \propto n^2$

$$(n+1)^2 - n^2 = (n-1)^2$$

$$n^2 + 1 + 2n - n^2 = n^2 + 1 - 2n$$

$$n^2 - 4n = 0$$

$\therefore \quad n = 4$

PART (C) : MATHEMATICS

61. **(C)**

$1+\sin\theta+\cos\theta$

$-2\le\sin\theta+\cos\theta\le\sqrt{2}$

Maximum value

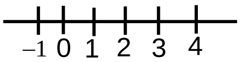
So Max. value of $1+\sin\theta+\cos\theta=1+\sqrt{2}$

62. **(B)**

$x^2-3x-4<0$

$(x-4)(x+1)<0$

$-1<x<4$



4 integers $\{0,1,2,3\}$

63. **(D)**

$\sin\theta_1+\sin\theta_2=2$

$\sin\theta_1=1$ $\quad$ $\sin\theta_2=1$ $\quad$ (Max. value of $\sin\theta_1+\sin\theta_2$)

$\cos\theta_1=\sqrt{1-\sin^2\theta_1}$ $\quad$ $\cos\theta_2=\sqrt{1-\sin^2\theta_2}$

$=\sqrt{1-1}$ $\quad$ $\cos\theta_2=\sqrt{1-1}$

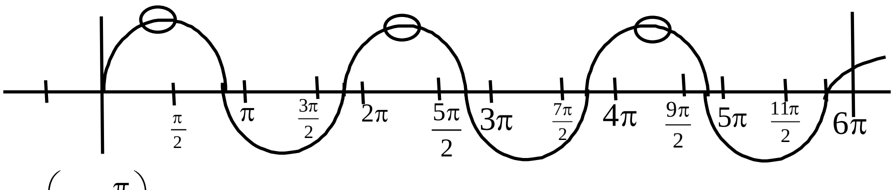
$=0$ $\quad$ $=0$

$\cos\theta_1+\cos\theta_2=0$

64. **(C)**

$\sin x+\cos x=\sqrt{2}$ $\quad$ $x\in[0,6\pi]$

$\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=1$



$\sin\left(x+\frac{\pi}{4}\right)=1$

65. **(A or C)**

$x^2(1+\lambda)+x(-6-4\lambda)+8+3\lambda=0 \quad \lambda\in R$

$D=b^2-4ac$

$=+(6+4\lambda)^2-4(1+\lambda)(8+3\lambda)$

$=36+16\lambda^2+48\lambda-4(3\lambda^2+11\lambda+8)$

$=16\lambda^2+48\lambda+36-12\lambda^2-44\lambda-32$

$= 4\lambda^2 + 4\lambda + 4$

$= 4(\lambda^2 + \lambda + 1)$

Always $+$ve of $\neq 0$

So $D > 0$

Real or unequal roots

66. **(C)**

$a > 0, b > 0, c > 0 \qquad ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a} = -ve$

$\alpha\beta = \frac{c}{a} + ve$

So booth roots are $-$ve

67. **(C)**

$= \frac{2\sin 15 \cos 15}{2}$

$= \frac{\sin 30°}{2}$

$= \frac{1}{4}$ rational no.

68. **(C)**

$x^2 - x + 1 = 0$

$\alpha^2 - \alpha + 1 = 0 \qquad \beta^2 - \beta + 1 = 0$

$\alpha^2 - \alpha = -1 \qquad \beta^2 - \beta = -1$

$(\alpha^2 - \beta)^3 + (\beta^2 - \beta)^3 = (-1)^3 + (-1)^3 = -1 - 1 = -2$

69. **(B)**

$(q + r - p)x^2 + (r + p - q)x + (p + q - r) = 0 \qquad q + p + r = 0$

$-2px^2 - 2qx - 2r = 0$

$px^2 + qx + r = 0$

$D = q^2 - 4pr \qquad q = -p - r$

$= p^2 + qr^2 + 2pr - 4pr$

$= (p - r)^2$

So real roots

70. **(B)**

$\cos\frac{2\pi}{9} + \cos\frac{\pi}{9} + \cos\frac{7\pi}{9} + \cos\frac{8\pi}{9}$

$\cos\frac{2\pi}{9} + \cos\frac{7\pi}{9} + \cos\frac{\pi}{9} + \cos\frac{8\pi}{9}$

$$2\cos\frac{\pi}{2}\cos\frac{5\pi}{18}+2\cos\frac{\pi}{2}\cos\frac{7\pi}{18}$$

$\cos\frac{\pi}{2} = 0$ (in both terms)

$=0$

71. **(B)**

$4x^2-2\sqrt{5}x+1=0$

Roots $=\dfrac{2\sqrt{5}\pm\sqrt{20-4\times4}}{2\times4}=\dfrac{\sqrt{5}\pm1}{4}$

$\cos 36^\circ, \sin 18^\circ$

72. **(B)**

Given $a,b\in R$, $ax^2+bx+5=0$

Since, the equation has no real roots $D<0$ for the question, so graph may lie entirely above or below *x*-axis, but

$\Rightarrow f(x)=ax^2+bx+5$

$\Rightarrow f(0)=5$

$\Rightarrow$ So, the graph of the equation would lie entirely above *x*-axis.

$\Rightarrow f(-1)=a-b+5>0$.

73. **(A)**

We have, cos 50° = $\cos^2 25°$ - $\sin^2 25°$
= (cos 25° + sin 25°) (cos 25° - sin 25°)
= k (cos 25° - sin 25°).
But (cos 25° + sin 25°) + (cos 25° - sin 25°)^2 = 2
$\Rightarrow$ cos 25° - sin 25° = $\sqrt{2-k^2}$
(As cos 25° - sin 25 is positive)
$\therefore$ cos 50° = $k\sqrt{2-k^2}$.

74. **(A)**

$$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{6\pi}{15}\cos\frac{8\pi}{15} \qquad \left(\frac{\pi}{15}=12^\circ\right)$$

$$=\left(\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right)\cos\frac{3\pi}{15}\cos\ 2\left(\frac{3\pi}{15}\right)$$

$$=\frac{\sin 2^4\left(\frac{\pi}{15}\right)}{2^4\sin\frac{\pi}{15}}\times\frac{1}{2^2}\frac{\sin 2^2\left(\frac{3\pi}{15}\right)}{\sin\left(\frac{3\pi}{15}\right)}$$

$$=\frac{\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}}\times\frac{1}{2^2}\frac{\sin\left(\frac{12\pi}{15}\right)}{\sin\frac{3\pi}{15}}$$

$$=\frac{1}{64}\frac{\sin\left(\pi+\frac{\pi}{15}\right)}{\sin\frac{\pi}{15}}\times\frac{\sin\left(\pi-\frac{3\pi}{15}\right)}{\sin\frac{3\pi}{15}}=\frac{1}{2^6}$$

75. **(B)**

We have, $\cos\theta\ .\ \cos 2\theta\ .\ \cos 2^2\theta\ .\ .\ .\ \cos 2^{n-1}\theta$

$$=\frac{1}{2\sin\theta}\left(2\sin\theta.\cos\theta.\cos 2\theta.\cos 2^2\theta...\cos 2^{n-1}\theta\right)$$

$$=\frac{1}{2\ \sin\theta}\ (\sin 2\theta.\ \cos 2\theta\ .\ \cos 2^2\theta\ .\ .\ .\ \cos 2^{n-1}\theta)$$

$$=\frac{1}{2^2\sin\theta}\left(2\sin 2\theta.\cos 2\theta.\cos 2^2\theta...\cos 2^{n-1}\theta\right)$$

$$=\frac{1}{2^2\sin\theta}\left(\sin 2^2\theta.\cos 2^2\theta...\cos 2^{n-1}\theta\right)$$

....

....

$$=\frac{1}{2^{n-1}\sin\theta}\sin 2^{n-1}\theta.\cos 2^{n-1}\theta$$

$$=\frac{1}{2^n\sin\theta}\left(2\sin 2^{n-1}\theta.\cos^{n-1}\theta\right)=\frac{1}{2^n\sin\theta}\sin 2^n\theta$$

$$=\frac{1}{2^n\sin\theta}\left[\sin(\pi-\theta)\right] \qquad \left[\because \theta=\frac{\pi}{2^n+1},\therefore 2^n\theta+\theta=\pi\right]$$

$$=\frac{1}{2^n\sin\theta}.\sin\theta=\frac{1}{2^n}.$$

76. **(D)**

$\cos A=\cos B\cos C \;\; A+B+C=\pi$

$\cos(B+C)=\cos(\pi-A) \qquad B+C=\pi-A$

$\Rightarrow \cos B\cos C-\sin B\sin C=-\cos A$

$\Rightarrow \cos B\cos C+\cos A=\sin B\sin C$

$\Rightarrow 2\cos B\cos C=\sin B\sin C$

$\Rightarrow \cot B\cot C=1/2$

77. **(B)**

Let α be the common root of $3x^2+ax+1=0$ and $2x^2+bx+1=0$

$\Rightarrow 3\alpha^2+a\alpha+1=0 \qquad (i)$

$\Rightarrow 2\alpha^2+b\alpha+1=0 \qquad (ii)$

$\Rightarrow a\alpha-\frac{3}{2}b\alpha+1-\frac{3}{2}=0$

$\Rightarrow \left(a-\frac{3}{2}b\right)\alpha=\frac{1}{2}$

$\Rightarrow \alpha=\left(\frac{1}{2a-3b}\right)$

$\therefore$ From (i), we get $3\left(\frac{1}{2a-3b}\right)^2+a\left(\frac{1}{2a-3b}\right)+1=0$

$\Rightarrow 3+a(2a-3b)+(2a-3b)^2=0$

$\Rightarrow 3+2a^2-3ab+4a^2+9b^2-12ab=0$

$\Rightarrow 3+6a^2+9b^2-15ab=0$

$\Rightarrow 1+2a^2+3b^2-5ab=0$

$\Rightarrow 5ab-2a^2-3b^2=1$

78. **(C)**

$x^2+ax-3x-(a+2)=0$ has real and distinct roots.

$\Rightarrow Disc.>0$

$\Rightarrow (a-3)^2+4(a+2)>0$

$\Rightarrow a^2-2a+17>0$

$\Rightarrow a^2-2a+1+16>0$

$\Rightarrow (a-1)^2+16>0$ which is true $\forall a\in\mathbb{R}$

$Now, \frac{a^2+1}{a^2+2}=1-\frac{1}{a^2+2}; a\in\mathbb{R}$

$Now, a^2+2\in[2,\infty)\Rightarrow\frac{1}{a^2+2}\in[\frac{1}{2},1)$

$\Rightarrow \quad -\frac{1}{a^2+2}\in(0,\frac{1}{2}]\Rightarrow 1-\frac{1}{a^2+2}\in[\frac{1}{2},1)$

$\Rightarrow$ Minimum value of $\frac{a^2+1}{a^2+1}$ $is\ \frac{1}{2}$

79. **(D)**

$(\sin 47+\sin 61)-(\sin 11+\sin 25)$

$=2\sin 54\cos 7-2\sin 18\cos 7$

$=2\cos 7(\sin 54-\sin 18)$

$=2\cos 7\left(\frac{\sqrt{5}+1}{4}-\frac{\sqrt{5}-1}{4}\right)$

$=2\cos 7\times\frac{1}{2}=\cos 7$

80. **(D)**

Given α,β,γ an roots of equation $x^3+P\gamma+q=0$

$\alpha+\beta+\gamma=0$

$\alpha\beta+\beta\gamma+\gamma\alpha=+P$

$\alpha\beta\gamma=-q$

$\frac{1}{\beta+\gamma}+\frac{1}{\gamma+\alpha}+\frac{1}{\alpha+\beta}=-\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)=-\frac{(\alpha\beta+\beta+\gamma\alpha)}{\alpha\beta\gamma}$

$=\frac{P}{q}$

81. **(4)**

Given, equation is $x^2+px+q=0$ has roots x_1 and x_2

$\Rightarrow \quad x_1+x_2=-p$ .....(1)

$\Rightarrow x_1x_2=q$ .....(2)

Also, $x_1=\frac{x_2+4}{2x_2-1}$

$\Rightarrow\ x_1-(2x_2-1)=x_2+4$

$\Rightarrow\ 2x_1x_2-x_1=x_2+4$

$\Rightarrow\ 2(x_1x_2)=(x_1+x_2)+4$

From equation (1) and (2), we get

$\Rightarrow\ 2(q)=-p+4$

$\Rightarrow\ 2q+p=4$

82. **(9)**

$f(\theta)=(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2$

$=\sin^2\theta+\cos^2\theta+\operatorname{cosec}^2\theta+\sec^2\theta+4$

$=5+\dfrac{1}{\sin^2\theta}+\dfrac{1}{\cos^2\theta}$

$=5+\dfrac{4}{\sin^2 2\theta}\ge 9$

83. **(6)**

$a,b\in R^+$

$x^2+ax+2b=0$ $\qquad$ $x^2+2bx+a=0$

$\because$ real roots. $\therefore\ D\ge 0$

$\Rightarrow a^2-8b\ge 0\ \&\qquad 4b^2-4a\ge 0$

$\Rightarrow\ a^2\ge 8b\qquad \&\qquad b^2\ge a$

squaring $a^4\ge 64b^2$

using $b^2\ge a$

$\Rightarrow\ a^4\ge 64a$

$a^3\ge 64\ (\because a>0)$

$\Rightarrow\ a\ge 4$

$\&\ b^2\ge a$

$b^2\ge 4$

$\Rightarrow b\ge 2$

84. **(0)**

$\sin^2A+\cos C(\cos A\cdot\cos B-\cos C)+\cos B(\cos A\cdot\cos C-\cos B)$

$=\sin^2A-\cos^2B-\cos^2C+2\cos A\cos B\cdot\cos C$

$=\sin^2A-\cos^2B-\cos^2C+(\cos(A+B)+\cos(A-B))\cdot\cos C$

$=\sin^1A-\cos^2B-\cos^2C+\cos^2C+\cos C\cdot\cos(A-B)$

$=\sin^2A-\cos^2B+\cos(A+B)\cdot\cos(A-B)$

$=\sin^2A-\cos^2B+\cos^2A-\sin^2B=0$

85. **(1)**

Given, $x^2+(2-\lambda)x-\lambda=0$ has roots α and β.

$\Rightarrow$ So, $\alpha+\beta=\lambda-2$

$\alpha\beta=-\lambda$

$\Rightarrow \left(\alpha^2+\beta^2\right)_{min} = \left((\alpha+\beta)^2-2\alpha\beta\right)_{min}$

$= \left((\lambda-2)^2-2(-\lambda)\right)_{min}$

$= \left(\lambda^2-4\lambda+4+2\lambda\right)_{min}$

$= \left(\lambda^2-2\lambda+4\right)_{min}$

$= \left((\lambda-1)^2+3\right)_{min}$

$\Rightarrow$ So, for minimizing this expression

$\Rightarrow (\lambda-1)^2=0$

$\Rightarrow \lambda=1$

86. **(2)**

Given equation is,

$\Rightarrow \sqrt{x^2+\sqrt{x^2+11}}+\sqrt{x^2-\sqrt{x^2+11}}=4$

$\Rightarrow$ Let $\sqrt{x^2+11}=a$

$\Rightarrow \sqrt{x^2+a}+\sqrt{x^2-a}=4$

$\Rightarrow \sqrt{x^2+a}=4-\sqrt{x^2-a}$

$\Rightarrow$ Squaring both the sides,

$\Rightarrow x^2+a=16+x^2-9-8\sqrt{x^2-a}$

$\Rightarrow 2a-16=-8\sqrt{x^2-a}$

$\Rightarrow a-8=-4\sqrt{x^2-a}$

$\Rightarrow$ Again, squaring both sides,

$\Rightarrow (a-8)^2=\left(-4\sqrt{x^2-a}\right)^2$

$\Rightarrow a^2-16a+64=16\left(x^2-a\right)$

$\Rightarrow a^2+64=16x^2$

Now, put the value of $a=\sqrt{x^2+11}$, in the above equations,

$\Rightarrow x^2+11+64=16x^2$

$\Rightarrow 75=15x^2$

$\Rightarrow 5=x^2$

$\Rightarrow x=\pm\sqrt{5}$

So, the given equation has two irrational solutions (i.e., $\sqrt{5},-\sqrt{5}$).

87. **(7)**

Given, $\alpha+\beta+\gamma=4, \alpha^2+\beta^2+\gamma^2=6, \alpha^3+\beta^3+\gamma^3=8$

$\Rightarrow$ We know, $\left(\alpha^2+\beta^2+\gamma^2\right)^2=\alpha^4+\beta^4+\gamma^4+2\left[(\alpha\beta)^2+(\beta\gamma)^2+(\gamma\alpha)^2\right]$

$\Rightarrow (6)^2=\alpha^4+\beta^4+\gamma^4+2\left(\Sigma(\alpha\beta)^2\right)$

$\Rightarrow 36-2\left(\Sigma(\alpha\beta)^2\right)=\alpha^2+\beta^4+\gamma^4 \quad .....(1)$

$\Rightarrow (\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow (4)^2=6+2(\Sigma\alpha\beta)$

$\Rightarrow (\Sigma\alpha\beta)=5 \quad .....(2)$

$\Rightarrow$ Squaring both sides of equation (2),

$\Rightarrow (\alpha\beta)^2+\left(\beta\gamma^2\right)+(\gamma\alpha)^2+2\left(\alpha\beta^2\gamma+\alpha\beta\gamma^2+\alpha^2\beta\gamma\right)=25$

$\Rightarrow \left(\Sigma(\alpha\beta)^2\right)+2\alpha\beta\gamma(\alpha+\beta+\gamma)=25$

$\Rightarrow \left(\Sigma(\alpha\beta)^2\right)+2\alpha\beta\gamma(4)=25$

$\Rightarrow \left(\Sigma(\alpha\beta)^2\right)=25-8\alpha\beta\gamma \quad .....(3)$

$\Rightarrow \alpha^3+\beta^3+\gamma^3=(\alpha+\beta+\gamma)\left(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha\right)+3\alpha\beta\gamma$

$\Rightarrow 8=(4)(6-\Sigma(\alpha\beta))+3\alpha\beta\gamma$

$\Rightarrow 8=4(6-5)+3(\alpha\beta\gamma)$

$\Rightarrow 8=4+3(\alpha\beta\gamma)$

$\Rightarrow \alpha\beta\gamma=\frac{4}{3} \quad .....(4)$

$\Rightarrow$ Putting value of $\alpha\beta\gamma$ from equation (4) in equation (3)

$\Rightarrow \Sigma(\alpha\beta)^2=25-8\left(\frac{4}{3}\right)$

$\Sigma(\alpha\beta)^2=\frac{75-32}{3}=\frac{43}{3}$

$\Rightarrow$ Putting the value of $\Sigma(\alpha\beta)^2$ in equation (1)

$\Rightarrow \alpha^4+\beta^4+\gamma^4=36-2\left(\Sigma(\alpha\beta)^2\right)$

$\Rightarrow \alpha^4+\beta^4+\gamma^2=36-\frac{2(43)}{3}$

$\Rightarrow \alpha^4+\beta^4+\gamma^4=\frac{108-86}{3}$

$\Rightarrow \alpha^2+\beta^4+\gamma^4=\frac{22}{3}$

$\Rightarrow \alpha^4+\beta^4+\gamma^4=7.333....$

So, $\left[\alpha^4+\beta^4+\gamma^4\right]=7$

88. **(1)**

Given equation,

$$x^2 - p(x+1) - q = 0$$
$$\Rightarrow x^2 - px - (p+q) = 0$$
$$\Rightarrow \alpha+\beta = p$$
$$\Rightarrow \alpha\beta = -(p+q)$$
$$\Rightarrow \alpha\beta = -(\alpha+\beta+q)$$
$$\Rightarrow \alpha\beta = -\alpha-\beta-q$$
$$\Rightarrow q = -\alpha\beta\left(\frac{1}{\beta}+\frac{1}{\alpha}-1\right)$$
$$\therefore \frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$$
$$=\frac{(\alpha+1)^2}{(\alpha+1)^2+q-1}+\frac{(\beta+1)^2}{(\beta+1)^2+q-1}$$

Let $\alpha+1=m$, and $\beta+1=n$

$$\therefore \frac{m^2}{m^2+q-1}+\frac{n^2}{n^2+q-1}$$
$$=\frac{1}{1+\frac{q-1}{m^2}}+\frac{1}{1+\frac{q-1}{n^2}}$$

Let $p(x)=0$

$\Rightarrow$ If a and b satisfies this equation then $(a+1)$ and $(b+1)$ will satisfy $p(x-1)$

$$\therefore p(x-1)=(x-1)^2\ p(x-1+1)-q=0$$
$$\Rightarrow x^2-(p+2)x-(q-1)=0$$

So, $mn=-(q-1)$

$$\frac{1}{1-\frac{n}{m}}+\frac{1}{1-\frac{m}{n}}$$
$$=\frac{m}{m-n}+\frac{n}{n-m}$$
$$=\frac{m-n}{m-n}$$
$$=1$$

89. **(5)**

$$4\operatorname{cosec}^2 225°-2\tan^2 120°+3\sec^2 720°+4\cot^2 90°$$
$$=4\operatorname{cosec}^2(180°+45°)-2\tan^2(180°-60°)+3\sec^2(360°+360°)+4\cot^2 90°$$
$$=4\operatorname{cosec}^2 45°-2\tan^2 60°+3\sec^2 360°+4\cot^2 90°$$
$$=4\left(\sqrt{2}\right)^2-2\left(\sqrt{3}\right)^2+3(1)+4(0)$$

$= 4(2) - 2(3) + 3$
$= 8 - 6 + 3$
$= 5$

90. **(4)**

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$

$= \dfrac{\sin 9°}{\cos 9°} + \dfrac{\cos 9°}{\sin 9°} - \left(\dfrac{\sin 27°}{\cos 27°} + \dfrac{\cos 27°}{\sin 27°}\right)$

$= \dfrac{2}{\sin 18°} - \dfrac{2}{\sin 54°}$

$= \dfrac{2(\sin 54° - \sin 18°)}{\sin 18° \cdot \sin 54°}$

$= \dfrac{4\cos 36° \cdot \sin 18°}{\sin 18° \cdot \sin 54°}$

$= \dfrac{4\cos 36° \cdot \sin 18°}{\sin 18° \cdot \cos 36°} = 4$