Main Booklet Solution

Electromagnetic Induction (EMI)

JEE Main Exercise

1. (B)

$$\left|\varepsilon_{avg}\right| = \left|\frac{\Delta\phi}{\Delta t}\right| = \left|\frac{0-2}{6}\right| = \frac{1}{3}\text{ V}$$

2. (D)

$$\varepsilon = -\frac{d\phi}{dt} = -3(3at^2 - 2bt)$$

$$i = \frac{\varepsilon}{R} = -(3at^2 - 2bt)$$

$$\frac{di}{dt} = -(6at - 2b) = 0$$

$$\Rightarrow \quad t = \frac{b}{3a}$$

At $t = \frac{b}{3a}$;

$$i_{\max} = -\left[3a\left(\frac{b}{3a}\right)^2 - 2b\left(\frac{b}{3a}\right)\right] = \frac{b^2}{3a} = 6\text{ A}$$

3. (C)

$$|\varepsilon| = \left|\frac{d\phi}{dt}\right| = \frac{d\left(B\pi R^2\right)}{dt} = B\pi 2R\frac{dR}{dt} = 2\pi(R_0 + t)B$$

As radius increases, magnetic flux through the loop increases.
To oppose that induced current will be anti-clockwise.

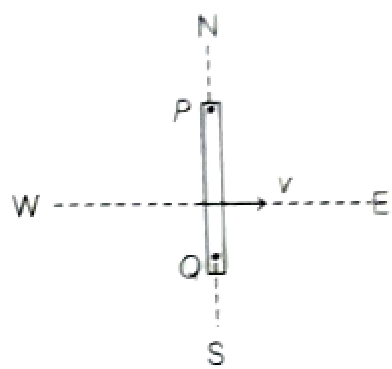
4. (A)

$$\varepsilon = Blv\sin 90^\circ$$

$$\Rightarrow V_P - V_Q = 4(0.3)(10)$$

$$\Rightarrow V_P - 0 = 12$$

$$\Rightarrow V_P = 12\text{ V}$$



5. (A)

$\varepsilon = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$

$= (2\hat{\mathbf{i}} \times (3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})) \cdot (5\cos 53° \hat{\mathbf{i}} + 5\sin 53° \hat{\mathbf{j}})$

$= (-8\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \cdot (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$= -32 \text{ V}$

6. (B)

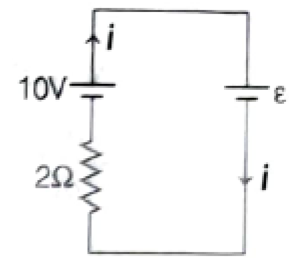
$F_{\text{ext}} = Bil$

$\Rightarrow \quad 0.5 = 0.5 \times i \times 0.25$

$\Rightarrow \quad i = 4 \text{ A}$

$i = \frac{10 - \varepsilon}{2} \Rightarrow 4 = \frac{10 - Blv}{2}$

$\Rightarrow \quad v = 16 \text{ m/s}$



7. (A)

Let velocity be v at any time.

$\varepsilon = Blv = 0.02 \times 0.5 \times v$

$\Rightarrow \varepsilon = 0.01v$

$i = \frac{1 - \varepsilon}{10} = \frac{1 - 0.01v}{10}$

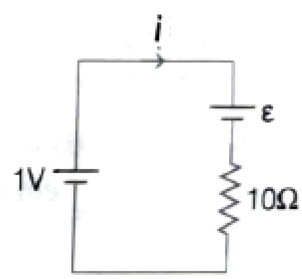
$F = Bil$

$\Rightarrow m\frac{dv}{dt} = Bil$

$\Rightarrow 0.002\frac{dv}{dt} = 0.02\left(\frac{1 - 0.01v}{10}\right)(0.5)$

$\Rightarrow \int_0^v \frac{dv}{100 - v} = \frac{1}{200}\int_0^t dt$

$\Rightarrow v = 100(10 - e^{-t/200})$



8. (D)

$\varepsilon = \int (\mathbf{v} \times \mathbf{B}) \cdot d\,\mathbf{l}$

$\mathbf{v} = \omega(x\sin\theta)\hat{\mathbf{k}}$

$\mathbf{B} = -B\,\hat{\mathbf{j}}$

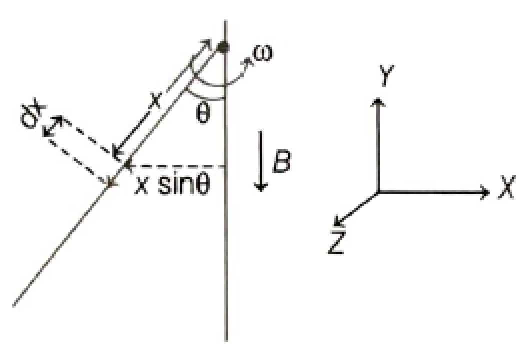
$\mathbf{dl} = -dx\sin\theta\,\hat{\mathbf{i}}$

$-dx\cos\theta\,\hat{\mathbf{j}}$

$\varepsilon = \int \left[\omega(x\sin\theta\hat{\mathbf{k}}) \times (-B\,\hat{\mathbf{j}})\right] \cdot (dx\sin\theta\hat{\mathbf{i}} - dx\cos\theta\hat{\mathbf{j}})$

$= \int \left[(B\omega x\sin\theta\hat{\mathbf{i}}) \cdot (-dx\sin\theta\,\hat{\mathbf{i}} - dx\cos\theta\,\hat{\mathbf{j}})\right]$

$= -B\omega\sin^2\theta\int_0^L x\,dx = \frac{-1}{2}B\omega L^2\sin^2\theta$



9. (A)

$$\varepsilon = \int Bvdx = \int_0^{L/2} B_0\left(v_0 + \frac{v_0}{L}x\right)dx = \frac{5}{8}B_0 v_0 L$$

10. (A)

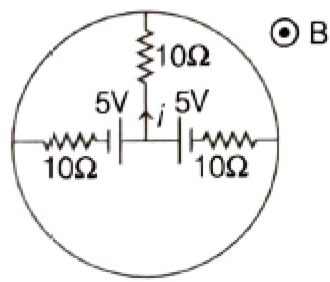
About instantaneous axis of rotation through Q, ring is in pure rotation.

So, $\varepsilon = \frac{1}{2}B\omega I^2 = \frac{1}{2}B\omega(\sqrt{2}R)^2 = B\omega R^2$

11. (C)

$$\varepsilon = \frac{1}{2}B\omega R^2 = \frac{1}{2}\times 50\times 20\times(0.1)^2$$

$$= 5\text{V}$$



$$\varepsilon_{eq} = \frac{\frac{5}{10}+\frac{5}{10}}{\frac{1}{10}+\frac{1}{10}} = 5\text{V};$$

$$r_{eq} = \frac{10(10)}{10+10} = 5\Omega$$

$$i = \frac{\varepsilon_{eq}}{R + r_{eq}} = \frac{5}{10+5} = \frac{1}{3}\text{A}$$

12. (C)

$$\varepsilon = \frac{1}{2}B\omega r^2, i = \frac{\varepsilon}{R} = \frac{1}{2}\frac{B\omega r^2}{R}$$

$$F_b = Bir$$

$$\sum\tau = 0$$

$$\Rightarrow \quad Fr - (Bir)\frac{r}{2} = 0$$

$$\Rightarrow \quad F = \frac{B^2\omega r^3}{4R}$$

13. (C)

For $r \le R$,

$$E(2\pi r) = \frac{d}{dt}\left(\int_0^r B(2\pi)dr\right)$$

$$\Rightarrow \quad Er\frac{d}{dt}\left(\frac{ktr^3}{3}\right)$$

$$\Rightarrow \quad E = \frac{kr^2}{3}$$

For $r > R$,

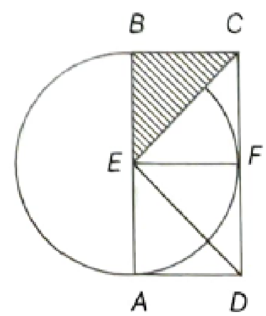
$$E(2\pi r) = \frac{d}{dt}\left(\int_0^R B2\pi r\, dr\right) \Rightarrow E = \frac{kR^3}{3r}$$

At $r = \frac{R}{2}, E_1 = \frac{kR^2}{12}$ and $r = 2R, E_2 = \frac{kR^2}{6}$

$\therefore \quad \frac{E_2}{E_1} = 2$

14. (C)

For loop EBC,



$$\phi = BA = B\left(\frac{R^2}{2}\left(\frac{\pi}{4}\right)\right)$$

$$|\varepsilon| = \left|\frac{d\phi}{dt}\right|$$

$$\Rightarrow \oint \mathbf{E}\cdot\mathbf{dl} = \left(\frac{\pi R^2}{8}\right)\frac{dB}{dt}$$

$$\Rightarrow \int_E^B \mathbf{E}\cdot\mathbf{dl} + \int_B^C \mathbf{E}\cdot\mathbf{dl} + \int_C^E \mathbf{E}\cdot\mathbf{dl} = \left(\frac{\pi R^2}{8}\right)\frac{dB}{dt}$$

$$\Rightarrow 0 + \int_E^B \mathbf{E}\cdot\mathbf{dl} + 0 = \left(\frac{\pi R^2}{8}\right)b = \varepsilon_{BC} = \frac{\pi R^2 b}{8}$$

$$\Rightarrow \varepsilon_{AB} = \int_A^B \mathbf{E}\cdot\mathbf{dl} = 0$$

$\therefore \quad \mathbf{E} \perp \mathbf{dl}$

15. (C)

$$L = \mu_0 n^2 V = \mu_0\left(\frac{N}{I}\right)^2 \pi r^2 I \text{ and } N = \frac{I}{2\pi r}$$

$$\Rightarrow L = \frac{\mu_0 I}{4\pi} \Rightarrow L \propto I$$

Since length is same, self-inductance will be same.

16. (B)

$$\phi = Mi \Rightarrow 1.8\times10^{-3} = M(2) \Rightarrow M = 9\times10^{-4}\text{ H}$$

$$\phi = Mi \Rightarrow \phi = (9\times10^{-4})(3) = 2.7\times10^{-3}\text{ Wb}$$

17. (C)

$$V_A - 5(1) - 15 - 5\times10^{-3}\left(-10^{-3}\right) = V_B$$

$$\Rightarrow V_B - V_A = -15\text{ V}$$

18. (B)

$$V_A - V_B = iR + L\frac{di}{dt}$$

$$140 = 5R + 10L \Rightarrow R + 2L = 28 \qquad \text{...(i)}$$

$$60 = 5R - 10L \Rightarrow R - 2L = 12 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$L = 4\,\text{H}$$

19. (C)

$$U = \frac{1}{2}Li^2 \Rightarrow \frac{dU}{dt} = \frac{1}{2}L\left(2i\frac{di}{dt}\right) = 2\times2\times4 = 16\ \text{J/s}$$

20. (A)

$$P = 8i^2 \Rightarrow \left(8\frac{di}{dt}\right)i = 8i^2$$

$$\Rightarrow \int_{i_0}^{2i_0}\frac{di}{i} = \int_0^t dt \Rightarrow t = \ln 2$$

21. (A)

Current in inductor branch will gradually increase to its steady state value.

22. (D)

$$(2L)\left(\frac{E}{R}\right) + L\left(-\frac{E}{R}\right) = (2L + L)i$$

$i = \dfrac{E}{3R}$, anti-clockwise

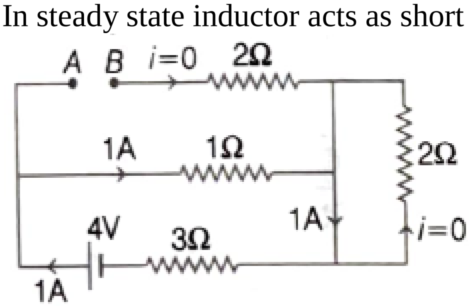
23. (A)

$$V = L\frac{di}{dt} = L\left(\frac{\varepsilon}{L}e^{-\frac{Rt}{L}}\right) = \varepsilon e^{-\frac{Rt}{L}}$$

At $t = \ln\sqrt{2}; V = 6e^{-\left(\frac{10\ln\sqrt{2}}{5}\right)} = 3\,\text{V}$

24. (B)

In steady state inductor acts as short circuit and capacitor acts as open circuit.



$$V_A - V_B = 1(1) = 1\,\text{V}$$

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)(1)^2 = \frac{1}{20}\text{ J}$$

$$U_C = \frac{1}{2}CV^2 = \frac{1}{2}\times 1\times 10^{-60}\times(1)^2$$

$$= \frac{10^{-6}}{2}\text{ J}$$

$$\Rightarrow \quad \frac{U_L}{U_C} = 10^5$$

25. (A)

Charge on capacitor in steady state $= C\varepsilon$

Current through inductor in steady state = 0

$$E = (C\varepsilon)\varepsilon = C\varepsilon^2$$

$$U = U_L + U_C = 0 + \frac{(C\varepsilon)^2}{2C} = \frac{1}{2}C\varepsilon^2$$

$$Q = E - U = \frac{1}{2}C\varepsilon^2$$

26. (C)

Using KVL,

$$+E - iR - L\frac{di}{dt} - \frac{q}{C} = 0$$

$$\Rightarrow \quad +4 - 2(1) - 1(4) - \frac{q}{3\times 10^{-6}} = 0$$

$$\Rightarrow \quad q = 6\,\mu\text{C}$$

27. (D)

$$i_{AB} = \frac{10}{2}\left(1 - e^{-\frac{2t}{2}}\right) = 5\left(1 - e^{-t}\right)$$

$$i_{CD} = \frac{10}{1}e^{-\frac{t}{(1)(1)}} = 10e^{-t}$$

$$i_{AB} = i_{CD} \Rightarrow 5\left(1 - e^{-t}\right) = 10e^{-t}$$

$$\Rightarrow \quad t = \ln 3\text{ s}$$

28. (D)

$$i = \frac{E}{R}e^{-\left(\frac{Rt}{L}\right)}$$

At $t = 0;\ i_1 = \frac{E}{R}$

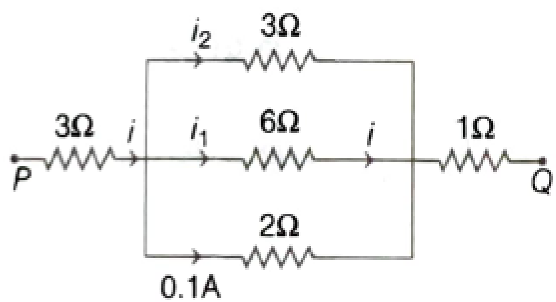
At $t = \frac{2L}{R};\ i_2 = \frac{E}{e^2R}$

Heat dissipated in resistor = Loss in energy of inductor

$$= \frac{1}{2}Li_1^2 - \frac{1}{2}Li_2^2$$

$$= \frac{LE^2}{2R^2}\left(\frac{e^4 - 1}{e^4}\right)$$

29. (5.00)



$i = i_1 + i_2 + 0.1$

$$= \frac{0.1}{3} + \frac{(0.1)2}{3} + 0.1 = 0.2\text{ A}$$

$$V_P - V_Q = i\left(3 + 1 + \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2}\right)^{-1}\right) = 1\text{V}$$

$V_P - V_Q = BIv = 1$

$\Rightarrow\ 2 \times 0.1 \times v = 1$

$\Rightarrow\ v = 5\text{ m/s}$

30. (8)

$i = \dfrac{BIv}{R},\ F = Bil = ma$

$$\Rightarrow\ \frac{B^2I^2v}{R} = -mv\frac{dv}{dx}$$

$$\Rightarrow\ \int_0^x dx = -\frac{mR}{B^2I^2}\int_{v_0}^0 dv$$

$$\Rightarrow\ x = \frac{mv_0R}{B^2I^2} = 8\text{ m}$$

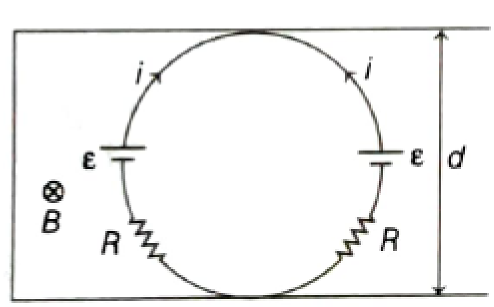
31. (4)

$\varepsilon = Bvd$ and $R = \lambda\left(\dfrac{\pi d}{2}\right)$

$$i = \frac{\varepsilon}{R} = \frac{2Bv}{\pi\lambda}$$

$$F = Bid + Bid = \frac{4B^2vd}{\pi\lambda}$$

So, $N = 4$

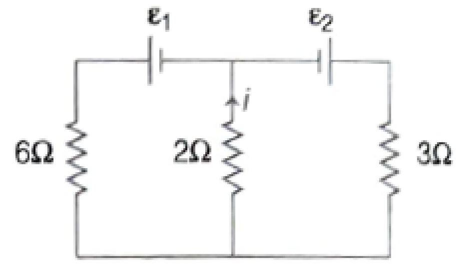


32. (2.36)

$$\varepsilon_1 = A_1 \frac{dB}{dt} = \pi(0.1)^2(100) = \pi \text{ V}$$

$$\varepsilon_2 = A_2 \frac{dB}{dt} = \pi(0.2)^2(100) = 4\pi \text{ V}$$

$$r_{eq} = \frac{6(3)}{6+3} = 2\Omega$$



$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$= \frac{\frac{\pi}{6} + \frac{4\pi}{3}}{\frac{1}{6} + \frac{1}{3}} = 3\pi \text{ V}$$

$$= i = \frac{\varepsilon_{eq}}{R + r_{eq}} = \frac{3\pi}{2+2} = \frac{3\pi}{4} \text{ A} = 2.36 \text{ A}$$

33. (3)

$$\phi = BA\cos\theta = \left(\frac{\mu_0 NIR^2}{2\left(R^2 + x^2\right)^{3/2}}\right)\pi r^2 \cos 0°$$

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \left(\frac{\mu_0 NIR^2 \pi r^2}{2}\right)\left(-\frac{3}{2}\frac{2x}{(R^2+x^2)^{5/2}}\frac{dx}{dt}\right)$$

To maximize $\varepsilon, \frac{d\varepsilon}{dt} = 0$

$$\Rightarrow \quad -\frac{3}{2}\mu_0 NIR^2 \, \pi r^2 v$$

$$\left(\frac{(R^2+x^2)^{5/2} v - \frac{5}{2}(R^2+x^2)^{3/2}(2x^2)v}{(R^2+x^2)^5}\right) = 0$$

$$\Rightarrow \quad R^2 + x^2 - 5x^2 = 0$$

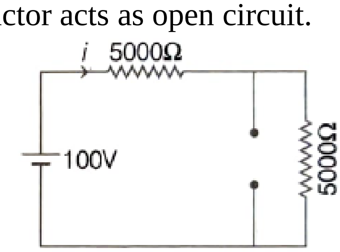
$$\Rightarrow \quad x = \frac{R}{2} = 3\text{m}$$

34. (1)

Just after closing the switch inductor acts as open circuit.

$$i = \frac{100}{(5000+5000)} = \frac{1}{100} \text{ A}$$

$$P = Vi = 100 = 1 \text{ W}$$



35. (100)

$$L = \mu_0 n^2 V = \mu_0 \left(\frac{N}{I}\right)^2 \pi r^2 I = \frac{\mu_0 (Nr)^2 \pi}{I}$$

$$1\times 10^{-3} = \frac{4\pi\times 10^{-7}(Nr)^2 \pi}{1} \Rightarrow N\pi r = 50$$

Length of wire $= N(2\pi r) = 2\times 50 = 100\text{ m}$

36. (1)

$$P = i^2 R = \left[\frac{V}{R}\left(1-e^{-t/\tau}\right)\right]^2 R$$

$$\frac{dP}{dt} = \frac{V^2}{R}\left[2\left(1-e^{-t/\tau}\right)\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)\right]$$

$$\frac{d^2P}{dt^2} = \frac{2V^2}{\tau R}\left[e^{-t/\tau}\left(-\frac{1}{\tau}\right) - e^{-2t/\tau}\left(-\frac{2}{\tau}\right)\right] = 0$$

$$\Rightarrow t = \tau \ln 2 = \frac{10}{6.93}(0.693) = 1\text{ s}$$

37. (3)

$$V_1 = L_1 \frac{di}{dt} \text{ and } V_2 = L_2 \frac{di}{dt}$$

$$\frac{V_1}{V_2} = \frac{L_1}{L_2} = \frac{\mu_0 N_1^2 A/I}{\mu_0 \mu_r N_2^2 A/I} = \left(\frac{600}{200}\right)^2 \left(\frac{1}{3}\right) = \frac{3}{1}$$

38. (2)

When only S_1 is closed, $T_1 = RC = 0.05$ ...(i)

When only S_2 is closed, $T_2 = \frac{L}{R} = 2$ ...(ii)

From Eqs. (i) and (ii), we get

$LC = 0.1$

When only S_3 is closed, $T = 2\pi\sqrt{LC}$

$= 2\sqrt{10}\sqrt{0.1} = 2\text{ s}$

39. (5)

$$i_1 = \frac{V}{R} e^{-\frac{t}{RC}} \text{ and } i_2 = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

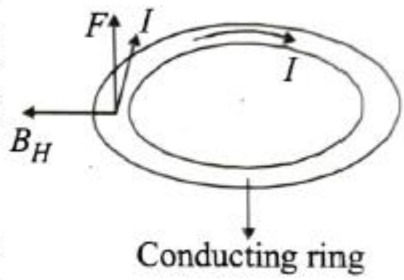
$$i = i_1 + i_2 = \frac{V}{R}\left(1 + e^{-\frac{t}{RC}} - e^{-\frac{Rt}{L}}\right)$$

Since i is constant $\Rightarrow e^{-\frac{t}{RC}} - e^{-\frac{Rt}{L}} = 0$

$$\Rightarrow R = \sqrt{\frac{L}{C}} = 5\,\Omega$$

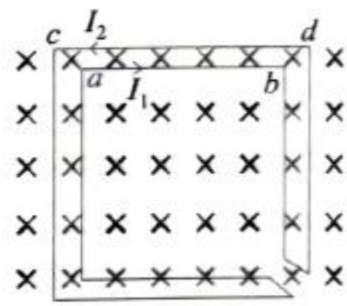
1. (A)

(a) The horizontal component of the magnetic field B_H interacts with the induced current produced in the conducting ring which produces an average force in the upward direction. This is an accordance with Fleming's left hand rule.



2. (D)

(d) The magnetic field is increasing in the downward direction. Therefore, according to Lenz's law the current I_1 will flow in the direction ab and I_2 in the direction dc.



3. (A)

(a) According to Faraday's law of electromagnetic induction, $\varepsilon = \frac{d\phi}{dt}$

Also, $\varepsilon = iR$

$\therefore \quad iR = \frac{d\phi}{dt} \quad \Rightarrow \quad \int d\phi = R\int idt$

Magnitude of change in flux ($d\phi$) = R × area under current vs time graph

or, $d\phi = 100 \times \frac{1}{2} \times \frac{1}{2} \times 10 = 250$ Wb

4. (A)

(a) As we know, Magnetic flux, $\phi = B.A$

[Trick: In such case we assume that current is flowing through bigger coil and flux due to magnetic field of bigger coil on smaller coil is determined.]

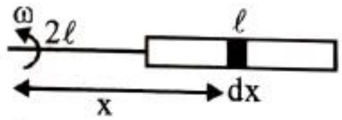
$$\frac{\mu_0(2)(20\times10^{-2})^2}{2[(0.2)^2+(0.15)^2]^{1.5}}\times\pi(0.3\times10^{-2})^2$$

On solving

$= 9.216 \times 10^{-11} = 9.2 \times 10^{-11}$ weber

5. (D)

(d) Here, induced e.m.f.



$de = (\omega x)Bdx$

$$e = \int_{2\ell}^{3\ell} (\omega x)Bdx = B\omega\frac{[(3\ell)^2 - (2\ell)^2]}{2} = \frac{5B\ell^2\omega}{2}$$

6. (C)

(c) Charge on he capacitor at any time t is given by $q = CV(1 - e^{-t/\tau})$

at $t = 2\tau$

$q = CV(1 - e^{-2})$

7. (A)

(a) Inside the sphere field varies linearly i.e., $E \propto r$ with distance and outside varies according to $E \propto \frac{1}{r^2}$

Hence the variation is shown by curve (a).

8. (A)

(a) **Given:** No. of turns $N = 1000$

Face area, $A = 4\text{ cm}^2 = 4 \times 10^{-4}\text{ m}^2$

Change in magnetic field,

$\Delta B = 10^{-2}\text{ wbm}^{-2}$

Time taken, $t = 0.01\text{ s} = 10^{-2}$ sec

Emf induced in the coil $e = ?$

Applying formula,

Induced emf, $e = \frac{-d\phi}{dt} = N\left(\frac{\Delta B}{\Delta t}\right)A\cos\theta$

$= \frac{1000 \times 10^{-2} \times 4 \times 10^{-4}}{10^{-2}} = 400\text{ mV}$

9. (B)

(b) Time in which left arm reach at x = 10 cm

$$t = \frac{0.1}{10} = \frac{1}{100} = 0.01 \sec$$

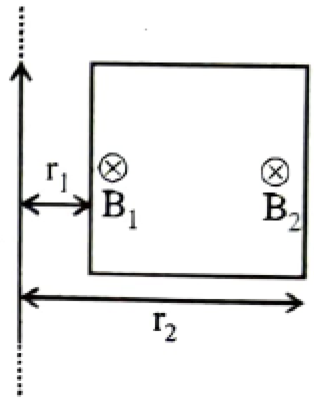
$r_1 = 10 \times 0.01 = 0.1$ m

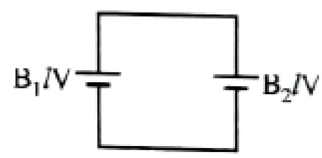
$e = lV(B_1 - B_2)$

$$= 0.1 \times 10\left(\frac{\mu_0 i}{2\pi r_1} - \frac{\mu_0 i}{2\pi r_2}\right)$$

$$= 0.1 \times 10 \times 2 \times 10^{-7} \times 1\left(\frac{1}{0.1} - \frac{1}{2 \times 0.1}\right)$$

$$= 2 \times 10^{-7}\left(\frac{2-1}{2 \times 0.1}\right) = 10^{-6} \text{V}$$



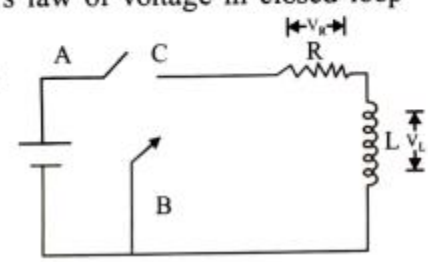


10. (C)

Hence damping will be faster for lesser self inductance.

(c) Applying Kirchhoff's law of voltage in closed loop

$$-V_R - V_C = 0 \Rightarrow \frac{V_R}{V_C} = -1$$



11. (D)

(d) According to Faraday's law of electromagnetic induction,

Induced emf, $e = \frac{Ldi}{dt}$

$$50 = L\left(\frac{5-2}{0.1 \sec}\right) \Rightarrow L = \frac{50 \times 0.1}{3} = \frac{5}{3} = 1.67 \text{ H}$$

12. (B)

(b) $I(0)=\frac{15\times100}{0.15\times10^3}=0.1A$

$I(\infty)=0$

$I(t)=[I(0)-I(\infty)]\ e^{\frac{-t}{L/R}}+i(\infty)$

$I(t)=0.1\ e^{\frac{-t}{L/R}}=0.1\,e^{\frac{R}{L}}$

$I(t)=0.1\ e^{\frac{0.15\times1000}{0.03}}=0.67mA$

13. (B)

(b) Electric flux is given by

$\phi=B.A$

$\phi=B_0\pi r^2e^{-t/\tau}$ $\quad\left(\because B=B_0e^{-t/\tau}\right)$

Induced E.m.f. $\varepsilon=\frac{d\phi}{dt}=\frac{B_0\pi r^2}{\tau^2}e^{-t/\tau}$

Heat $=\int_0^\infty\frac{\varepsilon^2}{R}=\frac{\pi^2r^4B_0^2}{2\tau R}$

14. (A)

(a) Given: Capacitance, $C=0.2\ \mu F=0.2\times10^{-6}$ F

Inductance L = 0.5 mH = 0.5×10^{-3} H

Current I = ?

Using energy conservation

$\frac{1}{2}CV^2=\frac{1}{2}CV_1^2+\frac{1}{2}LI^2$

$\Rightarrow\frac{1}{2}\times0.2\times10^{-6}\times10^2+0=\frac{1}{2}\times0.2\times10^{-6}\times5^2+\frac{1}{2}\times0.5\times10^{-3}I^2$

$\therefore\ I=\sqrt{3}\times10^{-1}\text{A}=0.17\text{ A}$

15. (A)

(a) $\phi=BA=(\mu_0n_i)A=\mu_0n\,(kt\,e^{-\alpha t})A$

$e=-\frac{d\phi}{dt}=-\mu_0nAk\frac{d}{dt}(te^{-\alpha t})$

$=-\mu_0nAk[t(-\alpha)e^{-\alpha t}+e^{-\alpha t}\times1]=-\mu_0nAk[e^{-\alpha t}(1-\alpha t)]$

$i=\frac{e}{R}=\frac{-\mu_0nAk}{R}[e^{-\alpha t}(1-t)]$

At $t=0$, $i\Rightarrow$ –ve

16. (B)

(b) According to faraday's law of electromagnetic induction, $e = \frac{-d\phi}{dt}$

$L \times \frac{di}{dt} = 25 \Rightarrow L \times \frac{15}{1} = 25$ or $L = \frac{5}{3} H$

Change in the energy of the inductance,

$$\Delta E = \frac{1}{2} L\left(i_1^2 - i_2^2\right) = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5 J$$

17. (None)

(None)

Net charge $Q = \frac{\Delta\phi}{R} = \frac{1}{10} A(B_f - B_i)$

$= \frac{1}{10} \times 3.5 \times 10^{-3} \left(0.4 \sin\frac{\pi}{2} - 0\right)$

$= \frac{1}{10}(3.5 \times 10^{-3})(0.4 - 0) = 1.4 \times 10^{-4}$

No option matches, So it should be a None.

18. (B)

(b) Induced emf,

$e = Bv\ell = 1 \times 10^{-2} \times 0.05 = 5 \times 10^{-4}$ V

Equivalent resistance,

$R = \frac{4 \times 2}{4 + 2} + 1.7 = \frac{4}{3} + 1.7 \simeq 3\,\Omega$

Current, $i = \frac{e}{R} = \frac{5 \times 10^{-4}}{3} \simeq 170\,\mu A$

19. (D)

Inductance $= \frac{\mu_0 N^2 A}{L}$

20. (A)

(a) Induced emf, $\varepsilon = Bv\ell$

$= 0.3 \times 10^{-4} \times 5 \times 10$

$= 1.5 \times 10^{-3}$ V

21. (D)

(d) The rate of mutual inductance is given by

$M = \mu_0 n_1 n_2 \pi r_1^2$ ...(i)

The rate of self inductance is given by

$L = \mu_0 n_1^2 \pi r_1^2$ ...(ii)

Dividing (i) by (ii)

$\Rightarrow \frac{M}{L} = \frac{n_2}{n_1}$

22. (C)

(c) $L = \mu_0 n^2 A l$

$L \propto l$

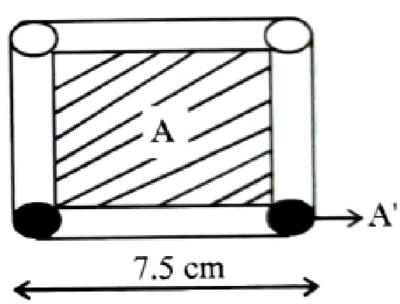
$\therefore \quad \frac{L_2}{L_1} = \frac{l_2}{l_1}$

$\Rightarrow \quad L_2 = 3L_1 \qquad [\because l_2 = 3l_1]$

23. (B)

(b) $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos 0°) = -A\frac{dB}{dt}$

$i = \frac{l}{R} = A\frac{dB}{dt} \times \frac{A'}{\rho l'}$



$= l^2 \frac{dB}{dt} \times \frac{\pi r^2}{Pl'}$

Here, $l = 7.5 \times 10^{-2}$ m

$l' = 0.30$ m

$r = 2 \times 10^{-3}$ m

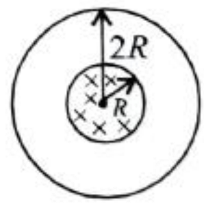
$\frac{dB}{dt} = 0.032$ T/s

Putting above value; we get

$i = 0.61$A

24. (D)

(d) According to question,



$I(t) = I_0 t(1-t)$

$\therefore I = I_0 t - I_0 t^2$

$\phi = B.A$

$\phi = (\mu_0 nI) \times (\pi R^2)$

$(\because\ B = \mu_0 nI \text{ and } A = \pi R^2)$

$$V_R = \frac{-d\phi}{dt}$$

$$V_R = \mu_0 n\pi R^2 (I_0 - 2I_0 t) \Rightarrow V_R = 0 \text{ at } t = \frac{1}{2}s$$

after $t > \frac{1}{2}$ sec, V_R is $-$Ve

i.e. current will change direction after $t > \frac{1}{2}$ sec.

25. (D)

(d) Magnetic field at a distance r from the wire

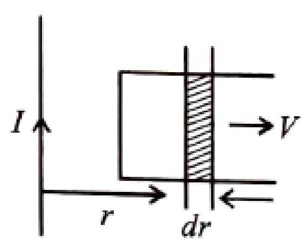
$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic flux for small displacement dr,

$\phi = B \cdot A = Bl dr$

$[\because A = l\,dr \text{ and } B.A = BA\cos 0°]$

$$\Rightarrow \phi = \frac{\mu_0 I}{2\pi r} l\,dr$$



Emf, $e = \frac{d\phi}{dt} = \frac{\mu_0 Il}{2\pi r} \cdot \frac{dr}{dt} \Rightarrow e = \frac{\mu_0}{2\pi} \cdot \frac{Ivl}{r}$

Induce current in the loop, $i = \frac{e}{R} = \frac{\mu_0}{2\pi} \cdot \frac{Ivl}{Rr}$

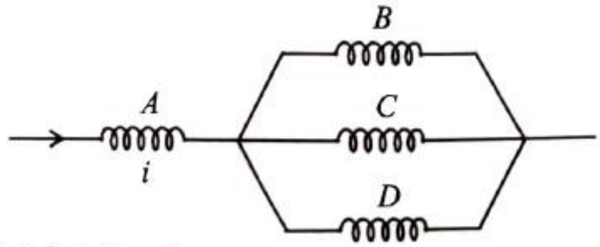
26. (A)

From Lenz law direction of induced current can be found, induced current will oppose the decrease in outward magnetic field, by producing its own outward magnetic field.

27. (C)

(c) From figure,

$B_A = 3T$



$I_B + I_C + I_D = I_A$

By symmetry, $I_B = I_C = I_D = i_0$ (let) $\Rightarrow i_0 = \frac{i}{3}$

We have $B_A = \mu_0 ni$

and, $B_C = \mu_0 ni_0 = \frac{\mu_0 ni}{3} = \frac{B_A}{3} = 1T$

28. (D)

(d) From question, $U = 25\%$ of U_0

$$U = \frac{1}{2}LI^2 = \frac{1}{4}\times\frac{1}{2}LI_0^2 \Rightarrow I = \frac{I_0}{2}$$

Also, $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right) \Rightarrow \frac{I_0}{2} = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\Rightarrow e^{\frac{-Rt}{L}} = \frac{1}{2} \Rightarrow (t)\frac{R}{L} = \ln 2 \quad \therefore t = \frac{L}{R}\ln 2$$

29. (C)

(c) Given,

Magnetic field, $B = 1T$

At $t = 0$s, the centre of circular ring will touch the boundary.

$\therefore$ Induced emf $= Blv$, l = length $\perp_r$ to $\vec{B}$ and $\vec{V}$.

$= 1.(2R).1 = 2V$.

30. (D)

(d) $P = \frac{e^2}{R} = \frac{\left(-NA\frac{dB}{dt}\right)^2}{\rho l} \times A_C,$

A = area of coil

A_c = area of cross section of wire used in coil

$P \propto NA_c$

$\frac{P_2}{P_1} = \frac{\frac{N}{2} \times 4A_c}{NA_c} = 2;\ P_2 = 2P_1$

31. (A)

$\phi = 5t^3 + 4t^2 + 2t - 5$

$|e| = 15t^2 + 8t + 2 \qquad \left[\because |e| = \frac{d\phi}{dt}\right]$

$R = 5\Omega$

So, $i = \frac{|e|}{R} = \frac{15t^2 + 8t + 2}{5} = 3t^2 + 1.6t + 0.4$

$= 3.2^2 + 1.6 \times 2 + 0.4 = 15.6\,\text{A} \ (\because t = 2\,\text{sec})$

32. (C)

(c) $\phi_2 \propto I_1$

$\phi_2 = M_{21} I_1$

$M_{21} = \frac{\phi_2}{I_1} = \frac{B_1 A_2}{I_1}$

Now, $B_1 = \frac{\mu_o I_1}{4\pi L/2}\ [\sin 45° + \sin 45°] \times 4$

$= \frac{\mu_o I_1}{2\pi L} \times \frac{8}{\sqrt{2}} = 2\sqrt{2}\frac{\mu_o I_1}{\pi L}$

So, $M_{21} = \frac{2\sqrt{2}\,\mu_o I_1}{\frac{\pi L}{I_1}} \ell^2 = \frac{\mu_o I_1 \ell^2}{\pi L} \times 2\sqrt{2}$

33. (D)

(d) If current in both inductor is in same direction then

$L_{eq} = L_1 + L_2 + 2M$

and, if current in both inductor is in opposite direction,

then $L_{eq} = L_1 + L_2 - 2M$

34. (B)

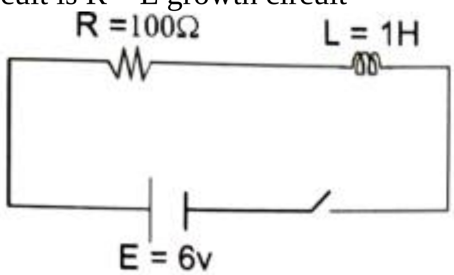
(b) emf induced between the two ends is given by

$$e = \frac{1}{2}B_H\omega l^2 = \frac{1}{2} \times 0.2 \times 10^{-4} \times 5 \times 1^2$$

$$= 0.1 \times 10^{-4} \times 5 = 5 \times 10^{-5} = 50\,\mu V$$

35. (C)

Given circuit is R – L growth circuit



$$i = \frac{E}{R}(1 - e^{-t/\tau}) = i_0(1 - e^{-t/\tau}), \text{ when } i = \frac{i_0}{2}$$

$$\Rightarrow \frac{E}{2R} = \frac{E}{R}(1 - e^{-t/\tau})$$

Solving we get, $t = \tau \ln 2$

$$\Rightarrow t = \frac{L}{R}\ln 2 = \frac{1}{100}0.693 = 0.00693 = 7\,\text{ms}$$

$$\text{Now, } i(15\,\text{ms}) = \frac{E}{R}\left(1 - e^{-\frac{15}{10}}\right) = \frac{E}{R}(1 - e^{-3/2})$$

$$\Rightarrow i = \frac{6}{100}\left(1 - \frac{1}{4}\right) = \frac{3}{4} \times \frac{6}{100} = \frac{18}{400} = \frac{9}{200}\,\text{A}$$

$$\text{As, } U = \frac{1}{2}LI^2 = \frac{1}{2} \times 1 \times \left(\frac{9}{200}\right)^2 \simeq 10^{-3}\,\text{J} = 1\,\text{mJ}$$

36. (10)

(10) Given $dI = 0.25 - 0 = 0.25$ A

$dt = 0.025$ ms

Induced voltage

$E_{ind} = 100$ v

Self-inductance, L = ?

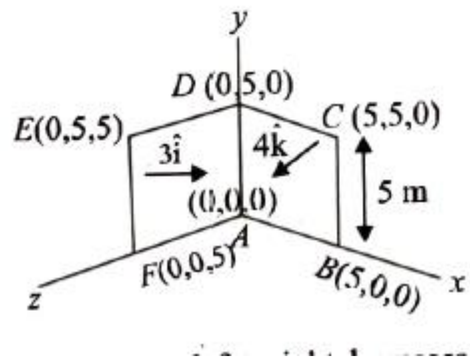
$$\text{Using, } E_{\text{ind}} = \frac{\Delta\phi}{\Delta t} \Rightarrow 100 = \frac{L(0.25 - 0)}{.025 \times 10^{-3}}$$

$\Rightarrow$ L = 0.01 H = 10 mH

37. (175.00)

(175.00)

Flux through the loop ABCDEFA,



$\phi = \vec{B}.\vec{A}$

$= (3\hat{i} + 4\hat{k}).(25\hat{i} + 25\hat{k})$

$\Rightarrow \phi = (3 \times 25) + (4 \times 25)$

$= 175$ weber

38. (60)

(60) Given,

Magnetic field, $B = 3 \times 10^{-2} T$

Angular speed of coil, $\omega = 50$ rad s^{-1}

Number of turns in coil, $n = 20$

Maximum emf, $\varepsilon = N\omega AB$

$\Rightarrow \varepsilon = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^2 = 60.28 \times 10^2$

Rounded off to nearest integer = 60

39. (12)

(12) Given,

Magnetic field, $B = (3t^3\hat{j} + 3t^2\hat{k})$

Magnetic flux, $\phi = \vec{B}.\vec{A}$

$= (3t^3\hat{j} + 3t^2\hat{k}).(\pi(1)^2\,\hat{k}) = 3t^2\pi$

Induced emf, $\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{d(3t^2\pi)}{dt} = 6t\pi$

$\therefore \quad \varepsilon_{t=2} = 6 \times 2 \times \pi = 12\pi$

40. (2)

(2) The magnetic flux given by $\phi = \frac{2}{3}(9 - t^2)$

when the flux becomes zero then $\phi = \frac{2}{3}(9 - t^2) = 0$

$t = 3$ sec

The emf will be

$e = \frac{-d\phi}{dt} = -\frac{2}{3}(0 - 2t) = \frac{4t}{3}$

Heat produced in 3 sec $= \int_0^t \frac{e^2}{r}dt = \int_0^3 \frac{16t^2}{9 \times 8}dt = 2$ J

41. (250)

(250) We have magnetic flux given as,

$\phi = 8t^2 - 9t + 5$

Inuced emf $= -\frac{d\phi}{dt} = -(16t - 9)$

At t = 0.25 s

Emf = $-[(16 \times 0.25) - 9] = 5V$

So, Current $= \frac{Emf}{Resistance} = \frac{5V}{20\Omega}$

$\Rightarrow \frac{1}{4}A = \frac{1000}{4}mA = -250\ mA$

42. (16)

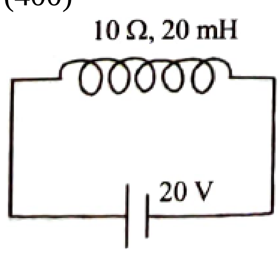
(16) The moving rod will cut the vertical component of magnetic field perpendicularly.

So, $e = B_v l V$

$= B_H \times l V \quad [\because Dip = 45° \Rightarrow B_v = B_H]$

$= 4 \times 10^{-3} \times 0.2 \times 20 = 16 \times 10^{-3} V$

43. (400)



$|e| = \frac{Ldi}{dt} = \frac{L(i_{initial} - 0)}{t} = \frac{LV}{Rt}$

$\therefore |e| = \frac{20 \times 10^{-3} \times 20}{10 \times 100 \times 10^{-6}} = 400\ V$

44. (4)

(4) Suppose at any instant, current through inductor is 'I'.
Now if in inductor when current change by dI in dt time then, work done by battery is given by
dw = change × P.difference

$$= I\,dt \times L\frac{dI}{dt} = LI\,dI$$

Now, $I = 2\sin(t^2)A$

If $I = 0 \Rightarrow t = 0$

If $I = 2A \Rightarrow 2 = 2\sin(t^2)$

$$\Rightarrow \sin t^2 = \sin \pi/2 \Rightarrow t = \sqrt{\frac{\pi}{2}}.$$

So, amount of total energy spent

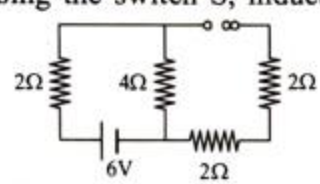
$$U = \int dw = \int LI\,dI = \int 2 \times 2\sin(t^2) \times 4t\cos(t^2)\,dt$$

$$= 8\int_0^{\sqrt{\pi/2}} t\sin(2t^2)\,dt = 8 \times \frac{1}{2} = 4\text{J}$$

45. (1)

(1) Just after closing the switch S, inductor behaves like an open circuit.

$$I = \frac{6}{2+4} = 1\text{ A}$$



46. (10)

(10) In steady state, capacitor is fully charged. Therefore current through it zero.
Charge $q = CV_{100\Omega}$

$$= (1.1 \times 10^{-6})\left(\frac{10}{R+r}R\right) \quad (\because\ V_{100} = IR_{100})$$

$$= 1.1 \times 10^{-6}\left(\frac{10}{110} \times 100\right) = 10\,\mu\text{C}$$