PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2025 **MAJOR TEST - 4** **DATE: 28/04/24**

ADVANCED **ANSWER KEY**

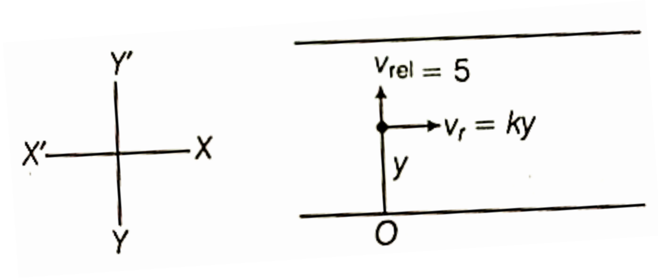
PAPER – 2 (Code – 21)

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	4	**19.**	2	**37.**	9
2.	5	**20.**	3	**38.**	4
3.	6	**21.**	8	**39.**	Bonus
4.	2	**22.**	3	**40.**	9
5.	8	**23.**	7	**41.**	5
6.	5	**24.**	7	**42.**	1
7.	AC	**25.**	BD	**43.**	ACD
8.	ACD	**26.**	AB	**44.**	AB
9.	ABCD	**27.**	ABC	**45.**	BD
10.	AC	**28.**	BC	**46.**	AB
11.	ABC	**29.**	ACD	**47.**	ABD
12.	BCD	**30.**	ABC	**48.**	ACD
13.	0.75	**31.**	Bonus	**49.**	4.00
14.	2.25	**32.**	23.52 – 23.68	**50.**	67.50
15.	0.50	**33.**	1.69	**51.**	0.11 – 0.14
16.	0.25	**34.**	6.90	**52.**	0.92
17.	23.94	**35.**	4.50	**53.**	1.32 – 1.35
18.	0.50	**36.**	1.2	**54.**	1.60

PART (A) : PHYSICS

1. (4)

$\therefore v_x = ky$ and $v_y = 5$



$\therefore y = 5t$, At middle point, $50 = 5t$

$\therefore t = 10s$

$\because v_x = ky$

At middle, $v_x = 2$ $\quad\therefore k = \frac{2}{50}$

$\therefore v_x = \frac{2}{50}y = \frac{y}{25} = \frac{5t}{25} = \frac{t}{5}$

$\Rightarrow \frac{dx}{dt} = \frac{t}{5}$

$\Rightarrow \int_0^{x_0} dx = \int_0^{10} \frac{t}{5} dt$

$\therefore x_0 = 10\,m$ upto middle

$\therefore$ Total drifting $= 2 \times 10 = 20\,m$

$\therefore 5n = 20, n = 4$

2. (5)

When the block A just starts to leave the surface, then

$kx_0 = Mg$

For minimum value of m, x_0 should be maximum elongation in the spring. At the instant of maximum elongation, the velocity of block B will be zero.

$\therefore$ Loss in gravitational potential energy of block B = Gain in elastic potential energy by spring

$\therefore mg\,x_0 = \frac{1}{2}k\,x_0^2$

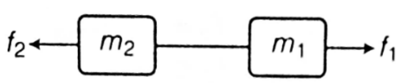
$\therefore x_0 = \frac{2mg}{k}$ or $\frac{Mg}{k} = \frac{2mg}{k} \Rightarrow m = \frac{M}{2} = 5\,kg$

3. (6)

$f_{1max} = 0.3 \times 20 \times 10 = 60\,N$

$f_{2max} = 0.3 \times 5 \times 10 = 15\,N$

For motion of m_2,



$f_1 \geq f_2$

But $f_1 = 2f_2 \Rightarrow f_1 > f_{2max}$ or $f_1 > 15N$

But $f_1 = 2f_2 \Rightarrow f_1 = 30N$

But $f_{1max} = 60N$

$\therefore \; f_1 < f_{1max}$

It means static friction comes into play between m_1 and M. Hence all blocks move together.

$\therefore \; F - f_{2max} = (m_1 + m_2 + M)a$

or $F - 15 = 75a_0$

For block M, $F - f_1 = 50a; F - 30 = 50a$

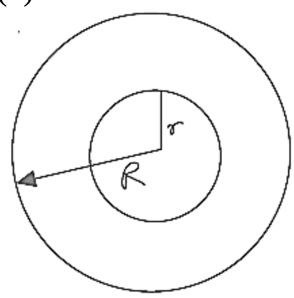
$$F - 30 = 50\left(\frac{F-15}{75}\right)$$

$\therefore \; F = 60N$

$\therefore \; 10n = 60$

$\therefore \; n = 6$

4. (2)



$$E_1 4\pi (R/2)^2 = \frac{\int_0^{\frac{R}{2}} (4\pi r^2 dr) kr^a}{\epsilon_0} = \frac{4\pi k}{\epsilon_0}\left(\frac{r^{3+a}}{3+a}\right)_0^{\frac{R}{2}}$$

$$E_2 \times 4\pi R^2 = \frac{\int_0^{R} (4\pi r^2 dr) kr^a}{\epsilon_0} = \frac{4\pi k}{\epsilon_0}\left(\frac{r^{3+a}}{3+a}\right)_0^{R}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{4\left(\frac{R}{2}\right)^{3+a}}{R^2(3+a)} \Bigg/ \frac{R^{3+a}}{(3+a)\times R^2}$$

$$\frac{E_1}{E_2} = \frac{4R^{(3+a)}}{2^{(3+a)} \times R^{(3+a)}} = \frac{4}{2^{3+a}} = \frac{1}{8}$$

$2^{3+a} = 32 = 2^5$

$\Rightarrow \; a = 2$

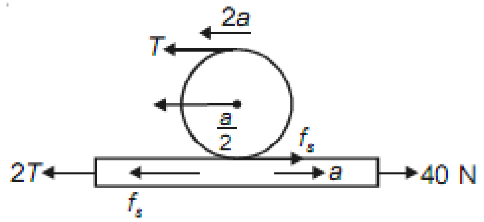
5. (8)

FBD of cylinder and block are as shown by Newton's laws

$40 - 2T - f_s = 8a$ (1)

$T - f_s = 4a/2$ (2)

Subtracting equation (2) from equation (1)
$40 - 3T = 6a$



$T = (40 - 6a)/3$
Also, by $\tau = I\alpha$, we get

$T \times R + f_s \times R = I\dfrac{3a}{2R}$

$\Rightarrow \quad T + f_s = 3Ia/2R^2$

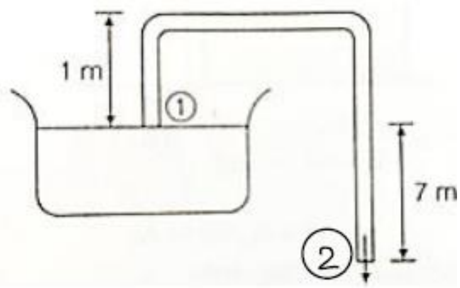
$\Rightarrow (40 - 6a)/3 + (40 - 12a)/3 = 3Ia/2R^2 \Rightarrow a = 80/(18 + 9I/2R^2)$

As $\tau = I\alpha = I \times (3a/2R)$

$$\therefore \quad \tau = \frac{3I}{2R}\frac{80}{\left(18+\frac{9I}{2R^2}\right)} \Rightarrow \tau = \frac{3\times 80}{18+9} = \frac{80}{9}\text{N}-\text{m}$$

6. (5)
Velocity gets interchanged.
$\therefore \left|\vec{V}_B\right|$ just after collision $= A\omega = 5\text{ m/s}$.

7. (AC)
Applying Bernoulli's theorem between points (1) and (2), we get



$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$p_1 = p_2 = p_{atm}$, $h_1 = 0, h_2 = -7\text{ m}$ and $v_1 \simeq 0$.

$$\therefore 1000\times 9.8\times 0 = \frac{1}{2}(1000)v_2^2 + 1000\times 9.8(-7)$$

$$\therefore v_2^2 = 2\times 9.8\times 7$$

$$\Rightarrow v_2 \simeq 11.7\text{ ms}^{-1}$$

Minimum pressure in the bend will be at A. Again, applying Bernoulli's theorem between points (A) and (1)

$$p_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1$$

Again, $v_1 \simeq 0, h_1 = 0, h_A = 1\text{m}, p_1 = 1.01\times 10^5\text{ Nm}^{-2}$

$p_A + \frac{1}{2}(1000)v_A^2 + (1000)(9.8)(1) = 1.01\times10^5 + 0 + 0$

$v_A = v_2$

$\therefore p_A + 500(11.7)^2 = 101000 - 9800$

$\Rightarrow p_A = 2.27\times10^4\ Nm^{-2}$

8. (ACD)

Mean KE $= \frac{1}{2}mv_{rms}^2 = \frac{1}{2}m\cdot\frac{3kT}{m}$

$= \frac{3}{4}m\cdot\left(\frac{2kT}{m}\right) = \frac{3}{4}mv_p^2$

$v_{rms} = \sqrt{\frac{3kT}{m}}, v_p = \sqrt{\frac{2kT}{m}}, \bar{v} = \sqrt{\frac{8}{\pi}\cdot\frac{kT}{m}}$

$\therefore\ v_{rms} > \bar{v} > v_p$

Also, $v_{rms} : \bar{v} : v_p :: \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$

$\Rightarrow v_{rms} : \bar{v} : v_p :: \sqrt{3} : \sqrt{2.5} : \sqrt{2}$

Also, KE of 1g of a gas $= \frac{3}{2}\frac{R}{M}\cdot T = \frac{3}{2}\frac{k}{m}T$

9. (ABCD)

$R_{eq} = R_1 + R_2 = \frac{l_1}{AK_1} + \frac{l_2}{AK_2}$

$H = \frac{dQ}{dt} = \frac{\Delta\theta}{R} = \frac{(T_1 - T_2)}{R_1 + R_2} = \frac{(T_1 - T_2)A}{\left(\frac{l_1}{K_1} + \frac{l_2}{K_2}\right)}$

Also, $\frac{T_1 - T_2}{R_1 + R_2} = \frac{T_1 - T}{R_1} \Rightarrow T = \frac{T_1\frac{l_2}{K_2} + T_2\frac{l_1}{K_1}}{\left(\frac{l_1}{K_1} + \frac{l_2}{K_2}\right)}$

And $R = R_1 + R_2$ so,

$K_{eq} = \frac{l_1 + l_2}{A(R_1 + R_2)} = \frac{l_1 + l_2}{\left(\frac{l_1}{K_1} + \frac{l_2}{K_2}\right)}$

10. (AC)

Heat received by ice $= mL + mC\Delta T = 10700$ cal

Heat given by container, then

$= -\int_{500}^{300} m_c(A + BT)dT = -m_c\left[AT + \frac{BT^2}{2}\right]_{500}^{300} = 21600\,m_c$

Heat lost = Heat gained

$\therefore 10700 = 21600\,m_c$

$\Rightarrow m_c = \dfrac{10700}{21600} \approx 0.495 \approx 0.5\,kg$

11. (ABC)

Comparing the given equation with standard velocity-displacement equation of SHM

or $\quad v = \omega\sqrt{A^2 - x^2}$

or $\quad v^2 = A^2\omega^2 - \omega^2 x^2$

We see that $\omega = 3$

or $\quad \dfrac{2\pi}{T} = 3$ or $T = \dfrac{2\pi}{3}$ units

Similar, amplitude of oscillation

$$A = \sqrt{\frac{A^2\omega^2}{\omega^2}} = \sqrt{\frac{144}{9}} = \frac{12}{9} = 4 \text{ units}$$

Acceleration of the particle at a distance x from the mean positive is given by

$a = \omega^2 x = (9)(3) \quad (x = 3 \text{ units})$

$= 27$ units

12. (BCD)

Potential at each point on y-z plane is zero. The electric field will be zero on y-z plane at a distance $\sqrt{2}a$ from origin.

13. (0.75)

$LHS = \left[L^3\right]$

$RHS = \left[M^{-1}L^3T^{-2}\right]^x \left[LT^{-1}\right]^y \left[ML^2T^{-1}\right]^z$

$= \left[M^{-x+z}L^{3x+y+2z}T^{-2x-y-z}\right]$

According to homogeneity principle,

$LHS = RHS$

$\because \; -x + z = 0$

$\therefore \; z = x \quad \ldots(i)$

$3x + y + 2z = 3$

$5x + y = 3 \quad \ldots(ii)$

And $-2x - y - z = 0$ or $-2x - y - x = 0$

or $-3x - y = 0$

or $3x + y = 0 \quad \ldots(iii)$

From Eqs. (ii) and (iii), we get

$2x = 3 \Rightarrow x = 3/2$

$\Rightarrow \dfrac{x}{2} = \dfrac{3}{4} = 0.75$

14. (2.25)

Strain produced in rods is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = \alpha_1\Delta\theta + \alpha_2\Delta\theta$$

or $\varepsilon = (\alpha_1 + \alpha_2)\Delta\theta$

$$= (10^{-5} + 2\times10^{-5})100 = 3\times10^{-3}$$

If σ is the stress induced by preventing this strain, then

$$\varepsilon = \frac{\sigma}{Y_1} + \frac{\sigma}{Y_2} = \sigma\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)$$

$$\sigma = \frac{\varepsilon}{\frac{1}{Y_1} + \frac{1}{Y_2}} = \frac{3\times10^{-3}}{\left(\frac{1}{3\times10^{10}} + \frac{1}{10^{10}}\right)} = 2.25\times10^{7}\ \text{Nm}^{-2}$$

15. (0.50)

Weight of the drop = Force due to surface tension + Buoyant force

$$\therefore \frac{4}{3}\pi r^3 \rho g = (2\pi r)T + \frac{1}{2}\left[\frac{4}{3}\pi r^3\right]\rho_L g$$

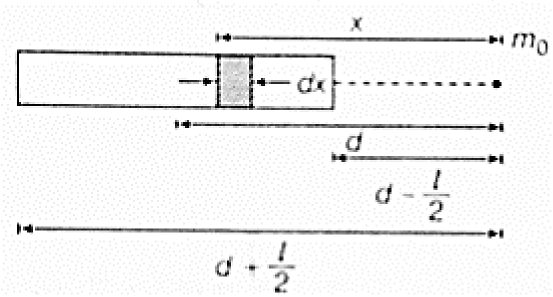
$$\frac{4}{3}\pi r^3 g\left[\rho - \frac{\rho_L}{2}\right] = 2\pi r \cdot T$$

$$\Rightarrow r^2 \frac{4}{3} g\left[\frac{2\rho - \rho_L}{2}\right] = 2T$$

$$\Rightarrow r^2 = \frac{3T}{g(2\rho - \rho_L)} \Rightarrow r = \sqrt{\frac{3T}{g(2\rho - \rho_L)}}$$

16. (0.25)

Consider an element as shown in the figure.



$$dm = \frac{m_0}{l}dx$$

Gravitational potential energy between dm and point mass is

$$dU = -\frac{Gm_0 dm}{x} = -\frac{Gm_0^2}{l}\cdot\frac{1}{x}dx$$

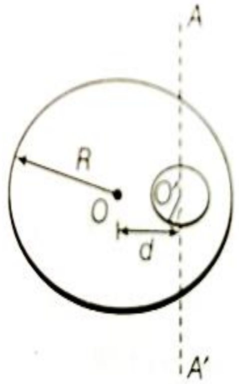
$$\text{Now, } U = -\frac{Gm_0^2}{l}\int_{d-\frac{l}{2}}^{d+\frac{l}{2}}\frac{1}{x}dx = -\frac{Gm_0^2}{l}\ln\left(\frac{d+\frac{l}{2}}{d-\frac{l}{2}}\right)$$

or $U = -\frac{Gm_0^2}{l}\ln\left(\frac{2d+l}{2d-l}\right)$

17. (23.94)

$(I_{AA'})_{\text{removed disc}} = \frac{mr^2}{2}$

Where, m is mass of disc of radius r.



$m = \frac{M}{R^2}r^2$

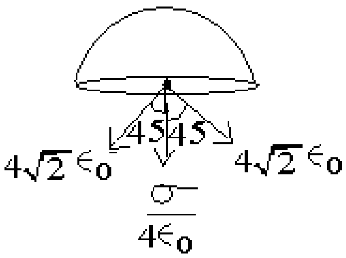
$\therefore (I_{AA'})_{\text{removed disc}} = \frac{M}{R^2}\cdot\frac{r^4}{2}$

$(I_{AA'})_{\text{original uncut disc}} = \frac{MR^2}{2} + Md^2$

$(I_{AA'})_{\text{residual disc}} = (I_{AA'})_{\text{original uncut disc}} - (I_{AA'})_{\text{removed disc}}$

$= \frac{MR^2}{2} + Md^2 - \frac{Mr^4}{2R^2} = \frac{M}{2}\left[R^2 + 2d^2 - \frac{r^4}{R^2}\right]$

18. (0.50)



PART (B) : CHEMISTRY

19. (2)

$n_{H_2SO_4} = 50 \times 0.02 = 1$

$n_{CaCO_3} = \frac{600}{10^6} \times 200 \times 10^3 \times \frac{1}{100} = 1.2$

$\because$ L.R. is H_2SO_4

$[H_2CO_3] = \frac{1}{250} = 4 \times 10^{-3} M$

$pH = -\log 2 \times 4 \times 10^{-3} = 3 - 3\log 2 = 3 - 0.903 = 2.1$

20. (3)

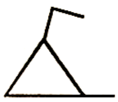
$f = e^{\frac{-Ea}{RT}} \Rightarrow \frac{10^{-3}}{100} = 10^{-5}$

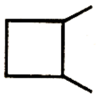
$x = Ae^{\frac{-Ea}{RT}}$

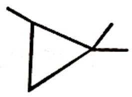
$3 \times 10^{-3} = A \times 10^{-5} \Rightarrow A = 300\, Min^{-1}$

Maximum value of rate constant is equal to A is

21. (8)


4 Optically Active Isomers


2 Optically Active


2 Optically Active

2 + 2 + 4 = 8

22. (3)

$$E_a = \frac{k_1 E_{a_1} + k_2 E_{a_2}}{k_1 + k_2} = \frac{2E_1 + 4E_2}{6} = \frac{E_1 + 2E_2}{3}$$

$x = 3$

23. (7)

$-COOH, -SO_3H$, Squaric acid.

24. (7)

$$\frac{1}{\lambda} = R_H (z)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

for $n_1 = 1 \quad n_2 = ?$

$$\frac{1}{30.4 \times 10^{-9}} = 1.09678 \times 10^7 \times 4\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$$

$n_2 = 2$

for $n_1 = 2$ (first excited state), $n_2 = ?$

$$\frac{1}{108.5 \times 10^{-9}} = 1.09678 \times 10^7 \times 4\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

$\boxed{n_2 = 5}$

sum of n_1 & $n_2 = 2 + 5 = 7$

25. (BD)
Bredt's Rule

26. (AB)

$$E_{Cell} = E^0_{Cell} - \frac{0.0591}{2}\log\frac{[Zn^{2+}]}{[Ag^+]^2}$$

$$= E^0_{Cell} - \frac{0.0591}{2}\log\frac{C_1}{(C_2)^2}$$

$E_{Cell} = E^0_{Cell}$, when $C_1 = C_2 = 1$ M or $C_2 = \sqrt{C_1}$

27. (ABC)

$$s = [Zn(OH)_2]_{aq} + [Zn(OH)]^+ + [Zn^{2+}] + [Zn(OH)_3]^- + [Zn(OH)_4]^-$$

$$= K_1 + \frac{K_2K_1}{[OH^-]} + \frac{K_1K_2K_3}{[OH^-]^2} + K_4[OH^-]K_1 + K_1K_4K_5[OH^-]^2$$

28. (BC)

$[A]=[A]_0-kt=[A]_0+(\tan 150)t=[A]_0-\frac{\sqrt{3}}{3}t=0.5\frac{-\sqrt{3}}{3}t$

Similarly $[C]=[C]_0-\sqrt{3}t$.........(1)

If $[B]=[A]\Rightarrow t=t_{1/2}=\frac{a}{2k}=\frac{0.5}{2\frac{\sqrt{3}}{3}}=0.25\sqrt{3}$

Now

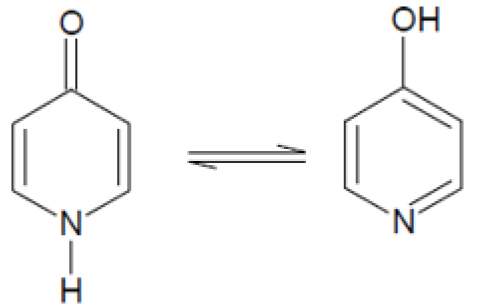
$[D]=[C]_0-[C]$

$=\sqrt{3}t \quad by\ (1)$

$=\sqrt{3}\times 0.25\times\sqrt{3}=0.75$

29. (ACD)

$R_2CHCH=NH \rightleftharpoons R_2C=CH-NH_2$



$CH_3CH_2NO \rightleftharpoons CH_3CH=NOH$

30. (ABC)

Cyclooctatetraene is tub shaped.

31. (Bonus)

$\sqrt{v}=a(Z-b)=aZ-ab$

Slope (a) = 1 ($\tan 45^o = 1$)

Also given intercept (ab) = 1

$\therefore \quad \sqrt{v}=1\times 52-1=51$

Or $\quad v=51^2=2601\,s^{-1}$

32. (23.52 - 23.68)

$H_2S+OH^-\rightarrow H_2O+HS^-; \qquad \Delta H=33.7kJ$

$H^++OH^-\rightarrow H_2O; \qquad \Delta H=-57.3kJ$

$\therefore\ H_2S\rightleftharpoons H^++HS^-; = +57.3-33.7=+23.6kJ$

33. (1.69)

$q = 0,\ w = -P(v_2 - v_1)$

$= -1(20-10)dm^3\ atm\ \ (\because\ 1\ atm\ dm^3 = 101.3\ J)$

$= -1013\ J$

$\Delta U = q + W \qquad (\text{from FLOT})$

$dU = -1013\ J$

$\Rightarrow \Delta U = nC_V\Delta T \text{ and } \Delta H = nC_P\Delta T$

$$\Rightarrow \frac{\Delta H}{\Delta U} = \frac{C_P}{C_V} = \frac{(C_V + R)}{C_V}$$

$$= \frac{5/2R}{3/2R}$$

$$\Delta H = \frac{5}{3}\Delta U \Rightarrow \Delta H = \frac{5}{3}\times(-1013)$$

$= -1688\ J$

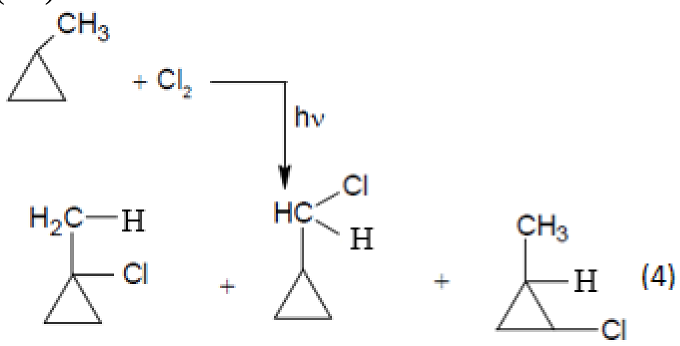
34. (6.90)

$10Al + 6KMnO_4 + 27H_2SO_4 \rightarrow 6KHSO_4 + 5Al_2(SO_4)_3 + 6MnSO_4 + 24H_2O$

35. (4.50)

7 H atoms can be exchanged with D so $(X+2)/2$ is 00004.50.

36. (1.2)



PART (C) : MATHEMATICS

37. (9)

For $x^2+y^2=9$, then centre $=(0,0)$ and the radius = 3

For $x^2+y^2-8x-6y+n^2=0$.

The centre = (4,3) and the radius $=\sqrt{4^2+3^2-n^2}$

$\therefore\ 4^2+3^2-n^2>0$ or $-5<n<5$

Circles should cut to have exactly two common tangents.

So, $r_1+r_2>d$ (distance between centres)

$\therefore 3+\sqrt{25-n^2}>\sqrt{4^2+3^2}$ or $\sqrt{25-n^2}>2$

Or $25-n^2>4$

$\therefore n^2<21$ or $-\sqrt{21}<n<\sqrt{21}$

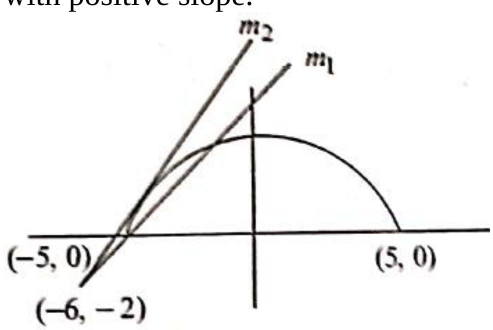
Therefore, common values of n should satisfy $-\sqrt{21}<n<\sqrt{21}$

But $n\in Z$, So, $n=-4,-3,....,4$

38. (4)

$M_1=\frac{2}{1}=2$

For the maximum value of slope in an extreme case that line should be the tangent to the semi circle with positive slope.



Tangent through $(-6,-2)$

$y+2=m(x+6)$

$y=mx+(6m-2)$

For tangent $(6m-2)^2=25m^2+25$

$11m^2-24m-21=0$

$m=\frac{12+\sqrt{375}}{11}$

$m\in\left[2,\frac{12+\sqrt{375}}{11}\right)$

So, $a = 2, b = \frac{12+\sqrt{375}}{11}$

$[a+b] = 4$

39. (Bonus)

$f(-x) = f(x), f(2+x) = f(2-x); f'(1) = -5$

Function is even, hence $f'(x) = 0$ at $x = 0$

Function is symmetric about the line $x = 2, f'(x) = 0$ at $x = 2$

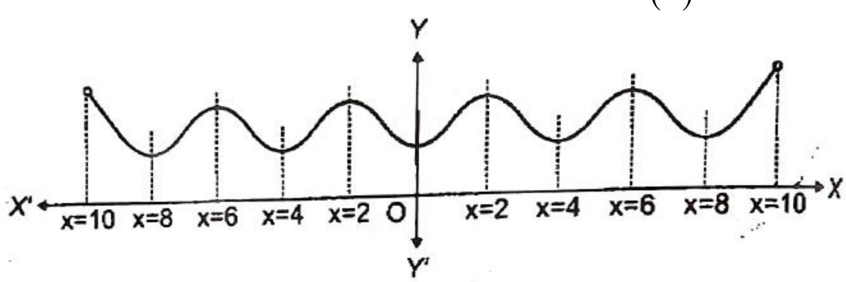
$f(2+x) = f(2-x)$ (Put $x = 2+t$)

$f(4+t) = f(-t)$

$f(4+t) = f(t)$

Thus, function has period $T = 4$

Periodic so it will be differentiable at $x = 4$ and $f'(x) = 0$ at 4 also.



$f'(x) = 0$ has 9 roots

$f''(x) = 0$ will have at least 8 roots

40. (9)

$$m = \frac{12+1}{1+\frac{2}{3}} = \frac{39}{5} = 7.8$$

So t_{7+1} is Numerically greatest & negative

$$\text{Now } \frac{t_7}{t_9} = \frac{{}^{12}C_6.2^6.3^6}{{}^{12}C_8.2^4.3^8} = \frac{4!.8!}{6!.6!}.\frac{4}{9} = \frac{112}{135} < 1$$

So, t_9 is A.G.T.

41. (5)

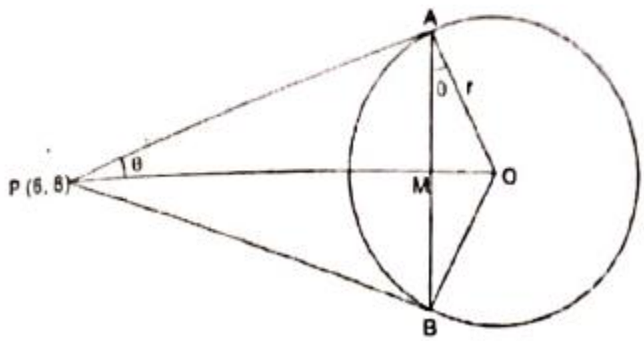
Let O be the centre of the circle $x^2 + y^2 = r^2$ and $P(6,8)$ be the point outside the circle. Let θ be the angle between the tangent AP and OP. Clearly, $\angle OAM = \theta$ so that $BM = r\cos\theta$ and

$$OM = r\sin\theta\left(0 < \theta < \frac{\pi}{2}\right)$$

Also, $OP = \sqrt{36+64} = 10$ and $r = 10\sin\theta$.

If A is the area of the triangle PAB, then

$A = 2(\text{Area of } \Delta PBM)$



$= 2\left(\left(\frac{1}{2}\right) PM \times BM\right) = (OP - OM) BM$

$= (10 - r\sin\theta) r\cos\theta$

$= \left(10 - 10\sin^2\theta\right)(10\sin\theta\cos\theta)$

$(\because r = \sin\theta)$

$= 100\cos^3\theta\sin\theta$

$= 50\sin 2\theta\cos^2\theta$

$= \frac{25}{2}[\sin 4\theta + 2\sin 2\theta]$

So, $\frac{dA}{d\theta} = 50(\cos 4\theta + \cos 2\theta)$

$= 100\cos 3\theta\cos\theta$

For maximum value of $\theta, \frac{dA}{d\theta} = 0$

$\Rightarrow \cos 4\theta + \cos 2\theta = 0$

$\Rightarrow 2\cos 3\theta\cos\theta = 0$

$\Rightarrow 3\theta = \frac{\pi}{2}\left(0 < \theta < \frac{\pi}{2}\right)$

$\Rightarrow \theta = \frac{\pi}{6}$

$\frac{d^2A}{d\theta^2} = -100(2\sin 4\theta + \sin 2\theta)$

$\left.\frac{d^2A}{d\theta^2}\right|_{\theta=\pi/6} = -100\left(\sqrt{3} + \frac{\sqrt{3}}{2}\right) < 0$

42. (1)

$\because (a+b+c).\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=9$

Equality holds in case a = b = c

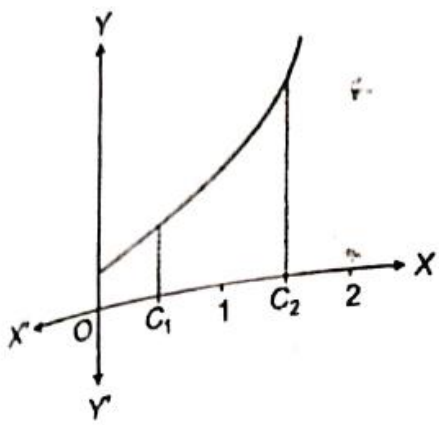
so $a=b=c$ & $\frac{bc}{a^2}=1$

43. (ACD)

$f''(x)>0$ in [0,2]

It means function is concave up. Also, $f'(x)$ is an increasing function.

Let $c_1 \in (0,1)$ and $c_2 \in (1,2)$



By LMVT; $f'(c_1)=\frac{f(1)-f(0)}{1-0}$

$f'(c_2)=\frac{f(2)-f(1)}{2-1}$

As $f'(x)$ is increasing so $c_1<c_2$

$\Rightarrow f'(c_1)<f'(c_2)$

$\Rightarrow f(1)-f(0)<f(2)-f(1)$

$\Rightarrow f(0)+f(2)>2f(1)$

Similarly, let $c_1 \in \left(0,\frac{2}{3}\right)$ and $c_2 \in \left(\frac{2}{3},2\right)$

By LMVT; $f'(c_1)=\dfrac{f\left(\frac{2}{3}\right)f(0)}{\frac{2}{3}-0};f'(c_2)=\dfrac{f(2)-f\left(\frac{2}{3}\right)}{2-\frac{2}{3}}$

$$f'(c_2) > f'(c_1) \Rightarrow \frac{f(2)-f\left(\frac{2}{3}\right)}{\frac{4}{3}} > \frac{f\left(\frac{2}{3}\right)-f(0)}{\frac{2}{3}}$$

$$\Rightarrow f(2)-f\left(\frac{2}{3}\right) > 2f\left(\frac{2}{3}\right)-2f(0)$$

$$\Rightarrow 2f(0)+f(2) > 3f\left(\frac{2}{3}\right)$$

44. (AB)

$(a+b)x+(a-b)y-2ab=0$ and $(a-b)x+(a+b)y-2ab=0$

$\Rightarrow$ equation of the angle bisectors are

$(a+b)x+(a-b)y-2ab=\pm((a-b)x+(a+b)y-2ab)$

$\Rightarrow 2bx-2by=0$ i.e, $x=y$ and

$2ax+2ay-4ab=0$ i.e $x+y=2b$

$\therefore$ equation of third side is give by (i) $x-y=k$

Satisfying the point $(b-a, a-b)$

$\therefore k=2b-2a$

$\therefore$ the line is $x-y=2(b-a)$ or

(ii) $x+y-2b=k$ passing through the point $(b-a, a-b)$

$\therefore k=-2b$

$\therefore$ the line is $x+y=0$

45. (BD)

Given that $P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{4}, P(E_3)=\frac{1}{4}$

$\therefore P(x)=\left\{P(E_1\cap E_2\cap \bar{E}_3)+P(E_1\cap \bar{E}_2\cap E_3)+P(\bar{E}_1\cap E_2\cap E_3)+P(E_1\cap E_2E_3)\right\}$

$=\left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}\right)+\left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}\right)$

$=\frac{1}{4}$

Now, (a) $P\left(X_1^c/X\right)$

$$=\frac{P\left(X_1^c\cap X\right)}{P(X)}=\frac{P\left(\bar{E}_1\cap E_2\cap E_3\right)}{P(X)}=\frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}}=\frac{1}{8}$$

(b) P (Exactly two engines of the ship are functioning)

$$= \frac{P\left(E_1 \cap E_2 \cap \bar{E}_3\right) + P\left(E_1 \cap \bar{E}_2 \cap E_3\right) + P\left(\bar{E}_1 \cap E_2 \cap E_3\right)}{P(X)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}} = \frac{7}{8}$$

(c) $P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)}$

$$= \frac{P(\text{Ship is operating with } E_2 \text{ function})}{P(X_2)}$$

$$= \frac{P\left(E_1 \cap E_2 \cap \bar{E}_3\right) + P\left(\bar{E}_1 \cap E_2 E_3\right) + P\left(E_1 \cap E_2 \cap E_3\right)}{P(E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}} = \frac{5}{8}$$

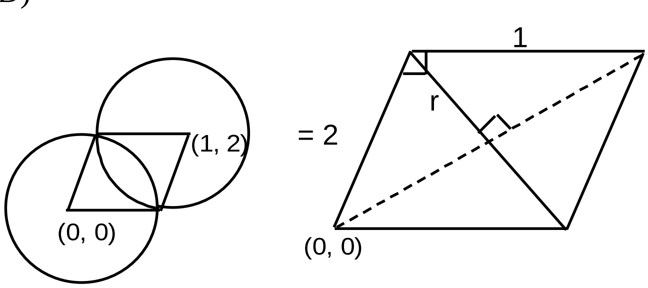
(d) $P(X/X_1) = \frac{P(X \cap X_1)}{P(X_1)}$

$$= \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$$

46. (AB)
Clearly 2x – 5 is one-one and onto i.e. objective

47. (ABD)

48. (ACD)



$2g_1g_2 + 2f_1f_2 = C_1 + C_2$

So, S_1 & S_2 intersect at 90^o

$\Rightarrow$ length of chord = $2x = \frac{4}{\sqrt{5}}$

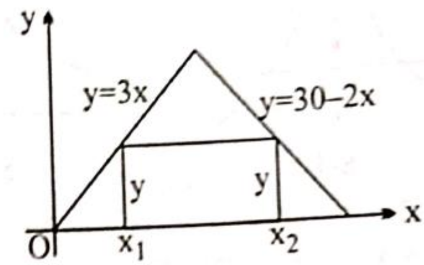
For $(2,3)$, the power of this point for both the circles is positive.

$S_1 = x^2 + y^2 - 4 = 0$

$S_2 = x^2 + y^2$

49. (4.00)

50. (67.50)



$A = (x_2 - x_1)y$

$y = 3x_1$ and $y = 30 - 2x_2$

$A(y) = \left(\frac{30-y}{2} - \frac{y}{3}\right)y$

$6A(y) = (90 - 3y - 2y)y = 90y - 5y^2$

$6A'(y) = 90 - 10y = 0$

$\Rightarrow y = 9; A''(y) = -10 < 0$

$x_1 = 3; x_2 = \frac{21}{2}$

$A_{max} = \left(\frac{21}{2} - 3\right)9 = \frac{15.9}{2} = \frac{135}{2}$

51. (0.11 – 0.14)

12 < 6G, 6R ⟶ 5 drawn

Let E is event as desired then

Red Box		Green Box	
5R	0G	1R	6G
4R	1G	2R	5G
3R	2G	3R	4G
2R	3G	4R	3G
1R	4G	5R	2G
0R	5G	6R	1G

$P(E) = \frac{{}^6C_0 \cdot {}^6C_5 + {}^6C_4 \cdot {}^6C_1}{{}^{12}C_5} =$

$$\frac{{}^6C_1 + {}^6C_4 \cdot {}^6C_1}{11\cdot 9\cdot 8} = \frac{6+90}{11\cdot 9\cdot 8} = \frac{96}{11\cdot 9\cdot 8} = \frac{4}{33}$$

52. (0.92)

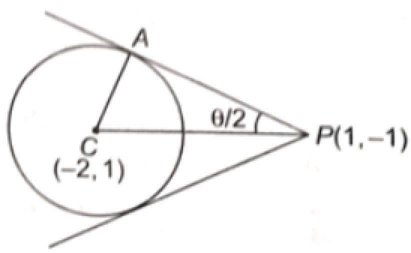
$$PA = \sqrt{S_1} = \sqrt{\left(3^2 + 2^2 - 9\right)} = 2$$

$$CA = r = 3$$

$$\therefore \tan\left(\frac{\theta}{2}\right) = \frac{CA}{PA} = \frac{3}{2}$$

and $\sin\theta = \dfrac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}$

$$= \frac{3}{1+9/4} = \frac{12}{13}$$



53. (1.32 – 1.35)

$$\because \quad \frac{x^2}{a^2} + \frac{y^2}{4} = 1 \qquad ...(i)$$

and $y^3 = 16x$ ...(ii)

From Eq. (i), $\dfrac{2x}{a^2} + \dfrac{2y}{4}\dfrac{dy}{dx} = 0$

$$\therefore \quad \frac{dy}{dx} = -\frac{4x}{a^2 y} \qquad ...(iii)$$

From Eq. (ii), $3y^2 \dfrac{dy}{dx} = 16$

$$\therefore \quad \frac{dy}{dx} = \frac{16}{3y^2} \qquad ...(iv)$$

$$\left(-\frac{4x}{a^2 y}\right)\left(\frac{16}{3y^2}\right) = -1$$

or $64x = 3a^2y^3$

$$= 3a^2(16x)$$

$$\therefore \quad a^2 = \frac{4}{3}.$$

54. (1.60)

We have $a+5d=2$ and $d>1$. Therefore

$a_1a_4a_5 = a(a+3d)(a+4d) = (2-5d)(2-2d)(2-d) = 2(2-5d)(1-d)(2-d)$

We need to find the maximum value of

$$f(d) = (2-5d)(1-d)(2-d)$$

Then,

$$f'(d) = -5(1-d)(2-d) - (2-5d)(2-d) - (2-5d)(1-d)$$
$$= -\left(15d^2 - 34d + 16\right)$$

and $f''(d) = -(30d-34)$

$f(d)$ is maximum when $f'(d)=0$ and $f''(d)<0$

$$15d^2 - 34d + 16 = 0$$

$\Rightarrow \quad d = \frac{2}{3}, \frac{8}{5}$

But $f''\left(\frac{2}{3}\right) = 14$ is positive, while $f''\left(\frac{8}{5}\right) = -14 < 0$,

So, the required value of d is $\frac{8}{5}$.