PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2025 **MAJOR TEST - 4** **DATE: 27/04/24**

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	C	**31.**	A	**61.**	A
2.	B	**32.**	B	**62.**	A
3.	B	**33.**	B	**63.**	B
4.	D	**34.**	B	**64.**	B
5.	B	**35.**	B	**65.**	D
6.	C	**36.**	B	**66.**	A
7.	C or D	**37.**	C	**67.**	D
8.	B	**38.**	A	**68.**	B
9.	B	**39.**	D	**69.**	C
10.	A	**40.**	B	**70.**	D
11.	C	**41.**	B	**71.**	A
12.	D	**42.**	B	**72.**	D
13.	D	**43.**	B	**73.**	C
14.	A	**44.**	A	**74.**	A
15.	C	**45.**	D	**75.**	C
16.	C	**46.**	D	**76.**	D
17.	C	**47.**	C	**77.**	B
18.	C	**48.**	C	**78.**	A
19.	D	**49.**	A	**79.**	D
20.	D	**50.**	C	**80.**	C
21.	4 or 5	**51.**	4	**81.**	19
22.	5	**52.**	Bonus	**82.**	2
23.	2	**53.**	2	**83.**	2
24.	3	**54.**	2	**84.**	4
25.	0.6	**55.**	12	**85.**	25
26.	9	**56.**	400	**86.**	33
27.	0.5	**57.**	8	**87.**	1
28.	5	**58.**	7	**88.**	2
29.	2	**59.**	3	**89.**	5
30.	5	**60.**	4	**90.**	1

PART (A) : PHYSICS

1. (C)

Logarithm has no dimensions.

$\therefore\ 1+\frac{\alpha l}{ma}=$ Dimensionless

$\therefore\ \frac{\alpha l}{ma}=\left[M^0L^0T^0\right]$

$\Rightarrow\ \alpha=\frac{ma}{l}=\frac{\left[MLT^{-2}\right]}{[L]}=\left[MT^{-2}\right]$

$\because$ Dimensions of $\phi=$ Dimensions of $\frac{ma}{\alpha}$

$=\frac{\left[MLT^{-2}\right]}{\left[MT^{-2}\right]}=[L]=\left[M^0LT^0\right]$

2. (B)

Case - B:

$-\frac{dv}{dt}=0.3t$

$-\int_{15}^{v}dv=\int_0^t 0.3t\cdot dt$

$v=15-0.3t^2/2$

v = 0 in t = 10 sec

$\frac{dx_2}{dt}=15-\frac{0.3t^2}{2}$

$x_2=15t-\frac{0.1t^3}{2}$

$x_2=150-\frac{0.1\times1000}{2}$

= 100 M

($x_2=$ distance by case – B before stopping)

Case - A:

$x_1=225-100=125+0$ just avoid collision

Final velocity $(V_A)=0$

$u_A=25\,m/s$

$a_A=?$

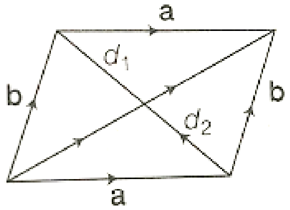
$v_A^2=u_A^2-2a_Ax_1$

$\Rightarrow\ 0^2=(25)^2-2a_A\times125$

$\Rightarrow\ a_A=\frac{625}{2\times125}=2.5\,m/s^2$

3. (B)

From figure, $a+b=d_2$ and $a+d_1=b$



$\therefore\ d_1=b-a$

$\therefore\ d_2=a+b$

$\therefore \quad d_2^2 = a^2 + b^2 + 2ab\cos\alpha$ ...(i)

Where, α is angle between a and b.

$\because \quad d_1 = a - b$

And $d_1^2 = a^2 + b^2 - 2ab\cos\alpha$ ...(ii)

Adding Eqs. (i) and (ii), we get

$$2\left(a^2 + b^2\right) = d_1^2 + d_2^2$$

$$\therefore \quad a^2 + b^2 = \frac{d_1^2 + d_2^2}{2}$$

4. (D)

Force applying on the block

$F = mg\sin\theta$ or $mg\sin\theta = ma$

$\therefore\ a = g\sin\theta$

Where, a is acceleration along the inclined plane.

$\therefore$ Where component of acceleration is $g\sin^2\theta$.

$\therefore$ Relative vertical acceleration of A with respects to B is

$$g\left(\sin^2 60 - \sin^2 30^\circ\right) = \frac{g}{2} = 4.9\,\text{ms}^{-2}$$ (in vertical direction)

5. (B)

The normal contact force on the block is 40 N.

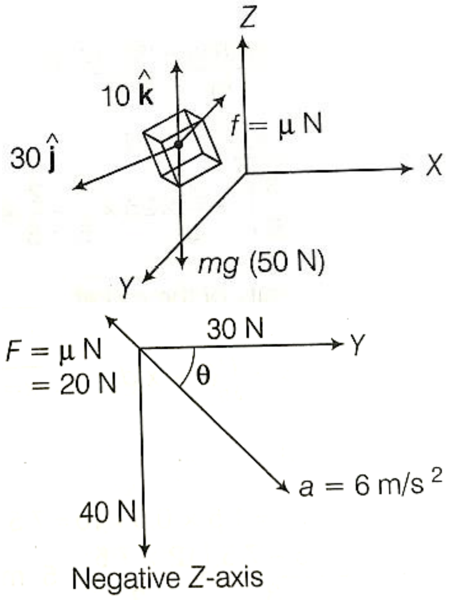
The resultant force on the block along the plane of wall is

$$\sqrt{(10-50)^2 + 30^2} = 50\text{N}$$

Thus, acceleration of the block is

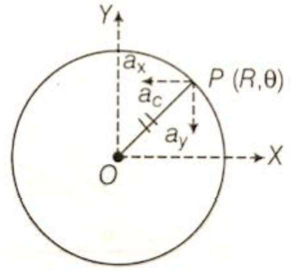
$$a = \frac{50 - \mu N}{m} = \frac{50 - 20}{5} = 6\text{m/s}^2$$

Here, $\tan\theta = \frac{40}{30} \Rightarrow \theta = 53^\circ$



$\therefore a = 6\cos 53^\circ\hat{j} - 6\sin 53^\circ k = 3.6\hat{j} - 4.8k$

6. (C)
For a particle in a uniform circular motion

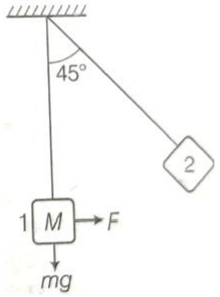


$a = \frac{v^2}{R}$ toward centre of circle [centripetal acceleration]

$\therefore \quad a = \frac{v^2}{R}\left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right)$

Or $\quad a = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

7. (C or D)
Apply W-E Theorem:



$\Delta K = 0 = W_F + W_{Mg} + W_{tension}$
(symbols have their usual meanings)

$W_F = F \times l \sin 45^\circ = \frac{Fl}{\sqrt{2}}$

$W_{Mg} = Mg\left(l - l\cos 45^\circ\right),$

$W_{tension} = 0$

$\therefore \quad F = Mg\left(\sqrt{2} - 1\right)$

8. (B)

$v_{COM} = \frac{mV_0 + M(0)}{m+M} = \frac{mV_0}{m+M}$ (this is constant)

$\therefore$ In centre of mass reference frame, relative velocity of mass m is

$$u_m = v_0 - \frac{mv_0}{m+M} = \frac{Mv_0}{m+M}$$

And of mass M, $\quad u_M = -\frac{mV_0}{m+m}$

$$K_i = \frac{1}{2}mu_m^2 + \frac{1}{2}Mu_M^2$$

$$\frac{1}{2}m\left(\frac{Mv_0}{m+M}\right)^2+\frac{1}{2}M\left(\frac{mv_0}{m+M}\right)^2 \text{ or } K_i=\frac{1}{2}.\frac{mMv_0^2}{m+M}$$

In centre of mass reference frame, at maximum compression, m and M are at rest.

$\therefore$ By conservation of energy, $K_f=0$

So, $\frac{1}{2}kx_{max}^2=\frac{1}{2}\cdot\frac{mMv_0^2}{m+M}$

$$x_{max}=\sqrt{\frac{mM}{k(m+M)}}v_0$$

9. (B)

$A_1=$ area of complete circle $=\pi a^2$

$A_2=$ area of removed part $=\pi\left(\frac{a}{2}\right)^2=\frac{\pi a^2}{4}$

Coordinates of COM of original uncut lamina = $(x_1,y_1)=(0,0)$

Coordinates of COM of removed portion = $(x_2,y_2)=\left(\frac{a}{2},0\right)$

For the residual part,

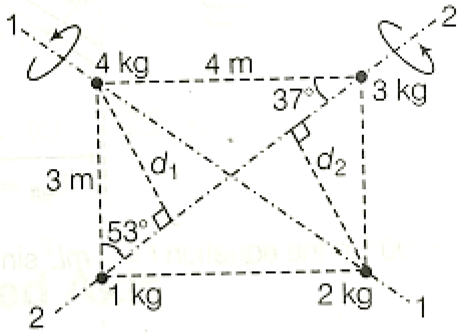
$$X_{COM}=\frac{A_1x_1-A_2x_2}{A_1-A_2}=\frac{\pi a^2(0)-\frac{\pi a^2}{4}\left(\frac{a}{2}\right)}{\pi a^2-\frac{\pi a^2}{4}}=-\frac{a}{6}$$

Similarly, you will get $Y_{COM}=0$

Hence, coordinates of COM of the uniform lamina = $\left(\frac{-a}{6},0\right)$

10. (A)

$l_{22}=1(0)^2+2(d_2)^2+3(0)^2+4(d_1)^2$



In the figure, $d_1\approx(\sin 53^o)\times3=\frac{12}{5}=2.4m$

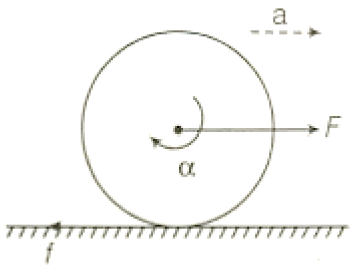
$\therefore$ $d_2=2.4m$

So, $l_{22}=2(2.4)^2+4(2.4)^2=34.56\text{ kg}-m^2$

Similarly, $l_{11}=1(2.4)^2+3(2.4)^2=23.04\text{ kg}-m^2$

11. (C)

For pure rolling take place on the ground



$a = R\alpha$

$F - f = Ma$

$I = MR^2$

$T = I\alpha = MR^2\alpha$

$fR = MR^2\alpha$

Using Eq. (i) in Eq. (iii), we get

$$f = Ma$$

Using this in Eq. (ii), we get

$$F - Ma = Ma$$

$\Rightarrow$ $$f = \frac{F}{2}$$

$\therefore$ $$f = \frac{F}{2}$$

$\therefore$ $$F - \frac{F}{2} = Ma \Rightarrow a = \frac{F}{2M}$$

12. (D)

Let the gravitational field is zero at a distance x from m_1.

$\therefore$ $$\frac{Gm_1}{x^2} = \frac{Gm_2}{(d-x)^2}$$

$\Rightarrow$ $$\frac{x}{d-x} = \sqrt{\frac{m_1}{m_2}}$$

$$x\sqrt{m_2} = d\sqrt{m_1} - x\sqrt{m_1}$$

$\Rightarrow$ $$x = \frac{d\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}}$$

$\therefore$ $$d - x = \frac{d\sqrt{m_2}}{\sqrt{m_1} + \sqrt{m_2}}$$

Net potential at this point is

$$V = -\frac{Gm_1}{x} - \frac{Gm_2}{(d-x)}$$

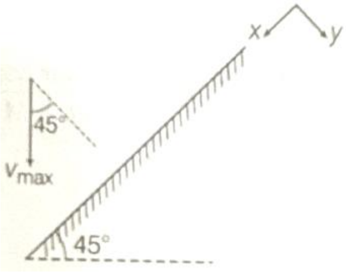
$$V = -G\left[\frac{m_1\left(\sqrt{m_1} + \sqrt{m_2}\right)}{d\sqrt{m_1}} + \frac{m_2\left(\sqrt{m_1} + \sqrt{m_2}\right)}{d\sqrt{m_2}}\right]$$

$$= -\frac{G}{d}\left(\sqrt{m_1} + \sqrt{m_2}\right)^2$$
$$= -\frac{G}{d}\left(m_1 + m_2 + 2\sqrt{m_1 m_2}\right)$$

13. (D)
Force on wire = Load carried
$$= m(g+a) = m\left(g + \frac{g}{2}\right) = \frac{3mg}{2}$$
Stress $\sigma \frac{F}{A}$ $\Rightarrow$ $A = \frac{F}{\sigma}$
Hence, $A_{min} = \frac{F}{\sigma_{max}}$
$$\frac{\pi d_{min}^2}{4} = \frac{\frac{3}{2}}{\sigma} \Rightarrow d_{min}^2 = \frac{\frac{3}{2}mg \times 4}{\sigma\pi}$$
$$d_{min} = \sqrt{\frac{6}{\pi}.\frac{mg}{\sigma}}$$

14. (A)
Centripetal force on the element considered is
$$(dp)A = (dm)x\omega^2$$
$$= (dxA\rho)x\omega^2$$
$$dp = \rho\omega^2 x dx$$
$$\int_{p1}^{p2} dp = \rho\omega^2 \int_0^L x dx$$
$$p_2 - p_1 = \frac{\rho\omega^2 L^2}{2}.,$$
Also, $p_2 - p_1 = \rho g h$
$$\therefore \quad \rho g h = \frac{\rho\omega^2 L^2}{2}$$
$$\Rightarrow \quad h = \frac{\omega^2 L^2}{2g}$$

15. (C)
Velocity of object = $V_{max}\frac{1}{\sqrt{2}}\hat{i} + V_{max}\frac{1}{\sqrt{2}}\hat{j}$



Velocity of image $= v_{max} = \frac{1}{\sqrt{2}}\hat{i} - v_{max}\frac{1}{\sqrt{2}}\hat{j}$

$\vec{v}_r = \vec{v}_{object,image} = \left(v_{max}\frac{1}{\sqrt{2}}\hat{i} + v_{max}\frac{1}{\sqrt{2}}\hat{j}\right) - \left(v_{max}\frac{1}{\sqrt{2}}\hat{i} - v_{max}\frac{1}{\sqrt{2}}\hat{j}\right)$

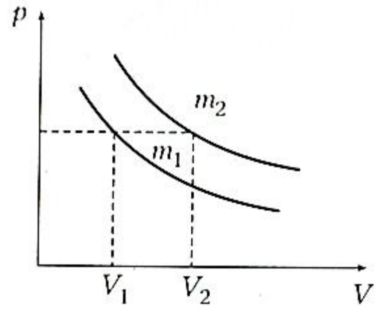
$= V_{max}\sqrt{2}\hat{j}$

$|\vec{v}_r| = \sqrt{2}v_{max} = \sqrt{2}A\omega \qquad = \sqrt{2}\,A\sqrt{\frac{k}{m}} \qquad (\because v_{max} = A\omega)$

$\left(\because \omega = \sqrt{\frac{K}{m}}\right)$

16. (C)

$pV = \propto RT = \frac{m}{M}RT$



For 1st plot, $\qquad p = \frac{m_1}{M}\frac{RT}{V_1}$

For 2nd plot, $\qquad p = \frac{m_2}{M}\frac{RT}{V_2}$

From Eqs. (i) and (ii), we get

$$\frac{m_1}{m_2} = \frac{V_1}{V_2} \Rightarrow m \propto V$$

As, $\qquad V_2 > V_1 \Rightarrow m_1 < m_2$

17. (C)

$V = -x^2y - xy^3 + 4$

The electric field is given by

$E = -\frac{\partial V}{\partial r} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}k\right]$

$= -\left[\left(-2xy - z^3\right)\hat{i} - \left(x^2\hat{j}\right) - \left(3z^2x\right)k\right]$

$= \left[\left(2xy + z^3\right)\hat{i} + \left(x^2\right)\hat{j} + \left(3xz^2\right)k\right]$

18. (C)

Increase in length $\Delta L = L\alpha(\Delta\theta)$

It is given that $L_1\alpha_a(\Delta\theta) = L_2\alpha_s(\Delta\theta)$ and $\Delta\theta = t$

$\therefore \frac{L_1}{L_2} = \frac{\alpha_s}{\alpha_a} \quad \therefore \frac{L_1}{L_1 + L_2} = \frac{\alpha_s}{\alpha_s + \alpha_a}$

19. (D)

Heat gained by water

$= \text{mass} \times \text{specific} \times \text{rise in temperature}$

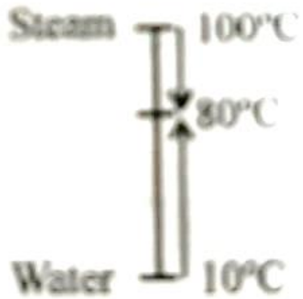
$= 20 \times 1 \times (80 - 10) = 1400 \text{ cal}$ ... (1)

And Heat lost by steam

$= \text{mass of steam}(m) \times L + ms_w \times \text{fall in temperature}$

$= m \times 540 + m \times 1 \times (100 - 80)$

$= 540m + 20m = 560m$ (2)



$\therefore$ Heat lost = Heat gained

$\therefore 560\,m = 1400 \qquad \therefore m = \frac{1400}{560} = 2.5$ gram

$\therefore$ Mass of water present at $80^\circ C = 20 + 2.5 = 22.5g$

20. (D)

For path ab, $(\Delta U)_{ab} = 7000J$

By using $\Delta U = nC_v\Delta T$

$7000 = n \times \frac{5}{2} R \times 700 \Rightarrow n = 4/R$

For path ca, $(\Delta Q)_{ca} = (\Delta U)_{ca} + (\Delta W)_{ca}$

$\because (\Delta U)_{ab} + (\Delta U)_{bc} + (\Delta U)_{ca} = 0$

$\therefore 7000 + 0 + (\Delta U)_{ca} = 0$

$\Rightarrow (\Delta U)_{ca} = -7000J$

Also, $(\Delta W)_{ca} = \rho_1(V_1 - V_2) = nR(T_1 - T_2)$

$= (4/R) \times R \times (300 - 1000)$

$= -2800 \text{ J}$

By solving Eqs.(i), (ii) and (iii), we get

$(\Delta Q)_{ca} = -700 - 2800 \quad = -9800 \text{ J}$

21. (4 or 5)

$$\therefore v=\sqrt{\frac{T}{\rho A}}$$

$$\therefore \frac{\Delta v}{v}=\frac{1}{2}\frac{\Delta T}{T}+\frac{1}{2}\frac{\Delta \rho}{\rho}+\frac{1}{2}\frac{\Delta A}{A}$$

Parentage error in velocity of transverse wave is

$$\frac{\Delta v}{v}\times 100=\frac{1}{2}\times\frac{\Delta T}{T}\times 100+\frac{1}{2}\frac{\Delta \rho}{\rho}\times 100+\frac{1}{2}\frac{\Delta A}{A}\times 100$$

$$=\frac{1}{2}\times 2+\frac{1}{2}\times 4+\frac{1}{2}\times 2$$

$$=1+2+1=4\%$$

22. (5)

First we find relation of magnification m and focal length f. By lens equation,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow\quad 1-\frac{v}{u}=\frac{v}{f}$$

$$\Rightarrow\quad 1-m=\frac{v}{f}\qquad\left[\because \frac{v}{u}=m\right]$$

$$\Rightarrow\quad m=1-\frac{v}{f}\qquad \text{.... (i)}$$

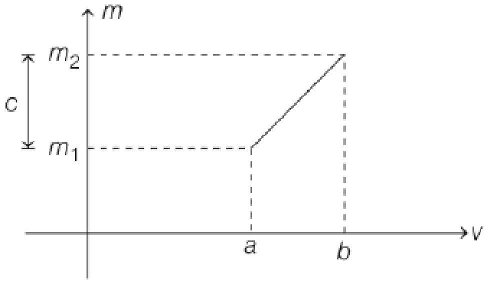
Now, from given graph and from Eq. (i),

At v = a, margination is

$$m_1=1-\frac{a}{f}\qquad \text{.... (ii)}$$

At $v=a+b$. magnification is

$$m_2=1-\frac{a+b}{f}\qquad \text{.... (iii)}$$



From graph, we can also say that

$$m_2-m_1=c\qquad \text{.... (iv)}$$

So, from Eqs. (ii),(iii) and (iv), we have

$$\left(1-\frac{(a+b)}{f}\right)-\left(1-\frac{a}{f}\right)=c$$

$$\Rightarrow\quad \frac{a-(a+b)}{f}-\left(1-\frac{a}{f}\right)=c$$

$$\Rightarrow\quad f=-\frac{b}{c}\ \text{or}\ |f|=\frac{b}{c}=\frac{10}{2}=5$$

23. (2)

The energy required for taking a satellite upto a height h from earth's surface is the difference between the energy at h height and energy at surface, then

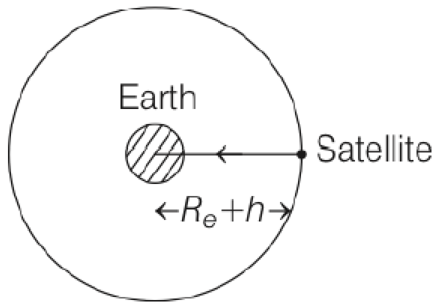
$\Rightarrow \quad E_1 = U_f - U_i$

$$E_1 = -\frac{GM_e m}{R_e + h} + \frac{GM_e m}{R_e} \qquad \ldots \text{ (i)}$$

(where, U = potential energy)

$\therefore$ Orbit velocity of satellite,

$$v_0 = \sqrt{\frac{GM_e}{(R_e + h)}} \quad \text{(where, } M_e = \text{mass of earth)}$$



So energy required to perform circular motion

$$\Rightarrow \quad E_2 = \frac{1}{2} m v_0^2 = \frac{GM_e m}{2(R_e + h)}$$

$$E_2 = \frac{GM_e m}{2(R_e + h)} \qquad \ldots \text{ (ii)}$$

According to the question,

$$E_1 = E_2$$

$$\therefore \frac{-GM_e m}{R_e + h} + \frac{GM_e m}{R_e} = \frac{GM_e m}{2(R_e + h)}$$

$$\Rightarrow \quad 3R_e = 2R_e + 2h$$

$$h = \frac{R_e}{2}$$

As radius of earth, $R_e \approx 6.4 \times 10^3 \text{km}$

Hence, $h = \frac{6.4 \times 10^3}{2} \text{km}$

Or $= 3.2 \times 10^3 \text{km}$

24. (3)

So, potential energy of stretched cord

= kinetic energy of stone

$$\Rightarrow \quad \frac{1}{2} Y \left(\frac{\Delta L}{L}\right)^2 A \cdot L = \frac{1}{2} m v^2$$

Here, $\Delta L = 20 \text{ cm} = 0.2\text{m}, L = 42 \text{ cm} = 0.42\text{m},$

$v = 20 \text{ ms}^{-1}, m = 0.02\text{kg}, d = 6\text{mm} = 6 \times 10^{3-} \text{m}$

$\therefore A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{6\times10^{-3}}{2}\right)^2$

$= \pi\left(3\times10^{-3}\right)^2 = 9\pi\times10^{-6}\,m^2$

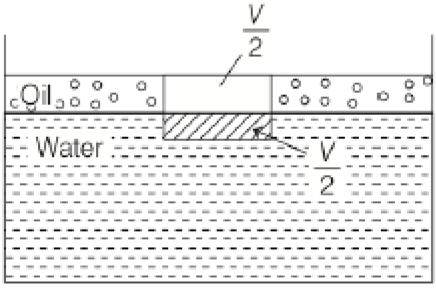
On substituting values, we get

$Y = \frac{mv^2L}{A(\Delta L)^2} = \frac{0.02\times(20)^2\times0.42}{9\pi\times10^{-6}\times(0.2)^2} \approx 3.0\times10^6\,Nm^{-2}$

25. (0.6)

For a floating body,

Up thrust = weight of the part of object, i.e. In first situation,



So, Wight of block of volume V = weight of oil of

Volume $\frac{v}{2}$ + weight of water of volume $\frac{v}{2}$.

$\Rightarrow \quad v\rho_o g = \frac{v}{2}\rho_o g + \frac{v}{2}\rho_w g$

Where, ρ_o = density of oil.

$\Rightarrow \quad 2\rho_b = \rho_o + \rho_w$

$\Rightarrow \quad \frac{2\rho b}{\rho_w} = \frac{\rho_o}{\rho_w} + 1$

$\Rightarrow \quad 2\times\frac{4}{5} = \frac{\rho_o}{\rho_w} + 1$ [using Eq. (i)]

$\Rightarrow \quad \rho_o / \rho_w = 8/5 - 1 = 3/5 = 0.6$

26. (9)

As we know, if other parameters remains constant, terminal velocity is proportional to square of radius of falling sphere.

i.e. $V_T \propto r^2$ … (i)

Now, when sphere of radius R is broken into 27 identical solid sphere of radius r, then Volume of sphere of radius R = 27× Volume of sphere of radius r

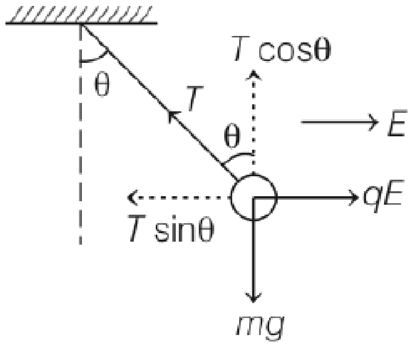
$\Rightarrow \quad \frac{4}{3}\pi R^3 = 27\times\frac{4}{3}\pi r^3$

$\Rightarrow \quad R = 3r \Rightarrow r = \frac{R}{3}$

So, from Eq. (i), we have

$$\frac{v_1}{v_2} = \frac{R^2}{\left(\frac{R}{3}\right)^2} = 9$$

27. (0.5)
Force on the bob are as shown



Fore equilibrium,

$T\cos\theta = mg$ … (i)

And $T\sin\theta = qE$ …. (ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\tan\theta = \frac{qE}{mg}$$

Here, $q = 5\times10^{-6}C, E = 2000V/m,$

$m = 2g = 2\times10^{-3}kg, g = 10\ ms^{-2}$

$$\therefore \quad \tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2} = 0.5$$

So, the angle made by the string of the pendulum with the vertical is

$\theta = \tan^{-1}(0.5)$

28. (5)
We know that,
Time period of a pendulum is given by

$T = 2\pi\sqrt{L/g_{eff}}$ … (i)

Here, L is the length of the pendulum and g_{eff} is the effective acceleration due to gravity in the respective medium in which bob is oscillating. Initially, when bob is oscillating in air , $g_{eff} = g$.

So, initial time period, $T = 2\pi\sqrt{\frac{L}{g}}$ … (ii)

Let ρ_{bob} be the density of the bob.

When this bob is dipped into a liquid whose density is given as

$$\rho_{liquid} = \frac{\rho_{bob}}{16} = \frac{\rho}{16} \quad \text{(given)}$$

$\therefore$ Net force on the bob is

$F_{net} = V\rho g - V\cdot\frac{\rho}{16}\cdot g$ … (iii)

(Where, V - volume of the bob = volume of displaced liquid by the bob when immersed in it).
If effective value of gravitational acceleration on the bob in this liquid is g eff, then net force on the bob can also be written as

$$F_{net} = V\rho g_{eff} \quad \ldots \quad (iv)$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{eff} = V_{\rho g} - V\rho g/16$$

$$\Rightarrow \quad g_{eff} = g - g/16 = \frac{15}{16}g \quad \ldots. \quad (v)$$

Substituting the value of g_{eff} from Eq. (v) in Eq.(i), the new time period of the bob will be

$$T'2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}}$$

$$\Rightarrow \quad T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}}$$

$$= \frac{4}{\sqrt{15}} \times T \quad \text{[using Eq.(ii)]}$$

29. (2)
Let x grams of water is evaporated. According to the principle of calorimetry, Heat lost by freezing water (that turns into ice) = Heat gained by evaporated water Given, mass of water = 150 g

$$\Rightarrow \quad (150 - X)\times 10^{-3} \times 3.36 \times 10^{5} = X \times 10^{-3} \times 2.10 \times 10^{6}$$

$$\Rightarrow \quad (150 - X)\times 3.36$$

$$21X$$

$$\Rightarrow \quad X = \frac{150}{7.25}$$

$$= 20.6$$

$$\therefore \quad X \approx 21$$

30. (5)
Root mean square (rms) velocity of the molecules of gas is given as

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Where, M is the atomic of the gas.

$$\Rightarrow \quad V_{rms} \propto \sqrt{\frac{1}{M}}$$

$$\therefore \quad \frac{V_{rms(helium)}}{V_{rms(argon)}} = \sqrt{\frac{M_{argon}}{M_{helium}}} \quad \ldots.. (i)$$

Given, $M_{argon} = 40u$ and $M_{helium} = 4u$

Substituting these values in Eq. (i), we get

$$\frac{V_{rms(helium)}}{V_{rms(argon)}} = \sqrt{\frac{40}{4}} = \sqrt{10}$$

PART (B) : CHEMISTRY

31. (A)

$2NH_3 + CO_2 \rightarrow NHCOONH_4$

$$\frac{^{n}NH_3}{^{n}CO_2} = \frac{2}{1}$$

$\Rightarrow \frac{W_{NH_3}}{W_{CO_2}} \times \frac{44}{17} = \frac{2}{1}$

$\Rightarrow \frac{W_{NH_3}}{W_{CO_2}} = \frac{17}{22}$

32. (B)

$HCl + NaOH \rightarrow NaCl + H_2O$

$0.5V \quad 0.25V \qquad 0.75V$

$0.25V \quad - \quad 0.75V + 0.25V = V$

$M = \frac{V}{3V} = \frac{1}{3}$

33. (B)

$n - l - 1 = 1; n - 1 = 3$

$\Rightarrow \quad n = 4, l = 2, L = \sqrt{l(l+1)} \cdot \frac{h}{2\pi}$

34. (B)

On decreasing pressure reaction will move forward and α will increase so answer is (b).

35. (B)

36. (B)

$\left[CN^-\right] = 0.1M$

$\underset{0.1(1-\alpha)}{CN^-} + H_2O \rightleftharpoons \underset{0.1\alpha}{HCN} + \underset{0.1\alpha}{OH^-}$

$\frac{k_w}{k_a} = 0.1\alpha^2 = 10^{-8.26}$

$\left(OH^-\right) = 0.1\alpha$

$pH = -\log\left[H^+\right]$

37. (C)

$k_b = c \cdot \alpha^2$

$\quad 0.05 \quad 0.025$

38. (A)

39. (D)

$$Cv(mix) = \frac{1\times 3R + 2\times \frac{3}{2}R}{3} = 2R, \Rightarrow C_p = 3R, \gamma = \frac{3}{2}$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow T_2 = \frac{320}{(4)^{0.5}} = 160K$$

$$\Delta U = \frac{-3\times R\times 160}{\frac{1}{2}} = -960R$$

40. (B)

$$[A]_t = \frac{2}{2^2} = 0.50M$$

41. (B)

$$A = 10^{5.4} = 2.5\times 10^5 \sec^{-1}$$

$$-\frac{E_a}{2.303R} = -100; \quad \frac{k}{A} = e^{-\frac{E_a}{RT}}$$

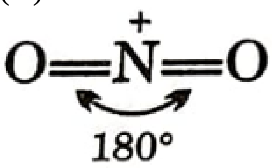
$$E_a = 460.6$$

42. (B)

$$E_{cell} = +0.44 - 0.33 - \frac{0.059}{1}\log\left(\frac{0.1}{0.2}\right) = +0.127\ V$$

43. (B)

44. (A)



45. (D)

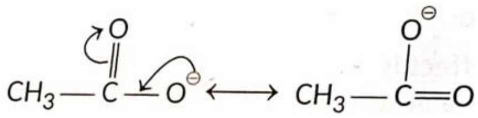
$$\overset{1}{\underset{\displaystyle F}{\underset{|}{C}}}$$

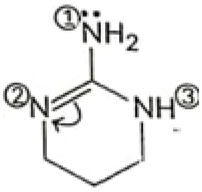
$$I - \overset{1}{C}(F) = \overset{2}{C}(Cl) - \overset{3}{C} \equiv \overset{4}{C} - OCH_3$$

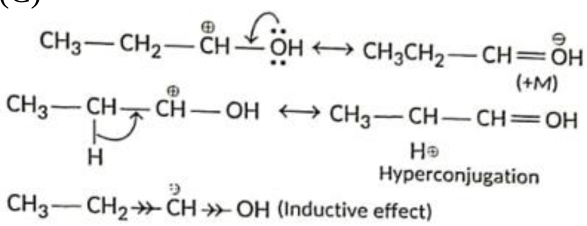
If position of double and triple bond is same then double bond in preferred.
IUPAC name is -2chooro-1-fluoro-1-iodo-4methoxbut-2-en-3yne

46. (D)
Both the compounds are identical having IUPAC name benzoic ethanoic anhydride.

47. (C)
Tautomerism is a type of structural isomers. In resonating structures, position of atoms does not change. But in tautomerism, hydrogen from α-carbon moves to the O-atom.

48. (C)

Most stable due to two equivalent resonating structures with negative charge on more electronegative O- atom.

49. (A)

2nd nitrogen is most basic as its lone pair is localised in addition to +M of rest two nitrogen atoms

50. (C)


51. (4)
1×4

52. (Bonus)
$\frac{1}{\lambda_1}=R_H Z^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$; $\frac{1}{\lambda_2}=R_H Z^1\left[\frac{1}{1^2}-\frac{1}{2^2}\right]$
$\lambda_1-\lambda_2=593\text{ Å}$ $\Rightarrow Z=3$

53. (2)
$nA \rightleftharpoons C+D$
$\Delta n_g=0 \Rightarrow n=2$

54. (2)
$9=-\log 5\times 10^{-10}+\log\left[\frac{5V}{20}\right]$

55. (12)
$Fe_2(C_2O_4)_3 \Rightarrow n\cdot f=6; 10\times 6=x\times 5$

56. (400)

57. (8)

58. (7)

59. (3)

There are three possible structural ether isomer of $C_4H_{10}O$.

$CH_3-CH_2-CH_2-O-CH_3$

Methoxypropane

$CH_3-\underset{\underset{CH_3}{|}}{C}H-O-CH_3$

2–methoxypropane

$CH_3-CH_2-O-CH_2-CH_3$

Ethoxy ethane

60. (4)

Total no. of stereoisomers = 2^n

N = no. of stereo centre

$CH_3-\underset{\underset{OH}{|}}{\overset{*}{C}}H-\underset{\underset{Br}{|}}{\overset{*}{C}}H-CH_3$

N = 2

Total stereoisomers $=2^2=4$

PART (C) : MATHEMATICS

61. (A)

$$\cos^2\theta = \frac{1}{6}\sin\theta.\tan\theta$$

$$6\cos^3\theta = 1-\cos^2\theta$$

$$6\cos^3\theta + \cos^2\theta - 1 = 0$$

$$(2\cos\theta-1)\left(3\cos^2\theta+2\cos\theta+1\right)=00$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n\in Z$$

62. (A)

If $0 < x < 1$, then $\text{LHS} < 0$

For $x > 1$

$$\frac{1}{\log_2 x}+\frac{1}{2\log_2 x} > 1 \Rightarrow \frac{3}{2\log_2 x} > 1$$

$$\Rightarrow 0 < \log_2 x < \frac{3}{2}$$

63. (B)

$$h\left(f\left(g(x)\right)\right) = h\left(f\left(\sqrt{x^2+1}\right)\right) = h\left(x^2\right)$$

$\because\ x^2 \geq 0$

$$h\left(x^2\right) = \begin{cases} 0, & x = 0 \\ x^2, & x \neq 0 \end{cases}$$

64. (B)

65. (D)

$$\lim_{x\to 0^-} \frac{-x}{2\tan^{-1}x - \left(-2\tan^{-1}x\right)} = -\frac{1}{4}\lim_{x\to 0^-}\frac{x}{\tan^{-1}x}$$

$$= -\frac{1}{4}$$

66. (A)

67. (D)

$$\frac{dy}{dx} = \frac{f_x}{f_y} = -\frac{n}{a}\left(\frac{x}{a}\right)^{n-1} + \frac{n}{b}\left(\frac{y}{b}\right)^{n-1}$$

$$\therefore (a,b)\frac{dy}{dx} = -\frac{b}{a}$$

It is independent of n.

$\therefore$ Tangent is $y-b=-\dfrac{b}{a}(x-a)$

or $bx+ay=2ab$

or $\dfrac{x}{a}+\dfrac{y}{b}=2$ for all values of n.

68. (B)

$f(x)=x^{25}(1-x)^{75},\ x\in[0,1]$

$f'(x)=x^{25}75(1-x)^{74}.(-1)+(1-x)^{75}.25x^{24}$

$=25x^{24}(1-x)^{74}(-3x+1-x)$

$=25x^{24}(1-x)^{74}(1-4x)$

$\therefore f(x)$ attains maximum at x = 1/4

69. (C)

$$\lim_{x\to-2^+}\frac{ae^{\frac{1}{|x+2|}}-1}{2-e^{\frac{1}{|x+2|}}}=-a$$

$$=\lim_{x\to-2^+}\sin\left(\frac{x^4-16}{x^5+32}\right)=\lim_{x\to-2^+}\sin\left(\frac{\frac{x^4-(-2)^4}{x-(-2)}}{\frac{x^5-(-2)^5}{x-(-2)}}\right)=\sin\left(-\frac{2}{5}\right)$$

or $a=\sin\dfrac{2}{5}$

70. (D)

(A) $f(x)$ is continuous everywhere because denominator never become zero.

(B) $f(x)=\tan^{-1}\left(\dfrac{6}{(3+|x-2|)}-1\right)$

$f(x)\infty\to$ not possible & it is an into function.

(C) graph is symmetric about the line $x=2$, because of $|x-2|$

71. (A)

When $t \ge 0$, $x = 2t - t = t$

$y = t^2 + t^2 = 2t^2$

Now relation between x & y

$y = 2x^2, t \ge 0, x \ge 0, y = 2x^2, x \ge 0$ when $t < 0$

$x = 2t - (-t) = 3t, y \Rightarrow t^2 + t(-t) = 0$

Now relation between x & y

$y = 0, t < 0, x < 0$

$y = 0, x < 0$ so that we can write $f(x) = \begin{cases} 2x^2 & 0 \le x \le 1 \\ 0 & -1 \le x < 0 \end{cases}$

clearly f(x) is continuous and diff. every where.

72. (D)

Point of discontinuity are

$\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{7}{3}, \frac{5}{2}, \frac{8}{3}$

73. (C)

74. (A)

The given equation is

$(\sqrt{3}-1)\sin\theta + (\sqrt{3}+1)\cos\theta = 2$

Let $(\sqrt{3}-1) = r\sin\alpha$ and $(\sqrt{3}+1) = r\cos\alpha$

$\therefore r = \sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2} = 2\sqrt{2}$

And $\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \tan(60° - 45°) = \tan 15°$

$\Rightarrow \alpha = 15° = \frac{\pi}{12}$

$\therefore r\sin\alpha\sin\theta + r\cos\alpha\cos\theta = 2$

$\Rightarrow 2\sqrt{2}\cos(\theta - \alpha) = 2$

$\Rightarrow \cos(\theta - \alpha) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

$\Rightarrow \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$

75. (C)

$f(k + x) = f(k - x) \Rightarrow f(2k - x) = f(x)$

(replacing x by k – x) so $f(x) = -f(2k + x)$ .... (1)

changing x to x + 2k, we have $f(x + 2k) = -f(x + 4k)$

So $f(x) = f(x + 4k)$. Thus

f is periodic Again $f(2k - x) = f(x) \Rightarrow f(2k + x) = f(-x)$.

Therefore equation (1) $\Rightarrow f(x) = -f(-x)$

Hence f is odd and periodic.

76. (D)

$f(x) = f\left(\frac{x+1}{x+2}\right)$ or $f(-x) = f(x)$

$\Rightarrow x = \frac{x+1}{x+2}$ or $-x = \frac{x+1}{x+2}$

$x^2 + 2x = x + 1$ or $-x^2 - 2x = x + 1$

$\Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$ or $x = \frac{-3 \pm \sqrt{5}}{2}$

$\Rightarrow$ total number of values of x = 4

77. (B)

$$f(x) = \begin{cases} \lim\limits_{n\to\infty} \dfrac{ax(x-1)\left(\cot\frac{\pi x}{4}\right)^n + (px^2+2)}{\left(\cot\frac{\pi x}{4}\right)^n + 1}, \\ 0, & x = 1 \end{cases}$$

$x \in (0,1) \cup (1,2)$

$$\therefore f(x) = \begin{cases} ax(x-1) & 0 < x < 1 \\ 0, & x = 1 \\ px^2 + 2, & 1 < x < 2 \end{cases}$$

$\therefore$ L.H.L. = R.H.L = f(1)

$\Rightarrow \quad p + 2 = 0 \Rightarrow p = -2$

$$\text{Also } f'(x) = \begin{cases} a(2x-1), & 0 < x < 1 \\ -4x, & x < 1 \end{cases}$$

$\because$ f(x) is differentiable at x = 1, so

$\because$ f(x), x = 1

L.H.D. = R.H.D. $\quad \Rightarrow a = -4$

Hence $|a + p| = 6$

78. (A)

Required probability = 1 – P(no ace of heart)

$= 1 - \frac{51}{52} \cdot \frac{51}{52} = \frac{(52)^2 - (51)^2}{52.52}$

$= \frac{(52-51)(52+51)}{2704} = \frac{103}{2704}$

79. (D)

Case-1 : Digits will be 9,9,9,9,7

$\therefore$ natural number formed $= \frac{5!}{4!} = 5$

Case-2 : Digits will be 9,9,9,8,8

$\therefore$ natural number formed $= \frac{5!}{3!2!} = 10$

$\therefore$ required natural number = 10 + 5 = 15

80. (C)

Let the equation of the circle be $x^2 + y^2 = r^2$.

The chord which subtends an angle $\frac{\pi}{4}$ at the circumference will subtend a right angle at the centre. So, Chord joining A(r, 0) and B(0, r) subtends a right angle at centre (0, 0). Mid point of AB is $C\left(\frac{r}{2}, \frac{r}{2}\right)$

$\therefore OC = \frac{r}{\sqrt{2}}$, which is radius of locus C.

81. (19)

$\because r_1 r_2 r_3 r_4 = \frac{5}{4}$ and $AM \geq GM$

$$\frac{1}{4}\left(\frac{r_1}{2} + \frac{r_2}{4} + \frac{r_3}{5} + \frac{r_4}{8}\right) \geq \left(\frac{r_1 r_2 r_3 r_4}{2.4.5.8}\right)^{1/4}$$

$$\Rightarrow \frac{1}{4} \geq \left(\frac{1}{64 \times 4}\right)^{1/4} = \frac{1}{4}$$

$$AM = GM \Rightarrow \frac{r_1}{2} = \frac{r_2}{4} = \frac{r_3}{5} = \frac{r_4}{8} = \frac{1}{4}$$

$$\frac{a}{4} = \Sigma r_1 \Rightarrow a = 19$$

82. (2)

$$S_1 = \frac{2\cot x}{1 - \sin^2 x} = \frac{2}{\sin x \cos x}$$

$$S_2 = \frac{\sin 2x}{1 - \sin^2 x} = \frac{2\sin x}{\cos x}$$

$$\therefore S_1 - S_2 = \frac{2}{\sin x \cos x} - \frac{2\sin x}{\cos x} = \frac{2\left(1 - \sin^2 x\right)}{\sin x \cos x}$$

$$= 2\cot x, \quad x \in \left(0, \frac{\pi}{4}\right]$$

$\therefore$ minimum value = 2

83. (2)

The given equation is

$(\cos x - 1)(12\cos^2 x + 5\cos x + 9) = 0$

$\Rightarrow \cos x = 1$

$x = 2n\pi,\ n \in Z$

84. (4)

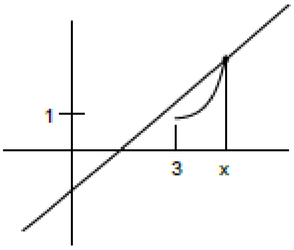
Let the equation of the chord be $x \pm y = \pm a$. The length of the perpendicular from the centre (0, 0) of the circle $x^2 + y^2 = 8$ must be less than the radius $\sqrt{8}$ of the circle.

$\Rightarrow \quad \frac{|\pm a|}{\sqrt{1+1}} < \sqrt{8} \Rightarrow |a| < 4 \Rightarrow a < 4$ [$\because$ a is +ve]

85. (25)
$S_1 - S_2 = -6x - 14y - 2\lambda = 0$ passes through the centre (1, –4) of the first circle.
$\therefore\ -6 + 56 - 2\lambda = 0 \Rightarrow \lambda = 25$

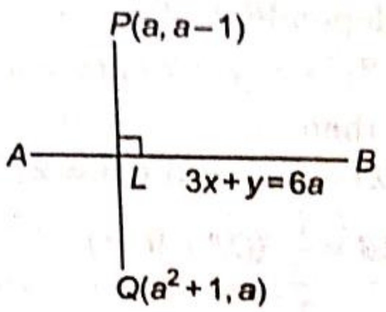
86. (33)
$S = \sum_{r=1}^{15} \frac{((r+2)-2)2^r}{(r+2)!}$
$S = \sum\left(\frac{2^r}{(r+1)!} - \frac{2^{r+1}}{(r+2)!}\right)$
$\therefore\ S = \frac{2}{2!} - \frac{2^{16}}{17!} = \frac{17! - 2^{16}}{17!}$

87. (1)
$f(x) = \pi^{x(x-3)} : [3, \infty) \to [1, \infty)$
Solution of $f(x) = f^{-1}(x)$ lies on line y = x
For $x \in [3, \infty)$ the function
$f(x) = \pi^{x(x-3)}$ is strictly increasing and intersect the line $y = x$ at only one point



$f(x) = f^{-1}(x)$ has only one solution

88. (2)
$L \equiv \left(\frac{a^2+a+1}{2}, \frac{2a-1}{2}\right)$



Since, L lies on line AB
$\therefore \quad 3\left(\frac{a^2+a+1}{2}\right) + \left(\frac{2a-1}{2}\right) = 6a$
$\Rightarrow \quad 3a^2 - 7a + 2 = 0$

$\Rightarrow \quad a = 2, \frac{1}{3}$

Also, $PQ \perp AB$

$\Rightarrow \quad -3 \cdot \left(\frac{1}{a^2 - a + 1}\right) = -1$

$\Rightarrow \quad a^2 - a - 2 = 0$

$\Rightarrow \quad a = 2, -1$

$\therefore$ Common value of a = 2.

89. (5)

90. (1)

$\because f'(x) = 4\cos 2x + 3\sin 2x - (a^2 + a - 7) \geq 0 \ \forall \ x \in R$

$\Rightarrow -(a^2 + a - 7) + (4\cos 2x + 3\sin 2x) \geq 0 \ \forall \ x \in R$

$\Rightarrow -(a^2 + a - 7) - 5 \geq 0$

$\Rightarrow a^2 + a - 7 + 5 \leq 0$

$a^2 + a - 2 \leq 0$

$a \in [-2, 1] \quad \therefore |p + q| = 1$