IIT – JEE: 2025 **TW TEST (ADV)** **DATE: 07/04/24**

TOPIC: GRAVITATION

Answer Key & Solution

1. (D)

By law of conservation of energy, we get

$$(U+K)_{surface} = (U+K)_{centre}$$

Now, for a solid sphere, we have

$$U_{surface} = -\frac{GMm}{R} \text{ and } U_{centre} = -\frac{3}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{R} - \left(-\frac{3}{2}\frac{GMm}{R}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \frac{v_c}{\sqrt{2}}$$

2. (C)

$$F = \int_h^{h+L} \frac{G\left(\frac{M}{L}dx\right)m}{x^2} = \frac{GMm}{L}\int_h^{h+L} x^{-2}\,dx$$

$$\Rightarrow \quad F = \frac{GMm}{L}\left(\left.\frac{x^{-2+1}}{-2+1}\right|_h^{h+L}\right)$$

$$\Rightarrow \quad F = -\frac{GMm}{L}\left(\frac{1}{h+L} - \frac{1}{h}\right)$$

3. (C)

By Law of Conservation of Energy, we have

$$(U+K)_{axis} = (U+K)_C$$

If m_0 be the mass of the particle, then

$$-\frac{Gmm_0}{\sqrt{r^2+r^2}} + \frac{1}{2}m_0(0)^2 = -\frac{Gmm_0}{r} + \frac{1}{2}m_0v^2$$

$$\Rightarrow \quad \frac{1}{2}m_0v^2 = \frac{Gmm_0}{r}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$$

4. (C)

$$dU = -\frac{Gmdm}{r}$$

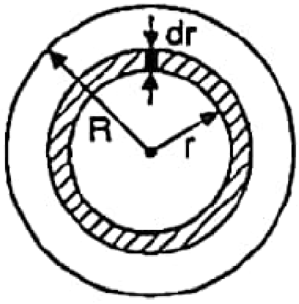
$$\Rightarrow \quad dU = -\frac{G\left(\frac{4}{3}\pi r^3\rho\right)\left(4\pi r^2 dr\rho\right)}{r}$$

$$\Rightarrow \quad dU = -\frac{16\pi^2 G\rho^2}{3}r^4 dr$$

$$\Rightarrow \quad U = -\frac{16}{3}\pi^3 G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2 \int_0^R r^4\,dr$$

$$\Rightarrow \quad U = \left(-\frac{16}{3}\pi^2 G\right)\left(\frac{M^2}{\frac{16}{9}\pi^2 R^6}\right)\left(\frac{R^5}{5}\right)$$

$$U = -\frac{3}{5}\frac{GM^2}{R}$$



5. (D)

$L = mvr$

Also, $\frac{mv^2}{r} = \frac{GMm}{r^2}$

From equations (1) and (2), we get

$L = m\sqrt{GMr}$

$\Rightarrow \quad L \propto r^{1/2}$

6. (AC)

$E_g = -\frac{dV}{dx}$

If $E_0 = 0$, then V = constant and this constant may also be zero.

7. (AD)

At two positions, when the planet is closest to the sun (perigee) and when it is farthest from the sun (apogee), velocity vector is perpendicular to force vector i.e., work done is zero. In one, complete revolution work done is zero.

8. (AD)

The field inside the shell is zero and so potential inside the shell is constant equal to the value that exists at the surface i.e. $-\frac{GM}{a}$.

9. (ABCD)

By Law of Conservation of Mechanical Energy, we get

$$(U+K)_{\infty} = (U+K)_r$$

$$\Rightarrow \quad 0+0 = \frac{-Gm(4m)}{r} + \frac{1}{2}\mu v_r^2$$

$$\Rightarrow \quad \frac{G(m)(4m)}{r} = \frac{1}{2}\mu v_r^2 \qquad ...(1)$$

Where, $\mu =$ reduced mass $= \frac{(m)(4m)}{m+4m} = \frac{4m}{5}$ and

$v_r =$ relative velocity of aaproach

Substituting (1), the total kinetic energy is

$$K = \frac{G(m)(4m)}{r}$$

$$\Rightarrow \quad K = \frac{Gm^2}{r}$$

Net torque of two equal and opposite forces acting on two objets is zero. Therfore, angular momentum will remain conserved. Intially both the objects were stationary i.e., angular momentum about any point was zero. Hence, angular momentum of both the particles about any point will be zero at all instants.

10. (ABD)

Kinetic energy, $KE = \frac{GMm}{2r}$

Potential energy, $PE = -\frac{GMm}{r}$ and

The total energy, $E = -\frac{GMm}{2r}$

Kientic energy is always positive and $KE \propto \frac{1}{r}$

Potential energy is negative and $|PE| \propto \frac{1}{r}$

Similarly total energy is also negative and $|E| \propto \frac{1}{r}$

Also, $|E| < |PE|$, so from the graph we observe that A is kinetic energy, B is potential energy and C is total energy of the satellite.

11. (ABC)

Both the stars will revolve about their centre of mass.

So, if the centre of mass be at a distance x from $2m$, then

$$x = \frac{2m(0) + mr}{3m} = \frac{r}{3}$$

So, $r_1 = \frac{2r}{3}$ and $r_2 = \frac{r}{3}$

ω and T will be same for both the stars, so

$$K_1 = \frac{1}{2} I_1 \omega^2 \text{ and } K_2 = \frac{1}{2} I_2 \omega^2$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{I_1}{I_2} = \frac{m\left(\frac{2r}{3}\right)^2}{2m\left(\frac{r}{3}\right)^2} = 2$$

$$L_1 = I_1 \omega \text{ and } L_2 = I_2 \omega$$

$$\Rightarrow \quad \frac{L_1}{L_2} = \frac{I_1}{I_2} = 2$$

12. (A)

Gravitational field is the acceleration due to gravity.

$$\text{So, } g = \begin{cases} \dfrac{GM}{r^2} & \underset{(\text{outside})}{r \geq R} \\ \dfrac{4}{3}\pi G \rho r & \underset{(\text{Inside})}{r < R} \end{cases}$$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} (\text{Outside})$$

Where $r_1 > R$ and $r_2 > R$ and $\frac{F_1}{F_2} = \frac{r_1}{r_2} (\text{Inside})$

Where $r_1 < R$ and $r_2 < R$

13. (AD)

$$U_i = \frac{-GMm}{R} = U\,(given)$$

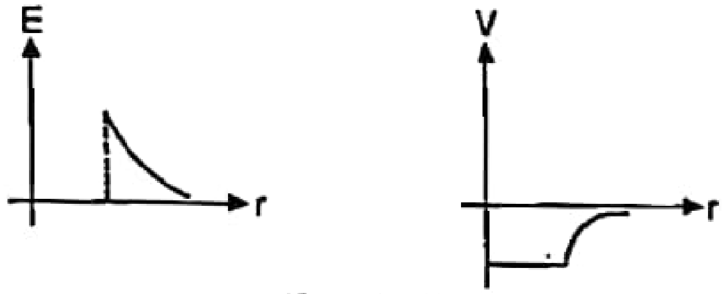
$$\Delta U = U_f - U_i = \frac{-GMm}{(R+R)} + \frac{GMm}{R}$$

$$\Rightarrow \quad \Delta U = \frac{GMm}{2R} = -\frac{U}{2}$$

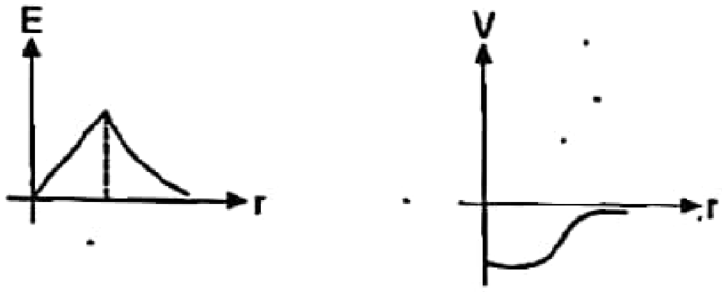
Same is the case with potential.

14. (ABC)

E-r and V-r graphs for a spherical shell and a solid sphere are shown here.



For a shell



For a solid sphere

15. (BCD)

$$T^2 = \frac{4\pi^2}{GM}\left(\frac{r_A + r_P}{2}\right)^3 \qquad \left\{\because r = \frac{r_A + r_P}{2}\right\}$$

$$\Rightarrow \; T^2 = \frac{\pi^2}{2GM}\left(r_A + r_P\right)^2 \text{k}$$

By Law of Conservation of Angular Momentum

$$mv_A r_A = mv_P r_P$$

$$\Rightarrow \; v_A r_A = v_P r_P$$

16. (3.00)

Since, $g_P = \frac{GM_\rho}{R_P^2} = \frac{4}{3}G\pi R_P \rho_P$

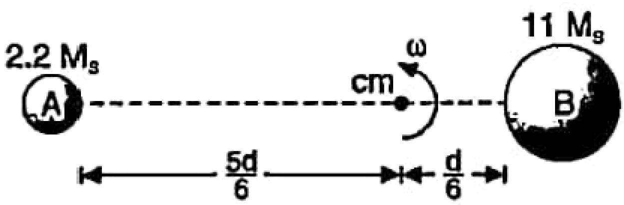
$$\Rightarrow \; \frac{g_P}{g_e} = \frac{R_\rho \rho_P}{R_e \rho_e}$$

Also, $v_e = \sqrt{2gR}$

$$\Rightarrow \; \frac{v_P}{v_e} = \sqrt{\frac{g_\rho R_P}{g_0 R_0}} = \left(\frac{g_P}{g_e}\right)\sqrt{\frac{\rho_e}{\rho_P}} = \frac{\sqrt{6}}{11} \times \sqrt{\frac{3}{2}}$$

$$\Rightarrow \; v_P = 3\,\text{kms}^{-1}$$

17. (6.00)



$$\frac{\text{Total angular momentum about cm}}{\text{Angular momentum of } B \text{ about cm}} = \frac{L}{L_B}$$

$$\Rightarrow \frac{L}{L_B} = \frac{(2.2\, M_S)\left(\omega\frac{5d}{6}\right)\left(\frac{5d}{6}\right) + (11M_S)\left(\omega\frac{d}{6}\right)\left(\frac{d}{6}\right)}{(11M_S)\left(\omega\frac{d}{6}\right)\left(\frac{d}{6}\right)} = 6$$

18. (7.00)

Let E be the gravitational field at x due to the complete sphere. If E_1 be the field due to hole and E_2 be the field due to the remaining portion, then we have

$$E = E_1 + E_2$$

$$\Rightarrow \quad E_2 = E - E_1$$

$$\Rightarrow \quad E_2 = \frac{GM}{x^2} - \frac{Gm}{\left(x - \frac{R}{2}\right)^2} \qquad ...(1)$$

Where, $M = \frac{4}{3}\pi R^3 \rho_0$ and $m = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho_0$

Substituting the values in equation (1), we get

$$E_2 = -\left(\frac{\pi G \rho_0 R^2}{6}\right)\left[\frac{1}{\left(x - \frac{R}{2}\right)^2} - \frac{8}{x^2}\right]$$

$$E_2 = -\left(\frac{\pi G \rho_0 R^3}{6}\right)\left[\frac{1}{\left(2R - \frac{R}{2}\right)^2} - \frac{8}{(2R)^2}\right]$$

$$E_2 = -\frac{\pi G \rho_0 R^3}{6}\left(\frac{4}{9R^2} - \frac{2}{R^2}\right)$$

$$E_2 = -\frac{\pi G \rho_0 R}{6}\left(\frac{4 - 18}{9}\right)$$

$$\Rightarrow \quad E_2 = \frac{14}{54}\pi G \rho_0 R$$

$$\Rightarrow \quad E_2 = \left(\frac{7}{27}\right)\pi G \rho_0 R$$

Since, $E = \left(\frac{a}{a+20}\right)\pi G \rho_0 R$

$$\Rightarrow \quad \frac{a}{a+20} = \frac{7}{27}$$

$$\Rightarrow \quad a = 7$$

19. (1.03)

$$\frac{v_{\max}}{v_{\min}} = \frac{1+e}{1+e} = \left[\frac{1+0.0167}{1-0.0167}\right] = 1.033$$

20. (10.00)

By Law of Conservation of Enerngy, we have

$$(U+K)_{\text{surface}} = (U+K)_{\text{at } \infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad -\frac{2GM}{R} + u^2 = v^2$$

Since, $v_e = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \quad -v_e^2 + u^2 = v^2$$

$$\Rightarrow \quad v^2 = -(11.2)^2 + (15)^2$$

$$\Rightarrow \quad v^2 = -125 + 225$$

$$\Rightarrow \quad v = 10\,\text{km}\,\text{s}^{-1}$$

IIT – JEE: 2025 **TW TEST (ADV)** **DATE: 07/04/24**

TOPIC: CHEMICAL KINETICS

Answer Key & Solution

21. (A)
$k_{obs} = k.k_c = 1.2 \times 10^{-1} \times 1.4 \times 10^{-2} = 1.68 \times 10^{-6}$ mole^{-1} L min^{-1}
Rate = k_{obs} $[NO]^2$ $[H_2]$ = $1.68 \times 10^{-6} \times 0.5^2 \times 0.5$
= 2.1×10^{-7} mole L^{-1} min^{-1}

22. (A)
Catalyst affect only activation energy. It brings down the activation energy of reaction.

23. (A)

A	$\rightarrow$ 2B	+ C
P	0	0
P – x	2x	x

At equilibrium
180 = P – x + 2x + x
180 = 90 + 2x
2x = 90, x = 45

$$K = \frac{2.303}{t}\log\frac{P}{P-x} = \frac{2.303}{10}\log\frac{90}{90-45} = \frac{2.303}{10}\log 2 = \frac{0.6932}{10}$$

$$= 0.6932 = \frac{0.06932}{60} = 1.1555 \times 10^{-3}\ \text{sec}^{-1}$$

24. (B)
Conceptual

25. (B)
$r_0 = K[A]^n\ [B]^m$
$r_1 = K[2A]^n\ [B/2]^m$
$r_2 = K\ 2^{n-m}\ [A]^n\ [B]^m$ or $r_1 = r \times 2^{n-m}$

26. (C)
Radioactivity follows first order kinetics

27. (ABCD)
Conceptual

28. (A)
Conceptual

29. (C)

$$K = \frac{2.303}{t}\log\frac{C_0}{C}$$

$$= \frac{2.303}{2\times10^4}\log\frac{800}{50}$$

$$= 1.38\times10^{-4}\ \sec^{-1}$$

30. (A)

Two reactants leads to bimolecular reaction may be of I or II order.

31. (A)

Rate constant 'K' is characteristic constant for a given reaction.

32. (ABD)

To increase the rate of reaction catalyst does

(i) decrease E_a

(ii) decreases E_a/RT

(iii) increases $-E_a/RT$

(iv) increases $e^{-E_a/RT}$

(v) increases k

33. (ACD)

$$-\frac{d[NH_3]}{dt} = \frac{k_1[NH_3]}{1+k_2[NH_3]} = \frac{k_1}{\frac{1}{[NH_3]}+k_2}$$

If $[NH_3]$ is very high $\frac{1}{[NH_3]}$ is very small than k_2

$\therefore\ \frac{-d[NH_3]}{dt} = \frac{k_1}{k_2}$ constant

i.e. order is zero if $[NH_3]$ is very low $\frac{1}{[NH_3]}$ is very high than k_2

$\therefore\ \frac{-d[NH_3]}{dt} = \frac{k_1}{1/[NH_3]}$ $\quad\therefore$ order is one.

34. (BC)

(A) It is $-k/2.303$

(B) It is of zero order

35. (ABC)

(A) $t_{x\%} \propto \frac{1}{a}$ for second order

(B) Rate = $kC_t = kC_0e^{-kt}$

$\ell n\left(\frac{dc}{dt}\right)$ = ℓn (rate) = $\ell n\, kC - kt \Rightarrow$ So, straight line

(C) k = $Ae^{-E_a/RT}$

Rate = k(conc.)^n = $Ae^{-E_a/RT}$ (conc.)^n

$\ell n\,(\text{rate}) = -\frac{E_a}{RT} + \text{constant}$

Slope = $\frac{-E_a}{R}$

(D) $\frac{t_{0.75}}{t_{0.5}}$ = ratio of time is constant with initial conc.

36. (60)

First order reaction

$$K=\frac{2.303}{t}\log\frac{a_0}{a_0-x}$$

$$K=\frac{2.303}{90}\log\frac{a_0}{0.25\,a_0} \qquad ...(1)$$

$$=0.0154$$

$$t=60\%=\frac{2.303}{K}\log\frac{a_0}{a_0} \qquad ...(2)$$

$$=\frac{2.303}{0.0154}\times(1-0.602)=59.51\,\text{min}\approx 60$$

37. (3)

After 2 seconds surface area becomes 1/4 th. Hence radius becomes 1/2 of initial therefore vol will become 1/8 th dissolved vol = 7/8

mass dissolved = 7/8 × 1/7 = 1/8 gm

molarity = $\frac{1}{8\times125}=\frac{1}{1000}=10^{-3}$

38. (2)

$t_{1/2}=20\,\text{min},\ t'_{1/2}=10\,\text{min}\ \&\ [A]_0'=2[A]_0$

$$\because t_{1/2}\,\alpha\,\frac{1}{[A]_0^{n-1}}\Rightarrow\frac{t'_{1/2}}{t_{1/2}}=\left[\frac{[A]_0'}{[A]_0}\right]^{n-1}$$

or $\frac{20}{10}=\left[\frac{2\times[A]_0^{n-1}}{[A_0]}\right]^{n-1}\rightarrow 2=2^{n-1}\rightarrow n-1=1\Rightarrow n=2$

39. (3)

$r=k[A]^x[B]^y$, write r_1,r_2,r_3. Divide one over another, get x & y

$x+y=3$

40. (4)

$\frac{2.303}{k}\log\frac{100}{100-99.99}=\frac{2.303}{k}\log\frac{100}{100-90}$ or, $\log 10^4=x\log10\Rightarrow x=4$

TOPIC: CIRCLE

SOLUTIONS

41. (A)

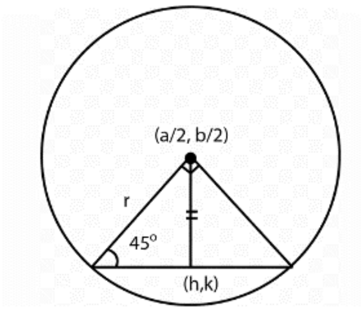
a, b, c are in A.P.So $ax + by + c = 0$ represents a family of lines passing through the point $(1, -2)$.

So, the family of circles (concentric) will be given by $x^2 + y^2 - 2x + 4y + c = 0$.

It interests given circle orthogonally.

$\Rightarrow 2(-1\times -2) + (2\times -2) = -1 + c \Rightarrow c = -3$

42. (C)



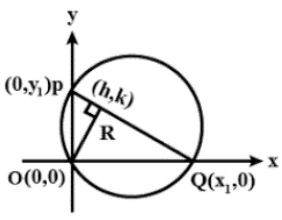
$$r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2}$$

$$\sin 45^\circ = \frac{\sqrt{\left(h - \frac{a}{2}\right)^2 + \left(h - \frac{b}{2}\right)^2}}{\frac{\sqrt{a^2 + b^2}}{2}}$$

$$\Rightarrow \frac{1}{2} = 4\left[\frac{\frac{(2h - a)^2}{4} + \frac{(2k - b)^2}{4}}{a^2 + b^2}\right]$$

Simplify to get locus $x^2 + y^2 - ax - by - \frac{a^2 + b^2}{8} = 0$.

43. (C)



Equation of line PQ is $y - k = \frac{-h}{k}(x - h)$

Or $hx+ky=h^2+k^2$

$\Rightarrow$ points $Q\left(\frac{h^2+k^2}{h},0\right)$ and $P\left(0,\frac{h^2+k^2}{k}\right)$

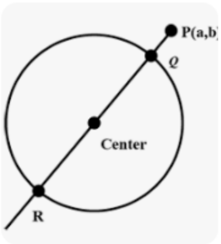
Also $2a=\sqrt{x_1^2+y_1^2}$

$\Rightarrow x_1^2+y_1^2=4a^2$

Eliminating x_1 and y_1 we have

$\left(x^2+y^2\right)^2\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=4a^2$

44. (A)



The given circle is $\left(x+1\right)^2=\left(y+2\right)^2=9,$ which has Radis = 3.

The points on the circle which are nearest and farthest to the point $P\left(a,b\right)$ are Q and R, respectively.

Thus, the circle centred at Q having radius PQ will be the smallest circle while the circle centred at R having Radis PR will be the largest required circle.

Hence, difference between their radii is $PR-PQ=QR=6$

45. (A)

Distance of given line from the centre of the circle is $|p|$.

Now the line subtends a eight angle at the centre.

Hence, radius = $\sqrt{2}|p|$

$\Rightarrow a=\sqrt{2}|p|$

$\Rightarrow a^2=2p^2$

46. (AC)

Equations of any tangent to the circle $x^2+y^2=25$ is of the form

$y=mx+5\sqrt{1+m^2}$,

[Where m is the slope]

Since, it passes through $\left(-2,11\right)$,

$11=-2m+5\sqrt{1+m^2}$,

$\Rightarrow\left(11+2m\right)^2=25\left(1+m^2\right)$

$\Rightarrow m=\frac{24}{7},-\frac{4}{3}$

Therefore, equation of the tangents are

$24x-7y+125=0$

Or $4x+3y=25$

47. (BC)

$x^2+y^2-8x-16y+60=0$ (1)

Equation of chord of contact from $(-2,0)$ is

$-2x-4(x-2)-8y+60=0$

$\Rightarrow 3x+4y-34=0$ (2)

Solving Eqs. (1) and (2),

$x^2+\left(\frac{34-3x}{4}\right)^2-8x-16\left(\frac{34-3x}{4}\right)+60=0$

$\Rightarrow 16x^2+1156-204x+9x^2-128x-2176+192x+960=0$

$\Rightarrow 5x^2-28x-12=0$

$\Rightarrow (x-6)(5x+2)=0$

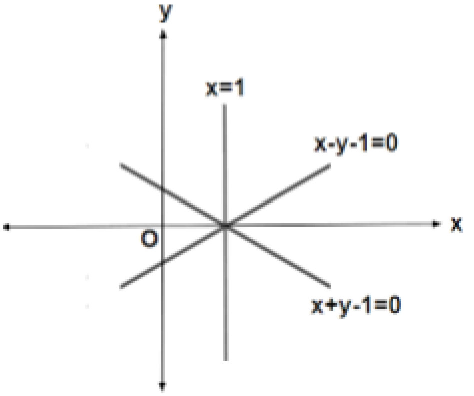
$\Rightarrow x=6,-\frac{2}{5}$

$\Rightarrow$ Point are $(6,4),\left(-\frac{2}{5},\frac{44}{5}\right)$

48. (BD)

Line pair is $(x-1)^2=0$, i.e., $x+y-1=0,\ x-y-1=0.$

Let the centre be $(\alpha,0)$, then its distance from $x+y=-1=0$ is



$\left|\frac{\alpha-1}{\sqrt{2}}\right|=2$ (Radius)

i.e., $\alpha=1\pm2\sqrt{2}$

Centre may be $(1+2\sqrt{2},0),(1-2\sqrt{2},0)$.

Now let the centre be $(1,\beta)$, then

$\left|\frac{1+\beta-1}{\sqrt{2}}\right|=2$

$\Rightarrow \beta=\pm\sqrt{2}$

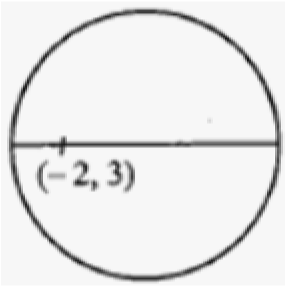
Centre may be $(1,2\sqrt{2}),(1,-2\sqrt{2})$.

49. (ACD)

$x^2+y^2+8x-10y-40=0$

Centre of the circle is $(-4,5)$. Its radius is 9.

Distance of the centre $(-4,5)$ from the point $(-2,3)$ is $\sqrt{4+4}=2\sqrt{2}$



$\therefore a = 2\sqrt{2} + 9$ and $b = -2\sqrt{2} + 9$

$\therefore a + b = 18$

$a - b = 4\sqrt{2}$

$ab = 81 - 8 = 73$

50. (ACD)

Coordinates of O are $(5,3)$ and radius = 2

Equation of tangent at $A(7,3)$ is $7x + 3y - 5(x+7) - 3(y+3) + 30 = 0$

i.e., $2x - 14 = 0$, i.e. $x = 7$

Equation of tangent at $B(5,1)$ is $5x + y - 5(x+5) - 3(y+1) + 30 = 0$, i.e. $y = 1$

Therefore, Coordinates of C are $(7,1)$

Therefore, Area of OACB is 4.

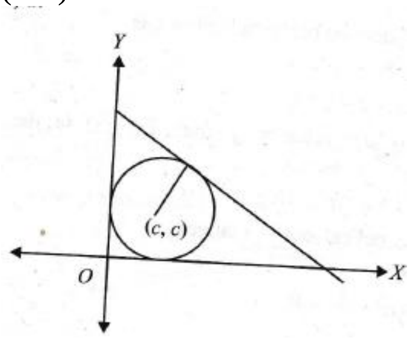
Equation of AB is $x - y = 4$ (radical axis)

Equation of the smallest circles is

$(x-7)(x-5) + (y-3)(y-1) = 0$

i.e., $x^2 + y^2 - 12x - 4y + 38 = 0$

51. (AD)



We must have $\left| \dfrac{\frac{c}{3} + \frac{c}{4} - 1}{\sqrt{\frac{1}{3^2} + \frac{1}{4^2}}} \right| = c$

$\Rightarrow c = 6, 1$

52. (ABCD)

Chords equidistance from the centre are equal

53. (ABCD)

$r_1 = 5, r_2 = \sqrt{15}, C_1C_2 = \sqrt{40}$

$\Rightarrow r_1 + r_2 > C_1C_2 > r_1 - r_2$

Hence, circles intersect in two distinct points.

There are two common tangents.

Also $2g_1g_1 + 2f_1f_2 = 2(1)(3) + 2(2)(-4) = -10$ and $c_1 + c_2 = -20 + 10 = -10$
Thus, two circle are orthogonal.

Length of common chord is $\dfrac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}} = 5\sqrt{\dfrac{3}{2}}$

Length of common tangent is

$\sqrt{C_1C_2^2 - (r_1 - r_2)^2} = 5\left(\dfrac{12}{5}\right)^{\frac{1}{4}}$

54. (BC)

Equation of pair of tangent by $SS' = T^2$ is $(ax + 0 - 1)^2 = (x^2 + y^2 - 1)(a^2 + 0 - 1)$

Or $(a^2 - 1)y^2 - x^2 + 2ax - a^2 = 0$

If θ be the angle between the tangents, then

$$\tan\theta \frac{2\sqrt{H^2 - AB}}{A + B}$$

$$= \frac{2\sqrt{-(a^2 - 1)}}{a^2 - 2}$$

$$\frac{2\sqrt{a^2 - 1}}{a^2 - 2}$$

If θ lies in second quadrant, then tan $\theta < 0$

$\Rightarrow \dfrac{2\sqrt{a^2 - 1}}{a^2 - 2} < 0$

$\Rightarrow a^2 - 1 > 0$

And $a^2 - 2 < 0$

$\Rightarrow |a| > 1$ and $|a| < 0$

$\Rightarrow a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$

55. (AB)
N = 81

56. (3)

Let $\sum_{i=1}^{6} x_1 = \alpha$ and $\sum_{i=1}^{6} y_i = \beta$

Let O be the orthocentre of the tringle made by $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)

$\Rightarrow$ O is $(x_1 + x_2 + x_3, y_1 + y_2 + y_3) \equiv (\alpha_1, \beta_1)$

Similarly le G be the centroid of the triangle made by other three points

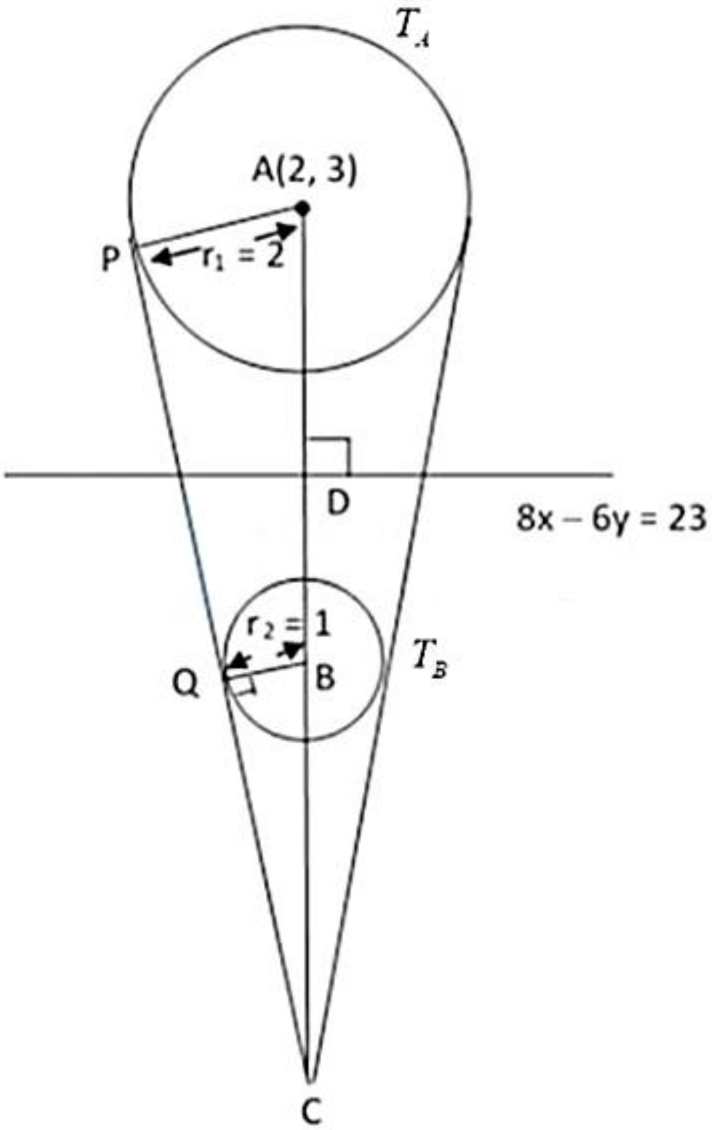
$\Rightarrow$ G is $\left(\dfrac{x_4 + x_5 + x_6}{3}, \dfrac{y_4 + y_5 + y_6}{3}\right)$

$\Rightarrow$ G is $\left(\dfrac{\alpha - \alpha_1}{3}, \dfrac{\beta - \beta_1}{3}\right)$

The point dividing OG in the ratio $3:1$ is $\left(\dfrac{\alpha}{4}, \dfrac{\beta}{4}\right) \equiv (2,1)$

$\Rightarrow h + k = 3$

57. (10)

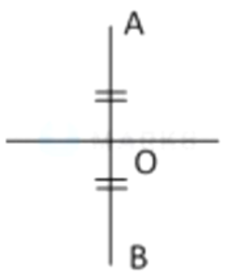


Clearly, ACP and BCQ are similar triangles.

Hence, $\frac{AC}{BC} = \frac{r_1}{r_2}$

$\frac{AC}{BC} = 2$

$AC = 2BC$ ...(i)



Now, since B, is image of A in line

$8x - 6y = 23$

$AD = BD$

$AB = 2AD$ ...(ii)

$AD =$ distance of A from line

$8x - 6y = 23$

$AD = \left|\frac{16-16-23}{10}\right| = \frac{5}{2}$

Now as $AC = 2BC$

$\Rightarrow AC = 2(AC - AB)$

$\Rightarrow AC = 2AB$

$\Rightarrow AC = 4AD$ (from (ii))

$AC = 4\times\frac{5}{2}$

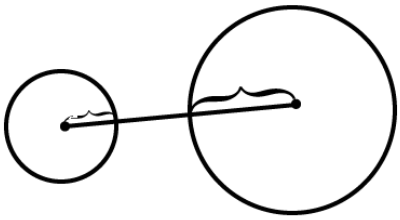
$AC = 10$

58. (9)

The given circles are

$(x-1)^2 + (y-2)^2 = 1$ (1)

And $(x-7)^2 + (y-10)^2 = 4$ (2)



Let $A \equiv (1, 2), B \equiv (7,10), r_1 = 1, r_2 = 2$.

Let $AB \equiv 10, r_1 + r_2 = 3$

Since $AB > r_1 + r_2$, the two circles are non-interesting.

Radii of the two circle at time t are $1+0.3t$ and $2+0.4t$.

For the two circles to touch each other,

$AB^2 = \left[(r_1 + 0.3t) \pm (r_2 + 0.4t)\right]^2$

$\Rightarrow 100 = \left[(1+0.3t) \pm (2+0.4t)\right]^2$

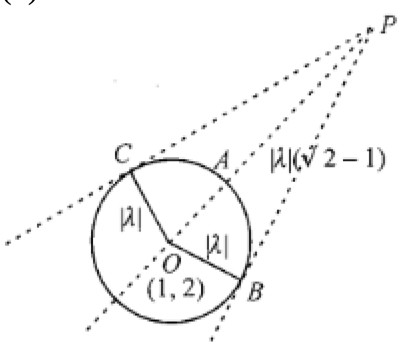
$\Rightarrow 100 = (3+0.7t)^2, \left[(0.1)t+1\right]^2$

$\Rightarrow 3+0.7t = \pm 10, 0.1t+1 = \pm 10$

$\Rightarrow t = 10, t = 90$ $(\because t > 0)$

The two circle will touch each other externally in 10 seconds and internally in 90 seconds.

59. (6)



Since the line $y = x + c$ is normal to the given circle, $c = 1$.

So the equation of line is $y = x + 1$ (1)

Also the radius of the circle is $|\lambda|$, Given $AP = |\lambda|\left(\sqrt{2}-1\right)$

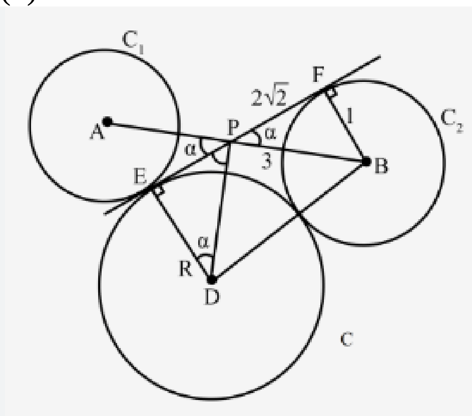
$\Rightarrow OP = \sqrt{2}|\lambda|.$

$\Rightarrow PC = |\lambda|$

$\Rightarrow$ Area of quadrilateral OBPC

$= 2\times\frac{1}{2}|\lambda|^2 = 36 \Rightarrow \lambda = \pm 6$

60. (8)



$\cos\alpha = \frac{2\sqrt{2}}{3}$

$\sin\alpha = \frac{1}{3}$

$\tan\alpha = \frac{2\sqrt{2}}{R}$

$\Rightarrow R = \frac{2\sqrt{2}}{\tan\alpha} = 8$ units