IN CHAPTER EXERCISE - 1

1. **Sol.** Order is 2 and degree is 2.(From the definition of order and degree of differential Equations).
Ans.[A]

2. **Sol.** Clearly, the given differential equation is not a polynomial in differential coefficients. So, its degree is not defined. **Ans.[D]**

3. **Sol.** A differential equation in which the dependent variable and its differential coefficient occur only in the first degree and are not multiplied together is called a linear differential equation.

Hence $y\frac{dy}{dx} + 4x = 0$ is non- linear differential equation. **Ans.[B]**

4. **Sol.** $(1 + y^2)\, dx + (1 + x^2)\, dy = 0$

$\Rightarrow \frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$

On integration, we get

$\tan^{-1} x + \tan^{-1} y = \tan^{-1} c$

$\Rightarrow \frac{x+y}{1-xy} = c \Rightarrow x + y = c\,(1- xy)$ **Ans.[C]**

5. **Sol.** $\frac{dy}{dx} + \frac{1+x^2}{x} = 0 \Rightarrow dy + \left(\frac{1}{x} + x\right) dx = 0$

On integrating, we get $y + \log x + \frac{x^2}{2} + c = 0$

Ans.[B]

6. **Sol.** We have: $(e^x + e^x)\frac{dy}{dx} = (e^x - e^x)$

$\Rightarrow \frac{dy}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Integrating, $y = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx + c$

Put $e^x + e^x = t$ so that $(e^x - e^{-x})\, dx = dt$

$\therefore\ y = \int \frac{dt}{t} + c = \log |t| + c$

Hence $y = \log |e^x + e^{-x}| + c$,

which is the reqd. general solution. **Ans.[A]**

7. **Sol.** We are given that $\frac{dy}{dx} = 1 + x + y + xy$

$\Rightarrow \frac{dy}{dx} = (1 + x) + y\,(1 + x)$

$\Rightarrow \frac{dy}{dx} = (1 + x)\,(1 + y)$

$\Rightarrow \frac{1}{1+y}\, dy = (1 + x)\, dx$

Integrating both sides, we get

$$\int \frac{1}{1+y}\, dy = \int (1 + x)\, dx$$

$\Rightarrow \log(1+ y) = x + \frac{x^2}{2} + C$, which is the required solution. **Ans.[B]**

8. **Sol.** $\frac{dy}{dx} = \frac{(1+x)y}{(y-1)x}$ can be written as

$$\Rightarrow \frac{y-1}{y} dy = \frac{(1+x)}{x}\, dx$$

$$\Rightarrow \left(1-\frac{1}{y}\right) dy = \left(1+\frac{1}{x}\right) dx$$

$\Rightarrow (y \ \ \log y) = (x + \log x) + c$

$\Rightarrow c = x \ \ y + \log xy$ **Ans.[C]**

9. **Sol.** Put $x + y = v$ or $1 + dy/dx = dv/dx$

$$\left(\frac{dv}{dx}-1\right) = \sec v \Rightarrow \frac{dv}{dx} = \sec v + 1$$

$$\Rightarrow \frac{dv}{\sec v+1} = dx \text{ or } \frac{\cos v\, dv}{\cos v+1} = dx$$

$$\Rightarrow \left(1-\frac{1}{\cos v+1}\right) dv = dx$$

$$\Rightarrow \left(1-\frac{1}{2\cos^2 v/2}\right) dv = dx$$

or $\left(1-\frac{1}{2}\sec^2\frac{v}{2}\right) dv = dx$

$v - \tan\frac{v}{2} = x + c$

or $x + y \ \tan\frac{x+y}{2} = x + c$

or $y - \tan\frac{x+y}{2} = c$ **Ans.[A]**

10. **Sol.** $x\frac{d^2y}{dx^2} = 1 \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \log x + c_1$

$\Rightarrow y = x \log x \ \ x + c_1\, x + c_2$

(on integrating twice)

Given $y = 1$ and $\frac{dy}{dx} = 0$ at $x = 1$

$\Rightarrow c_1 = 0$ and $c_2 = 2$

Therefore, the required solution is

$y = x \log x \ \ \ x + 2$ **Ans.[B]**

11. **Sol.** $\sin\left(\frac{dy}{dx}\right) = a \Rightarrow \frac{dy}{dx} = \sin^{-1} a \Rightarrow dy = \sin^{-1} a.\ dx$

On integration, we get $y = x \sin^{-1} a + c$

But it passes through (0, 1),

so $1 = 0 + c \Rightarrow c = 1$

Hence $y = x \sin^{-1} a + 1$

$\Rightarrow \frac{y-1}{x} = \sin^{-1} a$

$\Rightarrow \sin\left(\frac{y-1}{x}\right) = a$ **Ans.[C]**

12. **Sol.** We have $\frac{dx}{x} = \frac{y\,dy}{1+y^2}$, Integrating,

we get $\log |x| = \frac{1}{2} \log (1+ y^2) + \log c$

or $|x| = c\sqrt{(1+y^2)}$

But it passes through (1, 0), so we get c =1

$\therefore$ Solution is $x^2 = y^2 + 1$ or $x^2 - y^2 = 1$

Ans.[B]

13 **Sol.** We have $\frac{dy}{dx} = 1 - \frac{1}{x^2} \Rightarrow y = x + \frac{1}{x} + c$

This passes through $\left(2, \frac{7}{2}\right)$, therefore $\frac{7}{2} = 2 + \frac{1}{2} + c \Rightarrow c = 1$

Thus the equation of the curve is $y = x + \frac{1}{x} + 1$ or $xy = x^2 + x + 1$

Ans.[B]

14. **Solution.**

(i) $\left(\frac{d^2y}{dx^2}\right)^4 = y + \left(\frac{dy}{dx}\right)^6$

$\therefore$ order = 2, degree = 4

(ii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \ell n y$

$\therefore$ order = 2, degree = 1

(iii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \sin^{-1} y$

$\therefore$ order = 2, degree = 1

(iv) $e^{\frac{d^3y}{dx^3}} - x\frac{d^2y}{dx^2} + y = 0$

$\because$ equation can not be expressed as a polynomial in differential coefficients, so degree is not applicable but order is 3.

15. Find order and degree of the following differential equations.

(i) **Ans.** order = 1, degree = 2

(ii) **Ans.** order = 5, degree = not applicable.

(iii) **Ans.** order = 2, degree = 2

16. **Sol.** Family of straight lines passing through origin is y = mx where'm' is parameter.

Differentiating w.r.t. x

$$\frac{dy}{dx} = m$$

Eliminating 'm' from both equations

$$\frac{dy}{dx} = \frac{y}{x}$$

which is the required differential equation.

17. **Sol.** Equation of family of circles touching x-axis at the origin is

$x^2 + y^2 + \lambda y = 0$ ....(i) where λ is parameter

$$2x + 2y\frac{dy}{dx} + \lambda\frac{dy}{dx} = 0 \quad ....(ii)$$

Eliminating 'λ' from (i) and (ii)

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

which is required differential equation.

18. **Sol.** The given equation is:

$y = A\cos 2x + B\sin 2x$ ...(1)

Diff. w.r.t. x, $\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x$

Again diff. w.r.t.x,

$$\frac{d^2y}{dx^2} = -4A\cos 2x \quad 4B\sin 2x$$

$= -4(A\cos 2x + B\sin 2x) = -4y$ [Using (1)]

Hence $\frac{d^2y}{dx^2} + 4y = 0$, which is the required differential equation.

19. **Sol.** The given equation is $y = k\,e^{\sin^{-1}x} + 3$. ...(1)

Differentiating (1) w.r.t.x,

$$\frac{dy}{dx} = k\,e^{\sin^{-1}x}\,\frac{1}{\sqrt{1-x^2}} + 0$$

$$\Rightarrow \frac{dy}{dx} = (y-3)\,\frac{1}{\sqrt{1-x^2}} \quad \text{[Using (1)]}$$

$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = y - 3$, which is the required differential equation.

20.

Sol. Putting $4x + y + 1 = t$

$$4 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 4$$

Given equation becomes

$$\frac{dt}{dx} - 4 = t^2$$

$$\frac{dt}{t^2 + x} = dx \quad \text{(Variables are separated)}$$

Integrating both sides,

$$\int \frac{dt}{4 + t^2} = \int dx$$

$\Rightarrow \quad \frac{1}{2} \tan^{-1} \frac{t}{2} = x + c \qquad \Rightarrow \quad \frac{1}{2} \tan^{-1}\left(\frac{4x + y + 1}{2}\right) = x + c$

IN CHAPTER EXERCISE - 2

1. Solution. Taking $x = r\cos\theta,\ y = r\sin\theta$

$x^2 + y^2 = r^2$

$2x\,dx + 2y\,dy = 2r\,dr$

$x\,dx + y\,dy = r\,dr$ ....(i)

$\frac{y}{x} = \tan\theta$

$\frac{d\frac{dy}{dx} - y}{x^2} = \sec^2\theta \,.\, \frac{d\theta}{dx}$

$x\,dy - y\,dx = x^2 \sec^2\theta \,.\, d\theta$

$x\,dy - y\,dx = r^2\,d\theta$ ....(ii)

Using (i) & (ii) in the given differential equation then it becomes

$r\,dr = r\cos\theta .\ r^2\,d\theta$

$\frac{dr}{r^2} = \cos\theta\,d\theta$

$-\frac{1}{r} = \sin\theta + \lambda$

$-\frac{1}{\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}} + \lambda$

$\frac{y+1}{\sqrt{x^2+y^2}} = c$ where $-\lambda' = c$

$(y+1)^2 = c(x^2+y^2)$

2. Solution. Putting $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$2v + (v^2 - 1)\left(v + x\frac{dv}{dx}\right) = 0$

$v + x\frac{dv}{dx} = -\frac{2v}{v^2-1}$

$x\frac{dv}{dx} = \frac{-v(1+v^2)}{v^2-1}$

$\int \frac{v^2-1}{v(1+v^2)}\,dv = -\int \frac{dx}{x}$

$\int\left(\frac{2v}{1+v^2} - \frac{1}{v}\right)dv = -\ell n\,x + c$

$\ell n\,(1+v^2) - \ell n\,v = -\ell n\,x + c$

$\ell n\left|\frac{1+v^2}{v}.\,x\right| = c$

$\ell n\left|\frac{x^2+y^2}{y}\right| = c$

$x^2 + y^2 = yc'$ where $c' = e^c$

3. Solution. $\frac{dy}{dx} = -\frac{x^2 - y^2}{2xy}$

$y = vx$

$\frac{dy}{dx} = v + \frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = -\frac{1-v^2}{2v}$

$\int \frac{2v}{1+v^2}\, dv = -\int \frac{dx}{x}$

$\ell n\,(1 + v^2) = -\ell nx + c$

at $x = 1, y = 1 \quad \therefore \quad v = 1$

$\ell n\, 2 = c$

$\therefore \quad \ell n \left\{\left(1 + \frac{y^2}{x^2}\right) . x\right\} = \ell n_2$

$x^2 + y^2 = 2x$

4. Solution. Let $x = Y + h$, $y = Y + k$

$\frac{dy}{dx} = \frac{dy}{dY} . \frac{dY}{dX} . \frac{dX}{dx}$

$= 1. \frac{dY}{dX} . 1.$

$= \frac{dY}{dX}$

$\therefore \quad \frac{dY}{dX} = \frac{X + h + 2(Y + k) - 5}{2X + 2h + Y + k - 4}$

$= \frac{X + 2Y + (h + 2k - 5)}{2X + Y + (2h + k - 4)}$

h & k are such that $h + 2k - 5 = 0$ & $2h + k - 4 = 0$

$h = 1, k = 2$

$\therefore \quad \frac{dY}{dX} = \frac{X + 2Y}{2x + Y}$ which is homogeneous differential equation.

Now, substituting $Y = vX$

$\frac{dY}{dX} = v + X\frac{dv}{dX}$

$\therefore \quad X\frac{dv}{dX} = \frac{1+2v}{2+v} - v$

$\int \frac{2+v}{1-v^2}\, dv = \int \frac{dx}{X}$

$\int \left(\frac{1}{2(v+1)} + \frac{3}{2(1-v)}\right) dv = \ell n\, X + c$

$$\frac{1}{2}\ \ell n\ (v + 1) - \frac{3}{2}\ \ell n\ (1 - v) = \ell n\ X + c$$

$$\ell n \left|\frac{v+1}{(1-v)^3}\right| = \ell n X^2 + 2c$$

$$\frac{(Y+Y)}{(X-Y)^3}\ \frac{X^2}{X^2} = e^{2c}$$

$X + Y = c'(X - Y)^3$ where $e^{2c} = c^1$

$x - 1 + y - 2 = c'\ (-1- y + 2)^3$

$x + y - 3 = c'\ (x - y + 1)^3$

5. Solution. Putting $u = 2x + 3y$

$$\frac{du}{dx} = 2 + 3\ .\ \frac{dy}{dx}$$

$$\frac{1}{3}\left(\frac{du}{dx} - 2\right) = \frac{u-1}{2u-5}$$

$$\frac{du}{dx} = \frac{3u - 3 + 4u - 10}{2u - 5}$$

$$\int \frac{2u-5}{7u-13}\ dx = \int dx$$

$$\Rightarrow \quad \frac{2}{7}\int 1.du - \frac{9}{7}\int \frac{1}{7u-13}\ .\ du = x + c$$

$$\Rightarrow \quad \frac{2}{7}u - \frac{9}{7}\ .\ \frac{1}{7}\ \ell n\ (7u - 13) = x + c$$

$$\Rightarrow \quad 4x + 6y - \frac{9}{7}\ell n\ (14x + 21y - 13) = 7x + 7c$$

$$\Rightarrow \quad -3x + 6y - \frac{9}{7}\quad \ell n\ (14x + 21y - 13) = c'$$

6. Solution. Cross multiplying,

$2xdy + y\ dy - dy = xdx - 2ydx + 5dx$

$2\ (xdy + y\ dx) + ydy - dy = xdx + 5\ dx$

$2\ d(xy) + y\ dy - dy = xdx + 5dx$

On integrating,

$$2xy + \frac{y^2}{2} - y = \frac{x^2}{2} + 5x + c$$

$\Rightarrow \quad x^2 - 4xy - y^2 + 10x + 2y = c'$ where $c' = -2c$

7. Solution. $ydx + xdy = \dfrac{xdy - ydx}{x^2 + y^2}$

$d\ (xy) = d\ (\tan^{-1} y/x)$

Integrating both sides -

$xy = \tan^{-1} y/x + c$

8. Solution. The given equation can be written as -

$$\ell ny\ (2x)\ dx + x^2\left(\frac{dy}{y}\right) + 3y^2\ dy = 0$$

$\Rightarrow \quad \ell ny\ d\ (x^2) + x^2\ d\ (\ell ny) + d\ (y^3) = 0$

$\Rightarrow \quad d\ (x^2\ \ell ny) + d\ (y^3) = 0$

Now integrating each term, we get

$x^2 \ell ny + y^3 = c$

9. **Solution.** $\frac{dy}{dx} + Py = Q$

$P = \frac{3x^2}{1+x^3}$

$IF = e^{\int P.dx} = e^{\int \frac{3x^2}{1+x^3}dx} = e^{\ell n(1+x^3)} = 1 + x^3$

$\therefore$ General solution is

$y(IF) = \int Q(IF).dx + c$

$y(1 + x^3) = \int \frac{\sin^2 x}{1+x^3}(1 + x^3)\, dx + c$

$y(1 + x^3) = \int \frac{1-\cos 2x}{2}\, dx + c$

$y(1 + x^3) = \frac{1}{2}\, x - \frac{\sin 2x}{4} + c$

10. **Solution.** $\frac{dy}{dx} + \frac{1}{x\ell nx}\, y = \frac{2}{x}$

$P = \frac{1}{x\ell nx}$, $Q = \frac{2}{x}$

$IF = e^{\int P.dx} = e^{\int \frac{1}{x\ell nx}dx} = e^{\ell n(\ell nx)} = \ell n\, x$

$\therefore$ General solution is

$y.\,(\ell n\, x) = \int \frac{2}{x}.\ell nx.dx + c$

$y\,(\ell n\, x) = (\ell n\, x)^2 + c$

11. $t(1 + t^2)\, dx = (x + xt^2 - t^2)\, dt$ and it given that $x = -\pi/4$ at $t = 1$

Solution. $t(1 + t^2)\, dx = [x(1 + t^2) - t^2]\, dt$

$\frac{dx}{dt} = \frac{x}{t} - \frac{t}{(1+t^2)}$

$\frac{dx}{dt} - \frac{x}{t} = -\frac{t}{1+t^2}$

which is linear in $\frac{dx}{dt}$

Here, $P = -\frac{1}{t}$, $Q = -\frac{t}{1+t^2}$

$IF = e^{-\int \frac{1}{t}dt} = e^{-\ell nt} = \frac{1}{t}$

$\therefore$ General solution is -

$x \cdot \frac{1}{t} = \int \frac{1}{t}.\left(-\frac{t}{1+t^2}\right) dt + c$

$\frac{x}{t} = -\tan^{-1} t + c$

putting $x = -\pi/4$, $t = 1$

$-\pi/4 = -\pi/4 + c \Rightarrow c = 0$

$\therefore \quad x = -\, t \tan^{-1} t$

12. Solution. The given differential equation can be reduced to linear form by change of variable by a suitable subtitution.

Substituting $y^2 = z$

$$2y \frac{dy}{dx} = \frac{dz}{dx}$$

differential equation becomes

$$\frac{\sin x}{2}\frac{dz}{dx} + \cos x.z = \sin x \cos x$$

$\dfrac{dz}{dx} + 2 \cot x \,.\, z = 2 \cos x$ which is linear in $\dfrac{dz}{dx}$

$$\text{IF} = e^{\int 2\cot x\, dx} = e^{2\ell n \sin x} = \sin^2 x$$

$\therefore$ General solution is -

$$z. \sin^2 x = \int 2\cos x.\sin^2 x.dx + c$$

$$y^2 \sin^2 x = \frac{2}{3} \sin^3 x + c$$

13. Solution. Dividing both sides by y^2

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{xy} = \frac{1}{x^2} \qquad \text{..... (1)}$$

Putting $\dfrac{1}{y} = t$

$$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

$\therefore$ differential equation (1) becomes,

$$-\frac{dt}{dx} - \frac{t}{x} = \frac{1}{x^2}$$

$\dfrac{dt}{dx} + \dfrac{t}{x} = -\dfrac{1}{x^2}$ which is linear differential equation in $\dfrac{dt}{dx}$

$$\text{IF} = e^{\int \frac{1}{x} dx} = e^{\ell n x} = x$$

$\therefore$ General solution is -

$$t.\, x = \int -\frac{1}{x^2} \cdot x\, dx + c$$

$$tx = -\, \ell n x + c$$

$$\frac{x}{y} = -\, \ell n x + c$$

14. Solution. $(4x + 3y + 1)\, dx + (3x + 2y + 1)\, dy = 0$

$4xdx + 3\,(y\, dx + x\, dy) + dx + 2y\, dy + dy = 0$

Integrating each term,

$$2x^2 + 3\,xy + x + y^2 + y + c = 0$$

$$2x^2 + 3xy + y^2 + x + y + c = 0$$

which is the equation of hyperbola when $x^2 > ab$ & $\Delta \neq 0$.

Now, combined equation of its asymptotes is -

$$2x^2 + 3xy + y^2 + x + y + \lambda = 0$$

which is pair of straight lines

$\therefore \quad \Delta = 0$

$\Rightarrow \quad 2.1\,\lambda + 2\,.\,\frac{1}{2}.\frac{1}{2}.\frac{3}{2} - 2\,.\,\frac{1}{4} - 1\,.\,\frac{1}{4} - \lambda\,\frac{9}{4} = 0$

$\Rightarrow \quad \lambda = 0$

$\therefore \quad 2x^2 + 3xy + y^2 + x + y = 0$

$(x + y)\,(2x + y) + (x + y) = 0$

$(x + y)\,(2x + y + 1) = 0$

$x + y = 0 \quad$ or $\quad 2x + y + 1 = 0$

15. Solution. Let P (x, y) be any point on the curve

Equation of tangent at 'P' is -

$Y - y = m\,(X - x)$

$mX - Y + y - mx = 0$

Now,

$$\left(\frac{y - mx}{\sqrt{1+m^2}}\right) = x$$

$$y^2 + m^2x^2 - 2mxy = x^2\,(1 + m^2)$$

$$\frac{y^2 - x^2}{2xy} = \frac{dy}{dx}$$ which is homogeneous equation

Putting $y = vx$

$$\frac{dy}{dx} = v + x\,\frac{dv}{dx}$$

$$\therefore \quad v + x\,\frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$x\,\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$\int \frac{2v}{v^2 + 1}\,dv = -\int \frac{dx}{x}$$

$$\ell n\,(v^2 + 1) = -\ell n\,x + \ell n\,c$$

$$x\left(\frac{y^2}{x^2} + 1\right) = c$$

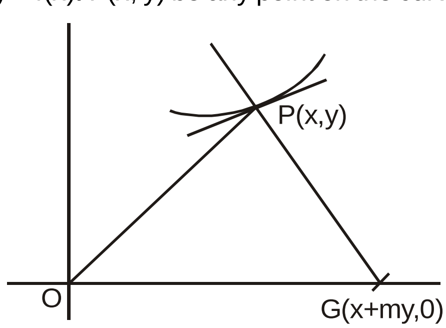
Curve is passing through (1, 1)

$\therefore \quad c = 2$

$x^2 + y^2 - 2x = 0$

16. Solution. Let the equation of the curve be $y = f(x)$. $P(x, y)$ be any point on the curve.

Slope of the tanget at $P(x, y)$ is $\frac{dy}{dx} = m$



$\therefore$ Slope of the normal at P is

$$m' = -\frac{1}{m}$$

Equation of the normal at 'P'

$$Y - y = -\frac{1}{m}\,(X - x)$$

Co-ordinates of G $(x + my, 0)$

Now, $OP^2 = PG^2$

$x^2 + y^2 = m^2y^2 + y^2$

$$m = \pm \frac{x}{y}$$

$$\frac{dy}{dx} = \pm \frac{x}{y}$$

Taking as the sign

$$\frac{dy}{dx} = \frac{x}{y}$$

$$y \,.\, dy = x \,.\, dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + \lambda$$

$$x^2 - y^2 = -2\lambda$$

$$x^2 - y^2 = c \qquad \text{(Rectangular hyperbola)}$$

Again taking as –ve sign

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$y\, dy = -x\, dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + \lambda'$$

$$x^2 + y^2 = 2\lambda'$$

$$x^2 + y^2 = c' \qquad \text{(circle)}$$

17. Sol. The given differential equation is

$x^2y\, dx \;\; (x^3 + y^3)\, dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2y}{x^3+y^3} \qquad \text{...(i)}$$

Since each of the function $x^2 y$ and $x^3 + y^3$ is a homogeneous function of degree 3, so the given differential equation is homogeneous.

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we get

$$v + x\frac{dv}{dx} = \frac{vx^3}{x^3+v^3x^3} \Rightarrow v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1+v^3} - v \Rightarrow x\frac{dv}{dx} = \frac{v-v-v^4}{1+v^3}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{v^4}{1+v^3}$$

$$\Rightarrow x(1+v^3)\, dv = -v^4\, dx$$

$$\Rightarrow \frac{1+v^3}{v^4}\, dv = -\frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{v^4}+\frac{1}{v}\right) dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\frac{v^{-3}}{-3} + \log v = -\log x + c$$

$$\frac{1}{3v^3} + \log v + \log x = c$$

$$\Rightarrow -\frac{1}{3}\frac{x^3}{y^3} + \log\left(\frac{y}{x}.x\right) = c \qquad [\because v = y/x]$$

$$\Rightarrow -\frac{1}{3}\frac{x^3}{y^3} + \log y = c,$$

which is the required solution.

Ans.[A]

18. **Sol.** Put x = X + h, y = Y + k

$$\frac{dy}{dx} = \frac{X+h+2(Y+k)-3}{2(X+h)+Y+K-3} = \frac{X+2Y+h+2k-3}{2X+y+2h+k-3}$$

Equating h + 2k – 3 = 0 and 2h + k – 3= 0

Solving we get 3k – 3 = 0, k = 1 & h = 1

$$\frac{dY}{dX} = \frac{X+2Y}{2X+Y} = \frac{1+2(Y/X)}{2+(Y/X)}$$

Put Y / X = v or Y = vX;

dY/ dX = v + X dv/dX

$$X\frac{dv}{dX} + v = \frac{1+2v}{2+v}$$

$$\text{or } X\frac{dv}{dX} = \frac{1+2v}{2+v} - v = \frac{1-v^2}{2+v}$$

$$\frac{dX}{X} = \frac{2+v}{1-v^2}dv = \frac{2+v}{(1-v)(1+v)}\ dv$$

$$= \left[\frac{1}{2}\left(\frac{1}{1+v}\right) + \frac{3}{2}\left(\frac{1}{1-v}\right)\right]dv$$

ℓn X = 1/2 ℓn (1 + v) – 3/2 ℓn(1 – v) + ℓnc

or ℓn X = ℓn $(1 + v)^{1/2}$ ℓn $(1 – v)^{3/2}$ + ℓnc

$$\ell n\ X = \ell n\frac{(1+v)^{1/2}}{(1-v)^{3/2}}c$$

$$\text{or } X = \frac{(1+v)^{1/2}}{(1-v)^{3/2}}c = \frac{(1+Y/X)^{1/2}}{(1-Y/X)^{3/2}}c$$

Put X = x – 1 and Y = y – 1

$$\Rightarrow x = 1 + \frac{\left(1+\frac{y-1}{x-1}\right)^{1/2}}{\left(1-\frac{y-1}{x-1}\right)^{3/2}}\ c$$

Ans.[C]

19.

Sol. The given differential equation is $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{x^2-1} y = \frac{1}{(x^2-1)^2} \qquad ...(i)$$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{2x}{x^2-1}$ and

$Q = \frac{1}{(x^2-1)^2}$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int 2x/(x^2-1) dx}$

$= e^{\log(x^2-1)} = (x^2 - 1)$

Multiplying both sides of (i) by I.F. $= (x^2 - 1)$, we get

$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$

Integrating both sides, we get

$$y(x^2 - 1) = \int \frac{1}{x^2-1} dx + c$$

[Using: y (I.F.) = $\int$ Q. (I.F.) dx + c]

$\Rightarrow y(x^2 - 1) = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c.$

This is the required solution. **Ans. [A]**

20. Sol. The given equation can be written as:

$$\frac{dx}{dy} + \frac{2}{y}x = 10y^2 \qquad ...(1)$$

[Linear Equation in x]

Here 'P' $= \frac{2}{y}$ and 'Q' $= 10y^2$.

I.F. $= e^{\int P.dy} = e^{\int \frac{2}{y} dy} = e^{2\log|y|}$

$= e^{\log y^2} = y^2$

Multiplying (1) by y^2, we get :

$y^2\frac{dx}{dy} + 2yx = 10y^4$

$\Rightarrow \frac{d}{dy}(x.y^2) = 10y^4$

Integrating, $xy^2 = 10\int y^4 dy + c$

$\Rightarrow xy^2 = 2y^5 + c$ which is required solution.

Ans.[C]

IN CHAPTER EXERCISE - 3

Ex.1 **Sol.** Area = $\int_1^2 y\,dx = \int_1^2 \frac{3}{x^2}\,dx$

$= \left[\frac{3}{x}\right]_1^2 = 3\left(\frac{1}{2}-1\right)$

= 3/2 **Ans.[A]**

Ex.2 **Sol.** Required area = $\int_0^{\pi/2} \sin^2 x\,dx$

$= \int_0^{\pi/2} \frac{1-\cos 2x}{2}\,dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{\pi}{4}$ **Ans.[C]**

Ex.3 **Sol.** Putting y = 0, we get,

$x^2 - 3x - 4 = 0$

$\Rightarrow (x-4)(x+1) = 0$

$\Rightarrow x = 1$ or $x = 4$

$\therefore$ required area = $\int_{-1}^{4} (4+3x-x^2)\,dx$

$= \left(4x + \frac{3x^2}{2} - \frac{x^3}{3}\right)_{-1}^{4} = \frac{125}{6}$ **Ans.[A]**

Ex.4 **Sol.** Area $= \int_0^3 x\,dy = \int_0^3 \frac{y^2}{4}\,dy$

$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{12}(27-0)$

= 9/4 units **Ans.[B]**

Ex.5 **Sol.** Given curve $\left(\frac{x}{a}\right)^{1/3} = \cos t$, $\left(\frac{y}{a}\right)^{1/3} = \sin t$

Squarring and adding $x^{2/3} + y^{2/3} = a^{2/3}$

Clearly it is symmetric with respect to both the axis, so whole area is

$= 4\int_0^a y\,dx$

$= 4\int_{\pi/2}^{0} a\sin^3 t\; 3a\cos^2 t\,(\sin t)\,dt$

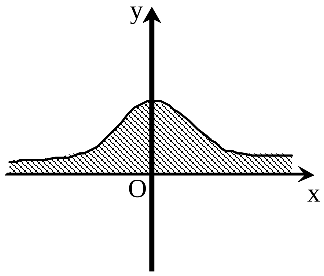
By given equation at x = 0; $t = \frac{\pi}{2}$ at x = a ; t = 0

$$= 12a^2 \int_0^{\pi/2} \sin^4 t \cos^2 t \, dt$$

$$= 12a^2 . \frac{3.1.1}{6.4.2} . \frac{\pi}{2} = \frac{3\pi a^2}{8}$$ **Ans.[C]**

Ex.6 **Sol.** Given curve is symmetrical about y-axis as shown in the diagram.

Reqd. area $= 2 \int_0^{\infty} \text{sech } x \, dx$

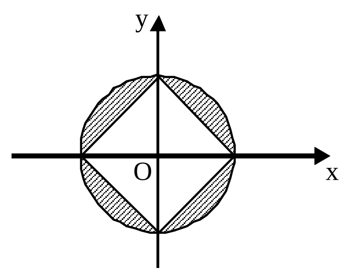


$$= 2\int_0^{\infty} \frac{2}{e^x + e^{-x}} dx = 4 \int_0^{\infty} \frac{e^x}{e^{2x} + 1} dx$$

$$= 4 \left[\tan^{-1}(e^x)\right]_0^{\infty} = 4\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \pi$$ **Ans.[B]**

Ex.7 **Sol.** By changing x as x and y as y, both the given equation remains unchanged so required area will be symmetric w.r.t both the axis, which is shown in the fig., so required area is

$$= 4 \int_0^1 \left[\sqrt{1-x^2} - (1-x)\right] dx$$



$$= 4 \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x - x + \frac{x^2}{2}\right]_0^1$$

$$= 4 \left[0 + \frac{1}{2}.\frac{\pi}{2} - 1 + \frac{1}{2}\right] = \pi - 2$$ **Ans.[A]**

Ex.8 **Sol.** $f(x) = y = \sin x$

when $x \in [0, \pi]$, $\sin x \geq 0$

and when $x \in [\pi, 2\pi]$, $\sin x \leq 0$

$$\therefore \text{ required area } = \int_0^{\pi} y \, dx + \int_{\pi}^{2\pi} (-y) \, dx$$

$$= \int_0^{\pi} \sin x \, dx + \int_{\pi}^{2\pi} (-\sin x) \, dx$$

$$= [\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$$

$$= (-\cos\pi + \cos 0) + (\cos 2\pi - \cos \pi)$$

$$= (1 + 1) + (1 + 1)$$

$= 4$ units **Ans.[A]**

Ex.9 **Sol.** The points of intersection of curves are $x = 0$ and $x = 1$.

$$\therefore \text{ required area } = \int_0^1 (\sqrt{x} - x) dx$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$ **Ans.[B]**

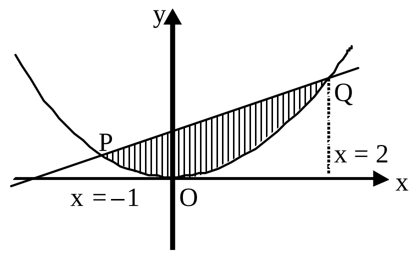
Ex.10 **Sol.** Solving the equation of the given curves for x, we get

$x^2 = x + 2$

$\Rightarrow (x \;\; 2)(x + 1) = 0$

$\Rightarrow x = \;\; 1, 2$

So, reqd. area

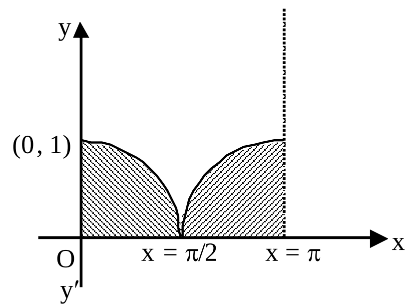


$$= \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right]dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}[(2 + 4 \;\; 8/3) \;\; (1/2 \;\; 2 + 1/3)] = 9/8$$ **Ans.[B]**

Ex.11 **Sol.** Required area = $\int_0^{\pi} \cos^2 x\, dx$



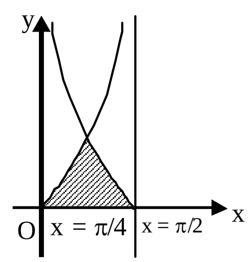
$$= \int_0^{\pi/2} \cos^2 x\, dx + \int_{\pi/2}^{\pi} \cos^2 x\, dx$$

$$= \frac{1}{2} \times \frac{\pi}{2} + \frac{1}{2}\int_{\pi/2}^{\pi}(1 + \cos 2x)\, dx$$

$$= \frac{\pi}{4} + \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_{\pi/2}^{\pi}$$

$$= \frac{\pi}{4} + \frac{1}{2}\left[\left(\pi - \frac{\pi}{2}\right)\right] = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$ **Ans.[C]**

Ex.12 **Sol.** From the fig. it is clear that



$$= \int_0^{\pi/4} \tan x\, dx \;\; \int_{\pi/4}^{\pi/2} \cot x\, dx$$

$$= [\log \sec x]_0^{\pi/4} + [\log \sin x]_{\pi/4}^{\pi/2}$$

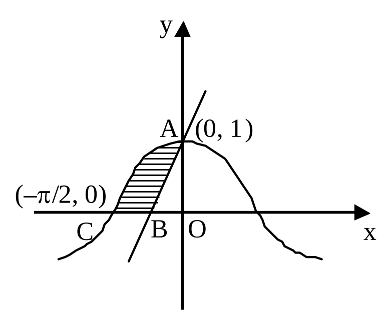
$$= \log\sqrt{2} - \log\frac{1}{\sqrt{2}}$$

$= \log 2$ **Ans.[A]**

Ex.13 **Sol.** Let the line y = x + 1, meets x-axis at the point A (0, 1). Also suppose that the curve y = cos x meets x-axis and y-axis respectively at the points C and A. From the adjoint figure it is obvious that

Required area = area of ABC

= area of OAC area of OAB



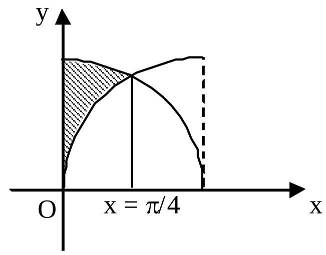
$$= \int_{-\pi/2}^{0} \cos x\, dx \;\; \frac{1}{2} \times OB \times OA$$

$$= [\sin x]_{-\pi/2}^{0} \;\; \frac{1}{2} \times 1 \times 1$$

= 1 – (1/2) = (1/2). **Ans. [D]**

Ex.14 Sol. In first quadrant sin x and cos x meet at $x = \pi/4$. The required area is as shown in the diagram. So

Required area = $\int_0^{\pi/4} (\cos x - \sin x)\, dx$

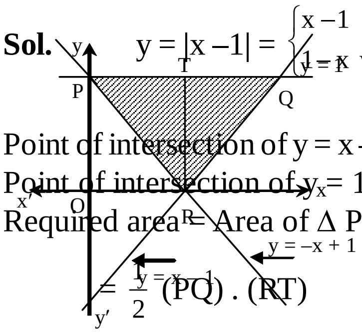


$= [\sin x + \cos x]_0^{\pi/4}$

$= (1/\sqrt{2} + 1/\sqrt{2})\ (0 + 1)$

$= \sqrt{2} - 1$ **Ans.[A]**

Ex.15 Sol. $y = |x - 1| = \begin{cases} x - 1 & \text{when } x \ge 1 \\ 1 - x & \text{when } x < 1 \end{cases}$



Point of intersection of y = x – 1, y = 1 is (2, 1)

Point of intersection of y = 1 – x, y = 1 is (0, 1)

Required area = Area of Δ PQR

$= \frac{1}{2}$ (PQ) . (RT)

$= \frac{1}{2}$. 2.1 = 1 **Ans.[A]**

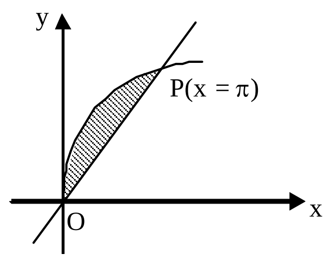
Ex.16 Sol. For the points of intersection of the given curves

x = x + sin x

⇒ sin x = 0

⇒ x = 0, π

∴ required area



$= \int_0^{\pi} [(x + \sin x) - x]\, dx$

$= \int_0^{\pi} \sin x\, dx = [\cos x]_0^{\pi} = 2$ **Ans.[A]**

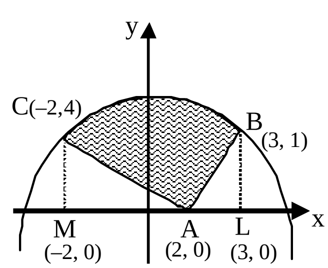
Ex.17 Sol. Here the first curve can be written in the following form

$x^2 = \frac{5}{3}\left(y - \frac{32}{5}\right)$

which is a parabola whose vertex lies on the y-axis.

Again second curve is given by

$y = \begin{cases} x - 2, x \ge 2 \\ -(x - 2), x < 2 \end{cases}$



which consists of two perpendicular lines AB and AC as shown in the fig.

These lines meet the parabola at B(3,1) and C(2,4) .

Hence the reqd. area A is given by

$A = \int_{-2}^{3} y\, dx\ \ \Delta\, ABL\ \ \Delta ACM$

$\int_{-2}^{3} \frac{1}{5}(32 - 3x^2)\, dx\ \ \frac{1}{2} . 1.1\ \ \frac{1}{2}(4 . 4)$

$$= \frac{1}{5}\left[32x - x^3\right]_{-2}^{3} \frac{17}{2} = \frac{1}{5}\,[69 + 56]\,\frac{17}{2} = \frac{33}{2}$$ **Ans.[C]**

18. Solution $y = \ell n\, x + \tan^{-1}x$

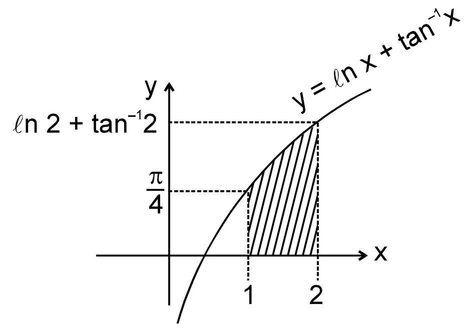
Domain $x > 0$ $\quad \frac{dy}{dx} = \frac{1}{x} + \frac{1}{1+x^2} > 0$

It is increasing function

$\underset{x\to\infty}{Lt}\, y = \underset{x\to\infty}{Lt}\,(\ell n\, x + \tan^{-1}x) = \infty$

$\underset{x\to 0^+}{Lt}\, y = \underset{x\to 0^+}{Lt}\,(\ell n\, x + \tan^{-1}x) = -\infty$

A rought sketch is as follows



$\therefore$ Required area $= \int_1^2 (\ell n\, x + \tan^{-1}x)\, dx$

$$= \left[x\,\ell n\, x - x + x\tan^{-1}x - \frac{1}{2}\,\ell n\,(1+x^2)\right]_1^2$$

$$= 2\,\ell n\, 2 - 2 + 2\tan^{-1}2 - \frac{1}{2}\,\ell n\, 5 - 0 + 1 - \tan^{-1}1 + \frac{1}{2}\,\ell n\, 2$$

$$= \frac{5}{2}\,\ell n\, 2 - \frac{1}{2}\,\ell n\, 5 + 2\tan^{-1}2 - \frac{\pi}{4} - 1$$

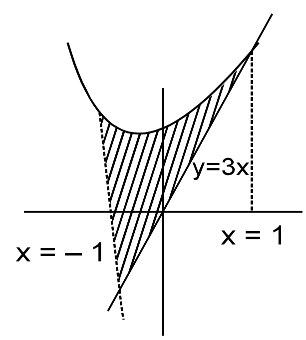
19. Solution. $\frac{dy}{dx} = 2x + 1$

$\frac{dy}{dx} = 3$ at $x = 1$

Equation of tangent is

$y - 3 = 3\,(x - 1)$

$y = 3x$



Required area $= \int_{-1}^{1} (x^2 + x + 1 - 3x)\, dx$

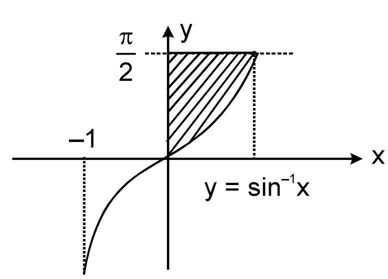
$$= \int_{-1}^{1} (x^2 - 2x + 1)\, dx = \left.\frac{x^3}{3} - x^2 + x\right]_{-1}^{1}$$

$$= \left(\frac{1}{3} - 1 + 1\right) - \left(-\frac{1}{3} - 1 - 1\right)$$

$$= \frac{2}{3} + 2 = \frac{8}{3}$$

20. Solution $y = \sin^{-1} x$

$\Rightarrow$ $x = \sin y$



Required area $= \int_0^{\frac{\pi}{2}} \sin y\, dy$

$$= -\cos y\Big]_0^{\frac{\pi}{2}} = -\,(0 - 1) = 1$$

21. **Sol.** $y = xe^{-x}$...(1) xy = 0

for point of inflection of curve (1)

$$\frac{d^2y}{dx^2} = 0 \quad ; \quad e^{-x} - (x-1)e^{-x} = 0 \Rightarrow x = 2$$

so Req Area $\int_0^2 y\,dx$; $= \int_0^2 xe^{-x}dx$ $= (-xe^{-x} - e^{-x})_0^2 = 1 - 3e^{-2}$

22. $x^2 + (2 - m)x - 4 = 0$

Let α,β be roots $\Rightarrow \alpha + \beta = m - 2$, $\alpha\beta = -4$

$$A(m) = \left|\int_\alpha^\beta (mx + 1 - x^2 - 2x + 3)dx\right|$$

$$= \left|\int_\alpha^\beta (-x^2 + (m-2)x + 4)dx\right|$$

$$= \left|\left(-\frac{x^3}{3} + (m-2)\frac{x^2}{2} + 4x\right)_\alpha^\beta\right|$$

$$= \left|\frac{\alpha^3 - \beta^3}{3} + \frac{m-2}{2}(\beta^2 - \alpha^2) + 4(\beta - \alpha)\right|$$

$$= |\beta - \alpha|.\left|-\frac{1}{3}(\beta^2 + \beta\alpha + \alpha^2) + \frac{(m-2)}{2}(\beta + \alpha) + 4\right|$$

$$= \sqrt{(m-2)^2 + 16}\left|-\frac{1}{3}\left((m-2)^2 + 4\right) + \frac{(m-2)}{2}(m-2) + 4\right|$$

$$= \sqrt{(m-2)^2 + 16}\left|\frac{1}{6}(m-2)^2 + \frac{8}{3}\right|$$

$$A(m) = \frac{1}{6}\left((m-2)^2 + 16\right)^{3/2}$$

Leas A(m) = $\frac{1}{6}(16)^{3/2} = \frac{32}{3}$.

EXERCISE 1(A)

1 [Hint: $\frac{dV}{dt} = -k4\pi r^2$ ....(1)

but $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ ....(2) ; hence $\frac{dr}{dt} = -K \Rightarrow$ (A)]

3 [Hint: $m^3 - 3m^2 - 4m + 12 = 0 \Rightarrow m = \pm 2, 3$; $m \in N$ hence $m \in \{2, 3\} \Rightarrow$ (C)]

4 [Sol. $y \sin 2x - \cos x + (1 + \sin^2 x)\frac{dy}{dx} = 0$ where $y = f(x)$

$$\frac{dy}{dx} + \left(\frac{\sin 2x}{1+\sin^2 x}\right)y = \frac{\cos x}{1+\sin^2 x}$$

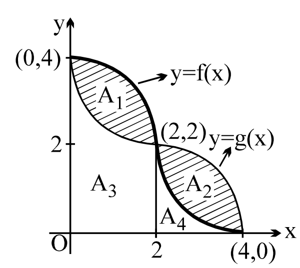
$$\text{I.F.} = e^{\int \frac{\sin 2x}{1+\sin^2 x}dx} = e^{\int \frac{dt}{t}} = e^{\ln(1+\sin^2 x)} = 1 + \sin^2 x \quad (\text{by putting } 1 + \sin^2 x = t)$$

$y(1 + \sin^2 x) = \int \cos x \, dx$

$y(1 + \sin^2 x) = \sin x + C$; $(y(0) = 0) \Rightarrow C = 0$

hence, $y = \frac{\sin x}{1+\sin^2 x}$; $y\left(\frac{\pi}{6}\right) = \frac{2}{5}$ **Ans.**]

5 [Sol. Given $\int_0^4 f(x)\,dx - \int_0^4 g(x)\,dx = 10$



$(A_1 + A_3 + A_4) - (A_2 + A_3 + A_4) = 10$
$A_1 - A_2 = 10$ ....(1)

again $\int_2^4 g(x)\,dx - \int_2^4 f(x)\,dx = 5$

$(A_2 + A_4) - A_4 = 5$
$A_2 = 5$ .....(2)

$\therefore$ (1) + (2)
$A_1 = 15$ Ans.]

6 [Sol. $\int y\,dy = \int (1 - x)\,dx$

$\frac{y^2}{2} = x - \frac{x^2}{2} + C$

Note : Family of concentric circles with (1,0) as the centre and variable radius

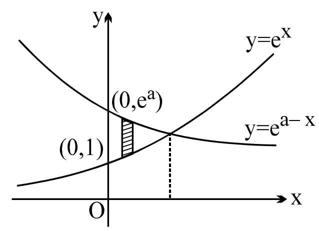
$x^2 + y^2 - 2x = C$
$(x - 1)^2 + y^2 = C + 1 = C \Rightarrow$ (B)]

7 [Sol. Solving $e^x = e^{a - x}$, we get

$e^{2x} = e^a \Rightarrow x = \frac{a}{2}$

$$S = \int_0^{a/2} (e^a \cdot e^{-x} - e^x)\,dx = \left[-(e^a \cdot e^{-x} + e^x)\right]_0^{a/2}$$

$$= (e^a + 1) - (e^{a/2} + e^{a/2}) = e^a - 2e^{a/2} + 1 = (e^{a/2} - 1)^2$$



$$\therefore \quad \frac{S}{a^2} = \left(\frac{e^{a/2}-1}{a}\right)^2 = \frac{1}{4}\left(\frac{e^{a/2}-1}{a/2}\right)^2$$

$$\therefore \quad \lim_{a\to 0} \frac{S}{a^2} = \frac{1}{4} \text{ Ans.]}$$

8 [Hint: $\frac{dy}{dt} = -k\sqrt{y}$; when t = 0; y = 4

$$\int_4^0 \frac{dy}{\sqrt{y}} = -k\int_0^t dt \; ; \qquad 2\sqrt{y}\Big|_4^0 = -kt = -\frac{t}{15} \; ; 0 - 4 = -\frac{t}{15} \;\Rightarrow\; t = 60 \text{ minutes} \;\Rightarrow\; (C) \;]$$

9 [Sol. $y \cdot e^{-2x} = Ax\, e^{-2x} + B$

$e^{-2x} \cdot y_1 - 2y\, e^{-2x} = A(e^{-2x} - 2x\, e^{-2x})$

Cancelling e^{-2x} throughout

$y_1 - 2y = A(1 - 2x)$ ....(1)

differentiating again

$$y_2 - 2y_1 = -2A \qquad \Rightarrow \qquad A = \frac{2y_1 - y_2}{2}$$

hence substituting A in (1)

$2(y_1 - 2y) = (2y_1 - y_2)(1 - 2x)$

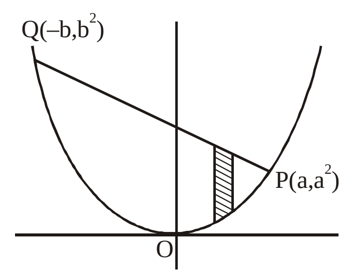
$2y_1 - 4y = 2y_1(1 - 2x) - (1 - 2x)y_2$

$$(1-2x)\frac{d}{dx}\left(\frac{dy}{dx} - 2y\right) + 2\left(\frac{dy}{dx} - 2y\right) = 0$$

hence k = 2 and $l = -2$ $\Rightarrow$ ordered pair $(k, l) \equiv (2, -2)$ **Ans.**]

10 [Sol. $m_{PQ} = \frac{a^2 - b^2}{a + b} = a - b$

equation of PQ



$$y - a^2 = \frac{a^2 - b^2}{a+b}(x - a)$$

or $y - a^2 = (a - b)(x - a)$

$y = a^2 + x(a - b) - a^2 + ab$

$y = (a - b)x + ab$

$$\therefore \quad S_1 = \int_{-b}^{a} ((a-b)x + ab - x^2)\,dx$$

which simplifies to $\frac{(a+b)^3}{6}$ ....(1)

Also $S_2 = \frac{1}{2}\begin{vmatrix} a & a^2 & 1 \\ -b & b^2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\left[ab^2 + a^2b\right] = \frac{1}{2}ab(a+b)$ ....(2)

$$\therefore \frac{S_1}{S_2} = \frac{(a+b)^3}{6} \cdot \frac{2}{ab(a+b)} = \frac{(a+b)^2}{3ab} = \frac{1}{3}\left[\frac{a}{b} + \frac{b}{a} + 2\right]$$

$$\therefore \left.\frac{S_1}{S_2}\right|_{min.} = \frac{4}{3}$$ **Ans.**]

11 [Sol. $\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x}$

$y = vx$

$V + x\frac{dv}{dx} = v - \cos^2 v$

$\int \frac{dv}{\cos^2 v} + \int \frac{dx}{x} = C$

$\tan v + \ln x = C$

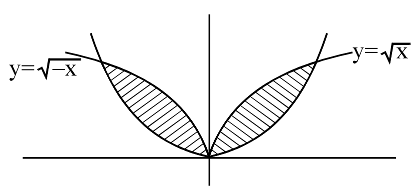
$\tan\frac{y}{x} + \ln x = C$

if $x = 1, y = \frac{\pi}{4} \Rightarrow C = 1$

$\tan\frac{y}{x} = 1 - \ln x = \ln\frac{e}{x}$

$y = x\tan^{-1}\left(\ln\frac{y}{x}\right) \Rightarrow$ A]

12 [Sol. $A = \left(\frac{16ab}{3}\right)\cdot 2$



$a = \frac{1}{4}$; $b = \frac{1}{4}$

$A = \frac{2}{3}$ Ans.]

13 [Sol. $Y - y = m(X - x)$

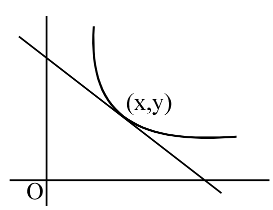
for X-intercept $Y = 0$

$X = x - \frac{y}{m}$

hence $x - \frac{y}{m} = y$

or $\frac{dy}{dx} = \frac{y}{x-y}$

put $y = Vx$



$$V + x\frac{dV}{dx} = \frac{V}{1-V}$$

$$x\frac{dV}{dx} = \frac{V}{1-V} - V = \frac{V - V + V^2}{1-V}$$

$$\int \frac{1-V}{V^2}\,dV = \int \frac{dx}{x}$$

$$-\frac{1}{V} - \ln V = \ln x + c$$

$$-\frac{x}{y} - \ln\frac{y}{x} = \ln x + c$$

$$-\frac{x}{y} = \ln y + c$$

$x = 1, y = 1 \;\Rightarrow\; c = -1$

$$1 - \frac{x}{y} = \ln y$$

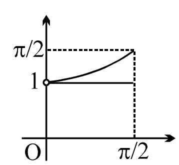
$$y = e \cdot e^{-x/y}$$

$$e^{-x/y} = \frac{e}{y}$$

$$y\,e^{x/y} = e \quad \Rightarrow \quad (A) \quad]$$

14 [Sol. $\sin x\frac{dy}{dx} + y\cos x = 1$

$$\frac{dy}{dx} + y\cot x = \text{cosec } x$$



$$\text{I.F.} = e^{\int \cot x\,dx} = e^{\ln(\sin x)} = \sin x$$

$$y\sin x = \int \text{cosec } x \cdot \sin x\,dx$$

$y\sin x = x + C$

if $x = 0$, y is finite

$\therefore$ $C = 0$

$$y = x(\text{cosec } x) = \frac{x}{\sin x}$$

Now $I < \frac{\pi^2}{4}$ and $I > \frac{\pi}{2}$

Hence $\frac{\pi}{2} < I < \frac{\pi^2}{4} \quad \Rightarrow \quad$ (A)]

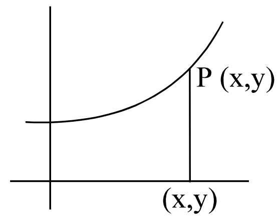
15 [Sol : $\int_1^b f(x)\,dx = (b-1)\sin(3b+4)$

Area function = $\int_1^x f(x)\,dx = (x-1)\sin(3x+4)$

differentiating

$\therefore$ $f(x) = \sin(3x+4) + 3(x-1)\,.\,\cos(3x+4) \Rightarrow$ C]

16 [Sol. $\int_0^x f(x)\,dx = y^3$



Differentiating

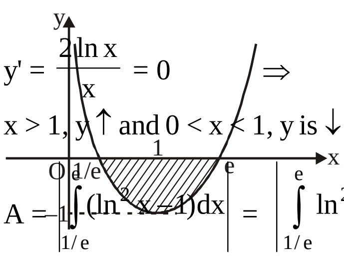
$f(x) = 3y^2.\dfrac{dy}{dx}$

$y = 3y^2\,\dfrac{dy}{dx} \quad \Rightarrow \quad y = 0$ (rejected)

or $3y\,dy = dx$

$\dfrac{3y^2}{2} = x + c \Rightarrow$ parabola $\Rightarrow$ C]

17 [Sol. $y = \ln^2 x - 1$



$y' = \dfrac{2\ln x}{x} = 0 \quad \Rightarrow \quad x = 1$

$x > 1$, $y \uparrow$ and $0 < x < 1$, y is $\downarrow$

$A = \left|\int_{1/e}^{e}(\ln^2 x - 1)\,dx\right| = \left|\int_{1/e}^{e}\ln^2 x\,dx - \int_{1/e}^{e}dx\right|$

$= \left| x\,\ln^2 x\Big]_{1/e}^{e} - 2\int_{1/e}^{e}\left(\dfrac{\ln x}{x}\right)\cdot x\,dx - \left(e - \dfrac{1}{e}\right)\right|$

$= \left|\left(e - \dfrac{1}{e}\right) - 2\int_{1/e}^{e}\left(\dfrac{\ln x}{x}\right)\cdot x\,dx - \left(e - \dfrac{1}{e}\right)\right|$

$= \left| -2\left[x\,\ln x\,\Big]_{1/e}^{e} - \int_{1/e}^{e}dx\right]\right| = \left| -2\left[\left(e + \dfrac{1}{e}\right) - \left(e - \dfrac{1}{e}\right)\right]\right| = \left|\dfrac{4}{e}\right| = \dfrac{4}{e}$ **Ans.**]

18 [Hint : $y = kx + b$; $\dfrac{dy}{dx} = k \quad \Rightarrow \quad kx + b \equiv k + xk^2 \quad \Rightarrow \quad k = k^2$ & $b = k$

$k = 0$ or $k = 1 \quad \Rightarrow$ result]

19 [Sol. $y = mx + c$; $\frac{dy}{dx} = m$; $\frac{d^2y}{dx^2} = 0$

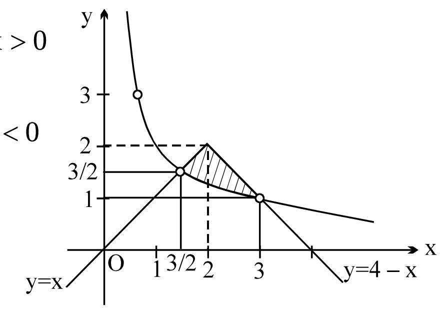
substituting in $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = -4x$

$$0 - 3m - 4(mx + c) = -4x$$
$$-3m - 4c - 4mx = -4x$$
$$-(3m + 4c) = 4x(m - 1) \quad ....(1)$$

(1) is true for all real x if

$m = +1$ and $c = -3/4$ $\Rightarrow$ (B)]

20 [Hint : $y = \begin{cases} 2-(2-x) \text{ if } x \le 2 \\ = x \\ 2-(x-2) \text{ if } x \ge 2 \\ = 4-x \end{cases}$; also $y = \begin{cases} \frac{3}{x} \text{ if } x > 0 \\ -\frac{3}{x} \text{ if } x < 0 \end{cases}$



$$A = \int_{3/2}^{2}\left(x - \frac{3}{x}\right)dx + \int_{2}^{3}\left((4-x) - \frac{3}{x}\right)dx$$

Now compute]

21 [Hint : $y = u^m \Rightarrow \frac{dy}{dx} = m\,u^{m-1}\frac{dy}{dx}$. Hence $2x^4 \cdot u^m \cdot m\,u^{m-1} \cdot \frac{du}{dx} + u^{4m} = 4x^6$.

$\frac{du}{dx} = \frac{4x^6 - u^{4m}}{2m\,x^4\,u^{2m-1}} \Rightarrow 4m = 6 \Rightarrow m = \frac{3}{2}$ and $2m - 1 = 2 \Rightarrow m = \frac{3}{2} \Rightarrow$ (C)]

22 [Hint: $\int_0^x f(x) = xe^x \Rightarrow f(x) = \frac{d}{dx}(xe^x) = xe^x + e^x$]

23 [Sol. S-1: $y = \sin kt$, $y' = k\cos kt$; $y'' = -k^2 \sin kt$

$\therefore$ $-k^2 \sin kt + 9 \sin kt = 0$

$\sin kt\,[9 - k^2] = 0 \Rightarrow k = 0,\ k = 3,\ k = -3$

S-2: $y = e^{kt}$, $y' = k\,e^{kt}$; $y'' = k^2\,e^{kt}$

$\therefore$ $k^2 e^{kt} + ke^{kt} - 6\,e^{kt} = 0$

$e^{kt}[k^2 + k - 6] = 0$

$(k + 3)(k - 2) = 0$

$k = -3$ or 2

common value is $k = -3$ Ans.]

24 [Hint: $\frac{dy}{dx} - \frac{y}{x^2}\tan\frac{1}{x} = -\sec\frac{1}{x}\cdot\frac{1}{x^2}$.   IF $= e^{-\int \frac{1}{x^2}\tan\frac{1}{x}dx} = \sec\frac{1}{x}$

$\Rightarrow y\,.\sec\frac{1}{x} = -\int \sec^2\left(\frac{1}{x}\right)\frac{1}{x^2}\,dx = \tan\frac{1}{x} + c$

if $y \to -1$ then $x \to \infty \Rightarrow c = -1 \quad \Rightarrow y = \sin\frac{1}{x} - \cos\frac{1}{x}$]

25 [Sol. Given $g(x) = 2x + 1$; $h(x) = (2x + 1)^2 + 4$

now $h(x) = f[g(x)]$

$(2x + 1)^2 + 4 = f(2x + 1)$

let $2x + 1 = t \Rightarrow f(t) = t^2 + 4$

$\therefore$ $f(x) = x^2 + 4$ ....(1)

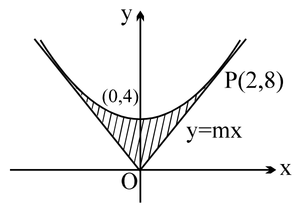
solving $y = mx$ and $y = x^2 + 4$

$x^2 - mx + 4 = 0$

put $D = 0$

$m^2 = 16 \Rightarrow m = \pm 4$

tangents are $y = 4x$ and $y = -4x$



$$A = 2\int_0^2 [(x^2+4) - 4x]dx = 2\int_0^2 [(x-2)^2 dx = \frac{2}{3}(x-2)^3\Big]_0^2 = \frac{16}{3} \text{ sq. units } \textbf{Ans.}]$$

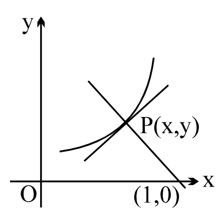
26 [Hint: $\frac{dM}{dt} = -KM \qquad M = c\,e^{-kt}$

when $t = 0$; $M = M_0 \Rightarrow c = M_0 \Rightarrow M = M_0 e^{-kt}$

when $t = 1$, $M = \frac{M_0}{2} \Rightarrow k = l\,n\,2$, hence $M = M_0 e^{-t\,ln\,2}$

when $M = \frac{M_0}{1000}$ then $t = \log_2 1000$]

28 [Sol. Slope of the normal $= \frac{y}{x-1}$

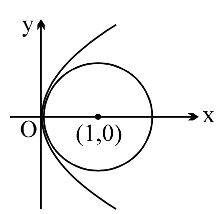


$\therefore \quad \frac{dy}{dx} = \frac{1-x}{y}$

$\frac{y^2}{2} = x - \frac{x^2}{2} + C$ .....(2)

(2) passes through (0, 0) hence C = 0

$x^2 + y^2 - 2x = 0$



now tangent to $y^2 = 4x$

$y = mx + \frac{1}{m}$ ....(3)

if it touches the circle

$x^2 + y^2 - 2x = 0$

then $\left|\dfrac{m+(1/m)}{\sqrt{1+m^2}}\right| = 1 \quad \Rightarrow \quad 1 + m^2 = m^2 \quad \Rightarrow \quad m \to \infty$

hence tangent is y axis i.e. $x = 0$ Ans.]

29 [Hint : $\ln c + \ln |x| = \dfrac{x}{y}$

diff. w.r.t. x, $\dfrac{1}{x} = \dfrac{y - x y_1}{y^2}$

$$\frac{y^2}{x} = y - x\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{x^2} \quad \Rightarrow \quad \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2} \quad \Rightarrow \quad D\ ]$$

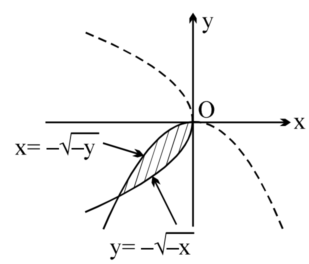
30 [Sol : $y = -\sqrt{-x} \Rightarrow y^2 = -x$ where x & y both (–) ve

$x = -\sqrt{-y} \Rightarrow x^2 = -y$ where x & y both (–) ve

Hence $A = \dfrac{16ab}{3}$

where $a = b = \dfrac{1}{4}$

$\therefore \quad A = \dfrac{1}{3} \Rightarrow$ (B)]



31 [Sol. $f'(x) - \dfrac{2x(x+1)}{x+1} f(x) = \dfrac{e^{x^2}}{(x+1)^2}$

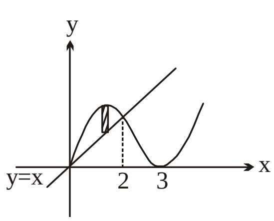
I. F. $= e^{\int -2x\,dx} = e^{-x^2}$

$\therefore \quad f(x).\ e^{-x^2} = \displaystyle\int \frac{dx}{(x+1)^2} \Rightarrow \quad f(x).\ e^{-x^2} = -\frac{1}{x+1} + C$

at $x = 0$, $f(0) = 5 \Rightarrow C = 6$

$\therefore \quad f(x) = \left(\dfrac{6x+5}{x+1}\right).e^{x^2}$ Ans]

32 [Sol. $A = \displaystyle\int_0^2 [x(x-3)^2 - x]\,dx$

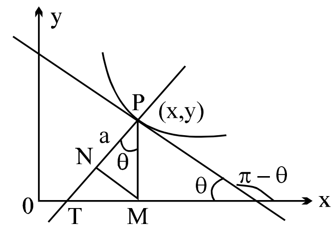


]

33 [Sol. Ordinate = PM. Let P ≡ (x, y)

Projection of ordinate on normal = PN

∴ PN = PM cosθ = a (given)



∴ $\frac{y}{\sqrt{1+\tan^2\theta}} = a \Rightarrow y = a\sqrt{1+(y_1)^2}$

⇒ $\frac{dy}{dx} = \frac{\sqrt{y^2-a^2}}{a} \Rightarrow \int \frac{a\,dy}{\sqrt{y^2-a^2}} = \int dx \Rightarrow a\,ln|y+\sqrt{y^2-a^2}| = x + c$]

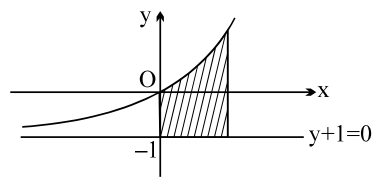
35 [Hint: $\frac{f''(x)}{f'(x)} = 1$

integrating, $ln\, f'(x) = x + c$, $f'(0) = 1 \Rightarrow c = 0$

$f'(x) = e^x$

$f(x) = e^x + k$, $f(0) = 0 \Rightarrow k = -1$

$f(x) = e^x - 1$



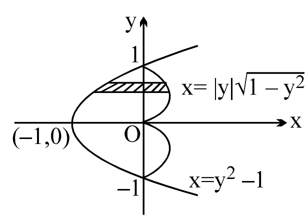
Area = $\int_0^1 (e^x - 1 + 1)\,dx = e^x\Big]_0^1 = e - 1$ Ans.]

36 [Sol. $\frac{dy}{dx} = 2ax = 2x \cdot \frac{y}{x^2}$; $\frac{dy}{dx} = \frac{2y}{x}$; now $m\frac{dy}{dx} = -1 \Rightarrow m = -\frac{x}{2y} \Rightarrow \frac{dy}{dx} = -\frac{x}{2y}$

$y^2 = -\frac{x^2}{2} + c$ Ans.]

37 [Hint: $A = 2\int_0^1 \left[y\sqrt{1-y^2} - (y^2-1)\right] dy$

$= 2$]



38 [Hint: I.F. = $e^{\int \tan x\,dx} = e^{ln \sec x\,dx} = \sec x$;

$y \cdot \sec x = \int (x\tan x + 1)\sec x\,dx = \int (x\sec x\tan x + \sec x)\,dx = x\sec x + c$

$y = x + c\cos x$

now $y = x + c\cos x \Rightarrow \frac{dy}{dx} = 1 - c\sin x$

∴ $\left.\frac{dy}{dx}\right|_{x=0} = 1 \Rightarrow$ (C)]

39 [Sol. diff. both sides

$x\,y(x) = 2x - y'(x)$

hence $\frac{dy}{dx} - xy = -2x$ $\quad (y'(x) = \frac{dy}{dx};\ y(x) = y)$

$\text{I.F} = e^{\int -x\,dx} = e^{\frac{-x^2}{2}}$

$y\,e^{\frac{-x^2}{2}} = \int -2x\,e^{\frac{-x^2}{2}}\,dx;$ $\quad e^{-\frac{x^2}{2}} = t \quad \Rightarrow \quad -x\,e^{-\frac{x^2}{2}}\,dx = dt;\ I = \int 2dt$

$y\,e^{\frac{-x^2}{2}} = 2e^{\frac{-x^2}{2}} + c$

$y = 2 + c\,e^{\frac{x^2}{2}}$

if $x = a \quad \Rightarrow \quad a^2 + y = 0 \quad \Rightarrow \quad y = -a^2$ (from the given equation)

hence $-a^2 = 2 + c\,e^{\frac{a^2}{2}};$ $\quad c\,e^{\frac{a^2}{2}} = -(2 + a^2);$ $\quad c = -(2 + a^2)\,e^{-\frac{a^2}{2}};$ $\ y = 2 - (2 + a^2)\,e^{\frac{x^2 - a^2}{2}}$]

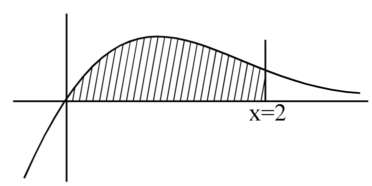
40 [Sol. $y = xe^{-x}$

$y' = e^{-x} - x\,e^{-x} = (1 - x)e^{-x} \uparrow$ for $x < 1$

$y'' = -e^{-x} - [e^{-x} - xe^{-x}] = e^{-x}[-1 - 1 + x]$

$= (x - 2)e^{-x}$

for point of inflection $y'' = 0 \quad \Rightarrow \quad x = 2$



$A = \int_0^2 xe^{-x}\,dx = -\,xe^{-x}\Big|_0^2 + \int_0^2 e^{-x}$

$= (-2\,e^{-2}) - \left(e^{-x}\right)_0^2$

$= -2\,e^{-2} - (e^{-2} - 1) = 1 - e^{-2} - 2e^{-2} = 1 - 3e^{-2}$ Ans.]

EXERCISE 1(B)

1 [Sol. $\int_0^1 f(tx)\,dt = n \cdot f(x)$ **[27-11-2005, 12th & 13th]**

put $tx = y \Rightarrow dt = \frac{1}{x}dy$

$\therefore \quad \frac{1}{x}\int_0^x f(y)\,dy = n\,f(x)$

$\therefore \quad \int_0^x f(y)\,dy = x \cdot n \cdot f(x)$

Differentiating

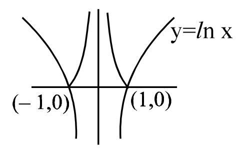
$f(x) = n[f(x) + x f'(x)]$

$f(x)(1-n) = n x f'(x)$

$\therefore \quad \frac{f'(x)}{f(x)} = \frac{1-n}{nx}$

Integrating $\quad \ln f(x) = \left(\frac{1-n}{n}\right)\ln cx = \ln (cx)^{\frac{1-n}{n}}$; $\quad \therefore \quad f(x) = c.x^{\frac{1-n}{n}}$ Ans.]

2 [Hint: $4\int_0^1 |\ln x|\,dx = -4\int_0^1 \ln x\,dx = 4$



]

3 [Hint: $(x^2z^{2\alpha} - 1)\,\alpha z^{\alpha-1}\,dz + 2x z^{3\alpha}\,dx = 0$

or $\alpha(x^2 z^{3\alpha-1} - z^{\alpha-1})dz + 2x z^{3\alpha}\,dx = 0$

for homogeneous every term must be of the same degree, $3\alpha + 1 = \alpha - 1 \Rightarrow \alpha = -1 \Rightarrow$ A]

4 [Sol. (a, 0) lies on the given curve

$\therefore \quad 0 = \sin 2a - \sqrt{3}\sin a \quad \Rightarrow \quad \sin a = 0$ or $\cos a = \sqrt{3}/2$

$\Rightarrow \quad a = \frac{\pi}{6}$ (as a > 0 and the first point of intersection with positive X-axis)

and $\quad A = \int_0^{\pi/6}(\sin 2x - \sqrt{3}\sin x)\,dx = \left(-\frac{\cos 2x}{2} + \sqrt{3}\cos x\right)_0^{\pi/6}$

$= \left(-\frac{1}{4} + \frac{3}{2}\right) - \left(-\frac{1}{2} + \sqrt{3}\right) = \frac{7}{4} - \sqrt{3} = \frac{7}{4} - 2\cos a$

$\Rightarrow \quad 4A + 8\cos a = 7$]

5
[Hint: differentiate $x\,y(x) = x^2 y'(x) + 2x\,y(x)$

or $x\,y(x) + x^2 y'(x) = 0$

$$x\frac{dy}{dx} + y = 0$$

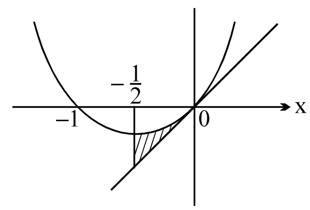
$\ln y + \ln x = \ln c$

$xy = c \Rightarrow$ (D)]

6 [Sol. $x = 1$; $y = 2$

$2 = a + b + c$ **....(1)**

$x = 0,\ y = 0 \Rightarrow c = 0 \Rightarrow a + b = 2$



now $\left.\frac{dy}{dx}\right|_{(0,0)} = 2a\,x + b = 1$

$\therefore$ $b = 1$; $a = 1$

Hence the curve is $y = x^2 + x$

$$A = \int_{-\frac{1}{2}}^{0}(x^2 + x - x)\,dx = \int_{-\frac{1}{2}}^{0}(x^2)\,dx = \frac{1}{24} \text{ sq. units }]$$

7 [Sol. $\int \frac{dy}{100 - y} = \int dx$

$-\ln(100 - y) = x + C$

$\ln(100 - y) = -x + C$

$x = 0, y = 50$ hence $C = \ln 50$

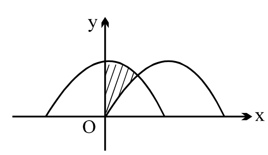
$x = \ln 50 - \ln(100 - y)]$

$$\ln\frac{50}{100 - y} = x \Rightarrow \frac{50}{100 - y} = e^x \Rightarrow 100 - y = 50e^{-x} \Rightarrow y = 100 - 50e^{-x} \Rightarrow \text{(B)]}$$

8 [Hint: I.F. $= e^{-x}$

$\therefore$ $ye^{-x} = \int e^{-x}(\cos x - \sin x)\,dx$ put $-x = t$

$= -\int e^{t}(\cos t + \sin t)\,dt$



$= -e^t \sin t + c$

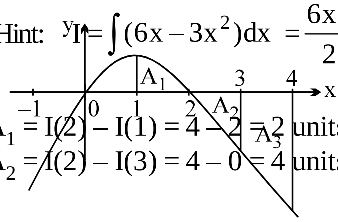
$y\,e^{-x} = e^{-x}\sin x + c$

since y is bounded when $x \to \infty \Rightarrow c = 0$

$\therefore$ $y = \sin x$

$$\text{Area} = \int_0^{\pi/4}(\cos x - \sin x)\,dx = \sqrt{2} - 1 \Rightarrow \text{(D)} \quad]$$

9 [Hint: $I = \int(6x - 3x^2)dx = \frac{6x^2}{2} - \frac{3x^3}{3} = 3x^2 - x^3 = x^2(3 - x)$



$A_1 = I(2) - I(1) = 4 - 2 = 2$ units

$A_2 = I(2) - I(3) = 4 - 0 = 4$ units

$A_3 = I(3) - I(4) = 0 - (-16) = 16$ units

$\Rightarrow$ one value of a will lie in (3, 4). Using symmetry, other will lie in (– 2, –1)]

[COMPREHENSION TYPE]

Paragraph for Question Nos. 10 to 12

12 [Hint: $f(x) = (x-1)(x^2 - 7x + 13)$

for f (x) to be prime at least one of the factors must be prime.

Hence $x - 1 = 1 \quad \Rightarrow \quad x = 2$ or

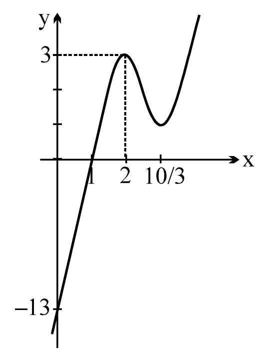
$x^2 - 7x + 13 = 1 \quad \Rightarrow \quad x^2 - 7x + 12 = 0$

$\Rightarrow \quad x = 3$ or 4

$\Rightarrow \quad x = 2, 3, 4 \quad \Rightarrow \quad$ (C)]

for Q.1 & 2 refer figure

$$A = \left| \int_0^1 f(x)\,dx \right| = \frac{65}{12}$$



]

Paragraph for Question Nos. 13 to 15

[Sol. $f(0) = 2$

$$f(x) = (e^x + e^{-x})\cos x - 2x - \left[x\int_0^x f'(t)\,dt - \int_0^x \underbrace{t}_{I}\,\underbrace{f'(t)}_{II}\,dt \right]$$

$$f(x) = (e^x + e^{-x})\cos x - 2x - \left[x\big(f(x) - f(0)\big) - \left\{ t\cdot f(t)\Big|_0^x - \int_0^x f(t)\,dt \right\} \right]$$

$$f(x) = (e^x + e^{-x})\cos x - 2x - x\,f(x) + 2x + \left[x\,f(x) - \int_0^x f(t)\,dt \right]$$

$$f(x) = (e^x + e^{-x})\cos x - \int_0^x f(t)\,dt \qquad ....(1)$$

differentiating equation (1)

$$f'(x) + f(x) = \cos x\,(e^x - e^{-x}) - (e^x + e^{-x})\sin x \qquad .....(2)$$

hence $\frac{dy}{dx} + y = e^x(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$ **Ans.(i)**

(ii) $f'(0) + f(0) = 0 - 2\cdot 0 = 0$ **Ans.(ii)**

(iii) I.F. of DE (1) is e^x

$$y\cdot e^x = \int e^{2x}(\cos x - \sin x)\,dx - \int (\cos x + \sin x)\,dx$$

$$y\cdot e^x = \int e^{2x}(\cos x - \sin x)\,dx - (\sin x - \cos x) + C$$

Let $\quad I = \int e^{2x}(\cos x - \sin x)\,dx = e^{2x}(A\cos x + B\sin x)$

solving $A = 3/5$ and $B = -1/5$ and $C = 2/5$

$$\therefore \quad y = e^{x}\left(\frac{3}{5}\cos x - \frac{1}{5}\sin x\right) - (\sin x - \cos x)e^{-x} + \frac{2}{5}e^{-x} \quad \textbf{Ans.(iii)}\ ]$$

Paragraph for Question Nos. 16 to 18

[Sol. $\quad \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = (1+x^2)$$

$$\therefore \quad y(1+x^2) = \int 4x^2\,dx = \frac{4x^3}{3} + C$$

passing through $(0, 0) \Rightarrow C = 0$

$$\therefore \quad y = \frac{4x^3}{3(1+x^2)}$$

$$\frac{dy}{dx} = \frac{4}{3}\left[\frac{(1+x^2)3x^2 - x^3\cdot 2x}{(1+x^2)^2}\right] = \frac{4}{3}\left[\frac{3x^2+x^4}{(1+x^2)^2}\right] = \frac{4x^2(3+x^2)}{3(1+x^2)^2}$$

hence $\frac{dy}{dx} > 0 \ \forall\ x \neq 0;$

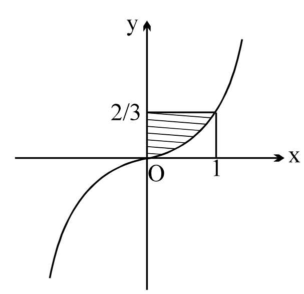
$\frac{dy}{dx} = 0$ at $x = 0$ and it does not change sign $\Rightarrow$ $x = 0$ is the point of inflection **Ans.**

$y = f(x)$ is increasing for all $x \in R$

$x \to \infty;\ y \to \infty \quad ; \quad x \to -\infty;\ y \to -\infty$

Area enclosed by $y = f^{-1}(x)$, x-axis and ordinate at $x = \frac{2}{3}$

$$A = \frac{2}{3} - \frac{4}{3}\int_0^1 \frac{x^3}{1+x^2}dx$$



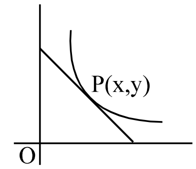
put $\quad 1 + x^2 = t \quad \Rightarrow \quad 2x\,dx = dt$

$$A = \frac{2}{3} - \frac{2}{3}\int_1^2 \frac{(t-1)}{t}dt = \frac{2}{3} - \frac{2}{3}\int_1^2 \left(1 - \frac{1}{t}\right)dt$$

$$= \frac{2}{3} - \frac{2}{3}\left[t - \ln t\right]_1^2 = \frac{2}{3} - \frac{2}{3}\left[(2 - \ln 2) - 1\right]$$

$$= \frac{2}{3} - \frac{2}{3}\left[1 - \ln 2)\right] = \frac{2}{3}\ln 2 \quad \textbf{Ans.}]$$

[REASONING TYPE]



19 [Hint: Equation of tangent

$Y - y = m(X - x)$

put $X = 0$, $Y = y - mx$

hence initial ordinate is

$y - mx = x - 1 \Rightarrow mx - y = 1 - x$

$\frac{dy}{dx} - \frac{1}{x}y = \frac{1-x}{x}$ which is a linear differential equation

Hence statement-1 is correct and its degree is 1

$\Rightarrow$ statement-2 is also correct. Since every 1st degree differential equation need not be linear hence statement-2 is not the correct explanation of statement-1.]

20 [Hint: S-1: order is 2.]

21 [Hint: Integral curves are

$y = cx - x^2$

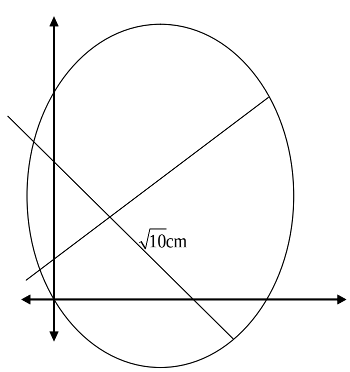
The DE does not represent all the parabolas passing through origin but it represents all parabolas through origin with axis of symmetry parallel to y-axis and coefficient of x^2 as -1, hence statement-1 is false. Statement-2 is universally **true**.]

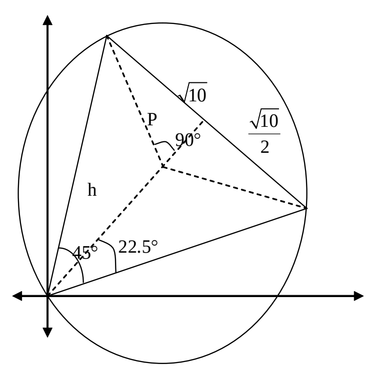
22 [Hint : $\frac{y\,dx - x\,dy}{y^2} \cdot \cot\frac{x}{y} = x\,dx$ or $\int \cot\frac{x}{y}\cdot d\left(\frac{x}{y}\right) = \int x\,dx$ or $\int \cot t\,dx = nx + c$

$ln(\sin t) = nx + c$; $\sin\frac{x}{y} = ce^{nx}$]

23 Sol $= -\int_0^1 x\log x\,dx = -\left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_0^1 = -\left[-\frac{1}{4}\right] = \frac{1}{4}$

24 Sol





$\tan 22.5° = \sqrt{2} - 1$

$\therefore \quad \sqrt{2} - 1 = \frac{\sqrt{10}}{2h}$

$\therefore \quad h = \frac{\sqrt{10}}{2}\left(\sqrt{2}+1\right)$

now $\quad P = \frac{\sqrt{10}}{2}$

$\therefore \quad r = h - p = \frac{\sqrt{20}}{2} = \sqrt{5}$

$\therefore \quad$ area $= 5\pi$

25 Sol $\quad \left(y - xy^2\right)dx + \left(x + x^2y^2\right)dy = 0$

$\therefore \quad x\,dy + y\,dx + x^2y^2dy - xy^2\,dx = 0$

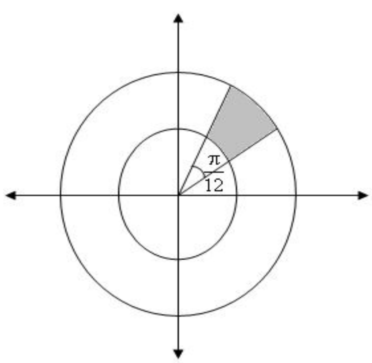
$$\left(x + x^2y^2\right)\frac{dy}{dx} + y - xy^2 = 0$$

$\therefore \quad \frac{dy}{dx} = \frac{xy^2 - y}{x + x^2y^2}$

26 Sol $\quad f\left(x\right) = \frac{x}{\sqrt{x^2+1}}$

$\therefore \quad f\left(\sqrt{8}\right) = \frac{\sqrt{8}}{3}$

27 Sol



$$A = \frac{\pi}{12} \times \frac{1}{2} \times \left(5^2 - 3^2\right)$$

$$= \frac{2\pi}{3}$$

28

Sol $\frac{v\,dv}{dx} = \sqrt{1+v^2}$

$\therefore \quad \frac{1}{2} \times \frac{1v\,dv}{\sqrt{1+v^2}} = dx$

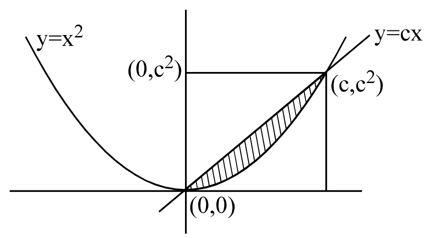
$\therefore \quad \frac{1}{2} \times 2\left[\sqrt{1+v^2}\right]_{\sqrt{3}}^{v} = x$

$\therefore \quad \sqrt{1+v^2} - 2 = x$

$\therefore \quad$ x at $v = \sqrt{48} = 5$

[MULTIPLE OBJECTIVE TYPE]

29 [Sol. Area (T) = $\frac{c \cdot c^2}{2} = \frac{c^3}{2}$



Area (R) = $\frac{c^3}{2} - \int_0^c x^2\,dx$

$= \frac{c^3}{2} - \frac{c^3}{3} = \frac{c^3}{6}$

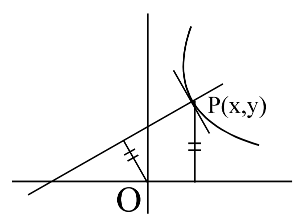
$\therefore \quad \lim_{c\to 0^+} \frac{\text{Area (T)}}{\text{Area (R)}} = \lim_{c\to 0^+} \frac{c^3}{2} \cdot \frac{6}{c^3} = 3$]

30 [Sol. Equation of normal

$Y - y = -\frac{1}{m}(X - x)$

$-my + my = X - x$

$X + my - (x + my) = 0$



perpendicular from (0, 0) = $\left|\frac{x+my}{\sqrt{1+m^2}}\right| = y$

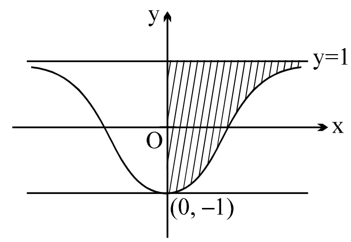
$x^2 + 2xym = y^2$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ $\Rightarrow$ homogeneous $\Rightarrow$ **(A)**

also $x \cdot 2y \cdot \frac{dy}{dx} - x^2 = y^2$ put $y^2 = t$; $2y\frac{dy}{dx} = \frac{dt}{dx}$; $x \cdot \frac{dt}{dx} + x^2 = t$

$\frac{dt}{dx} - \frac{1}{x}t = -x$ which is linear differential equation $\Rightarrow$ **(A) / (D)**]

32 [Sol. $y = f(x) = \frac{x^2-1}{x^2+1} = 1 - \frac{2}{x^2+1}$



$f'(x) = \frac{4x}{(x^2+1)^2}$

$x > 0$, f is increasing and $x < 0$ f is decreasing $\Rightarrow$ (B) is true

range is $[-1, 1)$ $\Rightarrow$ into $\Rightarrow$ (A) is false;

minimum value occurs at $x = 0$ and $f(0) = -1$ $\Rightarrow$ (C) is false

$A = 2\int_0^{\infty}\left(1-\frac{x^2-1}{x^2+1}\right)dx = 4\int_0^{\infty}\frac{dx}{x^2+1} = 4\cdot\tan^{-1}\Big|_0^{\infty} = 4\cdot\frac{\pi}{2} = 2\pi$ $\Rightarrow$ (D) is false]

33 [Hint: Make a Q.E. in $f'(x)$ and get $\frac{dy}{dx} = \left(-2 \pm \sqrt{3}\right)y$. Now integrate .]

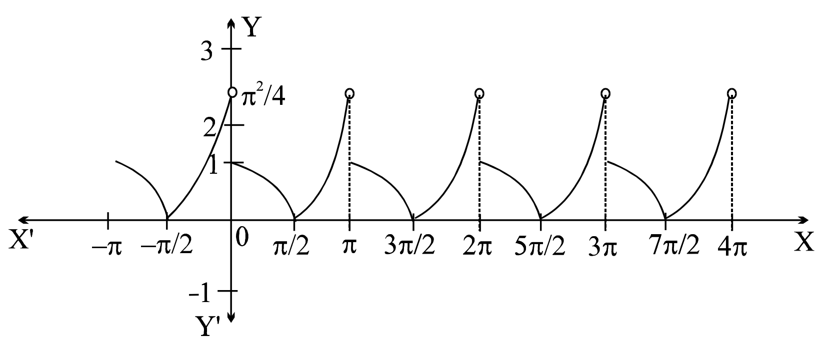
34 [Hint : $f(x) = \frac{\sin x}{x}$]

35 [Sol. Given $f(x) = \begin{cases} \cos x & 0 \le x < \frac{\pi}{2} \\ \left(\frac{\pi}{2} - x\right)^2 & \pi/2 \le x < \pi \end{cases}$ and f is periodic with period π

$\therefore$ Let us draw the graph of $y = f(x)$

From the graph, the range of the function is $\left[0, \frac{\pi^2}{4}\right)$ $\Rightarrow$ (A)

It is discontinuous at $x = n\pi,\ n \in I$. It is not differentiable at $x = \frac{n\pi}{2}$, $n \in I$.



Area bounded by $y = f(x)$ and the X-axis from $-n\pi$ to $n\pi$ for $n \in N$

$$= 2n\int_0^{\pi} f(x)\,dx = 2n\left[\int_0^{\pi/2}\cos x\ dx + \int_{\pi/2}^{\pi}\left(\frac{\pi}{2} - x\right)^2 dx\right] = 2n\left(1+\frac{\pi^3}{24}\right)\ ]$$

36 [Sol. $f(x) = \int_0^x \{f(t)\cos t - \cos(t-x)\}dx = \int_0^x f(t)\cos t\,dt - \int_0^x \cos(-t)\,dt$

$$\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$f(x) = \int_0^x f(t)\cos t\,dt - \sin x$$

differentiate both sides

$f'(x) = f(x)\cos x - \cos x$

let $f(x) = y$; $f'(x) = \frac{dy}{dx}$

$\frac{dy}{dx} - y\cos x = -\cos x$ (L.D.E.)

I.F. $= e^{-\int \cos x\,dx} = e^{-\sin x}$

hence, $y \cdot e^{-\sin x} = -\int e^{-\sin x}\cos x\,dx$; $y \cdot e^{-\sin x} = C + e^{-\sin x}$; $y = C\,e^{\sin x} + 1$

if $x = 0$; $y = 0$ (from the given relation)

$\Rightarrow$ $C = -1$

hence $f(x) = 1 - e^{\sin x}$

now minimum value $= 1 - e$ (when $x = \pi/2$)

maximum value $= 1 - e^{-1}$ (when $x = -\pi/2$)

$f'(x) = -e^{\sin x}\cos x$ hence $f'(0) = -1$

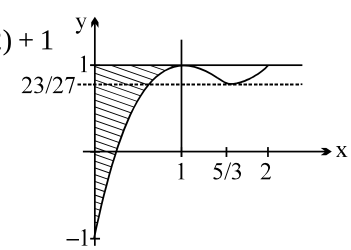
$f''(x) = -[\cos^2 x\, e^{\sin x} - e^{\sin x} \cdot \sin x]$

$f''\left(\frac{\pi}{2}\right) = e$ hence **(A)**, **(B)**, **(C)** are correct]

37 [Hint: The graph of $y = f(x) = (x-1)^2(x-2) + 1$

$f(1) = f(2) = 1$ and $f(0) = -1$

verify alternatives]



38 [Sol. (A) $32x + 2y\frac{dy}{dx} = 0$ $\Rightarrow$ $\frac{dy}{dx} = m_1 = -\frac{16x}{y}$

and $16y^{15}\frac{dy}{dx} = k$ $\Rightarrow$ $\frac{dy}{dx} = m_2 = \frac{k}{16y^{15}}$

$$m_1m_2 = -\frac{16x}{y}\cdot\frac{k}{16y^{15}} = -\frac{x}{y^{16}}\cdot k = -\frac{x}{y^{16}}\cdot\frac{y^{16}}{x} = -1 \quad \Rightarrow \quad \textbf{(A)} \text{ is correct}$$

(B) $\frac{dy}{dx} = 1 - ce^{-x} = 1 - (y - x) = -(y - x - 1)$ [using $ce^{-x} = y - x$]

and $\frac{dy}{dx} - k\cdot\frac{dy}{dx}\, e^{-y} = 1$

$\frac{dy}{dx}[1 - ke^{-y}] = 1$ or $[1 - (x + 2 - y)]\frac{dy}{dx} = 1$ [using $ke^{-y} = x - y + 2$]

$\frac{dy}{dx} = m_2 = \frac{1}{y - x - 1} \Rightarrow m_1 m_2 = -1 \Rightarrow$ **(B)** is correct

(C) $\frac{dy}{dx} = 2cx = 2x\cdot\frac{y}{x^2} = \frac{2y}{x} = m_1$

Also $2x + 4y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{2y} = m_2$

hence $m_1m_2 = -1 \Rightarrow$ **(C)** is correct

(D) $x^2 - y^2 = c$

$2x - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x}{y} = m_1$

$xy = k$

$x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} = m_2$

$\therefore$ $m_1m_2 = -1 \Rightarrow$ **(D)** is correct

$\Rightarrow$ (A), (B), (C), (D) all are correct]

39 [Sol. $\frac{dy}{dx} + y = f(x)$

I.F. $= e^x$

$ye^x = \int e^x f(x)\,dx + C$

now if $0 \le x \le 2$ then $ye^x = \int e^x e^{-x}\,dx + C \Rightarrow ye^x = x + C$

$x = 0$, $y(0) = 1$, $C = 1$

$\therefore$ $ye^x = x + 1$ ....(1)

$y = \frac{x+1}{e^x}$; $y(1) = \frac{2}{e}$ **Ans.** $\Rightarrow$ **(A)** is correct

$$y' = \frac{e^x - (x+1)e^x}{e^{2x}}$$

$y'(1) = \frac{e - 2e}{e^2} = \frac{-e}{e^2} = -\frac{1}{e}$ **Ans.** $\Rightarrow$ **(B)** is correct

if $x > 2$

$ye^x = \int e^{x-2}\,dx$

$ye^x = e^{x-2} + C$

$y = e^{-2} + Ce^{-x}$

as y is continuous

$\therefore \quad \lim_{x\to 2} \frac{x+1}{e^x} = \lim_{x\to 2} \left(e^{-2} + Ce^{-x}\right)$

$3e^{-2} = e^{-2} + Ce^{-2} \quad \Rightarrow \quad C = 2$

$\therefore$ for $x > 2$

$y = e^{-2} + 2e^{-x}$ hence $y(3) = 2e^{-3} + e^{-2} = e^{-2}(2e^{-1} + 1)$

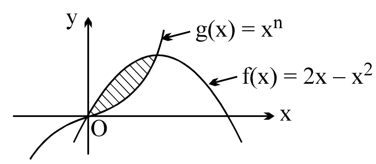
$y' = -2e^{-x}$

$y'(3) = -2e^{-3}$ **Ans.** $\Rightarrow$ **(D)** is correct]

40 [Sol. Solving $f(x) = 2x - x^2$ and $g(x) = x^n$

we have $2x - x^2 = x^n \quad \Rightarrow \quad x = 0$ and $x = 1$

$$A = \int_0^1 (2x - x^2 - x^n)\,dx = \left. x^2 - \frac{x^3}{3} - \frac{x^{n+1}}{n+1} \right]_0^1$$



$$= 1 - \frac{1}{3} - \frac{1}{n+1} = \frac{2}{3} - \frac{1}{n+1}$$

hence, $\frac{2}{3} - \frac{1}{n+1} = \frac{1}{2} \Rightarrow \quad \frac{2}{3} - \frac{1}{2} = \frac{1}{n+1}$

$\Rightarrow \quad \frac{4-3}{6} = \frac{1}{n+1} \quad \Rightarrow \quad n + 1 = 6 \Rightarrow n = 5$

Hence n is a divisor of 15, 20, 30 $\Rightarrow$ **B, C, D**]

EXERCISE 2(A)

1. Sol

(i) $x\frac{d^3y}{dx^3}=\frac{dy}{dx}+2$

Order $=3$, Degree $=1$

(ii) $x\frac{dy}{dx}+\frac{3}{\left(\frac{dy}{dx}\right)}=y^2$

$\therefore \quad x\left(\frac{dy}{dx}\right)^2+3=y^2\frac{dy}{dx}$

Order $=1$, Degree $=2$

(iii) $y=x\frac{dy}{dx}+a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Order 1 , Degree 2

(iv) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=5\frac{d^2y}{dx^2}$

Order 2 , Degree 2

2. Sol

(i) $y=kx+k^2+k^3$

$\therefore \quad \frac{dy}{dx}=k$

$\therefore \quad y=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2+\left(\frac{dy}{dx}\right)^3$

(ii) $y=-\lambda\sin x$

$\therefore \quad \frac{dy}{dx}-\lambda\cos x$

$\therefore \quad \lambda=-\sec x\frac{dy}{dx}$

$\therefore \quad y=\tan x\frac{dy}{dx}$

(iii) $y=ax+bx^2$

$\therefore \quad \frac{dy}{dx}=a+2bx$

$\therefore \quad \frac{d^2y}{dx^2}=2b$

$\therefore \quad b=\frac{1}{2}\frac{d^2y}{dx^2}$

$\therefore \quad a = x\frac{dy}{dx} - 2\times\frac{x}{2}\frac{d^2y}{dx^2} = \frac{dy}{dx} - x\frac{d^2y}{dx^2}$

$\therefore \quad y = x\frac{dy}{dx} - x^2\frac{d^2y}{dx^2} + \frac{x^2}{2}\frac{d^2y}{dx^2}$

$\therefore \quad y = x\frac{dy}{dx} - \frac{x^2}{2}\frac{d^2y}{dx^2}$

3. Sol

(i) Let the center of the circle be (a,b) and it's radius r.

$\because$ it lies entirely in 1st quadrant ,

$a > 0, b > 0, r > 0$ and $r < a, b$

$\because$ x-axis touches the circle the distance of (a,b) from the x-axis musst be r

i.e. $b = r$, $|||^{ly}$ $a = r$

Hence the equation of circle is,

$(x-r)^2 + (y-r)^2 = r^2$

i.e $x^2 + y^2 - 2r(x+y) + r^2 = 0$

$\Leftrightarrow (x+y)^2 - 2r(x+y) + r^2 = 2xy$

$\Leftrightarrow (x+y-r)^2 = 2xy$ .....(1)

differentiating w.r.t.x ,

$2(x+y-r)(1+y_1) = 2(y+xy_1)$ .....(2)

$\left(y_1 = \frac{dy}{dx}\right)$

Squaring (2) and dividing by (1) ,

$(1+y_1)^2 = \frac{(y+xy_1)^2}{2\times y}$

$2xy\left(1+\frac{dy}{dx}\right)^2 = \left(y + x\frac{dy}{dx}\right)^2$

(ii) $y = mx + c$ ...(General equation of line in x-y plane) (m,c are constants)

(Non verical lines $\Rightarrow$ inclination is not $\frac{\pi}{2}$)

$\Rightarrow y_1 = m$

Putting back in the given equation, i.e. $y = xy_1 + c$

differentiating w.r.t.x,

$\Rightarrow y_1 = y_1 + xy_{11}$ $\left(\begin{matrix} y_1 = \frac{dy}{dx} \\ y_{11} = \frac{d^2y}{dx^2} \end{matrix}\right)$

i.e. $\boxed{y_{11} = 0}$ Wich is the required differential equation.

4. Sol

(i) $\left(1-x^2\right)dy + xy\,dx = xy^2\,dx$

$\therefore \quad \dfrac{dy}{y^2-y} = \dfrac{x\,dx}{1-x^2}$

$\therefore \quad \dfrac{dy}{\left(y-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2} = -\dfrac{1}{2}\left(\dfrac{-2x\,dx}{1-x^2}\right)$

$\therefore \quad \ell n\left|\dfrac{y-1}{y}\right| = -\dfrac{1}{2}\ell n\left|1-x^2\right| + C$

$\therefore \quad \ell n\left|\dfrac{y-1}{y}\right| + \dfrac{1}{2}\ell n\left|1-x^2\right| = C$

(ii) $\dfrac{dy}{dx} = e^x e^y$

$\therefore \quad e^{-y}dy = e^x dx$

$\therefore \quad e^x + e^y = C$

(iii) $\sqrt{1+x^2+y^2+x^2y^2} + xy\dfrac{dy}{dx} = 0$

$\therefore \quad \sqrt{1+x^2}\sqrt{1+y^2} + xy\dfrac{dy}{dx} = 0$

$\therefore \quad \dfrac{\sqrt{1+x^2}}{x}dx + \dfrac{y}{\sqrt{1+y^2}}dy = 0$

$\dfrac{\sec^3\theta}{\tan\theta}d\theta + \dfrac{tdt}{t} = 0$

$\therefore \quad \int \sec^2\theta \operatorname{cosec}\theta + t = C$

$\therefore \quad \int \tan\theta \operatorname{cosec}\theta + \int \operatorname{cosec}\theta\, d\theta + t = C$

$\therefore \quad \tan\theta \operatorname{cosec}\theta + \ell n\left|\operatorname{cosec}\theta - \theta\right| + t = C$

$\therefore \quad \sqrt{1+x^2} + \ell n\left|\dfrac{\sqrt{1+x^2}-1}{x}\right| + \sqrt{1+y^2} = C$

(iv) $\sqrt{1-x^6}dy = x^2dx$

$\therefore \quad dy = \dfrac{x^2}{\sqrt{1-x^6}}dx$

$x^3 = t$

$\therefore \quad 3x^2dx = dt$

$\therefore \quad dy = \frac{1}{3}\frac{dt}{\sqrt{1-t^2}}$

$\therefore \quad y = \frac{1}{3}\sin^{-1} t + C \qquad \therefore \quad y = \frac{1}{3}\sin^{-1}\left(x^3\right) + C$

(v) $\frac{dy}{dx} - x\sin^2 x = \frac{1}{x\log x}$

$\therefore \quad \frac{dy}{dx} = x\sin x^2 x + \frac{1}{x\log x}$

$\therefore \quad dy = \left(x\sin^2 x + \frac{1}{x\log x}\right)dx$

$\therefore \quad dy = \left[\frac{x}{2}(1-\cos 2x)dx + \frac{1/x\,dx}{\log x}\right]$

$\therefore \quad y = \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{2x}{8} + \ell n(\ell n) + C$

(vi) $ydx - xdy = xy\,dx$

$\therefore \quad xdy = y(1-x)dx$

$\therefore \quad \frac{dy}{y} = \left(\frac{1}{x} - 1\right)dx$

$\therefore \quad \ell ny = \ell nx - x + C$

$\therefore \quad \ell n\left(\frac{y}{x}\right) + x = C$

(vii) $\left(e^x + e^{-x}\right)\frac{dy}{dx} = e^x - e^{-x}$

$\therefore \quad dy = \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

$\therefore \quad y = \ell n\left(e^x + e^{-x}\right) + C$

(viii) $\frac{dy}{dx} = \sin^3 x\cos^2 x + xe^x$

$\therefore \quad dy = \left(\sin^3 x\cos^2 x + xe^x\right)dx$

$= \sin^3 x\cos^2 xdx + xe^x dx$

$\cos x = t$

$\therefore \quad -\sin xdx = dt$

$\therefore \quad dy = -\left(1-t^2\right)t^2 dt + xe^x dx$

$\therefore \quad dy = \left(t^4 - t^2\right)dt + xe^x dx$

$\therefore \quad y = \frac{t^5}{5} - \frac{t^3}{3} + xe^x - e^x + C$

$\therefore \quad y = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + xe^x - e^x + C$

(ix) $\frac{dy}{dx} = (4x + y + 1)^2$

$\therefore \quad \frac{dy}{dx} = 16x^2 + y^2 + 1 + 8xy + 2y + 8x$

$\therefore \quad dy\left(16x^2 + y^2 + 1 + 8xy + 2y + 8x\right)dx$

(x) $\frac{dy}{dx} = \sin(x + y) = \sin x \cos y + \cos x \sin y$

$x + y = v$

$\therefore \quad 1 + \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \quad \frac{dv}{dx} = \sin v + 1$

$\therefore \quad \frac{dv}{\sin v + 1} = dx$

$\therefore \quad \frac{(1 - \sin v)dv}{\cos^2 v} = dx$

$\therefore \quad \sec^2 v - \sec v \tan v \, dv = dx$

$\therefore \quad \tan v - \sec v = x + C$

$\therefore \quad \tan(x + y) - \sec(x + y) = x + C$

(x) $\frac{dy}{dx} = (4x + y + 1)^2$

$4x + y = v$

$\therefore \quad 4 + \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \quad \frac{dv}{dx} - 4 = (v + 1)^2$

$\therefore \quad \frac{dv}{(v + 1)^2 + 4} = dx$

$\therefore \quad \frac{1}{4} \ell n \left| \frac{v - 1}{v + 3} \right| = x + C$

$\therefore \quad \frac{1}{4} \ell n \left| \frac{4x + y - 1}{4x + y + 3} \right| = x + C$

5.

Sol

(i) $x \frac{dy}{dx} - y = \sqrt{x^2 + y^2}$

$y = vx$

$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad x\left(v + x\frac{dv}{dx}\right) - vx = x\sqrt{1+v^2}$

$\therefore \quad \frac{dx}{x} = \frac{dv}{\sqrt{1+v^2}}$

$\therefore \quad \ell nx = \ell n\left|v + \sqrt{1+v^2}\right| + C$

$\therefore \quad x = e^c\left(\frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}\right)$

$\therefore \quad x^2 = e^c\left(y + \sqrt{x^2+y^2}\right)$

(ii) $x^2\frac{dy}{dx} = x^2 + xy + y^2$

$y = vx$

$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad x^2\left(v + x\frac{dv}{dx}\right) = x^2 + x^2v + x^2v^2$

$\therefore \quad \frac{dx}{x} = \frac{dv}{1+v^2}$

$\therefore \quad \ell nx = \tan^{-1} v + C$

$\therefore \quad \ell nx = \tan^{-1}\frac{y}{x} + C$

(iii) $x\frac{dy}{dx} + \frac{y^2}{x} = y$

$y = vx$

$\therefore \quad x\left(v + x\frac{dv}{dx}\right) + xv^2 = xv$

$\therefore \quad -\frac{dv}{v^2} = \frac{dx}{x}$

$\therefore \quad \frac{1}{v} = \ell nx + C$

$\therefore \quad \frac{x}{y} = \ell nx + C$

(iv) $2xy\frac{dy}{dx} = x^2 + 3y^2$

$\therefore \quad 2x^2v\left(v + x\frac{dv}{dx}\right) = x^2 + 3x^2v^2$

$\therefore \quad 2v^2 2vx\frac{dv}{dx} = 1 + 3v^2$

$$\therefore \quad 2vx\frac{dv}{dx} = 1 + v^2$$

$$\therefore \quad \frac{dx}{x} = \frac{2v\,dv}{1+v^2}$$

$$\therefore \quad \ell nx = \ell n\left(1+v^2\right) + C$$

$$\therefore \quad \ell nx = \ell n\left(\frac{x^2+y^2}{x^2}\right) + C$$

$$\therefore \quad \ell nx + 2\ell nx - \ell n\left(x^2+y^2\right) = C$$

$$\therefore \quad x^3 = e^c\left(x^2+y^2\right)$$

(v) $x^2\frac{dy}{dx} = \frac{xy}{2} + \frac{y^2}{2}$

$$\therefore \quad x^2\left(v + x\frac{dv}{dx}\right) = \frac{x^2v}{2} + \frac{x^2v^2}{2}$$

$$\therefore \quad x\frac{dv}{dx} = \frac{v^2 - v}{2}$$

$$\therefore \quad \frac{2dv}{v^2 - v + \frac{1}{4} - \frac{1}{4}} = \frac{dx}{x}$$

$$\therefore \quad \frac{2dv}{\left(v - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} = \frac{dx}{x}$$

$$\therefore \quad 2\ell n\left|\frac{v-x}{v}\right| = \ell nx + C$$

$$\therefore \quad 2\ell n\left|\frac{y-x}{y}\right| = \ell nx + C$$

$$\therefore \quad \left(\frac{y-x}{y}\right)^2 \times \frac{1}{x} = e^c$$

$$\therefore \quad \left(y-x\right)^2 = xy^2e^c$$

6. Sol

(i) $3x - 7y + 7dx + \left(7y - 3x + 3\right)dy = 0$

$$\therefore \quad \frac{dy}{dx} = \frac{3x-7y+7}{3x-7y-3}$$

$u = 3x - 7y$

$\therefore \quad \frac{dy}{dx} = 3 - \frac{7dy}{dx}$

$\therefore \quad \frac{dy}{dx} = \frac{1}{7}\left(3 - \frac{dy}{dx}\right)$

$\therefore \quad \frac{3}{7} - \frac{1}{7}\frac{dy}{dx} = \frac{y+7}{y-3}$

$\therefore \quad \frac{1}{7}\frac{dy}{dx} = \frac{3}{7} - \frac{y+7}{y-3}$

$= \frac{3y-9-7y-49}{7(y-3)}$

$\therefore \quad \frac{dy}{dx} = \frac{-4y-58}{y-3}$

$\therefore \quad \frac{(y-3)dy}{(4y+58)} = dx$

$\therefore \quad -\left(\frac{1}{4} + \frac{35}{2}\right)\frac{dy}{(4u+58)} = dx$

$\therefore \quad -\frac{y}{4} + \frac{35}{8}\ell n\left|6x - 14y + 26\right| + x = C$

(ii) $\frac{y-x+1}{y+x-5} = \frac{dy}{dx}$

$u = -x + y$

$\therefore \quad \frac{du}{dx} = \frac{dy}{dx} - 1$

$\therefore \quad \frac{dy}{dx} = 1 + \frac{du}{dx}$

$\therefore \quad 1 + \frac{du}{dx} = \frac{u+1}{u+2x-5}$

$\therefore \quad \frac{du}{dx} = \frac{u+1-(u+2x-5)}{u+2x-5} = \frac{-2x+6}{u+2x-5}$

$u = x + y$

$\therefore \quad \frac{du}{dx} = 1 + \frac{dy}{dx}$

$\therefore \quad \frac{du}{dx} - 1 = \frac{u-2x+1}{u-5}$

$\therefore \quad \frac{du}{dx} = \frac{u-2x-1}{u-5} + 1$

$= \frac{2u-2x-10}{u-5}$

$$= 2 - \frac{2x}{u-5}$$

$$\therefore \quad \frac{du}{dx} - 2 = \frac{-2x}{u-5}$$

$$\frac{dy}{dx} = \frac{y-x+1}{y+x-5}$$

$x = u + h$, $y = v + k$

$dy = dv, dx = du$

$$\therefore \quad \frac{dv}{du} = \frac{v+k-u-h+1}{v+u+k+h-5} = \frac{v-u}{v+u}$$

here, $k - h + 1 = 0$

$k + h - 5 = 0$

$\therefore \quad k = 2$ & $h = 3$

$v = uz$

$$\therefore \quad \frac{dv}{du} = z + u\frac{dz}{du}$$

$$\therefore \quad z + u\frac{dz}{du} = \frac{z-1}{z+1}$$

$$\therefore \quad u\frac{dz}{du} = \frac{z-1}{z+1} - z = \frac{z-1-z^2-z}{z+1}$$

$$\therefore \quad -\frac{du}{u} = \frac{(z+1)dz}{z^2+1}$$

$$\therefore \quad \ell n\, u = \frac{1}{2}\ell n\left|z^2+1\right| + \tan^{-1} z + C$$

$$\therefore \quad -\ell nu = \frac{1}{2}\ell n\left(\frac{v^2+u^2}{u^2}\right) + \tan^{-1} z + C$$

$$\therefore \quad -\ell nu = \frac{1}{2}\ell n\left(v^2+u^2\right) - \ell nu + \tan^{-1}\frac{v}{u} + C$$

$$\therefore \quad \frac{1}{2}\ell n\left((y-2)^2+(x-3)^2\right) + \tan^{-1}\left(\frac{y-2}{x-3}\right) = C$$

(iii) $$\frac{dy}{dx} = \frac{x+2y-3}{2x+y+3}$$

$x = u + h, y = v + k$

$$\therefore \quad \frac{dv}{dx} = \frac{u+2v+h+2k-3}{2u+v+2h+k+3} = \frac{u+2v}{2u+v}$$

$\therefore \quad h + 2k = 3$

$\therefore \quad h = -3$ & $k = 3$

$2h + k = -3$

$\therefore \quad h + k = 0$

$\therefore \quad h = -k$

$v = uz$

$\therefore \quad \left(z + u\dfrac{dz}{du}\right) = \dfrac{1+2z}{2+z}$

$\therefore \quad -\dfrac{du}{u} = \dfrac{(2+z)dz}{z^2-1}$

$\therefore \quad -\dfrac{du}{u} = +\dfrac{2dz}{z^2-1} + \dfrac{z\,dz}{z^2-1}$

$\therefore \quad -\ell nu + C = \ell n\left(\dfrac{v-u}{v+u}\right) + \dfrac{1}{2}\ell n\left(\dfrac{v^2-u^2}{u^2}\right)$

$\therefore \quad \ell n\left(\dfrac{y-x-6}{y+x}\right) + \dfrac{1}{2}\ell n\left((y-3)^2 + (x+3)^2\right) = C$

(iv) $(6x+3y+4)dy = (2x+y-1)dx$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{2x+y-1}{6x+3y+4}$

$2x + y = u$

$2 + \dfrac{dy}{dx} = \dfrac{du}{dx}$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{du}{dx} - 2$

$\therefore \quad \dfrac{du}{dx} - 2 = \dfrac{u-1}{3u+4}$

$\therefore \quad \dfrac{du}{dx} = \dfrac{u-1+6u+8}{3u+4} = \dfrac{7u+7}{3u+4}$

$\therefore \quad du\left(\dfrac{3u+4}{u+1}\right) = dx$

$\therefore \quad \dfrac{1}{7}\left[3 + \dfrac{du}{u+1}\right] = dx$

$\therefore \quad \dfrac{3}{7}u + \dfrac{1}{7}\ell n(u+1) = x + C$

$\therefore \quad \dfrac{3}{7}(2x+y) + \dfrac{1}{7}\ell n(2x+y+1) = x + C$

$\therefore \quad 6x + 3y + \ell n(2x+y+1) = 7x + C$

$\therefore \quad 3y - x + \ell n(2x+y+1) = C$

(v) $\dfrac{dy}{dx} = \dfrac{x+2y+1}{2x+4y+3}$

$u = x + 2y$

$\therefore \quad \dfrac{du}{dx} = 1 + 2\dfrac{dy}{dx}$

$\therefore \quad \frac{dy}{dx}=\frac{1}{2}\frac{du}{dx}-\frac{1}{2}$

$\therefore \quad \frac{1}{2}\frac{du}{dx}-\frac{1}{2}=\frac{u+1}{2u+3}$

$\therefore \quad \frac{dy}{dx}-1=\frac{2u+2}{2u+3}$

$\therefore \quad \frac{du}{dx}=\frac{4u+5u}{2u+3}$

$\therefore \quad dx=\left(\frac{2u+3}{4u+5}\right)du$

$\therefore \quad dx=\left(\frac{1}{2}+\frac{1/2}{4u+5}\right)du$

$\therefore \quad x+C=\frac{u}{2}+\frac{1}{8}\ell n(4u+5)$

$\therefore \quad x+C=\frac{x+2u}{2}+\frac{1}{8}\ell n(4u+5)$

$\therefore \quad 8x+C=4x+8y+\ell n(4x+8y+5)$

$\therefore \quad 8y-4x+\ell n(4x+8y+5)=C$

7. (i) $\left(e^{y}+1\right)\cos x dx+e^{y}\sin x dy=0$

$\therefore \quad \frac{d}{dx}\left(e^{y}\sin x\right)$

$=e^{y}\sin x\frac{dy}{dx}+e^{y}\cos x$

$\therefore \quad d\left(e^{y}\sin x\right)=e^{y}\sin x dy+e^{y}\cos x\,dx$

$\therefore \quad d\left(e^{y}\sin x\right)+\cos x\,dx=0$

$\therefore \quad e^{y}\sin x+\sin x=C$

$\therefore \quad \sin x\left(e^{y}+1\right)=C$

(ii) $x^{2}dx-2xydx-y^{2}dx-x^{2}dy-2xydy-y^{2}dy=0$

$\therefore \quad x^{2}dx-y^{2}dx-x^{2}dy=2xy(dx+dy)$

$\therefore \quad x^{2}dx=2xydx+x^{2}dy+2xydy+y^{2}dx$

$\therefore \quad \frac{x^{3}}{3}=x^{2}y+xy^{2}+\frac{y^{3}}{3}+C$

$\therefore \quad x^{3}=3x^{2}y+3xy^{2}+y^{3}+C$

(iii) $\left(x^{2}+y^{2}-a^{2}\right)xdx+\left(x^{2}-y^{2}-b^{2}\right)ydy=0$

$\therefore \quad \left(x^{2}+y^{2}\right)xdx+\left(x^{2}-y^{2}\right)ydy=a^{2}xdx+b^{2}ydy$

$y=vx$

$\therefore \quad dy = vdx + xdv$

$\therefore \quad \left(x^2 + v^2x^2\right)xdx + \left(x^2 - v^2x^2\right)vx\left(vdx + xdv\right) = a^2xdx + b^2vx\left(vdx + xdv\right)$

$\therefore \quad x^2\left(1+v^2\right)dx + x^2\left(1-v^2\right)v^2dx + x^3\left(1-v^2\right)vdv$

$= a^2dx + b^2v^2dx + b^2xvdv$

$x^2\left(1+2v^2-v^4\right)dx - \left(a^2 - b^2v^2\right)dx = b^2xvdv - x^3\left(1-v^2\right)vdv$

$\therefore \quad x^3dx + xy^2dx + x^2ydy - y^3dy = a^2xdx + b^2ydy$

$\therefore \quad x^3dx - y^3dy + \frac{1}{2}\left(2xy^2dx + 2x^2ydy\right) = a^2xdx + b^2ydy$

$\therefore \quad x^3dx - y^3dy + \frac{1}{2}d\left(x^2y^2\right) = a^2xdx + b^2ydy$

$\therefore \quad \frac{x^4}{4} - \frac{y^4}{4} + \frac{1}{2}x^2y^2 = \frac{a^2x^2}{2} + \frac{b^2y^2}{2} + C$

(iv) $\left(y^2e^x + 2xy\right)dx - x^2dy = 0$

$\therefore \quad y^2e^xdx + 2xydx - x^2dy = 0$

$\therefore \quad e^xdx + \frac{2xydx - x^2dy}{y^2} = 0$

$\therefore \quad e^xdx + d\left(\frac{x^2}{y}\right) = 0$

$\therefore \quad e^x + \frac{x^2}{y} = C$

(v) $y\left(2x^2y + e^x\right)dx - \left(e^x + y^3\right)dy = 0$

$\therefore \quad 2x^2y^2dx + e^x\left(ydx - dy\right) - y^3dy = 0$

$\therefore \quad 2x^2dx + \frac{e^xydx - e^xdy}{y^2} = ydy$

$\therefore \quad \frac{2x^3}{3} + \frac{e^x}{y} = \frac{y^2}{2} + C$

$\therefore \quad 4x^3y + 2e^x = 3y^2 + Cy$

C is a constant.

8. Sol

(i) $x\left(x-1\right)\frac{dy}{dx} - \left(x-2\right)y = x^3\left(2x-1\right)$

$\therefore \quad \frac{dy}{dx} - \frac{\left(x-2\right)}{x\left(x-1\right)}y = \frac{x^2\left(2x-1\right)}{x-1}$

$\ell n\, IF = \int \frac{x-2}{x\left(x-1\right)}dx = \int \left(\frac{x-2}{x^2-x}\right)dx$

$$=\frac{1}{2}\int\frac{2x-1}{x^2-x}dx-\frac{3}{2}\int\frac{dx}{x^2-x+\frac{1}{4}-\frac{1}{4}}$$

$$=\frac{1}{2}\ell n\left(x^2-x\right)-\frac{3}{2}\ell n\left(\frac{x-1}{x}\right)$$

$$=\frac{1}{2}\ell n\left(x\left(x-1\right)\right)-\frac{1}{2}\ell n\left(\frac{x-1}{x}\right)^3$$

$$=\frac{1}{2}\ell n\left(\frac{x\times x^3\left(x-1\right)}{\left(x-1\right)^2}\right)$$

$$=\ell n\left(\frac{x^2}{x-1}\right)$$

$\therefore\quad IF=\frac{x^2}{x-1}$

$\therefore\quad y\frac{x^2}{x-1}=\int\frac{x^2\left(2x-1\right)}{\left(x-1\right)}\times\frac{x^2}{\left(x-1\right)}dx$

$$=\int\frac{2x^5-x^4}{x^2-2x+1}dx=\int\left(2x^3+3x^2+4x+5\right)dx+\int\frac{\left(6x-5\right)}{\left(x-1\right)}$$

$\therefore\quad \frac{yx^2}{x-1}=\frac{x^4}{2}+x^3+2x^2+5x+3\int\frac{2\left(x-1\right)}{\left(x-1\right)^2}+\int\frac{\left(6x-5\right)}{\left(x-1\right)}$

$\therefore\quad \frac{yx^2}{x-1}=\frac{x^4}{2}+x^3+2x^2+5x+6\ell n\left(x-1\right)-\frac{1}{x-1}+C$

(ii) $x\left(x^2+1\right)\frac{dy}{dx}+y\left(x^2-1\right)=x^3\ \ell nx$

$\therefore\quad \frac{dy}{dx}+y\frac{\left(x^2-1\right)}{x\left(x^2+1\right)}=\frac{x^2\ell nx}{x^2+1}$

$\ell n\ IF=\int\frac{x^2-1}{x\left(x^2+1\right)}dx$

$$=\int\frac{x^2-1}{x^3+x}dx$$

$$=\frac{1}{3}\int\frac{3x^2+1}{x^3+x}dx-\frac{4}{3}\int\frac{dx}{x\left(x^2+1\right)}$$

$$=\frac{1}{3}\ell n\left(x^3+x\right)-\frac{4}{3}\int\left(\frac{A}{x}+\frac{Bx+C}{x^2+1}\right)dx$$

$A+B=0$, $A=1$, $C=0$, $B=-1$

$$= \frac{1}{3}\ell n\left(x^3 + x\right) - \frac{4}{3}\int \frac{dx}{x} - \frac{xdx}{x^2+1}$$

$$= \frac{1}{3}\ell n\left(x^3 + x\right) - \frac{4}{3}\ell nx + \frac{2}{3}\ell n\left(x^2+1\right)$$

$$= \frac{1}{3}\ell nx + \ell n\left(x^2+1\right) - \frac{4}{3}\ell nx = \ell n\left(\frac{x^2+1}{x}\right)$$

$$\therefore \quad y\frac{\left(x^2+1\right)}{x} = \int \frac{x\ell nx \times \left(x^2+1\right)}{x\left(x^2+1\right)} = \int x\ell nx$$

$$= \frac{x^2}{2}\ell nx - \frac{x^2}{4} + C$$

$$\therefore \quad y\frac{\left(x^2+1\right)}{x} = \frac{x^2}{2}\ell nx - \frac{x^2}{4} + C$$

(iii) $\left(x + 2y^3\right)\frac{dy}{dx} = y$

$$\therefore \quad y\frac{dx}{dy} = x + 2y^3$$

$$\therefore \quad \frac{dx}{dy} - \frac{x}{y} = 2y^2$$

$$IF = e^{\int -\frac{1}{y}dy} = e^{-\ell ny} = \frac{1}{y}$$

$$\therefore \quad \frac{x}{y} = y^2 + C$$

$$\therefore \quad x = y^2 + Cy$$

(iv) $\left(y\log x - 1\right)ydx = xdy$

$x = e^t$

$\therefore \quad dx = e^t dt$

$\therefore \quad \left(y^t - 1\right)ye^t dt = e^t dy$

$$\therefore \quad \frac{dy}{dt} = y^2 t - y$$

$$\therefore \quad \frac{dy}{dt} + y = ty^2 \qquad\qquad \frac{1}{y} = z$$

$$\therefore \quad \frac{1}{y^2}\frac{dy}{dt} + \frac{1}{y} = t \qquad\qquad \therefore \quad -\frac{1}{y^2}\frac{dy}{dt} = \frac{dz}{dt}$$

$$\therefore \quad -\frac{dz}{dt} - z = t$$

$$\therefore \quad IF = e^{\int -dt} = e^{-t}$$

$$\therefore \quad ze^{-t} = -te^{-t} - e^{-t} + C$$

$\therefore \quad \frac{e^{-t}}{y} + te^{-t} + e^{-t} = C$

$\therefore \quad \frac{1}{xy} + \frac{\ell nx}{x} + \frac{1}{x} = C$

$\therefore \quad 1 + y\ell nx + y = cxy$

(v) $ydx - xdy + 3x^2y^2e^{x^3}dx = 0$

$\therefore \quad \frac{ydx - xdy}{y^2} + 3x^2e^{x^3}dx = 0$

$\therefore \quad d\left(\frac{x}{y}\right) + d\left(e^{x^3}\right) = 0$

$\therefore \quad \frac{x}{y} + e^{x^3} = C$

$\therefore \quad x + ye^{x^3} = Cy$

9. Sol

(i) $\frac{dy}{dx} = \frac{2x(\log x + 1)}{\sin y + y\cos y}$ $\quad [x = 1, y = 0]$

$\therefore \quad (\sin y + y\cos y)dy = (2x\ell nx + 2x)dx$

$\therefore \quad -\cos y + y\sin y + \cos y = x^2 + x^2\ell nx - \frac{x^2}{2} + C$

$\therefore \quad 0 = 1 - \frac{1}{2} + C$

$\therefore \quad C = -\frac{1}{2}$

$\therefore \quad y\sin y = \frac{x^2}{2} + x^2\ell nx - \frac{1}{2}$

$\therefore \quad 2y\sin y = x^2 + 2x^2\ell nx - 1$

(ii) $2x\frac{dy}{dx} = 3y$

$\therefore \quad 2\frac{dy}{y} = 3\frac{dx}{x}$ $\quad y(1) = 4$

$\therefore \quad 2\ell ny = 3\ell nx + C$

$\therefore \quad 2\ell n4 = C$

$\therefore \quad 2\ell ny - 2\ell n4 = 3\ell nx$

$\therefore \quad \left(\frac{y}{4}\right)^2 = x^3$

$\therefore \quad y^2 = 16x^3$

(iii) $\frac{dy}{dx} + 2y\tan x = \sin x$

$\therefore \quad IF = e^{\int 2\tan x dx} = e^{2\ell n|\sec x|} = \sec^2 x$

$\therefore \quad y\sec^2 x = \int \sec x \tan x dx$

$\therefore \quad y\sec^2 x = \sec + C \qquad\qquad y\left(\frac{\pi}{3}\right) = 0$

$\therefore \quad 0 = 2 + C$

$\therefore \quad C = -2$

$\therefore \quad y\sec^2 x = \sec x - 2$

$\therefore \quad y = \cos x - 2\cos^2 x$

(iv) $\frac{dy}{dx} = \frac{1-2x+3x^2}{1+2y-3y^2} \qquad\qquad y(1) = 2$

$\therefore \quad y + y^2 - y^3 = x - x^2 + x^3 + C$

$\therefore \quad 2 + 4 - 8 = 1 - 1 + 1 + C$

$\therefore \quad -2 = 1 + C$

$\therefore \quad C = -3$

$\therefore \quad x^3 + y^3 - x^2 - y^2 + x - y - 3 = 0$

(v) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 6x \qquad\qquad t = \frac{dy}{dx}$

$x\frac{dt}{dx} + 2t = 6x$

$\therefore \quad \frac{dt}{dx} + \frac{2t}{x} = 6 \qquad\qquad IF = e^{\int \frac{2}{x}dx} = x^2$

$\therefore \quad tx^2 = 2x^3 + C_1$

$\therefore \quad x^2\frac{dy}{dx} = 2x^3 + C_1$

$\therefore \quad dy\left(2x + \frac{C_1}{x^2}\right)dx$

$\therefore \quad y = \frac{3x^2}{2} - \frac{C_1}{x} + C_2$

$\therefore \quad y = x^2 - \frac{C_1}{x} + C_2$

10. Sol

(i) $\left(\frac{d^3y}{dx^3}\right)^{2/3} = \frac{dy}{dx} + C$

$\therefore \quad \left(\frac{d^3y}{dx^3}\right)^2 = \left(\frac{dy}{dx} + C\right)^3$

$\therefore$ Order 3 degree 2

(ii) $\frac{d^2y}{dx^2} = x\ell n\frac{dy}{dx}$

Order 2 degree undefined

(iii) $\sqrt{\frac{d^2y}{dx^2}} = \sqrt[3]{\frac{dy}{dx}+3}$ $\therefore$ $\left(\frac{d^2y}{dx^2}\right)^3 = \left(\frac{dy}{dx}+3\right)^2$ $\therefore$ Order 2 Degree 3

(iv) $\frac{d^2y}{dx^2} = \sin\left(\frac{dy}{dx}\right)$

Order 2 Degree undefined

(v) $\frac{dy}{dx} = \sqrt{3x+5}$

Order 1 Degree 1

(vi) $y(c_1+c_2)e^x + c_3e^{x+c_4}$

$y = Ae^x + Be^x$ i.e. $y = ce^x$

$\therefore$ $\frac{dy}{dx} = y$

$\therefore$ Order 1 Degree 1

11. Sol

(i) $y = Ae^{2x} + Be^{-2x}$

$\therefore$ $\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$

$\therefore$ $\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y$

$\therefore$ $\frac{d^2y}{dx^2} = 4y$

(ii) $V = \frac{A}{r} + B$

$\therefore$ $\frac{dV}{dr} = -\frac{A}{r^2}$ $A = -r^2\frac{dV}{dr}$

$\therefore$ $\frac{d^2V}{dr^2} = \frac{2A}{r^3}$

$\therefore$ $\frac{d^2V}{dr^2} = \frac{2}{r_r^3} \times -r^2\frac{dV}{dr}$

$\therefore$ $A = \frac{r^3}{2}\frac{d^2V}{dr^2}$

$\therefore$ $\frac{dV}{dr} = -\frac{r^3}{2r^2}\frac{d^2V}{dr^2}$

$\therefore$ $\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = 0$

(iii) $Ax^2 + By^2 = 1$

$\therefore \quad 2Ax + 2By\frac{dy}{dx} = 0 \qquad\qquad A = -\frac{By}{x}\frac{dy}{dx}$

$\therefore \quad A + B\left(\frac{dy}{dx}\right)^2 + By\frac{d^2y}{dx^2} = 0$

$\therefore \quad \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = \frac{y}{x}\frac{dy}{dx}$

$\therefore \quad x\left\{\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right\} = y\frac{dy}{dx}$

(iv) $x^2 + (y-a)^2 = a^2$

$\therefore \quad x^2 + y^2 - 2ay = 0$

$\therefore \quad 2x + 2y\frac{dy}{dx} - 2x\frac{dy}{dx} = 0$

$\therefore \quad x\frac{dx}{dy} + y = a$

$\therefore \quad x^2 + y^2 - 2y\left(x\frac{dx}{dy} + y\right) = 0$

$\therefore \quad x^2 - y^2 - 2xy\frac{dx}{dy} = 0$

$\therefore \quad \left(x^2 - y^2\right)\frac{dy}{dx} = 2xy$

(v)

(a) $y = A\cos(x+3)$

$\frac{dy}{dx} = -A\sin(x+3) \qquad\qquad A = \frac{y}{\cos(x+3)}$

$\therefore \quad \frac{dy}{dx} = -y\tan(x+3)$

$\therefore \quad \frac{dy}{dx} + y\tan(x+3) = 0$

(b) $y = x\sin(x+A)$

$\therefore \quad \frac{dy}{dx} = \sin(x+A) + x\cos(x+A)$

$\therefore \quad x\frac{dy}{dx} = x\sin(x+A) + x^2\cos(x+A)$

$= y + x\sqrt{x^2 - y^2}$

(vi) $y = ax^2 + bx + c$

$\therefore \quad \frac{d^3y}{dx^3} = 0$

12. Sol **(i)** $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$\therefore \quad (x+a)\frac{dy}{dx} = y = ay^2$

$\therefore \quad \frac{dy}{y - ay^2} = \frac{dx}{x+a}$

$\therefore \quad \frac{1}{a}\frac{dy}{\left(\frac{1}{4a^2} - \frac{1}{4a^2} + \frac{y}{a} - y^2\right)} = \frac{dx}{x+a}$

$\therefore \quad \frac{1}{a}\frac{dy}{\left(\frac{1}{4a}\right)^2 - \left(y - \frac{1}{2a}\right)^2} = \frac{dx}{x+a} = \frac{1}{a} \times a\ \ell n\left|\frac{ay-1}{a-y}\right| = \ell n(x+a) + C$

$(x+a)\frac{dy}{dx} = y - ay^2$

$\therefore \quad \frac{dx}{x+a} + \frac{dy}{ay^2 - y} = 0$

$\therefore \quad \ell n(x+a) + \frac{1}{a}\int \frac{dy}{y^2 - \frac{y}{a} + \frac{1}{4a^2} - \frac{1}{4a^2}} = C$

$\therefore \quad \ell n(x+a) + \frac{1}{a} \times \frac{a}{a^2}\ell n\left|\frac{ay-1}{ay}\right| = C$

$\therefore \quad a^2\ell n(x+a) + \ell n\left(\frac{ay-1}{ay}\right) = C$

(ii) $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

$\therefore \quad e^y dy = \left(e^x + x^2\right)dx$

$\therefore \quad e^y = e^x + \frac{x^3}{3} + C$

(iii) $\sec^2 x \tan y dy + \sec^2 y \tan x dx = 0$

$\therefore \quad \cos^2 y \tan y dy + \cos^2 x \tan x dx = 0$

$\therefore \quad \sin 2y dy + \sin 2x dx = 0$

$\therefore \quad \cos 2x + \cos 2y = C$

(iv) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

$x + y = v$

$\therefore \quad \frac{dy}{dx} = \frac{dv}{dx} - 1 = \sin v + \cos v$

$\therefore \quad \frac{dv}{\sin v + \cos v + 1} = dx$

$\therefore \quad \frac{\left(1 + \tan^2 \frac{v}{2}\right) dv}{2 \tan \frac{v}{2} + 2} = dx \qquad \tan \frac{v}{2} = t$

$\therefore \quad \frac{2dt}{2t + 2} = dx$

$\therefore \quad \ell n(t+1) = x + c$

$\therefore \quad \ell n\left(1 + \tan\left(\frac{x+y}{2}\right)\right) = x + c$

(v) $\sqrt{1+x^2}\sqrt{1+y^2}dx + xydy = 0$

$x = \tan\theta, y = \tan\phi$

$\sec^3\theta \sec\phi d\theta + \tan\theta \tan\phi \sec^2\phi d\phi = 0$

$\therefore \quad \sec^3\theta d\theta + \tan\theta \sec\phi \tan\phi d\phi = 0$

$\therefore \quad \sec^2\theta \operatorname{cosec}\theta d\theta + \sec\phi \tan\phi d\phi = 0$

$\therefore \quad \tan\theta \operatorname{cosec}\theta + \int \operatorname{cosec}\theta d\theta + \sec\phi = C$

$\therefore \quad \sec\theta + \ell n|\operatorname{cosec}\theta - \cot\theta| + \sec\phi = C$

$\therefore \quad \sqrt{1+x^2} + \ell n\left|\frac{\sqrt{1+x^2} - 1}{x}\right| + \sqrt{1+y^2} = C$

13. Sol

(i) $y(ydx - xdy) - x\sqrt{x^2+y^2}dy = 0$

$\therefore \quad y^2dx - xydy - x\sqrt{x^2+y^2}dy = 0$

$\therefore \quad y^2 - \left(xy + x\sqrt{x^2+y^2}\right)\frac{dy}{dx} = 0$

$\therefore \quad v^2x^2 - \left(x^2v + x^2\sqrt{1+v^2}\right)\left(v + x\frac{dv}{dx}\right) = 0$

$\therefore \quad v^2 - v^2 - v\sqrt{1+v^2} - vx\frac{dv}{dx} - x\sqrt{1+v^2}\frac{dv}{dx} = 0$

$\therefore \quad -v\sqrt{1+v^2} = x\frac{dv}{dx}\left(v + \sqrt{1+v^2}\right)$

$$\therefore \quad \left(\frac{v+\sqrt{1+v^2}}{-v\sqrt{1+v^2}}\right)dv = \frac{dx}{x}$$

$$\therefore \quad \left(-\frac{1}{\sqrt{1+v^2}}-\frac{1}{v}\right)dv = \frac{dx}{x}$$

$$\therefore \quad -\ell n\left|v+\sqrt{1+v^2}\right| - \ell nv + C = \ell nx + C$$

$$\therefore \quad \ell n(v) + \ell n(x) + \ell n\left(v+\sqrt{1+v^2}\right) = C$$

$$\therefore \quad \ell n\left(xv\left(v+\sqrt{1+v^2}\right)\right) = C$$

$$\therefore \quad \ell n\left(y\left(\frac{y}{x}+\sqrt{\frac{y^2+x^2}{x^2}}\right)\right) = C$$

$$\therefore \quad \ell n\left(\frac{y^2+y\sqrt{y^2+x^2}}{x}\right) = C$$

(ii) $x\cos\left(\frac{y}{x}\right)(ydx+xdy) = y\sin\left(\frac{y}{x}\right)(xdy-ydx)$

$$\therefore \quad x\cos\left(\frac{y}{x}\right)\left(y+x\frac{dy}{dx}\right) = y\sin\left(\frac{y}{x}\right)\left(x\frac{dy}{dx}-y\right)$$

$y = vx$

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \quad x\cos v\left(vx+x\left(v+x\frac{dv}{dx}\right)\right) = \sin v\left(x\left(v+x\frac{dv}{dx}\right)\right) - vx$$

$$\therefore \quad 2v\cos v + x\cos v\frac{dv}{dx} = v\sin v \times x\frac{dv}{dx}$$

$$\therefore \quad 2v\cos v = (v\sin v - \cos v)x\frac{dv}{dx}$$

$$\therefore \quad \frac{dx}{x} = \left(\frac{v\sin v - \cos v}{2v\cos v}\right)dv$$

$$\therefore \quad \frac{dx}{x} = \left(\frac{\tan v}{2} - \frac{1}{2v}\right)dv$$

$$\therefore \quad \ell nx = \frac{\ell n|\sec v|}{2} - \frac{1}{2}\ell n|v| + C$$

$$\therefore \quad 2\ell nx = \frac{\ell n\left(\sec\frac{y}{x}\right)}{2} - \frac{1}{2}\ell ny + \frac{1}{2}\ell nx + C$$

$\therefore \quad \ell n xy = \ell n\left(\sec \frac{y}{x}\right) + C$

$\therefore \quad xy = C\left(\sec \frac{y}{x}\right)$

(iii) $\left(y^3 - 2yx^2\right)dx + \left(2xy^2 - x^3\right)dy = 0$

$\therefore \quad y^3dx + 2xy^2dy = 2x^2ydx + x^3dy$

$\therefore \quad xy^4dx + 2x^2y^3dy = 2x^3y^2dx + x^4ydy$

$\therefore \quad \frac{1}{2}\left(2xy^4dx + 4x^2y^3dy\right) = \frac{1}{2}\left(4x^3y^2dx + 2x^2ydy\right)$

$\therefore \quad d\left(x^2y^4\right) = d\left(x^4y^2\right)$

$\therefore \quad x^2y^4 = x^4y^2 + C$

(iv) $x\frac{dy}{dx} = y\left(\log y - \log x + 1\right) \qquad y = vx$

$\therefore \quad x\left(v + x\frac{dv}{dx}\right) = vx\left(\log v + 1\right)$

$\therefore \quad x\frac{dv}{dx} = v\log v$

$\therefore \quad \frac{dv}{v\log v} = \frac{dx}{x}$

$\therefore \quad \ell n\left(\ell n v\right) = \ell n x + c$

$\therefore \quad \ell n v = cx$

$\therefore \quad \ell n y = cx + \ell n x \qquad \Rightarrow \qquad y = x \cdot e^{cx}$

(v) $x\frac{dy}{dx} = y + x\tan\left(\frac{y}{x}\right) \qquad y = vx$

$\therefore \quad x\left(v + x\frac{dv}{dx}\right) = vx + x\tan v$

$\therefore \quad x\frac{dv}{dx} = \tan v$

$\therefore \quad \cot v dv = \frac{dx}{x}$

$\therefore \quad \ell n\left|\sin v\right| = \ell n x + c$

$\therefore \quad \sin\left(\frac{y}{x}\right) = cx$

14.
Sol

(i) $\frac{dy}{dx}=\frac{2x-6y+z}{x-3y+4}$

$x-3y=u$

$\therefore \quad 1-3\frac{dy}{dx}=\frac{du}{dx}$

$\therefore \quad \frac{dy}{dx}=\frac{1}{3}-\frac{1}{3}\frac{du}{dx}$

$\therefore \quad \frac{1}{3}-\frac{1}{3}\frac{du}{dx}=\frac{2u+7}{u+4}$

$\therefore \quad \frac{1}{3}\frac{du}{dx}=\frac{1}{3}-\frac{2u+7}{u+4}=\frac{u+4-6u-21}{3(u+4)}$

$\therefore \quad \frac{du}{dx}=\frac{-5u-17}{u+4}$

$\therefore \quad \left(\frac{u+4}{5u+17}\right)du=-dx$

$\therefore \quad \frac{1}{5}du+\frac{3}{5}\frac{du}{5u+17}=-dx$

$\therefore \quad \frac{4}{5}-\frac{3}{25}\ell n(5u+17)+x=C$

$\therefore \quad \frac{x-3y}{5}-\frac{3}{25}\ell n(5x-15y+17)+x=C$

$\therefore \quad \frac{6x-3y}{5}-\frac{3}{25}\ell n(5x+15y+17)=C$

$\therefore \quad 2x-y-\frac{1}{5}\ell n(5x+15y+17)=C$

(ii) $\frac{dy}{dx}=\frac{4x+6y+5}{3y+2x+4}$

$2x+3y=u$

$\therefore \quad 2+3\frac{dy}{dx}=\frac{du}{dx}$

$\therefore \quad \frac{dy}{dx}\frac{1}{3}\frac{du}{dx}-\frac{2}{3}$

$\therefore \quad \frac{1}{3}\frac{du}{dx}=\frac{2u+5}{u+4}+\frac{2}{3}$

$\therefore \quad \frac{1}{3}\frac{du}{dx}=\frac{6u+15+2u+8}{3(u+4)}$

$$\therefore \quad \frac{du}{dx}=\frac{8u+23}{u+4}$$

$$\therefore \quad \frac{u+4}{8u+23}du=dx$$

$$\therefore \quad \left(\frac{1}{8}+\frac{9}{8}\frac{du}{8u+23}\right)=dx$$

$$\therefore \quad \frac{4}{8}+\frac{9}{64}\ell n(8u+23)=x+C$$

$$\therefore \quad 2x+3y+\frac{9}{8}\ell n(16x+24y+23)=8x+C$$

$$\therefore \quad 3y-6x+\frac{9}{8}\ell n(16z+24y+23)=C$$

$$\therefore \quad y-2x+\frac{3}{8}\ell n(16x+24y+23)=C$$

(iii) $\frac{dy}{dx}=\frac{-3x-2y+5}{3x+2y-5}=-1$

$$\therefore \quad y+x=C$$

15. Sol

(i) $xdx+ydy=\frac{a^2(xdy-ydx)}{x^2+y^2}$

$$(x^2+y^2)\left(x+y\frac{dy}{dx}\right)=a^2\left(x\frac{dy}{dx}-y\right)$$

$$x^3dx+y^3dy+x^2ydy+y^2xdx=a^2xdy-ydx$$

$\therefore \quad \frac{(x^2+y^2)}{2}d(x^2+y^2)=a^2(xdy-ydx)$ (Now, solve yourself)

(ii) $\cos(\cos x-\sin a\sin y)dx+\cos y(\cos y-\sin a\sin x)dy=0$

$$\therefore \quad \cos^2 xdx+\cos^2 ydy=\sin a(\cos x\sin ydx+\sin x\cos ydy)$$

$$\therefore \quad \frac{1}{2}\left(2x+\frac{\sin 2x}{2}\right)+\frac{1}{2}\left(2y+\frac{\sin 2y}{2}\right)=\sin a\sin x\sin y+C$$

$$\therefore \quad 2x+2y+\sin 2x+\sin 2y=4\sin a\sin x\sin y+C$$

(iii) $y^2e^{xy^2}dx+\left(2xye^{xy^2}-3y^2\right)dy=0$

$$\therefore \quad de^{xy^2}=3y^2dy$$

$$\therefore \quad e^{xy^2}=y^3+C$$

(iv) $\left(12x^2y+2xy^2+4x^3-4y^3+2ye^{2x}-e^y\right)dx+\left(2x^2y+4x^3-12xy^2+3y^2-xe^y+e^{2x}\right)dy=0$

$$\therefore \quad 12x^2ydx-12xy^2dy+2xy^2dx+2x^2ydy+4x^3dx+3y^2dy$$

$$-4y^3dx+4x^3dy+2ye^{2x}dx+e^{2x}dy-e^ydx-xe^ydy=0$$

(v) $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

$$\left(1+e^{x/y}\right)\frac{dx}{dy}+e^{x/y}\left(1-\frac{x}{y}\right)=0$$

$x=vy$

$$\therefore \quad \frac{dx}{dy}=v+y\frac{dv}{dy}$$

$$\therefore \quad \left(1+e^{v}\right)\left(v+y\frac{dv}{dy}\right)=e^{v}\left(1-v\right)$$

$$\therefore \quad v+y\frac{dv}{dy}+ve^{v}+vy\frac{dv}{dy}=e^{v}-ve^{v}$$

$$\therefore \quad \left(v+1\right)y\frac{dv}{dy}=e^{v}\left(1-2v\right)-v$$

$$\therefore \quad \frac{dy}{y}=\frac{\left(v+1\right)dv}{e^{v}\left(1-2v\right)-v}$$

(vi) $xdy=ydx$

$$\therefore \quad \ell ny=\ell nx+c$$

$$\therefore \quad y=cx$$

(vii) $ydx-xdy+\left(1+x^{2}\right)dx+x^{2}\sin ydy=0$

$$\therefore \quad \left(-\frac{ydx+xdy}{x^{2}}\right)+\frac{\left(1+x^{2}\right)}{x^{2}}dx+\sin y\,dy=0$$

$$\therefore \quad -d\left(\frac{y}{x}\right)+\frac{dx}{x^{2}}+dx+\sin y\,dy=0$$

$$\therefore \quad -\frac{y}{x}-\frac{1}{x}+x-\cos y=c$$

$$\therefore \quad y+1-x^{2}+x\cos y=cx$$

$$\therefore \quad x^{2}-y-1-x\cos y=cx$$

(viii) $\left(x^{2}+y^{2}+x\right)dx-\left(2x^{2}+2y^{2}-y\right)dy=0$

$$\therefore \quad x^{2}dx+xdx+y^{2}dx-2x^{2}dy-2y^{2}dy+ydy=0$$

$$\therefore \quad x^{2}dx+y^{2}dx-2x^{2}dy-2y^{2}dy=-\left(xdx+ydy\right)$$

16. Sol

(i) $x\cos x\dfrac{dy}{dx}+y\left(x\sin x+\cos x\right)=1$

$\therefore \quad \dfrac{dy}{dx}y\left(\tan x+\dfrac{1}{x}\right)=\dfrac{1}{x\cos x}$

$IF = e^{\int\left(\tan x+\frac{1}{x}\right)dx} = e^{\ell n\left(x\sec x\right)} = x\sec x$

$\therefore \quad xy\sec x=\int \sec^2 x dx$

$\therefore \quad xy\sec x=\tan x+C$

(ii) $\dfrac{dy}{dx}+\dfrac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$

$\ell n\, IF=\int\dfrac{dx}{(1-x)\sqrt{x}} \qquad x=t^2$

$=\int\dfrac{2t\,dt}{\left(1-t^2\right)t}=\dfrac{2}{2}\ell n\left|\dfrac{1+t}{1-t}\right|$

$\therefore \quad IF=\left(\dfrac{1+t}{1-t}\right)$

$\therefore \quad y\left(\dfrac{1+\sqrt{x}}{1-\sqrt{x}}\right)=x+\dfrac{2}{3}x^{3/2}+C$

(iii) $(1+y^2)\dfrac{dx}{dy}+x=e^{-\tan^{-1}y}$

$\therefore \quad \dfrac{dx}{dy}+\dfrac{x}{1+y^2}=\dfrac{e^{-\tan^{-1}y}}{1+y^2}$

$\therefore \quad IF=e^{\tan^{-1}y}$

$\therefore \quad xe^{\tan^{-1}y}=\tan^{-1}y+C$

(iv) $\dfrac{dy}{dx}+\dfrac{y}{\left(1-x^2\right)^{3/2}}=\dfrac{x+\sqrt{1-x^2}}{\left(1-x^2\right)^2}$

$\therefore \quad \ell n\, IF=\int\dfrac{dx}{\left(1-x^2\right)^{3/2}}$

$=\int\dfrac{\cos\theta\, d\theta}{\cos^2\theta} \qquad x=\sin\theta$

$=\tan\theta$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$\therefore \quad ye^{\frac{x}{\sqrt{1-x^2}}} = \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x+\sqrt{1-x^2}}{\left(1-x^2\right)^2}\right)$$

$$= \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x}{\left(1-x^2\right)^2} + \frac{1}{\left(1+x^2\right)^{3/2}}\right)$$

$$ye^{\frac{x}{\sqrt{1-x^2}}} = \frac{x}{\left(1-x^2\right)^2} + C$$

(v) $\dfrac{dy}{dx} + \dfrac{3x^2}{1+x^3}y = \dfrac{\sin^2 x}{1+x^3}$

$IF = 1 + x^3$

$$\therefore \quad y\left(1+x^3\right) = \int\left(\frac{1}{2} - \frac{\cos 2x}{2}\right)dx$$

$$\therefore \quad y\left(1+x^3\right) = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

17. **Sol**

(i) $x\dfrac{dy}{dx} + y = y^2 \log x$

$$\therefore \quad \frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = \frac{\log x}{x}$$

$$\therefore \quad \frac{1}{y} = z$$

$$\therefore \quad -\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\therefore \quad +\frac{dz}{dx} - \frac{z}{x} = -\frac{\log x}{x}$$

$$\therefore \quad IF = e = \frac{1}{x}$$

$$\therefore \quad \frac{z}{x} = \int -\frac{\ell nx}{x^2}dx$$

$$= -\left[-\frac{\ell nx}{x} + \int\frac{dx}{x^2}\right]$$

$$= \frac{\ell nx + 1}{x} + C$$

$$\therefore \quad \frac{1}{xy} = \frac{\ell nx + 1}{x} + C$$

(ii) $\sec^2 y\dfrac{dy}{dx}+x\tan y=x^3$ $\qquad \tan y=z$

$\therefore \quad \dfrac{dz}{dx}+xz=x^3$

$\therefore \quad ze^x=\int x^3e^x dx$

$\therefore \quad ze^x=x^3e^x-3x^2e^x+6xe^x-6e^x+C$

$\therefore \quad \tan y e^x=x^3e^x-3x^2e^x+6xe^x-6e^x+C$

(iii) $\dfrac{dy}{dx}=e^{x-y}\left(e^x-e^y\right)$

$\therefore \quad e^y\dfrac{dy}{dx}=e^x\left(e^x-e^y\right)$ $\qquad e^y=z$

$\therefore \quad \dfrac{dz}{dx}=e^x\left(e^x-z\right)$

$\therefore \quad \dfrac{dz}{dx}+e^xz=e^{2x}$

$\therefore \quad ze^{e^x}=\int e^{2x}e^{e^x}dx=\dfrac{e^{2x}}{2}e^{e^x}-\int\dfrac{e^{3x}}{2}e^{e^x}dx$

(iv) $\left(1+x^2\right)\dfrac{dy}{dx}=x^2y^3-xy$

$\therefore \quad \dfrac{dy}{dx}=\dfrac{x^2y^3}{1+x^2}-\dfrac{xy}{1+x^2}$

$\therefore \quad \dfrac{1}{y^2}\dfrac{dy}{dx}=\dfrac{-x}{y^2\left(1+x^2\right)}+\dfrac{x^3}{1+x^2}$ $\qquad \dfrac{1}{y^2}=z$

$\therefore \quad -\dfrac{2}{y^3}\dfrac{dy}{dx}=\dfrac{dz}{dx}$ $\qquad \therefore \quad -\dfrac{1}{2}\dfrac{dz}{dx}+\dfrac{xz}{1+x^2}+\dfrac{x^3}{1+x^2}$

$\therefore \quad \dfrac{dz}{dx}-\dfrac{2xz}{1+x^2}=\dfrac{-2x^3}{1+x^2}$ $\qquad \therefore \quad IF=\int\dfrac{-2x}{1+x^2}dx=e^{\ell n\left|1+x^2\right|}=\dfrac{1}{1+x^2}$

$\therefore \quad z\left(1+x^2\right)=\int\dfrac{-2x^3}{\left(1+x^2\right)^2}dx$ $\qquad x=\tan\theta$

$$=\int\frac{-2\tan^2\theta\sec^2\theta\, d\theta}{\sec^2\theta}=\int -2\tan\theta\sin^2\theta\, d\theta=\int -2\tan\theta\left(1-\cos 2\theta\right)d\theta$$

$$=-\ell n\sec\theta+\int\tan\theta\cos 2\theta\, d\theta=-\ell n\sec\theta+\tan\theta\frac{\sin 2\theta}{2}-\int\tan\theta\, d\theta$$

$$=-2\ell n\sec\theta+\sin^2\theta+C$$

$$=-2\ell n\sqrt{1+x^2}+\frac{x^2}{1+x^2}+C$$

$\therefore \quad \dfrac{1+x^2}{y^2}=-2\ell n\sqrt{1+x^2}+\dfrac{x^2}{1+x^2}+C$

(v) $\frac{dy}{dx}\left(x^2y^3+xy\right)=1$

$\therefore \quad \frac{dx}{dy}=x^2y^3+xy$

$\therefore \quad \frac{1}{x^2}\frac{dx}{dy}=y^3+\frac{y}{x}$

$\frac{1}{x}=z$

$\therefore \quad -\frac{dz}{dy}=y^3+yz$

$\therefore \quad \frac{dz}{dy}+zy=y^3$

$IF=e^{\frac{y^2}{2}}$

$\therefore \quad ze^{\frac{y^2}{2}}=\int y^3e^{\frac{y^2}{2}}dy \qquad y^2=t \;\therefore\; 2ydy=dt$

$$=\frac{1}{2}\int te^{\frac{t}{2}}dt=\frac{1}{2}\left[\frac{te^{\frac{t}{2}}}{2}-\frac{e^{\frac{t}{2}}}{4}\right]$$

$\therefore \quad ze^{\frac{y^2}{2}}=\frac{1}{4}y^2e^{\frac{y^2}{2}}-\frac{1}{8}e^{\frac{y^2}{2}}+C$

$\therefore \quad \frac{e^{\frac{y^2}{2}}}{z}=\frac{1}{4}y^2e^{\frac{y^2}{2}}-\frac{1}{8}e^{\frac{y^2}{2}}+C$

EXERCISE 2(B)

Q.1 **Sol.** $\frac{dy}{dx} - y\ell n2 = 2^{\sin x}.(\cos x - 1)\ \ell n2$

$$\text{I.F.} = e^{\int -\ell n2 dx = e^{-\ell n2 \int dx} = e^{-\ell n2.x}}$$
$$= e^{\ell n 2^{-x} = 2^{-x}}$$

$\Rightarrow 2^{-x}\frac{dy}{dx} - y.2^{-x}\ell n2 = 2^{-x}.\ 2^{\sin x}(\cos x - 1)\ell n2$

$\Rightarrow \frac{d}{dx}(y.2^{-x}) = 2^{\sin x - x}(\cos x - 1)\ \ell n2$

$\Rightarrow d(y.2^{-x}) = 2^{\sin x - x}(\cos x - 1)\ \ell n2\ dx$

$\Rightarrow \int d(y.2^{-x}) = \ell n2 \int 2^{\sin x - x}.(\cos x - 1)\,dx$

$\Rightarrow y.\ 2^{-x} = \ell n2\ \frac{2^{\sin x - x}}{\ell n2} + c$

$\Rightarrow y = 2^{\sin x} + c2^{x}$

$\because \lim_{x\to\infty} y = \text{finite (it's possible only when } c = 0)$

$\therefore\ y = 2^{\sin x}$

Q.2 **Sol.** $\frac{dy}{dx} = y + \int_0^1 y\ dx$ ...(i)

$\Rightarrow \frac{dy}{dx} = y + k_1$ $\left(\text{let } k_1 = \int_0^1 y\,dx\right)$

$\Rightarrow \int \frac{dy}{y + k_1} = \int dx$

$\Rightarrow \ell n(y + k_1) = x + c$

$\Rightarrow y + k_1 = k_2 e^{x}$ ...(ii)

$(0, 1) \Rightarrow 1 + k_1 = k_2$ ....(iii)

putting the value of y (from equation (ii) in equation (i))

$$\frac{dy}{dx} = (k_2e^x - k_1) + \int_0^1 (k_2e^x - k_1)dx$$

$$\Rightarrow \frac{dy}{dx} = (k_2 e^x - k_1) + [k_2e^x - k_1x]_0^1$$

$$\Rightarrow \frac{dy}{dx} = k_2e^x - k_1 + k_2e - k_1 - k_2$$

$$\Rightarrow dy = (k_2e^x + k_2e - 2k_1 - k_2)dx$$

$$\Rightarrow \int_1^y dy = \int_0^x (k_2e^x + k_2e - 2k_1 - k_2)\,dx$$

$$\Rightarrow y - 1 = [k_2e^x + (k_2e - 2k_1 - k_2)x]_0^x$$

$$\Rightarrow y - 1 = k_2e^x + (k_2 e - 2k_1 - k_2)x - k_2 \quad ...(ii)$$

since equation (i) & (iv) are similar

$$\therefore\ k_2e - 2k_1 - k_2 = 0\ \&\ 1 - k_2 = -k_1$$

$$\Rightarrow k_2e - k_2 = 2k_1 \quad ...(v)$$

solving equation (iii) & (v) we get,

$$k_2 = \frac{2}{3-e},\ k_1 = \frac{e-1}{3-e}$$

from equation (ii),

$$y + k_1 = k_2e^x$$

$$\Rightarrow y = \frac{2}{3-e}e^x - \frac{e-1}{3-e}$$

$$\therefore\ y = \frac{1}{3-e}(2e^x - e + 1)$$

Q.3 **Sol.** Equation of tangent at point $p(x_1, y_1)$

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

$$\Rightarrow y = y_1 + \frac{dy}{dx}(x - x_1)$$

$$Q\left(0, y_1 - x_1\frac{dy}{dx}\right)$$

A/c to question.

$$PQ = 2 \Rightarrow PQ^2 = 4$$

$$\Rightarrow x_1^2 + \left(y_1 - y_1 + x_1\frac{dy}{dx}\right)^2 = 4$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{4 - x_1^2}{x_1^2}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \pm\sqrt{\frac{4-x_1^2}{x_1^2}}$$

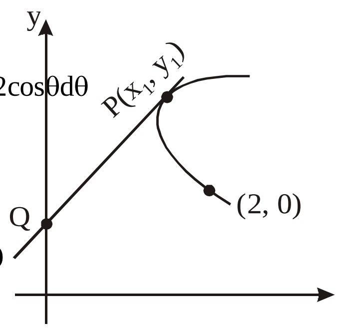
$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)} = \pm\sqrt{\frac{4-x^2}{x^2}}$$

$$\Rightarrow \int dy = \pm\int\sqrt{\frac{4-x^2}{x^2}}dx$$

$$\text{let } I = \int\sqrt{\frac{4-x^2}{x^2}}dx$$

$x = 2\sin\theta$

$\Rightarrow dx = 2\cos\theta d\theta$



$$= \int\sqrt{\frac{4-4\sin^2\theta}{(2\sin\theta)^2}}\ 2\cos\theta d\theta$$

$$= \int\frac{2\cos\theta}{2\sin\theta}2\cos\theta d\theta$$

$\delta\theta \to \delta\theta$

$$= 2\int\frac{\cos^2\theta}{\sin\theta}d\theta = 2\int\frac{1-\sin^2\theta}{\sin\theta}d\theta$$

$$= 2\int(\text{cosec}\theta - \sin\theta)\,d\theta$$

$$= 2\int\text{cosec}\theta\, d\theta - 2\int\sin\theta\, d\theta$$

$$= 2\ell n\,(\text{cosec}\theta - \cot\theta) + 2\cos\theta + c$$

$$= 2\ell n\left(\frac{2}{x} - \frac{\sqrt{4-x^2}}{x}\right) + 2\frac{\sqrt{4-x^2}}{2} + c$$

$$\int_0^y dy = \pm\int_2^x\sqrt{\frac{4-x^2}{x^2}}dx$$

$$\Rightarrow y = \pm\left[\sqrt{4-x^2} + 2\ell n\left(\frac{2-\sqrt{4-x^2}}{x}\right)\right]_2^x$$

$$\therefore\ y = \pm\left[\sqrt{4-x^2} + 2\ell n\left(\frac{2-\sqrt{4-x^2}}{x}\right)\right]$$

Q.4 **Sol.** $$x\,dy + y\,dx + \frac{xdy - ydx}{x^2+y^2} = 0$$

$$\Rightarrow d(xy) + \frac{\frac{xdy-ydx}{x^2}}{\frac{x^2+y^2}{x^2}} = 0$$

$$\Rightarrow d(xy) + \frac{d\left(\frac{y}{x}\right)}{1+\frac{y^2}{x^2}} = 0$$

$$\Rightarrow \int d(xy) + \int \frac{d\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2} = 0$$

$$\Rightarrow xy + \tan^{-1}\left(\frac{y}{x}\right) = c$$

Q.5 **Sol.** $$\frac{ydx - xdy}{(x-y)^2} = \frac{dx}{2\sqrt{1-x^2}}$$

$$\Rightarrow \frac{\frac{ydx - xdy}{y^2}}{\frac{(x-y)^2}{y^2}} = \frac{dx}{2\sqrt{1-x^2}}$$

$$\Rightarrow \frac{d\left(\frac{x}{y}\right)}{\left(\frac{x}{y}-1\right)^2} = \frac{dx}{2\sqrt{1-x^2}}$$

$$\Rightarrow \int \frac{d\left(\frac{x}{y}\right)}{\left(\frac{x}{y}-1\right)^2} = \int \frac{dx}{2\sqrt{1-x^2}}$$

$$\Rightarrow \frac{-1}{\frac{x}{y}-1} = \frac{\sin^{-1}x}{2} + k_1$$

$$\Rightarrow \frac{y}{x-y}+\frac{\sin^{-1}x}{2}=k_1 \quad ...(i)$$

$$(1, 2) \Rightarrow \frac{2}{1-2}+\frac{\sin^{-1}1}{2}=k_1$$

$$\Rightarrow k_1=\frac{\pi}{4}-2$$

putting value of k_1 in equation (1)

$$\frac{y}{x-y}+\frac{1}{2}\sin^{-1}x = \frac{\pi}{4}-2$$

Q.6 **Sol.** $$\frac{dy}{dx}+P(x)y=Q(x) \quad ...(i)$$

(i) $\because$ $y=u(x)$ & $y=v(x)$ is a solution of above equation

$$\frac{d\,u(x)}{dx}+p(x)\,u(x)=Q(x) \qquad ...(ii)$$

$$\frac{d(u(x))}{dx}+p(x)\,v(x)=Q(x) \qquad ...(iii)$$

from equation (i) – (ii)

$$\frac{d}{dx}(y-u(x))+(y-u(x))\,p(x)=0 \qquad ...(iv)$$

from equation [(ii) – (iii)]

$$\frac{d}{dx}(u(x)-v(x))+(u(x)-v(x))\,p(x)=0$$

from equation (iv) & (v)

$$\frac{\frac{d}{dx}(y-u(x))}{\frac{d}{dx}(u(x)-v(x))}=\frac{y-u(x)}{u(x)-v(x)}$$

$$\Rightarrow \int\frac{d(y-u(x))}{y-u(x)}=\int\frac{d(u(x)-v(x))}{u(x)-v(x)}$$

$$\Rightarrow \ell n(y-u(x))=\ell n(u(x)-v(x))$$

$\therefore\ y = u(x) + k(u(x) - v(x));\ k \in R$

(ii) $\because\ y = \alpha\ u(x) + \beta\ v(x)$ is a solution of above equation

$\therefore\ \alpha u(x) + \beta v(x) = u(x) + k(u(x) - v(x))$

comparing coeff. of u(x) & v(x)

$\alpha = 1 + k\ ,\ \ \beta = -k$

$\therefore\ \alpha + \beta = 1$

(iii) Again $y = w(x)$ is also a solution of above equation

$\therefore\ w(x) = u(x) + k(u(x) - v(x))$

$\Rightarrow w(x) - u(x) = k[u(x) - v(x)]$

$$\Rightarrow \frac{u(x) - v(x)}{w(x) - u(x)} = \frac{1}{k}$$

$$\therefore \frac{v(x) - u(x)}{w(x) - u(x)} = -\frac{1}{k}$$ which is a constant no.

Q.7 **Sol.** Equation of normat on given at point $P(x_1, y_1)$ is

$$y - y_1 = \frac{-1}{\frac{dy}{dx}}\ (x - x_1)$$

$$\Rightarrow Q\left(\frac{dy}{dx}.y_1 + x_1, 0\right)$$

$\because$ middle point of segment PQ, R lies on curve $2y^2 = x$

$$R\left(\frac{x_1 + \frac{dy}{dx}y_1 + x_1}{2}, \frac{y_1}{2}\right)$$

$$\therefore\ 2\left(\frac{y_1}{2}\right)^2 = \frac{2x_1 + \frac{dy}{dx}y_1}{2}$$

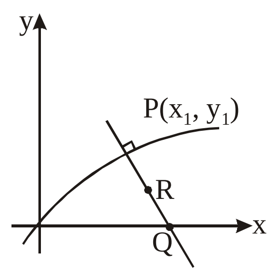
$$\Rightarrow\ y_1^2 = 2x_1 + \frac{dy}{dx}y_1$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x_1,y_1)} y_1 = y_1^2 - 2x_1$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)} y = y^2 - 2x$$

$$\Rightarrow \frac{dy}{dx} y = y^2 - 2x$$

$$\boxed{\begin{aligned} \text{Let. } y^2 = t \Rightarrow 2y\frac{dy}{dx} &= \frac{dt}{\partial x} \\ \Rightarrow \frac{dy}{dx} y &= \frac{1}{2}\frac{dt}{dx} \end{aligned}}$$



$$\Rightarrow \frac{1}{2}\frac{dt}{dx} = t - 2x \quad \Rightarrow \frac{dt}{dx} - 2t + 4x = 0$$

$$\boxed{IF = e^{\int -2dx} = e^{-2x}}$$

$$\Rightarrow e^{-2x} \frac{dt}{dx} - 2e^{-2x} t + 4xe^{-2x} = 0$$

$$\Rightarrow \frac{d}{dx}(e^{-2x} .t) + 4xe^{-2x} = 0$$

$$\Rightarrow d(e^{-2x} . t) + 4xe^{-2x} dx = 0$$

$$\boxed{\begin{aligned} I &= \int xe^{-2x}dx = x\int e^{-2x} - \int\left(\frac{dx}{\partial x}\int e^{-2x}\partial x\right)\partial x \\ &= \frac{xe^{-2x}}{-2} + \frac{1}{2}\int e^{-2x}\partial x \\ &= \frac{-1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} \end{aligned}}$$

$$\Rightarrow \int d(e^{-2x}t) + \int 4xe^{-2x}dx = 0$$

$$\Rightarrow e^{-2x}t + 4\left[\frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4}\right] = k$$

$\Rightarrow t = 2x + 1 + ke^{2x}$

$\Rightarrow y^2 = 2x + 1 + ke^{2x}$

above curve passes through point (0, 0)

$0 = 0 + 1 + k$

$\Rightarrow k = -1$

hence equation of curve is

$y^2 = 2x + 1 - e^{2x}$

Q.8 **Sol.** $\int_0^x t\, f(x-t)dt = \int_0^x f(t)dt + \sin x + \cos x - x - 1$ apply Leibnitz's rule,

$$\int_0^x tf'(x-t)dt + xf(x-x) = f(x) + \cos x - \sin x - 1$$

$$\Rightarrow \int_0^x t\,f'(x-t)dt + xf(0) = f(x) + \cos x - \sin x - 1$$

$$\left[\int t\ f'(x-t)dt = t\int f'(x-t)dt - \int\left(\frac{dt}{dt}\int f'(x-t)\right) = t\ f(x-t) - \int f(x-t)dt\right]$$

$$\Rightarrow [t\ f(x-t)]_0^x - \int_0^x f(x-t)\,dt \pm x\,f(0)$$

$$= f(x) + \cos x - \sin x - 1$$

$$\Rightarrow xf(0) - \int_0^x f(x-t)\,dt + xf(0) = f(x) + \cos x - \sin x - 1$$

$$\Rightarrow 2xf(0) - \int_0^x f(x-t)\,dt = f(x) + \cos x - \sin x - 1$$

Again apply Leibnity rule.

$\Rightarrow 2f(0) - f(0) = f'(x) - \sin x - \cos x$

$$\Rightarrow \frac{dy}{dx} = \sin x + \cos x + f(0)$$

Q.9 **Sol.** $(1-x^2)dy + (x\sqrt{1-x^2} - x - \sqrt{1-x^2})dx = 0$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{(1-x^2)\sqrt{1-x^2}} = \frac{x+\sqrt{1-x^2}}{(1-x^2)^2}$$

$$\text{I.F.} = e^{\int \frac{dx}{(1-x^2)\sqrt{1-x^2}}}$$

$$I = \int \frac{dx}{(1-x^2)\sqrt{1-x^2}} \qquad \text{Let } x = \sin\theta$$

$$\Rightarrow dx = \cos d\theta$$

$$= \int \frac{\cos\theta d\theta}{\cos^2\theta.\cos\theta}$$

$$= \int \sec^2\theta d\theta = \tan\theta$$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$\therefore \text{ I.F.} = e^{\frac{x}{\sqrt{1-x^2}}}$$

$$\Rightarrow e^{\frac{x}{\sqrt{1-x^2}}}\frac{dy}{dx} + \frac{e^{\frac{x}{\sqrt{1-x^2}}}}{(1-x^2)^{3/2}}\, y = e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x+\sqrt{1-x^2}}{(1-x^2)^2}\right)$$

$$\Rightarrow \frac{d}{dx}\left(y e^{\frac{x}{\sqrt{1-x^2}}}\right) = e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x+\sqrt{1-x^2}}{(1-x^2)^2}\right)$$

$$\Rightarrow \int d\left(y e^{\frac{x}{\sqrt{1-x^2}}}\right) = \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x+\sqrt{1-x^2}}{(1-x^2)}\right)dx$$

$$I = \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x+\sqrt{1-x^2}}{(1-x^2)^2}\right)dx$$

$$\text{let } t = \frac{x}{\sqrt{1-x^2}}$$

$$\Rightarrow dt = \left(\frac{1}{\sqrt{1-x^2}} - \frac{-2x.x}{2(1-x^2)^{3/2}}\right)dx$$

$$\Rightarrow dt = \left(\frac{1}{\sqrt{1-x^2}} + \frac{x^2}{(1-x^2)^{3/2}}\right)dx$$

$$\Rightarrow dt = \frac{1-x^2+x^2}{(1-x^2)^{3/2}}dx$$

$$\therefore\ dt = \frac{dx}{(1-x^2)^{3/2}}$$

$$\therefore\ I = \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)\ \frac{dx}{(1-x^2)^{3/2}}$$

$$= \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x}{\sqrt{1-x^2}}+1\right)\frac{dx}{(1-x^2)^{3/2}}$$

$$= \int e^t (t+1)dt$$

$$= te^t = \frac{x}{\sqrt{1-x^2}}.e^{\frac{x}{\sqrt{1-x^2}}}$$

$\because$ Differential equation is,

$$\int d\left(y\, e^{\frac{x}{\sqrt{1-x^2}}}\right) = \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x+\sqrt{1-x^2}}{(1-x^2)^2}\right)dx$$

$$\Rightarrow y.e^{\frac{x}{\sqrt{1-x^2}}} = \frac{x}{\sqrt{1-x^2}}\ \ e^{\frac{x}{\sqrt{1-x^2}}+k}$$

$$\Rightarrow y = \frac{x}{\sqrt{1-x^2}} + ke^{-\frac{x}{\sqrt{1-x^2}}}\ ;\ k \in R$$

Q.10 Sol. $3x^2y^2 + \cos(xy) - xy\sin(xy) + \frac{dy}{dx}\ \{2x^3y - x^2\sin(xy)\} = 0$

Let

$$xy = t \Rightarrow y = \frac{t}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}\frac{dt}{dx} - \frac{t}{x^2}$$

$$\Rightarrow 3x^2y^2 + \cos(xy) - xy\sin(xy) + x^2 \frac{dy}{dx}(2xy - \sin(xy)) = 0$$

$$\Rightarrow 3t^2 + \cos t - t\sin t + x^2\left(\frac{1}{x}\frac{dt}{dx} - \frac{t}{x^2}\right)(2t - \sin t) = 0$$

$$\Rightarrow 3t^2 + \cos t - t\sin t + \left(x\frac{dt}{dx} - t\right)(2t - \sin t) = 0$$

$$\Rightarrow x\frac{dt}{dx} - t + \frac{t^2 + \cos t + 2t^2 - t\sin t}{2t - \sin t} = 0$$

$$\Rightarrow x\frac{dt}{dx} - t + \frac{t^2 + \cos t}{2t - \sin t} + t = 0$$

$$\Rightarrow x\frac{dt}{dx} + \frac{t^2 + \cos t}{2t - \sin t} = 0$$

$$\Rightarrow \int \frac{(2t - \sin t)}{t^2 + \cos t}dt + \int \frac{dx}{x} = 0$$

$$\Rightarrow \ell n(t^2 + \cos t) + \ell n x = k_1$$

$$\Rightarrow x(t^2 + \cos t) = k$$

$$\Rightarrow x(x^2y^2 + \cos(xy)) = k;\ k \in R$$

Q.11 Sol.

Equation of tangent at point P.

$$y - y_1 = \left.\frac{dy}{dx}\right|_{(x_1, y_1)}(x - x_1)$$

$$\Rightarrow y - y_1 = f'(x_1)(x - x_1)$$

$$R \equiv \left(x_1 - \frac{y_1}{f'(x_1)}, 0\right)$$

Equatio of tangent at Q.

$$y - y_2 = \left.\frac{dy}{dx}\right|_{(x_1, y_1)}(x - x_1)$$

$$\Rightarrow y - y_2 = f'(x_1)(x - x_1)$$

$$S \equiv \left(x_1 - \frac{y_2}{f(x_1)}, 0\right)$$

A/c to questin,

$R \equiv S$

$$\Rightarrow x_1 - \frac{y_1}{f'(x_1)} = x_1 - \frac{y_2}{f(x_1)}$$

$$\Rightarrow \frac{f'(x_1)}{f(x_1)} = \frac{y_1}{y_2}$$

$$\Rightarrow \frac{f'(x_1)}{f(x_1)} = \frac{f(x_1)}{\int_{-\infty}^{x_1} f(t)dt} \quad \begin{bmatrix} y_1 = f(x_1) \\ y_2 = \int_{-\infty}^{x_1} f(t)dt \end{bmatrix}$$

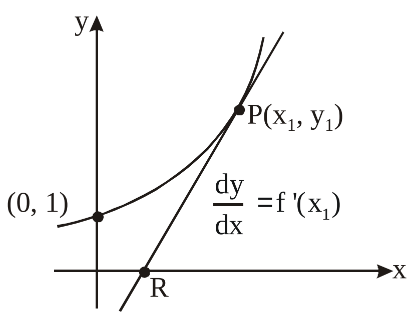
$$\Rightarrow \int_{-\infty}^{x_1} f(t)dt = \frac{f^2(x_1)}{f'(x_1)}$$

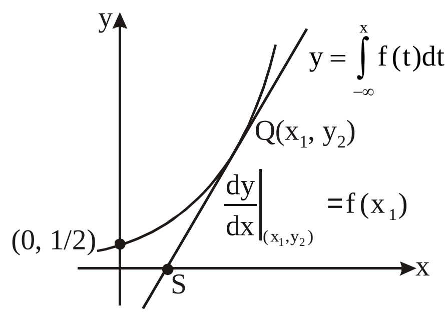
On Generative, we set

$$\Rightarrow \int_{-\infty}^{x} t(t)dt = \frac{f^2(x)}{f^1(x)}$$

Differentiating both sides w.r. to x

$$f(x) = 2f(x)\frac{f'(x)}{f'(x)} - f^2(x)\frac{f''(x)}{(f'(x))^2}$$





$$\Rightarrow 1 = 2 - f(x)\frac{f''(x)}{(f'(x))^2}$$

$$\Rightarrow f(x)\frac{f''(x)}{(f''(x))^2} = 1 \Rightarrow \int \frac{f''(x)}{f'(x)}dx = \int \frac{f'(x)}{f'(x)}dx$$

$$\Rightarrow \ell n\, f'(x) = \ell n\, f(x) + k_1$$

$$\Rightarrow f'(x) = k_2 f(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = k_2 \Rightarrow \int \frac{f'(x)}{f(x)}dx = \int k_2 dx$$

$$\Rightarrow \ell n f(x) = k_2 x + k_3 \qquad \Rightarrow f(x) = k_4 \cdot e^{k_2 x}$$

$$2^{nd} \text{ curve } \; y = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{x} k_4 e^{k_2 x} dx = \frac{k_4}{k_2}\left[e^{k_2 x}\right]_{-\infty}^{x}$$

$\Rightarrow y = \frac{k_4}{k_2} e^{k_2 x}$

1st curve

$y = k_4 e^{k_2 x}$

of passes through (0, 1)

$1 = k_4 e^0$

2nd curve,

$y = \frac{k_4}{k_2} e^{k_2 x}$

at passes through $\left(0, \frac{1}{2}\right)$

$\frac{1}{2} = \frac{1}{k_2} e^0 \Rightarrow \boxed{k_2 = 2}$

Hence equation of 1st curve is $\boxed{y = e^{2x}}$

Q.12 Sol. $x(1 - x\ell ny)\frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx}(x - x^2 \ell n\, y) + y = 0$

$\Rightarrow x - x^2 \ell ny + \frac{dx}{dy} y = 0$

$\Rightarrow \frac{dx}{dy} y + x = x^2 \ell ny$

Let

$xy = t \Rightarrow y\frac{dx}{dy} + x = \frac{dt}{dy}$

$\Rightarrow \frac{dt}{dy} = \frac{t^2}{y^2} \ell n\, y \Rightarrow \int \frac{dt}{t^2} = \int \frac{\ell ny}{y^2} dy$

$I = \int \frac{\ell ng}{y^2} dy$, $u = \ell ny$

$\Rightarrow dx = \frac{dy}{y}$

$= \int \frac{u}{e^u} du$

$= \int ue^{-u} du = ue^{-u} + \int e^{-u} du = -ue^{-u} - e^{-u}$

$= -e^{\ell ny}(\ell ny + 1) = -y(\ell ny + 1)$

$\Rightarrow -\frac{1}{t} = -y(\ell ny + 1) + k_1 \Rightarrow \frac{1}{xy} = y(\ell ny + 1) + k_1$

Above curve passes through (1, 1/e)

$e = 1/e(\ell n, 1/e + 1) + k_1 \Rightarrow \boxed{k_1 = e}$

$\therefore xy[y(\ell ny + 1) + e] = 1$

Q.13 Sol. Equation of tangent at point $P(x_1, y_1)$ on curve,

$y - y_1 = \left.\frac{dy}{dx}\right|_{(x_1, y_1)} (x - x_1)$

Above curve also passes through point R

$\frac{y_1}{2} - y_1 = \left.\frac{dy}{dx}\right|_{(x_1, y_1)} (0 - x_1) \Rightarrow -\frac{y_1}{2} = \left.\frac{dy}{dx}\right|_{(x_1, y_1)} x_1$

on generalise

$\frac{y}{2} = \frac{dy}{dx} x$

$\Rightarrow \int \frac{dx}{x} = 2\int \frac{dy}{y} \Rightarrow \ell nx = 2\ell ny + k_1$ $\qquad \because$ R is the mid point of segment PQ

$\Rightarrow \ell nx = \ell ny^2 + k_1$

$y^2 = kx, \; k \in R$

Q.14 Sol. Equation of tangent at point $P(x_1, y_1)$

$y - y_1 = \left.\frac{dy}{dx}\right|_{(x_1, y_1)} (x - x_1)$

$\Rightarrow y - y_1 = f'(x_1)(x - x_1)$

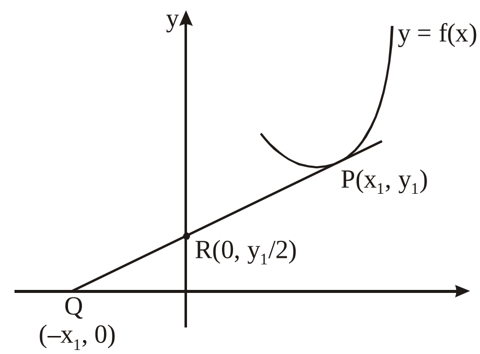
$\because AQ = a$

$\Rightarrow \frac{|f'(x_1)(x_1 - x_1) - 0 + y_1|}{\sqrt{(f'(x_1))^2 + 1}} = a$

$$\Rightarrow y_1 = a\sqrt{(f'(x_1))^2+1} \Rightarrow y_1^2 = a^2\left[\left(\frac{dy}{dx}\right)^2+1\right]$$

On generative

$$\frac{y^2}{a^2} = \left(\frac{dy}{dx}\right)^2+1$$



$$\Rightarrow \frac{dy}{dx} = \pm\sqrt{\frac{y^2-a^2}{a^2}}$$

$$\Rightarrow \int\frac{a\,dy}{\sqrt{y^2-a^2}} = \pm\int dx$$

$$\Rightarrow a\,\ell n\,(y+\sqrt{y^2-a^2}\,) = \pm x$$

$$\Rightarrow y+\sqrt{y^2-a^2} = e^{\pm x/a}$$

$$\Rightarrow y - e^{\pm x/a} = \sqrt{y^2-a^2}$$

$$\Rightarrow y^2 - 2e^{\pm x/a}y + e^{\pm 2x/a} = y^2 - a^2$$

$$\Rightarrow 2e^{\pm x/a}y = e^{\pm 2x/a} + a^2$$

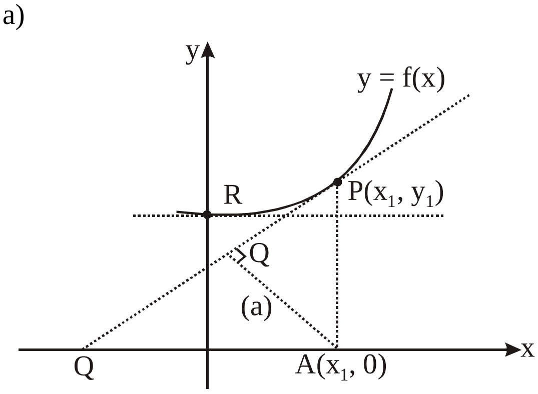
$$y = \frac{e^{\pm x/a}+a^2e^{\pm x/a}}{2}$$

considering targent at point R_1

$y = \pm a$ ($\because$ since distance from origin is a)

Q.15

Sol. Equation of tangent at point P



$$y - y_1 = \left.\frac{dy}{dx}\right|_{(x_1,y_1)}(x-x_1)$$

$$Q\left(0, y_1 - x_1\left.\frac{dy}{dx}\right|_{(x_1,y_1)}\right)$$

SR = length of sub normal

sub tangent $= \dfrac{y_1}{f'(x_1)}$

$$= y_1 \left.\frac{dy}{dx}\right|_{(x_1,y_1)}$$

A/c to questions,

$$\frac{\left(y_1 - x_1\frac{dy}{dx}\right)^2}{y_1\frac{dy}{dx}} = \frac{x_1 y_1}{f'\left(\frac{dy}{dx}\right)^2 \cdot \frac{y_1}{f'(x_1)}}$$

On generalise

$$\frac{(y - x\ f'(x))^2}{y f'(x)} = \frac{x.f'(x)}{(f'(x_1))^2}$$

$$\Rightarrow (y - x\ f'(x))^2 = xy$$

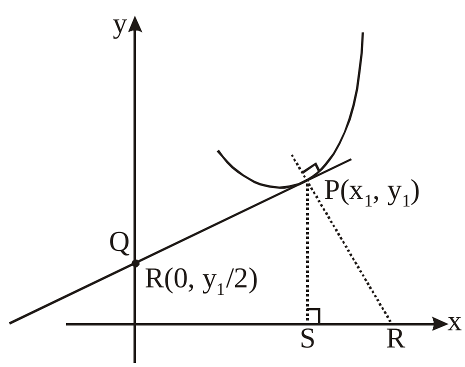
$$\Rightarrow y - x\ f'(x) = \pm\sqrt{xy}$$

$$\Rightarrow x\frac{dy}{dx} - y = \pm\sqrt{xy}$$

$$\Rightarrow \frac{xdy - ydx}{dx} = \pm\sqrt{xy}$$

$$\Rightarrow \frac{xdy - ydx}{y^2} = \pm\sqrt{xy}\ \frac{dx}{y^2}$$

$$\Rightarrow d\left(\frac{x}{y}\right) = \pm\sqrt{\frac{x}{y}}.\frac{dx}{y} = \pm\sqrt{\frac{x}{y}}.\frac{x}{y}\ \frac{dx}{x}$$



$$\Rightarrow \int\frac{d\left(\frac{x}{y}\right)}{\left(\frac{x}{y}\right)^{3/2}} = \pm\int\frac{dx}{x}$$

$$\Rightarrow \frac{\left(\frac{x}{y}\right)^{1-3/2}}{1-\frac{3}{2}} = \pm\ \ell nx$$

$$\Rightarrow \left(\frac{y}{x}\right)^{1/2} = \pm \frac{1}{2} \ell n\, x$$

$$\Rightarrow \ell nx = \pm 2\left(\frac{y}{x}\right)^{1/2}$$

$$\therefore \quad x = e^{\pm 2(y/x)^{1/2}}$$

Q.16 Sol. A/c to question,

$$\frac{dA}{dt}\ \alpha - A \qquad\qquad \frac{dB}{dt}\alpha - B$$

$$\Rightarrow \frac{dA}{dt} = -k_1 A$$

$$\Rightarrow \int_{A_0}^{A} \frac{dA}{A} = \int_0^t -k_1 dt$$

$$\Rightarrow \ell n \frac{A}{A_0} = -k_1 t$$

$$\Rightarrow \frac{A}{A_0} = -\ e^{k_1 t}$$

$\therefore A = A_0\ e^{-k_1 t}$; $k_1 \in R$

Similarly

$B = B_0\ e^{-k_2 t}$; $k_2 \in R$

At $t = 0 \Rightarrow A = 2B \Rightarrow \boxed{A_0 = 2B_0}$

At $t = 1$ hr A = 3/2 B

$\Rightarrow A_0\ e^{-k_1} = 3/2\ B_0\ e^{-k_2}$

$$\Rightarrow e^{k_2 - k_1} = \frac{3}{2}\frac{B_0}{A_0}$$

$$\Rightarrow e^{k_2 - k_1} = \frac{3}{4} \Rightarrow k_2 - k_1 = \ell n\, 3/4$$

Let at $t = t_1$, both reserroirs A & B has equal quantity of water.

$A_1 = A_0\ e^{-k_1 t_1}$

$B_1 = B_0\ e^{-k_2 t_1}$

$\because\ A_1 = B_1$

$\Rightarrow A_0\ e^{-k_1 t_1} = B_0\ e^{-k_2 t_2}$

$\Rightarrow e^{(k_2 - k_1)t_1} = B_0/A_0$

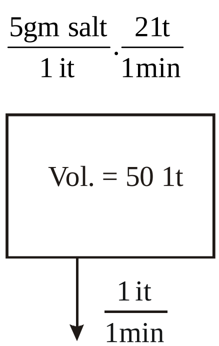
$\Rightarrow (k_2 - k_1)\ t_1 = \ell n\ (B_0/A_0)$

$$\Rightarrow t_1 = \frac{\ell n \frac{1}{2}}{\ell n \frac{3}{4}} = \frac{\ell n 2}{\ell n \frac{4}{3}}$$

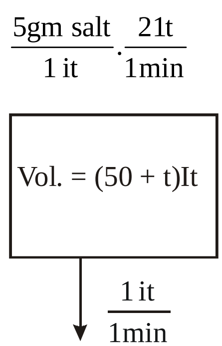
$\therefore\ t_1 = \log^{2}_{4/3}$

Q.17 **Sol.** At time

t = 0



t = t



Since amount salt at time t = t = m

Initial volume of tenk = 50 lit.

volume of tenk at time t = (50 + 2t – t)It

= (50 + t) It

Rate of charge of salt

= rate of salt coming inside the tenk – rate of salt coming outside the tank

$$\Rightarrow \frac{dm}{dt} = \frac{5gm}{1lg} . \frac{2lt}{1min} - \frac{mgm}{(50+t)1t} . \frac{1\ lt}{1min}$$

$$\Rightarrow \frac{dm}{dt} = 10 - \frac{m}{50+t}$$

$$\Rightarrow \frac{dm}{dt} + \frac{m}{50+t} = 10$$

$$\boxed{\begin{aligned} I.F. &= e^{\int \frac{dt}{50+t}} = e^{\ell n|50+t|} \\ &= (50+t) \end{aligned}}$$

$$\Rightarrow (50 + t)\frac{dm}{dt} + m = 10(50 + t)$$

$$\Rightarrow \frac{d}{dt}((50+t)m) = 10(50 + t)\ dt$$

$$\Rightarrow \int d((50+t)m) = \int 10(50+t)dt$$

$$\Rightarrow (50 + t)m = 10\left(50t + \frac{t^2}{2}\right) + c$$

At $t = 0$, $m = 0$

$\Rightarrow c = 0$

$\therefore (50 + t)m = 5(100t + t^2)$

$$\Rightarrow m = \frac{5t(100+t)}{50+t}$$

$$= 5t\left(\frac{50+50+t}{50+t}\right)$$

$$\therefore m = 5t\left(1 + \frac{50}{50+t}\right) \text{ gms}$$

At, $t = 10$ min.

$$m = 5 \times 10\left(1 + \frac{50}{50+10}\right)$$

$$= 50\left(1+\frac{5}{6}\right) = 50\left(\frac{6+5}{6}\right)$$

$$= 50 \times \frac{11}{6}$$

$$= 91\frac{2}{3} \text{ gm}$$

Q.18Sol. Equation of tangent at point $P(x_1, y_1)$

$$y - y_1 = \left.\frac{dy}{dx}\right|_{(x_1,y_1)} (x - x_1)$$

$$Q = \left(0, y_1 - x_1 \left.\frac{dy}{dx}\right|_{(x_1,y_1)}\right)$$

A/c to question,

$$y_1 - x_1 \left.\frac{dy}{dx}\right|_{(x_1,y_1)} \alpha\ x_1^3$$

$$\Rightarrow y_1 - x_1 \left.\frac{dy}{dx}\right|_{(x_1,y_1)} = k_1 x_1^3 \qquad (k_1 \in R)$$

on Genalise,

$$y - x\frac{dy}{dx} = k_1 x^3 \qquad \Rightarrow \frac{dy}{dx} - \frac{y}{x} + k_1 x^2 = 0$$

$$\text{I.F.} = e^{\int \frac{-dx}{x}} = e^{-\ell nx} = e^{\ell n 1/x} = \frac{1}{x}$$

$$\Rightarrow \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} + k_1 x = 0$$

$$\Rightarrow \frac{d}{dx}\left(\frac{y}{x}\right) + k_1 x = 0$$

$$\Rightarrow d\left(\frac{y}{x}\right) + k_1 x\, dx = 0$$

$$\Rightarrow \int d\left(\frac{y}{x}\right) + \int k_1 x dx = 0$$

$$\Rightarrow \frac{y}{x} + k_1 \frac{x^2}{2} = k_2 \qquad (k_2 \in R)$$

$$\Rightarrow 2y + k_1 x^3 = 2k_2 x$$

$\therefore\ 2y + k_1x^3 = k_3x \qquad (k_3 \in R)$

Q.19 Sol. (i) $y = ax^2$ ...(i)

$$\Rightarrow \frac{dy}{dx} = 2ax$$

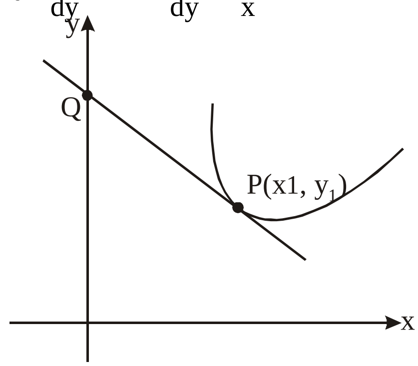
$$\Rightarrow ax = \frac{1}{2}\frac{dy}{dx}$$

putting value of an in equation (i)

$$y = x.\frac{1}{2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y}{x}$$

for orthogonal trajectory replace $\frac{dy}{dx}$ by $\frac{-dx}{dy}$ $\quad -\frac{dx}{dy} = \frac{dy}{x}$



$\Rightarrow 2ydy + xdx = 0$

$\Rightarrow \int 2ydy + \int xdx = 0$

$$\Rightarrow y^2 + \frac{x^2}{2} = k_1$$

$\therefore\ 2y^2 + x^2 = k, \qquad k \in R$

(iii) $x^k + y^k = a^k$

$$\Rightarrow kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0$$

for orthogonal trajectory equation

$$x^{k-1} - y^{k-1}\frac{dx}{dy} = 0$$

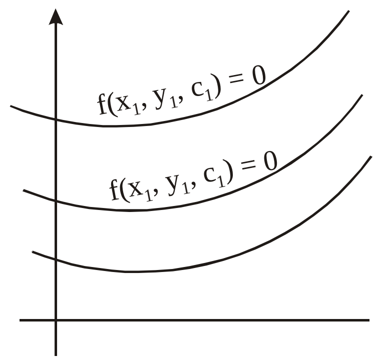
(iv) since angle between both curve is 45° (slope of 1st curve = m & slope of 2nd curve = m_2)

$$\tan 45^\circ = \left|\frac{m - m_2}{1 + mm_2}\right| \Rightarrow \frac{m - m_2}{1 + mm_2} = \pm 1 \Rightarrow m_2 = \frac{m-1}{m+1}, \frac{1+m}{1-m}$$

given equation is

$x^2 - y^2 = a^2 \Rightarrow 2x - 2y \dfrac{dy}{dx} = 0 \Rightarrow x - y \dfrac{dy}{dx} = 0$

Q.20 Sol. Let, we have an integral homogeneous curve $f(x, y_1 c) = 0$. When we get charge the valyue of arbitary constant only, then we have different curves but all curves are symmetrical in nature (i.e. all curves are parallel to each other)



21. $x + yf'(x_0) = x_0 + y_0 f'(x_0)$

$A \equiv (x_0 + y_0 f'(x_0), 0)$

$B \equiv \left(0, y_0 + \dfrac{x_0}{f'(x_0)}\right)$

$\dfrac{1}{OA} + \dfrac{1}{OB} = 1$

$\Leftrightarrow \dfrac{1}{|x_0 + y_0 f'(x_0)|} + \dfrac{|f'(x_0)|}{|x_0 + y_0 f'(x_0)|} = 1$

$\Leftrightarrow 1 + |f'(x_0)| = |x_0 + y_0 f'(x_0)|$

$\pm(1 + |f'(x_0)|) = x_0 + y_0 f'(x_0)$

or $\pm(1 \pm f'(x_0)) = x_0 + y_0 f'(x_0)$

Integrating, we get,

$x^2 + y^2 \pm 2x \pm 2y = c$

($c \in$ Real constant ($y = f(x)$))

$(5, 4)$ lies on $y = f(x)$

Hence, possible curves are,

$(x-1)^2+(y-1)^2=25$ or

$(x-1)^2+(y+1)^2=41$ or

$(x+1)^2+(y+1)^2=61$ or

$(x+1)^2+(y-1)^2=45$.

22. Sol 6 Let the amount of salf in the vessel be c kg at time t,

balt enters at rate 1 kg / min and leaves at the rate $\frac{c}{100}$ kg / min.

i.e. $\frac{dc}{dt}=\left(1-\frac{c}{100}\right)$

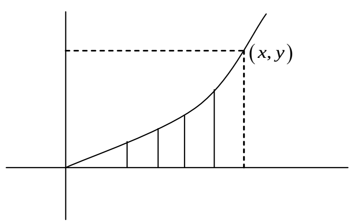
$$\left(\frac{dc}{dt}\right)=\left(\frac{100-c}{100}\right)$$

$$\frac{dc}{100-c}=\frac{dt}{100}$$

Integrating from $c=0$ to $c=50$,

$\text{In}\,2' = \frac{t}{100} \quad \Rightarrow \boxed{t=100\,\text{In}\ 2\ \text{min}}$.

23. Sol



$$\int_0^x f(x).dx=\frac{1}{3}x\,f(x) \qquad (x>0)$$

Differentiating, w.r.t. x,

$$f(x)=\left(\frac{1}{3}\right)\left(f(x)+xf'(x)\right)$$

$$2f(x)=xf'(x)$$

$$2\frac{dx}{x}=\frac{dy}{y}$$

Integrating, $2\,Inx=In\,y\ +c$

$\Rightarrow \boxed{x^2=ky}$ ($k\in$ constant).

24. Sol $-\frac{mvdv}{dx}=\frac{GMm}{x^2}$

$\Rightarrow -vdv = GM\,\frac{dx}{x^2}$

As $x\to\infty$, Vshould approach 0,

Intergrating from initial velocity $v_0\left(x=R\right)$ to final velocity

v_∞ (same x_∞)

$$\frac{v_0^2-v_\infty^2}{2}=GM\left(\frac{1}{R}-\frac{1}{x_\infty}\right)$$

as $x_\infty\to\infty, v_\infty\to 0$

Hence, $v_{esc}^2=\frac{2GM}{R}$

$$\Leftrightarrow \boxed{v_{esc}=\sqrt{\frac{2GM}{R}}} \qquad \left(v_{esc}>0\right).$$

25. Sol Velocity of swimmer at any y is,

$V_{(y)}=y\left(a-y\right)\hat{i}+V_0\,\hat{j}$

$\Rightarrow \boxed{y=V_0 t}$ (t is time)

$$\frac{dx}{dt}=\left(V_0 t\right)\left(a-V_0 t\right)$$

$$dx=\left(V_0\right)\left(at-V_0 t^2\right)\cdot dt$$

Intefrating from $t=0\left(x=0\right)$ to $t=a/V_0$ $\left(x_{\text{final}}\right)$

$$x_{\text{final}}=V_0\left(\frac{a}{2}\left(\frac{a^2}{V_0^2}\right)-\left(\frac{V_0}{3}\right)\left(\frac{a^3}{V_0^3}\right)\right)$$

$$=\left(V_0\right)\left(\frac{a^3}{V_0^2}\right)\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$=\left(\frac{a^3}{6V_0}\right)$$

Hence, the final co-ordinats are, $\left(\frac{a^3}{6V_0},a\right)$

EXERCISE 3

1. Sol $y^2 = 4ax$

$\therefore$ $y-$coordinate of LR is $\pm 2a$.

$\therefore$ $x = \dfrac{y^2}{4a}$ x coordinate is a

$y = \sqrt{4ax}\,a$

$\therefore$ $A = 2\int_0^a \sqrt{4ax}\,dx$

$$= 2\sqrt{4a} \times \frac{2\left[x^{3/2}\right]_0^a}{3}$$

$$= \frac{8}{3}\sqrt{a} \times a\sqrt{a} = \frac{8a^2}{3}$$

2. Sol $A = \int_1^4 x^3 dx$

$$= \left[\frac{x^4}{4}\right]_1^4 = 64 - \frac{1}{4} = \frac{255}{4} \text{ sq units.}$$

3. Sol $\int_0^{\frac{\pi}{3}} \sin x\,dx$

$$= \left[-\cos x\right]_0^{\frac{\pi}{3}}$$

$$A = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\int_0^{\frac{\pi}{3}} \sin 2x\,dx$$

$$A_2 = \frac{1}{2}\left[-\cos 2x\right]_0^{\frac{\pi}{3}} = \frac{1}{2} \times \left[1 + \frac{1}{2}\right] = \frac{3}{4}$$

$$\frac{A_1}{A_2} = \frac{2}{3}$$

4. Sol $A = 2\int_0^5 \sqrt{y+4}\,dy$

$$= 2\int_2^3 t \times 2t\,dt$$

$$= 2 \times \left[\frac{4^2 t^3}{3}\right]_2^3 = \frac{4}{3}\left[27 - 8\right] = \frac{76}{3}$$

5. Sol $y=\sqrt{a^2-x^2}$

$\therefore$ $\text{Area}=4\int_0^a \sqrt{a^2-x^2}\,dx$ $\qquad\qquad x=a\sin\theta$

$=4\int_0^{\frac{\pi}{2}} a\cos\theta\times a\cos\theta\, d\theta$

$=4a^2\int_0^{\frac{\pi}{2}}\left(\frac{1+2\cos^2\theta}{2}\right)d\theta$

$=4a^2\times\left[\frac{\pi}{4}\right]=\pi a^2$

6. Sol $\frac{x^4}{16a^2}=4ax$

$\therefore$ $x=4a$

$\therefore$ $A=\int_0^{4a}\left(\sqrt{4ax}-\frac{x^2}{4a}\right)dx$

$=\left[\sqrt{4a}\times\frac{2}{3}x^{\frac{3}{2}}-\frac{x^3}{12a}\right]_0^{4a}$

$=2\sqrt{a}\times\frac{2}{3}\times a\sqrt{a}-\frac{64a^2}{12a}$

$=\frac{16a^2}{3}$

7. Sol $A=2\int_0^{\frac{1}{2}}\sqrt{1-y^2}dx+2\int_{\frac{1}{2}}^{1}\sqrt{1-(x-1)^2}dx$

$=2\int_0^{\frac{\pi}{6}}\cos^2\theta\, d\theta+2\int_{-\frac{\pi}{6}}^{0}\cos^2\alpha\, dx$

$=2\int_0^{\frac{\pi}{6}}\left[\frac{1+\cos 2\theta}{2}\right]+2\int_{-\frac{\pi}{6}}^{0}\left[\frac{1+\cos 2\alpha}{2}\right]d\alpha$

$=2\left[\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right]^{\frac{\pi}{6}}+2\left[\frac{\alpha}{2}+\frac{\sin 2\alpha}{4}\right]_{-\frac{\pi}{3}}^{0}$

$=2\left[\frac{\pi}{12}+\frac{\sqrt{3}}{8}\right]+2\left[\frac{\pi}{12}+\frac{\sqrt{3}}{8}\right]$

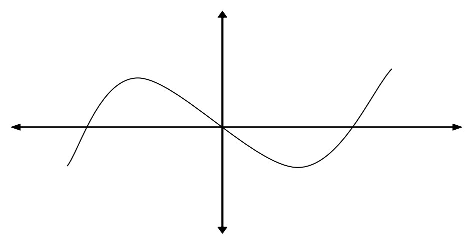
$=\frac{\pi}{3}+\frac{\sqrt{3}}{2}$

8. Sol $A' = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$

$= [\sin x + \cos x]_0^{\frac{\pi}{4}}$

$= \sqrt{2} - 1$

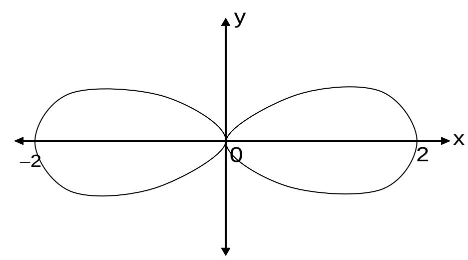
9. Sol $y = x^3 - 4x = x(x+2)(x-2)$



$A = 2\int_{-2}^{0} (x^3 - 4x)\,dx$

$= 2\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} = 2[8-2] = 8$

10. Sol



$A = 4\int_0^2 \sqrt{4x^2 - x^4}\,dx$

$= 4\int_0^2 x\sqrt{4 - x^2}\,dx$

$x = 2\sin\theta \quad dx = 2\cos\theta\,d\theta$

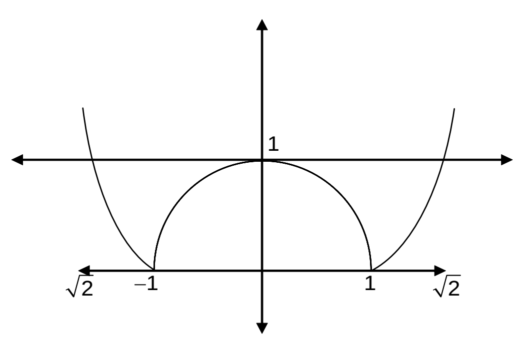
$= 4\int_0^{\frac{\pi}{2}} 2\sin\theta \times 2\cos\theta \times 2\cos\theta\,d\theta$

$= 32\int_0^{\frac{\pi}{2}} \sin\theta\cos^2\theta\,d\theta \qquad \cos\theta = t$

$= 32\int_0^1 t^2\,dt$

$= \frac{32}{3}$

11. Sol $A = 2\left[\int_0^1 x^2 + \int_1^{\sqrt{2}} (2 - x^2)\,dx\right]$



$= 2\left(\left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}\right)$

$= 2\left(\frac{1}{3} + 2\sqrt{2} - \frac{2\sqrt{2}}{3} - 2 + \frac{1}{3}\right)$

$= \frac{2}{3}\left(2 + 6\sqrt{2} - 2\sqrt{2} - 6\right)$

$= \frac{8}{3}\left(\sqrt{2} - 1\right)$

12. Sol $A_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^n x\,dx$

$$= \int_0^{\frac{\pi}{4}} \tan^{n-2} x\left(\sec^2 x - 1\right)dx$$

$$\left[\tan^{n-1} x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (n-2)\tan^{n-2} x \sec^2 x - A_{n-2}$$

$\therefore \quad A_n + A_{n-2} = I$

$$= 1 - (n-2)I$$

$\therefore \quad (n-1)I = 1$

$\therefore \quad I = \frac{1}{n-1}$

$\therefore \quad A_n + A_{n-2} = \frac{1}{n-1}$

Now as n increases A_n decreases

$\therefore \quad A_n < \frac{A_n + A_{n-2}}{2}$

$\therefore \quad A_n > \frac{A_n + A_{n+2}}{2}$

$\therefore \quad A_n > \frac{1}{2n+2}$

13. Sol $\int_1^t \left[f(x) - x\right]dx = \left(t + \sqrt{1+t^2}\right) - \left(1 + \sqrt{2}\right)$

$\therefore \quad \int_1^t f(x)\,dx - \left[\frac{x^2}{2}\right]_1^t = \left(t + \sqrt{1+t^2}\right) - \left(1 + \sqrt{2}\right)$

$\therefore \quad \int_1^t f(x)\,dx = t + \sqrt{1+t^2} - \left(1 + \sqrt{2}\right) + \frac{t^2}{2} - \frac{1}{2}$

$\therefore \quad \int_1^t f(x)\,dx = \frac{t^2}{2} + t + \sqrt{1+t^2} - \left(\frac{1^2}{2} + 1 + \sqrt{1+(1)^2}\right)$

$\therefore \quad f(x) = x + 1 + \frac{x}{\sqrt{1+x^2}}$

14. Sol $y \log x$ and $y = (\log x)^2$

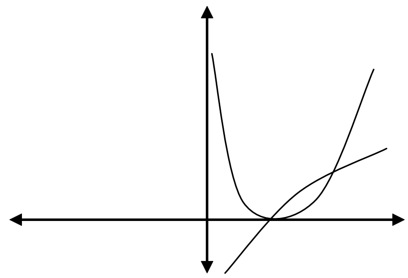
$A = \int_1^e (\log x) - (\log x)^2 \, dx$

$\ell n x = t$

$\therefore \quad x = e^t$

$\therefore \quad dx = e^t dt$

$\therefore \quad A = \int_0^1 (t - t^2) e^t dt$



$= \int_0^1 e^t (-t^2 - 2t) dt + \int_0^1 e^t (3t) dt$

$= \left[e^t (-t^2) \right]_0^1 + 3 (te^t - e^t)_0^1$

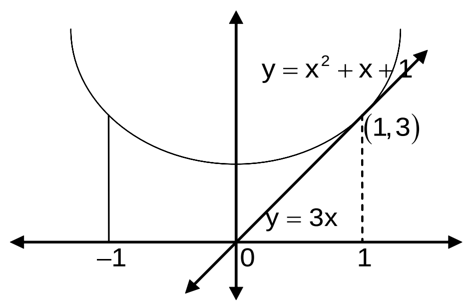
$= -e + 3(1) = 3 - e$

15. Sol $\frac{dy}{dx} = 2x + 1 = m$ at $(1,3), m = 3$

$\therefore \quad y - 3 = 3(x - 1)$

$\therefore \quad y = 3x$

$\therefore \quad A = \int_{-1}^{0} (x^2 + x + 1) dx + \int_0^1 (x^2 - 2x + 1) dx$



$= \left[\frac{x^3}{3} + \frac{x^2}{2} + x \right]_{-1}^{0} + \left[\frac{x^3}{3} - x^2 + x \right]_0^1$

$= \frac{1}{3} + \frac{1}{2} + 1 + \frac{1}{3} = \frac{4+3}{6} = \frac{7}{6}$

16. Sol $y = 1 + 4x - x^2$

$y\left(\frac{3}{2}\right) = 1 + 6 - \frac{9}{4} > 0$

$A = \int_0^{\frac{3}{2}} (1 + 4x - x^2) dx$

$= \left[x + 2x^2 - \frac{x^3}{3} \right]_0^{\frac{3}{2}}$

$= \frac{3}{2} + \frac{9}{2} - \frac{9}{8} = \frac{48 - 9}{8} = \frac{39}{8}$

$y = mx$

$\therefore \quad mx = 1 + 4x - x^2$

$\therefore \quad x^2 + (m - 4)x - 1 = 0$

$\therefore \quad x = \frac{4 - m + \sqrt{m^2 - 8m + 20}}{2}$

$$\int_0^{\frac{3}{2}}\left(1+(4-m)x-x^2\right)dx=\frac{39}{16}$$

$\therefore \quad \left[x+\frac{(4-m)x^2}{2}-\frac{x^3}{3}\right]_0^{\frac{3}{2}}=\frac{39}{16}$

$\therefore \quad \frac{3}{2}+\frac{(4-m)9}{8}-\frac{9}{8}=\frac{39}{16}$

$\therefore \quad +\frac{3}{8}+\frac{9(4-m)}{8}=\frac{39}{16}$

$\therefore \quad 6+72-18m=39$

$\therefore \quad 18m=39$

$\therefore \quad m=\frac{13}{6}$

17. Sol $f\left(\frac{x}{y}\right)=f(x)-f(y)$

$\lim\limits_{x\to 0} f\frac{(1+x)}{x}=3$

$f'(1+x)=3$

$f\left(\frac{x+1}{y}\right)=f(x)-f(y)$

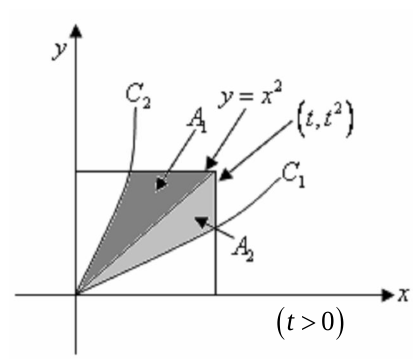
$\therefore \quad \frac{1}{y}f'(x+1)=f'(x+1)$

$\therefore \quad$ Function is $3\ell n x$

$A=\int_0^e(3-3\ell n x)dx$

$=[3x]_0^e-3[x\ell n x-x]_0^e \quad =3e-3[e-e]=3e$

18. Sol $\quad A_1=A_2 \quad$ (Given)



$\Leftrightarrow \quad \int_0^{t^2}\left(\sqrt{y}-C_2^{-1}(y)\right)\cdot dy=\int_0^t\left(x^2-\frac{x^2}{2}\right)\cdot dx$

$\Leftrightarrow \quad \left(\frac{2}{3}\right)(t^3)-\int_0^{t^2}C_2^{-1}(y)\cdot dy=\left(\frac{1}{2}\right)\left(\frac{t^3}{3}\right)$

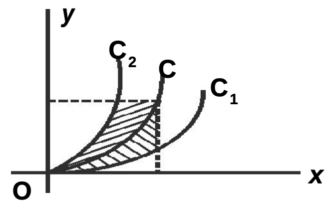
$\Leftrightarrow \quad \frac{t^3}{2}=\int_0^{t^2}C_2^{-1}(y)\cdot dy$

Differentiating both sides w.r.t. t,

$$\frac{3t^2}{2} = 2tC_2^{-1}\left(t^2\right)$$

$$C_2\left(\frac{3t}{4}\right) = t^2 = \left(\frac{3t}{4}\right)^2 \times \left(\frac{16}{9}\right)$$

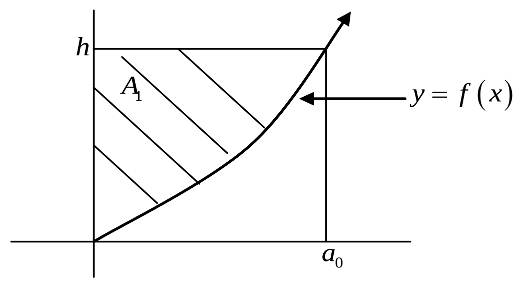
$$\Leftrightarrow \quad C_2(t) = \frac{16t^2}{9}.$$



19. Sol Let $h = f(a_0)$ $\quad a_0 \in [0,a]$

$$\underbrace{\int_0^h f^{-1}(x).dx}_{A_1} + \int_0^{a_0} f(x)\cdot dx$$

$=$ Area of rectangle $= a_0 h$



Hence, $\int_0^h f^{-1}(x) + \int_0^{a_0} f(x).dx + a_0 h$

$= a_0 h + \int_{a_0}^{a} f(x).dx$

$\because \quad f(x)$ is an function

$f(x) > f(0) = 0 \qquad \forall x \in [a_0, a]$

Hence, the integral $\int_{a_0}^{a} f(x)\cdot dx \geq 0.$

(Equality occure when $a = a_0$)

Thus,

$\int_0^a f(x)\cdot dx + \int_0^h g(x)\cdot dx \geq a_0 f(a_0) \qquad \left(a_0 \in [0,a] f(a_0) \in [f(0), f(a)]\right)$

Equality occurs when $(a_0 = a)$

Now, $\int_0^{\frac{\pi}{2}} \sin x.dx + \int_0^a \sin^{-1} x.dx \;\; a \in [0,1] \geq a \sin^{-1} a \qquad$ (Equality when $a = 1$)

Hence, $\int_0^{\frac{\pi}{2}} \sin x.dx + \int_0^1 \sin^{-1} x.dx = \frac{\pi}{2}$

20. Now, $\int_{2}^{a}\left(1+\frac{8}{x^2}\right)dx = 2$

$\therefore \quad a-\frac{8}{a}-2+4=2$

$\therefore \quad a-\frac{8}{a}=0$

$\therefore \quad a^2=8 \qquad \therefore \quad a=2\sqrt{2}$

21. Sol $x-bx^2=\frac{1}{b}x^2$

$\therefore \quad x=\left(b+\frac{1}{b}\right)x^2$

$\therefore \quad x=\frac{b}{b^2+1}$

$$A=\int_{0}^{\frac{b}{b^2+1}}\left(x-bx^2-\frac{x^2}{b}\right)dx$$

$$\therefore \quad A=\left[\frac{x^2}{2}-\frac{bx^3}{3}-\frac{x^3}{3b}\right]_0^{\frac{b}{b^2+1}}$$

$$=\frac{b^2}{2\left(b^2+1\right)^2}-\frac{b^4}{3\left(b^2+1\right)^3}-\frac{b^2}{3\left(b^2+1\right)^3}$$

$$=\frac{3b^2\left(b^2+1\right)-2b^4-b^2}{6\left(b^2+1\right)^3}$$

$$=\frac{b^4-2b^2}{6\left(b^2+1\right)^3}$$

$$\therefore \quad \frac{dA}{db}=\frac{6\left(b^2+1\right)^3\left[4b^3-4b\right]-\left(b^4-2b^2\right)\left[18\left(b^2+1\right)^2\times 2b\right]}{6\left(b^2+1\right)^6}$$

$\therefore \quad \left(b^2+1\right)^3\left(4b^3-4b\right)=\left(b^4-2b^2\right)\left(3\left(b^2+1\right)^2\times 2b\right)$

$\therefore \quad 4b\left(b^2+1\right)\left(b^2-1\right)=b^2\left(b^2-2\right)\times 6b$

$\therefore \quad 2b^4-2=3b^4-6b^2$

$\therefore \quad 3b^4-8b^2+2=0$

$\therefore \quad b^2=\frac{8\pm\sqrt{64-24}}{6}$

$$= \frac{8 \pm \sqrt{40}}{6} = \frac{4 \pm \sqrt{10}}{3}$$

$\therefore \quad b = \sqrt{\frac{4 \pm \sqrt{10}}{3}}$

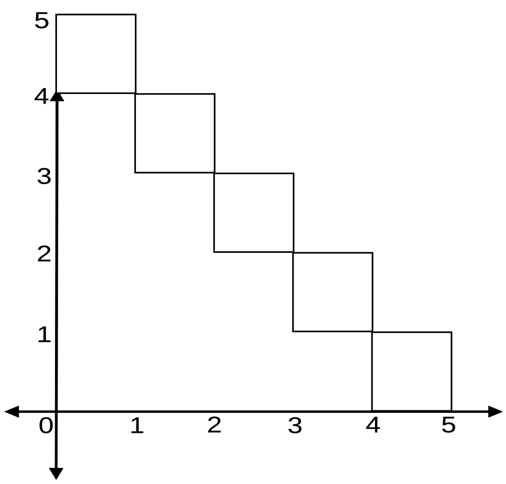
22.

Sol $\int_1^a f(x)\,dx = \sqrt{1+a^2} - \sqrt{2}$

$$= \sqrt{1+a^2} - \sqrt{1+(1)^2}$$

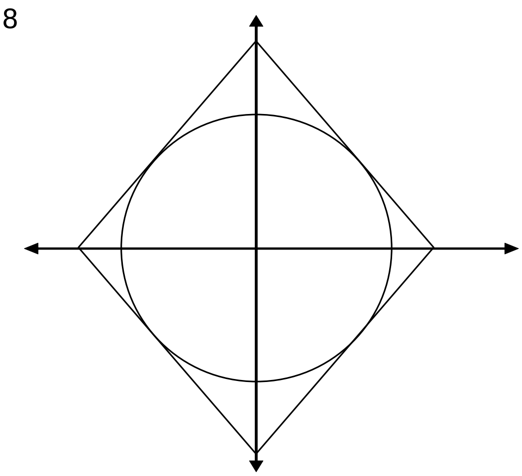
$\therefore \quad f(x) = \frac{x}{\sqrt{1+x^2}}$

23. Sol $[x]+[y]=4$



$A = 5$

24. Sol $A = 8$



25. Sol $f'(x+y)=f'(x)-8y$

$g'(x+y)=g'(x)+3y(x+y)+3xy$

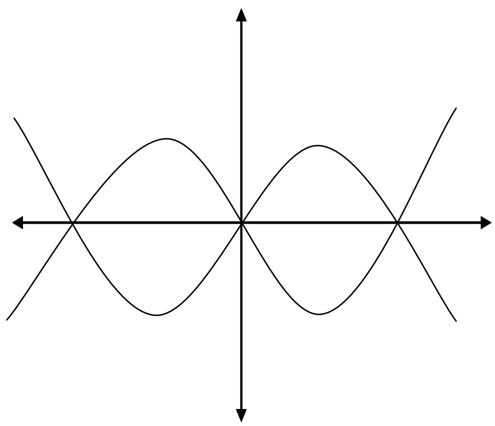
$\therefore \quad f'(y)=8-8y \qquad f(0)=0$

$\therefore \quad g'(y)=-4+3y^2 \qquad g(0)=0$

$\therefore \quad f(x)=8x-4x^2+C_1$

$g(x)=y^3-4y+C_2$

$\therefore \quad C_2=0$



$8x-4x^2=4x-x^2$

$\therefore\ 8-4x=4-x^2$

$\therefore\ x^2-4x+4=0$

$x=2$

$$\therefore \quad A=\int_0^2\left(8x-4x^2+x^3-4x\right)dx$$

$$=\int_0^2\left(x^3-4x^2+4x\right)dx$$

$$=\left[\frac{x^4}{3}-\frac{4x^3}{3}+2x^2\right]_0^2$$

$$=\frac{16}{3}-\frac{32}{3}+8$$

$$=\frac{8}{3}=\frac{4}{3}$$

26. Sol $|x-2y|+|x+2y|\le 8$

If $x>2y$ & $x>-2y$ $\quad\therefore\quad 2x<8 \quad\therefore\quad x<4$

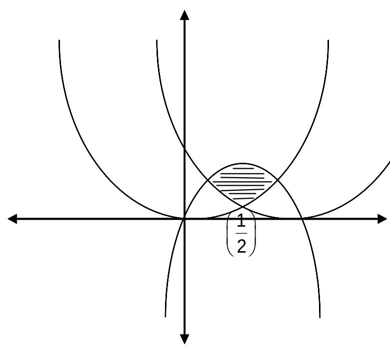
27. Sol $y = \tan x$ tangent at $x = \frac{\pi}{4}$ and x axis

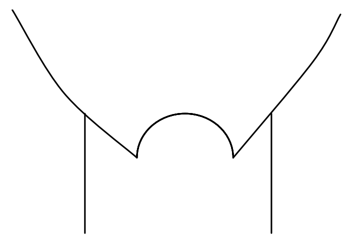
$\therefore \quad \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = 2 \qquad \therefore \quad y - 1 = 2x - \frac{\pi}{2} \qquad y = 2x + 1 - \frac{\pi}{2}$

$y = 0 \qquad \Rightarrow \qquad x = \frac{\pi}{4} - \frac{1}{2}$

$$A = \int_{\frac{\pi}{4}-\frac{1}{2}}^{\frac{\pi}{4}} \left(\tan x - 2x - 1 + \frac{\pi}{2}\right) dx = \left(\ell n|\sec x) - x^2 - x + \frac{\pi}{2}x\right)_{\frac{\pi}{4}-\frac{1}{2}}$$

28. Sol





$x^2 = 2x - 2x^2$

$\therefore \quad 3x = 2 \qquad x = \frac{2}{3} \qquad x = \frac{1}{3}$

$$\therefore \quad A = \int_0^{\frac{1}{3}} (1-x)^2 dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left(2x - 2x^2\right) dx + \int_{\frac{2}{3}}^{1} x^2 dx$$

$$= \left[x - x^2 + \frac{x^3}{3}\right]_0^{\frac{1}{3}} + \left[x^2 - \frac{2x^3}{3}\right]_{\frac{1}{3}}^{\frac{2}{3}} + \left[\frac{x^3}{3}\right]_{\frac{2}{3}}^{1}$$

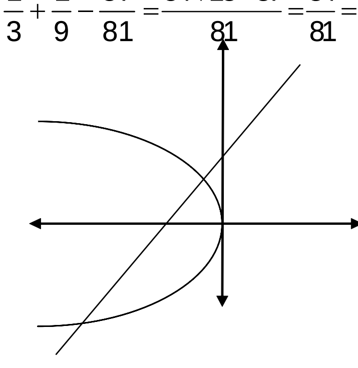
$$= \frac{1}{3} - \frac{1}{9} + \frac{1}{81} + \frac{4}{9} - \frac{32}{81} - \frac{1}{9} + \frac{2}{81} + \frac{1}{3} - \frac{8}{81} = \frac{2}{3} + \frac{2}{9} - \frac{37}{81} = \frac{54+18-37}{81} = \frac{34}{81} = \frac{17}{27}$$

29. Sol $x = -y^2$

$x = y - 6$

$\therefore \quad y - 6 = -y^2$

$\therefore \quad y = 2$ or $y = -3$



$\therefore \quad A=\int_{-3}^{2}\left(y^2-y+6\right)dy$

$=\left[\frac{y^3}{3}-\frac{y^2}{2}+6y\right]_{-3}^{2}$

$=\left|\frac{8}{3}-2+12-9+\frac{9}{2}-18\right|=\left|\frac{8}{3}+\frac{9}{2}-17\right|=\left|\frac{102-43}{6}\right|=\frac{61}{6}$

30. Sol $y=x^2-6x^2+8x$

$=x(x-2)(x-4)$

$A=\int_0^2\left(x^3-6x^2+8x\right)dx+\int_2^4\left(6x^2-x^3-8x\right)dx$

$=\left[\frac{x^4}{4}-3x^3+4x^2\right]_0^2+\left[2x^3-\frac{x^4}{4}-4x^2\right]_2^4$

$=4-16+16+128-64-64-16+4+16=8$

31. Sol $y=x^{-p}, x=1 \;\&\; x=b$

$S=\int_1^b x^{-p}dx$

$=\left[\frac{x^{1-p}}{1-p}\right]_1^b$

$=\frac{b^{1-p}}{1-p}-\frac{1}{1-p}=\frac{1-b^{-(p-1)}}{p-1}$

As $b\to\infty \qquad (p>1)$

$S=\frac{1}{p-1}$

$\lim_{b\to\infty} S=\frac{1}{p-1}$

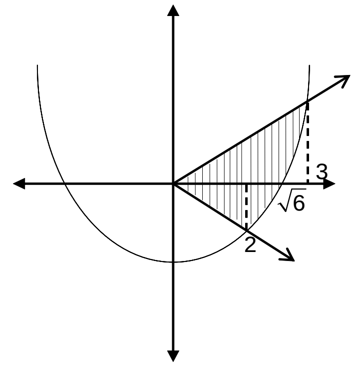
32. Sol $x^2-6=-x$

$\therefore \quad x^2+x-6=0 \qquad \therefore \quad x=2$

$A=\frac{1}{2}\times 2\times 2-\int_2^{\sqrt{6}}\left(x^2-6\right)dx+\frac{1}{2}\times\sqrt{6}\times\sqrt{6}3+\int_{\sqrt{6}}^{3}\left(x-x^2+6\right)dx$

$=5-\left[\frac{x^3}{3}-6x\right]_2^{\sqrt{6}}+\left[\frac{x^2}{2}-\frac{x^3}{3}+6x\right]_{\sqrt{6}}^{3}$

$$= 5 - \left[2\sqrt{6} - 6\sqrt{6} - \frac{8}{3} + 12\right] + \left[\frac{9}{2} - 9 + 18 - 3 + \sqrt{6} - 6\right]$$

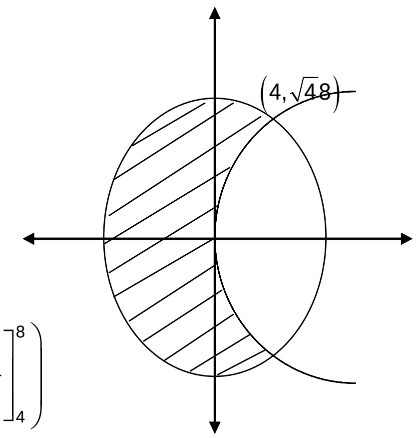


$$= 5 - \left[-4\sqrt{6} + \frac{28}{3}\right] + \left[6 + \frac{9}{2} - 4\sqrt{6}\right]$$

$$= 5 - \frac{28}{3} + \frac{21}{2}$$

$$= \frac{30 - 56 + 63}{6} = \frac{37}{6}$$

33. Sol $x^2 + y^2 \le 64$ and $y^2 \ge 12x$



$12x + x^2 = 64$

$\therefore \quad x^2 + 12x - 64 = 0$

$\therefore \quad x = 4$

$$A = \pi(8)^2 - 2\left[\int_0^4 \sqrt{12x}\,dx + \int_4^8 \sqrt{64 - x^2}\,dx\right]$$

$$= 64\pi - 2\left(\left[\sqrt{12}\left[x^{3/2}\right]_0^4 \times \frac{2}{3}\right]_0^4 + \left[32\sin^{-1}\frac{x}{8} + x\frac{\sqrt{64 - x^2}}{2}\right]_4^8\right)$$

$$= 64\pi - 2\left[\left(2\sqrt{3} \times 8 \times \frac{2}{3}\right) + 32\left(\frac{\pi}{3}\right) - 2 \times 4\sqrt{3}\right]$$

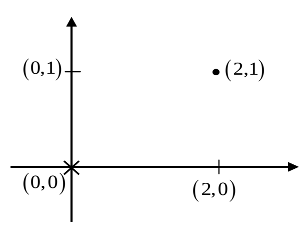
$$= 64\pi - 2\left[\frac{32\sqrt{3}}{3} + \frac{32\pi}{3} - 8\sqrt{3}\right]$$

$$= 64\pi - 2\left[\frac{8\sqrt{3}}{3} + \frac{32\pi}{3}\right]$$

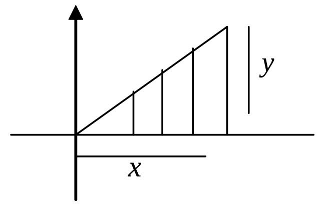
$$= 64\pi - \frac{16\sqrt{3}}{3} - \frac{64\pi}{3}$$

$$= \frac{128\pi - 16\sqrt{3}}{3}$$

34. Sol As you must be knowing,

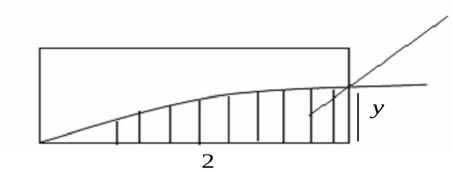


The Area under a parabola from it's



vertex along is axis as shown is $\frac{2}{3}$ xy.

Case 1 :-



$\left(\frac{2}{3}\right)(2)(y)=1 \quad \Rightarrow \quad \boxed{y=\frac{3}{4}}$

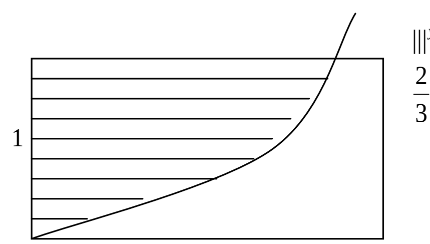
$y^2 = 4ax$ $(a \in R)$ (any parabola with axis as x-axis and vertex at 0)

It passes through $\left(2,\frac{3}{4}\right)$ Hence, $\frac{9}{16}=(4a)(2) \Rightarrow a=\frac{9}{128}$

$\boxed{y^2=\frac{9x}{32}}$

Case 2:- $x^2 = 4ay$

passes Through $\left(\frac{3}{2},1\right)$



$\frac{9}{4}=(4a)(1)$

$4a=\left(\frac{9}{4}\right)$

$\boxed{x^2=\frac{9y}{4}}$

Hecne, the 2curves are , $x^2=\frac{9y}{4}$ and $y^2=\frac{9a}{32}$.

35. Sol $y = x(x-1)^2$

$x^3-2x^2+x-2=0$

$\therefore \quad (x^2+1)(x-2)=0$

$\therefore \quad A=\int_0^2 (2-x^3+2x^2-x)dx$

$$= \left[2x - \frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} \right]_0^2$$

$$= 4 - 4 + \frac{16}{3} - 2$$

$$A = \frac{10}{3} \cdot$$