SOLUTIONS

1. (C)

$2015 = 5 \times 13 \times 31$, where 5, 13 and 31 are prime numbers.

$\therefore\ 2015 = 1 \times 1 \times 2015$ (3 arrangements)

$= 1 \times 5 \times 403$ (3! = 6 arrangements)

$= 1 \times 13 \times 155$ (3! = 6 arrangements)

$= 1 \times 31 \times 65$ (3! = 6 arrangements)

$= 5 \times 13 \times 31$ (3! = 6 arrangements)

$\therefore$ there are **27** possible triples (x, y, z).

2. (B)

Since the number 2, in the ratio 4 : 2 : 3, is not divisible by 3, we use the equivaletn ratio 12 : 6 : 9 because 6 is divisible by 3.
Let the area of Δ DEF be 6 units.
Area of shaded region $= \frac{1}{3} \times 6 = 2$ units.
Area of trapezium ABCD = 12 units
Area of parallelogram GHJK = 9 units
Area of trapezum ABCD that is unshaded = 12 – 2 = 10 units
Area of parallelogram GHJK that is unshaded = 9 – 2 = 7 units
Total area of unshaded regions = 10 + 7 = 17 units
$\therefore$ ratio of area of shaded region to total area of unshaded regions = **2 : 17**

3. (D)

If a number is divisible by 24 (= 3 × 8), then it is also divisible by 3 and 8, since 3 and 8 are relatively prime.
Using the divisibility test for 8, the last three digits 56Y is also divisible by 8. But there are still 10 possibilities for Y.
If a number is divisible by 8, then the number is also divisible by any factor of 8, i.e. X56Y is also divisible by 2 and by 4.
Since X56Y is divisible by 2, then Y must be even, i.e. Y = 0, 2, 4, 6 or 8.
Since X56Y is divisible by 4, using the divisibility test for 4, the last two digits 6Y is also divisible by 4. Since only 62 and 66 are not divisible by 4, then Y = 0, 4 or 8.
Now we use the divisibility test for 8. Since only 564 is not divisible by 8, then Y = 0 or 8.
Using the divisiblity test for 3, X + 5 + 6 + Y = X + Y + 11 is also divisible by 3.
If Y = 0, then X + Y + 11 = X + 11 is also divisible by 3, i.e. X = 1, 4 or 7.
If Y = 8, then X + Y + 11 = X + 19 is also divisible by 24 are: **1560, 4560, 7560, 2568, 5568, 8568.**
$\therefore$ No. of four-digit numbers of the form X56Y that are divisible by 24 = **6**

4. (C)

For each pile to have a difference number of toys, and the biggest pile to have the smallest possible number of toys, put 1 toy in the 1st pile, 2toys in the 2nd pile, 3 toys in the 3rd pile, 4 toys in the 4th pile and 5 toys in the 5th pile. So the biggest pile is the 5th pile, but there are only 1 + 2 + 3 + 4 + 5 = 15 toys.
The 16th toy will have to go to the pile so that each pile will have a different number of toys.
The 17th toy cannot go to the 5th pile because we want to find the smallest possible number of toys in the biggest pile, so the 17th toy will have to go to the 4th pile. Similarly, the 18th, 19th and 20th toys will go to the 3rd, 2nd and 1 st piles respectively.
The 21st toy will then go to the 5th pile again, and the 22nd, 23rd, 24th and 25th toys will go to the 4th,

3^{rd}, 2^{nd} and 1^{st} piles respectively.
Similarly, the 26^{th} to 30^{th} toys will go to the 5^{th} to 1^{st} piles respectively.
Similarly, the 31^{st} to 35^{th} toys will also go to the 5^{th} to 1^{st} piles respectively.
$\therefore$ the largest pile (which is the 5^{th} pile) will contain 5 + 1 + 1 + 1 + 1 = **9 toys.**

5. (A)
From the identity $4ab = (a+b)^2 - (a-b)^2$, one sees that when $a+b$ is fixed, the product is the largest when $|a-b|$ is the smallest.
Now let $a < b < c$ be 3 distinct positive integers such that $a+b+c = 3n$ and the product abc is the biggest possible. If a and b differ more than 2, then by increasing a by 1 and decreasing b by 1, the product abc will become bigger. Hence a and b differ by at most 2. Similarly b and c differ by at most 2.
On the other hand, if $c = b+2 = a+4$, then one can increase the value of abc by decreasing c by 1 and increasing a by 1. So this case is ruled out.
Since $a+b+c$ is divisible by 3, it is not possible that $c = b+1 = a+3$ or $c = b+2 = a+3$.
So we must have $c = b+1 = a+2$. Hence the three numbers are $n-1$, n and $n+1$.
Thus, their product is $(n-1)n(n+1) = n^3 - n$.

6. (C)
Observe that the stop numbers $1, 9, 25, 49, \ldots$ are at the lower right corners. The point $(0, -n)$ is at the stop number $(2n+1)^2 - (n+1) = 4n^2 + 3n$. When $n = 22$, we have $4(22^2) + 3(22) = 2002$. So the point $(0, -22)$ is the 2002-th stop. Thus the point $(3, -22)$ is the 2005-th stop.

7. (A)
Observer that $\left((b-c)+(c-a)\right)^2 - 4(b-c)(c-a) = 0$.
Hence $\left((b-c)-(c-a)\right)^2 = 0$.
Thus, $b-c = c-a$.

8. (B)
Since $m \neq n$ and $m^2 - n^2 = (n-m)$, we get $m+n = -1$.
Thus $m^2 + n^2 = (m+n) + 4 = 3$ and $mn = \frac{1}{2}\left[(m+n)^2 - m^2 - n^2\right] = -1$.
Thus $4mn - m^3 - n^3 = 4mn - (m+n)\left(m^2 + n^2 - mn\right) = -4 - (-1)\left(3 - (-1)\right) = 0$.

9. (B)
Since $x = 200600 + y - 20\sqrt{2006y}$ is an integer, $y = 2006u^2$ for some positive integer *u*.
Similarly $x = 2006v^2$ for some positive integer *v*. Thus $u + v = 10$.
There are 11 pairs in total, namely $(0,10), (1,9), \ldots\ldots, (10,0)$.

10. (A)
Observer that $\Delta ABD \cong \Delta EBD$. Thus, $BE = AB = 4$, $AD = DE$.
Hence, area of $\Delta ACE = 2\times$ Area of ΔDCE.

Since, $BC = 3, EC = -BC = 4 - 3 = 1$. Thus,
Area of $\Delta ABC = 3\times$ Area of ΔACE.
$= 6\times$ Area of ΔDCE.
Thus, Area of $ABCD$ = Area of $\Delta ABC +$ Area of ΔACD
$= 6\times$Area of ΔDCE + Area of DCE
$= 7\times$ Area of ΔDCE

11. (B)
Let $AB = CD = a, AD = BC = b$ and $AX = x$.
Then triangles AXY and BXC are similar.
Thus $\frac{b}{AY} = \frac{a-x}{x}$ and $AY = \frac{bx}{a-x}$. Now, $DY = AY + b = \frac{bc}{a-x} + b = \frac{ab}{a-x}$.
Also, $BX = a - x$ and hecne $BX \times DY = ab$, the area of the rectangle $ABCD$.

12. (B)
Cubing both sides of $\sqrt[3]{x} + \sqrt[3]{y} = 4$ we get

$$x + y + 12(xy)^{\frac{1}{3}} = 64$$

Putting $x + y = 28$ in the above, we get $xy = 27$.
Hence, x, y are roots of $t^2 - 28t + 27 = 0$ and $(x, y) = (1, 27)$ or $(27, 1)$.

13. (A)

$$x + \sqrt{x^2 + \sqrt{x^3 + 1}} = 1 \quad \Rightarrow \quad x^2 + \sqrt{x^3 + 1} = (1 - x)^2$$
$$\Rightarrow \quad \sqrt{x^3 + 1} = 1 - 2x$$
$$\Rightarrow \quad x^3 + 1 = 1 - 4x + 4x^2$$
$$\Rightarrow \quad x^3 - 4x^2 + 4x = 0$$
$$\Rightarrow \quad x(x^2 - 4x + 4) = 0$$
$$\Rightarrow \quad x = 0, 2, 2$$

When $x = 2$, the given equation is not satisfied.
Hence $x = 0$ is the only solution.

14. (B)
Let S denote the sum of all numbers.
The middle number is $\frac{S}{9}$. The sum of the five largest number is $5 \times 68 = 340$. The sum of the five smallest numbers is $5 \times 44 = 220$. If we add these two, we would have added the middle numbers twice.
Thus, $S + \frac{S}{9} = 340 + 220 = 560$
Hence, $S = 504$.

15. (D)
Let $QR = b$. Note that the differnce between the areas A and B can be obtianed as the differnce between the area of the triangle PQR minus the area of the semi circle on PQ as diameter.

Area of the semi circle $=\frac{1}{2}\times\frac{22}{7}\times 21\times 21=693\text{ cm}^2$ and area of the triangle $PQR=\frac{1}{2}\times 42\times b=21b$.
Since $21b-693=357$, we have $b=50\text{ cm}$.

16. (B)
Clearly $\alpha_3=\alpha_6=45°$.
We observer $\alpha_1=\alpha_2$ and $\alpha_4=\alpha_5$.
Considering the right triangles contianing α_1 and α_5 we find that $\alpha_1+\alpha_5=90°$.
Similarly, $\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6=(\alpha_3+\alpha_6)+(\alpha_1+\alpha_5)+(\alpha_2+\alpha_4)$
$=270°$

17. (C)
Let a, b, c be the uniform speeds of A, B, C respectively.

$$\frac{x}{a}=\frac{x-30}{b} \quad ...(5)$$

$$\frac{x}{b}=\frac{x-20}{c} \quad ...(6)$$

$$\frac{x}{a}=\frac{x-48}{c} \quad ...(7)$$

From (6) and (7), we get

$$\frac{a}{b}=\frac{x-20}{x-48} \quad ...(8)$$

And from (5), we get

$$\frac{a}{b}=\frac{x}{x-30} \quad ...(9)$$

Hence from (8) and (9), we get

$$\frac{x}{x-30}=\frac{x-20}{x-48}$$

Solving for x, we get $x=300$.

18. (B)

$$\frac{a\sqrt{a}+b\sqrt{b}}{(\sqrt{a}+\sqrt{b})(a-b)}+\frac{2\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\frac{\sqrt{ab}}{a-b}$$

$$=\frac{a\sqrt{a}+b\sqrt{b}+2\sqrt{b}(a-b)-\sqrt{ab}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(a-b)}$$

$$=\frac{a\sqrt{a}-b\sqrt{b}+a\sqrt{b}-b\sqrt{a}}{(\sqrt{a}+\sqrt{b})(a-b)}$$

$$=\frac{(\sqrt{a}+\sqrt{b})(a-b)}{(\sqrt{a}+\sqrt{b})(a-b)}$$

$$=1$$

19. (A)
There are total 100 numbersm, out of which
50 numbers are divisibel by 2,
33 numbers are divisibel by 3,
20 numbers are divisibel by 5

Following are counted twice above
16 numbers are divisible by both 2 and 3
10 numbers are divisible by both 2 and 5
6 numbers are divisible by both 3 and 5

Following is counted thrice above
3 numbers are divisble by all 2, 3 and 5

So total numbers divisible by 2, 3 and 5 are
= 50 + 33 + 20 – 16 – 10 – 6 + 3
= 103 – 29
= 74
So probability that a numbers is number is not divisibel by 2, 3 and 5 $=(100-74)/100=0.26$

20. (C)
$\angle ACD=\angle CBD$, being the angel in the alternate segemtn and $CD = DB$ sicne D is the mid point of the arc BC. Thus $\angle BCD=\angle DBC$. Thus CD biects the angle $\angle ACB$. Similarly, BD bisects the angel $\angle ABC$. Consequently, D is the inceter of the triangle ABC.

21. (D)
There is no way to reduce the cuts to fewer than 6: Just consider the middle cube (tha one which has no exposed surfaces int eh beginning), each of the its sides requires at least one cut.

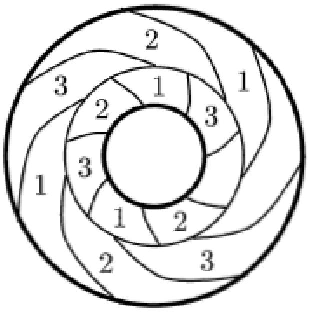
22. (A)
$$\frac{(1\times2\times3)+(2\times4\times6)+(3\times6\times9)+....+(335\times670\times1005)}{(1\times3\times6)+(2\times6\times12)+(3\times9\times18)+....+(335\times1005\times2010)}$$
$$=\frac{(1\times2\times3)\left[1^3+2^3+3^3+....+335^3\right]}{(1\times3\times6)\left[1^3+2^3+3^3+....+335^3\right]}$$
$$=\frac{1\times2\times3}{1\times3\times6}=\frac{1}{3}.$$

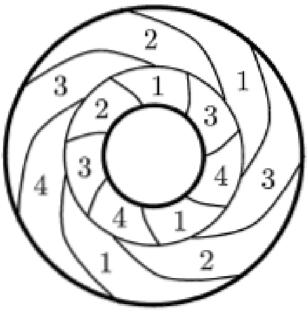
23. (D)
From the given equations, we obtain
$$a+b+c+d=a(r+1),\ a+b+c+d=b(r+1),$$
$$a+b+c+d=c(r+1), a+b+c+d=d(r+1).$$
Adding these four equations gives
$$4(a+b+c+d)=(a+b+c+d)(r+1),$$
that is,
$$(3-r)(a+b+c+d)=0.$$
Thus $r=3$, or $a+b+c+d=0$. If $a+b+c+d=0$, then we see from the original given equaitons

that $r=-1$. Hence the value of r is either 3 or –1.

24. (B)
Let $K=N_2+....+N_{2010}$. Then $X=(N_1+K)(K+N_{2011})$ and $Y=(N_1+K+N_{2011})K$.
$X-Y=(N_1K+K^2+N_1N_{2011}+KN_{2011})-(N_1K+K^2+N_{2011}K)=N_1N_{2011}>0$.

25. (B)
The left shows that 3 colours are not enough. The right is a painting using 4 colours.





26. (B)
Only the last statemetn is correct: $ac^2<bc^2$ implies $c^2>0$, hence $a<c$. For other statemetns, counterexamples can be take as $a=-2, b=-1; a=0$ and $a=0$ respectively.

27. (D)
Use Inclusion and Exclusion Principle.

28. (D)
Just count: Lable the "center" O. There are 6 triangles like ΔAFO; 3 like ΔAOB; 6 like ΔABD and 1 like ΔABC. Total: 16.

29. (C)
$x+y+z=9+\frac{S}{2}$.
So, $x=1+\frac{S}{2}, y=9-\frac{S}{2}$ and $z=-1+\frac{S}{2}$.
Since, $x, y, z\geq 0$, we have $2\leq S\leq 18$.

30. (A)
Assume $p=\frac{a}{b}$, then $q=\frac{a+10}{b+10}$, since $b>a,$ it implies $p<q$.