IIT – JEE: 2025	**TW TEST (MAIN)**	**DATE: 16/03/24**
TIME: 1 Hr.	**TOPIC: SIMPLE HARMONIC MOTION**	**MARKS: 100**

Answer Key & Solution

1. (A)

In $y = A\sin\omega t + B$, the oscillating part is $A\sin\omega t$, so amplitude of motion is A.

2. (C)

Given, $y_1 = 0.1\sin(100\pi t + \pi/3)$

And $y_2 = 0.1\sin(\pi t + \pi/2)$

So $\phi = \phi_1 - \phi_2 = \pi/3 - \pi/2 = -\pi/6$

3. (B)

$a = -bx$, on comparing with $a = -\omega^2 x$

We get $\omega = \sqrt{b}$.

$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{b}}$

4. (C)

At $T = 2\pi\sqrt{\frac{\ell}{g}}$, so time period becomes double when length becomes four times.

5. (B)

When some mercury is drained off, the centre of gravity of the bob moves down and so length of the pendulum increases, which result increase in time period.

6. (B)

$g = \frac{GM}{R^2}$. On the planet $g' = \frac{G(2M)}{(2R)^2} = \frac{g}{2}$.

$T = 2\pi\sqrt{\ell/g}$ and $T' = 2\pi\sqrt{\ell/(g/2)} = \sqrt{2}T$

$= \sqrt{2} \times 2 = 2\sqrt{2}$.

7. (C)

$T = 2\pi\sqrt{\frac{\ell}{g}}$,

$$T' = 2\pi\sqrt{\frac{\ell}{g+a}} = 2\pi\sqrt{\frac{\ell}{\left(g+\frac{g}{4}\right)}} = \frac{2}{\sqrt{5}}T.$$

8. (D)

$$\omega_1 A_1 = \omega_2 A_2$$

Or $$\sqrt{\frac{k_1}{m}}A_1 = \sqrt{\frac{k_2}{m}}A_2$$

Or $$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}.$$

9. (A)

$$T_1 = \frac{T}{12} \text{ and } T_2 = \frac{T}{6}$$

Clearly $T_2 = 2T_1$

10. (A)

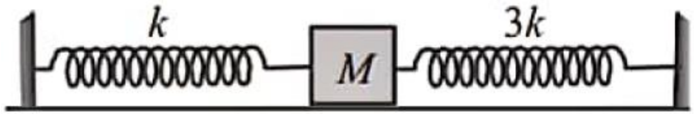
$$x = \sin\omega t - \cos\omega t$$

$\therefore$ $$\frac{d^2x}{dt^2} = -\omega^2(\sin\omega t - \cos\omega t)$$

$=$ $-\omega^2 x$. So it represents SHM.

11. (B)

The equivalent system is shown in figure.



The equivalent force constant of which

$$k_e = k + 3k = 4k$$

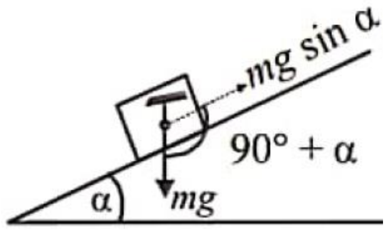
$\therefore$ $$\omega = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$$

And $$f = \frac{\omega}{2\pi} = \frac{2}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{\pi}\sqrt{\frac{k}{m}}.$$

12. (A)

$$a_{net} = \sqrt{g^2 + (g\sin\alpha)^2 + 2g(g\sin\alpha)\cos(90^o + \alpha)}$$

$$= g\cos\alpha.$$



$\therefore \qquad T = 2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$

13. (D)

The half of the oscillation is completed with one spring and other half oscillation with two springs and so

$$T' = \left[\frac{T}{2}\right]_{\text{one spring}} + \left[\frac{T}{2}\right]_{\text{Two springs in parallel}}$$

$$= \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{2k}}$$

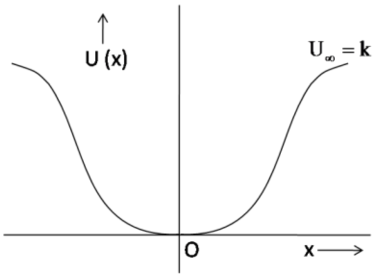
14. (D)

Shows the plot of U(x) versus x. At x = 0, potential energy U(0) = k[1 – exp (0)] = k(1 – 1) = 0 and it has a maximum value = k at x = $\pm\infty$ since

$$U(\pm\infty) = k\left[1 - \exp(-\pm\infty)^2\right] = k(1-0) = k$$

Since the total mechanical energy has a constant value = (k/2), the kinetic energy will be maximum at x = 0 and minimum at $x = \pm\infty$. At x = 0

$$\left(\frac{dU}{dx}\right)_{x=0} = \left[2kx\exp(-x^2)\right]_{\text{at } x=0} = 0$$



Hence the particle is in stable equilibrium at x = 0 (origin) and would oscillate abut x = 0 (for small displacements) simple harmonically. Hence (D) is the only correct choice.

15. (C)

The reduced mass of the system

$$\mu = \frac{mm}{m+m} = \frac{m}{2}$$

And $\quad k_e = \dfrac{k \times 2k}{k + 2k} = \dfrac{2k}{3}$

Time period $T = 2\pi\sqrt{\dfrac{\mu}{k_e}} = 2\pi\sqrt{\dfrac{m/2}{2k/3}} = 2\pi\sqrt{\dfrac{3m}{4k}}$

16. (C)

We can write

$$\frac{d^2y}{dt^2} + \frac{9}{4}y = 0$$

On comparing with $a = -\omega^2 y$, we get

$$\omega = \frac{3}{2}.$$

17. (C)

Given, $x = 3\sin \pi t + 4\cos \omega t$

The general equation of SHM can be written as

$$x = A\sin(\omega t + \phi)$$

Or $\quad x = A\sin \omega t \cos\phi + A\cos \omega t \sin\phi$

On comparing two equations, we get

$A\sin\phi = 4$ and $A\cos\phi = 3$

So, $\quad x = 5\sin(\pi t + \phi)$

Also $\quad \omega = \pi$

Or $\quad f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}$ Hz.

18. (B)

Given, $1 = A\cos(\omega x_0 + \theta)$

Or $\quad A\cos\theta = 1 \quad$ ……….. (i)

Also velocity $v = \omega A\sin(\omega t + \theta)$

Or $\quad \pi = \pi A\sin(\omega \times 0 + \theta)$

Or $\quad A\sin\theta = 1 \quad$ ……….. (ii)

Squaring equations (i) and (ii) and adding, we get

$$A = \sqrt{2}$$

19. (A)

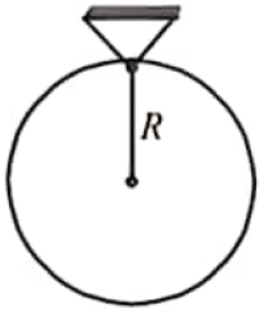
When m_1 is removed, the unbalanced upward force is $= m_1 g$.

So amplitude of motion $= \frac{m_1 g}{k}$.

20. (B)

The restoring torque (for small displacement),

$\tau_{rest} = -mg(R\theta)$



$\therefore \quad \alpha = \frac{\tau_{rest}}{1} = \frac{mgR}{2mR^2}(-\theta) = \frac{g}{2R}$

$\therefore \quad T = 2\pi\sqrt{\frac{2R}{g}}$

The length of equivalent pendulum

$\ell = 2R$.

21. (4)

Time taken by particle to travel from mean position to given position $= 4 - 2 = 2\,\text{sec}$.

$x = A\sin\omega t$

$= A\sin\frac{2\pi}{16}\times 2$

$= \frac{A}{\sqrt{2}}$

$\therefore \quad V = \omega\sqrt{A^2 - x^2}$

$= \frac{\omega A}{\sqrt{2}} = \frac{2\pi}{16}\times\frac{32\sqrt{2}}{\pi}\times\frac{1}{\sqrt{2}} = 4\,\text{m/s}$.

22. (5)

$KE = \frac{1}{2}m\omega^2A^2$

$\Rightarrow \quad 18 = \frac{1}{2}\times 1\times\omega^2\times 36\times 10^{-4}$

$\Rightarrow \quad \omega = \sqrt{10000}$

$= 100\,\text{rad/s}$

23. (2)

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\,\text{sec}$.

In 2.5 sec, it completes $\frac{5}{4}$ oscillation

Total distance $4a + a$

$= 5a$

24. (0)

During one line period, change in velocity becomes zero.

$\therefore$ Average acceleration is zero.

25. (4)

Time period of simple pendulum is independent of amplutide.

26. (1)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow 2 = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow 1 = \pi^2 . \frac{\ell}{g} \Rightarrow \boxed{\ell = 1\text{m}}$$

27. (2)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow 8 = 2\pi\sqrt{\frac{m}{k}} \quad \ldots\ldots\ldots\ldots (1)$$

&, $T^1 = 2\pi\sqrt{\frac{m}{4k}}$

$$= \frac{2\pi}{2} . \sqrt{\frac{m}{4k}}$$

$$= \frac{8}{2} \quad \text{(from (1))}$$

$$= 4 \text{sec}$$

28. (27)

$$\frac{KE}{PE} = \frac{\frac{1}{2} m.\omega^2 \left(A^2 - \frac{A^2}{4} \right)}{\frac{1}{2} m\omega^2 A^2/4} = \frac{3}{1}$$

$\therefore$ $n = 27$

29. (4)

$$F = \frac{-dU}{dx}$$

$$\Rightarrow F = -2(x - 4)$$

At mean position $F = 0$

$$\Rightarrow x - 4 = 0$$

$$\Rightarrow x = 4 \text{m}$$

30. (0)

Total mechanical energy is constant.

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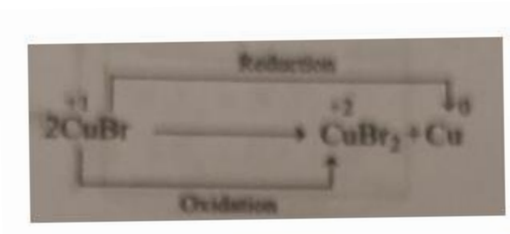
TOPIC: VOLUMETRIC ANALYSIS

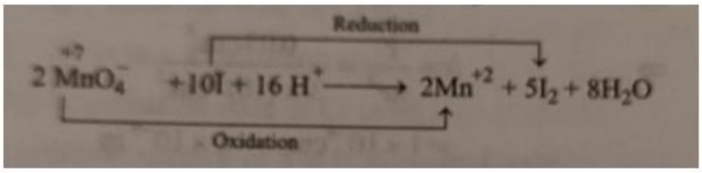
Answer Key & Solution

31. (D)

In disproportionation reaction, same element undergoes oxidation as well as reduction
e.g



Here, CuBr get oxidised to $CuBr_2$ and also it get reduced to Cu. Other given reactions and their types are given below.



In the given reaction, MnO_4^- get oxidised to Mn^{2+} and I^- get reduced to I_2. It is an example of redox reaction. The reaction takes place in acidic medium.

$2KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$

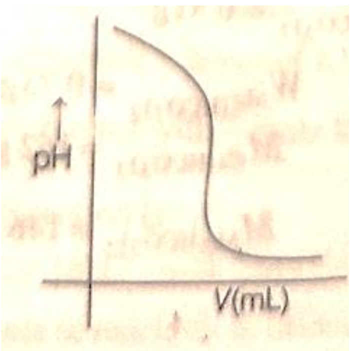
The given reaction is an example of decomposition reaction. Here, one compound split into two or more simpler compounds atleast one of which must be elemental form.

$2NaBr + Cl_2 \longrightarrow 2NaCl + Br_2$

The given reaction is an example of displacement reaction. In this reaction, an atom (or ion) replaces the ion (or atom) of another element from a compound.

32. (B)

The graph that shows the correct change of pH of the titration mixture in the experiment is



In this case, both the titrants are completely ionized

$HCl + NaOH \longrightarrow NaCl + H_2O$

As H^+ is added to a basic solution $\left[OH^+\right]$ decreases and $\left[H^+\right]$ increases. Therefore pH goes on decreasing. As the equivalence point to reached $\left[OH^+\right]$ is rapidly reduced. After this point $\left[OH^+\right]$ decreases rapidly and pH of the solution remains fairly constant. Thus, there is an inflexion point at the equivalence point.

The difference in the volume of NaOH solution between the end point and the equivalence point is not significant for most of the commonly used indicators as there is a large change in the pH value around the equivalence point. Most of them change their colour across this pH change.

33. (A)

The oxidation of a mixture of one mole of each of $FeC_2O_4, Fe_2\left(C_2O_4\right)_3$ $FeSO_4$ and $Fe_2\left(SO_4\right)_3$ in acidic medium with $KMnO_4$ is as follows:

$FeC_2O_4 + KMnO_4 \longrightarrow Fe^{3+} + CO_2 + Mn^{2+}$ …..(i)

$Fe_2\left(C_2O_4\right)_3 + KMnO_4 \longrightarrow Fe^{3+} + CO_2 + Mn^{2+}$ ……(ii)

$FeSO_4 + KMnO_4 \longrightarrow Fe^{3+} + SO_4^{2-} + Mn^{2+}$ ……..(iii)

Change in oxidation number of Mn is 5. Change in oxidation number of Fe in (i), (ii) and (iii) are $+3, +6, +1$, respectively.

$n_{eq} KMnO_4 = n_{eq}\left[FeC_2O_4 + Fe\left(C_2O_4\right)_3 + FeSO_4\right]$

$n \times 5 = 1 \times 3 + 1 \times 6 + 1 \times 1$

$\therefore n = 2$

34. (D)

Given, $W_{Ca(HCO_3)_2} = 0.81 g$

$W_{Mg(HCO_3)_2} = 0.73 g$

$M_{Ca(HCO_3)_2} = 162\, g\, mol^{-1}$,

$M_{Mg(HCO_3)_2} = 146\, mol^{-1}$

$V_{H_2O} = 100\, mL$

Now, $n_{eq}\left(CaCO_3\right) = n_{eq}\left[Ca\left(HCO_3\right)_2\right] + n_{eq}\left[Mg\left(HCO_3\right)_2\right]$

$$\frac{W}{100} \times 2 = \frac{0.81}{162} \times 2 + \frac{0.73}{146} \times 2$$

$$\therefore \frac{W}{100} = 0.005 + 0.005$$

$W = 0.01 \times 100 = 1$

Thus, hardness of water sample $= \frac{1}{100} \times 10^6 = 10,000 ppm$

35. (A)

The reaction tales place as follows,

$H_2C_2O_4 + 2NaOH \longrightarrow Na_2C_2O_4 + 2H_2O$

Now, 50 mL of 0.5 M $H_2C_2O_4$ is needed to neutralize 25 mL of NaOH

$\therefore$ Meq of $H_2C_2O_4$ = Meq of NaOH

$50 \times 0.5 \times 2 = 25 \times M_{NaOH} \times 1$

$M_{NaOH} = 2M$

Now, molarity $= \dfrac{\text{Number of moles}}{\text{Volume of solution (in L)}}$

$= \dfrac{\text{Weight / molecular mass}}{\text{Volume of solution (in L)}}$

$2 = \dfrac{w_{NaOH}}{40} \times \dfrac{1000}{50}$

$w_{NaOH} = \dfrac{2 \times 40 \times 50}{1000} = 4g$

36. (C)

The reaction of HCl with Na_2CO_3 is as follows:

$2HCl + Na_2CO_3 \longrightarrow 2NaCl + H_2O + CO_2$

We know that, M_{eq} of HCl = M_{eq} of Na_2CO_3

$\dfrac{25}{1000} \times 1 \times M_{HCl} = \dfrac{30}{1000} \times 0.1 \times 2$

$M_{HCl} = \dfrac{30 \times 0.2}{25} = \dfrac{6}{25} M$

The reaction of HCl with NaOH is as follows:

$NaOH + HCl \longrightarrow NaCl + H_2O$

Also, M_{eq} of HCl = M_{eq} of NaOH

$\dfrac{6}{25} \times 1 \times \dfrac{V}{1000} = \dfrac{30}{1000} \times 0.2 \times 1$

$V = 25mL$

37. (C)

Reaction of oxalate with permanganate in acidic medium

$5C_2O_4^{2-} + 2MnO_4^- \longrightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$

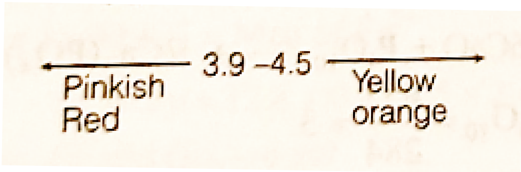
n-factor: $(4-3) \times 2 = 2$ $(7-2) = 5$

Number of moles 5 2 10

$\Rightarrow 5C_2O_4^{2-}$ ions transfer $10e^-$ to produce 10 molecules of CO_2. So, number of electrons involved in producing 10 molecules of CO_2 is 10. Thus, number of electrons involved in producing 1 molecule of CO_2 is 1.

38. (C)

Methyl orange show Pinkish colour towards more acidic medium and yellow orange colour towards basic or less acidic media. Its working pH range is



Weak base have the pH range greater than 7. When methyl orange is added to this weak base solution it shows yellow orange colour.

Now when this solution is titrated against strong acid the pH move towards more acidic range and reaches to end point near 3.9 where yellow orange colour of methyl orange changes to Pinkish red resulting to similar change in colour of solution as well.

39. (D)

n-factor of dichromate is 6.

Also, n-factor of Mohr's salt is 1 as:

$$FeSO_4\left(NH_4\right)_2SO_4.6H_2O \xrightarrow{OH} Fe^{3+}$$

$\because$ 1 mole of dichromate =6 equivalent of dichromate

$\therefore$ 6 equivalent of Mohr's salt would be required

Since , n-factor of Mohr's salt is 1,6 equivalent of its would also the equal to 6 moles.

Hence,1 mole of dichromate will oxidise 6 moles of Mohr's salt.

40. (B)

The following reaction occur between $S_2O_3^{2-}$ and $Cr_2O_7^{2-}$:

$$26H^+ + 3S_2O_3^{2-} + 4Cr_2O_7^{2-} \longrightarrow 6SO_4^{2-} + 8Cr^{3+} + 13H_2O$$

Change in oxidation number of $Cr_2O_7^{2-}$ per formula units is 6 (it is always fixed for $Cr_2O_7^{2-}$)

Hence, equivalent weight of $K_2Cr_2O_7 = \dfrac{\text{Molecular weight}}{6}$

41. (C)

It is an example of disproportionation reaction because the same species $\left(ClO^-\right)$ is being oxidised to ClO_3^- as well as reduced to Cl^-.

42. (A)

Oxalic acid dihydrate $H_2C_2O_4.2H_2O$: mw $= 126$

It is a dibasic acid, hence equivalent weight $= 63$

$\Rightarrow$ Normality $= \dfrac{6.3}{63} \times \dfrac{1000}{250} = 0.4N$

$\Rightarrow$ $N_1V_1 = N_2V_2$

$\Rightarrow$ $0.1 \times V_1 = 0.4 \times 10$

Hence, $V_1 = 40mL$

43. (D)

In MnO_4^- oxidation state of Mn is $+7$

In $Cr\left(CN\right)_6^{3-}$, oxidation state of Cr is +3

In NiF_6^{2-} ,Ni is in +4 oxidation state.

In CrO_2Cl_2, oxidation state of Cr is $+6$

44. (A)

In S_8, oxidation number of S is 0, elemental state.

In S_2F_2, F is in -1 oxidation state, hence S is in $+1$ oxidation state.

In H_2S, H is in +1 oxidation state, hence S is in -2 oxidation state.

45. (B)

The balanced redox reaction is:

$$3MnO_4^- + 5FeC_2O_4 + 24H^+ \longrightarrow 3Mn^{2+} + 5Fe^{3+} + 10CO_2 + 12H_2O$$

$\because$ 5 moles of FeC_2O_4 require 3 moles of $KMnO_4$

$\therefore$ 1 mole of FeC_2O_4 will require $\frac{3}{5}$ mole of $KMnO_4$

46. (A)

The balanced chemical reaction is:

$$2MnO_4^- + 5SO_3^{2-} + 6H^+ \longrightarrow 2Mn^{2+} + 5SO_4^{2-} + 3H_2O$$

$\because$ 5 moles SO_3^{2-} reacts with 2 moles of $KMnO_4$

$\therefore$ 1 mole of SO_3^{2-} will react with $\frac{2}{5}$ mole $KMnO_4$

47. (A)

The balanced redox reaction is:

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 16H_2O$$

Hence, the coefficients of reactants in balanced reaction are 2,5 and 16 respectively

48. (B)

Volume strength of H_2O_2 = Normality $\times 5.6 = 1.5 \times 5.6 = 8.4V$

49. (C)

In $Ba(H_2PO_2)_2$, oxidation number of Ba is $+2$. Therefore ,

$H_2PO_2^- : 2\times(+1) + x + 2\times -2 = -1$

$\Rightarrow \quad x = +1$

50. (B)

Equivalent weight in redox system is defined as:

$$E = \frac{\text{Molar mass}}{\text{n} - \text{factor}}$$

Here n-factor is the net change in oxidation number per formula unit of oxidising or reducing agent. In the present case, n-factor is 2 because equivalent weight is half of molecular weight. Also,

n-factor		
	$MnSO_4 \longrightarrow \frac{1}{2}Mn_2O_3$	$1(+2 \longrightarrow +3)$
	$MnSO_4 \longrightarrow MnO_2$	$2(+2 \longrightarrow +4)$
	$MnSO_4 \longrightarrow MnO_4^-$	$5(+2 \longrightarrow +7)$

$$MnSO_4 \longrightarrow MnO_4^{2-} \qquad 4(+2 \longrightarrow +6)$$

Therefore, $MnSO_4$ converts to MnO_2.

51. (32)

$$\overset{+4}{S}O_2(g) + 2H_2O \longrightarrow \overset{+2}{H_2}\overset{+6-8}{SO_4} + 2H^+ + 2e^-$$

Change in oxidation number of sulphur $= 6 - 4 = 2$

Equivalent mass of $SO_2 = \frac{\text{Molecular mass}}{2} = \frac{64}{2} = 32$

52. (49.0)g

(a) Molecular mass of $H_3PO_4 = (3+31+64) = 98\,g$. H_3PO_4 when neutralised to HPO_4^{2-}, two H^+ ions have been replaced.

Thus, eq. mass $= \frac{\text{Mol.mass}}{\text{No.of replaceable hydrogen atoms}}$

$= \frac{98}{2} = 49.0\ g$

53. (6)

$$\underset{1\,mol}{Cr_2O_7^{2-}} + 14H^+ + \underset{6\,mol}{6e^-} \longrightarrow 2Cr^{3+} + 7H_2O$$

$\therefore 0.01\,M\,Cr_2O_7^{2-} \equiv 0.06\,N\,Cr_2O_7^{2-}$

Number of millimoles $= M \times V = 0.01 \times 100 = 1$

Number of milliequvalents $= N \times V = 0.06 \times 100 = 6$

54. (5)

We know that,

$M_1V_1 + M_2V_2 = M_R(V_1 + V_2)$

$x \times 250 + y \times 500 = 1.6(2000)$

$x + 2y = 1.6 \times 8$

$x + 2y = 12.8$

$\frac{x}{y} + 2 = \frac{12.8}{y}$

$\frac{5}{4} + 2 = \frac{12.8}{y}$

$\frac{5}{4} + 2 = \frac{12.8}{y}$

$\frac{13}{4} = \frac{12.8}{y}$

$y = \frac{12.8 \times 4}{13} = 3.94$

$x = 4.92$

55. (2)

A^{n+} is oxidised to AO_3^-

Change in oxidation number $= 5\left(\text{in } AO_3^-\right) - n\left(\text{in } A^{n+}\right)$

$= 5 - n$ ......(i)

2.68×10^{-3} mol of A^{n+} ion react with 1.6×10^{-3} mol of MnO_4^- ions.

$\therefore$ 1 mol of A^{n+} ion will react with $\dfrac{1.6\times10^{-3}}{2.68\times10^{-3}}$ mol of MnO_4^- ions

$= 0.579\,\text{mol}$ of MnO_4^- ions

$$2K\overset{+7}{Mn}O_4 + 3H_2SO_4 \longrightarrow K_2SO_4 + 3H_2O + 5[O]$$

Number of equivalents of MnO_4^- used in oxidation of A^{n+} to

$AO_3^- = 0.597\times5 = 2.985 \approx 3$

Thus, from equation (i), $5 - n = 3$

$n = 2$

56. (37)

MnO_4^-	$\longrightarrow$	Mn^{2+}
(Oxidaiton number of Mn $= +7$)		(Oxidaiton number of Mn $= +2$)

Equivalent mass of $KMnO_4 = \dfrac{\text{Molecular mass}}{\text{Change in Oxidation number}}$

$= \dfrac{158}{5} = 31.6$

57. (63.00)

$$\underset{(\text{NaOH})}{N_1V_1} = \underset{(\text{Acid})}{N_2V_2}$$

$\dfrac{1}{10}\times25 = N_2\times20$

$N_2 = \dfrac{25}{10\times20} = 0.125$

$N_2 = \dfrac{25}{10\times20} = 0.125$

Strength = Noramlity $\times$ Eq. mass

Eq. mass of the acid $= \dfrac{7.875}{0.125} = 63.00$

58. (2)

25 mL of $\dfrac{N}{15}$ NaOH solution $\equiv$ 25mL of $\dfrac{N}{15}$ oxalic acid solution

Mass of oxalic acid present in 25 mL of $\dfrac{N}{15}$ oxalic acid solution

$= \dfrac{N\times E\times V}{1000} = \dfrac{1\times(90+18x)\times25}{15\times2\times1000}$

$= \frac{(90+18x)}{1200} g$

Actually $\frac{(90+18x)}{1200}$ g oxalic acid is present in 16.68 mL

250 mL of the solution contains oxalic acid

$= \frac{(90+18x)\times 250}{1200\times 16.68} = 1.575(\text{given})$

Or $90+18x = \frac{1.575\times 1200\times 16.68}{250} = 126$

Or $18x = 126-90 = 36$

$x = 2$

59. (30.0)

60 mL 0.5 N $Na_2S_2O_3 \equiv 60 mL\, 0.5 N\, I_2$

$\equiv 60 mL\, 0.5 N\, Cl_2$

Amount of chlorine $= \frac{35.5\times 0.5}{1000}\times 60 = 1.065 g$

% of available chlorine $= \frac{1.965}{3.55}\times 100 = 30.0$

60. (3)

Number of moles of $KMnO_4 = \frac{MV}{1000} = \frac{0.145\times 46.9}{1000}$

$= 6.8\times 10^{-3}$

Number of moles of $H_2O_2 = 6.8\times 10^{-3}\times 2.5 = 0.017$

Mass of $H_2O_2 = 0.017\times 34 = 0.578$

Mass % of $H_2O_2 = \frac{0.578}{20}\times 100 \approx 2.9$

IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 16/03/24**

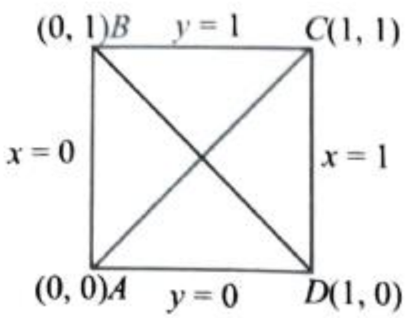
TOPIC: STRAIGHT LINES & POSL

SOLUTIONS

61. (B)
Equation of straight line perpendicular to 3x – 4y – 7 = 0 is 4x + 3y + c = 0
It distance of origin is
$$\frac{|c|}{\sqrt{3^2+4^2}}=10 \Rightarrow |c|=50$$
$c=\pm 50$

62. (C)
Required ratio is
$$-\left(\frac{3+4-10}{9+16-10}\right)=\frac{3}{15}=\frac{1}{5}$$

63. (A)
Coordinates of the vertices of the square are A(0, 0), B(0, 1), C(1, 1) and D(1, 0).



Now the equation of AC is y = x
and that of BD is
$$y-1=-\frac{1}{1}(x-0)$$
$$\Rightarrow \quad x+y=1$$

64. (B)
The equation of the line joining the points (2, –1) and (5, –3) is given by
$$y+1=\frac{-1+3}{2-5}(x-2)$$
or $2x+3y-1=0$ … (1)
since, $(x_1, 4)$ and $(-2, y_1)$ lie on $2x+3y-1=0$, therefore

$$2x_1 + 12 - 1 = 0 \Rightarrow x_1 = -\frac{11}{2} \text{ and } -4 + 3y_1 - 1 = 0 \Rightarrow y_1 = \frac{5}{3}$$

Thus, (x_1, y_1) satisfies 2x + 6y + 1 = 0

65. (B)

The given lines

$$ax \pm by \pm c = 0$$

$$\Rightarrow \frac{x}{\pm c/a} + \frac{y}{\pm c/b} = 1$$

the vertex at A(c/a, 0), C(–c/a, 0), B(0, c/b), D(0, –c/b).

Therefore, the diagonals AC and BD of quadrilateral ABCD are perpendicular.

Hence, it is a rhombus whose area is given by

$$\frac{1}{2} \times AC \times BD = \frac{1}{2} \times \frac{2c}{a} \times \frac{2c}{b} = \frac{2c^2}{ab}$$

66. (C)

The given inequality is equivalent to the following system of inequalities.

$2x + 3y \le 6$, where $x \ge 0, y \ge 0$

$2x - 3y \le 6$, when $x \ge 0, y \le 0$

$-2x + 3y \le 6$, when $x \le 0, y \ge 0$

$-2x - 3y \le 6$, where $x \le 0, y \le 0$

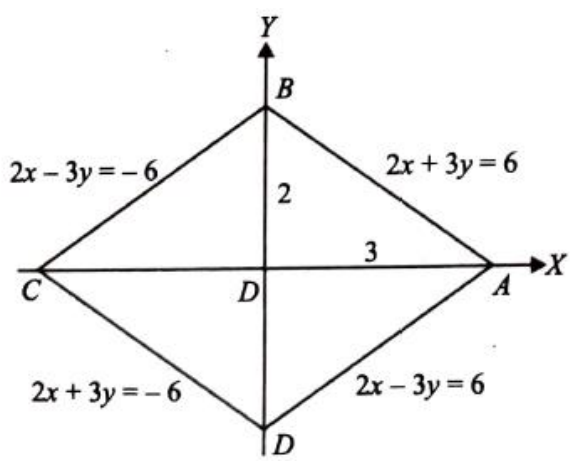
Which represents a rhombus with sides

$2x \pm 3y = 6$ and $2x \pm 3y = -6$

Length of the diagonals is 6 and 4 units along x- and y-axes.

Therefore, the required area is

$\frac{1}{2} \times 6 \times 4 = 12$ sq. units.



67. (A)

Extremities of the given diagonal are (4, 0) and (0, 6).

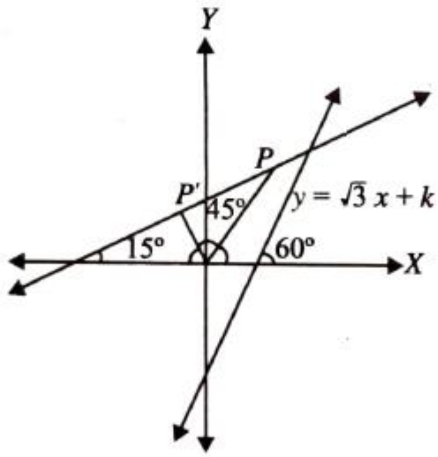
Hence, slope of this diagonal is –3/2 and slope of other diagonal is 2/3.

The equation of the other diagonal is

$$\frac{x-2}{\frac{3}{\sqrt{13}}} = \frac{y-3}{\frac{2}{\sqrt{13}}} = r$$

For the extremities of the diagonal, $r = \pm\sqrt{13}$.
Hence, $x - 2 = \pm 3,\ y - 3 = \pm 2$
$x = 5, -1$ and $y = 5, 1$
Therefore, the extremities of the diagonal are (5, 5) and (–1, 1).

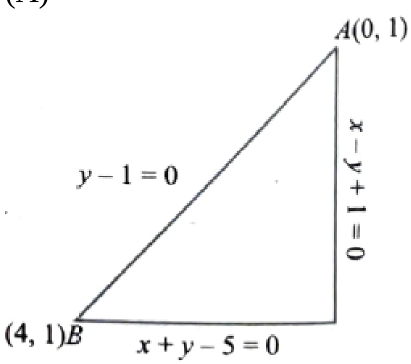
68. (D)



Angle between both the lines is 45°.

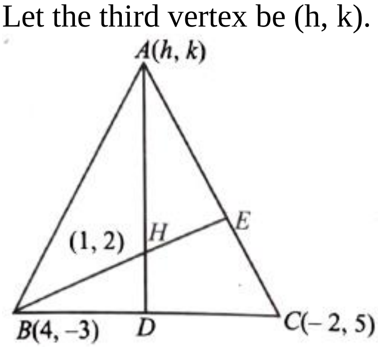
Hence $OP = OP'\sqrt{2} = \left(\frac{5}{\sqrt{2}}\right) \times \sqrt{2} = 5$.

69. (A)



Since the triangle is right angled, so the cirumcentre will be the middle point of hypotenuse, i.e. (2, 1).

70. (B)
Let the third vertex be (h, k).



Now slope of AD is (k – 2)/(h – 1) slope of BC is (5 + 3)/(–2 – 4) = –4/3, slope of BE is (–3 – 2)/(4 – 1) = 5/3 and slope of AC is (k – 5)/(h + 2).
Since AD ⊥ BC, so

$$\frac{k-2}{h-1}\times\frac{-4}{3}=-1$$

$\Rightarrow\ 3h-4k+5=0 \qquad ...(1)$

Again since $BE\perp AC$, so

$$-\frac{5}{3}+\frac{k-5}{h+2}=-1$$

$\Rightarrow\ 3h-3k+31=0 \qquad ...(2)$

On solving (1) and (2) we get h = 33, k = 26.
Hence, the third vertex is (33, 26).

71. (C)
For any point P(x, y) that is equidistant from given line, we have

$$x+y-2=-\left(x+y-2\sqrt{2}\right)$$

$\Rightarrow\ 2x+2y-3\sqrt{2}=0$

72. (D)
All value of a.

73. (B)
The set of lines is 4ax + 3by + c = 0, where a + b + c = 0.
Eliminating c, we get

$$4ax+3by-(a+b)=0$$

$\Rightarrow\ a(4x-1)+b(3y-1)=0$

This passes through the intersection of the lines
4x – 1 = 0 and 3y – 1 = 0, i.e., x = ¼, y = 1/3, i.e., (1/4, 1/3).

74. (B)
The line passing through (2, 3) and perpendicular to –y + 3x + 4 = 0 is

$$\frac{y-3}{x-2}=-\frac{1}{3}$$

or $3y+x-11=0$

Therefore, foot is x = –1/10, y = 37/10.

75. (A)
We have,

$3x+5y=2007\ \Rightarrow\ x+\frac{5y}{3}=669$

Clearly, 3 must divide 5y and so y = 3k, for some $k\in N$.
Thus, $x+5k=669$

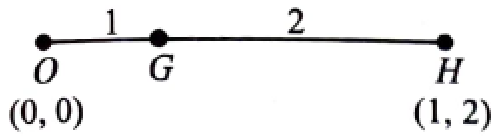
$\Rightarrow\ 5k\le 668$

$\Rightarrow\ k\le\frac{668}{5}\Rightarrow k\le 133$

76. (A)
The point (4, 5) lies on the given line 7x – 3y – 13 = 0.

The locus of the point equidistant from the given point and the line is a line perpendicular to 7x – 3y – 13 = 0 at (4, 5).

77. (B)



$$G \equiv \left(\frac{1\times1+2\times0}{3}, \frac{2\times1+2\times0}{3}\right)$$

$$= \left(\frac{1}{3}, \frac{2}{3}\right)$$

78. (D)
The point Q is (–b, –a) and the point R is (–a, –b).
Therefore, the midpoint of PR is (0, 0).

79. (A)

$$9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$$

$$\Rightarrow (3x - 4y + 2)(3x - 4y - 6) = 0$$

Hence, distance between lines is $\frac{|6-(-2)|}{5} = \frac{8}{5}$.

80. (D)
Here $my(y - mx) + x(y - mx) = 0$, i.e., $(y - mx)(my + x) = 0$.
So the lines are y = mx and y = (–1/m)x.
Bisectors between the lines xy = 0 and y = x and y = –x.
Therefore, m = 1 or –1.

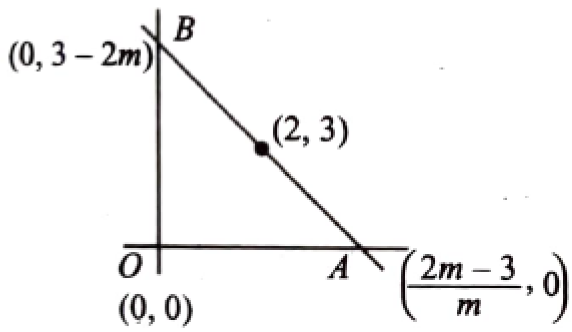
81. (3)
Equation of any line through (2, 3) is

$$y - 3 = m(x - 2)$$

$$\Rightarrow \quad y = mx - 2m + 3$$

From the figure, area of ΔOAB is ± 12.

That is, $\frac{1}{2}\left(\frac{2m-3}{m}\right)(3 - 2m) = \pm 12$

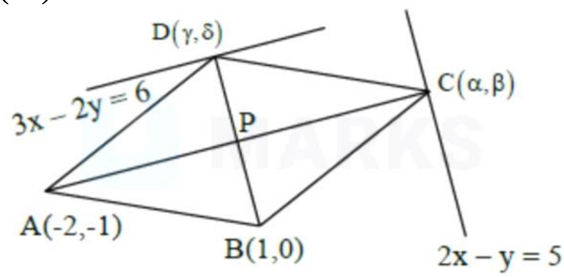


Taking positive sign, we get $(2m+3)^2 = 0$.

This gives one value of m as –3/2. Taking negative sign, we get $4m^2 - 36m + 9 = 0 (D > 0)$

This is a quadratic in m which gives two values of m.
Hence, three straight lines are possible.

82. (32)



$P \equiv \left(\frac{\alpha-2}{2}, \frac{\beta-1}{2}\right) \equiv \left(\frac{\gamma+1}{2}, \frac{\delta}{2}\right)$

$\frac{\alpha-2}{2} = \frac{\gamma+1}{2}$ and $\frac{\beta-1}{2} = \frac{\delta}{2}$

$\Rightarrow \alpha - \gamma = 3$ ...(1)

$\beta - \delta = 1$ ...(2)

Also, $((\gamma, \delta)$ lies on $3x - 2y = 6$

$3\gamma - 2\delta = 6$ ...(3)

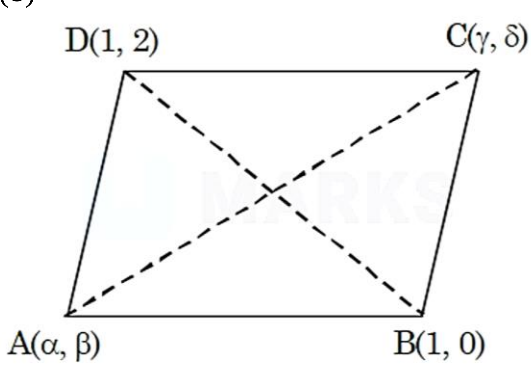
and (α, β) lies on $2x - y = 5$

$\Rightarrow 2\alpha - \beta = 5$ ...(4)

Solving (1), (2), (3), (4)
$\alpha = -3, \beta = -11, \gamma = -6, \delta = -12$

$|\alpha + \beta + \gamma + \delta| = 32$

83. (8)



Let E is mid point of diagonals

$\frac{\alpha+\gamma}{2} = \frac{1+1}{2}$ & $\frac{\beta+\delta}{1} = \frac{2+0}{2}$

$\alpha + \gamma = 2$ $\beta + \delta = 2$

$2(\alpha + \beta + \gamma + \delta) = 2(2+2) = 8$

84. (14)

Locus of point P(x, y) whose distance from given x + 2y + 7 = 0 & 2x – y + 8 = 0 are equal is

$$\frac{x+2y+7}{\sqrt{5}}=\pm\frac{2x-y+8}{\sqrt{5}}$$

$$(x+2y+7)^2-(2x-y+8)^2=0$$

Combined equation of lines

$$(x-3y+1)(3x+y+15)=0$$

$$3x^2-3y^2-8xy+18x-44y+15=0$$

$$x^2-y^2-\frac{8}{3}xy+6x-\frac{44}{3}y+5=0$$

$$x^2-y^2+2hxy+2gx^2+2fy+c=0$$

$$h=\frac{4}{3},\ g=3,\ f=-\frac{22}{3},\ c=5$$

$$g+c+h-f=3+5-\frac{4}{3}+\frac{22}{3}=8+6=14$$

85. (529)

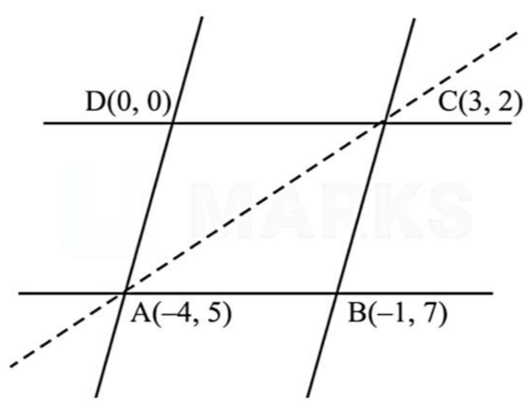
The equations of two adjacent sides of a parallelogram ABCD be 2x – 3y = –23 and 5x + 4y = 23,

So, $AB\equiv 2x-3y=-23$ and $BC\equiv 5x+4y=23$

Also given, $AC\equiv 3x+7y=23$

Solving the above lines we get,

$A(-4,5),\ B(-1,7),\ C(3,2)$



We know that,

Diagonal of parallelogram have same midpoint,

So AC and BD have same mid-point and let point D be (x, y),

$$\frac{x-1}{2}=\frac{-4+3}{2}\Rightarrow x=0 \text{ and } \frac{y+7}{2}=\frac{2+5}{2}\Rightarrow y=0$$

Hence, point D is (0, 0)

Now equation of BD will be 7x + y = 0

Now finding the distance of A(–4, 5) form 7x + 5 = 0 we get,

$$d=\left|\frac{7(-4)+5}{\sqrt{7^2+1^2}}\right|=\frac{23}{\sqrt{50}}$$

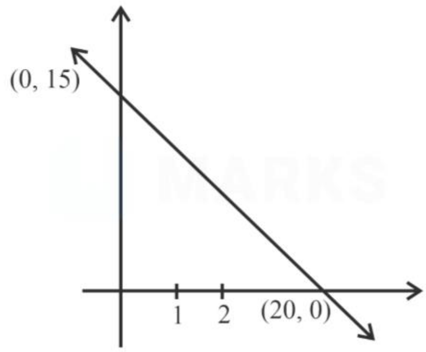
Hence, $50d^2=23^2=529$

86. (31)

Given:

$3x + 4y = 60$

$\Rightarrow \frac{x}{20} + \frac{y}{15} = 1$



If $x = 1, y = \frac{57}{4} = 14.25$

So, points are

(1, 1) (1, 2) – (1, 14) $\Rightarrow$ 14 points.

If $x = 2, y = \frac{27}{2} = 13.5$

So, points are

(2, 2) (2, 4) … (2, 12) $\Rightarrow$ 6 points.

If $x = 3, y = \frac{51}{4} = 12.75$

So, points are (3, 3) (3, 6) – (3, 12) $\Rightarrow$ 4 points.

So, points are

(4, 4) (4, 8) $\Rightarrow$ 2 points.

If $x = 5, y = \frac{45}{4} = 11.25$

So, points are

(5, 5), (5, 10) $\Rightarrow$ 2 points.

If $x = 6, y = \frac{21}{2} = 10.5$

So, point is

$(6, 6) \Rightarrow 1$ points

If $x = 7, y = \frac{39}{4} = 9.75$

So, point is

$(7, 7) \Rightarrow 1$ point

If x = 8, y = 9

So, point is

$(8, 8) \Rightarrow 1$ point

If $x = 9 \Rightarrow y = \frac{33}{4} = 8.25 \Rightarrow$ no point

Total points inside the triangle = 31 points

87. (5)

P must be centroid of ΔABC

$$\therefore \ P\left(\frac{17}{6},\frac{8}{3}\right) \Rightarrow PQ=\sqrt{\left(\frac{24}{6}\right)^2+\left(\frac{9}{3}\right)}=5 \text{ units}$$

88. (8)

Eliminating x and y from the three equations, we get

$$-2=m(a+m)$$

$$\Rightarrow \ m^2+am+2=0.$$

Since $m \in R$,

Discriminant ≥ 0

$$\Rightarrow a^2-8\geq 0$$

$$\Rightarrow \ a^2 \geq 8$$

89. (1)

The equations of the line L in the two coordinates systems are

$$\frac{x}{a}+\frac{y}{b}=1 \text{ and } \frac{x}{p}+\frac{y}{q}=1$$

Where (x, y) are the new coordinates of a point (x, y) when the axes are rotated through a fixed angle, keeping the origin fixed.

As the length of the perpendicular from the origin has not changed,

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$$

$$\Rightarrow \ \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{p^2}$$

$$\Rightarrow \ \frac{\frac{1}{a^2}+\frac{1}{b^2}}{\frac{1}{p^2}+\frac{1}{q^2}}=1$$

90. (81)

$$\frac{3}{a}=\frac{a}{27} \ \Rightarrow \ a^2=81$$