IIT – JEE: 2025 **TW TEST (ADV)** **DATE: 17/03/24**

TOPIC: SIMPLE HARMONIC MOTION

Answer Key & Solution

1. (B)
Amplitude of the first simple harmonic motion is
$$A_1 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{2}\right)^2} = a$$
Amplitude of the second motion is
$$A_2 = \sqrt{a^2 + a^2} = \sqrt{2}a$$
$$\therefore \quad \frac{A_1}{A_2} = \frac{1}{\sqrt{2}}.$$
Hence the correct choice is (B).

2. (B)
Assuming the collision lasts for a small interval. We can apply conservation of momentum and get the common velocity $= \frac{\upsilon}{2}$
$$K.E = \frac{1}{2}(2m)\left(\frac{\upsilon}{2}\right)^2 = \frac{1}{4}m\upsilon^2$$
This is also the total energy of system as the spring is unstretched at this moment. If the amplitude is A, total energy $= \frac{1}{2}kA^2$
$$\therefore \frac{1}{2}kA^2 = \frac{1}{4}m\upsilon^2$$
$$\therefore A = \sqrt{\frac{m}{2k}}.\upsilon$$

3. (D)
$$\frac{A}{2} = A\sin\phi_1$$
And $\frac{-\sqrt{3}}{2}A = A\cos\phi_2$
$$\Rightarrow \phi_1 = \frac{\pi}{4}$$
And $\phi_2 = \frac{2\pi}{3}$

So, $\phi_2 - \phi_1 = \frac{11\pi}{12}$

4. (C)

The angular frequency of the system is

$$\omega = \left[\frac{k}{(M+m)}\right]^{1/2} \qquad (1)$$

The upper block of mass m will not slip over the lower block of mass M if the maximum force on the upper block f_{max} does not exceed the frictional force μmg between the two blocks. Now

$$f_{max} = ma_{max} = m\omega^2 A_{max} \qquad (2)$$

Where a_{max} is the maximum acceleration and A_{max} is the maximum amplitude. Using (1) in (2), we get

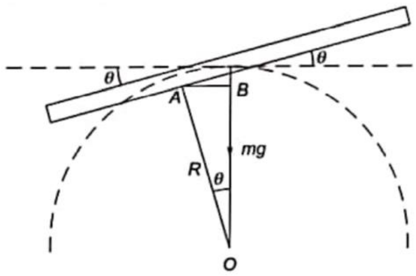
$$f_{max} = \frac{mk\,A_{max}}{(M+m)}$$

For no slipping , $f_{max} = \mu mg$

Or $\frac{mk\,A_{max}}{(M+m)} = \mu mg$ or $A_{max}\ \frac{\mu(M+m)g}{k}$,

Which is choice (C)

5. (C)



Refer to Figure. The magnitude of the restoring torque= force $\times$perpendicular distance

$= mg \times AB = mg \times R\sin\theta$

Since θ is small, $\sin\theta \simeq \theta$ here θ is expressed in radian. The equation of motion of the scale is

$$I\frac{d^2\theta}{dt^2} = -mgR\theta$$

Or $\frac{d^2\theta}{dt^2} = \left(-\frac{mgR}{I}\right)\theta$

$$\therefore \omega = \sqrt{\frac{mgR}{I}} \text{ or } \frac{2\pi}{T} = \sqrt{\frac{mgR}{I}} \text{ or } T = 2\pi\sqrt{\frac{I}{mgR}}$$

Now $I = \frac{mL^2}{12}$. Hence

$$T = \frac{\pi L}{\sqrt{3gR}}$$

Using the values $L = 1m, g = 10ms^{-2}$ and $R = 0.3m$, we get $T = \pi/3$ second. Hence the correct choice is (C)

6. (A,D)

(A) Friction is always opposite relative velocity

(D) Velocity of platform = V_P = dy(t)/dt = (Aωcosωt)j

Velocity of block = V_B = dx(t)/dt = (-Aωsinωt)i

Velocity of block relative to platform = $V_B - V_P$ = (-Aωsinωt)i – (Aωcosωt)j

Friction on block is in the direction opposite to this vector.

7. (A,B,C,D)

$$y = A\sin\frac{2\pi}{T}t$$

$$y\left(t=\frac{T}{8}\right) = A\sin\frac{\pi}{4} = \frac{4}{\sqrt{2}}(C) \text{ and (D)}$$

$$\frac{dy}{dt} = \frac{2\pi}{T}A\cos\frac{2\pi}{T}t$$

$$\Rightarrow \frac{dy}{dt}\left(t=\frac{T}{8}\right) = \frac{2\pi}{T}A\cos\frac{\pi}{t} = \frac{2\pi}{T}\frac{A}{\sqrt{2}} = \frac{\upsilon_{max}}{\sqrt{2}}$$

8. (B,C,D)

Acceleration $a = \frac{-1}{m}\frac{dU(x)}{dx}$

$$= \frac{-1}{2}\times 15(2x-2)$$

$$= -16(x-1)\,m/s^2$$

The particle executes S.H.M.

$$\omega^2 = 16$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}s$$

At $x = 1m, F(x) = 0$ (i.e) corresponds to equilibrium position

$$\omega A = 2m/s \Rightarrow A = 0.5m$$

The particle describes oscillatory motion from

$$x_1 = 0.5\,m \text{ to } x_2 = 1.5m$$

9. (A,D)

The only horizontal force acting on the coin is the force of friction F. Hence the horizontal acceleration is always in the direction of F and its magnitude is $\frac{F}{m}$. The magnitude and the direction of F can thus be obtained from the magnitude and direction of the acceleration.

10. (A,C)

Total energy = translational K.E.+ rotational K.E.+ P.E. stored in spring

$$= \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}m\upsilon^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{\upsilon}{R}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow E = \frac{3}{4}m\upsilon^2 + \frac{1}{2}kx^2 \qquad (1)$$

Now, the total energy of the system must remain constant, i.e. $\frac{dE}{dt} = 0$

Differentiating Eq. (1) with respect to time t and setting $\frac{dE}{dt} = 0$, we have

$$\frac{dE}{dt} = 0 = \frac{3}{4}m\left(2\upsilon\frac{d\upsilon}{dt}\right) + \frac{1}{2}k\left(2x\frac{dx}{dt}\right)$$

Now acceleration $a = \frac{d\upsilon}{dt}$ and velocity $\upsilon = \frac{dx}{dt}$.

Therefore, $\frac{3}{2}m\upsilon a + k\upsilon x = 0$

Or $\upsilon\left(\frac{3}{2}ma + kx\right) = 0$

Since $\upsilon \neq 0$, we have

$\frac{3}{2}ma + kx = 0$

Or $a = -\left(\frac{2}{3}\frac{k}{m}\right)x = -\omega^2 x \qquad (2)$

Where $\omega = \sqrt{\frac{2k}{3m}}$. Hence $T = 2\pi\sqrt{\frac{2k}{m}}$

The correct choices are (A) and (C)

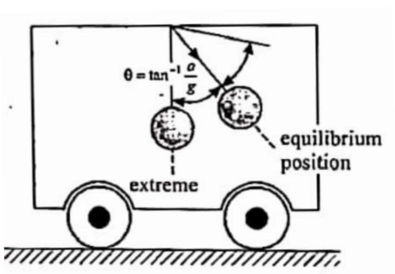
11. (A,B)

Comparing the graph with the relation in velocity and displacement $v = \left[(k/m)\left(A^2 - x^2\right)\right]^{1/2}$ we get option (A) and (C) are correct and maximum acceleration of the particle is given by $a_{max} = kA/m$ which gives option(B) is also correct.

12. (B,C)

Bob will oscillate about equilibrium position with amplitude $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ for any value of a.

If $a << g$, motion will be SHM, and then



Time period will be $2\pi\sqrt{\frac{l}{\sqrt{a^2 + g^2}}}$

13. (B, C)

(B) Amplitude can be found using $v^2 = \omega^2(A^2 - x^2)$ since v, ω and x are known. φ can be found using

$x = A\sin(\omega t + \varphi)$ applied at t=0.

(C) ω can be found first using $v^2 = \omega^2A^2 - \omega^2x^2$ since $A\omega$ is given (maximum speed). Then $V_{max} = A\omega$ gives amplitude A. Finally $x = A\sin(\omega t + \varphi)$ applied at t=0 gives φ.

14. (A,C)

When 3 kg mass is released the amplitude of its oscillations is 2m and at a distance 1m from the equilibrium position we can find the speed of it using the relation $v = \left[(k/m)(A^2 - x^2)\right]^{1/2}$ then by conservation of momentum we can find the resulting speed of the combined mass and the new amplitude using the above relation which gives options (A) and (C) are correct.

15. (A,D)

When block is displaced downwards by x, it experiences an upward force of

$$F = -(d_2Ax - d_1Ax)$$

$$F = -xA(d_2 - d_1)$$

This force is proportional to x, and hence block executes SHM. Displacement will be symmetric about equilibrium position.

16. (1.20)

Given, $y = kt^2; \therefore a = \frac{d^2y}{dt^2} = 2k = 2\times1 = 2\,m/s^2$

Thus $T_1 = 2\pi\sqrt{\frac{\ell}{g}}$ and $T_2 = 2\pi\sqrt{\frac{\ell}{(g+2)}}$. For $g = 10$,

$$\frac{T_1^2}{T_2^2} = \frac{6}{5}$$

17. (1.6)

$$T = 2\pi\sqrt{\frac{m}{K}}$$

1st case $2 = 2\pi\sqrt{\frac{m}{K}} \Rightarrow 4 = 4\pi^2\times\frac{m}{K}$ (1)

2nd case $3 = 2\pi\sqrt{\frac{m+2}{K}} \Rightarrow 9 = 4\pi^2\times\frac{m+2}{K}$ (2)

Solving (1) & (2)

$$\frac{9}{4} = \frac{m+2}{m} \Rightarrow m = \frac{8}{5} = 1.6\text{kg}$$

18. (6)

The maximum normal reaction from the floor occurs when block A is at its lower extreme position because the spring is maximally compressed in this state. At this moment, block A has an acceleration $a\omega^2$ upwards. If the upwards force by spring on block A at this moment be F, then by Newton's second law for A,

$$F - m_Ag = m_A(a\omega^2)$$

$$F = m_Ag + m_A(a\omega^2)$$

The same spring force F acts downwards on block B at this moment. Normal from the floor on B acts upwards and its weight m_Bg acts downwards. The acceleration of B is zero. By Newton's second law for B,

$F + m_B g = N_{max}$ since the acceleration of B is zero

19. (6)

$$K.E. = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

Here, K.E. $= \frac{3}{4}$ T.E.

$$\frac{1}{2} m\omega^2 A^2 \cos^2 \omega t = \frac{3}{4} \times \frac{1}{2} m\omega^2 A^2$$

$$\cos \omega t = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \omega t = \frac{\pi}{6}$$

$$\Rightarrow \frac{2\pi}{2} t = \frac{\pi}{6} = \frac{1}{x}$$

$$\Rightarrow x = 6s$$

20. (5)

At $t_1 = T/4$, it is at mean

At $t_2 = 3T/8$, it is $A/\sqrt{2}$ above mean

Now, amplitude $= x_0 = mg/K = \frac{\sqrt{2}}{20}$ m

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TOPIC: VOLUMETRIC ANALYSIS

Answer Key & Solution

21. **(C)**
Oxidation state of Cr in CrO_5 is +6.

22. **(B)**
24 x 0.1 x 2 = 16 x 0.1 x (7-X)
X = +4,
hence MnO_2

23. **(C)**
Let charge on O_2 group be X.
(+3) + X + (4 x 0) + (1 x 0) + (-1) x 2 = 0
Hence, X = -1
O_2^- is superoxo.

24. **(D)**
3 As_2S_3 + **10** HNO_3 + **4** H_2O → **6** H_3AsO_4 + **10** NO + **9** S

25. **(B)**
n-factor for Trona using phenolphthalein = 1
n-factor for Trona using methyl orange = 3
Volume of HCl solution used in the two cases must be in 1:3 ratio.

26. **(A, D)**
Let 2A millimoles of XO and A millimoles of X_2O_3 be present in the initial sample.
2A x 5 + A x 2 x 4 = 0.015 x 1000 x 6
A = 5, 2A = 10

27. **(B)**
Average oxidation state of S-atoms in $S_2O_7^{2-}$ as well as in $S_2O_8^{2-}$ is +6.

28. **(A, C, D)**
$Ba_2\mathbf{Xe}O_6$ → oxidation state of Xe is +8
$K\mathbf{Au}Cl_4$ → oxidation state of Au is +3
$Rb_4Na[H\mathbf{V}_{10}O_{28}]$ → oxidation state of V is +5
$Na_4\mathbf{P}_2O_7$ → oxidation state of P is +5

29. **(A, C, D)**
KIO_3, NaOCl and PbO_2 are reduced by H_2O_2 in their respective reactions.

30. **(B, C, D)**
$\mathbf{O}_2[PtF_6] \rightarrow$ **(+1/2)** K[**Co**$(CO)_4] \rightarrow$ **(-1)** **N**$H_2OH \rightarrow$ **(-1)** $(\mathbf{N}_2H_5)_2SO_4 \rightarrow$ **(-2)**

31. **(C)**
Step 1: $H_2C_2O_4 \rightarrow H_2O + CO_2 + CO$
Step 2: $CO + I_2O_5 \rightarrow CO_2 + I_2$
Step 3: $I_2 + Na_2S_2O_3 \rightarrow NaI + Na_2S_4O_6$
In $\mathbf{Na_2S_4O_6}$**,** average oxidation state of S-atoms is **+2.5**
In step 3,
n-factor of I_2 = 2
g.eq. of I_2 reacted = g.eq. of $Na_2S_2O_3$
(moles of I_2 reacted) x 2 = 0.2 x 0.2
Moles of I_2 reacted = 0.02
In step 2,
n-factor of I_2 = 10
Moles of I_2O_5 reacted = Moles of I_2 produced = 0.02
g.eq. of CO = g.eq. of I_2O_5
(moles of CO) x 2 = 0.02 x 10
Mole of CO = 0.1
In step 1,
Moles of $H_2C_2O_4$ reacted = moles of CO produced = 0.1
Mass of $H_2C_2O_4$ reacted = 0.1 x 90 g = 9 g

32. **(A, C, D)**
3 Cu + **8** $HNO_3 \rightarrow$ **3** $Cu(NO_3)_2$ + **2** NO + **4** H_2O **(6e$^-$)**
n-factor for HNO_3 = 6/8 = 0.75

33. **(A, B)**
n-factor for $KMnO_4$ = 5
n-factor for $K_2Cr_2O_7$ = 6
n-factor for I_2 (in all reactions) = 2
n-factor for $Na_2S_2O_3$ = 1
n-factor for $Na_2S_4O_6$ = 2

34. **(A)**
Oxidation number of Cl in HOCl is +1.

35. **(A, B, C, D)**
n-factor for $KMnO_4$ = 5
n-factor for $K_2Cr_2O_7$ = 6
(Moles of $KMnO_4$ used) x 5 = (Moles of $K_2Cr_2O_7$) x 6
Same mass of H_2O_2 will release same mass of O_2 in the two reactants.

36. **(2)**
$(7-4) \times 2 = (4-n) \times 3$
$n = 2$

37. **(2)**
$1.5 \times 10^{-3} \times [(5 - n) \times 2] = 1.8 \times 10^{-3} \times 5$
Hence, $n = 2$

38. **(9)**
$Cl_2 + \mathbf{2}\ ClO_2^- \rightarrow \mathbf{2}\ Cl^- + \mathbf{2}\ ClO_2$
With 90% yield, moles of ClO_2 produced = (5 x 2) x (1/1) x (90/100) = 9

39. **(5)**
Moles of electrons lost = 0.5 x 3 x [6 – (8/3)] = 5

40. **(2)**
$\mathbf{6}\ KI + \mathbf{8}\ HNO_3 \rightarrow \mathbf{6}\ KNO_3 + \mathbf{2}\ NO + \mathbf{3}\ I_2 + \mathbf{4}\ H_2O$ **($6e^-$)**
$I_2 + \mathbf{2}\ Na_2S_2O_3 \rightarrow Na_2S_4O_6 + \mathbf{2}\ NaI$ **($2e^-$)**
$V_1 \times 0.2 \times (6/8) = V_2 \times 0.3 \times 1$
Hence, $V_1/V_2 = 2$

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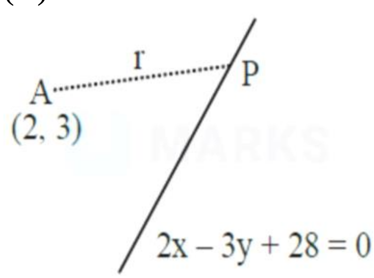
TOPIC: STRAIGHT LINE & POSL

SOLUTIONS

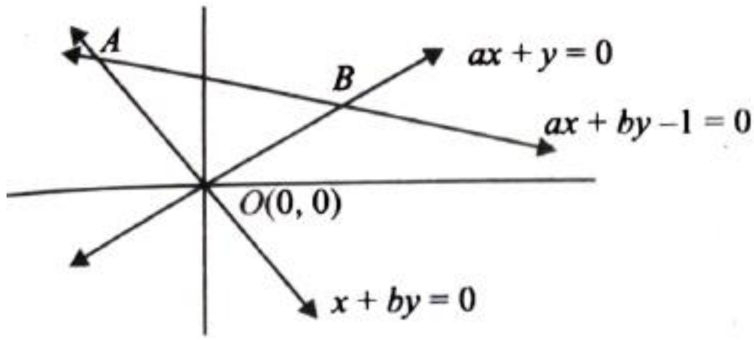
41. (C)
A(a, b), B(3, 4), C(–6, –8)
(–6, –8) (a, b) (3, 4)
$\Rightarrow$ $a = 0, b = 0$ $\Rightarrow P(3, 5)$
Distance from P measured along
$x - 2y - 1 = 0$
$\Rightarrow x = 3 + r\cos\theta,\ y = 5 + r\sin\theta$
Where $\tan\theta = \frac{1}{2}$
$r\left(2\cos\theta + 3\sin\theta\right) = -17$
$\Rightarrow$ $r = \left|\frac{-17\sqrt{5}}{7}\right| = \frac{17\sqrt{5}}{7}$

42. (D)



Writing P in terms of parametric co-ordinates 2 + r
$\cos\theta, 3 + r\sin\theta$ as $\tan\theta = \sqrt{3}$
$P\left(2 + \frac{r}{2}, 3 + \frac{\sqrt{3}r}{2}\right)$
P must satisfy $2x - 3y + 28 = 0$
So, $2\left(2 + \frac{r}{2}\right) - 3\left(3 + \frac{\sqrt{3}r}{2}\right) + 28 = 0$
We find $r = 4 + 6\sqrt{3}$

43. (B)
Lines ax + y = 0 and x + by = 0 intersect at O(0, 0).



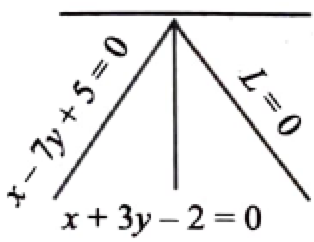
Hence, if AB subtends right angle at O(0, 0), then ax + y = 0 and x + by = 0 are perpendicular to each other. So, $(-a)\left(-\frac{1}{b}\right)=-1$

44. (C)

The circumcentre of the triangle is (0, 0) as all the vertices lie on the circle $x^2+y^2=5$.
So the orthocentre will be (sum of x coordinates, sum of y coordinates).

45. (C)

Family of line through the given lines is
$L \equiv x-7y+5+\lambda(x+3y-2)=0$ ...(1)



For line L = 0 in the diagram, distance of any point say (2, 0) on the line x + 3y – 2 = 0 from the line –7x + 5 = 0 and the line L = 0 must be same.

$\Rightarrow \left|\frac{2+5}{\sqrt{50}}\right|=\left|\frac{2+2\lambda+5-2\lambda}{\sqrt{(1+\lambda)^2+(3\lambda-7)^2}}\right|$

$\Rightarrow 10\lambda^2-40\lambda=0$

$\Rightarrow \lambda=4$ or 0

Hence, $L=0, \lambda=4$

$\Rightarrow$ Required line is $5x+5y-3=0$

46. (AB)

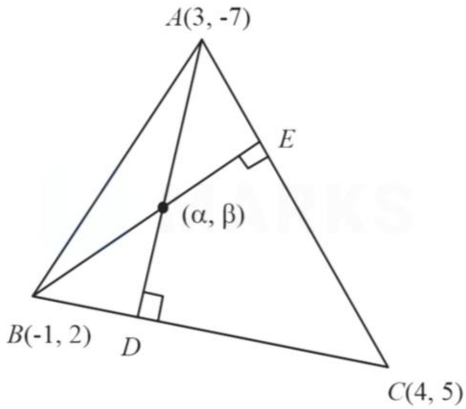
Solving lines $L_1(3x+2y=14)$ and $L_2(5x-y=6)$ to get A(2, 4) and solving lines $L_3(4x+3y=8)$ and $L_4(6x+y=5)$ to get $B\left(\frac{1}{2}, 2\right)$.

Finding Eqn. of AB : 4x – 3y + 4 = 0
Calculate distance PM

$\Rightarrow \left|\frac{4(5)-3(-2)+4}{5}\right|=6$

47. (ACD)

The required diagram will be:



Equation of $AD: y+7=\frac{-5}{3}(x-3)$

$\Rightarrow 3y+21=-5x+15$

$\Rightarrow 5x+3y+6=0 \qquad ...(i)$

Equation of $BE: y-2=\frac{-1}{12}(x+1)$

$\Rightarrow 12y-24=-x-1$

$\Rightarrow x=23-12y \qquad ...(ii)$

By (ii) 115 – 60y + 3y + 6 = 0

$\Rightarrow 57y=121$

$y=\frac{121}{57}, x=23-12\times\frac{121}{57}$

$\therefore 9\alpha-6\beta+60=9\times23-108\times\frac{121}{57}-6$

$=207-242+60=25$

48. (AB)

A, 8, b are in H.P.

$\Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{1}{4}$

$\Rightarrow b=\frac{4a}{a-4}$

$\Rightarrow$ area, $A=\frac{4a^2}{2(a-4)}$

A is minimum at a = 8.

Hence, minimum value of A is 32 sq. units.

49. (AB)

Being a pair of lines, $abc+2fgh-af^2-bg^2-ch^2=0$.

This gives m = 4.

Now, find angle between lines.

50. (AC)

We have, $6x^2 - xy + 12y^2 = 0$

$\Rightarrow (2x-3y)(3x+4y) = 0$ ...(1)

and $15x^2 + 14xy - 8y^2 = 0$

$\Rightarrow (5x-2y)(3x+4y) = 0$ ...(2)

Equation of the line common to (1) and (2) is

$3x + 4y = 0$ ...(3)

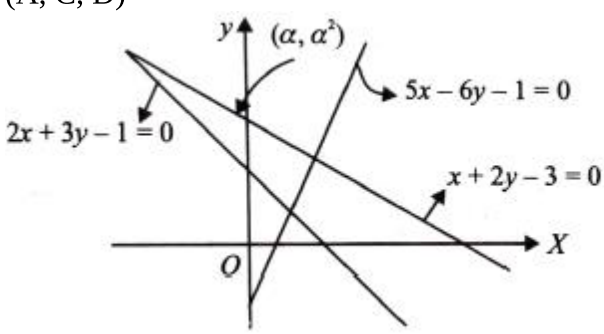
Equion of any line parallel to (3) is

$3x + 4y = k$

Since its distance from (3) is 7, so

$\left|\frac{k}{\sqrt{3^2+4^2}}\right| = 7 \Rightarrow k = \pm 35$

51. (A, C, D)



O and the point (α, α^2) lie on the opposite sides w.r.t. $2x + 3y - 1 = 0$. Hence,

$\Rightarrow 2\alpha + 3\alpha^2 - 1 > 0$ ...(1)

O and the point (α, α^2) lie to the same side w.r.t. $x + 2y - 3 = 0$. Hence,

$\Rightarrow \alpha + 2\alpha^2 - 3 < 0$ ...(2)

Again O and the point (α, α^2) lie to the same side w.r.t. $5x - 6y - 1 = 0$. Hence,

$5\alpha - 6\alpha^2 - 1 < 0$

$\Rightarrow 6\alpha^2 - 5\alpha + 1 > 0$

52. (A, C)

The equation represents a pair of straight lines.

Hence, $1 \times (-2)(-1) + 2\left(\frac{3}{2}\right) \times 0 \times \frac{m}{2} - 1 \times \left(\frac{3}{2}\right)^2 - (-2) \times 0^2 - 1(-1) \times \left(\frac{m}{2}\right)^2 = 0$

$\Rightarrow m = 1, -1$

The points of intersection of the pair of lines are obtained by solving

$\frac{\partial S}{\partial x} \equiv 2x + my = 0$ and $\frac{\partial S}{\partial x} \equiv mx - 4y + 3 = 0$

When m = 1, the required point is the intersection of 2x + y = 0, x – 4y + 3 = 0. When m = –1, the required point is the intersection of $2x - y = 0$, –x – 4y + 3 = 0

53. (A, B, D)

Since, the given point lies on the line lx + my + n = 0, so a, b, c are the roots of the equation

$$l\left(\frac{t^3}{t-1}\right)+m\left(\frac{t^2-3}{t-1}\right)+n=0 \text{ or}$$

$$l t^3+mt^2+nt-(3m+n)=0 \qquad ...(1)$$

Hence, $a+b+c=-\frac{m}{l}$

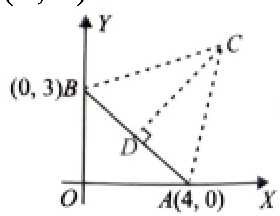
$$ab+bc+ca=\frac{n}{l} \qquad ...(2)$$

$$abc=\frac{3m+n}{l} \qquad ...(3)$$

So from Eqs. (1), (2) and (3), we get

$$abc-(bc+ca+ab)+3(a+b+c)=0$$

54. (A, D)



$$AB=5,\ D\equiv\left(2,\frac{3}{2}\right)$$

$CD=5\times\frac{\sqrt{3}}{2}=\frac{5\sqrt{3}}{2}$, slope of AB is –3/4, slope of CD is 4/3.

If $C\equiv(h,k)$, then $\frac{h-2}{3/5}=\frac{k-3/2}{4/5}=\pm\frac{5\sqrt{3}}{2}$

$$\Rightarrow\ h=2\left(1-\frac{3\sqrt{3}}{4}\right),\ k=\frac{3}{2}\left(1-\frac{4}{\sqrt{3}}\right)$$

or $\quad h=2\left(1+\frac{3\sqrt{3}}{4}\right),\ k=\frac{3}{2}\left(1+\frac{4}{\sqrt{3}}\right)$

55. (A, B)

Vertices $(a\cos\theta_1, a\sin\theta_1), (a\cos\theta_2, a\sin\theta_2)$, and $(a\cos\theta_3, a\sin\theta_3)$ are equidistant from origin (0, 0). Hence, the origin is circumcentre (centroid) of circumcircle. Therefore, the coordinates of centroid are $\left(\frac{a(\cos\theta_1+\cos\theta_2+\cos\theta_3)}{3}, \frac{a(\sin\theta_1+\sin\theta_2+\sin\theta_3)}{3}\right)$

But the centroid is the origin (0, 0), $\cos\theta_1+\cos\theta_2+\cos\theta_3=0$ and $\sin\theta_1+\sin\theta_2+\sin\theta_3=0$.

56. (32)

$$2x-y+3=0$$
$$6x+3y+1=0$$
$$\alpha x+2y-2=0$$

will not form a Δ if $\alpha x+2y-2=0$ is concurrent with $2x-y+3=0$ and $6x+3y+1=0$ or parallel to either or them so

Case-1: Concurrent lines

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix}=0 \Rightarrow \alpha=\frac{4}{5}$$

Case-2: Parallel lines

$$-\frac{\alpha}{2}=\frac{-6}{3} \text{ or } -\frac{\alpha}{2}=2$$

$\Rightarrow \alpha=4$ or $\alpha=-4$

$\Rightarrow \alpha=4$ or $\alpha=-4$

$$P=16+16+\frac{16}{25}$$

$$[P]=\left[32+\frac{16}{25}\right]=32$$

57. (348)

Given,

$L_1: 3y-2x=3$ is angular bisector of

$L_2=x-y+1=0$ and

$L_3=\alpha x+\beta y+17=0$

Now finding point of intersection of L_1 & L_2

We get (0, 1)

And point will lie on L_2, so

$\alpha\times 0+\beta+1+17=0$

$\Rightarrow \beta=-17$

Any point, say $\left(\frac{-3}{2}, 0\right)$ on L_1 should be equidistance from lines L_2 & L_3

Now using the formula of distance of a point from the line we get,

$$\Rightarrow \left|\frac{\frac{-3}{2}-0+1}{\sqrt{1^2+1^2}}\right|=\left|\frac{\frac{-3\alpha}{2}+0+17}{\sqrt{\alpha^2-(17)^2}}\right|$$

$\Rightarrow (\alpha-7)(\alpha-17)=0$

Now, for $\alpha=17, L_2$ & L_3 coincides

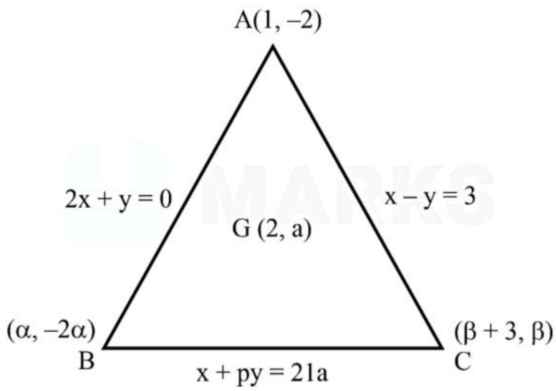
So, $\alpha=7$

Now putting the value of α and β in given expression we get,

$$\alpha^2+\beta^2-\alpha-\beta=(-17)^2+7^2-7+17=348$$

58. (122)

We have,



Since, $G(2, a)$, so

$\frac{1+\alpha+\beta+3}{3}=2$ and $\frac{-2-2\alpha+\beta}{3}=\alpha$

$\Rightarrow\ \alpha+\beta=2$ and $-2\alpha+\beta=3a+2$

So, $\alpha=-a, \beta=2+\alpha$

So, $\alpha=-a, \beta=2+a$

So, $B\equiv(-a, 2a), C\equiv(5+a, 2+a)$

Now, B and C lies on the line x + py = 21a,

Now, B and C lies on the line x + py = 21a,

So, $-a+2pa=21a$

$\Rightarrow\ p=11$

Also,

$5+a+11(2+a)=21a$

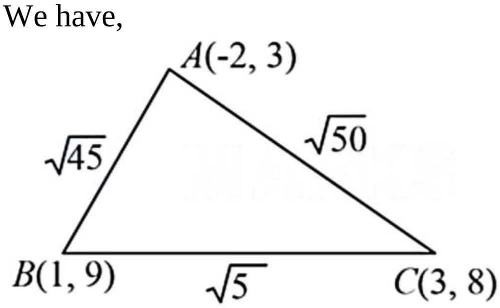
$\Rightarrow\ 27=9a \Rightarrow a=3$

So, $B\equiv(-3, 6), C\equiv(8, 5)$

So, $BC^2=121+1=122$

59. (9)

We have,



$AB=\sqrt{(-2-1)^2+(3-9)^2}=\sqrt{45}$

$AC=\sqrt{(-2-3)^2+(3-8)^2}=\sqrt{50}$

$BC=\sqrt{(3-1)^2+(8-9)^2}=\sqrt{5}$

Here, $AC^2=AB^2+BC^2$

$\Rightarrow\ \left(\sqrt{50}\right)^2=\left(\sqrt{45}\right)^2+\left(\sqrt{5}\right)^2$

Hence, $\angle B = 90°$, so circumcentre is the mid-point of hypotenusei i.e., AC.

Circumcentre $\equiv \left(\frac{1}{2}, \frac{11}{2}\right)$

Mid-point of $BC = \left(2, \frac{17}{2}\right)$

Equation of line L is

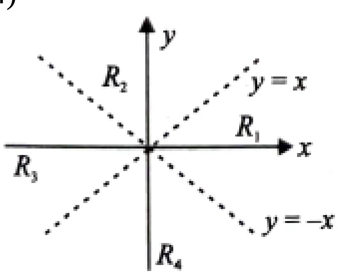
$$\left(y - \frac{11}{2}\right) = \left(\frac{\frac{11}{2} - \frac{17}{2}}{\frac{1}{2} - 2}\right)\left(x - \frac{1}{2}\right)$$

$$\Rightarrow \ y = 2x + \frac{9}{2}$$

Passing through $\left(0, \frac{\alpha}{2}\right)$, so

$$\frac{\alpha}{2} = \frac{9}{2} \Rightarrow \alpha = 9$$

60. (4)



$d(P, L_1) = \frac{|x - y|}{\sqrt{2}}$ and $d(P, L_2) = \frac{|x + y|}{\sqrt{2}}$

We have $2 \le d(P, L_1) + d(P, L_2) \le 4$

$\Rightarrow \ 2\sqrt{2} \le |x - y| + (x + y) \le 4\sqrt{2}$ (1)

Let us consider the four regions, namely, R_1, R_2, R_3 and R_4 in the liens L_1 and L_2 dividing the coordinates plane.

In R_1, we have $y < x, y > -x$. In R_2, we have $y > x$, $y > -x$.

Similarly in R_3, we have $y > x, y < -x$.

Finally, in R_4 we have $y > x, y < -x$.

Thus for R_1, equation (1) becomes

$2\sqrt{2} \le x - y + x + y \le 4\sqrt{2} \Rightarrow \sqrt{2} \le x \le 2\sqrt{2}$

Similarly for R_2, equation (1) becomes

$2\sqrt{2} \le y - x + x + y \le 4\sqrt{2} \Rightarrow \sqrt{2} \le y \le 2\sqrt{2}$

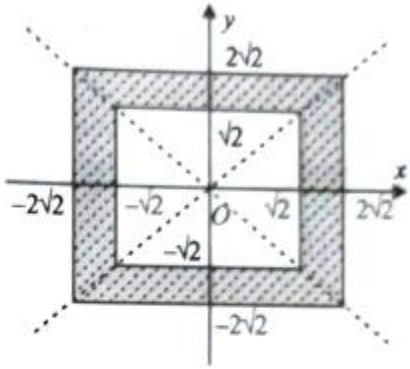
For R_3, equation (1) becomes

$2\sqrt{2} \le y - x - y \le 4\sqrt{2} \Rightarrow -\sqrt{2} \le y \le -2\sqrt{2}$

For R_4, equation (1) becomes

$2\sqrt{2} \le x - y - y \le 4\sqrt{2} \Rightarrow -\sqrt{2} \le y \le -2\sqrt{2}$

Thus region R will be between concentric squares formed by the lines $x = \pm 2\sqrt{2},\ y = \pm 2\sqrt{2}$ and $x = \pm 2\sqrt{2},\ y = \pm\sqrt{2}$.



Thus the required area is $\left(4\sqrt{2}\right)^2 - \left(2\sqrt{2}\right)^2 = 24$ sq. units.