Elasticity

1. (B)

$$\Delta l = \frac{Fl}{\pi r^2 Y} \Rightarrow \frac{\Delta l_1}{\Delta l_2} = \left(\frac{F_1}{F_2}\right)\left(\frac{l_1}{l_2}\right)\left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \frac{l}{\Delta l_2} = \left(\frac{F}{2F}\right)\left(\frac{L}{2L}\right)\left(\frac{2r}{r}\right)^2$$

$$\Rightarrow \Delta l_2 = l$$

2. (C)

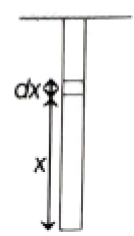
Slope $= \frac{F}{\Delta l} = \frac{AY}{l} \Rightarrow$ Slope $\propto A$

Slope is least for wire *A*. So, wire *A* will be thinnest.

3. (A)

$$\Delta l = \frac{Fl}{AY},$$

Slope $= \frac{\Delta l}{F} = \frac{(4-1)\times 10^{-4}}{80-20}$



$$\Rightarrow \frac{l}{AY} = \frac{1}{20}\times 10^{-4}$$

$$\Rightarrow \frac{1}{10^{-6}Y} = \frac{1}{20}\times 10^{-4}$$

$$\Rightarrow Y = 2\times 10^{11}\ \mathrm{N/m^2}$$

4. (B)

$$\Delta l = \frac{Fl}{AY}$$

$$\Rightarrow \frac{\Delta l_1}{\Delta l_2} = \left(\frac{F_1}{F_2}\right)\left(\frac{l_1}{l_2}\right)\left(\frac{r_2}{r_1}\right)^2\left(\frac{Y_2}{Y_1}\right)$$

$$= \left(\frac{3mg}{2mg}\right)(a)\left(\frac{1}{b}\right)^2\left(\frac{1}{c}\right) = \frac{3a}{2b^2c}$$

5. (C)

Let natural length be *l*.

$$F = kx$$

$$\Rightarrow T_1 = k(l_1 - l) \qquad ...(\mathrm{i})$$

and $T_2 = k(l_2 - l) \qquad ...(\mathrm{ii})$

Dividing Eq. (i) by Eq. (ii), we get

$$l = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$$

6. (A)

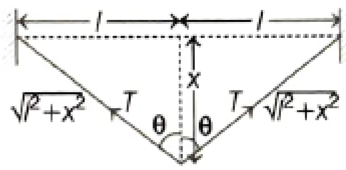
Breaking stress depends only on the material of wire.

7. (C)

$$T=\frac{2m_1m_2g}{m_1+m_2}=\frac{4mg}{3}$$

Breaking stress $=\frac{T}{A}\Rightarrow S=\frac{\frac{4mg}{3}}{\pi r^2}\Rightarrow r=\sqrt{\frac{4mg}{3\pi S}}$

8. (C)



Strain $=\frac{\Delta l}{l}=\frac{2\sqrt{l^2+x^2-2l}}{2l}=\left(1+\frac{x^2}{l^2}\right)^{\frac{1}{2}}-1$

$$=1+\frac{x^2}{2l^2}-1 \qquad \{x<<l\}$$

$$=\frac{x^2}{2l^2}$$

9. (D)

$2\cos\theta=w$

$$\Rightarrow\ T=\frac{w}{2\cos\theta}=\frac{w\sqrt{l^2+x^2}}{2x}$$

Stress $=\frac{T}{A}=\frac{w\sqrt{l^2+x^2}}{2xA}=\frac{wl}{2xA}\qquad \because\{x<<l\}$

10. (A)

$$\Delta L=\frac{FL}{A\eta}=\frac{500\times0.04}{\left(4\times16\times10^{-4}\right)\left(2\times10^{6}\right)}=0.156\times10^{-2}\text{ m}$$

= 0.156 cm

11. (D)

Bulk strain $=\frac{\Delta V}{V}=3\left(\frac{\Delta l}{l}\right)=3\left(\frac{2}{100}\right)=0.06$

12. (B)

$$B=\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)}\Rightarrow B=\frac{\rho gh}{\left(\frac{\Delta V}{V}\right)}=\frac{10^3\times10\times200}{\left(\frac{0.1}{100}\right)}$$

$= 2\times10^{9}\ \text{N/m}^2$

13. (B)

$$p = p_0\,\rho^{\alpha V} \Rightarrow \frac{dp}{dV} = p_0 e^{\alpha V}(\alpha)$$

$$B = \frac{dp}{\left(\frac{dV}{V}\right)} = V\left(p_0 e^{\alpha V}\alpha\right) = \alpha p V$$

14. (D)

$$B = \frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta p}{B}$$

$$\Rightarrow\ \gamma\Delta T = \frac{p}{B} \Rightarrow\ \Delta T = \frac{p}{B\gamma}$$

15. (C)

Steel has the highest value of modulus of elasticity among the given materials.

16. (A)

$$\text{Longitudinal strain} = \frac{\text{Lateral strain}}{\sigma} = \frac{0.01\times10^{-3}}{0.4}$$

$$= 2.5\times10^{-5}$$

$$\text{Longitudinal stress} = \frac{F}{A} = \frac{100}{0.025} = 4\times10^{3}\ \text{N/m}^2$$

$$Y = \frac{4\times10^{3}}{2.5\times10^{-5}} = 1.6\times10^{8}\ \text{Nm}^{-2}$$

17. (A)

$$\frac{\Delta V}{V} = (1-2\sigma)\frac{\Delta l}{l} = (1-2(0.2))\left(4\times10^{-3}\right) = 0.24\times10^{-2}$$

$$\frac{\Delta V}{V}\times100 = \left(0.24\times10^{-2}\right)\times100 = 0.24\%$$

18. (C)

$$\sigma_x = \sigma_y = \sigma_z = \frac{F}{A}$$

In x-direction, Strain $= \dfrac{\sigma_x}{Y} - \sigma\left(\dfrac{\sigma_y}{Y}\right) - \sigma\left(\dfrac{\sigma_z}{Y}\right)$

$$\Rightarrow\ \frac{\Delta l}{1} = \frac{F}{AY}(1-2\sigma)$$

$$\Rightarrow\ \Delta l = \frac{F}{AY}(1-2\sigma)$$

19. (A)

Energy per unit volume

$$=\frac{1}{2}Y(\text{Strain})^2=\frac{1}{2}\times 2\times 10^{10}\times\left(\frac{0.06}{100}\right)^2$$

$$=3600\ \text{Jm}^{-3}$$

20. (D)

$$\sigma=\frac{F}{A}=\frac{F}{\pi r^2}\ \Rightarrow\ \sigma\propto\frac{1}{r^2}\Rightarrow\frac{\sigma_1}{\sigma_2}=\left(\frac{r_2}{r_1}\right)^2$$

$$U=\frac{(Stress)^2}{2Y}\Rightarrow U\propto\sigma^2$$

$$\Rightarrow\ \frac{U_1}{U_2}=\left(\frac{\sigma_1}{\sigma_2}\right)^2=\left(\frac{r_2}{r_1}\right)^4$$

$$=\left(\frac{2r}{r}\right)^4=16:1$$

21. (B)

$$k_{\text{eq}}=k_1+k_2\Rightarrow\frac{(2A)Y_{\text{eq}}}{L}=\frac{AY_1}{L}+\frac{AY_2}{L}$$

$$\Rightarrow\ Y_{\text{eq}}=\frac{Y_1+Y_2}{2}$$

22. (D)

For no change in the lengths of individual rods, the compressive forces in both rods due to thermal expansion should be equal. So,

$$A_1Y_1\alpha_1\Delta T=A_2Y_2\alpha_2\Delta T$$

$$\Rightarrow\ \frac{A_1}{A_2}=\frac{\alpha_2Y_2}{\alpha_1Y_1}$$

23. (B)

$$E=\frac{\text{Stress}}{\text{Strain}}\qquad\Rightarrow\ E=\frac{\frac{F}{A}}{\left(\frac{2\pi R-2\pi r}{2\pi r}\right)}$$

$$\Rightarrow\ F=AE\left(\frac{R-r}{r}\right)$$

Surface Tension

24. (D)

For equilibrium of wire *ab*,

$$2Tl=mg$$

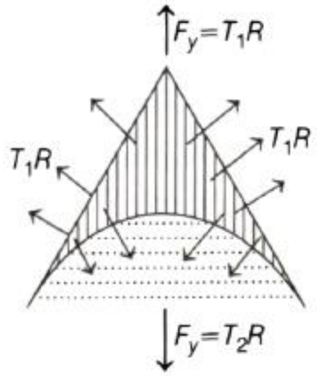
$$\Rightarrow\ m=\frac{2Tl}{g}=\frac{2\times 25\times 10^{-3}\times 0.1}{10}=0.5\ \text{g}$$

25. (B)

$F_y = T \times$ Projected length

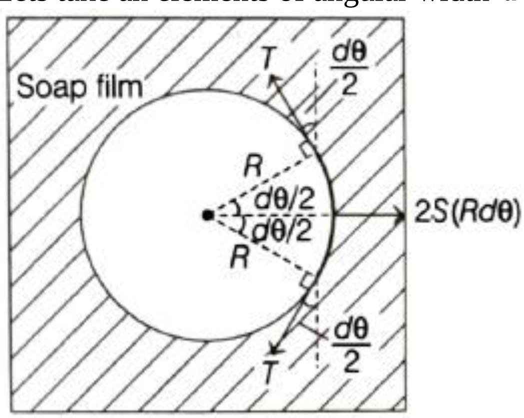
Net force $= T_1R - T_2R$

$= (T_1 - T_2)R$ in +y-direction



26. (D)

Lets take an elements of angular width $d\theta$ on the thread.



Force on the element due to surface tension

$= 2S(Rd\theta)$

For equilibrium of element,

$$2T\sin\left(\frac{d\theta}{2}\right) = 2SRd\theta$$

$$\Rightarrow \quad 2T\left(\frac{d\theta}{2}\right) = 2SRd\theta \Rightarrow T = 2SR$$

27. (B)

Let radius of single large drop be R.

For volume conservation, $2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$$\Rightarrow \quad R = 2^{1/3}r$$

$$U_1 = T\left(4\pi r^2\right)\times 2$$

$$U_2 = T\left(4\pi R^2\right)$$

$$\frac{U_2}{U_2} = \frac{T\left(8\pi r^2\right)}{T\left(4\pi r^2\right)} = \frac{2r^2}{\left(2^{1/3}r\right)^2} = 2^{1/3}:1$$

28. (B)

$$U_1 = S\left[2\times 4\pi\left(\frac{d}{2}\right)^2\right], \qquad U_2 = S\left[2\times 4\pi\left(\frac{2d}{2}\right)^2\right]$$

$$W = \Delta U = U_2 - U_1 = 6\pi d^2 S$$

29. (D)

$$W = T\left(4\pi r^2 \times 2\right)$$

$$= T\left[4\pi\left\{\left(\frac{3V}{4\pi}\right)^{1/3}\right\}^2 \times 2\right] \qquad \left\{\because V = \frac{4}{3}\pi r^3 \Rightarrow r = \left(\frac{3V}{4\pi}\right)^{1/3}\right\}$$

$$\Rightarrow \quad W \propto V^{2/3}$$

$$\Rightarrow \quad \frac{W_1}{W_2} = \left(\frac{V_1}{V_2}\right)^{2/3} \Rightarrow \frac{W}{W_2} = \left(\frac{V}{2V}\right)^{2/3}$$

$$\Rightarrow \quad W_2 = 2^{2/3}W = \sqrt[3]{4}\,W$$

30. (B)

From volume conservation, $\frac{4}{3}\pi R^3 = n\left(\frac{4}{3}\pi r^3\right)$

$$\Rightarrow \quad r = \frac{R}{n^{1/3}}$$

$$U_1 = T\left(4\pi R^2\right)$$

$$U_2 = nT\left(4\pi T^2\right) = n4\pi T\left(\frac{R}{n^{1/3}}\right)^2 = n^{1/3}T\left(4\pi R^2\right)$$

$$W = \Delta U = U_2 - U_1 = \left(n^{1/3} - 1\right)T\left(4\pi R^2\right)$$

31. (A)

$$U = T\left(8\pi r^2\right)$$

$$P = \frac{dU}{dt} = T\left(8\pi\right)\left(2r\frac{dr}{dt}\right)$$

$$\Rightarrow \quad P \propto r \qquad \because \left\{\frac{dr}{dt} \text{ is constants}\right\}$$

So, P versus r graph will be straight line passing through origin.

32. (C)

Air bubble inside water has only one free surface.

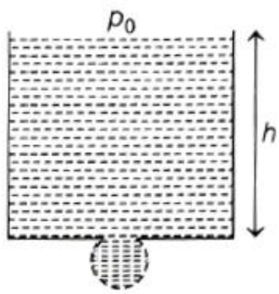
So, excess pressure will be $\frac{2T}{r}$.

$$p = (p_0 + ghd) + \frac{2T}{r}$$

33. (A)

Pressure difference on two sides of a curved surface is inversely proportional to the radius of curvature.

34. (D)



$$p_0 + \rho gh - \frac{2T}{r} = p_0$$

$$\Rightarrow \quad h = \frac{2T}{\rho gr} = \frac{2 \times 70}{1 \times 1000 \times 0.005} = 28.0 \text{ cm}$$

35. (D)

$$h = \frac{2T\cos\theta}{\rho g_{\text{eff}} R}$$

Fore freely falling elevator, $g_{\text{eff}} = 0 \Rightarrow h \to \infty$

So, capillary will be completely filled with water.

36. (B)

$$H = \frac{2T\cos\theta}{\rho gR} \Rightarrow H \propto \frac{1}{R}$$

So, when radius is double, H will be halved.

$$M' = \rho\left[\pi(2R)^2\left(\frac{H}{2}\right)\right] \text{ and } M = \rho(\pi R^2 H)$$

$$\frac{M'}{M} = 2 \Rightarrow M' = 2M$$

37. (D)

$$T(2\pi r)\cos\theta = w$$

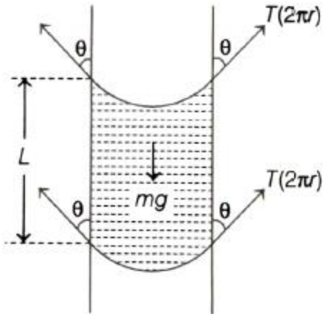
$$\Rightarrow \quad \text{Circumference } = 2\pi r = \frac{w}{T\cos\theta}$$

$$= \frac{75 \times 10^{-4}}{6 \times 10^{-2}\cos 0°} = 12.5 \times 10^{-2} \text{ m}$$

38. (A)

For equilibrium of water column,

$$T(2\pi r)\cos\theta + T(2\pi r)\cos\theta = mg$$

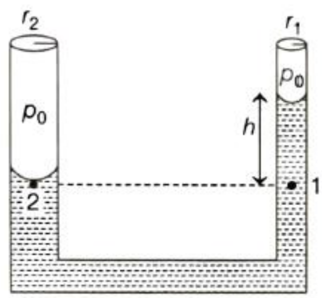


$$\Rightarrow \quad T(4\pi r)\cos 0° = \left(\rho\pi r^2 L\right)g \Rightarrow O = \frac{4T}{\rho g r}$$

39. (A)

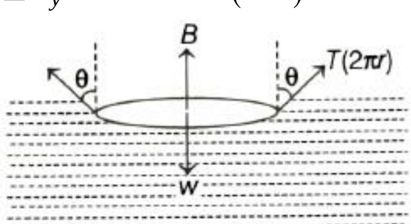
$p_1 = p_2$

$$\Rightarrow \quad p_0 - \frac{2T}{r_2} = p_0 - \frac{2T}{r_1} + \rho g h \Rightarrow T = \frac{\rho g h r_1 r_2}{2(r_2 - r_1)}$$



40. (C)

$$\sum F_y = 0 \Rightarrow B + T(2\pi r)\cos\theta = w_{\text{disc}}$$



$$\Rightarrow \quad w_{\text{disc}} = w + 2\pi r T\cos\theta$$

Viscosity

41. (A)

$$v_T = \frac{2}{9}\frac{(\sigma)r^2 g}{\eta}$$

$$\Rightarrow v_T = \frac{2}{9} \times \frac{10^3 \times \left(0.3\times 10^{-3}\right)^2 \times 9.8}{1.8\times 10^{-5}}$$

$$= 10.9 \text{ m/s}$$

42. (B)

$$m = \sigma\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \sigma \propto \frac{1}{r^3} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{r_2^3}{r_1^3}$$

$$\Rightarrow \sigma_2 = 8\sigma_1$$

$$v_T = \frac{2}{9}\frac{\sigma r^2 g}{\eta}$$

$$\Rightarrow \frac{v_1}{v_2} = \left(\frac{\sigma_1}{\sigma_2}\right)\left(\frac{r_1}{r_2}\right)^2$$

$$\Rightarrow \frac{v}{v_2} = \left(\frac{\sigma_1}{8\sigma_1}\right)\left(\frac{r_1}{r_1/r_2}^{2}\right)$$

$$\Rightarrow v_2 = 2v$$

43. (C)

$$mg - B - 6\pi\eta r v = ma$$

So, *a versus v* graph will straight line with negative slope.
Hence, option (C) in incorrect.

44. (B)

From volume conservation,

$$2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad R = (2)^{1/3} r$$

$$v_T \propto r^2 \Rightarrow \frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow \frac{v}{v_2} = \left(\frac{r}{2^{1/3} r}\right)^2$$

$$\Rightarrow \quad v_2 = (4^{1/3})v$$

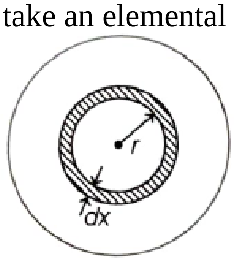
45. (D)

$$2\eta A\frac{dv}{dx} = mg \Rightarrow 2\eta A\frac{dv}{dx} = (\rho A d)g$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{\rho d g}{2\eta}$$

46. (C)

Lets take an elemental ring of radius *r* and thickness *dr* on the disc.



$$F=\eta A\frac{dv}{dx}\Rightarrow dF=\eta(2\pi rdr)\left(\frac{\omega r-0}{t}\right)$$

$$\int d\tau=\int_0^R\eta(2\pi rdr)\left(\frac{\omega r}{t}\right)r$$

$$\Rightarrow\quad \tau\propto R^4$$

47. (C)

$$R_e=\frac{\rho vd}{\eta}=\frac{1\times6\times2}{0.01}=1200$$

48. (B)

$$v_T=\frac{2}{9}\frac{(\sigma-\rho)r^2g}{\eta}$$

$$\Rightarrow\quad 8=\frac{2}{9}\frac{(10.5-1.5)(0.2)\times980}{\eta}$$

$$\Rightarrow\quad \eta=9.8\text{ poise}$$

Numerical Value Answer

Elasticity

49. (200)

$$F=\sigma A=\sigma(2\pi rt)$$

$$=3.45\times10^8\left(2\times3.14\times0.73\times10^{-2}\times1.24\times10^{-2}\right)=200\text{ kN}$$

50. (8)

$$k=\frac{(\pi r^2)Y}{L}\Rightarrow\frac{k_1}{k_2}=\left(\frac{r_1}{r_2}\right)^2=\frac{1}{4}$$

$$\Delta l_1+\Delta l_2=10\text{ mm}\qquad ...(i)$$

$$k_1\Delta l_1=k_2\Delta l_2\Rightarrow\frac{\Delta l_1}{\Delta l_2}=\frac{k_2}{k_1}=4\qquad ...(ii)$$

Solving Eqs. (i) and (ii), we get

$$\Delta l_1=8\text{ mm}$$

51. (1025)

Consider 1m^3 of water at the surface. Let us calculate change in volume ΔV of this water when it is at a pressure of 501 atm.

$$\Delta V=\frac{(\Delta p)V}{B}=\left(\frac{501-1\times10^5\times1}{2\times10^9}\right)=0.025\text{ m}^3$$

$$\text{New density}=\frac{m}{V-\Delta V}=\frac{1000}{1-0.025}=1025\text{ kg m}^{-3}$$

52. (3)

$$\sigma = \frac{T}{A} = \frac{m(g+a)}{A}$$

$$\text{Strain} = \frac{\sigma}{Y} = \frac{m(g+a)}{AY}$$

$$= \frac{10(10+2)}{2\times10^{-4}\times2\times10^{11}} = 3\times10^{-6}$$

53. (4)

$$F = \left(\frac{AY}{l}\right)\Delta l \Rightarrow \text{Force constant } = \frac{AY}{l}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{AY}{ml}} = \sqrt{\frac{4.9\times10^{-7}Y}{0.1\times1}} = 140$$

$$\Rightarrow \quad Y = 4\times10^{9}\ \text{Nm}^{-2}$$

Surface Tension

54. (4)

$$B = \frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta p}{B} \Rightarrow \frac{3\Delta R}{R} = \frac{2T}{BR}$$

$$\Rightarrow \quad \Delta R = \frac{2T}{3B} = \frac{2\times0.075}{3\times1.25\times10^{8}} = 4\times10^{-10}\ \text{m} = 4\,\text{Å}$$

55. (450)

$$h = \frac{2T\cos\theta}{\rho g R}$$

$$\Rightarrow \quad 3 = \frac{2T\cos0^\circ}{1.5\times1000\times0.25\times10^{-1}}$$

$$\Rightarrow \quad T = 56.25\ \text{dyne / cm}$$

$$\Delta p = \frac{4T}{r} = \frac{4\times56.25}{0.5} = 450\ \text{dyne}/\text{cm}^2$$

56. (3)

For an isothermal situation,

$$p_A V_A + p_B V_B = p_c V_c$$

$$\Rightarrow \quad \left(p_0 + \frac{5T}{a}\right)\left(\frac{4}{3}\pi a^3\right) + \left(p_0 + \frac{4T}{b}\right)\left(\frac{4}{3}\pi b^3\right)$$

$$= \left(p_0 + \frac{4T}{c}\right)\left(\frac{4}{3}\pi c^3\right)$$

$$\Rightarrow \quad p_0\left(\frac{4}{3}\pi a^3 + \frac{4}{3}\pi b^3 - \frac{4}{3}\pi c^3\right) + \frac{4T}{3}\left(4\pi a^2 + 4\pi b^2 - 4\pi c^2\right) = 0$$

$$\Rightarrow \quad p_0 V + \left(\frac{4T}{3}\right)S = 0 \quad \Rightarrow \quad 3p_0 V + 4TS = 0$$

So, $\lambda = 3$

57. (2.4)

$$p = p_0 + \rho g h + \frac{2T}{R}$$

$$= 10^5 + 10^3 \times 10 \times 6 + \frac{2 \times 0.07}{3.5 \times 10^{-6}} = 2 \times 10^5 \text{ Pa}$$

$$\rho = \frac{pM}{RT} \Rightarrow \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}$$

$$\Rightarrow \frac{1.2}{\rho_2} = \frac{10^5}{2 \times 10^5}$$

$$\Rightarrow \rho_2 = 2.4 \text{ kg/m}^3$$

58. (9)

Initial pressure inside bubble $= p_0 + \dfrac{4T}{r}$

Final pressure inside bubble $= p_{\text{new}} + \dfrac{4T}{(r/2)}$

For isothermal condition,

$$p_1 V_1 = p_2 V_2$$

$$\Rightarrow \left(p_0 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(p_{\text{new}} + \frac{8T}{r}\right)\frac{4}{3}\pi\left(\frac{r}{2}\right)^3$$

$$\Rightarrow p_{\text{new}} = 8p_0 + \frac{24T}{r} = 8 \times 10^5 + \frac{24 \times 0.08}{1.92 \times 10^{-5}}$$

$$= 9 \times 10^5 \text{ N/m}^2$$

Viscosity

59. (20.4)

$$v_T = \frac{2}{9}\frac{(\sigma - \rho) r^2 g}{\eta} = \sqrt{2gh}$$

$$\Rightarrow \frac{2}{9}\frac{\left(10^4 - 10^3\right)\left(1 \times 10^{-4}\right)^2 (9.8)}{9.8 \times 10^{-6}} = \sqrt{2 \times 9.8\, h}$$

$$\Rightarrow h = 20.4 \text{ m}$$

60. (9)

$F_1 =$ Viscous force on upper surface

$$= \eta A \frac{dv}{dx} = \eta A\left(\frac{v - 0}{d/2}\right) = 2\eta \frac{Av}{d}$$

$F_2 =$ viscous force on lower surface $= \dfrac{2\eta A v}{d}$

$$F_{\text{ext}} = F_1 + F_2 = \frac{4\eta A v}{d}$$

$$= \frac{4\left(150\times10^{-1}\right)(0.1)^2\left(3\times10^{-2}\right)}{2\times10^{-3}} = 9\text{ N}$$

61. (50)

In fluid frame, ball is moving with constant velocity and resultant of all forces acting on it is zero.

$$v_T = \frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}$$

$$= \frac{2}{9}\frac{(1000-100)(0.05)^2(10)}{0.1}$$

$$= 50\text{ m/s}$$

62. (5)

Rate of production of heat $= P = Fv_T = \left(6\pi\eta r V_T\right)v_T$

$P \propto r v_T^2$

$P \propto r^5 \because \left\{v_T \propto r^2\right\}$

1. (D)

(d) $Y_c \times (\Delta L_c / L_c) = Y_s \times (\Delta L_s / L_s)$

$$\Rightarrow \quad 1\times10^{11}\times\left(\frac{1\times10^{-3}}{1}\right)=2\times10^{11}\times\left(\frac{\Delta L_s}{0.5}\right)$$

$$\therefore \quad \Delta L_s=\frac{0.5\times10^{-3}}{2}=0.25\text{ mm}$$

Therefore, total extension of the composite wire

$= \Delta L_c + \Delta L_s = 1\text{ mm} + 0.25\text{ m} = 1.25\text{ m}$

2. (C)

(c) According to questions,

$$\frac{\ell_s}{\ell_b}=a,\ \frac{r_s}{r_b}=b,\ \frac{y_s}{y_b}=c,\ \frac{\Delta\ell s}{\Delta\ell_b}=?$$

As, $y=\frac{F\ell}{A\Delta\ell}\Rightarrow\Delta\ell=\frac{F\ell}{Ay}$

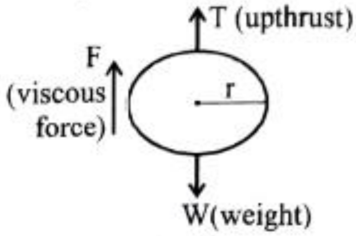
$$\Delta\ell_s=\frac{3mg\ell_s}{\pi r_s^2\cdot y_s}\qquad[\because F_s=(M+2M)g]$$

$$\Delta\ell_b=\frac{2Mg\ell_b}{\pi r_b^2\cdot y_b}\qquad[\because F_b=2Mg]$$

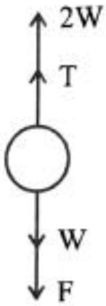
$$\therefore \quad \frac{\Delta\ell_s}{\Delta\ell_b}=\frac{\frac{3Mg\ell_s}{\pi r_s^2\cdot y_s}}{\frac{2Mg.\ell_b}{\pi r_b^2\cdot y_b}}=\frac{3a}{2b^2C}$$

3. (C)

(c) Initially



W = T + F



Finally

Here

2W + T = W + F

$\Rightarrow$ W = T + F

So in both case W = T + F

So, in both case state of body will be same.

4. (B)

(b) $F = \eta A \frac{dv}{dx}$

$$\frac{F}{A} = \eta \frac{dv}{dx} = 10^{-2} \times \frac{5}{5} = 10^{-2} \text{ N/m}^2$$

5. (B)

(b) According to Toricelli's theorem,

Velocity of efflex,

$$V_{eff} = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} \cong 9.8 \text{ ms}^{-1}$$

6. (C)

(c) Poisson's ratio, $\sigma = \frac{\text{lateral strain } (\beta)}{\text{longitudinal strain } (\alpha)}$

For material like copper, $\sigma = 0.33$

And, $Y = 3k(1 - 2\sigma)$ Also, $\frac{9}{Y} = \frac{1}{k} + \frac{3}{\eta}$

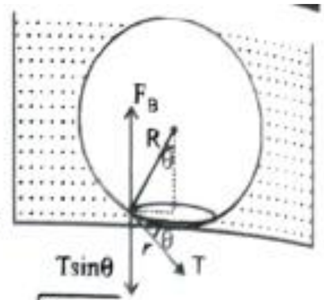
$Y = 2\eta(1 + \sigma)$ Hence, $\eta < Y < k$

7. (A)

(a) When the bubble gets detached,

Buoyant force = force due to surface tension from diagram

$$\sin\theta = \frac{r}{R}$$



$$\rho_{\omega} Vg = (T\sin\theta)\times(2\pi r)$$

$$\Rightarrow \rho_{\omega}\times\frac{4}{3}\pi R^3 g = T\cdot\frac{r}{R}\times 2\pi r$$

$$\Rightarrow \rho_{\omega}\frac{4}{3}R^3 g = \frac{2T}{R}r^2 \Rightarrow r = R^2\sqrt{\frac{2\rho_{\omega}g}{3T}}$$

8. (A)

Inside a drop or an air bubble, towards concave side

$$P = P_0 + \frac{2T}{R} \Rightarrow \Delta P = \frac{2T}{R}$$

Here, the figure shows that the water between the plates formed a curved surface, similar to that formed by air bubble inside the water. So, pressure in water between the plates is lowered by $\frac{2T}{R}$.

9. (C)

When a point mass is falling vertically in a viscous medium, the medium or viscous fluid exerts drag force on the body to oppose its motion and at one stage body falling with constant terminal velocity.

10. (A)

Young's modulus $Y = \frac{F}{A}\Big/\frac{\Delta\ell}{\ell}$

$$Y = \frac{F\ell}{\pi r^2\Delta\ell}$$

Given, radius r = 5 mm, force $F = 50\pi k$ N,

$$\frac{\ell}{\Delta\ell} = 0.01\,\text{mm}$$

$$\therefore\ Y = \frac{F}{\pi r^2}\frac{\ell}{\Delta\ell} = 2\times10^{14}\ \text{N}/\text{m}^2$$

11. (D)

$$\text{Stress} = \frac{\text{Normal force}}{\text{Area}} = \frac{N}{A} = \frac{N}{(2\pi a)b}$$

$$V = \pi a^2 b$$

$$\Delta V = 2\pi ab\,\Delta a$$

Stress = $B\times$ strain

$$\frac{N}{(2\pi a)b} = B\frac{2\pi a\Delta a\times b}{\pi a^2 b}\left[\because\ \text{Strain} = \frac{\Delta V}{V}\right]$$

$$\Rightarrow\ N = B\frac{(2\pi a)^2\,\Delta ab^2}{\pi a^2 b}$$

For needed to push the cork.

$$f=\mu N=\frac{\mu\times B\times 4\pi^2 a^2 b^2 \Delta a}{\pi a^2 b}=(4\pi\mu Bb)\Delta a$$

12. (C)

(c) Bulk modulus, $K=\frac{\text{volumetric stress}}{\text{volumetric strain}}$

$K=\frac{mg}{a\left(\frac{dV}{V}\right)}$ $\left[\because \text{ Stress}=\frac{F}{A}=\frac{mg}{a}\right]$

$\Rightarrow \quad \frac{dV}{V}=\frac{mg}{Ka}$ ....(i)

volume of sphere, $V=\frac{4}{3}\pi r^3$

Fractional change in volume $\frac{dV}{V}=\frac{3dr}{r}$ ...(ii)

Using eq. (i) & (ii) $\frac{3dr}{r}=\frac{mg}{Ka}$

$\therefore \quad \frac{dr}{r}=\frac{mg}{3Ka}$ (fractional decrement in radius)

13. (B)

(b) As we know that

$\frac{2T\cos\theta}{r\rho g}=R\ h$ or $\frac{T_{Hg}}{T_{Water}}=7.5$

$\frac{\rho_{Hg}}{\rho_W}=13.6\ \&\ \frac{\cos\theta_{Hg}}{\cos\theta_W}=\frac{\cos 135°}{\cos 0°}=\frac{1}{\sqrt{2}}$

$\frac{R_{Hg}}{R_{Water}}=\left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$

$=7.5\times\frac{1}{13.6}\times\frac{1}{\sqrt{2}}=0.4=\frac{2}{5}$

14. (D)

(d) We have, $h=\frac{2T\cos\theta}{r\rho g}$

Mass of the water in the capillary

$m=\rho V=\rho\times\pi r^2 h=\rho\times\pi r^2\times\frac{2T\cos\theta}{r\rho g}\Rightarrow m\ \alpha\ r$

$\therefore \frac{m_1}{m_2}=\frac{r}{2r}$ or $m_2=2m_1=2m.$

15. (A)

(a) $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$ or $r = \frac{R}{3}$.

Terminal velocity, $v \propto r^2$

$$\therefore \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2} \text{ or } v_2 = \left(\frac{r_2}{r_1}\right)^2 v_1 = \left(\frac{R/3}{R}\right)^2 v_1 = \frac{1}{9}$$

or $\frac{v_1}{v_2} = 9.$

16. (C)

(c) $\Delta_{temp} = \Delta_{force}$

or $L\alpha(\Delta T) = \frac{FL}{AY}$ $\therefore \alpha = \frac{FL}{AYT} = \frac{F}{\pi r^2 YT}$

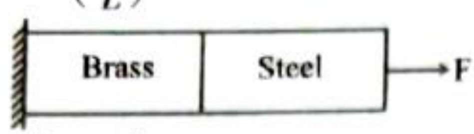
Coefficient of volume expression

$$r = 3\alpha = \frac{3F}{\pi r^2 YT}.$$

17. (None)

(None) Young modulus, $Y = \frac{\text{Stress}}{\left(\frac{\Delta l}{L}\right)}$

Let σ be the stress.



Total elongation $\Delta l_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$

$$\Delta l_{net} = \sigma\left[\frac{1}{Y_1} + \frac{1}{Y_2}\right] \quad [\because L_1 = L_2 = 1\text{m}]$$

$$\sigma = \Delta l\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right) = 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9$$

$$= 8 \times 10^6 \frac{N}{m^2}$$

18. (B)

(b) Stress $= \frac{F}{A} = \frac{400 \times 4}{\pi d^2} = 379 \times 10^6 \text{ N/m}^2$

$$\Rightarrow d^2 = \frac{400 \times 4}{379 \times 10^6 \pi} \Rightarrow d = 1.15 \text{ mm}$$

19. (C)

(c) $\Delta_1 = \Delta_2$

or $\dfrac{Fl_1}{\pi r_1^2 y_1} = \dfrac{Fl_2}{\pi r_2^2 y_2}$ or $\dfrac{2}{R^2 \times 7} = \dfrac{1.5}{2^2 \times 4}$

$\therefore \quad R = 1.75\,\text{mm}$

20. (A)

(a) When a catapult is stretched up to length l, then the stored energy in it = Δk. E $\Rightarrow \frac{1}{2}.\left(\frac{YA}{L}\right)(\Delta l)^2 = \frac{1}{2}mv^2$

$$\Rightarrow \quad y = \frac{mv^2 L}{A(\Delta l)^2}$$

$m = 0.02\,\text{kg}, v = 20\,\text{ms}^{-1}, L = 0.42\,\text{m}, A = (\pi d^2)/(4)$

$d = 6 \times 10^{-3}\,\text{m}, \Delta l = 0.2\,\text{m}$

$$y = \frac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04} = 2.97 \times 10^6\,\text{N/m}^2$$

So, order is 10^6.

21. (A)

(a) If force F acts along the length L of the wire of cross-section A, then energy stored in unit volume of wire is given by

Energy density = $\frac{1}{2}$ stress × strain

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{F}{AY} \quad \left(\because \text{ stress} = \frac{F}{A} \text{ and strain} = \frac{F}{AY}\right)$$

$$= \frac{1}{2}\frac{F^2}{A^2 Y} = \frac{1}{2}\frac{F^2 \times 16}{(\pi d^2)^2 Y} = \frac{1}{2}\frac{F^2 \times 16}{\pi d^4 Y}$$

If u_1 and u_2 are the densities of two wires, then

$$\frac{u_1}{u_2} = \left(\frac{d_2}{d_1}\right)^4 \Rightarrow \frac{d_1}{d_2} = (4)^{1/4} \Rightarrow \frac{d_1}{d_2} = \sqrt{2} : 1$$

22. (C)

(c) According to question, pressure inside, 1st soap bubble,

$$\Delta P_1 = P_1 - P_0 = 0.01 = \frac{4T}{R_1} \quad ...(i)$$

And $\Delta P_2 = P_2 - P_0 = 0.02 = \frac{4T}{R_2} \quad ...(ii)$

Dividing, equation (ii) by (i),

$$\frac{1}{2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$$

Volume $V = \frac{4}{3}\pi R^3 \Rightarrow \frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = \frac{8}{1}$

23. (B)

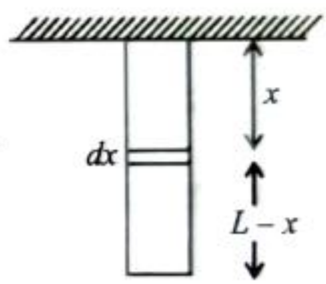
(b) Given, Weight of uniform heavy rod. $W = 10\ \text{kg ms}^{-2}$

Length of rod, $L = 20\ \text{cm} = 0.2\ \text{m}$

Consider an element dx at distance x from top.

Force acting on part below dx

$$= \frac{(L-x)W}{L}$$



Elongation for small element dx, dl

$$= \left(\frac{L-x}{L}\right)\frac{Wdx}{AY}$$

$$\int_0^{\Delta l} dl = \int\left(\frac{L-x}{L}\right)\frac{w\,dx}{AY} \Rightarrow \Delta \ell = \frac{WL}{2AY} = 5 \times 10^{-9}\text{m}$$

24. (B)

(b) Force exerted on each column $F = \frac{mg}{4}$

$$\because y = \frac{F}{A} \bigg/ \frac{\Delta \ell}{\ell}$$

$$\therefore \quad \text{Strain} \left(\frac{\Delta \ell}{\ell}\right) = \frac{mg}{4AY}$$

$$= \frac{50 \times 10^3 \times 9.8}{4 \times \left[\pi \times 1^2 - \pi(.5)^2\right]} = 2.6 \times 10^{-7} \quad \left[\because A = \pi r^2\right]$$

25. (C)

(c) Here, $m_1 = 3kg, m_2 = 5\,kg$

Tension in the metal wire.

$$T = \frac{2m_1m_2g}{m_1 + m_2} = \frac{2 \times 3 \times 5 \times 10}{8} = \frac{75}{2}$$

$$\text{Stress } = \frac{T}{A} = \frac{T}{\pi r^2}$$

$$\Rightarrow \frac{24}{\pi} \times 10^2 = \frac{75}{2 \times \pi r^2} \Rightarrow r^2 = \frac{75}{2 \times 24 \times 100} = \frac{3}{8 \times 24}$$

$$\Rightarrow r = 0.125\text{ m} = 12.5\text{ cm}$$

26. (A)

(a) Bulk modulus (K) is given by $K = \dfrac{P}{\left(-\dfrac{\Delta V}{V}\right)}$

Density, $\rho = \dfrac{m}{V}$

So, $\dfrac{\Delta\rho}{\rho} = \dfrac{+\Delta V}{V}$ $\therefore K = \dfrac{P}{\left(\dfrac{\Delta\rho}{\rho}\right)} \Rightarrow \dfrac{\Delta\rho}{\rho} = \dfrac{P}{K} \Rightarrow \Delta\rho = \dfrac{\rho P}{K}$

27. (D)

(d) We know that

$Y = 3K(1 - 2\sigma)$

Here,

σ = Poisson's ratio

$$\sigma = \frac{1}{2}\left(1 - \frac{Y}{3K}\right)$$

Also,

$Y = 2\eta(1 + \sigma)$

$$\sigma = \frac{Y}{2\eta} - 1$$

From equation (i) and equation (ii),

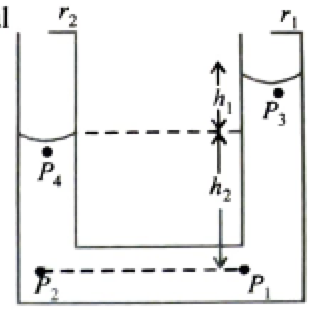
$$\frac{1}{2}\left(1 - \frac{Y}{3K}\right) = \frac{Y}{2\eta} - 1$$

$$\Rightarrow 1 - \frac{Y}{3K} = \frac{Y}{\eta} - 2 \Rightarrow \frac{Y}{3K} = 3 - \frac{Y}{\eta} \Rightarrow \frac{Y}{3K} = \frac{3\eta - Y}{\eta}$$

$$\Rightarrow \frac{\eta Y}{3K} = 3\eta - Y \Rightarrow K = \frac{\eta Y}{9\eta - 3Y}$$

28. (B)

(b) Pressure at same horizontal level is equal



So, $P_1 = P_2$

$$P_3 = P_{atm} - \frac{2T}{r_1}$$

$$\& \; P_4 = P_{atm} - \frac{2T}{r_2}$$

$$P_2 = P_4 + \rho g h_2 = P_{atm} - \frac{2T}{r_2} + \rho g h_2$$

$$\& \; P_1 = P_3 + \rho g (h_1 + h_2) = P_{atm} - \frac{2T}{r_1} + \rho g (h_1 + h_2)$$

$$\therefore P_{atm} - \frac{2T}{r_1} + \rho g (h_1 + h_2) = P_{atm} - \frac{2T}{r_2} + \rho g h_2$$

$$\therefore \rho g h_1 = 2T \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$= 2 \times 7.3 \times 10^{-2} \left[\frac{1}{2.5 \times 10^{-3}} - \frac{1}{4 \times 10^{-3}} \right]$$

$$\Rightarrow h_1 = 2.19 \times 10^{-3} \text{ m} = 2.19 \text{ mm}$$

29. (A)

(a) As volume remain same i.e volume of two smaller drops will be equal to volume of one big drop.

$$2 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \Rightarrow 2r^3 = R^3$$

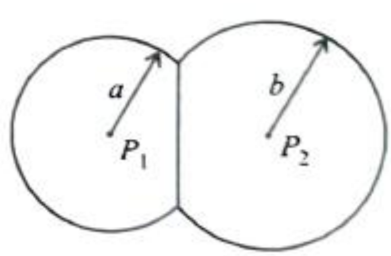
$$\Rightarrow 2 = \left(\frac{R}{r} \right)^3 \Rightarrow \frac{R}{r} = 2^{\frac{1}{3}}$$

$$\frac{U_i}{U_f} = \frac{T \times 2 \times (4\pi r^2)}{T \times 4\pi R^2} = \frac{2r^2}{2^{\frac{2}{3}} R^2} = 2^{\frac{1}{3}}$$

30. (A)

(a) Let R be the radius of curvature of common surface

$$P_1 = P_0 + \frac{4T}{a} \text{ and}$$



$$P_2 = P_0 + \frac{4T}{b}$$

$$\text{And } P_1 - P_2 = \frac{4T}{R}$$

$$\left(P_0 + \frac{4T}{a} \right) - \left(P_0 + \frac{4T}{b} \right) = \frac{4T}{R}$$

$$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{R} \qquad \therefore \; R = \frac{ab}{(b-a)}$$

31. (C)

(c) As volume remains unchanged

$$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow nr^3 = R^3$$

Work done = T × Increment in surface area

$$W = T[4\pi nr^2 - 4\pi R^2] \quad ...(i)$$

By conservation of energy

Loss in thermal energy = work done against surface tension

$$JQ = W$$

Where J = mechanical equivalent of heat

$$Q = \frac{W}{J} = \frac{4\pi T(nr^2 - R^2)}{J} \qquad \text{[Using (i)]}$$

Heat energy per unit volume

$$Q = \frac{4\pi T}{Jn \cdot \frac{4}{3}\pi r^3}[nr^2 - R^2] = \frac{4\pi T}{J \times \frac{4}{3}\pi}\left[\frac{1}{r} - \frac{R^2}{nR^3}\right]$$

$$= \frac{3T}{J}\left[\frac{1}{r} - R^2\right] = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

32. (A)

Young modulus depends on material of wire not on length and area of wire's cross section.

33. (C)

(c) Force $F = YA\frac{\Delta l}{l}$ Y = young's modulus of the wire

$$= 2 \times 10^{11} \times 10^{-4}\left(\frac{2l - l}{l}\right) = 2 \times 10^7\,\text{N}$$

34. (D)

(d) Young's modulus,

$$\text{Y} = \frac{\text{F/A}}{\frac{\Delta\ell}{\ell}} = \frac{\text{F}\ell}{\text{A}\Delta\ell} \Rightarrow \Delta\ell = \frac{\text{F}\ell}{\text{AY}}$$

Net elongation, $\Delta\ell = \Delta\ell_1 + \Delta\ell_2$.

$$\Rightarrow \Delta\ell = \frac{\text{F}\ell_1}{\text{A}_1\text{Y}_s} + \frac{\text{F}\ell_2}{\text{A}_2\text{Y}_c}$$

$$\Rightarrow \text{F} = \frac{\Delta\ell}{\frac{\ell_1}{\text{A}_1\text{Y}_s} + \frac{\ell_2}{\text{A}_2\text{Y}_c}} = \frac{\Delta\ell\text{A}}{\frac{\ell_1}{\text{Y}_s} + \frac{\ell_2}{\text{Y}_c}} \quad (\because \text{A}_1 = \text{A}_2 = \text{A})$$

$$= \frac{1.4 \times 10^{-3} \times \pi \times (1.4 \times 10^{-3})^2}{\frac{3.2}{2 \times 10^{11}} + \frac{4.4}{1.1 \times 10^{11}}} = 1.54 \times 10^2 = 154\,\text{N}$$

35. (A)

(a) Since breaking stress (Maximum lifting capacity) is the property of material so it will remain same.

$$\text{breaking stress} = \frac{\text{Maximum lifting capacity}}{\text{Area of cross-section of rope}}$$

$$\Rightarrow \frac{10}{2.5\times10^{-4}} = \frac{25}{A}$$

$$\Rightarrow A = \frac{25\times2.5\times10^{-4}}{10} = 625\times10^{-6} = 6.25\times10^{-4}\,m^2$$

36. (C)

(c) When length 'L' is hanging from fixed support

$$\Delta L = \frac{mgL}{\gamma A} \Rightarrow \Delta L \propto m$$

$$\Rightarrow \frac{\Delta L_1}{\Delta L_2} = \frac{m_1}{m_2} = \frac{1}{2} \Rightarrow \frac{L_1 - L}{L_2 - L} = \frac{1}{2}$$

$$\Rightarrow 2L_1 - 2L = L_2 - L \Rightarrow 2L_1 - L_2 = L$$

$$\therefore L = 2L_1 - L_2$$

37. (C)

(c) At equilibrium, $\vec{F} = 0$

$$\Rightarrow \frac{-du}{dr} = 0 \Rightarrow \frac{-d}{dr}\left(\frac{A}{r^{10}} - \frac{B}{r^5}\right) = 0$$

$$\Rightarrow +\frac{A}{r^{11}}\times10 - \frac{5B}{r^6} = 0 \Rightarrow \frac{1}{r^6}\left[\frac{10A}{r^5} - \frac{5B}{1}\right] = 0$$

$$\Rightarrow \frac{10A}{r^5} = 5B \Rightarrow r^5 = \frac{10A}{5B} \Rightarrow r = \left(\frac{2A}{B}\right)^{1/5}$$

38. (C)

(c) Bulk modulus, $B = -\frac{\Delta P}{\frac{\Delta V}{V}} \Rightarrow \Delta P = -B\frac{\Delta V}{V}$

$$|\Delta P| = +3\times10^{10}\times0.02 = 6\times10^8$$

39. (D)

(d) We know that terminal velocity is given by

$$V_T = \frac{2gr^2}{g\eta}(\rho - \rho_\ell)$$

Here, we have no involvement of buoyant force. So remove ρ_ℓ.

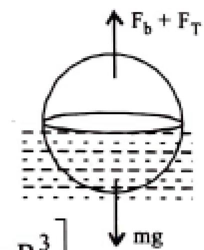
Then, $v_T = \frac{2gr^2\rho}{9\eta} = \frac{2\times10\times10^{-12}\times10^3}{9\times1.8\times10^{-5}} = 123.4\times10^{-6}$ m/s.

40. (A)

(a) As liquid drop is in equilibrium.

So $F_{net} = 0$

Boyant force + surface tension = mg



$$\sigma\frac{V}{2}g + 2\pi RT = \rho Vg$$

$$\Rightarrow 2\pi RT = \frac{(2\rho-\sigma)}{2}\frac{4}{3}\pi R^3 g \quad \left[\because V = \frac{4}{3}\pi R^3\right]$$

$$\Rightarrow R^3 = \frac{3T}{(2\rho-\sigma)g} \Rightarrow R = \sqrt{\frac{3\times7.5\times10^{-2}\,N-m^{-1}}{(2\rho-\sigma)\times10}}$$

$$\Rightarrow R = \frac{3}{20\sqrt{(2\rho-\sigma)}}m = \frac{15}{\sqrt{2\rho-\sigma}}cm$$

41. (A)

(a) $E_i = 0^- A_i 0^-$ = surface energy per unit area

$= T a_i \; [\because 0^- = T] = T.4\pi r_i^2$

Now, $V_i = V_f \Rightarrow \frac{4}{3}\pi r_i^3 = 64\times\frac{4}{3}\pi r_f^3 \Rightarrow r_i^3 = 64 r_f^3$

$\Rightarrow r_i = 4r_F$

So, $E_f = 0^- A_f = T\times64\times\ 4\pi r_f^2 = 256T\times\pi\frac{r_i^2}{16} = 16\pi T r_i^2$

So, $\Delta E = E_f - E_i = 12\pi + Tr_i^2 = 12\pi\times0.075\times0.1^2$

$= 2.82\times10^{-4}$ J

42. (101)

(101) Given : Radius of capillary tube,

$r = 0.015\text{ cm} = 15 \times 10^{-5}\text{ mm}$

$h = 15\text{ cm} = 15 \times 10^{-2}\text{ mm}$

Using, $h = \dfrac{2T\cos\theta}{\rho g r}$ $[\cos\theta = \cos 0° = 1]$

Surface tension,

$$T = \frac{rh\rho g}{2} = \frac{15\times10^{-5}\times15\times10^{-2}\times900\times10}{2} = 101$$

milli newton m^{-1}

43. (4)

(4) $T = Ml\omega^2$

$$\sigma = \frac{T}{A} = \frac{ml\omega^2}{A}$$

$$\frac{ml\omega^2}{A} \le 48\times10^7 \Rightarrow \omega^2 \le \frac{(48\times10^7)A}{ml}$$

$$\Rightarrow \omega^2 \le \frac{(48\times10^7)(10^{-6})}{10\times3} = 16 \Rightarrow \omega_{max} = 4\text{ rad/s}$$

44. (40)

(40) Let m be mass that can be placed in the pan. For wire W_1

$$\text{Stress} = \frac{\text{Maximum weight}}{\text{Area}} = \frac{(m+30)g}{8\times10^{-7}}$$

$\Rightarrow \quad m + 30 = 1.25 \times 10^9 \times 8 \times 10^{-7}$

$\Rightarrow \quad m + 30 = 100 \Rightarrow m = 70\text{ kg}$

For wire W_2

$$\text{Stress} = \frac{(m+10)g}{4\times10^{-7}} = 1.25\times10^9$$

$\Rightarrow \quad m + 10 = 50 \Rightarrow m = 40\text{ kg}$

Maximum mass that can be placed = 40 kg.

45. (20)

Energy stored in stretched catapult is converted into kinetic energy of stone

$$\frac{1}{2}\cdot\frac{YA}{L}\cdot x^2=\frac{1}{2}mv^2$$

$$\frac{0.5\times10^9\times10^{-6}\times(0.04)^2}{0.1}=\frac{20}{1000}v^2\Rightarrow v^2=400$$

$\therefore\ v=20\,\text{m/s}$

46. (2)

As $\Delta l=\frac{F\cdot l}{Y\cdot\pi r^2}\Rightarrow\Delta l\propto\frac{l}{r^2}$

$$\Delta l_2=\left(\frac{l_2}{l_1}\right)\left(\frac{r_1}{r_2}\right)^2\Delta l_1=(2)\left(\frac{1}{2}\right)^2(0.04)\,\text{m}=2\,\text{cm}$$

47. (500)

(500) Bulk modulus, $B=-\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$$\Rightarrow B=-\frac{V}{dV}(\rho gh)\Rightarrow h=-\frac{B\frac{\Delta V}{V}}{\rho g}$$

$$\therefore\ h=\frac{9.8\times10^8\times0.5}{100\times10^3\times9.8}=500\,\text{m.}$$

48. (2)

(2) Excess pressure inside bigger bubble $=\frac{4T}{r_2}$

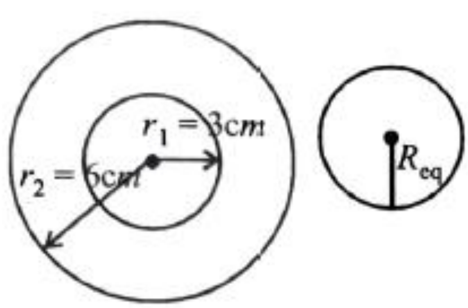
So, Excess pressure inside the smaller soap bubble

$$\Delta P_1=\frac{4T}{r_1}+\frac{4T}{r_2}$$

The excess pressure inside the equivalent soap bubble.

$$\Delta P_2=\frac{4T}{R_{eq}}$$

$$\Delta P_1=\Delta P_2$$



$$\Rightarrow\ \frac{4T}{R_{eq}}=\frac{4T}{r_1}+\frac{4T}{r_2}$$

$$\Rightarrow\ R_{eq}=\frac{r_1r_2}{r_1+r_2}$$

Putting $r_1=3\text{cm}$, $r_2=6\text{ cm}\Rightarrow R_{eq}=2\text{ cm}$

49. (50)

(50) Given, length of metal wire, $\ell = 0.5$ m

Cross-sectional area, $A = 10^{-4}\,m^2$

Breaking stress $= 5 \times 10^8\,Nm^{-2}$

Mass of block $m = 10$ kg

T_{max} = Breaking stress × Area

$$\frac{mv^2}{\ell} = 5 \times 10^8 \times 10^{-4} = 5 \times 10^4$$

$$\frac{10\,v^2}{0.5} = 5\times10^4 \Rightarrow v = \sqrt{\frac{0.5\times5\times10^4}{10}} = 50\text{ m/s}$$

50. (30)

$$\textbf{(30) Strain} = F/AY = \frac{mg + \frac{mv^2}{R}}{AY}$$

$$= \frac{20 + \frac{2(5)^2}{0.5}}{3\times10^{-6}\times10^{11}} = 30\times10^{-5}$$

51. (2)

$$\textbf{(2)}\ \text{Slope} = \frac{\text{Extension/Load}}{\text{Length of wire}} = \frac{\Delta l/w}{L}$$

$$\text{Young's modulus, } Y = \frac{mg/A}{\Delta\ell/L} = \frac{wL}{\Delta\ell A}$$

$$\therefore\ Y = \frac{1}{(\text{slope})A} \Rightarrow Y = \frac{1}{2\times10^{-6}(0.25\times10^{-5})}$$

$$\Rightarrow Y = 2\times10^{11}\ N/m^2$$

52. (25)

Given mass of rod $m = 20$ kg

Cross-section area, $A = 0.4\ m^2$

Length $\ell = 20$ m

Let extension is dy in length dx.

$$Y = \frac{stress}{strain} = \frac{\frac{T}{A}}{\frac{dy}{dx}} = \frac{T}{A}\cdot\frac{dx}{dy} \Rightarrow dy = \frac{Tdx}{AY}$$

Tension at a distance x from lower end0020 $= \frac{mg}{\ell}x$

$$\int_0^{\Delta\ell} dy = \int_0^{\ell}\frac{mg}{\ell}x\frac{dx}{AY} \Rightarrow \Delta\ell = \frac{mg}{\ell AY}\left[\frac{x^2}{2}\right]_0^{\ell}$$

$$\Delta \ell = \frac{mg\ell}{2AY} = \frac{20 \times 10 \times 20}{2 \times 0.4 \times 2 \times 10^{11}} = 25 \times 10^{-9} \text{ m}$$

Compare with $x \times 10^{-9}$ we have, $x = 25$

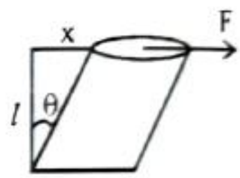
53. (5)

(5) $\Delta \ell = \frac{F\ell}{\gamma A} \Rightarrow \Delta \ell \propto \frac{F\ell}{r^2}$

$$\Rightarrow \Delta \ell_2 = \Delta \ell_1 \left(\frac{F_2}{F_1}\right)\left(\frac{\ell_2}{\ell_1}\right)\left(\frac{r_1}{r_2}\right)^2 = 5 \times 4 \times 4 \times \frac{1}{16} = 5 \text{ cm}$$

54. (48)

(48) $\eta = \frac{F}{A \tan\theta} = \frac{F}{A\left(\frac{x}{\ell}\right)}$



$$\frac{F}{A} = \eta \frac{x}{\ell} \Rightarrow \frac{F\ell}{A\eta} = x$$

$$\Rightarrow \quad x = \frac{18 \times 10^4 \times 60 \times 10^{-2}}{60 \times 10^{-2} \times 15 \times 10^{-2} \times 25 \times 10^2}$$

$= 48 \times 10^{-6} \text{m} = 48 \, \mu\text{m}$

55. (25)

(25) $y = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain} = \frac{1}{y}\text{stress}$

$$\therefore \frac{1}{y} = \text{slope} = \frac{(2-1) \times 10^{-10}}{40 - 20} = \frac{1}{20} \times 10^{-10} \Rightarrow y = 20 \times 10^{10}$$

So, energy density $= \frac{1}{2} \times \text{stress} \times \text{strain}$

$$= \frac{1}{2} \times y \times (\text{strain})^2 = \frac{1}{2} \times 20 \times 10^{10} \times (5 \times 10^{-4})^2$$

$= 10^{11} \times 25 \times 10^{-8} = 25 \times 10^3 = 25 \text{ kJ/m}^3$

56. (25)

(25) $F_V + F_B = mg$ (V = constant)

$F_V = mg - F_B = \rho_B Vg - \rho_L Vg = (\rho_B - \rho_L)Vg$

$$= (8 - 1.3) \times 10^{+3} \times \frac{0.3 \times 10^{-3}}{8 \times 10^3} \times 10 = 25 \times 10^{-4} \text{ N}$$

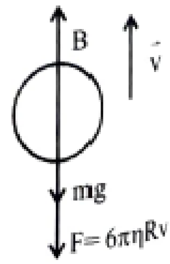
Hence, the value of viscous force acting on ball will be 25×10^{-4} N.

Compair with $x \times 10^{-4}$, we get $x = 25$

57. (11)

(11) When bubble is rising steadily the net force acting on it will be zero.

So, buoyant force (B) = drag force (F)



$$\Rightarrow \frac{4\pi}{3}R^3\rho g = 6\pi\eta R v$$

$$\therefore \quad \eta = \frac{4}{18}\frac{R^2\rho g}{v}$$

$$\text{or,} \quad \eta = \frac{4}{18}\frac{(10^{-3})^2 \times 1.75 \times 10^3 \times 10}{0.35 \times 10^{-2}}$$

$$\therefore \quad \eta = \frac{10}{9} = 1.11 = 11 \text{ poise}$$

58. (20)

(20) Speed after falling 'h' height = Terminal velocity

$$\Rightarrow \sqrt{2gh} = \frac{2r^2 g}{g\eta}(\rho - \rho_f)$$

$$\Rightarrow \sqrt{2 \times 10 \times h} = \frac{2 \times (0.1 \times 10^{-3})^2 \times 10}{9 \times 10^{-5}} \times (10^4 - 10^3)$$

$$\Rightarrow \sqrt{20h} = \frac{2 \times 10^{-8} \times 10}{9 \times 10^{-5}} \times 9 \times 10^3$$

$$\Rightarrow \sqrt{20h} = 20 \Rightarrow 20h = 400 \Rightarrow h = 20 \text{ m}$$

59. (500)

(500) $P = P_0 + \frac{2T}{R} \Rightarrow P - P_0 = \frac{2T}{R}$

$$500 = \frac{2 \times T}{2 \times 10^{-3}}$$

$T = 500 \times 10^{-3}$

So, $x = 500$