JEE Main Exercise

1. (A)

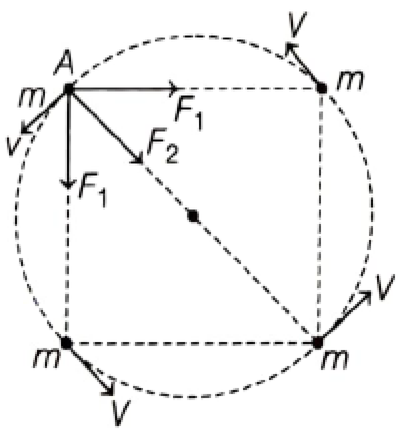
$$F_1 = \frac{GM^2}{a^2} \text{ and } F_2 = \frac{GM^2}{(\sqrt{2}a)^2}$$

Net force on $A = F_2 + 2F_1 \cos 45°$

$$\Rightarrow \left(\frac{1}{2} + \sqrt{2}\right)\frac{GM^2}{a^2}$$

$$\Rightarrow F_{\text{net}} = \frac{Mv^2}{r} = \left(\frac{2\sqrt{2}+1}{2}\right)\frac{GM^2}{a^2}, \text{ where } r = \frac{a}{\sqrt{2}}$$

$$\Rightarrow v = \sqrt{\frac{GM(2\sqrt{2}+1)}{2\sqrt{2}a}}$$



2. (D)

$$F = mE = m\left(\frac{\sqrt{2}G\lambda}{R}\right), \text{ where } \lambda = \frac{m}{(\pi R/2)} \Rightarrow F = \frac{2\sqrt{2}Gm^2}{\pi R^2}$$

3. (D)

$$F_1 = \frac{GMm}{(2R)^2}, F_2 = \frac{GMm}{(2R)^2} - \frac{G(M/8)m}{(3R/2^2)} = \frac{7GMm}{36R^2}$$

So, $\frac{F_2}{F_1} = \frac{7}{9}$

4. (C)

Let distance of neutral point from smaller mass be x.

$$E = \frac{GM}{x^2} - \frac{G(4M)}{(6R-x)^2} = 0 \Rightarrow x = 2R$$

5. (B)

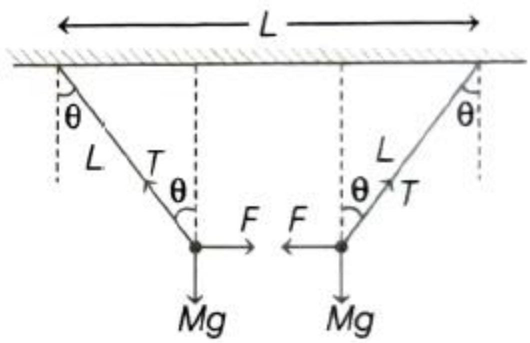
$$F = \frac{GM^2}{(L - 2L\sin\theta)^2} = \frac{GM^2}{L^2} \qquad [\because (\theta \to 0)]$$

$$T\sin\theta = F \qquad ...(i)$$

$$T\cos\theta = Mg \qquad ...(ii)$$

Dividing Eqs. (i) by (ii)

$$\theta = \tan^{-1}\left(\frac{GM}{gL^2}\right)$$



6. (A)

When body is taken above the earth's surface,

$$\frac{\Delta g}{g} \times 100 = -\frac{2h}{R} \times 100$$

$$\Rightarrow \quad -1\% = -\frac{2h}{R} \times 100$$

$$\Rightarrow \quad \frac{h}{R} \times 100 = 0.5\%$$

When body is taken below the earth's surface,

$$\frac{\Delta g}{g} \times 100 = -\frac{h}{g} \times 100 = -0.5\%$$

So, weight decreases by 0.5%.

7. (B)

On earth, $T = 2\pi\sqrt{\frac{l}{g}} = 2$

On planer, $g' = \frac{G(2M)}{(2R)^2} = \frac{g}{2}$

$$T' = 2\pi\sqrt{\frac{l}{(g/2)}} = \sqrt{2}T = 2\sqrt{2}\text{ s}$$

8. (C)

$$g_{\text{app}} = g - \omega^2 R\cos^2\lambda = 0$$

$$\Rightarrow \quad \omega = 2\sqrt{\frac{g}{R}}, T = \frac{2\pi}{\omega} = \pi\sqrt{\frac{R}{g}}$$

9. (D)

$$V = V_1 + V_2 + V_3 + ...$$
$$= -\frac{Gm}{1} - \frac{Gm}{2} - \frac{Gm}{4} - \frac{Gm}{8} ....$$
$$= -Gm\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right)$$
$$= -\frac{Gm(1)}{1 - \frac{1}{2}} = -2Gm$$

10. (B)

$$dV = -E\,dr$$
$$\Rightarrow \int_{v_i}^{v} dV = +\int_{d_i}^{r} \frac{k}{r} dr$$
$$\Rightarrow V - V_i = +k \ln\left(\frac{r}{d_i}\right)$$
$$\Rightarrow V = V_i + k \ln\left(\frac{r}{d_i}\right)$$

11. (C)

$$V = -\frac{Gm}{R} = -\frac{G\sigma\left(4\pi R^2\right)}{R} = -G\sigma 4\pi R$$
$$\frac{V_A}{V_B} = \frac{R_A}{R_B} = \frac{3}{4}$$

When shell A and B coalesce into single shell,

$$\sigma 4\pi R_A^2 + \sigma 4\pi R_B^2 = \sigma 4\pi R_C^2$$
$$\Rightarrow R_C = \sqrt{R_A^2 + R_B^2}$$
$$V_C = -G\sigma 4\pi R_C = -G\sigma 4\pi\sqrt{R_A^2 + R_B^2}$$
$$\frac{V_C}{V_A} = \frac{\sqrt{R_A^2 + R_B^2}}{R_A} = \sqrt{1 + \left(\frac{R_B}{R_A}\right)^2} = \frac{5}{3}$$

12. (B)

$$W = \Delta U = -\frac{Gm^2 \times 3}{2a} - \left(-\frac{Gm^2}{a} \times 3\right) = \frac{3Gm^2}{2a}$$

13. (C)

$K_1 + U_1 = K_2 + U_2$

$\Rightarrow\ 0 - \dfrac{GMm}{R_0} = \dfrac{1}{2}mv^2 - \dfrac{GMm}{R} \ \Rightarrow\ v = \sqrt{2GM\left(\dfrac{1}{R} - \dfrac{1}{R_0}\right)}$

14. (A)

$V_A = \dfrac{-GM}{3R}$ $\qquad V_B = \dfrac{-GM}{2R^3}\left[3R^2 - \left(\dfrac{R}{2}\right)^2\right]$

$= -\dfrac{11GM}{8R}$

$0 + M\left(\dfrac{-GM}{3R}\right) = \dfrac{1}{2}MV_B^2 - M\left(\dfrac{11GM}{8R}\right)$

15. (C)

$K_1 + U_1 = K_2 + U_2$

$\Rightarrow\ \dfrac{1}{2}m(kv_e)^2 - \dfrac{GMm}{R} = 0 - \dfrac{GMm}{r}$

$\Rightarrow\ \dfrac{1}{2}M\left(K\sqrt{\dfrac{2GM}{R}}\right)^2 - \dfrac{GMm}{R} = -\dfrac{GMm}{r}$

$\Rightarrow\ r = \dfrac{R}{1-k^2}$

16. (C)

$K_1 + U_1 = K_2 + U_2$

$\Rightarrow\ \dfrac{1}{2}m(2v_e)^2 - \dfrac{GMm}{R} = \dfrac{1}{2}mv^2 + 0$

$\Rightarrow\ v = \sqrt{4v_e^2 - v_e^2} = \sqrt{3}v_e = 11.2\sqrt{3}\,\text{km/s}$

17. (A)

$v = \omega R$

$g_{\text{app}} = g - \omega^2 R = \dfrac{g}{2} \ \Rightarrow\ g = 2\omega^2 R$

$v_e = \sqrt{2gR} = \sqrt{2\left(2\omega^2 R\right)R}$

$= 2\omega R = 2v$

18. (D)

$F = \dfrac{GMm}{r^m} = m\omega^2 r \ \Rightarrow\ \omega = \dfrac{1}{r^{(m+1)/2}}$

$T = \dfrac{2\pi}{\omega} \ \Rightarrow\ T \propto r^{(m+1)/2}$

19. (C)

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{G\rho\left(\frac{4}{3}\pi R^3\right)}} = 2\pi\sqrt{\frac{3}{4G\rho\pi}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{\rho_2}{\rho_1}} \Rightarrow 1 = \frac{\rho_2}{\rho_1} \Rightarrow \frac{\rho_1}{\rho_2} = 1$$

20. (C)

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = \frac{2\pi}{\omega_{earth}}$$

$$2\pi\sqrt{\frac{r_1^3}{GM}} = \frac{2\pi}{(2\omega_{earth})}$$

On solving, we get

$$r_1 = \frac{r}{(4)^{1/3}}$$

21. (C)

$$\frac{dA}{dt} = \frac{A}{t_1} = \frac{A}{t_2} \Rightarrow t_1 = t_2$$

22. (D)

Angular momentum is conserved.

So, $L_1 = L_2$

$mv_1r_1 = mv_2r_2 \Rightarrow v_2 > v_1$ $(\because r_2 < r_1)$

$\Rightarrow K_2 > K_1$

23. (A)

$$\frac{dA}{dt} = \frac{A}{T}$$

$$\Rightarrow \frac{L}{2m} = \frac{A}{T} \Rightarrow L = \frac{2mA}{T}$$

24. (A)

$$r_p = a(1-e);\ r_a = a(1+e)$$

$$mv_pr_p = mv_ar_a \Rightarrow \frac{v_p}{v_a} = \frac{1+e}{1-e}$$

25. (C)

$mv_1r_1 = mv_2r_2$ ….. (i)

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2} \quad \text{..... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1 = \sqrt{\frac{2GMr_2}{r_1(r_1 + r_2)}}$$

So, $E = K_1 + U_1 = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1}$

$$= -\frac{GMm}{2a} = \text{constant}$$

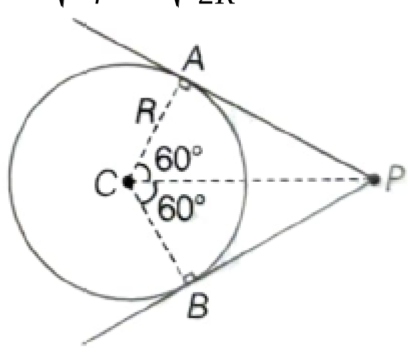
Now, $\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$

$$\Rightarrow \quad v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

26. (2)

$$CP = \frac{R}{\cos 60°} = 2R,$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}}$$

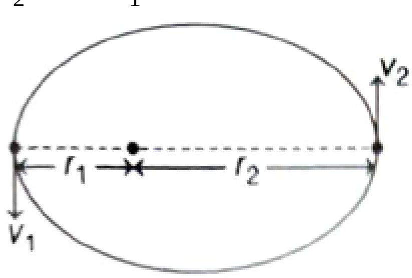


27. (40)

$r_1 = 1.8 \times 10^{12}$ m

$a = 2 \times 10^{12}$ m

$r_2 = 2a - r_1 = 2.2 \times 10^{12}$ m



$$\frac{dA}{dt} = \frac{L}{2m} = \frac{v_2 r_2}{2}$$

$$\Rightarrow \quad 4.4\times10^{16} = \frac{v_2\left(2.2\times10^{12}\right)}{2} \Rightarrow v_2 = 40\,\text{km/s}$$

28. (0.01)

$$g' = g\left(1-\frac{h}{R}\right) = g\left(1-\frac{64}{6400}\right) = 0.99\,g$$

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta g}{g}$$

$$\Rightarrow \quad \frac{\Delta T}{2} = \frac{1}{2}\left(\frac{g-0.99\,g}{g}\right)$$

$$\Rightarrow \quad \Delta T = 0.01\,\text{s}$$

29. (4)

Areal velocity, $= \dfrac{L}{2m} = \dfrac{vr}{2} = \left(\sqrt{\dfrac{Gm}{r}}\right)\dfrac{r}{2} \propto \sqrt{r}$

$$\frac{(\text{Areal velocity})_1}{(\text{Areal velocity})_2} = \frac{\sqrt{r_1}}{\sqrt{r_2}} = 2$$

$$\Rightarrow \quad \frac{r_1}{r_2} = 4$$

30. (5)

$$K_1 + U_1 = K_2 + U_2$$

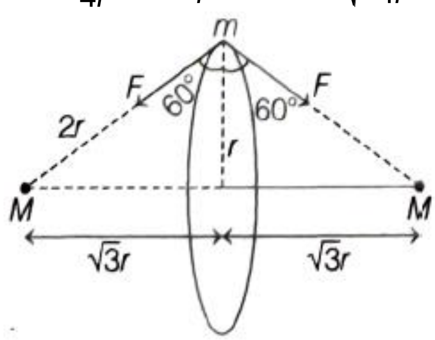
$$\Rightarrow \quad \frac{1}{2}m_0\left(\sqrt{\frac{5GM_e}{4R}}\right)^2 - \frac{GM_e m_0}{R} = 0 - \frac{GM_e m_0}{r}$$

$$\Rightarrow \quad r = \frac{8R}{3}$$

$$h = r - R = \frac{8R}{3} - R = \frac{5R}{3}$$

31. (4)

$$F = \frac{GMm}{4r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{4r}}$$



$$\frac{GMm}{4r^2}=\frac{mv^2}{r} \Rightarrow v=\sqrt{\frac{GM}{4r}}$$

32. (8)

Let C be the midpoint of the line joining C_1 and C_2.

Gravitational field Intensity at C is zero. So, we just need to make the point mass reach C with negligible velocity. Once it crosses C, it will be pulled by the second sphere.

Applying energy conservation between A and C

$$K_1+U_1=K_2+U_2$$

$$\Rightarrow \frac{1}{2}mv_0^2+m\left(\frac{-GM}{R}-\frac{GM}{9R}\right)=0+m\left(\frac{-GM}{5R}-\frac{GM}{5R}\right)$$

$$\Rightarrow v_0=\frac{8}{3}\sqrt{\frac{GM}{5R}}$$

33. (7)

Let speed of the ball just before first collision be v_1.

$$K_1+U_1=K_2+U_2$$

$$\Rightarrow 0-\frac{GMm}{R}=\frac{1}{2}mv_1^2-\frac{3GM}{2R}$$

$$\Rightarrow v_1=\sqrt{\frac{GM}{R}}$$

Speed of ball just after Ist collision,

$$v_2=ev_1=\frac{1}{5}\sqrt{\frac{GM}{R}}$$

Lets take maximum height reached to be h after first collision,

$$K_1+U_1=K_2+U_2$$

$$\frac{1}{2}m\left(\frac{1}{5}\sqrt{\frac{GM}{R}}\right)^2-\frac{3GMm}{2R}=0-\frac{GMm}{2R^3}\left(3R^2-h^2\right)$$

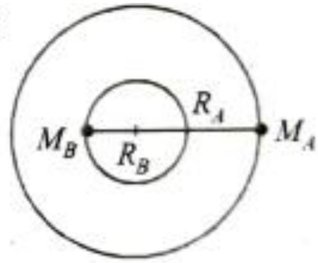
$$\Rightarrow h=\frac{R}{5}$$

Total distance travelled before second collision

$$=R+2h=\frac{7R}{5}$$

1. (D)

(d) Incase of binary star system, the gravitational force of attraction between the stars will provide the necessary centripetal forces.



So angular velocity ω of both stars is the same. Therefore time period $T = \frac{2\pi}{\omega}$ remains the same.

2. (A)

(a) As we know,

Gravitational potential energy = $\frac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2}-1\right) = \frac{-GMm}{6R} \Rightarrow E_i = \frac{-GMm}{R} + K$$

$E_i = E_f$

Therefore minimum required energy, $K = \frac{5GMm}{6R}$

3. (C)

(c) Let mass of smaller sphere (which has to be removed) is m

Radius = $\frac{R}{2}$ (from figure)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3} \Rightarrow m = \frac{M}{8}$$

Mass of the left over part of the sphere

$$M' = M - \frac{M}{8} = \frac{7}{8}M$$

Therefore gravitational field due to the left over part of the sphere

$$E=\left(-\frac{GM}{x^2}+\frac{GM}{8\left(x+\frac{R}{2}\right)^2}\right)$$

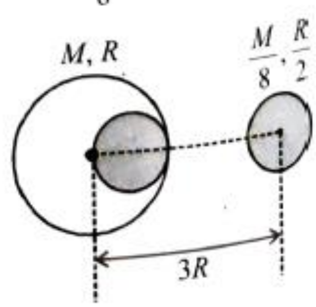
$$=+GM\left[\frac{1}{8\left(x+\frac{R}{2}\right)^2}-\frac{1}{x^2}\right]=GM\left[\frac{x^2-8\left(x+\frac{R}{2}\right)^2}{8\left(x+\frac{R}{2}\right)^2x^2}\right]$$

$$=GM\left[\frac{x^2-8x^2}{8x^4}\right]=\frac{GM}{8x^2}[-7]=\frac{-7GM}{8x^2}\Rightarrow|\vec{E}|=\frac{7GM}{8x^2}$$

4. (A)

(a) Let M' be the mass of removed section

Then, $\frac{M}{\frac{4}{3}\pi R^3}=\frac{M'}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}\Rightarrow M'=\frac{M}{8}$



$$F=\frac{GM.\frac{M}{8}}{(3R)^2}-\frac{G\frac{M}{8}\frac{M}{8}}{\left(\frac{5R}{2}\right)^2}$$

$$=\frac{41}{3600}\frac{GM^2}{R^2}$$

5. (D)

(d) Gravitational field, $I=\left(5\hat{i}+12\hat{j}\right)$ N/kg

$$I=-\frac{dv}{dr}$$

$$\Delta v=-\left[\int_0^x I_x dx+\int_0^y I_y dy\right]$$

$$=-\left[I_x.x+I_y.y\right]=-\left[5(7-0)+12(-3-0)\right]$$

$$=-\left[35+(-36)\right]=1\,J/kg$$

i.e., change in gravitational potential 1 J/kg.

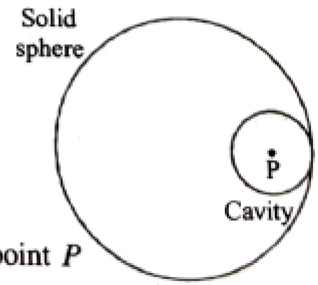
$\Delta U=m\Delta v=1\times1=1\,J$

Hence change in gravitational potential energy 1 J

6. (D)

(d) Due to complete solid sphere, potential at point P

$$V_{sphere} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$



$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$

Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}} = -\frac{3GM}{8R}$$

So potential at the centre of cavity

$$= V_{sphere} - V_{cavity} = -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

7. (C)

(c) As, $V = -\frac{GM}{2R^3}(3R^2 - r^2)$

Graph (c) most closely depicts the correct variation of $v(r)$.

8. (A)

(a) Due to infinite wire of mass 'm' at 'r' distance

$$E_g = \frac{G\lambda}{r} \quad \text{so } F_g = mE_g$$

So force on star $= \frac{Gm\lambda}{r} = \frac{mv^2}{r} \Rightarrow v = \sqrt{G\lambda}$

as $T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{G\lambda}} \Rightarrow T \propto r$

9. (A)

(a) As we know, Gravitational force of attraction,

$$F = \frac{GMm}{R^2}$$

$$F_1 = \frac{GM_e m}{r_1^2} \text{ and } F_2 = \frac{GM_e M_s}{r_2^2}$$

$$\Delta F_1 = -2\frac{GM_e m}{r_1^3}\Delta r_1 \text{ and } \Delta F_2 = -2\frac{GM_e M_s}{r_2^3}\Delta r_2$$

$$\frac{\Delta F_1}{\Delta F_2} = \frac{m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s \Delta r_2} = \left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$$

Using $\Delta r_1 = \Delta r_2 = 2\,R_{earth}$; $m = 8 \times 10^{22}$ kg;
$M_s = 2 \times 10^{30}$ kg
$r_1 = 0.4 \times 10^6$ km and $r_2 = 150 \times 10^6$ km

$$\frac{\Delta F_1}{\Delta F_2} = \left(\frac{8\times10^{22}}{2\times10^{30}}\right)\left(\frac{150\times10^6}{0.4\times10^6}\right)^3 \times 1 \cong 2$$

10. (D)

(d) With rotation of earth or latitude, acceleration due to gravity vary as $g' = g - \omega^2 R \cos^2 \phi$
Where ϕ is latitude, there will be no change in gravity at poles as $\phi = 90°$
At all other points as ω increases g' will decreases hence, weight, W = mg decreases.

11. (C)

(c) Initial gravitational potential energy, $E_i = -\frac{GMm}{2R}$
Final gravitational potential energy,

$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R}$$

$$= -\frac{4GMm}{6R} = -\frac{2GMm}{3R}$$

$\therefore$ Difference between initial and final energy,

$$E_f - E_i = \frac{GMm}{R}\left(-\frac{2}{3}+\frac{1}{2}\right) = -\frac{GMm}{6R}$$

12. (C)

(c) Areal velocity; $\frac{dA}{dt}$

$$dA = \frac{1}{2}r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

Also, $L = mvr = mr^2\omega$ $\quad \therefore \frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$

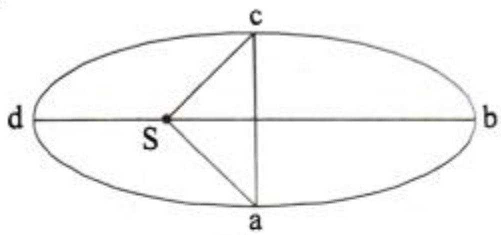
13. (C)

(c) Let area of ellipse abcd = x

Area of SabcS = $\frac{x}{2}+\frac{x}{4}$ (i.e., ar of abca + SacS)

(Area of half ellipse + Area of triangle)

$= \frac{3x}{4}$



Area of SadcS = $x - \frac{3x}{4} = \frac{x}{4}$

$\frac{\text{Area of SabcS}}{\text{Area of SadcS}} = \frac{3x/4}{x/4} = \frac{t_1}{t_2}$

$\frac{t_1}{t_2} = 3$ or, $t_1 = 3t_2$

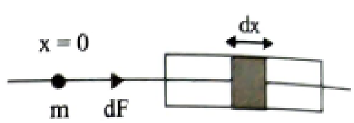
14. (D)

(d) Given $\lambda = (A + Bx^2)$,

Taking small element dm of length dx at a distance x from $x = 0$

so, $dm = \lambda\, dx$

$dm = (A + Bx^2)dx$



$dF = \frac{Gmdm}{x^2}$

$\Rightarrow \quad F = \int_a^{a+L} \frac{Gm}{x^2}(A + Bx^2)dx$

$= Gm\left[-\frac{A}{x} + Bx\right]_a^{a+L}$

$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

15. (D)

(d) $\frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{9}{4}$ or $\frac{g_e}{g_p} = \frac{9}{4}$

or $\frac{GM/R^2}{G(M/9)/R_p^2} = \frac{9}{4} \therefore \quad R_p = \frac{R}{2}$

16. (A)

(a) $g_{\text{eff}} = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$\Rightarrow \dfrac{g}{2} = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} \Rightarrow \sqrt{2} = 1+\dfrac{h}{R} \Rightarrow \dfrac{h}{R} = \sqrt{2}-1$

$\Rightarrow h = (\sqrt{2}-1)\times 6400\times 10^3 \text{ m} = 2.6\times 10^6 \text{ m}$

17. (C)

$E_g = \dfrac{GM}{(3a)^2} + \dfrac{G(2M)}{(3a)^2} = \dfrac{GM}{3a^2}$

18. (B)

(b) $AC = a\sqrt{2} \quad \because r = \dfrac{AC}{2} = \dfrac{a\sqrt{2}}{2} = \dfrac{a}{\sqrt{2}}$

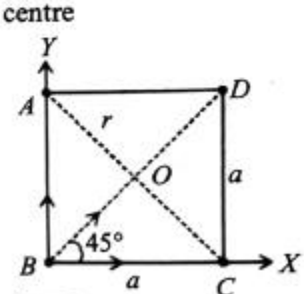
Resultant force on the body

$B = \dfrac{GM^2}{a^2}\hat{i} + \dfrac{GM^2}{a^2}\hat{j} + \dfrac{GM^2}{(a\sqrt{2})^2}(\cos 45°\hat{i} + \sin 45°\hat{j})$

$\Rightarrow |F| = \dfrac{GM^2}{a^2}(\sqrt{2}) + \dfrac{GM^2}{2a^2}$

$\dfrac{Mv^2}{r}$ = Resultant force towards centre

$\therefore \dfrac{Mv^2}{\left(\dfrac{a}{\sqrt{2}}\right)} = \dfrac{GM^2}{a^2}\left(\sqrt{2}+\dfrac{1}{2}\right)$



$\Rightarrow v^2 = \dfrac{GM}{a}\left(1+\dfrac{1}{2\sqrt{2}}\right)$

$\Rightarrow v = \sqrt{\dfrac{GM}{a}\left(1+\dfrac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\dfrac{GM}{a}}$

19. (A)

(a) $F = \dfrac{GMm}{r} = \int a\dfrac{\rho(dV)m}{r^2}$

$= mG\int_0^R \dfrac{k}{r^2}\dfrac{4\pi r^2 dr}{r^2} = -4\pi kGm\left(\dfrac{1}{r}\right)_0^R = -\dfrac{4\pi kGm}{R}$

Using Newton's second law, we have

$\dfrac{mv_0^2}{R} = \dfrac{4\pi kGm}{R}$

or v_0 = C (const.)

Time period, $T = \dfrac{2\pi R}{v_0} = \dfrac{2\pi R}{C}$ or $\dfrac{T}{R}$ = constant.

20. (B)

(c) $U_{surface} + E_1 = U_h$

[$\because$ E_1 is minimum, at height 'h', K.E. = 0]

$$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re+h)}$$

$$\Rightarrow E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right) \Rightarrow E_1 = \frac{GM_e m}{(R_e+h)} \times \frac{h}{R_e}$$

Gravitational attraction

$$F_G = ma_c = \frac{mv^2}{(R_e+h)} = \frac{GM_e m}{(R_e+h)^2} \Rightarrow mv^2 = \frac{GM_e m}{(R_e+h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e+h)}$$

$E_1 = E_2$

Clearly, $\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200\text{km}$

21. (D)

14. (d) $K_E = \frac{1}{2}m \times \frac{2GM_e}{R_e} = \frac{GmM_e}{R_e}$

as, $64\ V_m = V_e \Rightarrow 64 \times \frac{4}{3}\pi R_m^3 = \frac{4}{3}\pi R_e^3 \Rightarrow 4R_m = R_e$

and, $M \propto V$

So, $M_e = 64M_m$

$$K_{moon} = \frac{1}{2}m \times \frac{2GM_m}{R_m}$$

$$= \frac{GmM_m}{R_m} = \frac{GmM_e/64}{\frac{R_e}{4}} = \frac{GmM_e}{16R_m} = \frac{K_e}{16}$$

22. (B)

(b) Orbital, velocity, $v = \sqrt{\frac{GM}{r}}$

Kinetic energy of satellite A,

$T_A = \frac{1}{2}m_A V_A^2$

Kinetic energy of satellite B,

$$T_B = \frac{1}{2}m_B V_B^2 \Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{GM}{R}}{2m \times \frac{GM}{2R}} = 1$$

23. (D)

(d) For a satellite orbiting close to the earth, orbital velocity is given by

$$v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$$

Escape velocity (v_e) is

$$v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR} \quad [\because h << R]$$

$$\Delta v = v_e - v_0 = (\sqrt{2}-1)\sqrt{gR}$$

24. (B)

(b) At height r from center of earth, orbital velocity

$$v = \sqrt{\frac{GM}{r}}$$

By principle of energy conservation

$$\text{KE of 'm'} + \left(-\frac{GMm}{r}\right) = 0 + 0 \quad (\because \text{At infinity, PE} = \text{KE} = 0)$$

$$\text{or} \quad \text{KE of 'm'} = \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$$

25. (D)

(d) Let M is mass of star m is mass of meteroite

By energy convervation between 0 and ∞.

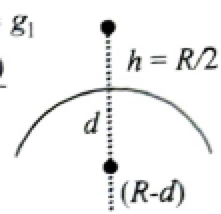
$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV_\infty^2 = 0 + 0$$

$$\therefore \quad v = \sqrt{\frac{4GM}{r}} = \sqrt{\frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}}$$

$$= 2.8 \times 10^5 \text{ m/s}$$

26. (B)

(b) According to question, $g_h = g_d = g_1$



$$g_h = \frac{GM}{\left(R + \frac{R}{2}\right)^2} \text{ and } g_d = \frac{GM(R-d)}{R^3}$$

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3} \Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$$

$$\Rightarrow 4R = 9R - 9d \Rightarrow 5R = 9d$$

$$\therefore \frac{d}{R} = \frac{5}{9}$$

27. (A)

(a) Value of g at equator, $g_A = g \cdot - R\omega^2$

Value of g at height h above the pole,

$$g_B = g \cdot \left(1 - \frac{2h}{R}\right)$$

As object is weighed equally at the equator and poles, it means g is same at these places.

$$g_A = g_B$$

$$\Rightarrow g - R\omega^2 = g\left(1 - \frac{2h}{R}\right)$$

$$\Rightarrow R\omega^2 = \frac{2gh}{R} \Rightarrow h = \frac{R^2\omega^2}{2g}$$

28. (A)

(a) Given : Gravitational field,

$$E_G = \frac{Ax}{(x^2 + a^2)^{3/2}}, V_\infty = 0$$

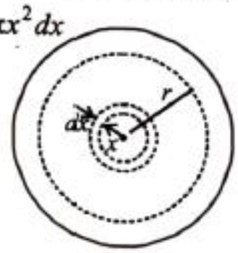
$$\int_{V_\infty}^{V_x} dV = -\int_{\infty}^{x} \vec{E}_G \cdot \vec{d}_x \Rightarrow V_x - V_\infty = -\int_{\infty}^{x} \frac{Ax}{(x^2 + a^2)^{3/2}} dx$$

$$\therefore V_x = \frac{A}{(x^2 + a^2)^{1/2}} - 0 = \frac{A}{(x^2 + a^2)^{1/2}}$$

29. (D)

(d) Mass of small element of planet of radius x and thickness dx.

$$dm = \rho \times 4\pi x^2 dx = \rho_0 \left(1 - \frac{x^2}{R^2}\right) \times 4\pi x^2 dx$$



Mass of the planet

$$M = 4\pi\rho_0 \int_0^r \left(x^2 - \frac{x^4}{R^2}\right) dx$$

$$\Rightarrow M = 4\pi\rho_0 \left|\frac{r^3}{3} - \frac{r^5}{5R^2}\right|$$

Gravitational field,

$$E = \frac{GM}{r^2} = \frac{G}{r^2} \times 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$\Rightarrow E = 4\pi G\rho_0 \left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$$

E is maximum when $\frac{dE}{dr} = 0$

$$\Rightarrow \frac{dE}{dr} = 4\pi G\rho_0 \left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0 \Rightarrow r = \frac{\sqrt{5}}{3} R$$

30. (B)

(b) Gravitation field at the surface

$$E = \frac{Gm}{r^2}$$

$$\therefore E_1 = \frac{Gm_1}{r_1^2} \text{ and } E_2 = \frac{Gm_2}{r_2^2}$$

From the diagram given in question,

$$\frac{E_1}{E_2} = \frac{2}{3} \ (r_1 = 1\text{m}, R_2 = 2m \text{ given})$$

$$\therefore \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2 \left(\frac{m_1}{m_2}\right) \Rightarrow \frac{2}{3} = \left(\frac{2}{1}\right)^2 \left(\frac{m_1}{m_2}\right)$$

$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \frac{1}{6}$$

31. (A)

(a) Orbital speed of the body when it revolves very close to the surface of planet

$$V_0 = \sqrt{\frac{GM}{R}} \quad \text{...(i)}$$

Here, G = gravitational constant

Escape speed from the surface of planet

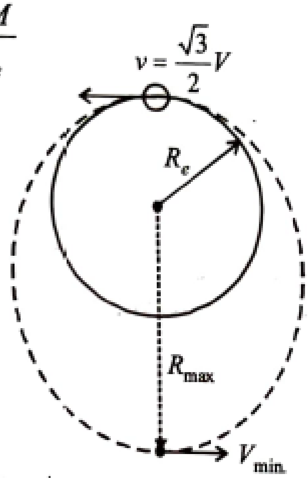
$$V_e = \sqrt{\frac{2GM}{R}} \quad \text{...(ii)}$$

Dividing (i) by (ii), we have

$$\frac{V_0}{V_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$$

32. (C)

(c) Orbital velocity, $V_0 = \sqrt{\frac{GM}{R_e}}$



From energy conversation,

$$-\frac{GMm}{R_e} + \frac{1}{2}m\left(\sqrt{\frac{3}{2}}V\right)^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mV_{min}^2 \quad \text{...(1)}$$

From angular momentum conversation

$$\sqrt{\frac{3}{2}}VR_e = V_{min}R_{max} \quad \text{...(2)}$$

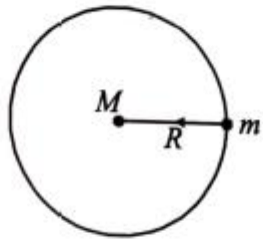
Solving equation (1) and (2) we get,

$$R_{max} = 3R_e$$

33. (A)

(a) According to question, mass density of a spherical galaxy varies as $\frac{k}{r}$.

Mass, $M = \int \rho dV$



$$\Rightarrow M = \int_0^{r=R_0} \frac{k}{r} 4\pi r^2\, dr$$

$$\Rightarrow M = 4\pi k \int_0^{R_0} r\, dr$$

$$\text{or, } M = \frac{4\pi k R_0^2}{2} = 2\pi k R^2 \Rightarrow F_G = \frac{GMm}{R_0^2} = m\omega_0^2 R \; (= F_C)$$

$$\Rightarrow \frac{G\frac{4\pi k R^2}{2}}{R^2} = \omega_0^2 R \Rightarrow \omega_0 = \sqrt{\frac{2\pi KG}{R}} \quad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\frac{2\pi R}{KG}} \Rightarrow T^2 = \frac{2\pi R}{KG}$$

$\because 2\pi$, K and G are constants $\therefore T^2 \propto R$.

34. (D)

(d) From law of conservation of momentum, $\vec{p}_i = \vec{p}_f$

$$m_1 u_1 + m_2 u_2 = MV_f$$

$$\Rightarrow v_f = \frac{\left(mv + \frac{mv}{4}\right)}{\frac{3m}{2}} = \frac{5v}{6}$$

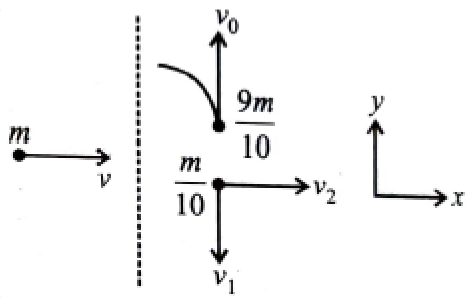
Clearly, $v_f < v_i$ $\quad\therefore$ Path will be elliptical

35. (B)

(b) Let v be the speed of satellite just before ejection of rocket then,

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$\Rightarrow \quad v^2 = u^2 - \frac{GM}{R} \quad \text{..... (i)}$$



Along x :

$$(P_i)_x = (P_f)_x \Rightarrow mv = \frac{m}{10}v_2$$

$$\Rightarrow \quad v_2 = 10v \Rightarrow v_2^2 = 100\left(u^2 - \frac{GM}{R}\right) \quad \text{[From (i)]}$$

Along y :

$$(P_i)_y = (P_f)_y \Rightarrow 0 = \frac{9m}{10}v_0 - \frac{m}{10}v_1$$

$$\Rightarrow \quad v_1 = 9v_0 \Rightarrow v_1^2 = 81\frac{GM}{2R}$$

$$\therefore \quad (\text{K.E.})_R = \frac{1}{2}\frac{m}{10}\left(v_1^2 + v_2^2\right) = 5m\left[u^2 - \frac{119}{200}\frac{GM}{R}\right]$$

36. (B)

(b) According to Kepler's law, when a planet revolves around the sun, its areal velocity is constant.

$$\frac{dA}{dt} = \text{constant}$$

37. (C)

(c) We have given $F \propto \frac{1}{R^3} \Rightarrow F = \frac{K}{R^3}$

Here K is a constant. This force will provide the required centripetal force to the particle for revolution.

$$\frac{mv^2}{R} = \frac{K}{R^3} \Rightarrow v \propto \frac{1}{R}$$

Time period of revolution,

$$T = \frac{2\pi R}{v} \quad \therefore T \propto R^2$$

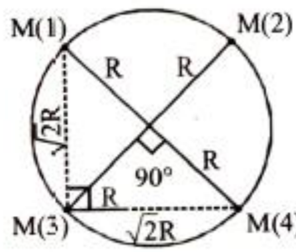
38. (B)

(b) Centripetal Force $F_{net} = \frac{Mv^2}{R}$

Gravitational force between two masses $= \frac{GM_1M_2}{d^2}$

So, $F_{12} = F_{13} = \frac{GM^2}{(\sqrt{2}R)^2}$

$(\because M_1 = M_2 = M)$



Resultant of these two forces

$= \sqrt{2}\frac{GM^2}{2R^2}$

Combining all forces and equating with centripetal force we get

$$\sqrt{2}\frac{GMM}{(\sqrt{2}R)^2} + \frac{GMM}{(2R)^2} = \frac{Mv^2}{R} \Rightarrow \frac{GM}{R}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = v^2$$

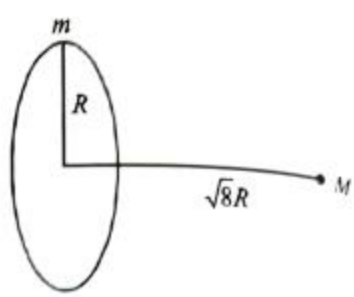
$$\Rightarrow \frac{GM}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right) = v^2 \Rightarrow v = \sqrt{\frac{GM(4+\sqrt{2})}{R4\sqrt{2}}}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{GM(2\sqrt{2}+1)}{R}}$$

39. (A)

(c) Gravitational field of ring

$$E = \frac{-Gmx}{(R^2+x^2)^{3/2}}$$



Force between sphere and ring

$$F = \frac{GMm\sqrt{8}R}{[R^2+8R^2]^{3/2}}$$

$$\Rightarrow F = \frac{\sqrt{8}GMm}{27R^2}$$

40. (C)

(c) As density is same, $2M_E = M_P$

$$\Rightarrow 2\rho \times \frac{4}{3}R_E^3 = \rho \times \frac{4}{3}\pi R_P^3 \Rightarrow R_P = 2^{1/3}R_E$$

Acceleration due to gravity on the surface of planet,

$$g_P = \frac{GM_P}{R_P^2} \Rightarrow g_p = \frac{G2M_E}{(2^{1/3}R_E)^2} = \frac{G2M_E}{2^{2/3}R_E^2}$$

$\Rightarrow g_P = 2^{1/3} g_e$

Weight on planet = $2^{1/3}$ Weight on earth

$\Rightarrow \quad W_P = 2^{1/3} W$

41. (B)

(b) Weight of body at pole = mg = 49 N

Weight of body at equator due to rotation,

$$g_e = g - R\omega^2$$

so $W_e = mg_e = m(g - R\omega^2)$

$\therefore \; W_P > W_e \qquad W_P = 49$ N

So, $W_e = 48.83$ N. $\qquad W_e < 49$ N.

42. (D)

(d) The gravitational potential at a point A due to mass of the centre is

$$V_1 = -\frac{GM}{r}$$

Gravitational potential at A due to shell is

$$V_2 = -\frac{GM}{R}$$

$$V_A = V_1 + V_2 \Rightarrow V_A = \left[-\frac{GM_1}{r} - \frac{GM_2}{R}\right]$$

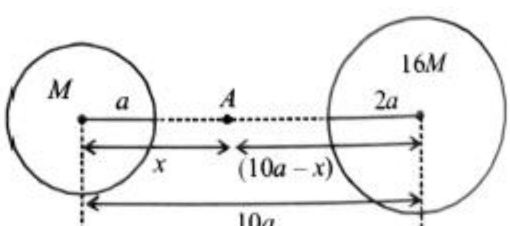
$$= \left[-\frac{50}{25}G - \frac{100}{50}G\right] = -4G$$

43. (A)

Inside uniform spherical shall, field is zero & hence potential is constant same as on surface.

44. (D)

(d)



Let A be the point where gravitation field of both planets cancel each other i.e. zero. After this field due to small mass will dominate and 'm' will easily reach small mass surface.

$$\frac{GM}{x^2} = \frac{G(16M)}{(10a - x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{4}{(10a - x)} \Rightarrow 4x = 10a - x \Rightarrow x = 2a \qquad \text{...(i)}$$

Using conservation of energy, we have

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$KE = GMm\left[\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right]$$

$$\Rightarrow KE = GMm\left[\frac{1 + 64 - 4 - 16}{8a}\right]$$

$$\Rightarrow \frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right] \Rightarrow v = \sqrt{\frac{90GM}{8a}} \quad \Rightarrow v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

45. (B)

(b) Total energy at middle point

= K.E + P.E of M_1 & m + P.E of M_2 & m

To get escape velocity total energy should be zero.

$$\frac{1}{2}mV^2 - \frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = 0$$

$$\Rightarrow \frac{1}{2}mV^2 = \frac{2Gm}{r}(M_1 + M_2) \;\therefore V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$$

46. (B)

(b) Speed of satellite, $v = \sqrt{\frac{GM}{r}}$

Time, $T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = 2\pi\sqrt{\frac{r^3}{GM}}$

$$\therefore T_B - T_A = \frac{2\pi}{\sqrt{GM}}\left[r_B^{3/2} - r_A^{3/2}\right]$$

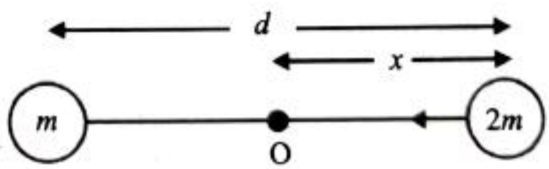
$$= \frac{2\pi}{\sqrt{GM}}\left[(8 \times 10^6)^{3/2} - (7 \times 10^6)^{3/2}\right]$$

$$= \frac{2\pi}{\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}}} \times 10^9[8^{3/2} - 7^{3/2}]$$

≈ 1300 s

47. (B)

(b)



For point O to be the centre of mass of the system, moment about O should be zero.

$\therefore\ 2mx = m(d-x)$

$\Rightarrow 3mx = md \Rightarrow x = \dfrac{d}{3}$

For equilibrium,

$F_{gravitational} = F_{centripetal}$

$\therefore\ F = \dfrac{G(2m)m}{d^2} = (2m)\omega^2\left(\dfrac{d}{3}\right)$

$\Rightarrow \dfrac{Gm}{d^2} = \omega^2 \dfrac{d}{3} \Rightarrow \omega^2 = \dfrac{3Gm}{d^3} \Rightarrow \omega = \sqrt{\dfrac{3Gm}{d^3}}$

$\therefore$ Period of revolution, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{d^3}{3Gm}}$

48. (B)

(b) By Kepler's law

$T^2 \propto R^3$

$T_2^2 = T_1^2 \times \left(\dfrac{R_2}{R_1}\right)^3 \Rightarrow T_2 = T_1\left(\dfrac{R_2}{R_1}\right)^{\frac{3}{2}}$

$= 7(3)^{\frac{3}{2}} = 7 \times 3\sqrt{3} = 21\sqrt{3}$ hours $\simeq 36$ hours

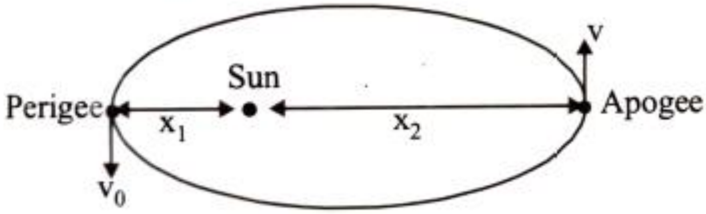
49. (C)

(c) By kepler's law $T^2 \propto r^3$

$\Rightarrow \left(\dfrac{T_A}{T_B}\right)^2 = \left(\dfrac{r_A}{r_B}\right)^3 \Rightarrow 2^2 = \left(\dfrac{r_A}{r_B}\right)^3 \Rightarrow r_A^3 = 4r_B^3$

50. (C)

(c) When distance of the planet from the sun is maximum i.e., x at apogee so velocity is minimum and vice-versa.



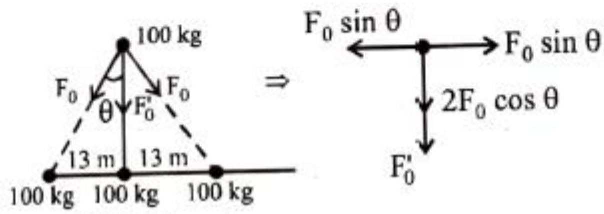
By angular momentum conservation

$$mv_0x_1 = mvx_2$$

$$\Rightarrow \quad v = \frac{v_0x_1}{x_2}$$

51. (B)

(b)



So, $F_{net} = F_o' + 2F_o \cos\theta$

$$= \frac{G \times 100^2}{(13)^2} + \frac{2G(100)^2}{(13\sqrt{2})^2} \cdot \frac{13}{13\sqrt{2}}$$

$$= \frac{G100^2}{13^2}\left(1 + \frac{1}{\sqrt{2}}\right) = 100\,G$$

52. (A)

(a) Given, radius of earth = 6400 km

We have, $g' = g\left(1 - \frac{2d}{R}\right)$

The percentage decrease in the weight,

$$\frac{g'-g}{g} = \frac{-2d}{R} = \frac{2 \times 32 \times 100}{6400} = 1\%$$

53. (D)

(d) Since $g = \frac{GM}{R^2} \Rightarrow g + \frac{G}{R^2} \times \rho \times \frac{4}{3}\pi R^3 \Rightarrow g = \rho R$

So, $\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2} - \frac{R_1}{R_2} = \frac{3}{5} \cdot \frac{2}{3} = \frac{2}{5}$

54. (A)

(a) Law of gravitation is universal law so it is valid for any two pair of bodies.
At centre, $d = R$

So, $g = \left(1 - \frac{d}{R}\right) = \left(1 - \frac{R}{R}\right) = 0$

55. (A)

(a) The expression for acceleration due to gravity is as shown,

$$\vec{g} = -\frac{GMr}{R^3}(\hat{r}), \quad r < R$$

$$= -\frac{GM}{r^2}\hat{r}, \qquad r > R \Rightarrow \quad |g| \propto r, \; r < R$$

$$\propto \frac{1}{r^2}, \quad r > R$$

56. (B)

(b) $W_h = \frac{W_{surface}}{3} \Rightarrow Mg' = \frac{Mg}{3}$

$$\Rightarrow \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{3} \Rightarrow \left(1+\frac{h}{R}\right) = \sqrt{3}$$

$$\Rightarrow \frac{h}{R} = \sqrt{3}-1 \Rightarrow h = (\sqrt{3}-1)R$$

$$\Rightarrow h = 0.732 \times 6400 \simeq 4685 \text{ km}$$

57. (C)

(c) From $mg = \frac{GMm}{R^2}$

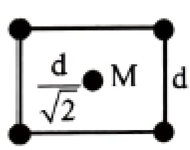
$$\Rightarrow \quad g = \frac{GM}{R^2} \text{ and } g' = \frac{GM}{(0.99R)^2}$$

$\because$ Radius of the earth shrinks by 1%

$$\therefore \quad \frac{g'}{g} = \left(\frac{R}{0.99R}\right)^2 \Rightarrow g' > g$$

58. (A)

(a)



$$U_{net} = -\frac{Gmm}{d} \times 4 - \frac{GMm}{d} \times 4\sqrt{2}$$

$$-\frac{Gmm}{\sqrt{2}d} \times 2 = -\frac{Gm}{d}\left[(4+\sqrt{2})\text{m} + 4\sqrt{2}M\right]$$

59. (B)

(b) As, $E = -\frac{GMm}{2r}$

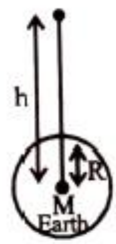
$E \propto \frac{M}{r}$

$\frac{E_A}{E_B} = \frac{M_A}{M_B} \times \frac{r_B}{r_A} = \frac{4}{3} \times \frac{4r}{3r} = \frac{16}{9}$

60. (B)

(b) Initial kinetic energy

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(\lambda v_e)^2$$



Initial potential energy,

$$U_i = \frac{GMm}{R}$$

Final potential energy,

$$U_f = \frac{-GMm}{h}$$

Using law of conservation of energy

$$K_i + U_i = K_f + U_f$$

$$-\frac{GMm}{R} + \frac{1}{2}m\lambda^2 V_e^2 = \frac{GMm}{h} \quad (\because \text{Final kinetic energy} = 0)$$

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}\lambda^2 \frac{2GMm}{R} = -\frac{GMm}{h}$$

$$\Rightarrow \frac{\lambda^2}{R} - \frac{1}{R} = \frac{-1}{h} \Rightarrow \frac{1}{h} = \frac{1-\lambda^2}{R} \Rightarrow h = \frac{R}{1-\lambda^2}$$

61. (A)

(a) Escape velocity, $v_e = \sqrt{\frac{2Gm}{R}}$

from conservation of energy

$$\frac{-GMm}{R} + \frac{1}{2}m\frac{V_e^2}{9} = -\frac{GMm}{R+h}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{V_e^2}{18} \Rightarrow \frac{GM}{R+h} = \frac{GM}{R} - \frac{GM}{9R}$$

$$\frac{GM}{R+h} = \frac{8GM}{9R} \Rightarrow \frac{1}{R+h} = \frac{8}{9R}$$

$9R = 8R + 8h$

The maximum height attained by the body,

$$h = \frac{R}{8} \Rightarrow \frac{6400}{8} \Rightarrow 800 \text{ km}$$

62. (A)

(a) We know that, escape velocity is given as

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G \times \rho \times \frac{4}{3}\pi R^3}{R}} \Rightarrow v_e \propto \sqrt{\rho R^2}$$

So, $v_{e_2} = v_{e_1}\sqrt{\frac{\rho^2}{\rho_1} \times \left(\frac{R_2}{R_1}\right)^2} = 12\sqrt{4 \times \left(\frac{1}{2}\right)^2} = 12$ km/s.

63. (16.00)

(16.00) Using law of conservation of energy

Total energy at height 10 R = total energy at earth

$$-\frac{GM_E m}{10R} + \frac{1}{2}mV_0^2 = -\frac{GM_E m}{R} + \frac{1}{2}mV^2$$

$$\left[\because \text{Gravitational potential energy} = -\frac{GMm}{r}\right]$$

$$\Rightarrow \frac{GM_E}{R}\left(1 - \frac{1}{10}\right) + \frac{V_0^2}{2} = \frac{V^2}{2} \Rightarrow V^2 = V_0^2 + \frac{9}{5}gR$$

$$\Rightarrow V = \sqrt{V_0^2 + \frac{9}{5}gR} \approx 16 \ km/s \quad [\because V_0 = 12 \text{ km/s given}]$$

64. (04.00)

(04.00) At the surface of earth, $g = \frac{GM}{R^2}$

At point C $g_c = \frac{GM}{\left(R + \frac{R}{2}\right)^2} = \frac{4}{9}g$

At point A $g_A = g\left(1 - \frac{d}{R}\right)$ or, $g_A = g\left(1 - \frac{AB}{R}\right)$

From question,

$$g_A = g_C \Rightarrow \frac{4}{9}g = g\left(1 - \frac{AB}{R}\right) \Rightarrow AB = \frac{5R}{9}$$

$$\therefore \quad OA = OB - AB = R - \frac{5}{9}R = \frac{4R}{9}$$

$$\therefore \quad \frac{OA}{AB} = \frac{x}{y} = \frac{\frac{4R}{9}}{\frac{5R}{9}} = \frac{4}{5} \quad \therefore \quad x = 04.00$$

65. (2)

18. (2) $U = -G\left[\frac{(M-m)m}{a}\times 4 + \frac{m^2}{\sqrt{2}a} + \frac{(M-m)^2}{\sqrt{2}a}\right]$

$$= -\frac{G}{a}\left[4Mm - 4m^2 + \frac{m^2}{\sqrt{2}} + \frac{(M-m)^2}{\sqrt{2}}\right]$$

$$\frac{dU}{dm} = 4M - 8m + \frac{2m}{\sqrt{2}} - \frac{2(M-m)}{\sqrt{2}}$$

For maximum 'U' $\frac{dU}{dm} = 0$

$\Rightarrow \quad 0 = 4M - 8m + 2\sqrt{2}m - \sqrt{2}M$

$\Rightarrow \quad 0 = M(4-\sqrt{2}) - 2m(4-\sqrt{2}) \Rightarrow M = 2m \Rightarrow \frac{M}{m} = 2$

66. (3)

(3) Binding energy of uniform sphere $= \frac{3}{5}\frac{GM^2}{R}$

Energy given

$$E = U_f - U_i = 0 - \left(\frac{3}{5}\frac{GM^2}{R}\right) - \frac{3}{5}\frac{GM^2}{R} \quad \therefore x = 3$$

67. (64)

(64) Escape velocity,

$$v_e = \sqrt{\frac{2GM}{R}}$$

Let R' be the radius so that escape velocity is increased 10 times.

$$v' = 10v_e = \sqrt{\frac{2GM}{R'}} \Rightarrow 10\times\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R'}}$$

$$\therefore \quad R' = \frac{R}{100} = \frac{6400}{100} = 64 \text{ km}$$

68. (10)

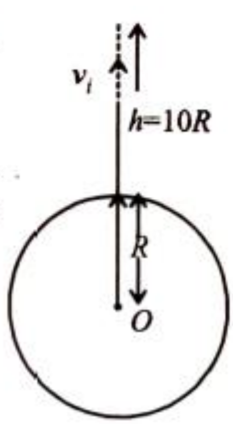
(10) From energy conservation,

$$-\frac{GM_e m}{R} + \frac{1}{2}mv_i^2 = -\frac{GM_e m}{11R}$$

$$v_i = \sqrt{\frac{20}{11}\frac{GM_e}{R}} \quad \because v_e = \sqrt{\frac{2GM_e}{R}}$$

$$\therefore v_i = \sqrt{\frac{10}{11}}V_e$$

$\therefore x = 10.$



69. (2)

(2) At '*h*' height above the ground ($h \ll R$)

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

At depth '*d*' below the surface of earth

$$g_d = g\left(1 - \frac{d}{R}\right)$$

Given, $g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$

$$\Rightarrow \frac{2h}{R} = \frac{d}{R} \Rightarrow 2h = d \Rightarrow \alpha = 2$$

70. (6)

(6) Enlongation of wire due to its own weight is given by

$\Delta l = \frac{mgl}{yA}$ or, $\Delta l \propto g$

$$\Rightarrow \frac{\Delta l_e}{\Delta l_p} = \frac{g_e}{g_p} \Rightarrow g_p = g_e \frac{\Delta l_p}{\Delta l_e}$$

$$\Rightarrow g_p = \frac{10 \times 6 \times 10^{-5}}{10^{-4}} = 6 \text{ m/s}^2$$

71. (2)

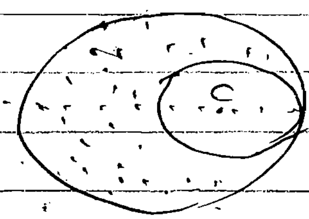
(2) We know that orbital velocity is given as

$$V = \sqrt{\frac{GM}{x}} \therefore V \propto \frac{1}{\sqrt{x}} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$$

Gravitation

Only one option is Correct

(1)



$$E_C = \frac{G\left(\frac{4}{3}\pi R^3 d\right)\left(\frac{R}{2}\right)}{R^3}$$

$$E_C = \frac{2G\pi dR}{3}$$

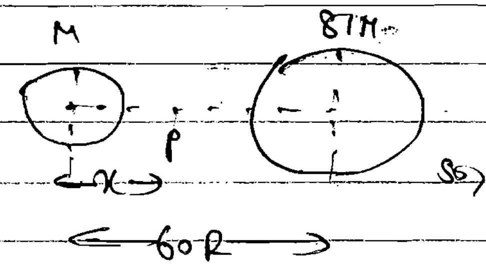
$$F = \left(\frac{2G\pi dR}{3}\right)m = \frac{2\pi RGmd}{3} \quad (A)$$

(2)

$$g' = \frac{g}{4} \Rightarrow \frac{GM}{(R+h)^2} = \frac{1}{4}\frac{GM}{R^2}$$

$$\therefore h = R \quad (B)$$

(3)



$$E_P = 0$$

$$\frac{GM}{x^2} = \frac{G\,81M}{(60-x)^2}$$

$$\therefore 60 - x = 9x$$

$$\therefore x = 6R \quad (A)$$

(4) By Conservation of energy

$$\frac{1}{2}m_{reduced}v_{rel}^2 = \frac{Gm_1m_2}{d}$$

$$\frac{1}{2}\frac{m_1m_2}{m_1+m_2}v_{rel}^2 = \frac{Gm_1m_2}{d}$$

$$\therefore v_{rel} = \sqrt{\frac{2G(m_1+m_2)}{d}} \quad (C)$$

(5) (0,0) — (1,0) (2,0) (4,0) ... ∞

$$E_{origin} = \frac{G(1)}{(1)^2} + \frac{G(1)}{2^2} + \frac{G(1)}{4^2} + \ldots \infty$$

$$= G\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots \infty\right) = \frac{4G}{3}$$

(B)

(6) $$F_A = \frac{G(M_1 + M_2)m}{P^2}$$

$$F_B = \frac{GM_1m}{q^2}$$

$$F_C = 0$$

(D)

Only one option is Correct!

(1) $V_e = \sqrt{\frac{2GM}{R}} = V_0$ given

$V_{planet(e)} = \sqrt{\frac{2G\,2M}{R/2}} = \sqrt{4\left(\frac{2GM}{R}\right)} = 2V_0$ (B)

(2) By CoE!

for A: $\frac{1}{2}mv_1^2 = \frac{GMm}{R}$ $\therefore v_1 = \sqrt{\frac{2GM}{R}}$

for B: $\frac{1}{2}mv_2^2 - \frac{GMm}{R} = -\frac{GmM}{7R}$

$\frac{v_2^2}{2} = \frac{GM}{R} - \frac{GM}{7R} = \frac{6GM}{7R}$ $\therefore v_2 = \sqrt{\frac{12GM}{7R}}$

$\frac{v_1}{v_2} = \sqrt{\frac{7}{6}}$ (D)

(3) $\frac{mv_0^2}{R} = \frac{GMm}{R_0^2}$ $\therefore v_0^2 = \frac{GM}{R_0} \Rightarrow \left(\frac{\sqrt{2gR}}{2}\right)^2 = \frac{GM}{R_0}$

$\Rightarrow \frac{2gR}{4} = \frac{GM}{R_0} \Rightarrow \frac{GM}{2R} = \frac{GM}{R_0}$ $\therefore h = R$ (B)

(4) $\frac{g_1}{g_2} = \frac{GM}{(4R)^2} \Big/ \frac{GM}{(5R)^2} = \frac{25}{16}$ (B)

(5) By CLM, $2Mv_1 = Mv_2$ $\therefore v_2 = 2v_1$

$\Rightarrow \int v_2\,dt = 2\int v_1\,dt$

$\Rightarrow d_2 = 2d_1$

Also, $d_1 + d_2 = H \Rightarrow d_1 = H/3 = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2H}{3g}} \quad (C)$$

(6) for H_{max}, COE

$$-\frac{GMm}{R} + \frac{1}{2} m \frac{4gR}{3} = -\frac{GMm}{H}$$

$$\Rightarrow -\frac{GMm}{R} + \frac{2mR}{3} \frac{GM}{R^2} = -\frac{GMm}{H}$$

$$\Rightarrow -\frac{GMm}{3R} = -\frac{GMm}{H} \quad \therefore H = 3R$$

So H_{max} from surface $= 2R$

Part 2 By COE

$$\frac{1}{2} m \frac{4gR}{3} - \frac{GMm}{R} = -\frac{GMm}{2R} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{GMm}{6R^2 R} = \frac{1}{2}mv^2 \quad \therefore v = \sqrt{\frac{gR}{3}}$$

(B) (9)

(7) $V_o < V < V_e$ for an orbit

$$V_{net} = n\sqrt{\frac{GM}{R}} < \sqrt{\frac{2GM}{R}}$$

$$n < \sqrt{2} \quad (B)$$

(8) $U_i = -\frac{GMm}{R}$, for $r = 2R$

$$\frac{GMm}{(2R)^2} = \frac{mv^2}{2R}$$

$$v = \sqrt{\frac{GM}{2R}}$$

$$U_f = -\frac{GMm}{2R} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{2R} + \frac{1}{2}m\frac{GM}{2R}$$

$$U_f = \frac{GMm}{4R} - \frac{GMm}{2R} = -\frac{GMm}{4R}$$

So $E_{supply} = U_f - U_i = \frac{GMm}{R} - \frac{GMm}{4R}$

$$= \frac{3GMm}{4R} = \frac{3}{4}mRg = \frac{3}{4}mgR$$

(D)

(9) $$\frac{(V_e)_e}{(V_e)_m} = \sqrt{\frac{2g_1R_1}{2g_2R_2}} = \sqrt{\frac{g_1R_1}{g_2R_2}}$$

~~$\sqrt{\frac{M_1 R_2}{R_1 M_2}}$~~ $= \sqrt{10 \times 6} = \sqrt{60}$

(D)

Exercise #1

(1) As $\vec{F} = 0$ then $\vec{V}$ = const (B)

(2) $g_{planet} = \frac{GM_p}{R_p^2} = \frac{G\left(\frac{4}{3}\pi R_p^3\right)(8\rho_e)}{R_p^2}$

$M_e = \frac{4}{3}\pi R_e^3 \rho_e \qquad = \left(\frac{\frac{4}{3}\pi R_p^3\, G}{R_p^2}\right)\frac{8 M_e}{\frac{4}{3}\pi R_e^3}$

$= \frac{(2R_e)^3\, G\, 8 M_e}{2R_e(2R_e)\, R_e^3}$

$= \frac{16 R_e^3\, G M_e}{R_e^5} = 16\left(\frac{GM_e}{R_e^2}\right)$

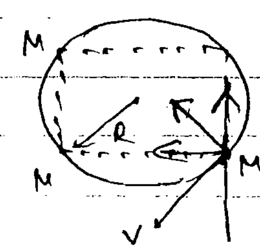
$g_{planet} = 16g$

So, $S = \frac{1}{2}gt^2 = \frac{1}{2}10(1)^2 = 5\,m$

for planet: $S = \frac{1}{2}16gt^2 \therefore t = \frac{1}{4}$ sec

(A)

(3)



$\frac{Mv^2}{R} = \frac{\sqrt{2}GM^2}{(\sqrt{2}R)^2} + \frac{GM^2}{4R^2}$

$\frac{Mv^2}{R} = \frac{GM^2}{4R^2}\left(2\sqrt{2}+1\right)$

$v = \sqrt{\frac{GM}{R}\frac{(2\sqrt{2}+1)}{4}}$ (A)

(4) $\frac{GM}{(R+h)^2} = \frac{1}{100}\frac{GM}{R^2}$

$\therefore 10R = R+h \therefore h = 9R$ (A)

(5)

$\text{Center of gravity} = \frac{x_1F_1 + x_2F_2}{F_1+F_2}$

$= \frac{\frac{GMm}{R^2}R + 0}{\frac{GMm}{R^2}} = R$

$\text{distance} = \frac{R}{2}$ (B)

(6) $\frac{GMm}{r^2} = m\omega_0^2 r \therefore \omega_0^2 r^3 = GM$

$g = \frac{GM}{R^2} = \left(\frac{\omega_0^2 r^3}{R^2}\right)$ (D)

(7) $F_1 = \frac{GMm}{9R^2} = \frac{Gm}{9R^2}\frac{4}{3}\pi R^3\rho$

$= \frac{4}{27}Gm\rho\pi R$

$F_2 = \frac{4}{27}Gm\rho\pi R - \frac{2}{75}G\pi\rho Rm$

$= \frac{Gm\rho\pi R(100-18)}{27\times 25} = \frac{82(Gm\rho\pi R)}{27\times 25}$

$\frac{F_1}{F_2} = \frac{4}{27}\frac{27\times 25}{82} = \frac{50}{41} \therefore \frac{F_2}{F_1} = \frac{41}{50}$

(B)

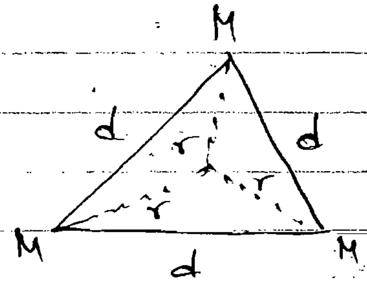
(8) $g_{planet} = \frac{G\,2M}{(2R)^2} = \left(\frac{GM}{R^2}\right)\frac{1}{2} = \frac{g}{2}$

$T = 2\pi\sqrt{\frac{L}{g/2}} = \sqrt{2} \times 2 = 2\sqrt{2}$ seconds (B)

(14)

(9) (D) from basic E/V formulae

(10)



$F_{net} = \frac{2GM^2}{d^2}\cos 30$ (15)

$= \frac{Mv^2}{d/\sqrt{3}}$

$\left(\text{Put } r = \frac{d}{\sqrt{3}}\right)$

$\frac{2GM^2}{d^2}\frac{\sqrt{3}}{2} = \frac{Mv^2\sqrt{3}}{d}$ $v = \sqrt{\frac{GM}{d}}$

$\therefore d = \frac{GM}{v^2}$ (D)

(11) $\left(\frac{GMx}{R^3}\right)m = \frac{1}{4}\left(\frac{GMm}{4R^2}\right)$

$\frac{x}{R} = \frac{1}{16}$ $\therefore x = \frac{R}{16}$ depth $= R - \frac{R}{16}$ (16)

$= \frac{15R}{16}$ (B)

(12) (C) $t = \pi/\omega = \frac{T}{2} = 12\,H$

(13) By COE! (17)

$-\frac{GMm}{\sqrt{2}R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$ (18)

$\Rightarrow \frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{\sqrt{2}R}$

$$\Rightarrow \frac{v^2}{2} = \frac{GM}{R}\frac{(\sqrt{2}-1)}{\sqrt{2}}$$

$$\therefore v = \sqrt{\frac{GM(2-\sqrt{2})}{R}} \quad (D)$$

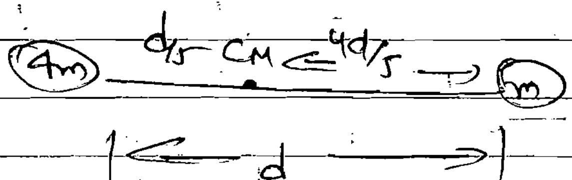
(14) $(g - \omega^2 R_e) = \frac{1}{2} g \therefore \omega^2 R_e = \frac{g}{2}$ — (1)

$$V = \omega R_e \text{ — (2)}$$

$$V_e = \sqrt{2gR_e} = \sqrt{2 \times 2\omega^2 R_e^2}$$

$$= 2v \quad (A)$$

(15)



$$\frac{KE_1}{KE_2} = \frac{\frac{1}{2} I_1 \omega_1^2}{\frac{1}{2} I_2 \omega_2^2} = \frac{m_1 \left(\frac{d}{5}\right)^2}{m_2 \left(\frac{4d}{5}\right)^2}$$

(Here $\omega_1 = \omega_2$)

$$= \frac{4m}{m} \frac{1}{16} = \frac{1}{4} \quad (A)$$

(16) $$\frac{V_A}{V_B} = \sqrt{\frac{2g_A R_A}{2g_B R_B}} = \sqrt{\frac{GM_A}{R_A}\frac{R_B}{GM_B}} = \sqrt{\frac{R_A^2}{R_B^2}} = \frac{R_A}{R_B}$$

$$= 2 \quad (A)$$

(17) (D) Package will move circularly.

(18) By COE!

$$W_{ext} = \Delta K + \Delta U$$

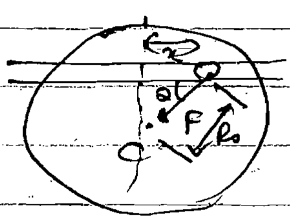
$$-3 = V \times 1 + \frac{1}{2} 1 (2)^2 \therefore V = -5 \text{ J/kg}$$

(C)

(19) $E = \frac{\frac{GMm}{R}}{m} = \frac{GM}{R}$ (23

$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2E}$ (A)

(20)



~~$F = F\cos\theta = \frac{GM}{R^3}(R_0^2+x^2)^{1/2} \times \frac{x}{R_0}$~~ (24

$F_{net} = F\cos\theta$

$= \frac{GM}{R^3} R_0 \times \frac{x}{R_0} = \frac{GMx}{R^3}$

$\boxed{F \propto x}$ (D)

(21) $V_i = -\frac{GM}{R}$, $V_f = -\frac{2GM}{R}$

$V_i - V_f = \frac{GM}{R} > 0 \therefore V_i > V_f$ (B)

(22) $W = U_f - U_i$ (25,

$= \left(-\frac{G2m^2}{a} - \frac{G6m^2}{a} - \frac{G3m^2}{a}\right)$

$-\left(-\frac{G2m^2}{a} - \frac{G3m^2}{a} - \frac{G6m^2}{\sqrt{2}a}\right)$

$= -\frac{6Gm^2}{a} + \frac{6Gm^2}{\sqrt{2}a}$

$= -\frac{6Gm^2}{a}\left(1 - \frac{1}{\sqrt{2}}\right)$ (C)

(23) For circular motion

$$V_C = \left(GM/R\right)^{1/2}$$

Due to collision $V_C < \sqrt{\frac{GM}{R}}$

So (C)

(24) By COE (Before st)

$$-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$V = \sqrt{\frac{GM}{R}}$$

Next: (After coll^n)

$$-\frac{GMm}{3R/2} = -\frac{GMm}{R} + \frac{1}{2}mv_0^2$$

$$v_0 = \sqrt{\frac{2}{3}}\sqrt{\frac{GM}{R}} = \sqrt{\frac{2}{3}}\,V$$

So $e = \frac{v_0}{V} = \sqrt{\frac{2}{3}}$ (B)

(25) $\Delta U = \left(-\frac{GMm}{4R}\right) - \left(-\frac{GMm}{R}\right)$

$$= \frac{GMm}{R} - \frac{GMm}{4R} = \frac{3GMm}{4R} \times \frac{R}{R}$$

$$= \frac{3}{4}\left(\frac{GM}{R^2}\right)mR = \frac{3}{4}mgR$$ (B)

(26) $\frac{GMm}{r^2} = m\omega^2 r$ — (I)

$\frac{GMm}{(2r)^2} = m\omega_1^2 2r$ — (II)

(I)/(II)

$4 = \frac{\omega^2}{2\omega_1^2}$ $\therefore \omega_1 = \sqrt{\frac{\omega^2}{8}}$

$\therefore \omega_1 = \frac{\omega}{2\sqrt{2}}$ (A)

(27) (C) Kepler's second law

(28)

$T^2 \propto R^3$

$\frac{T_1}{T_2} = \left(\frac{R}{1.02R}\right)^{3/2}$

$T_2 = T_1(1.02)^{3/2} = T_1(1+0.03)$

$\% = \frac{T_2 - T_1}{T_1} \times 100 = 3\%$ (B)

(29) $V_{initial} = \sqrt{\frac{GM_0}{R}}$ $P_i = 5MV_{initial}$

$P_f = -MV_M + 4MV_{4M}$

By CLM $4MV_{4M} = MV_M + 5MV_i$

$\therefore MV_M + 4MV_{4M} = 5MV_i$

for M: $V_M = \sqrt{\frac{GM_0}{R}} = V_i$ $\therefore V_{4M} = 1.5V_i = 1.5\sqrt{\frac{GM_0}{R}}$

So $V_{4M} > V_{escape}$ (B)

(30)

(31)

$\frac{V_1}{V_2}$

(32)

(33)

(30) $L = mvR$

$= m\omega R^2$

$= m\frac{2\pi R^2}{T} = \frac{2mA}{T}$ (A)

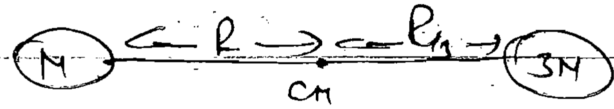
(31) 1st orbit: $\frac{mv^2}{R_1} = \frac{GmM}{R_1^2}$ $v_1 = \sqrt{\frac{GM}{R_1}}$

2nd orbit: $\frac{mv^2}{R_2} = \frac{GMm}{R_2^2}$ ∴ $v_2 = \sqrt{\frac{GM}{R_2}}$

$\frac{v_1}{v_2} = \frac{1}{2} = \sqrt{\frac{R_2}{R_1}}$ $\frac{T_1}{T_2} = \left(\frac{2\pi R_1}{v_1}\right)\left(\frac{v_2}{2\pi R_2}\right) = \left(\frac{R_1}{R_2}\right)\left(\frac{v_2}{v_1}\right)$

∴ $\frac{R_1}{R_2} = 4$ $\frac{T_1}{T_2} = 4 \times 2 = 8$ ∴ $T_2 = \frac{T}{8}$ (D)

(32)



$\frac{GM3M}{(4R/3)^2} = M\omega^2 R$

$\omega = \sqrt{\frac{9GM\times 3}{16R^3}} = \sqrt{\frac{27GM}{16R^3}} = \frac{2\pi}{T}$

∴ $T = 2\pi\sqrt{\frac{16R^3}{27GM}}$ (D)

(33) $E_1 = U_1 + K_1$

$= -\frac{GMm}{2R_e} + \frac{1}{2}mv_1^2$

$= -\frac{GMm}{2R_e} + \frac{1}{2}\frac{GMm}{2R_e}$

$= \frac{GMm}{4R_e} - \frac{GMm}{2R_e} = -\frac{GMm}{4R_e}$

$E_f = U_f + K_f$

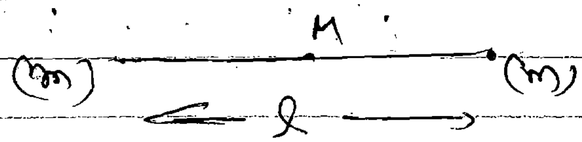
$E_f = -\frac{GMm}{6R_e}$

$\Delta E = E_f - E_i$ (1)

$= \frac{GMm}{4R_e} - \frac{GMm}{6R_e} = \frac{GMm}{12R_e}$

(D) (2)

(34)



$P.E = -\frac{GMm}{l/2} \times 2 = -\frac{4GMm}{l}$

$\frac{1}{2} M v_e^2 = \frac{4GMm}{l}$

$\therefore v_e = \sqrt{\frac{8Gm}{l}} = 2\sqrt{\frac{2GM}{l}}$

(D)

(35) By COE (3)

$-\frac{GMm}{R} = -\frac{3GMm}{2R} + \frac{1}{2}mv^2$ (4)

$\Rightarrow \frac{3GMm}{2R} - \frac{2GMm}{2R} = \frac{1}{2}mv^2$

$\Rightarrow \frac{GMm}{2R} = \frac{1}{2}mv^2$

$\therefore v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{2GM}{R\sqrt{2}}}$

$= v_e/\sqrt{2}$ (B)

Exercise # II

(1) $g(\text{in}) = \frac{GMr}{R^3}$ $g(\text{out}) = \frac{GM}{r^2}$

(A) (D)

(2) For 1st : $-\frac{GMm}{R} + \frac{1}{2}m\frac{2gR}{3}$

$= -\frac{GMm}{H}$

$\Rightarrow -\frac{GMm}{R} + \frac{mR}{3}\frac{GM}{R^2} = -\frac{GMm}{H}$

$\Rightarrow -\frac{GMm}{R} + \frac{GMm}{3R} = -\frac{GMm}{H}$

$\Rightarrow +\frac{2}{3R} = +\frac{1}{H}$ $\therefore H = \frac{3R}{2}$

$R + h_1 = H$ $\therefore h_1 = \frac{R}{2}$ Similarly $h_2 = R$

$h_3 = 2R$

(C) (D)

(3) $V = -\frac{GM}{R}$ so (A), (B), (C)

(4) $\frac{V_1}{V_2} = \sqrt{\frac{2g_1R_1}{2g_2R_2}} = \sqrt{\frac{R_2}{R_1}} = \frac{1}{\sqrt{2}}$ (D)

As m same so $\rho \frac{4}{3}\pi R_1^3 = 8\rho\frac{4}{3}\pi R_2^3$

$R_1 = 2R_2$

$\frac{g_1}{g_2} = \left(\frac{R_2}{R_1}\right)^2 = \frac{1}{4}$ (B)

(5) (B) (D)

(6) $U = -\frac{GMm}{r}$

$U_i = -\frac{GMm}{r_1}$, $U_f = -\frac{GMm}{r_2}$

$U_f - U_i = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) > 0$

(A)

$V_{orbital} = \sqrt{\frac{GM}{R}}$ As $R \uparrow$ $V_{orbital} \downarrow$

$\omega = \frac{V}{R} \Rightarrow$ as $R \uparrow$ $V \downarrow$ $\omega \downarrow$

$a_c = \frac{V^2}{R}$ as $V \downarrow$ $R \uparrow$ $a_c \downarrow$

so (B) (C) (D) are wrong

(7) $T^2 \propto R^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \left(\frac{32}{256}\right)^2 = \left(\frac{x}{R_2}\right)^3$

$\Rightarrow \frac{1}{64} = \left(\frac{x}{R_2}\right)^3$ ∴ $R_2 = 4x$

(B)

$E = -\frac{GMm}{r}$ so (C)

(8) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{1}{4}\right)^3$

$\frac{T_1}{T_2} = \frac{1}{8}$ (A)

$V = \omega R \propto \frac{R}{T}$ (B)

$\frac{V_1}{V_2} = \left(\frac{R_1}{T_1}\right)\left(\frac{T_2}{R_2}\right) = \frac{1}{4} \times \frac{8}{1} = 2$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_2 v_2 R_2} = \frac{2}{1} \times \frac{1}{4} = \frac{1}{2}$$

(C) Wrong

(9) (A)

(10) $v \propto \frac{1}{\sqrt{r}}$ (A) Correct $T^2 \propto R^3$ (B) Correct

$T.E = -\frac{GMm}{2r}$ (C) is Correct

Exercise III

(1) $t_1 > t_2$ (B)

In region $V_{(ADB)} > V_{(ACB)}$

(2) $\frac{mv_1^2}{r} = \frac{GMm}{r^2}$ ∴ $\frac{1}{2}mv^2 = \left(\frac{GMm}{2r}\right)$

∴ $K = \frac{|U|}{2}$ ∴ $|U| = 2K$

(C)

(3) $F = \frac{GMm x}{R^3} = m\omega^2 x$ ∴ $\omega = \sqrt{\frac{GM}{R^3}} = \frac{2\pi}{T}$

∴ $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$

$T_{required} = \frac{T}{2} = \pi\sqrt{\frac{R}{g}} = 3.14\sqrt{\frac{6400000}{10}}$

∴ $T_{req} = \frac{3.14 \times 800}{60}$ min

$= 42$ min (B)

(4) By CAM! $v_1 r_1 = v_2 r_2$

$V_p > V_a$ (B)

(5) $V_{orbit} = \sqrt{\frac{GM}{r}}$

As P.E const so circular orbit

$V_{new} = \frac{V_0}{\sqrt{2}}$ (B)

(6) (C)

(7) $TE = -\frac{GMm}{2r} = -\left(\frac{1}{2}\right) mv^2$ (A)

(8) $Ma_1 = ma_2$

$\frac{a_1}{a_2} = \frac{m}{M}$ (B)

(9) $KE = \frac{mv^2}{2} = +\frac{GMm}{2r}$

$PE = -\frac{GMm}{r}$

$TE = K + P = -\frac{GMm}{2r}$

$V_O = \sqrt{\frac{GM}{r}}$

(A) → (iv) (B) → (iii) (C) → (i)
(D) → (ii)

(10) (A) → (iii) (B) → (i) (C) → (iv) (D) → (ii)

(11) (A)

(12) (C)

(13) (B)

(14) (B) $g_{pole} = g$, $g_{equator} = g - \omega^2 R$ $\left(\omega = \frac{2\pi}{T}; T^2 \propto R^3\right)$

(15) (D)

Exercise IV

(1) $$\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$$

$$\frac{Gm}{4r^2} = \frac{v^2}{r} \therefore v = \sqrt{\frac{Gm}{4r}}$$

(2) $$F = \int_r^{r+l} \frac{GM}{x^2}\frac{m}{l}dx = \frac{GMm}{l}\left(\frac{1}{r} - \frac{1}{r+l}\right)$$

$$= \frac{GMm}{l}\frac{l}{r(r+l)}$$

$$F = \frac{GMm}{r(r+l)}$$

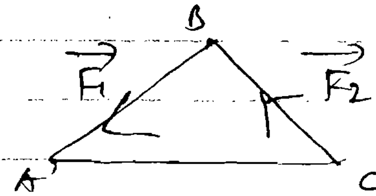
(3) $$U = -\int_{a-l/2}^{a+l/2} \frac{Gm}{x}\frac{m}{l}dx$$

$$U = -\frac{Gm^2}{l}\ln\left(\frac{a+l/2}{a-l/2}\right)$$

$$U = -\frac{Gm^2}{l}\ln\left(\frac{2a+l}{2a-l}\right)$$

$$U = \frac{Gm^2}{l}\ln\left(\frac{2a-l}{2a+l}\right).$$

(4)



$$\vec{F_1} = \frac{G\left(\frac{4\pi R^3 \rho}{3}\right)r}{R^3}$$

$$\vec{F_1} = \frac{4G\rho\vec{r}_A}{3}$$

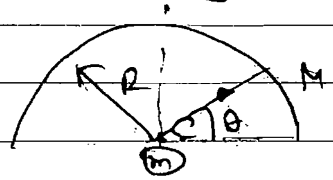
$$\vec{F_2} = \frac{G \frac{4}{3}\pi r^3 \rho}{r^3}\,\vec{r_0}$$

$$\vec{F_2} = \frac{4G\pi\vec{r_0}\rho}{3}$$

$$F_{net} = \vec{F_1} + \vec{F_2}$$
$$= \frac{4G\pi l}{3}(\vec{r_0} + \vec{r}) = \frac{4G\pi\rho l}{3}$$

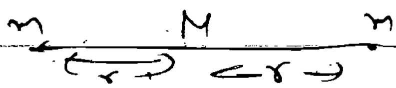
$$F_{net} = \frac{4\pi G\rho l}{3} \quad \overleftarrow{r_2}$$

(5)



$$F = \int_0^{\pi} \frac{G}{R^2}\left(\frac{M\,d\theta}{\pi}\right) m \sin\theta$$
$$= \frac{2GMm}{\pi R^2}$$

(6)



$$\frac{mv^2}{r} = \frac{GMm}{r^2} + \frac{Gm^2}{4r^2}$$

$$v^2 = \frac{GM}{r} + \frac{Gm}{4r}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r} + \frac{Gm}{4r}}}$$

$$= \frac{2\pi r}{\sqrt{\frac{G(4M+m)}{4r}}} = \frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$$

(7)

$$F = \int_d^\infty \frac{Gm}{x^2}\frac{\lambda}{x}\,dx = \frac{G\lambda m}{-2}\left[\frac{1}{x^2}\right]_d^\infty$$

$$= \frac{G\lambda m}{2d^2}$$

(8)

$$F_{net} = \frac{2}{3}\pi G m \rho R \quad [Q(4)]$$

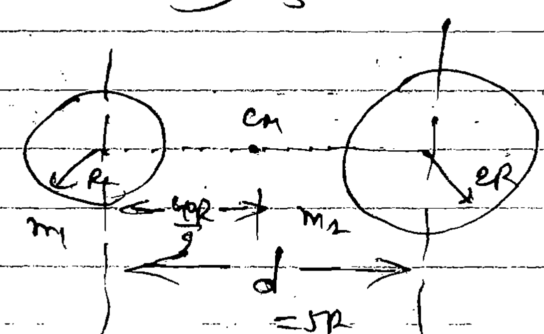
$$a = \frac{2}{3}\pi G\rho R$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2\,\frac{2}{3}\pi G\rho R\,\frac{R}{2}$$

$$v = \sqrt{\frac{2\pi G\rho R^2}{3}}$$

(9)



$$m_1 = \frac{4}{3}\pi R^3\rho = m_0$$

$$m_2 = 8\left(\frac{4}{3}\pi R^3\rho\right) = 8m_0$$

$$\frac{G m_1 m_2}{(5R)^2} = m_1\omega^2\frac{40R}{9}$$

$$\omega = \left(\frac{9Gm_2}{1000R^3}\right)^{1/2} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{1000R^3}{9GM_e}}$$

$$= 2\pi\sqrt{\frac{1000R^3}{9G \times \frac{4\pi R^3 \rho}{3}}}$$

$$= 2\pi\sqrt{\frac{1000}{3G\rho(32\pi)}}$$

$$= \sqrt{\frac{1000 \times 4\pi \times \pi}{3G\rho \times 32\pi}} = \sqrt{\frac{125\pi}{3G\rho}}$$

$$\therefore T = 5\sqrt{\frac{5\pi}{3G\rho}}$$

(10) $R_{max} = \frac{v_0^2}{g} \quad \therefore v_0 = \sqrt{bg}$

Let radius of planet $= R$

$$M_{planet} = \frac{4}{3}\pi R^3 2\rho_e$$

$$\therefore M_{planet} = \frac{4}{3}\pi R^3 \, 2\left(\frac{M_e}{\frac{4}{3}\pi R_e^3}\right) = \frac{2M_e R^3}{R_e^3}$$

$$(V_e)_{planet} = \sqrt{\frac{2GM}{R}} = v_0 = \sqrt{bg}$$

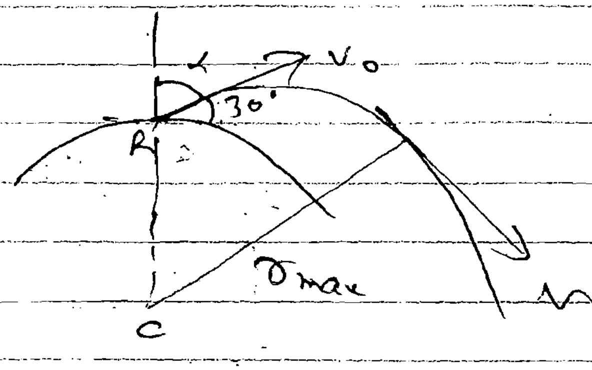
$$\frac{2GM}{R} = b\frac{GM_e}{R_e^2}$$

$$R = \frac{2MR_e^2}{M_e b} = \frac{2R_e^2}{M_e b} \times \frac{2M_e R^3}{R_e^3}$$

$$R = \frac{4R^3}{bR_e} \quad \therefore R = \frac{1}{2}\sqrt{bR_e}$$

$$R = \sqrt{6.4}\ \text{km}$$

(11)



By CAM about C;

$$\cancel{m} v_0 \sin\alpha \, R = \cancel{m} v \, r_{max}$$

$$v_0 \sin\alpha \, R = v \, r_{max} \quad \text{— (1)}$$

By COE:

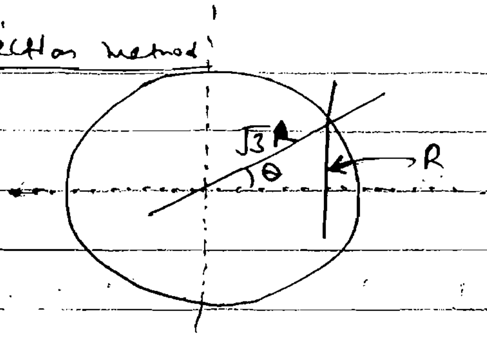
$$\frac{1}{2} m v_0^2 - \frac{GMm}{R} = \frac{1}{2} m v^2 - \frac{GMm}{r_{max}}$$

$$\frac{1}{2} m\left(\frac{1.5\,GM}{R}\right) - \frac{GMm}{R} = \frac{1}{2} m\left(\frac{v_0 \sin 60 \, R}{r_{max}}\right)^2 - \frac{GMm}{r_{max}}$$

Solving $r_{max} = \frac{\sqrt{7}R}{2} + 2R$

So h_{max} (from surface) $= \left(\frac{\sqrt{7}}{2} + 1\right) R$

(12) Projection method:



$$V = \omega (A^2 - x^2)^{1/2}$$

$$\frac{2GM}{R} = \omega^2 (A^2 - R^2) \quad \text{— (I)}$$

By COE

$$E_{center} = E_{surface}$$

$$\Rightarrow -\frac{3GMm}{2R} + \frac{1}{2} m v^2 = \frac{1}{2} \times 2gR - \frac{GMm}{R}$$

$$\therefore v = \sqrt{\frac{3GM}{R}} = A\omega \quad \text{— (II)}$$

By (I) & (II)

$$\frac{2GM}{R} = \frac{3GM}{RA^2}(A^2 - R^2)$$

$$2 = \frac{3}{A^2}(A^2 - R^2) \Rightarrow 2A^2 = 3A^2 - 3R^2$$

$$\therefore A = \sqrt{3} R$$

$$A\omega = \sqrt{\frac{3GM}{R}}$$

$$\sqrt{3} R\omega = \sqrt{\frac{3GM}{R}}$$

$$\omega = \sqrt{\frac{9GM}{3R^3}} = \sqrt{g/R}$$

From fig. 1 $\sin\theta = \frac{1}{\sqrt{3}}$

$$\omega t = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$t = \frac{1}{\omega} \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\therefore t = \sqrt{\frac{R}{g}} \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

IITJEE QUESTIONS

Objective Question (I)

① $g = \frac{GM}{R^2}$ $\therefore g \propto \frac{1}{R^2}$ (M const)

ⓒ

② $U_f = -\frac{GMm}{2R}$

$U_i = -\frac{GMm}{R}$

$U_f - U_i = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$

$\Delta U = \frac{1}{2}\left(\frac{GM}{R^2}\right)mR = \frac{1}{2}mgR$ ⓐ

③ $\frac{GMm}{R^{5/2}} = m\omega^2 R$

$\omega = \sqrt{\frac{GM}{R^{7/2}}} = \frac{2\pi}{T}$

$T = 2\pi\sqrt{\frac{R^{7/2}}{GM}}$ $\therefore T^2 \propto R^{7/2}$ ⓑ

④ $T^2 \propto r^3$

$T \propto r^{3/2}$

$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$ $\therefore T_2 = \left(\frac{365}{2\sqrt{2}}\right)$

$T_2 = 129$ ⓑ

(5) (a)

(6) (c) $T \propto \frac{1}{\sqrt{g}}$ ∴ $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

⇒ $\frac{T_1}{T_2} = \sqrt{\frac{R^3}{4R^2}} = \frac{1}{2}$ ∴ $\frac{T_2}{T_1} = 2$ (d)

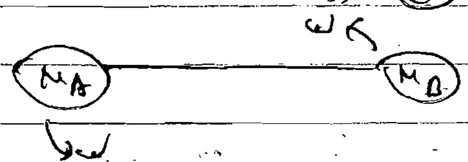
(7) $T_{\text{very close to earth}} = 2\pi\sqrt{\frac{R}{g}} = 84.6\ \text{min}$

As particle few km above surface

$T > 84.6\ \text{min}$

so (c)

(8)



$\omega_A = \omega_B$ so $T_A = T_B$ (d)

(9) $\frac{GMm}{R^3} r = \frac{mv^2}{r}$ ∴ $v^2 \propto r^2$

$v \propto r$ $\quad r \le R$

so (c)

Assertion & Reason:

(1) (A)

Objective Question II :

(1) By Symmetry $E = 0$ (a)

(c) & (d)

As on yz plane all circle have equal potential.

(2) $F_1 = \frac{GM}{r_1^2}$

$F_2 = \frac{GM}{r_2^2}$

$\frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2$ $\quad r_1 > R$, $r_2 > R$

(b)

Also $F_1 = \frac{GMm\, r_1}{R^3}$

$F_2 = \frac{GMm\, r_2}{R^3}$ $\quad \frac{F_1}{F_2} = \frac{r_1}{r_2}$ (a)

Fill in the blanks:

① $g = \omega^2 R$ $\qquad \omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6400\times10^3}}$

$\therefore \omega = 1.24\times10^{-3}$ rad/sec

② Angular momentum

③ $T^2 \propto r^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{7R}{3.5R}\right)^3$

$\therefore T_2 = \frac{T_1}{2\sqrt{2}} = \frac{24}{2\sqrt{2}} = 8.48H$

④ $\frac{1}{2}mv_e^2 = \left(\frac{GM_1m}{d/2} + \frac{GM_2m}{d/2}\right)$

$\frac{v_e^2}{2} = \frac{2G(M_1+M_2)}{d} \therefore v_e = 2\sqrt{\frac{G(M_1+M_2)}{d}}$

⑤ $\frac{dA}{dt} = \frac{L}{2m} \therefore A = \frac{L}{2m}T$

$= \frac{1}{2}\times4.4\times10^{+15}\times365\times86400$

$= 6.94\times10^{22}\,m^2$

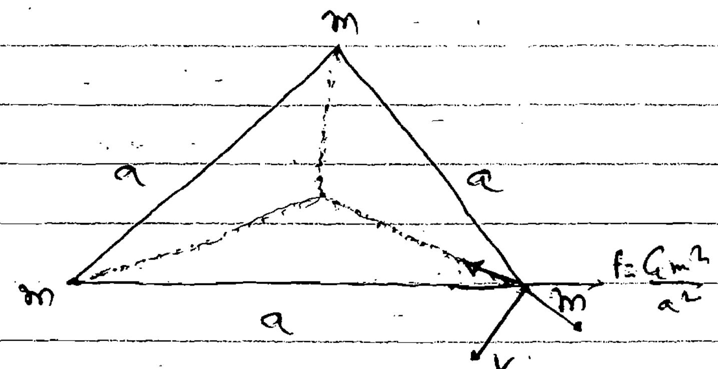
⑥ By COE

$-\frac{GMm}{R} + \frac{1}{2}\left(\frac{1}{2}mv_e^2\right) = -\frac{GMm}{H}$

$\therefore H = 2R \quad \therefore h_{surface} = R$

Analytical & Descriptive question

①



$\therefore r = a/\sqrt{3}$

$F_{net} = 2F\cos 30 = \frac{2Gm^2}{a^2}\frac{\sqrt{3}}{2} = \frac{mv^2}{a/\sqrt{3}}$

$\frac{Gm^2\sqrt{3}}{a^2} = \frac{mv^2\sqrt{3}}{a} \quad \therefore v = \sqrt{\frac{Gm}{a}}$

$T = \frac{2\pi r}{v} = 2\pi\frac{a}{\sqrt{3}} \Big/ \sqrt{\frac{Gm}{a}}$

$= 2\pi\sqrt{\frac{a^3}{3Gm}}$

② (a) $v_0 = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{GM}{R+h}} = \frac{v_e}{2}$

$\Rightarrow \sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{2gR}$

$\Rightarrow \frac{GM}{R+h} = \frac{1}{4}\,2\,\frac{GM}{R} = \frac{1}{2R}$

$h = R = 6400\,km$

(b) By CoE

$$-\frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$$

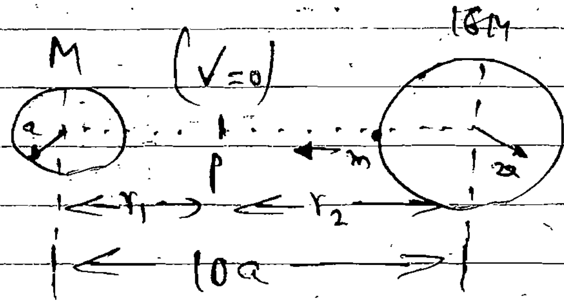
$$\therefore v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

$$= \sqrt{9.8 \times 6400 \times 10^3}$$

$$= 7919 \text{ m/s}$$

$$= 7.9 \text{ km/sec}$$

(3)



$$E_p = 0 \; ; \; \frac{GM}{r_1^2} = \frac{G\,16M}{r_2^2} \Rightarrow r_2 = 4r_1$$

$$r_1 + r_2 = 10a$$

$$\therefore r_1 = 2a, \; r_2 = 8a$$

If Particle reach P then after P $F_M > F_{16M}$

so particle will reach its destination.

By CoE:

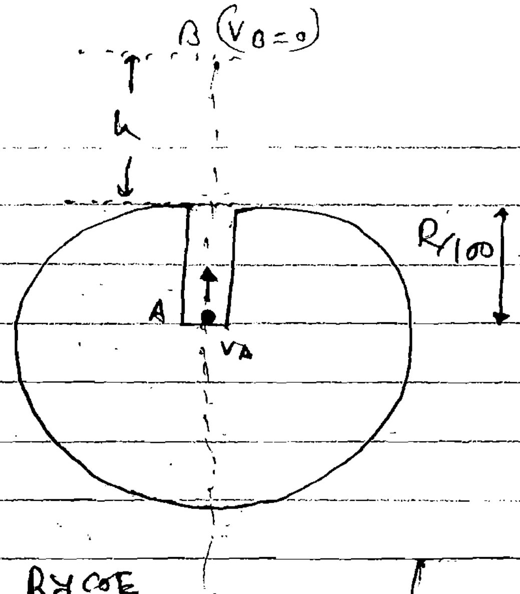
$$\frac{1}{2}mv_{min}^2 + \left[-\frac{G\,16Mm}{2a} - \frac{GMm}{8a}\right]$$

$$= -\frac{GMm}{r_1} - \frac{16GMm}{r_2}$$

$$\frac{1}{2}mv_{min}^2 = \frac{45GMm}{8a} \Rightarrow v_{min} = \sqrt{\frac{45GM}{4a}}$$

$$= \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

(4)



By COE

$$v_A = \sqrt{2GM/R}$$

$$\frac{1}{2} m v_A^2 - \frac{GMm}{R^3}\left(1.5R^2 - 0.5\left(R - \frac{R}{100}\right)^2\right) = -\left(\frac{GMm}{R+h}\right)$$

$$\frac{GM}{R} - \frac{GM}{R^3}\left(1.5R^2 - 0.5\left(R - \frac{R}{100}\right)^2\right) = -\frac{GM}{R+h}$$

$$\therefore \boxed{h = 99.5R}$$

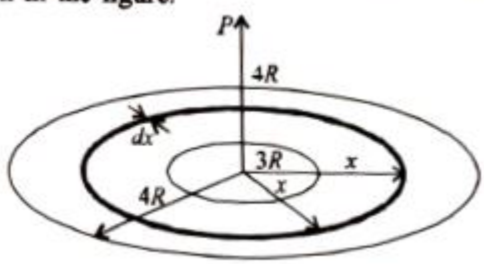
Only One Option Correct

1. (A)

(a) Mass per unit area of the shaded part.

$$\sigma = \frac{\text{mass}}{\text{area}} = \frac{M}{\pi((4R)^2 - (3R)^2)} = \frac{M}{7\pi R^2}$$

Let us consider a ring of radius x and thickness dx as shown in the figure.



Mass of the ring, $dM = \sigma 2\pi x dx = \frac{2\pi M x dx}{7\pi R^2}$

Potential at point P due to shaded part

$$V_P = \int_{3R}^{4R} -\frac{G dM}{\sqrt{(4R)^2 + (x)^2}} = -\frac{GM 2\pi}{7\pi R^2} \int_{3R}^{4R} \frac{x dx}{\sqrt{16R^2 + x^2}}$$

Solving, we get

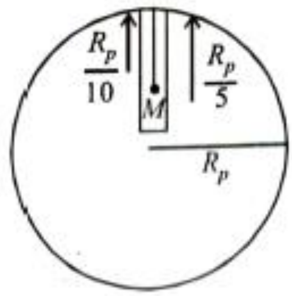
$$V_P = -\frac{GM 2\pi}{7\pi R^2}\left[\sqrt{16R^2 + x^2}\right]_{3R}^{4R} = -\frac{2GM}{7R}(4\sqrt{2} - 5)$$

Workdone in moving a unit mass from P to $\infty = V_\infty - V_P$

or $W_{P\infty} = 0 - \left(\frac{-2GM}{7R}(4\sqrt{2} - 5)\right) = \frac{2GM}{7R}(4\sqrt{2} - 5)$

2. (B)

(b) The mass of the wire
$= 10^{-3} \times 1.2 \times 10^{5} = 120$ kg
Acceleration due to gravity at the surface of the planet and earth



$$g_p = \frac{4}{3}\pi\rho G R_P \,;$$

and $g_e = \frac{4}{3}\pi\rho G R_E$

$$\therefore \quad \frac{g_p}{g_e} = \frac{R_p}{R_E} = \frac{1}{10} \Rightarrow g_p = \frac{10}{10} = 1 \text{ ms}^{-2}$$

Let g_{pM} be the acceleration due to gravity at point M which is the mid point of the wire and is at a depth of $\frac{R_p}{10}$.

$$g_{pM} = g_p\left[1 - \frac{R_p/10}{R_p}\right] = 1[1-0.1] = 0.9 \text{ ms}^{-2}$$

$\therefore$ Force = mass of wire × g_{pM} = 120 × 0.9 = 108 N

3. (B)

(b) Applying energy conservation

$$\frac{1}{2}mV_s^2 - \frac{GM_e m}{R_e} = \frac{GM_e m \times 3 \times 10^5}{2.5 \times 10^4 R_e}$$

$$\frac{V_s^2}{2} = \frac{GM_e}{R_e}\left[1 + \frac{3\times10^5}{2.5\times10^4}\right]$$

$$V_s = \sqrt{13\left(\frac{2GM_e}{R_e}\right)} \left(\because V_e = \sqrt{\frac{2Gm_e}{R_e}} = 11.2 \text{ km/s}\right)$$

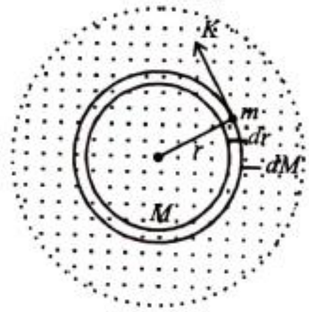
$\therefore \quad V_s = \sqrt{13} \times 11.2 \approx 42$ m/s

4. (B)

(b) Gravitational pull of the mass 'M' present in the sphere of radius 'r'. Provide the required centripetal force of particle of mass 'm' to revolve in a circular path.

$$\frac{mv^2}{r}=\frac{GMm}{r^2}\Rightarrow\frac{1}{2}mv^2=\frac{GMm}{2r}\Rightarrow K=\frac{GMm}{2r}$$

$\therefore \quad M=\frac{2Kr}{Gm}$



Differentiating the above equation w.r.t 'r' we get

$$\frac{dM}{dr}=\frac{2K}{Gm}$$

or $\quad dM=\frac{2K}{Gm}dr$

$\therefore \quad 4\pi r^2 dr\,\rho=\frac{2K}{Gm}dr \quad\Rightarrow\quad \rho=\frac{K}{2\pi r^2 mG}$

$\therefore \quad \frac{\rho}{m}=\frac{K}{2\pi r^2 m^2 G} \quad$ or $\quad \frac{\rho(r)}{m}=\frac{K}{2\pi r^2 m^2 G}$

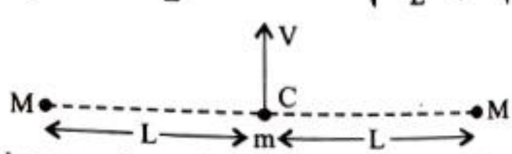
One or More than One Option Correct

1. (B, D)

From conservation of mechanical energy,

$$\frac{-GMm}{L}-\frac{GMm}{L}+\frac{1}{2}mv^2=0+0$$

or, $\frac{1}{2}mv^2=\frac{2GMm}{L} \quad \therefore \quad V=\sqrt{\frac{4GM}{L}}=2\sqrt{\frac{GM}{L}}$



Total energy of mass 'm' is conserved as there is no external force involved.

Matrix-Match Type

1. (B)

(b) $\because$ Orbital velocity,

$$V = \sqrt{\frac{GM}{R}}, \text{ or, } V \propto \frac{1}{\sqrt{R}} \quad \therefore \frac{V_1}{V_2} = \sqrt{\frac{R_2}{R_1}} = \frac{2}{1}$$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_1 v_2 R_2} = \frac{2\times 2\times 1}{1\times 1\times 4} = \frac{1}{1}$$

Kinetic energy, $K = \dfrac{GMm}{2R}$.

$$\therefore \frac{k_1}{k_2} = \frac{m_1}{m_2}\times\frac{R_1}{R_2} = \frac{2\times 4}{1\times 1} = \frac{8}{1}$$

From Kepler's law of planetary motion.

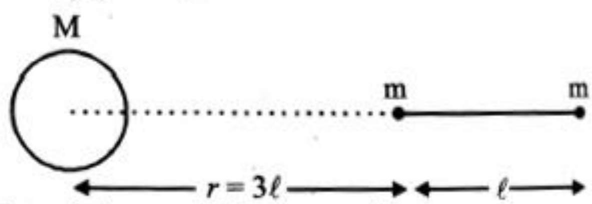
$$T^2 \propto R^3 \quad \therefore \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \frac{1}{8}$$

Integer / Numerical Answer Type

1. (7)

(7) For point mass at distance $r = 4l$

$$\frac{GMm}{(4l)^2} + \frac{Gm^2}{l^2} = ma$$



For point mass at distance $r = 3l$

$$\frac{GMm}{(3l)^2} - \frac{Gm^2}{l^2} = ma$$

$$\therefore \quad \frac{GMm}{(4\ell)^2} + \frac{Gmm}{\ell^2} = \frac{GMm}{(3\ell)^2} - \frac{Gmm}{\ell^2}$$

$$\therefore \quad 2m = M\left[\frac{1}{9} - \frac{1}{16}\right] \Rightarrow m = \frac{7M}{288} \quad \therefore K = 7$$

2. (2)

(2) Let h be the height to which the bullet rises with the height acceleration due to gravity varies as

$$g^1 = g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow \frac{g}{4} = g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow h = R$$

We know escape speed, $v_e = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$ (given) ...(i)

Now applying conservation of energy principle

Loss of kinetic energy = gain in gravitaional potential energy

$$\therefore \quad \frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$\therefore \quad v = \sqrt{\frac{GM}{R}} \qquad ...(ii)$$

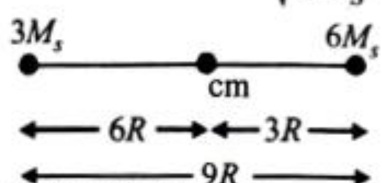
Comparing eq. (i) & (ii) we get $N = 2$

3. (9)

(9) The centre of mass lies at a distance $6R$ from lighter mass

In circular orbit,

Time period, $T = 2\pi\sqrt{\frac{R^3}{GM_S}}$



Binary stars system

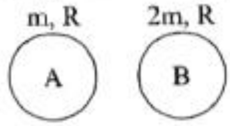
$$nT = 2\pi\sqrt{\frac{(9R)^3}{G(3M_S + 6M_S)}}$$

$$\text{or,} \quad n \times 2\pi\sqrt{\frac{R^3}{GM_S}} = 9 \times 2\pi\sqrt{\frac{R^3}{GM_S}}$$

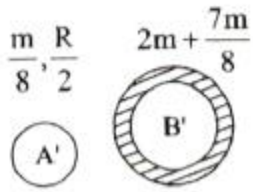
$$\left[\because T = 2\pi\sqrt{\frac{R^3}{GM_S}}\right] \therefore n = 9$$

4. (2.3)

(2.3) Initially, let $M_A = m$. Then, $M_B = 2m$



Finally,



Now, $V_e = \sqrt{\dfrac{2GM}{R}}$

$$\frac{V_B}{V_A} = \sqrt{\frac{M'_B}{M'_A} \times \frac{R'_A}{R'_B}} = \sqrt{\frac{2m + \frac{7m}{8}}{\frac{m}{8}} \times \frac{\frac{R}{2}}{R'_B}}$$

Now, $\rho \times \frac{4}{3}\pi\left((R'_B)^3 - R^3\right) = \frac{7}{8} \times \rho \times \frac{4}{3}\pi R^3$

$$\Rightarrow (R'_B)^3 - R^3 = \frac{7}{8}R^3 \Rightarrow (R'_B)^3 = \frac{15}{8}R^3$$

$$\Rightarrow R'_B = \frac{15^{1/3}}{2}R$$

So, $\dfrac{V_B}{V_A} = \sqrt{\dfrac{\frac{23m}{8}}{\frac{m}{8}} \times \dfrac{\frac{R}{2}}{\frac{15^{1/3}R}{2}}} = \sqrt{\dfrac{\frac{23m}{8}}{\frac{m}{8}} \times \dfrac{1}{15^{1/3}}}$

$$= \sqrt{\frac{23}{15^{1/3}}} = \sqrt{\frac{10 \times 2.3}{15^{1/3}}}$$

Therefore, $n = 2.3$