IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 11/02/24**

TOPIC: HEAT & THERMODYNAMICS
ROTATIONAL DYNAMICS

SOLUTIONS

1. (B)
Using parallel axex theorem.

2. (A)
$$I = \frac{ML^2}{12}$$

3. (D)
$$I = \frac{2}{5} + MR^2 = \frac{7}{5}MR^2$$

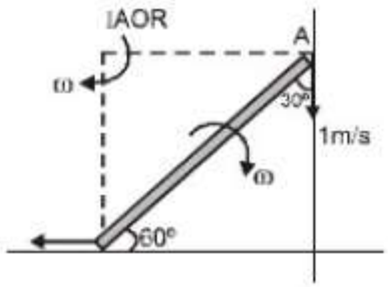
4. (B)
$$I = \frac{ML^2}{12} + m\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$$

5. (A)
$$\frac{\tau_1}{\tau_2} = \frac{I_1}{I_2} = \frac{Mr^2}{2m\left(\frac{r}{2}\right)^2} = 2$$
$$\tau_2 = \frac{\tau}{2}$$

6. (A)
$$\tau = I\alpha$$
$$I = \frac{31.4}{4\pi} = 2.5\ \text{kg-m}^2$$

7. (C)
$$KE_R = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$
$$L = \sqrt{2IKE_R} = 10\ \text{joule-sec}$$

8. (A)



$1 = (1\cos 60^{o})\omega$

$\omega = 2\,\text{rad/s}$

9. (C)

Using angular momentum conservation

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + 2m\frac{L^2}{4}\right)\omega$$

$$\omega = \frac{M\omega_0}{M+6m}$$

10. (B)

11. (D)

Power radiated $\propto$ (surface area) $(T)^4$. The radius is halved, hence, surface area will become $\frac{1}{4}$ times.

Temperature is doubled, therefore, T^4 becomes 16 times.

New power $= (450)\left(\frac{1}{4}\right)(16) = 1800\,\text{W}$

12. (C)

$1 \to 2$ and $3 \to 4$ are isochric process.

Therefore, work done is zero.

$$\therefore \quad W_{\text{total}} = W_{23} + W_{41}$$
$$= P_2(V_3 - V_2) + P_4(V_1 - V_4)$$
$$= nR(T_3 - T_2) + nR(T_1 - T_4)$$
$$= nR(T_3 - T_2 + T_1 - T_4)$$
$$= 800\,nR = 2400\,R$$

13. (B)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \therefore\ v_{\text{rms}} \propto \sqrt{T}$$

v_{rms} is to reduce two times *i.e.,* temperature of the gas will have to reduce four times or

$$\frac{T'}{T} = \frac{1}{4}$$

During adiabatic process

$$TV^{\gamma-1} = T'V'^{\gamma-1} \text{ or } \frac{V'}{V} = \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}}$$

$$= (4)^{\frac{1}{1.5-1}} = (4)^2 = 16$$

$\therefore \quad V' = 16V$

14. (C)

Rate of flow of heat $\frac{dQ}{dt}$ or H is equal throughout the rod.
Temperature difference is given by
Temperature difference = (H) (thermal resistance)
or Temperature difference $\propto$ thermal resistance (R)

where, $R = \frac{I}{KA}$

or $\quad R \propto \frac{1}{A}$

Area across CD is less. Therefore, temperature difference across CD will be more.

15. (D)

$\rho = \frac{PM}{RT}$ and $\rho \propto \frac{1}{V}$

During AB, ρ and hence V is constant.

Therefore, work done is zero.

During BC, $P \propto \rho$ *i.e.*, T and hence, U is constant.

16. (D)

The rate at which energy leaves the object is

$$\frac{\Delta Q}{\Delta t} = e\sigma AT^4$$

Since, $\Delta Q = mC\Delta T$, we get

$$\frac{\Delta T}{\Delta t} = \frac{e\sigma AT^4}{mC}$$

Also, since $m = \frac{4}{3}\pi r^3 \rho$ for a sphere, we get

$$A = 4\pi r^2 = 4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}$$

Hence, $\frac{\Delta T}{\Delta t} = \frac{e\sigma T^4}{mC}\left[4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right] = K\left(\frac{1}{m}\right)^{1/3}$

For the given two bodies

$$\frac{(\Delta T/\Delta t)}{(\Delta T/\Delta t)_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

17. (B)

From Wein's displacement law

$$\lambda mT = \text{constant}$$

or $T = \frac{1}{\lambda_m}$

$$\because \quad \frac{T_{\text{sun}}}{T_{\text{north star}}} = \frac{(\lambda_m)_{\text{north star}}}{(\lambda_m)_{\text{sun}}} = \frac{350}{510} \approx 0.69$$

18. **(D)**

From $PV = nRT$ we have

$$V = nR\left(\frac{T}{P}\right) \text{ or } V \propto \frac{T}{P}$$

$\frac{T}{P}$ is maximum at *d*. Therefore volume is maximum at *d*.

19. **(C)**

In Carnots cycle,

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\therefore \quad \frac{Q_2}{400} = \frac{250}{500} \qquad (Q_1 = W_1 = 400 \text{ J})$$

$$\therefore \quad Q_2 = 200 \text{ J}$$

20. **(B)**

At, $V = V_0, P = \frac{P_0}{2}$

$$\therefore \quad T_i = \frac{PV}{nR} = \frac{\left(\frac{P_0}{2}\right)(V_0)}{R} = \frac{P_0V_0}{2R} \qquad (n = 1)$$

and at $V = 2V_0, P = \frac{4P_0}{5}$

$$\therefore \quad T_f = \frac{PV}{nR} = \frac{(2V_0)\left(\frac{4P_0}{5}\right)}{R} = \frac{8P_0V_0}{5R}$$

$$\therefore \quad \Delta T = T_f - T_i = \left(\frac{8}{5} - \frac{1}{2}\right)\frac{P_0V_0}{R} = \frac{11P_0V_0}{10R}$$

21. **(1)**

$N_2 : f_R = 2$

$$\therefore \quad U_R = \frac{nf_R}{2} RT$$

$$= \frac{(n)(2)}{(2)} RT = nRT$$

22. **(4)**

PT = constant

or $P(PV)$ = constant

or $PV^{1/2}$ = constant

or $x = ½$

$$W = n\left(\frac{R}{1-x}\right)\Delta T$$

$$= (2)\left(\frac{R}{1-1/2}\right)(T_0) = 4RT_0$$

23. **(5)**

Net work done = Area *ABC* – Area *AED*

$$= \frac{1}{2}\times 2\times 1 - \frac{1}{2}\times 1\times 1 = \frac{1}{2}\text{ J}$$

24. **(5)**

$$H = \frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{I}$$

or $H \propto \frac{A}{I}$

or $H_2 = 2H_1$

25. **(4)**

Initially the rods are in parallel

$$\frac{Q}{t} = \frac{(\theta_1 - \theta_2)}{R}$$

$$\Rightarrow \left(\frac{\Theta}{\tau}\right)_1 = \frac{\mu A}{\tau}$$

$$= q_1 L = \frac{(100-0)}{R/2} \qquad ...(1)$$

Finally when rods are in series

$$\Rightarrow \left(\frac{Q}{t}\right)_2 = \frac{mL}{t} = q_2 L = \frac{(100-0)}{2R} \qquad ...(2)$$

From equation (1) and (2),

$$\frac{q_1}{q_2} = \frac{4}{1}$$

26. (5)

$\mathrm{W} = 25\times 2 = 50\,\mathrm{N}$

27. (3)

$\tau = \mathrm{I}\alpha \Rightarrow 10\times 0.3 + 9\times 0.3 - 12\times 0.05 = 5100\,\alpha$

28. (3)

$$\mathrm{I} = \int_0^R \mathrm{kr}^2 \cdot 2\pi \mathrm{rdr} \cdot \mathrm{r}^2$$

$$\mathrm{M} = \int_0^R \mathrm{kr}^2\, 2\pi \mathrm{rdr}$$

29. (2)

$$I_{OO'} = m(O)^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 = \frac{ma^2}{2}$$

30. (4)

Apply $\tau = I\alpha$ about bottom most point

$$F \times h = I\alpha = \frac{3}{2}mR^2 \ \alpha$$

$$\alpha R = \frac{2Fh}{3mR}$$

For translational motion

$$a = \alpha R = \frac{F}{m} \Rightarrow h = \frac{3R}{2}$$

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TOPIC: GOC & IONIC EQUILIBRIUM

SOLUTIONS

31. (B)

32. (D)
Only electrons are allowed to move, positions of nuclei don’t change.

33. (C)
No empty orbital in N.

34. (A)

$$CH_2=CH-\overset{\oplus}{C}H-\overset{\ominus}{C}H-\ddot{N}H_2$$

35. (B)
$-NH_2$ is electron releasing group via resonance.

36. (A)
Only electrons are allowed to move, positions of nuclei don’t change.

37. (B)
Maximum +M effect.

38. (D)
$-CH_3$ is a positive inductive effect group.

39. (C)
$-NH_2$ is a negative inductive effect group.

40. (B)
FCH_2COOH

41. (C)
$pH = 3. \therefore \left[H^+\right] = 10^{-3}; pH = 6 \therefore \left[H^+\right] = 10^{-6}$
Hence $\left[H^+\right]$ reduced by 10^{-3} times.

42. (C)

$HCl + NH_4Cl$. HCl is strong acid hence not used in buffer.

43. (C)

$$pH = pK_a + \log\frac{[SALT]}{[ACID]}$$

$$5.5 = 4.5 + \log\frac{[SALT]}{0.1}.$$

$$[SALT] = 1M$$

44. (D)

$$pOH = -\log K_b + \log\frac{[SALT]}{[BASE]}$$

$$= -\log K_b \left(\text{since}\frac{SALT}{BASE} = 1\right)$$

$$= -\log 2\times10^{-5} = 4.7. \quad \therefore pH = 9.3$$

45. (C)

Given, $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$;

$K_{a_1}, = 1.5\times10^{-5}$ ……… (i)

$HCN \rightleftharpoons H^+ + CN^-; K_{a_2} = 4.5\times10^{-10}$

Or $H^+ + CN^- \rightleftharpoons HCN$;

$K'_{a_2} = \frac{1}{K_{a_2}} = \frac{1}{4.5\times10^{-10}}$ ……. (ii)

$\therefore$ From (i) and (ii), we find that the equilibrium constant (K_a) for the reaction,

$CN^- + CH_3COOH \rightleftharpoons CH_3COO^- + HCN$, is

$$K_a = K_{a_1} \times K'_{a_2} = \frac{1.5\times10^{-5}}{4.5\times10^{-10}} = \frac{1}{3}\times10^5 = 3.33\times10^4$$

46. (D)

$Ba(OH)_2(s) \rightarrow Ba^{2+}(aq) + 2OH^-(aq)$

$pH = 12$ or $pOH = 2$

$[OH^-] = 10^{-2}M$

$Ba(OH)_2 \rightarrow Ba^{2+} + 2OH^-$

$0.5\times10^{-2} \quad 10^{-2}$

[$\therefore$ Concentration of Ba^{2+} is half of OH^-]

$K_{sp} = [Ba^{2+}][OH^-]^2$

$=\left[0.5\times10^{-2}\right]\left[1\times10^{-2}\right]^2$

$=0.5\times10^{-6}=5\times10^{-7}\,M^3$

47. (B)

$[H_3O]^+$ for a solution having $pH=3$ is given by

$[H_3O]^+=1\times10^{-3}$ moles/litre

$\left[\therefore [H_3O]^+=10^{-pH}\right]$

Similarly for solution having pH = 4,

$[H_3O]^+=1\times10^{-4}$ moles/litre and for $pH=5$

$\left[H_3O^+\right]=1\times10^{-5}$ moles/litre

Let the volume of each solution in mixture be IL, then total volume of mixture solution $L=(1+1+1)L=3L$

Total $[H_3O]^+$ ion present in mixture solution $=\left(10^{-3}+10^{-4}+10^{-5}\right)$ moles

Then $[H_3O]^+$ ion concentration of mixture solution

$=\dfrac{10^{-3}+10^{-4}+10^{-5}}{3}M=\dfrac{0.00111}{3}M$

$=0.00037M=3.7\times10^{-4}M.$

48. (D)

$K_a\times K_b=K_w$

$\therefore 1.8\times10^{-5}\times K_a=1\times10^{-14}\ \therefore k_a=5.56\times10^{-10}$

49. (A)

$HCOONH_4$ is a salt of weak acid and weak base;

$pH=1/2\,pK_w+1/2\,pK_a-1/2\,pK_b$

$\therefore pH=\frac{1}{2}\times14+\frac{1}{2}\times3.8-\frac{1}{2}\times4.8;\ pH=6.5$

50. (C)

$Na_2CO_3+2H_2O\rightleftharpoons 2NaOH+H_2CO_3$

$CO_3^-+2H_2O\rightleftharpoons 2OH^-+H_2CO_3$ Anionic hydrolysis

51. (5)

$-OH$, $-NH_2$, $-NO_2$, $-CN$, $-CHO$

52. (4)

$-CH_3$, $-CH_2CH_3$, $-\bar{O}$, $-COO\bar{O}$

53. (6)

$-\bar{O}$, $-OH$, $-NH_2$, $-NHCOCH_3$, $-OCOCH_3$, $-NHCOC_2H_5$

54. (3)

–CHO, $–NO_2$, –CN

55. (2)

$$-CH_2-\overset{\oplus}{C}H_2, \quad -CH_2-\overset{-}{C}H_2$$

56. (11.50)

NaCN is a salt of strong base and weak acid; pH

$$=7+\frac{1}{2}pK_a+\frac{1}{2}\log C$$

$$pK_a \text{ for } HCN=14-4.70=9.30$$

$$\therefore pH=7+\frac{1}{2}\times 9.30+\frac{1}{2}\log 0.5; pH=11.5$$

57. (2.00)

Let the volume of KCN to be added is Vml; Conc. of KCN

$=S\times V$ and

Conc. of $HCN=10\times 2; pH=9, \therefore H^+=10^{-9}$

$$H^+=\frac{K_a[ACID]}{[SALT]};10^{-9}=\frac{5\times 10^{-10}\times 10\times 2}{5\times V};V=2ml$$

58. (11.30)

$$\left[OH^-\right]=\frac{(20\times 0.02)-(30\times 0.01)}{50}=0.002M$$

$$-\log\left[OH^-\right]=pOH=-\log .002=2.7$$

$$\therefore pH=14-2.7=11.3$$

59. (0.02)

$$Fe(OH)_2 \rightleftharpoons Fe^{++}+2\overline{O}H$$

$$pH=8,\left[H^+\right]=10^{-8},\left[OH^-\right]=10^{-6}$$

$$Fe^{++}=\frac{K_{sp}}{\left[OH^-\right]^2}=\frac{1.6\times 10^{-14}}{\left[10^{-6}\right]^2}=0.016$$

60. (1.30)

$$NaCN(aq)+HCl(aq)\rightarrow HCN(aq)+NaCl(aq).$$

Millimoles of $NaCN=50\times 0.1=5$;

Millimoles of HCl $=50\times 0.2=10$

Excess millimoles of HCl $=5$ in 100ml; $\left[H_3\overset{+}{O}\right]=0.05$

$$pH=-\log 0.05=1.30$$

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TOPIC: LCD

Answer & Solution

61. (D)

Let $x = 2 + h$ $x \to 2$, $h \to 0$

$$= \lim_{h\to 0} \frac{\sin(e^h - 1)}{\ell n(1+h)} = \lim_{h\to 0} \frac{\sin(e^h - 1)}{(e^h - 1)} \times \frac{(e^h - 1)}{h} \times \frac{1}{\frac{\ell n(1+h)}{h}} = 1,1,1 = 1$$

62. (D)

$$\lim_{x\to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}\ ;\ \lim_{x\to 1} \frac{\sqrt{2\sin^2(x-1)}}{x-1}$$

$$\lim_{x\to 1} \frac{\sqrt{2}\left|\sin(x-1)\right|}{x-1};\ \text{LHL} = \lim_{x\to 1} \frac{\sqrt{2}\sin h}{-h} = -\sqrt{2}$$

$$\text{RHL} = \lim_{h\to 0} \frac{\sqrt{2}\ \sin h}{h} = \sqrt{2}$$

$\because$ L.H.L $\neq$ R.H.L

So limit does not exist.

63. (D)

$$\lim_{x\to 0} \frac{\sqrt{2} - \sqrt{1+\cos x}}{\sin^2 x} = \lim_{x\to 0} \frac{\sqrt{2} - \sqrt{2}\cos\left(\frac{x}{2}\right)}{\sin^2 x} = \lim_{x\to 0} \frac{2\sqrt{2} - \sin^2\left(\frac{x}{4}\right)}{16.\frac{x^2}{16}\ .\ \sin^2 x} = \frac{\sqrt{2}}{8}$$

64. (B)

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x)}{\sqrt{x}} = \lim_{h\to 0^+} \frac{\cos^{-1}[1-(0+h)}{\sqrt{0+h}} = \lim_{h\to 0^+} \frac{\cos^{-1}(1-h)}{\sqrt{h}}$$

Let $1 - h = \cos\theta$

$$\sin\theta = \sqrt{1-(1-h^2)}$$

$$\therefore \theta = \sin^{-1}\sqrt{2h-h^2} = \lim_{h\to 0} \frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{h}} = \lim_{h\to 0} \frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{2h-h^2}} \cdot \frac{\sqrt{2h-h^2}}{\sqrt{h}}$$

$$= 1 \times \sqrt{2} = \sqrt{2}$$

65. (C)

$\sin h < h < \tan h, \quad h \in \left(0, \frac{\pi}{2}\right)$

$$\frac{h}{\sin h} > 1 \quad \Rightarrow \frac{-h}{\sin h} < -1$$

$$\text{LHL} = \lim_{h \to 0}\left[\frac{\frac{\pi}{2} - h - \frac{\pi}{2}}{\cos\left(\frac{\pi}{2} - h\right)}\right] \lim_{h \to 0}\left[\frac{-h}{\sin h}\right] = -2$$

$$\text{RHL} = \lim_{h \to 0}\left[\frac{\frac{\pi}{2} + h - \frac{\pi}{2}}{\cos\left(\frac{\pi}{2} + h\right)}\right] \lim_{h \to 0}\left[\frac{-h}{\sin h}\right] = -2$$

$\therefore \text{LHL} = \text{RHL} = -2$

66. (A)

$$\lim_{x \to \infty}\left(x - x^2 \, \ell n\left(1 + \frac{1}{x}\right)\right) = \lim_{x \to \infty} x - x^2\left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \ldots\ldots\right)$$

$$\lim_{x \to \infty} x - x + \frac{1}{2} - \frac{1}{3x} + \ldots\ldots = \frac{1}{2}$$

67. (C)

$$\lim_{x \to 0}\frac{e^{-\frac{x^2}{2}} - \cos x}{x^3 \sin x} = \lim_{x \to 0}\frac{\left[1 - \frac{x^2}{2} + \frac{x^4}{4 \,.\, 2!} - \ldots\ldots\right] - \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right]}{x^3\left[x - \frac{x^3}{3!} + \ldots\right]}$$

$$= \lim_{x \to 0}\frac{x^4\left[\frac{1}{8} - \frac{1}{4!}\right] - x^6\left[\frac{1}{8 \,.\, 3!} - \frac{1}{6!}\right] + \ldots\ldots}{x^4\left[1 - \frac{x^2}{3!} + \ldots\ldots\right]} = \left[\frac{1}{8} - \frac{1}{24}\right] = \frac{1}{12}$$

68. (A)

$$\lim_{x \to \infty}\left(\frac{x+2}{x-2}\right)^{x+1} \quad (1^{\infty} \text{ form})$$

$$= e^{\lim_{x \to \infty}\left(\frac{x+2}{x-2} - 1\right) . (x+1)} = e^{\lim_{x \to \infty}\frac{4(x+1)}{(x+2)}} = e^{\lim_{x \to \infty}\frac{4\left(1 + \frac{1}{x}\right)}{\left(1 - \frac{2}{x}\right)}} = e^4$$

69. (B)

$$\lim_{x \to \frac{\pi}{4}}\left(1 + [x]\right)^{\frac{1}{\ell n(\tan x)}} = \lim_{x \to \frac{\pi}{4}} (\text{exact } 1)^{\frac{1}{\ell n(\tan x)}} = 1$$

70. (D)

$$\lim_{n\to\infty} \frac{1}{n^4}\left([1^3\ x] + [2^3\ x] + ... + [n^3\ x]\right)$$

$$1^3\ x - 1 < [1^3\ x]\, 1^3\ x$$

$$2^3\ x - 1 < [2^3\ x]\, 2^3\ x$$

. . .

. . .

. . .

$$n^3\ x - 1 > [n^3\ x]\, n^3\ x$$

Adding all these inequilities

$$(1^3 + 2^3 + 3^3 .... + n^3)x - n < [1^3x] + [2^3x] + ...[n^3x]$$

$$\frac{\frac{n^2(n+1)^2}{4}x - n}{n^4} < \frac{[1^3\ x] + [2^3\ x] + ... + [n^3\ x]}{n^4} \le \frac{\left(\frac{n(n+1)}{2}\right)}{n^4}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 \frac{x}{4} - \frac{1}{n^3} < \lim_{n\to\infty} \frac{[1^3\ x] + [2^3\ x] + ... + [n^3\ x]}{n^4} \le \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 \frac{x}{4}$$

$$\frac{x}{4} < \lim_{n\to\infty} \frac{[1^3\ x] + [2^3\ x] + ... + [n^3\ x]}{n^4} \le \frac{x}{4}$$

$$\therefore \lim_{n\to\infty} \frac{[1^3\ x] + [2^3\ x] + ... + [n^3\ x]}{n^4} = \frac{x}{4}$$

71. (B)

$$\lim_{x\to 1^+} x \sin\left(\frac{\pi}{2}(x+2)\right) = \lim_{x\to 1^+}(-x)\sin\left(\frac{\pi}{2}\right) = -1 \quad \lim_{x\to 1^-} x \sin\left(\frac{\pi}{2}(x+0)\right) = 1$$

72. (A)

$$\text{LHL } (x = 0) = f(0) = \text{RHL } (x = 0);\ \text{LHL} = \lim_{x\to 0^-} \frac{\sqrt{1+P(x)} - \sqrt{1-Px}}{x} = \frac{2P}{2} = p$$

RHL = –1

73. (B)

Let f(x) = 2 tan x + 5x – 2

$f(0) = -2$ $f(\pi/4) = 2 \tan \frac{\pi}{4} + \frac{5\pi}{4} - 2 = \frac{5\pi}{4}$

Now, $f(x) \in \left[-2, \frac{5\pi}{4}\right]$ and f(x) is continuous and increasing on $[0, \pi/4]$

$\therefore$ By intermediate value theorem $f(c) = 0$ for exactly one $c \in [0, \pi/4]$.

$\therefore$ (B) is correct.

74. (B)

$f(x)$ is continuous when $x = 1 - x \Rightarrow x = \frac{1}{2}$

75. (B)

$f(x) = \dfrac{x}{\sqrt{x+1} - \sqrt{x}}$ $f(0) = 0$ Domain $x \ge 0$

$\because \lim_{x \to 0^+} f(x) = 0 \quad \therefore f(x)$ is continuous

$$\because \text{RHD}(x = 0) = \lim_{h \to 0^+} \frac{\dfrac{h}{\sqrt{h+1} - \sqrt{h}} - 0}{h} = 1$$

$\therefore$ f(x) is differentiable at x = 0

76. (B)

$$f(x) = \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}\cos x = \begin{cases} \dfrac{\pi}{2} + x\ ,\ x \in [0, \pi] \\ \dfrac{\pi}{2} - x\ ,\ x \in [-\pi, 0] \end{cases}$$

Continuous but not differentiable at x = 0

77. (D)

$f\left(0^-\right) = \lim_{x \to 0^-} f(x) = 0 + 1 + 0.\sin(1) = 1$

$f(0^+) = \lim_{x \to 0^+} f(x) = 0 + 0 + 0.\sin 0 = 0$

$f(2^+) = 2 + 0 + 2\sin 0 = 2$

78. (B)

If ' f ' is differentiable then |f| is differentiable at each point x, where $f(x) \neq 0$

if $f(\alpha) = 0$ and $f'(\alpha) = 0$, then |f| is differentiable at x = α

if $f(\alpha) = 0$ and $f'(\alpha) \neq 0$,, then |f| is not differentiable at x = α

$\Rightarrow$ If f is differentiable then |f| may or may not be differentiable, [option A, C, D not necessarily true]

Now $|f|^2 = f^2 \ \left(f^2\right)' = 2.f.f'$

since f is differentiable

$\therefore f^2$ is also differentiable

79. (A)

$$f(x) = \begin{cases} \dfrac{x}{1-x}\ ,\ x < 0 \\ \dfrac{x}{1+x}\ ,\ x \ge 0 \end{cases}$$

f(x) is continuous and differentiable for $x \in R$

80. (C)

$$\text{If } f[g(x)] = \text{sgn}(g(x) = \text{sgn}(x(x^2 - 5x + 6)) = \text{sgn}(x(x-2)(x-3)) = \begin{cases} 1;\ x > 3,\ 1 < x < 2 \\ 0;\ x = 0,\ 2, 3 \\ -1:\ 2 < x < 3;\ x < 1 \end{cases}$$

f(g(x)) is discontinuous at 3 points (0, 2 and 3)

81. (9)

$$\lim_{x\to 3}\frac{(x^3+27)\,\ell n(x-2)}{(x^2-9)}=\lim_{x\to 3}\frac{(x+3)(x^2-3x+9)\,\ell n[1+(x-3)]}{(x-3)(x+3)}=\lim_{x\to 3}(x^2-3x+9)$$

$$\frac{\ell n[1+(x-3)]}{(x-3)}$$

$$(9-9+9)\,(1)=9$$

82. (1)

$$\lim_{x\to 0}\frac{\sin(\ell n(1+x))}{\ell n(1+\sin x)}$$

$$=\lim_{x\to 0}\frac{\sin(\ell n(1+x))}{\ell n(1+\sin x)}\cdot\frac{\sin x}{\ell n(1+\sin x)}\cdot\frac{\ell n(1+\sin x)}{x}\cdot\frac{x}{\sin x}=1.1.1.1=1$$

83. (1)

$$\lim_{x\to\infty}\frac{x^3\sin\frac{1}{x}+x+1}{x^2+x+1}=\lim_{x\to\infty}\frac{\frac{\sin\frac{1}{x}}{\frac{1}{x}}+\frac{1}{x}+\frac{1}{x^2}}{1+\frac{1}{x}+\frac{1}{x^2}}=\frac{1+0+0}{1+0+0}=1$$

84. (0)

$$\lim_{n\to\infty}\frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}},\lim_{n\to\infty}\frac{5.5^n+3^n-4^n}{5^n+2^n+27^n}=\lim_{n\to\infty}\frac{5\,.\left(\frac{5}{9}\right)^n+\left(\frac{3}{9}\right)^n-\left(\frac{4}{9}\right)^n}{\left(\frac{5}{9}\right)^n+\left(\frac{2}{9}\right)^n+27}=\frac{0+0-0}{0+0+27.9^n}=0$$

85. (0.5)

$$\lim_{x\to 1}\left(\frac{2}{1-x^2}+\frac{1}{x-1}\right)=\lim_{x\to 1}\left(\frac{1}{1-x}+\frac{1}{1+x}+\frac{1}{x-1}\right)=\lim_{x\to 1}\left(\frac{1}{x+1}\right)=\frac{1}{2}$$

86. (1)

$$\lim_{x\to 0}(\cos x)^{\frac{1}{\sin x}}\quad(1^\infty\text{ form})$$

$$=e^{\lim_{x\to 0}\left(\frac{\cos x-1}{\sin x}\right)}=e^{\lim_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}}=e^{\lim_{x\to 0}-\tan\left(\frac{x}{2}\right)}=e^0=1$$

87. (0)

$$\lim_{x\to 0}=\frac{\cos(\sin x)-\cos x}{x^2}=a\Rightarrow\frac{2}{x^2}\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)=a$$

$$\Rightarrow a = \lim_{x\to 0} = 2.\frac{\sin\left(\frac{\sin x + x}{2}\right)}{\frac{\sin x + x}{2}} \cdot \frac{\sin\left(\frac{x - \sin x}{2}\right)}{\frac{x - \sin x}{2}} \cdot \frac{1}{4}\left(\frac{\sin x + x}{x}\right)\left(\frac{x - \sin x}{x}\right)$$

$$= 2.1.1\frac{1}{4}(1 - 1)(1 - 1) = 0$$

88. (3)

$y = \frac{1}{t^2 + t - 2}$ where $t = \frac{1}{x - 1}$, $y = f(x)$ is discontinuous at $x = 1$, where t is discontinuous and

$y = \frac{1}{(t + 2)(t - 1)}$ at $t = -2$ and $t = 1$

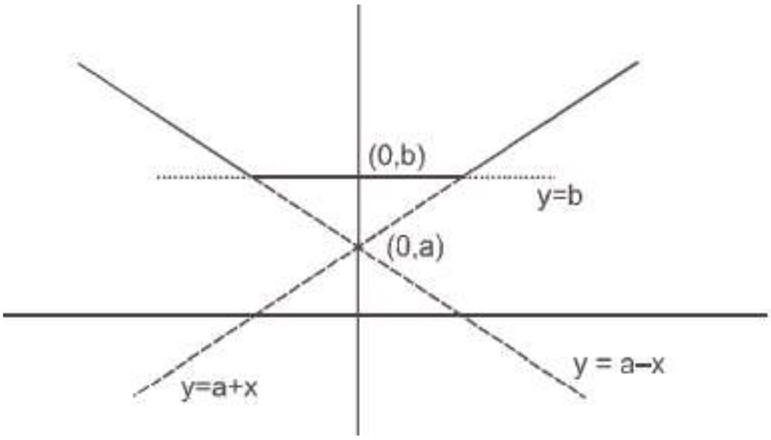
$\Rightarrow \frac{1}{x - 1} = -2 \Rightarrow -2x + 2 = 1, \quad x = \frac{1}{2}$

$1 - \frac{1}{x - 1} = 1 \Rightarrow x = 2$

$\therefore$ f(x) is discontinuous at $x = \frac{1}{2}, 2, 1$

89. (2)

$y = f(x) = \max\{a - x, a + x, b\}$, $0 < a < b$



f(x) is non-differentiable at 2 points.

90. (3)

$$\lim_{h\to 0}\frac{6}{4} = \lim_{h\to 0}\frac{f(2h + 2 + h^2) - f(2)}{(2h + 2 + h^2) - (2)}\,\frac{(h - h^2 + 1) - (1)}{f(h - h^2 + 1) - f(1)}\,\frac{(2h + 2 + h^2) - (2)}{(h - h^2 + 1) - (1)}$$

$$= \lim_{h\to 0}\frac{f'(2)}{f'(1)} \cdot \frac{h(2 + h)}{h(1 - h)} = 3$$